Lattice calculations for TMDs and DPDs

G. Bali, L. Castagnini, P. Bruns, M. Diehl, M. Engelhardt, J. Gaunt, B. Gläßle, M. Göckler, X. Ji, B. Musch, A. Sternbeck, A. Vladimirov, J.-H. Zhang, S, Zhao, C. Zimmermann, W. Wang, CLS, RQCD, TMDC, LP^3, ...

- TMDs on the lattice
- The relevance of DPDs
- DPDs on the lattice "Two-current correlations in the pion on the lattice" arXiv:1807.03073
- Conclusion

A few basic facts relevant for this talk

QCD is contained in the generating functional:

$$Z[J^a_\mu, \bar{\eta}^i, \eta^i] = \int \mathcal{D}[A^{a\mu}, \bar{\psi}^i, \psi^i] \exp\left(\mathrm{i} \int d^4x \left[\mathcal{L}_{\mathrm{QCD}} - J^a_\mu A^a_\mu - \bar{\psi}^i \eta^i - \bar{\eta}^i \psi^i \right] \right)$$

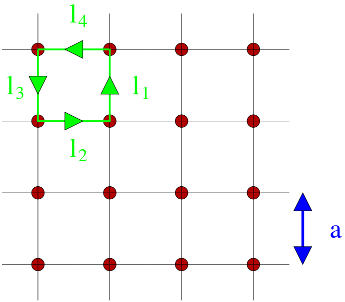
A numerical integration is made possible by analytic continuation to imaginary time:

$$t \leftrightarrow -\mathrm{i}\tau$$

$$S = \int d^4x (T - V) \leftrightarrow \mathrm{i} \int d^4x_E (T + V) = \mathrm{i} S_E$$

$$e^{\mathrm{i}S} \leftrightarrow e^{-S_E}$$

Discretized space time ⇒ e.g. the Wilson action



$$U(l_1) = \exp\left(-igA^b(l_1)\,\frac{\lambda^b}{2}\,a\right)$$

$$W_\square = \mathrm{Tr}\{U(l_1)U(l_2)U(l_3)U(l_4)\}$$

$$\sum_\square \frac{2}{g^2}(3 - \mathcal{R}e\; W_\square) = \frac{1}{4}\int d^4x\left(F^a_{\mu\nu}F^a_{\mu\nu} + O(a^2)\right)$$

- The $U(\ell)$ are the building blocks of Lattice QCD $\Rightarrow$ Investigating gauge links should be simple
  $\int \mathcal{D}[\psi, \bar{\psi}]... \quad \Rightarrow \quad Det[D]$
- However, non-locality in time cannot be treated numerically $\Rightarrow$ Mellin moments; quasi PDFs
- The continuum limit $a \to 0$ is the numerical challenge large topological autocorrelation times for $a < 0.05$ fm. Usually $0.05\ \mathrm{fm} < a < 0.1$ fm
- For distances of a few lattice spacings discretization errors are typically large $\Rightarrow$ $1/[O(3)a]$ must be larger than any physically relevant momentum.
- Simulations with open, not periodic boundary conditions M. Lüscher and S. Schaefer $\Rightarrow$ CLS, a collaboration od collaborations

Hadronic 2- and 3- Point functions

One needs combinations of field operators which have the wanted quantum numbers, e.g. for the nucleon ($C = i\gamma^2\gamma^4 = C^{-1}$):

$$\hat{B}_\alpha(t,\vec{p}) = \sum_{\vec{x}} e^{\mathrm{i}\vec{p}\cdot\vec{x}} \epsilon_{ijk} \hat{u}^i_\alpha(x)\ \hat{u}^j_\beta(x)(C^{-1}\gamma_5)_{\beta\gamma}\hat{d}^k_\gamma(x)$$

$$\begin{aligned}\langle 0|T\left\{\hat{B}(y_4)\hat{A}(x_4)\right\}|0\rangle &= e^{-(T-y_4+x_4)E_B}\langle B|\hat{B}(0)|0\rangle\langle 0|\hat{A}(0)|B\rangle \\ &+ e^{-(y_4-x_4)E_A}\langle 0|\hat{B}(0)|A\rangle\langle A|\hat{A}(0)|0\rangle\end{aligned}$$

$\hat{B}$ generates the antiparticle of $\hat{A}$. One has (anti)periodic boundary conditions.

To get the hadron masses one simply has to determine the slopes.

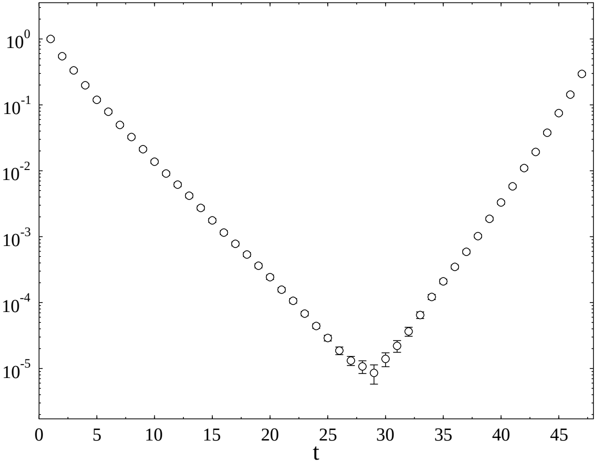
$$e^{-(y_4-x_4)M_N}\langle 0|\hat{N}^\dagger(0)|N\rangle\langle N|\hat{N}(0)|0\rangle$$

$$\begin{aligned} |B\rangle \quad &\sim \quad c_0|N\rangle \;+\; c_1|N'\rangle \;+\; c_2|N\pi\rangle \;+\; \ldots \\ &\Rightarrow \quad c_0 e^{-E_N t}|N\rangle \;+\; c_1 e^{-E_{N'} t}|N'\rangle \;+\; c_2 e^{-E_{N\pi} t}|N\pi\rangle \;+\; \ldots \end{aligned}$$

Note: A quark propagator is the inverse of the Dirac operator on the lattice, which is just a large matrix.

$$\begin{aligned} &\langle B_\alpha(t,\vec{p})\bar{B}_\beta(0,\vec{p})\rangle \\ &= \sum_{\substack{x \\ x_4=t}} \sum_{\substack{y \\ y_4=0}} \mathrm{e}^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})} \epsilon_{ijk}\epsilon_{i'j'k'}(C^{-1}\gamma_5)_{\alpha'\alpha''}(\gamma_5 C)_{\beta'\beta''} \\ &\left\langle G^{ki'}_{\alpha''\beta'}(x,y)\left(G^{jj'}_{\alpha'\beta''}(x,y)G^{ik'}_{\alpha\beta}(x,y) - G^{ij'}_{\alpha\beta''}(x,y)G^{jk'}_{\alpha'\beta}(x,y)\right)\right\rangle_g \end{aligned}$$

A nucleon 2-point function



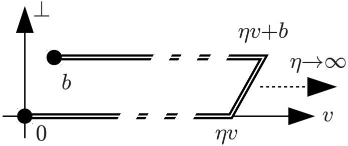
Once the propagation in imaginary time has projected the original source onto the physical wave function on can calculate physical correlators from

$$\frac{\tilde{\Gamma}_{\alpha\beta}\langle B_\beta(t,\vec{p})\mathcal{O}\bar{B}_\alpha(0,\vec{p})\rangle}{\Gamma_{\alpha\beta}\langle B_\beta(t,\vec{p})\bar{B}_\alpha(0,\vec{p})\rangle}$$

TMDs arXiv:1706.03406 M. Engelhardt, P. Hägler, B. Musch, et al.

TMDs are related to correlators of the type

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \equiv \frac{1}{2}\langle P, S|\, \bar{q}(0)\, \Gamma\, \mathcal{U}[0, \eta v, \eta v + b, b]\, q(b)\, |P, S\rangle$$



We simulate for spatial, not light-like separations, but the limit $\hat{\zeta} \to \infty$ of

$$\hat{\zeta} := \frac{v \cdot P}{\sqrt{v^2}\sqrt{P^2}}$$

reproduces the light-cone behavior.

We used RBC/UKQCD (domain wall) and W&M (Clover) ensembles, $N_f = 2 + 1$

ID	Clover	DWF
Fermion Type	Clover	Domain-wall
Geometry	$32^3 \times 96$	$32^3 \times 64$
$a(\mathrm{fm})$	0.11403(77)	0.0840(14)
$m_\pi(\mathrm{MeV})$	317(2)(2)	297(5)
# confs.	967	533
# meas.	23208	4264

only connected diagrams, i.e. $u - d$

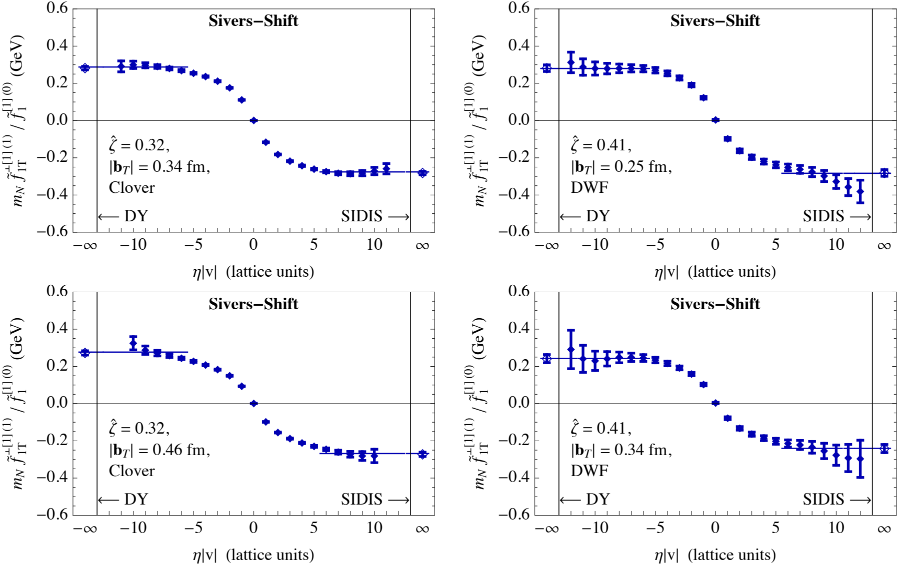
$$
\begin{aligned}
\widetilde{\Phi}^{[\Gamma]}_{\text{subtr.}}(b,P,S,\ldots) &= \widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots)\cdot\mathcal{S}\cdot Z_{\text{TMD}}\cdot Z_2 \\
\Phi^{[\Gamma]}(x,\boldsymbol{k}_T,P,S,\ldots) &= \int\frac{d^2\boldsymbol{b}_{\text{T}}}{(2\pi)^2}\int\frac{d(b\cdot P)}{2\pi P^+}e^{ix(b\cdot P)-i\boldsymbol{b}_{\text{T}}\cdot\boldsymbol{k}_{\text{T}}}\,\widetilde{\Phi}^{[\Gamma]}_{\text{subtr.}}\Big|_{b^+=0} \\
\Phi^{[\gamma^+]} &= f_1-\frac{\epsilon_{ij}\boldsymbol{k}_i\boldsymbol{S}_j}{m_N}f_{1T}^{\perp} \\
\Phi^{[\gamma^+\gamma^5]} &= \Lambda g_1+\frac{\boldsymbol{k}_{\text{T}}\cdot\boldsymbol{S}_{\text{T}}}{m_N}g_{1T} \\
\Phi^{[i\sigma^{i+}\gamma^5]} &= \boldsymbol{S}_i h_1+\frac{(2\boldsymbol{k}_i\boldsymbol{k}_j-\boldsymbol{k}_{\text{T}}^2\delta_{ij})\boldsymbol{S}_j}{2m_N^2}h_{1T}^{\perp}+\frac{\Lambda\boldsymbol{k}_i}{m_N}h_{1L}^{\perp}+\frac{\epsilon_{ij}\boldsymbol{k}_j}{m_N}h_1^{\perp} \\
\tilde{f}^{[m](n)}(\boldsymbol{b}_{\text{T}}^2,\ldots) &= n!\left(-\frac{2}{m_N^2}\partial_{\boldsymbol{b}_{\text{T}}^2}\right)^n\int_{-1}^{1}dx\,x^{m-1}\int d^2\boldsymbol{k}_{\text{T}}e^{i\boldsymbol{b}_{\text{T}}\cdot\boldsymbol{k}_{\text{T}}}f(x,\boldsymbol{k}_{\text{T}}^2) \\
\langle\vec{k}_y\rangle_{TU}(\boldsymbol{b}_{\text{T}}^2;\ldots) &= m_N\frac{\tilde{f}_{1T}^{\perp[1](1)}(\boldsymbol{b}_{\text{T}}^2;\ldots)}{\tilde{f}_1^{[1](0)}(\boldsymbol{b}_{\text{T}}^2;\ldots)}
\end{aligned}
$$

the generalized tensor charge

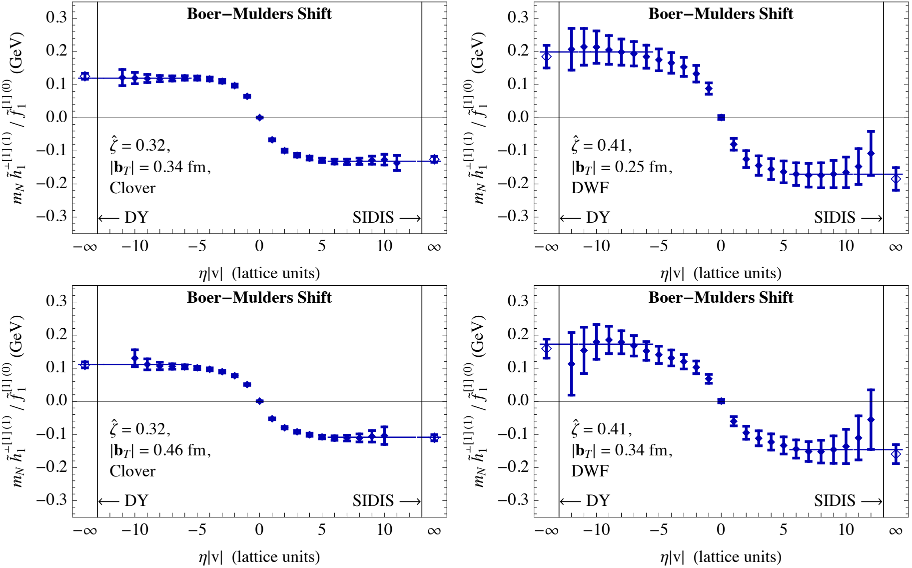
$$g_T^{u-d} = \begin{array}{c} \tilde{h}_1^{[1](0)}(\boldsymbol{b}_{\mathrm{T}}^2;\ldots)/\tilde{f}_1^{[1](0)}(\boldsymbol{b}_{\mathrm{T}}^2;\ldots) \\ \int dx\, d^2\boldsymbol{k}_{\mathrm{T}}\, h_1(x,\boldsymbol{k}_{\mathrm{T}}^2) = \tilde{h}_1^{[1](0)}(\boldsymbol{b}_{\mathrm{T}}^2 = 0) \end{array}$$

limits:

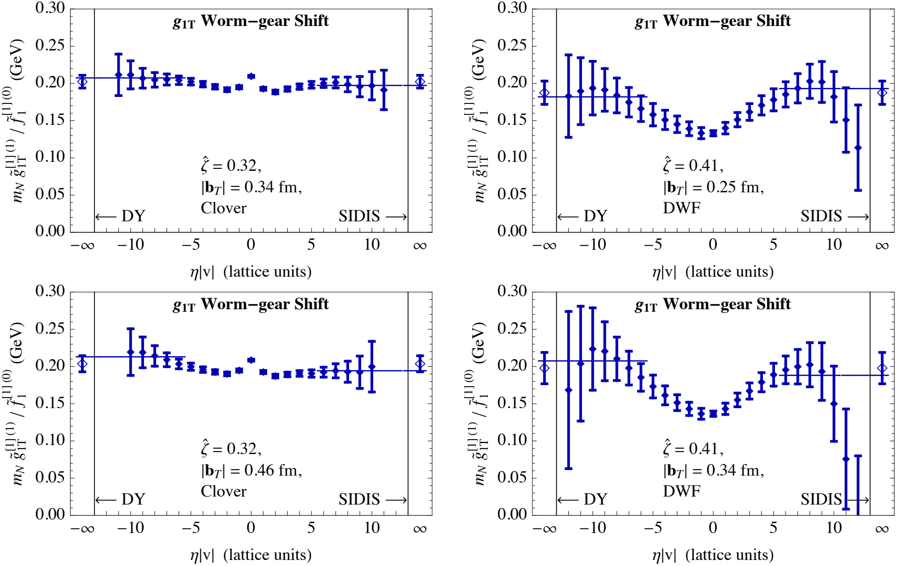
$$\begin{aligned} \eta|v| &\to \infty \\ b_T &\gg a \\ \hat{\zeta} &\to \infty \end{aligned}$$



Sivers shift



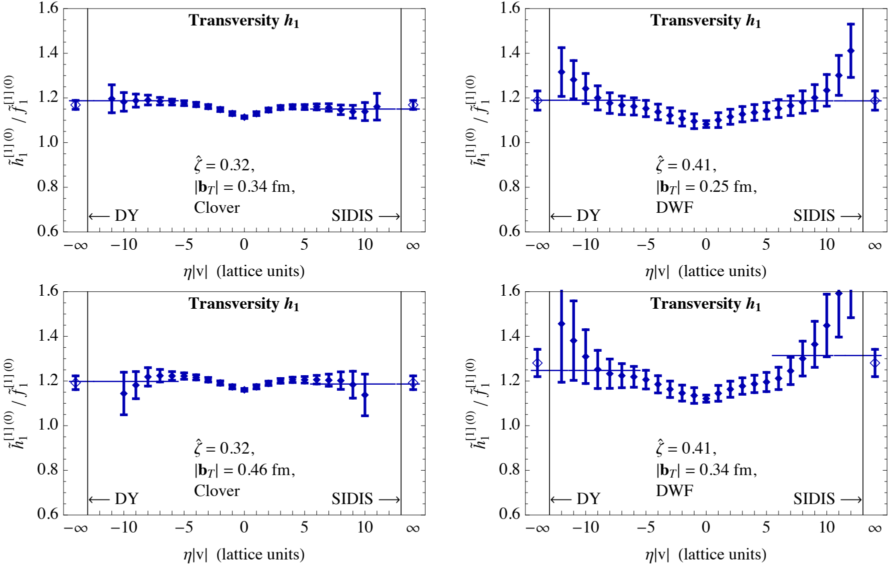
Boer-Mulders shift



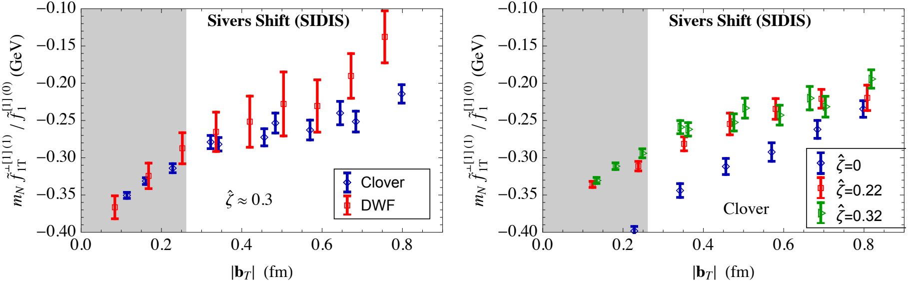
Worm-gear shift, for g_{1T}

note

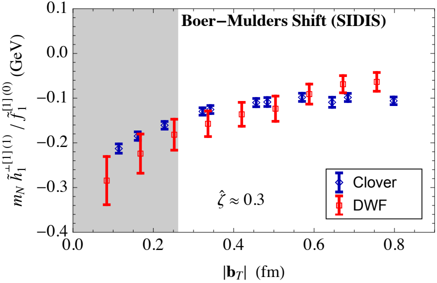
- Discrepancy between Clover and DWF for large η and small b_T
- Very funny behaviour for $\eta = 0$, i.e. for a straight gauge link. Not relevant for TMDs but perhaps a hint for an explanation of the first observation

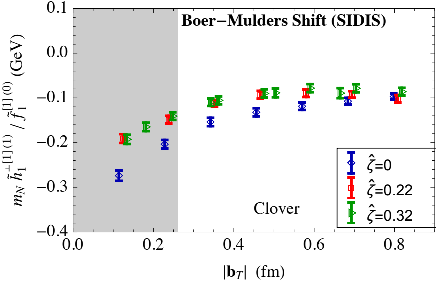


transversity ratio

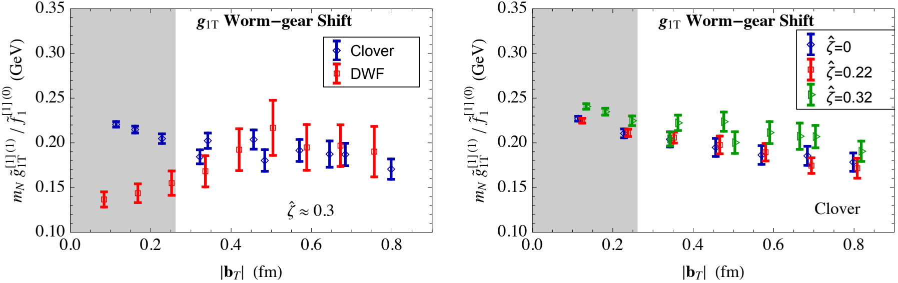


Sivers shift





Boer-Mulders shift



Worm-gear shift, for g_{1T}

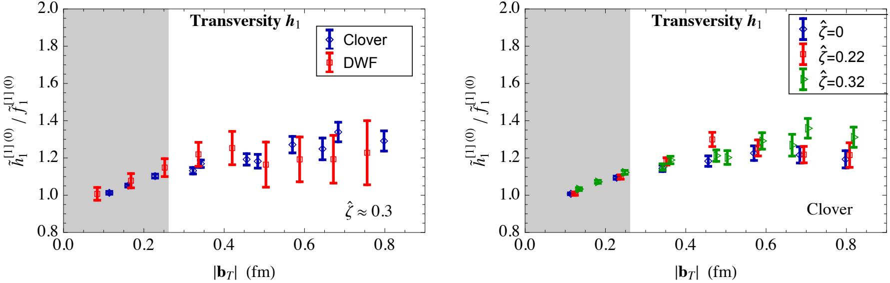
M.Constantinou, H. Panagopoulos arXiv:1705.11193 operator mixing

Operator-mixing under renormalization for the worm-gear shift and Clover-Wilson fermions

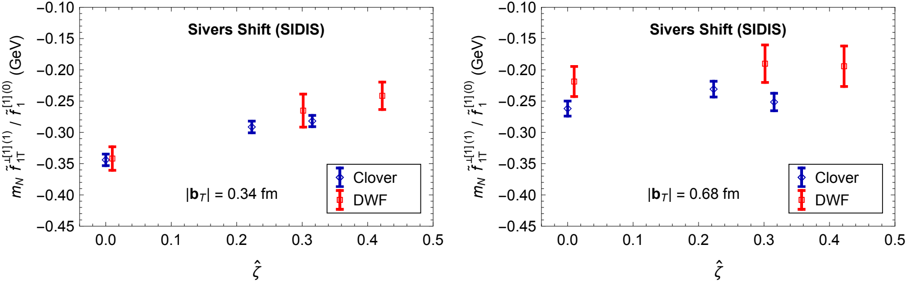
There exists a continuum analysis of I. Scimemi and A. Vladimirov arXiv:1804.08148

$$\frac{g_{1T}^{(0)}(\vec{b})}{f_1^{(0)}(\vec{b})} \sim 0.13 + \frac{\Delta T^{(2,1)}}{2 f_1^{(0)}} + O(\alpha_s) + O(\vec{b}^2) \sim 0.13$$

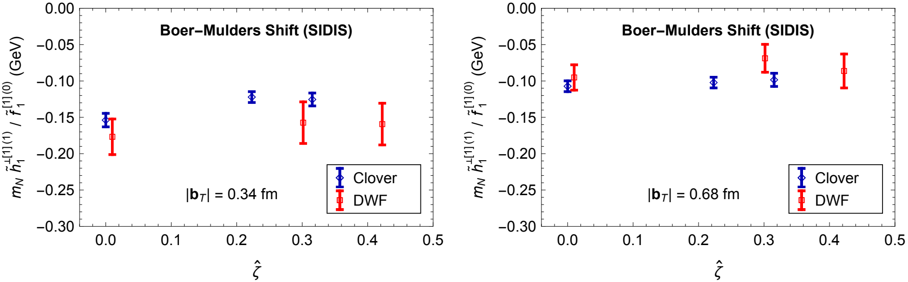
if $\Delta T^{(2,1)}$ as well as all higher order and large distance effects are set equal to zero, to be compared to the lattice value of roughly 0.2.



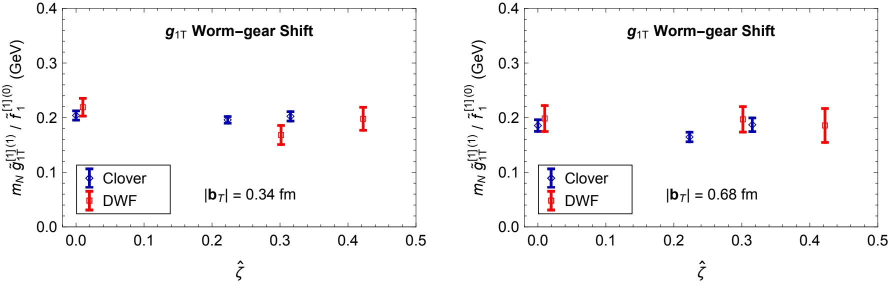
transversity ratio



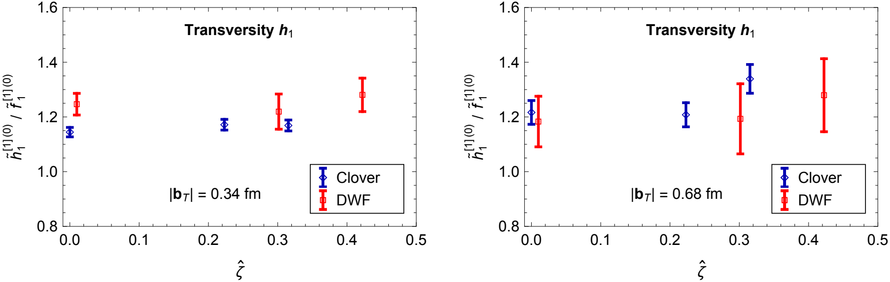
Sivers shift



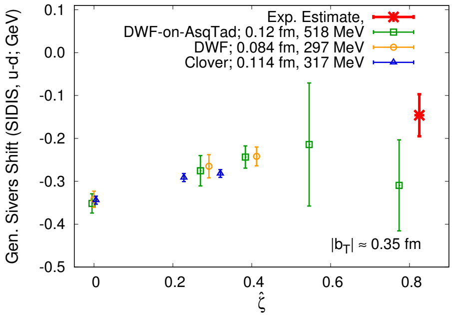
Boer-Mulders shift

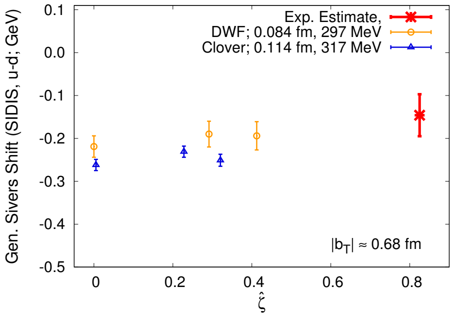


Worm-gear shift, for g_{1T}



transversity ratio



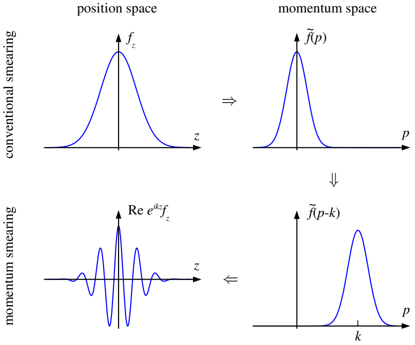


Comparison with experiment

Improvement strategy (for a decade ??)

- physical masses, finer lattices, more statistics etc. straight forward but numerically extremely demanding
- Simulating for larger ζ, i.e. larger momentum

momentum dependent smearing: B. Musch et al., 1602.05525

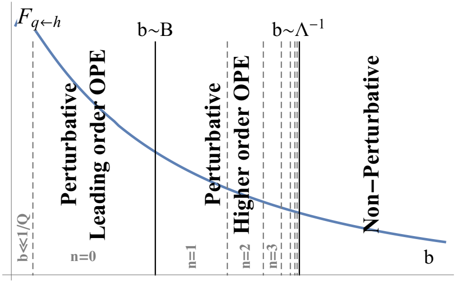


Something new: TMD evolution

Various groups have produced fits to existing data. Some results from

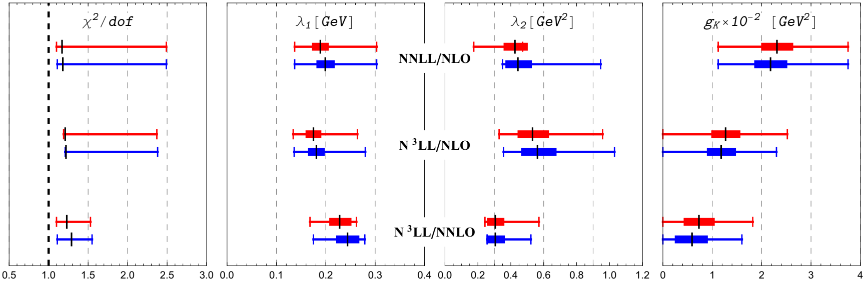
arXiv:1706.01473 I. Scimemi and A. Vladimirov "Analysis of vector boson production within TMD factorization"

$$
\begin{aligned}
F_{q\leftarrow N}(x,\vec{b};\zeta,\mu) &= \frac{Z_q(\zeta,\mu)R_q(\zeta,\mu)}{2} \\
&\times \sum_X \int \frac{d\xi^-}{2\pi} e^{-ixp^+\xi^-} \langle N| \left\{ T[\bar{q}_i \tilde{W}_n^T]_a\Big(\frac{\xi}{2}\Big) \,|X\rangle \right. \\
&\times \left. \gamma_{ij}^+ \langle X| \bar{T}[\tilde{W}_n^{T\dagger} q_j]_a\Big(-\frac{\xi}{2}\Big) \right\} |N\rangle, \\
F(x,\vec{b};\zeta_f,\mu_f) &= \mathcal{R}(\vec{b};\zeta_f,\mu_f,\zeta_i,\mu_i)F(x,\vec{b};\zeta_i,\mu_i) \\
\mathcal{R}(\vec{b};\zeta_f,\mu_f,\zeta_i,\mu_i) &= \exp\left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu}\gamma\Big(\alpha_s(\mu),\ln\frac{\zeta_f}{\mu^2}\Big)\right\}\Big(\frac{\zeta_f}{\zeta_i}\Big)^{-\mathcal{D}(\mu_i,\vec{b})}
\end{aligned}
$$



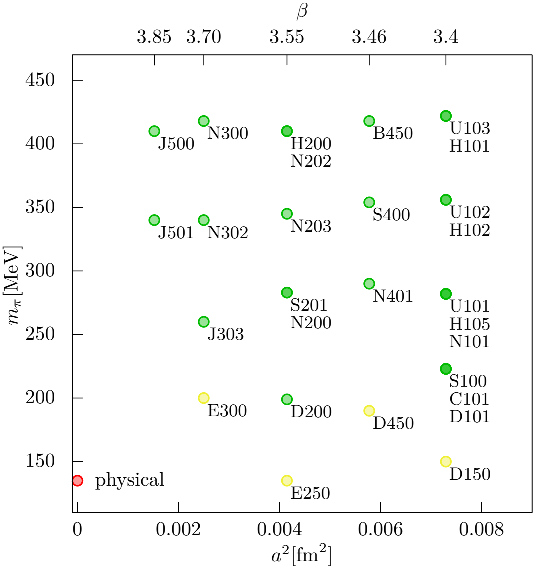
TMDs themselves as well as their evolution contain unsuppressed non-perturbative parts, which have to be parameterized.

$$
\begin{aligned}
F_{q\leftarrow h}(x,\vec{b};\mu,\zeta) &= \int_x^1 \frac{dz}{z}\sum_f C_{q\leftarrow f}(z,\vec{b};\mu,\zeta) f_{f\leftarrow h}\Big(\frac{x}{z},\mu\Big) f_{NP}(z,\vec{b}) \\
f_{NP}(\vec{b}) &= e^{-\lambda_1 b}(1+\lambda_2 b^2) \\
\mathcal{D}^f(\mu,\vec{b}) &= \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\Gamma^f + \mathcal{D}^f_{\text{pert}}(\mu_0,\vec{b}) + g_K \vec{b}^2
\end{aligned}
$$



experimental (fat) and theoretical (thin; from just varying μ and the coefficient function in reasonable bounds) uncertainties for TMDs from DY

What next? study the non-perturbative part of evolution This requires many different $a = 1/\mu \Rightarrow$ CLS



continuum QCD predicts that all TMDs have the same scaling, which, therefore, can be obtained from all the ratios

$$
\begin{aligned}
\frac{f^{[m,n]}(\zeta)}{f^{[m,n]}(\zeta')} &= \frac{f^{[m,n]}(\zeta)}{f^{[m',n]}(\zeta')} = \left(\frac{\zeta}{\zeta'}\right)^{-(D_{pert}+D_{NP})(\mu,\vec{b}_T)} \\
D(\mu,\vec{b}_T)_{pert} &= C^f \sum_{n=1}^{\infty} a_s^n \sum_{k=0}^{n} L_\mu^k d^{(n,k)} \\
L_\mu &:= \ln \frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}
\end{aligned}
$$

The $d^{(n,k)}$ are known. This tests also the assumption that on the lattice one has multiplicative renormalisation factors which cancle in ratios.

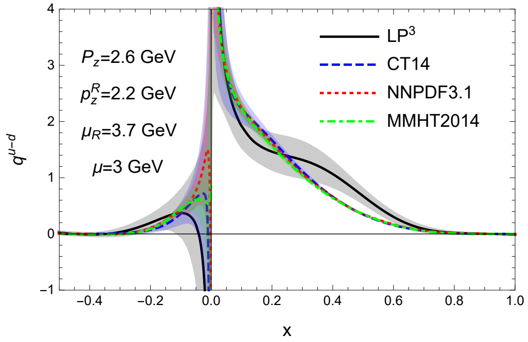
quasi PDFs, quasi TMDs, ... Xiangdong Ji: PRL **110** (2013) 262002, arXiv:1305.1539

Calculate correlators non-local in space on the lattice and relate them to pdfs, DAs etc.

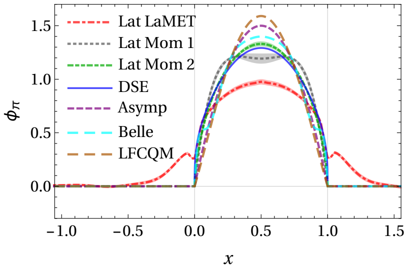
$$
\begin{aligned}
q(x,\mu^2) &= \int \frac{d\xi^-}{4\pi}\, e^{-\mathrm{i}x\xi^- P^+}\, \langle P|\bar{\psi}(\xi^-)\gamma^+ \,\exp\left(-\mathrm{i}g\int_0^{\xi^-} d\eta^-\, A^+(\eta^-)\right)\, \psi(0)|P\rangle \\
q(x,\mu^2,P^z) &= \int \frac{dz}{4\pi}\, e^{\mathrm{i}zk^z}\, e^{-\mathrm{i}x\xi^-}\, \langle P|\bar{\psi}(z)\gamma^z \,\exp\left(-\mathrm{i}g\int_0^{z} dz'\, A^z(z')\right)\, \psi(0)|P\rangle \\
&+ \mathcal{O}\left((\Lambda_{QCD}/P^z)^2, (M/P^z)^2\right) \\
q(x,\mu^2,P^z) &= \int_x^1 \frac{dy}{y}\, Z\left(\frac{x}{y},\frac{\mu}{P^z}\right)\, q(y,\mu^2) \\
Z(x,\mu/P^z) &= \delta(x-1) + \frac{\alpha_s}{2\pi} Z^{(1)}(x,\mu/P^z) + \ldots
\end{aligned}
$$

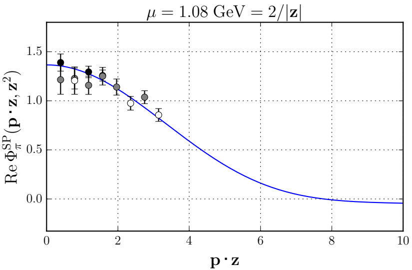
Note: One has to fulfill $\mu = 1/x \geq 1$ GeV and $P^z/\mu = P^z x \sim 1$.

The idea is great, but how large are the systematic errors? The LP^3 collaboration and others explore quark quasi-distribution functions systematically. This works nice for the nucleon quark PDFs arXiv:1807.06566



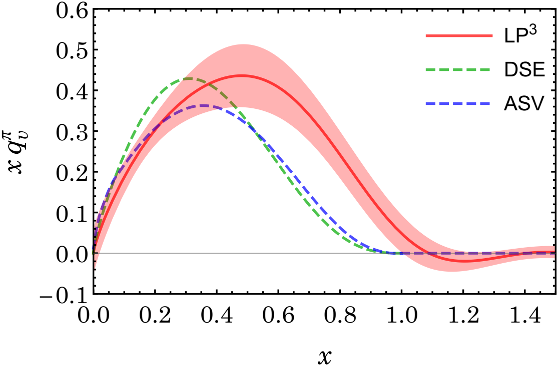
is more difficult for DAs arXiv:1712.10025





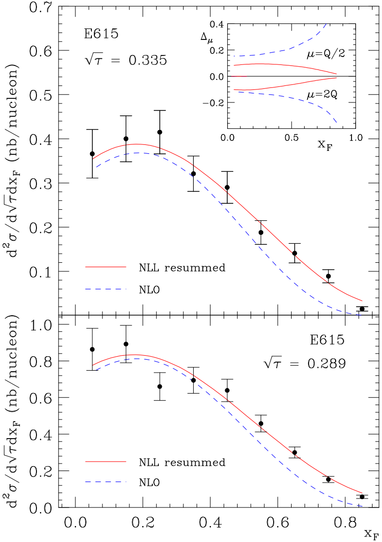
Left: Comparison of ϕ_π to previous determinations in literature.
Right: Converted of ϕ_π to the scheme of Braun and Mueller

is not very successful for the pion PDFs arXiv:1804.01483



There was/is (?) a big debate about the pion quark PDF

ASV is Aicher, AS, **Vogelsang** arXiv:1009.2481

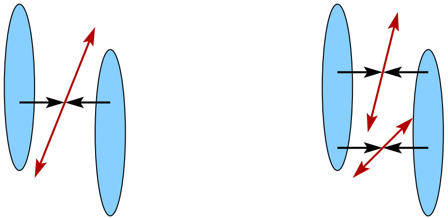


E615
$\sqrt{\tau}$ = 0.335
Δ_μ
μ=Q/2
μ=2Q
x_F
NLL resummed
NLO
E615
$\sqrt{\tau}$ = 0.289
NLL resummed
NLO
$d^2\sigma/d\sqrt{\tau}dx_F$ (nb/nucleon)
x_F

My very personal opinion: While it is nice to explore ever more quantities, the real challenge is to understand the systematic errors of the quasi PDF/DA/TMD approach.
Most puzzleing: Why does the method work much better for the nucleon than for the pion?

In addition there are still conceptual questions to be solved, e.g. renormalization for quasi gluon PDFs
J. H. Zhang, X. Ji, AS, W. Wang and S. Zhao, arXiv:1808.10824
Independently the same was done with different techniques but similar results by Z. Y. Li, Y. Q. Ma and J. W. Qiu, arXiv:1809.01836

What are Double Parton Distributions (DPDs) ?
Why are they related to TMDs ?
LHC: DPIs (more generally MPIs) look unproblematic at first sight



Single parton interaction   Double parton interaction

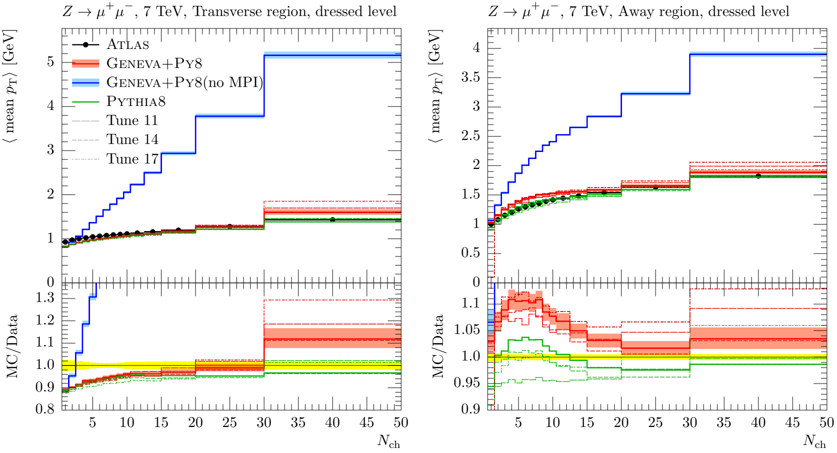
but they are not **!!!**

- collinear physics $\Rightarrow$ light-cone gauge $\Rightarrow$ gauge links can be neglected. **Here they can not; connection to TMDs**
- high multiplicity $=$ enhanced MPI contributions

Alioli, Bauer, Guns, Tackmann 1605.07192 mean charged particle p_T as function of N_{ch}.

Events with large N_{ch} have an even higher MPI contribution.

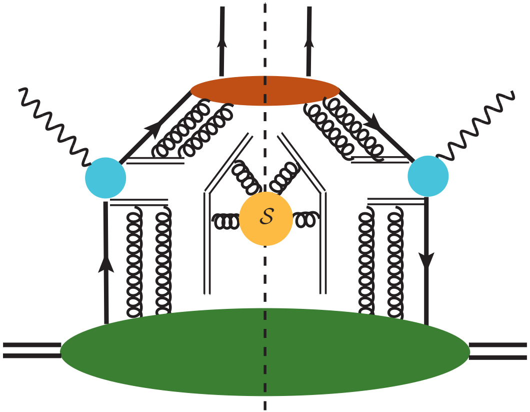
MPIs produce many particles with p_T of O(1 GeV).



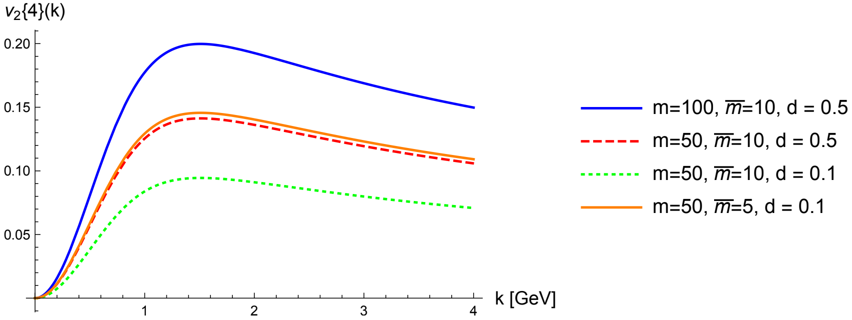
The connection: A. Vladimirov arXiv:1608.04920

Abstract: We show at NNLO that the soft factors for double parton scattering (DPS) for both integrated and unintegrated kinematics, can be presented entirely in the terms of the soft factor for single Drell-Yan process, i.e. the transverse momentum dependent (TMD) soft factor ...

Factorizing the soft factor



Blok, Jäkel, Strikman and Wiedemann, "Collectivity from interference", arXiv:1708.08241



The fourth order cummulant

$$v_n\{4\} = \left(\left\langle\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle_c\right)^{1/4} \quad \text{for p+p.}$$

"These findings indicate that the no-interaction baseline including QCD interference effects can make a sizeable if not dominant contribution to the measured v_n coefficients in pp collisions."

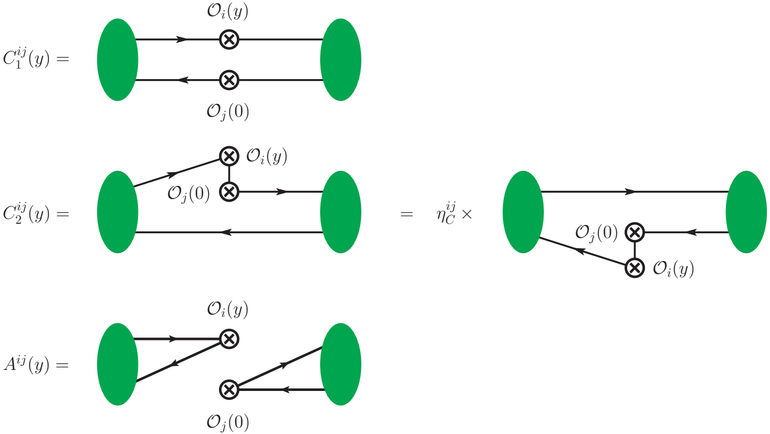
A large DESY (M. Diehl)-Regensburg project: Moments of DPD's on the lattice ch. 4.2 in arXiv: 1111.0910

$$
\begin{aligned}
M^{n_1,n_2}_{a_1,a_2}(\boldsymbol{y}^2) &= \int_0^1 dx_1\, x_1^{n_1-1} \int_0^1 dx_2\, x_2^{n_2-1} \Big[{}^1F_{a_1,a_2}(x_1,x_2,\boldsymbol{y}) + (-1)^{n_1}\, \sigma_{a_1}\, {}^1F_{\bar{a}_1,a_2}(x_1,x_2,\boldsymbol{y}) \\
&+ (-1)^{n_2}\, \sigma_{a_2}\, {}^1F_{a_1,\bar{a}_2}(x_1,x_2,\boldsymbol{y}) + (-1)^{n_1+n_2}\, \sigma_{a_1}\sigma_{a_2}\, {}^1F_{\bar{a}_1,\bar{a}_2}(x_1,x_2,\boldsymbol{y}) \Big] \\
&= \frac{1}{2}(p^+)^{1-n_1-n_2} \int dy^- \langle p|\, \mathcal{O}^{+\cdots+}_{a_1}(0)\, \mathcal{O}^{+\cdots+}_{a_2}(y) |p\rangle_{y^+=0} \\
\mathcal{O}^{\mu_1\cdots\mu_n}_{q}(y) &= \underset{(\mu_1\cdots\mu_n)}{\mathrm{T}}\ \underset{(\mu_1\cdots\mu_n)}{\mathrm{S}}\ \bar{q}(y)\, \gamma^{\mu_1}\, i\overleftrightarrow{D}^{\mu_2}(y) \cdots i\overleftrightarrow{D}^{\mu_n}(y)\, q(y) \\
\mathcal{O}^{\mu_1\cdots\mu_n}_{\Delta q}(y) &= \underset{(\mu_1\cdots\mu_n)}{\mathrm{T}}\ \underset{(\mu_1\cdots\mu_n)}{\mathrm{S}}\ \bar{q}(y)\, \gamma^{\mu_1}\gamma_5\, i\overleftrightarrow{D}^{\mu_2}(y) \cdots i\overleftrightarrow{D}^{\mu_n}(y)\, q(y) \\
\mathcal{O}^{\lambda\mu_1\cdots\mu_n}_{\delta q}(y) &= \underset{(\lambda\mu_1\cdots\mu_n)}{\mathrm{T}}\ \underset{(\lambda\mu_1)}{\mathrm{A}}\ \underset{(\mu_1\cdots\mu_n)}{\mathrm{S}}\ \bar{q}(y)\, i\sigma^{\lambda\mu_1}\gamma_5\, i\overleftrightarrow{D}^{\mu_2}(y) \cdots i\overleftrightarrow{D}^{\mu_n}(y)\, q(y) \\
\overleftrightarrow{D}^{\mu}(y) &= \tfrac{1}{2}(\overrightarrow{\partial}^{\mu} - \overleftarrow{\partial}^{\mu}) + igA^{\mu}(y)
\end{aligned}
$$

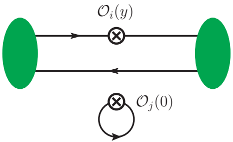
arXiv:1807.03073 $n = 1$, charge-charge correlations in the pion; hopefully the first of many papers

Correlations of interest are linear combinations of quantities calculated separately on the lattice ⇒ High numerical accuracy is needed. Therefore, we started with the pions.

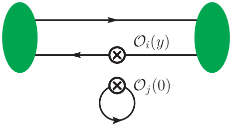
$$
\begin{aligned}
\langle\pi^+|\mathcal{O}_i^{uu}(y)\mathcal{O}_j^{dd}(0)|\pi^+\rangle &= C_1 + \left[2S_1 + D\right] \\
\langle\pi^+|\mathcal{O}_i^{uu}(y)\mathcal{O}_j^{uu}(0)|\pi^+\rangle &= \left[2C_2 + S_2\right] + \left[2S_1 + D\right] \\
\langle\pi^0|\mathcal{O}_i^{uu}(y)\mathcal{O}_j^{dd}(0)|\pi^0\rangle &= \left[2S_1 + D\right] - A \\
\langle\pi^0|\mathcal{O}_i^{uu}(y)\mathcal{O}_j^{uu}(0)|\pi^0\rangle &= C_1 + \left[2S_1 + D\right] + \left[2C_2 + S_2\right] + A \\
\langle\pi^0|\mathcal{O}_i^{du}(y)\mathcal{O}_j^{du}(0)|\pi^0\rangle &= -C_1 + \left[2C_2 + S_2\right] \\
\langle\pi^-|\mathcal{O}_i^{du}(y)\mathcal{O}_j^{ud}(0)|\pi^+\rangle &= 2C_1 + 2A \\
\langle\pi^+|\mathcal{O}_i^{du}(y)\mathcal{O}_j^{du}(0)|\pi^+\rangle &= 2C_2^{ij}(y) + S_2 + A^{ij}(y) \\
\sqrt{2}\langle\pi^0|\mathcal{O}_i^{du}(y)\mathcal{O}_j^{uu}(0)|\pi^+\rangle &= C_1 + \left[C_2^{ij}(y) - C_2^{ji}(-y)\right] + A^{ij}(y)
\end{aligned}
$$


$C_1^{ij}(y) =$
$\mathcal{O}_i(y)$
$\mathcal{O}_j(0)$
$C_2^{ij}(y) =$
$\mathcal{O}_i(y)$
$\mathcal{O}_j(0)$
$= \quad \eta_C^{ij} \times$
$\mathcal{O}_j(0)$
$\mathcal{O}_i(y)$
$A^{ij}(y) =$
$\mathcal{O}_i(y)$
$\mathcal{O}_j(0)$

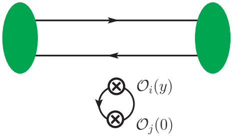
$S_1^{ij}(y) =$



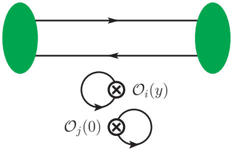
$= \eta_C^{ij} \times$



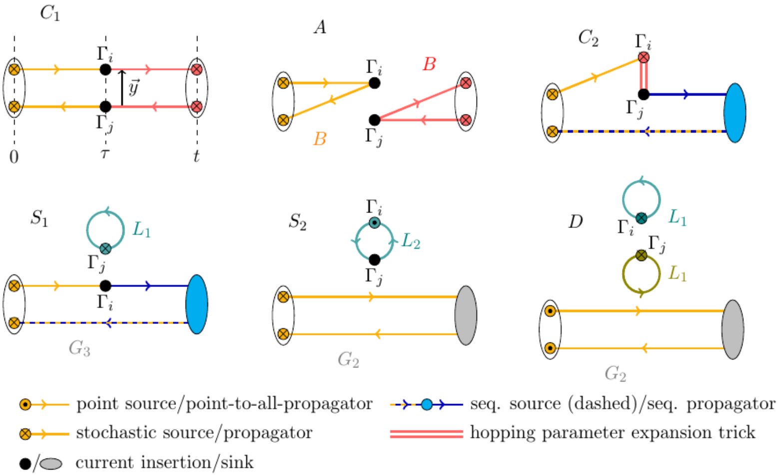
$S_2^{ij}(y) =$



$D^{ij}(y) =$



On the lattice this is calculated like this



We have started meanwhile also simulations for the nucleon, which is the case relevant for LHC.

All stays the same except for C_1

ensemble	β	a [fm]	κ	$L^3 \times T$	m_π [MeV]	Lm_π	N_{full}	N_{used}	N_{sm}
IV	5.29	0.071	0.13632	$32^3 \times 64$	294.6(14)	3.42	2023	960	400
V	5.29	0.071	0.13632	$40^3 \times 64$	288.8(11)	4.19	2025	984	400

The used $N_f = 2$ ensembles.

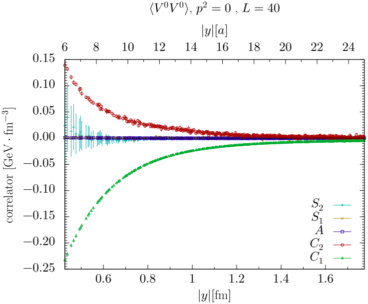
Renormalization:

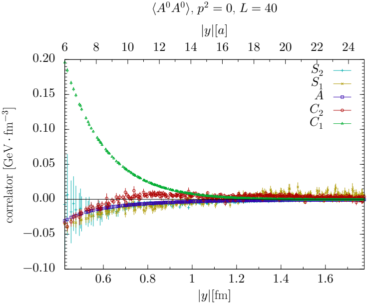
$$R_{ij}^{\overline{\rm MS}} = \tilde{Z}_i \, \tilde{Z}_j \, R_{ij}^{\rm lat} \quad \text{with} \quad \tilde{Z}_i = Z_i^{\overline{\rm MS}} \, (1 + a m_q \, b_i)$$

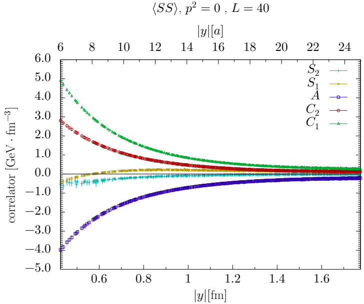
	S	P	V	A	T
$Z^{\overline{\rm MS}}$	0.6153(25)	0.476(13)	0.7356(48)	0.76487(64)	0.8530(25)
$b^{\rm pert}$	1.3453	1.2747	1.2750	1.2731	1.2497
$b^{\rm np}$	1.091(55)		1.586(32)		
$b^{\rm resc}$	1.673	1.586	1.586	1.584	1.555

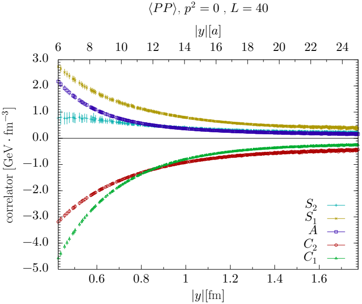
Renormalisation factors $Z_i^{\overline{\rm MS}}$ from Bali *et al.* arXiv:1412.7336 in the $\overline{\rm MS}$ scheme at $\mu = 2\,\text{GeV}$. For P, A and T we rescale the perturbative value according to V. The S case shows that this procedure contributes a significant systematic error.

very few results

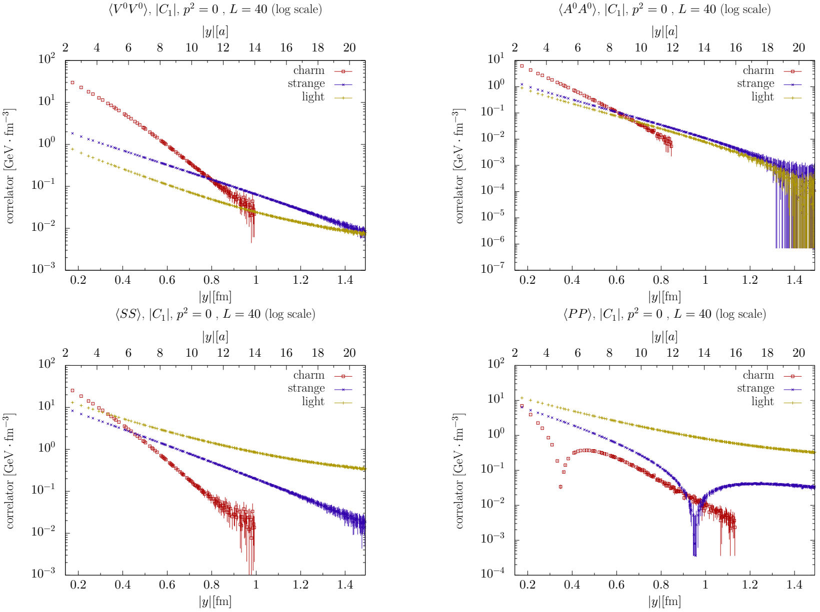








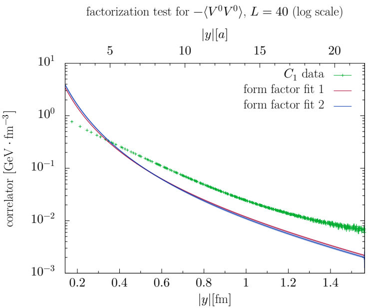
quark mass dependence

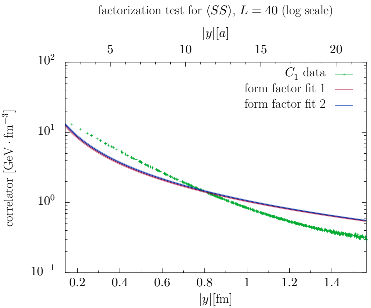


The strength of correlation effects

$$
\begin{aligned}
\mathcal{M}_{ii}(\vec{q}^2) &= \int d^3y\, e^{i\vec{y}\cdot\vec{q}}\, \langle \pi^+(p)|\mathcal{O}_i^{uu}(y)\mathcal{O}_i^{dd}(0)|\pi^+(p)\rangle \\
&\stackrel{?}{=} \int d^3y\, e^{i\vec{y}\cdot\vec{q}} \int \frac{d^3p'}{(2\pi)^3 2p'^0} \langle \pi^+(p)|\mathcal{O}_i^{uu}(y)|\pi^+(p')\rangle \\
&\times \langle \pi^+(p')|\mathcal{O}_i^{dd}(0)|\pi^+(p)\rangle \\
&= \frac{\eta_C^i}{2E_q} \left|\langle \pi^+(E_q,-\vec{q})|\mathcal{O}_i^{uu}(0)|\pi^+(p)\rangle\right|^2
\end{aligned}
$$

$$
\begin{aligned}
-\mathcal{M}_{V^0V^0}(\vec{q}^2) &\stackrel{?}{=} \frac{(m_\pi + E_q)^2}{2E_q} \left[F_V(2m_\pi E_q - 2m_\pi^2)\right]^2 \\
\mathcal{M}_{SS}(\vec{q}^2) &\stackrel{?}{=} \frac{1}{2E_q} \left[\, F_S(2m_\pi E_q - 2m_\pi^2)\,\right]^2
\end{aligned}
$$





The Fourier transformation of both sides of the factorization test equations.

$\Rightarrow$ The correlations are sizeable and must be treated exactly
$\Rightarrow$ Understanding transverse correlation in hadron structure could become crucial for LHC; work for decades

Conclusions

- Lattice QCD does not provide magic bullets. Progress is steady but very tedious
- Calculating double moments of TMDs is in good shape but needs improvement (physical masses, continuum limit, larger ζ, more statistics) and will never give more than a few Mellin moment.
- quasi-TMDs are a long term possibility to go beyond moments IF all systematic and conceptual problems can be controlled
- MPIs are important for LHC. It is claimed that they can be perfectly described by event generators. I would like to see Lattice QCD checks. This is again very tedious and long term, but first results suggest significant quark correlations in the pion. Is there a difference between pions and nucleons?