Higher Order calculation of SIDIS Y term

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INT Program
Transverse spin and TMDs
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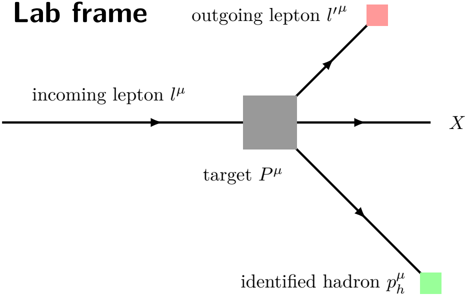
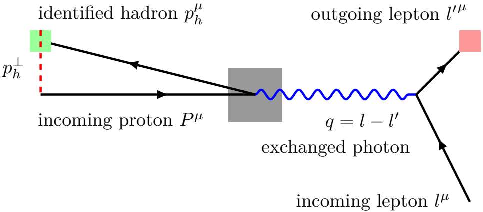
SIDIS overview

Semi inclusive deep inelastic scattering (SIDIS)

Lab frame

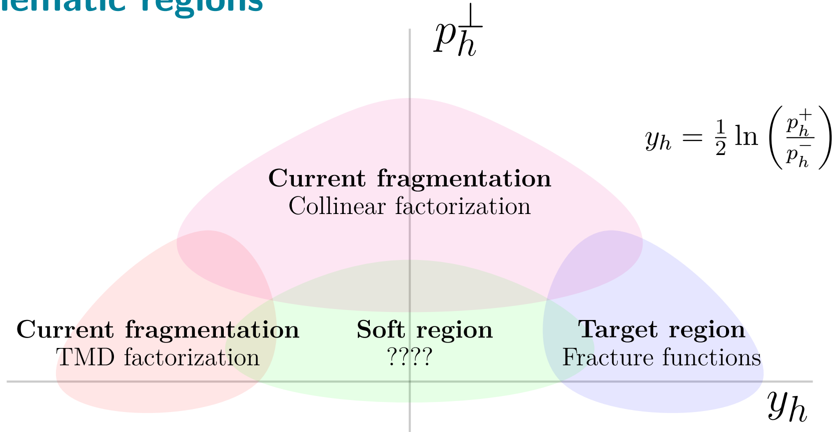


Breit frame



Key question :
How is $p_h^{\perp}$ generated at short distances?

Kinematic regions



- **Different regions** are sensitive to distinct physical mechanisms

Factorization in the current region

- q_{T} **integrated** cross sections
  + 2 non perturbative ingredients: $f_1(\xi)$, $d_1(\zeta)$

- q_{T} **differential** cross sections
  + 4 non perturbative ingredients: $f_1(\xi)$, $d_1(\zeta)$, $f_1(\xi, k_\perp)$, $d_1(\zeta, k_\perp)$

- How to relate the two methods?
  $\rightarrow$ Collins, Gamberg, Prokudin, Rogers, NS, Wang

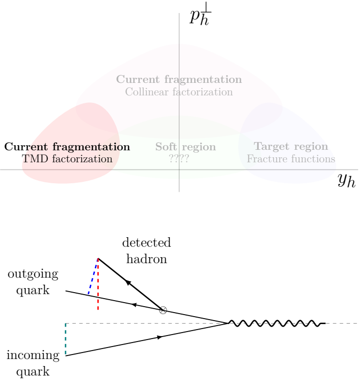
$$\frac{d\sigma}{dxdzdQ^2} = \int dq_{\mathrm{T}} \frac{d\sigma}{dxdzdQ^2 dq_{\mathrm{T}}}$$

- Can we **validate** the formalism in nature?

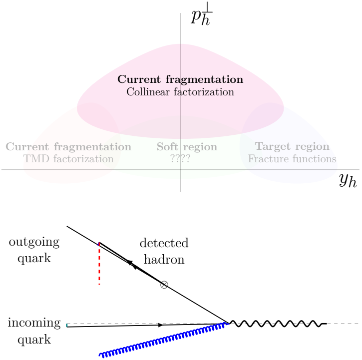
The formalism for q_{T} differential cross section

Theory framework for current fragmentation

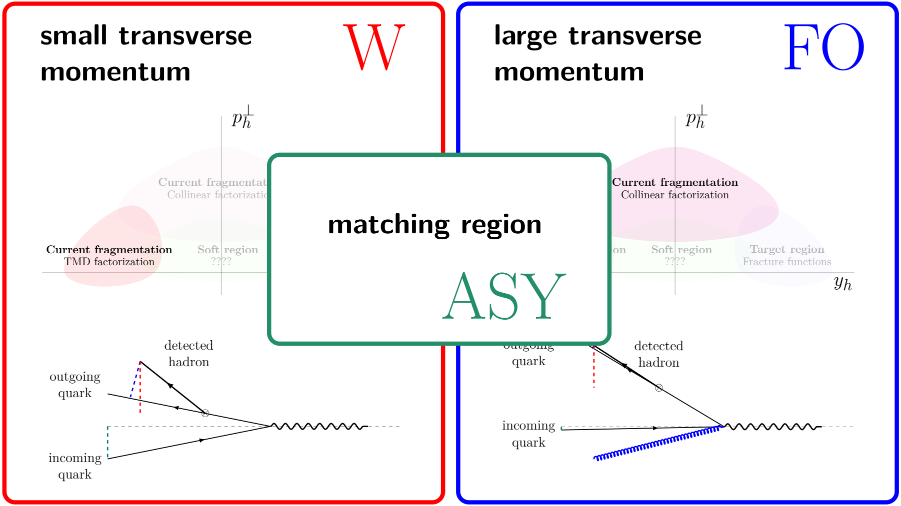
small transverse momentum W



large transverse momentum FO



Theory framework for current fragmentation



Theory framework for current fragmentation

- The formulation of is based on a scale separation governed by the ratio

$$q_{\mathrm{T}}/Q$$

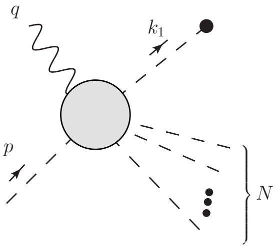
- where

$$z = \frac{P \cdot p_h}{P \cdot q}, \qquad q_{\mathrm{T}} = p_h^{\perp}/z$$

- The cross section is built as

$$\frac{d\sigma}{dx dQ^2 dz dp_h^{\perp}} = \mathrm{W} + \mathrm{FO} - \mathrm{ASY} + \mathcal{O}(m^2/Q^2)$$
$$\sim \mathrm{W} \quad \text{for } q_{\mathrm{T}} \ll Q$$
$$\sim \mathrm{FO} \quad \text{for } q_{\mathrm{T}} \sim Q$$

Why q_T/Q ? (J. Gonzalez-Hernandes, T.C Rogers, NS, B. Wang)



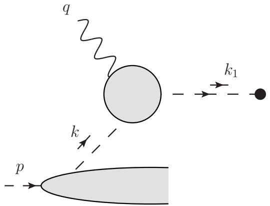
- Lets define

$$k \equiv k_1 - q$$

- Propagators in the blob

$$\frac{1}{k^2 + O(\Lambda_{\rm QCD}^2)}, \qquad \frac{1}{k^2 + O(Q^2)}$$

- Two extreme regions
  - $|k^2| \sim \Lambda_{\rm QCD}^2 \to k$ is part of PDF
  - $|k^2| \sim Q^2 \to k$ is part of hard blob



- $|k^2|/Q^2$ is the relevant Lorentz invariant measure of transverse momentum size

Why q_{T}/Q ? (J. Gonzalez-Hernandes, T.C Rogers, NS, B. Wang)

- In terms of partonic variables

$$\left|\frac{k^2}{Q^2}\right| = (1-\hat{z}) + \hat{z}\frac{q_{\mathrm{T}}^2}{Q^2}$$

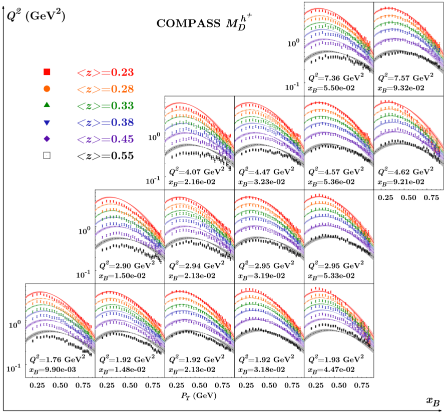
- For $q_T < Q$ one can write

$$\frac{q_{\mathrm{T}}^2}{Q^2} < \left|\frac{k^2}{Q^2}\right| < 1 - z\left(1 - \frac{q_{\mathrm{T}}^2}{Q^2}\right)$$

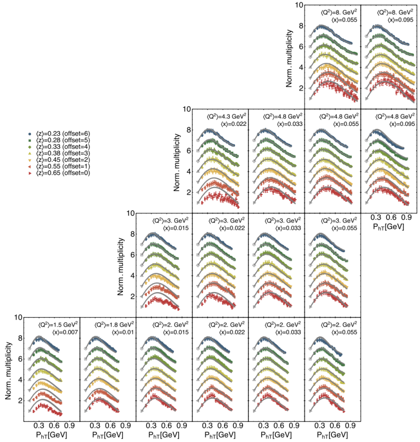
- One can conclude that
  - $q_{\mathrm{T}} \ll Q$ signals the onset of TMD region
  - $q_{\mathrm{T}} \sim Q$ signals the large transverse momentum region

Phenomenology

Existing phenomenology



Anselmino et al



Bacchetta et al

- These analyzes used only W (Gaussian, CSS)
- Samples with $q_{\mathrm{T}}/Q \sim 1.63$ has been included
- **BUT TMDs are only valid for $q_{\mathrm{T}}/Q \ll 1$!**

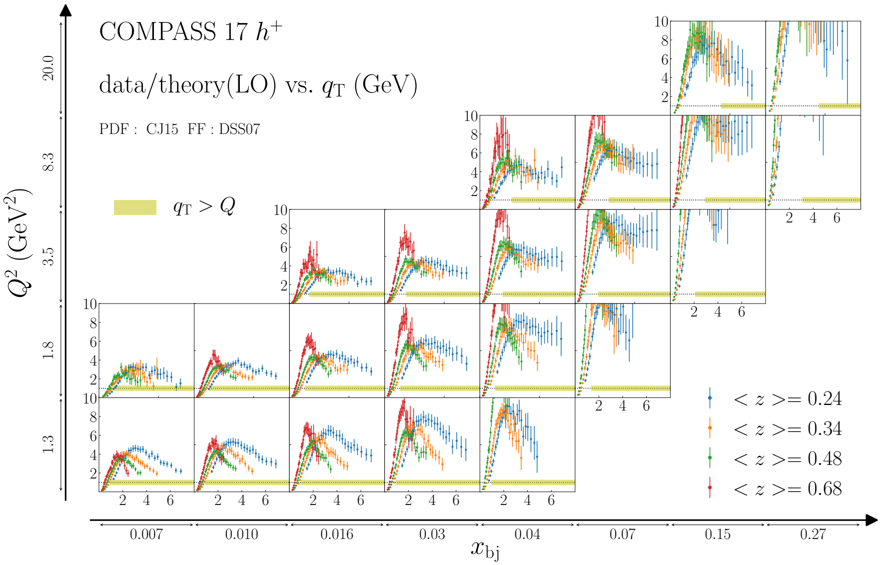
Large p_{T} SIDIS phenomenology

- At LO:

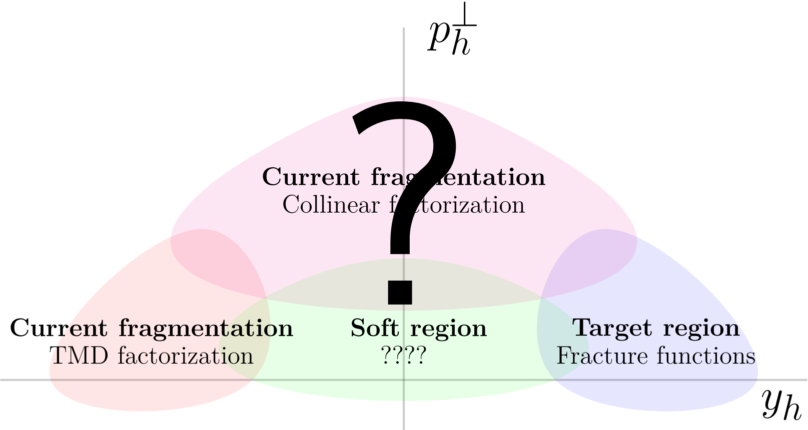
$$\frac{d\sigma}{dx dQ^2 dz dp_{\mathrm{T}}} \sim \sum_q e_q^2 \int_{\frac{q_{\mathrm{T}}^2}{Q^2}\frac{xz}{1-z}+x}^{1} \frac{d\xi}{\xi - x} f_q(\xi,\mu)\ d_q(\zeta(\xi),\mu)\ H(\xi)$$

- For collinear distributions we use
  - PDFs: CJ15
  - FFs: DSS07

FO @ LO predictions (DSS07)



The large p_{T} puzzle



$$\underbrace{\frac{d\sigma}{dx\,dz\,dQ^2}}_{?} \overset{?}{=} \int dq_{\mathrm{T}} \underbrace{\left[\underbrace{\color{red}{\mathrm{W}}}_{\checkmark} + \underbrace{\color{blue}{\mathrm{FO}}}_{?} - \underbrace{\color{teal}{\mathrm{ASY}}}_{?}\right]}_{?} + \mathcal{O}(m^2/Q^2)$$

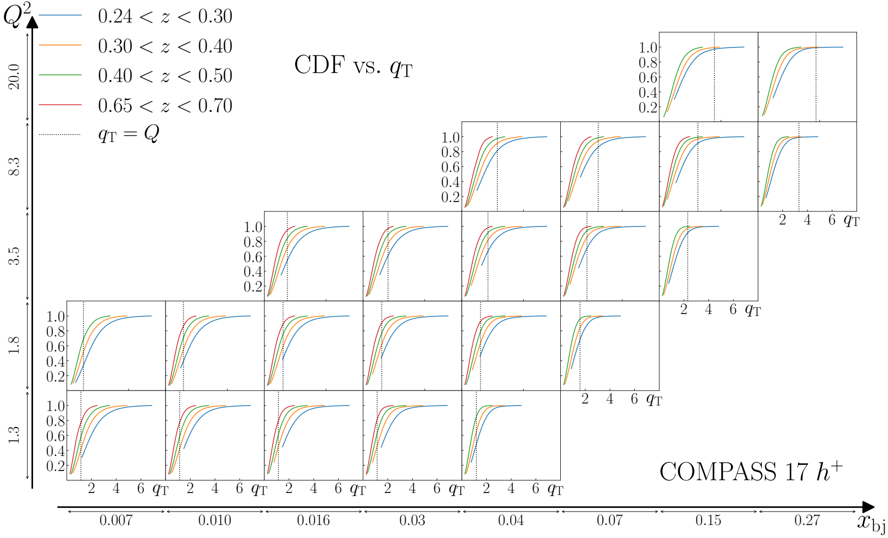
Question

- How important is the $P_{\rm T}$ tail for the integrated SIDIS multiplicities?

- Consider the cumulative distribution function (CDF)

$$\mathrm{CDF} = \int_0^{P_{\rm T}^2} dP_{\rm T}^2 \frac{1}{M(x,z)} \frac{dM}{dP_{\rm T}^2}(x,z,P_{\rm T}^2)$$

From q_T differential to q_T integrated



Revisiting charged hadron FFs (JAM)

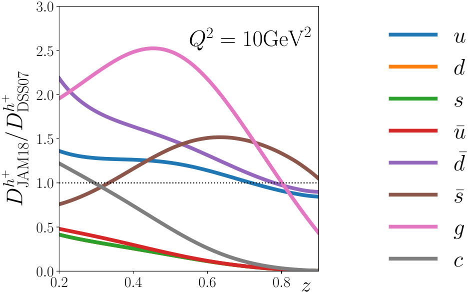
Revisiting charged hadron FFs (in JAM)

- For q_{T} integrated cross section @ NLO:

$$\frac{d\sigma}{dx dQ^2 dz} = \sum_q H_q \otimes f_q \otimes d_q(x,z)$$

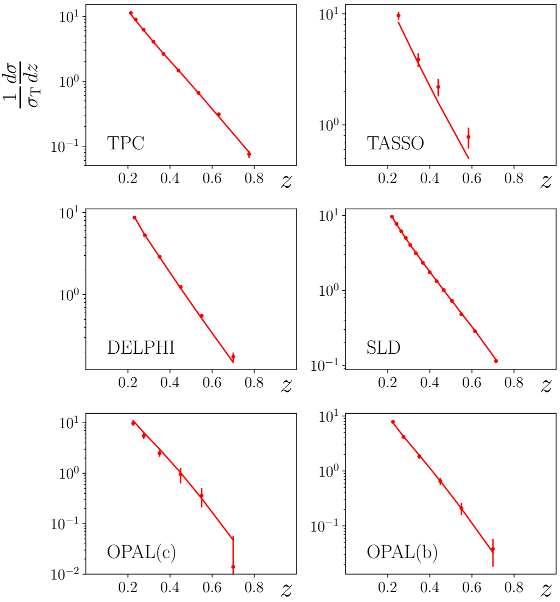
- Data sets:
  - SIDIS(h^+, h^-) q_{T} integrated data from COMPASS
  - $e^+e^- \to h^\pm + X$ (work with the $0.2 < z < 0.8$ samples)
  - PDFs: JAM18

Revisiting charged hadron FFs (in JAM)



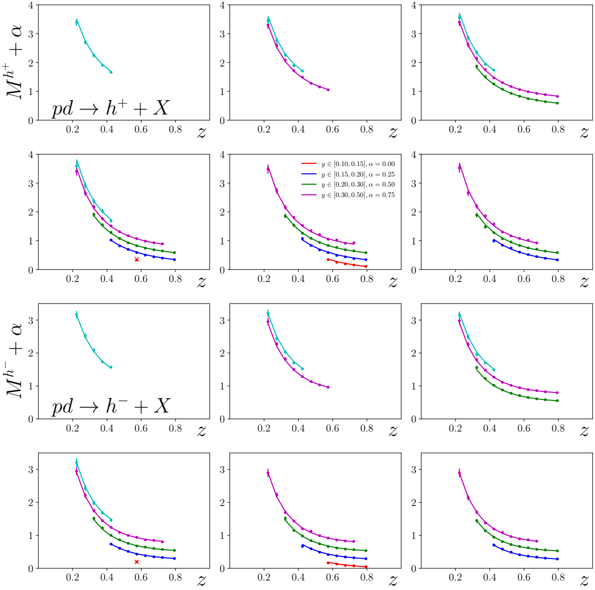
- The gluon fragmentation is significantly different $\rightarrow$ recently observed by the NNPDF

Revisiting charged hadron FFs (in JAM)



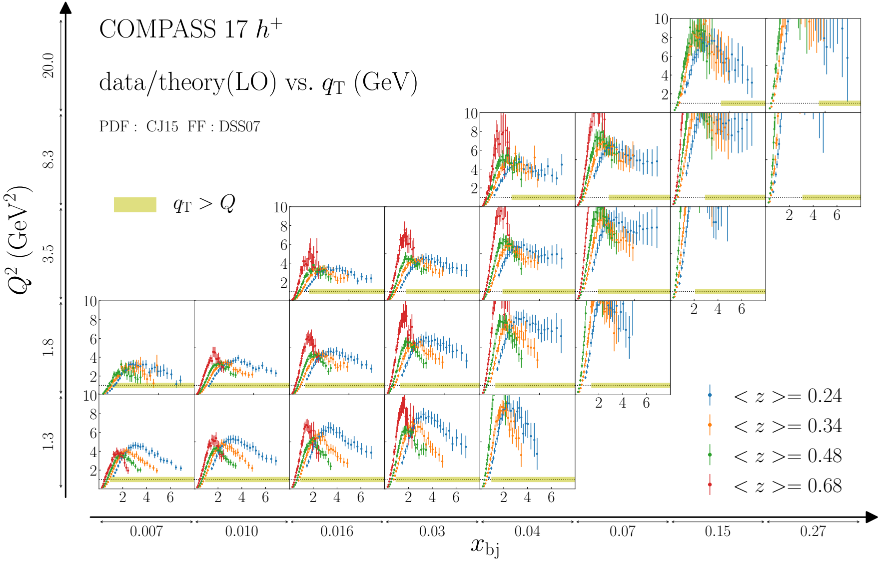
$\chi^2/\text{npts} = 0.53$

Revisiting charged hadron FFs (in JAM)

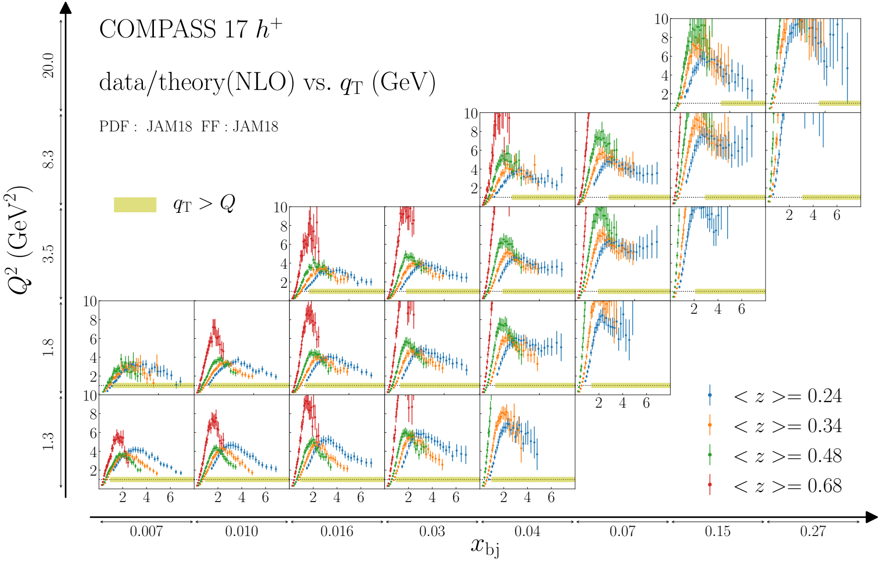


$$\chi^2/\text{npts} = 0.48$$

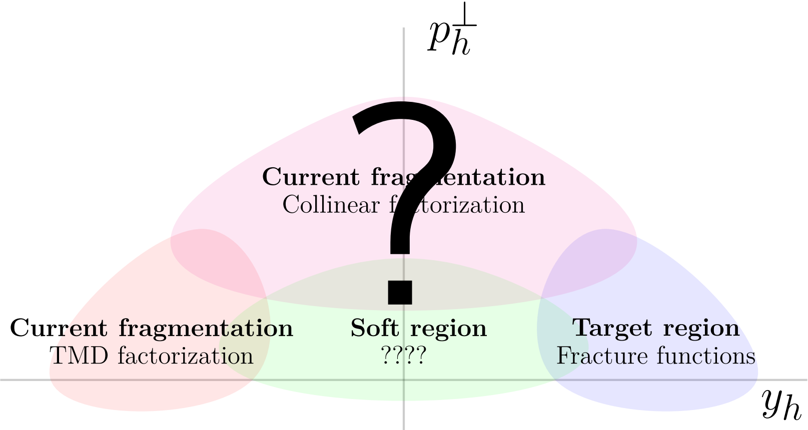
FO @ LO predictions (DSS07)



FO @ LO predictions (JAM18)

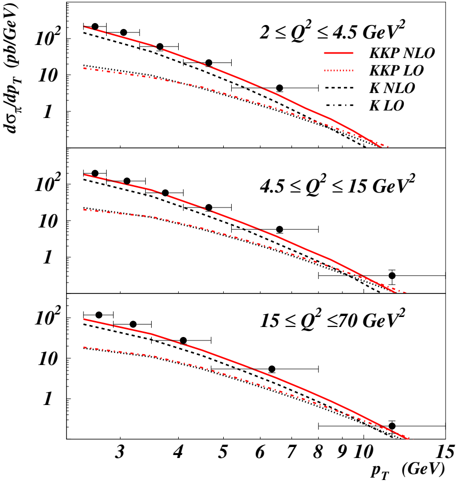


The large p_T puzzle



$$\underbrace{\frac{d\sigma}{dx\,dz\,dQ^2}}_{\checkmark} \stackrel{?}{=} \int dq_\mathrm{T} \underbrace{\left[\underbrace{\mathrm{W}}_{\checkmark} + \underbrace{\mathrm{FO}}_{?} - \underbrace{\mathrm{ASY}}_{?} \right]}_{?} + \mathcal{O}(m^2/Q^2)$$

order α_S^2 corrections to FO



Daleo,et al. (2005)
PRD.71.034013

- There are strong indications that order α_S^2 corrections are very important
- An order of magnitude of corrections at small p_T.
- As a sanity check, we need to have an independent calculation

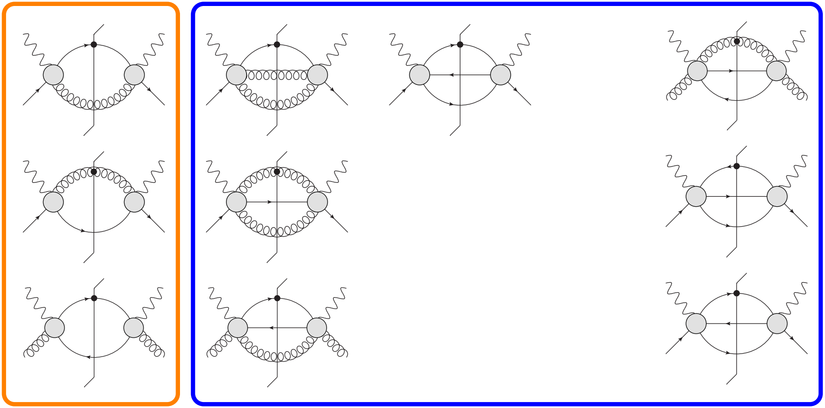
$O(\alpha_S^2)$ calculation (J. Gonzalez-Hernandes, T.C Rogers, NS, B. Wang)

$$W^{\mu\nu}(P,q,P_H) = \int_{x-}^{1+} \frac{d\xi}{\xi} \int_{z-}^{1+} \frac{d\zeta}{\zeta^2} \hat{W}_{ij}^{\mu\nu}(q, x/\xi, z/\zeta) f_{i/P}(\xi) d_{H/j}(\zeta)$$

$$\begin{aligned} \{\mathrm{P}_g^{\mu\nu}\hat{W}_{\mu\nu}^{(N)}; \mathrm{P}_{PP}^{\mu\nu}\hat{W}_{\mu\nu}^{(N)}\} &\equiv \frac{1}{(2\pi)^4}\int \{|M_g^{2\to N}|^2; |M_{pp}^{2\to N}|^2\}\, \mathrm{d}\Pi^{(N)} - \text{Subtractions} \\ &\equiv \{\mathrm{P}_g^{\mu\nu}\hat{W}_{\mu\nu}^{(N)}; \mathrm{P}_{PP}^{\mu\nu}\hat{W}_{\mu\nu}^{(N)}\}_{\text{unsub}} - \text{Subtractions} \end{aligned}$$

- ✓ Compute $2 \to 2$ virtual graphs (Passarino-Veltman)
- ✓ Compute $2 \to 3$ real graphs
- ✓ Integrate 3-body PS analytically using dim reg
- ✓ Cancel double and single IR poles

Our setup for $O(\alpha_S^2)$ contribution

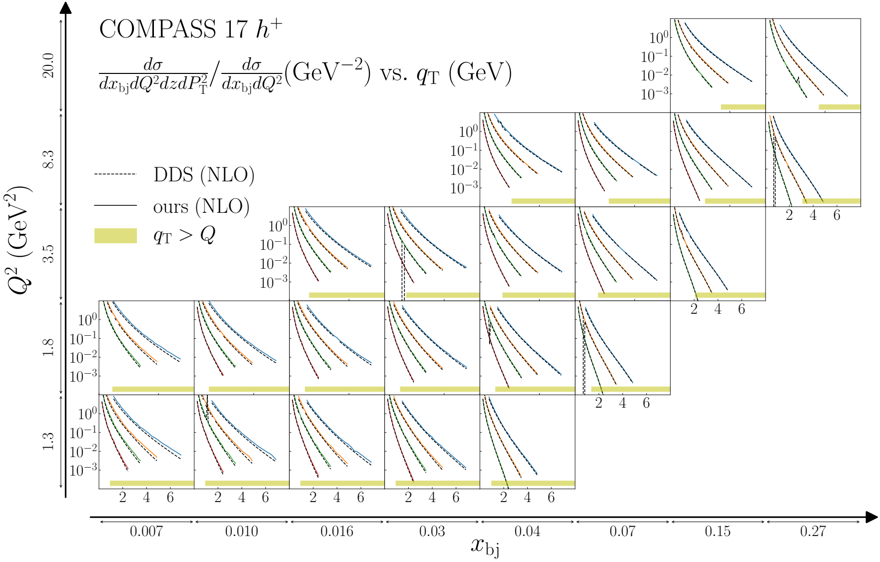


Born/Virtual

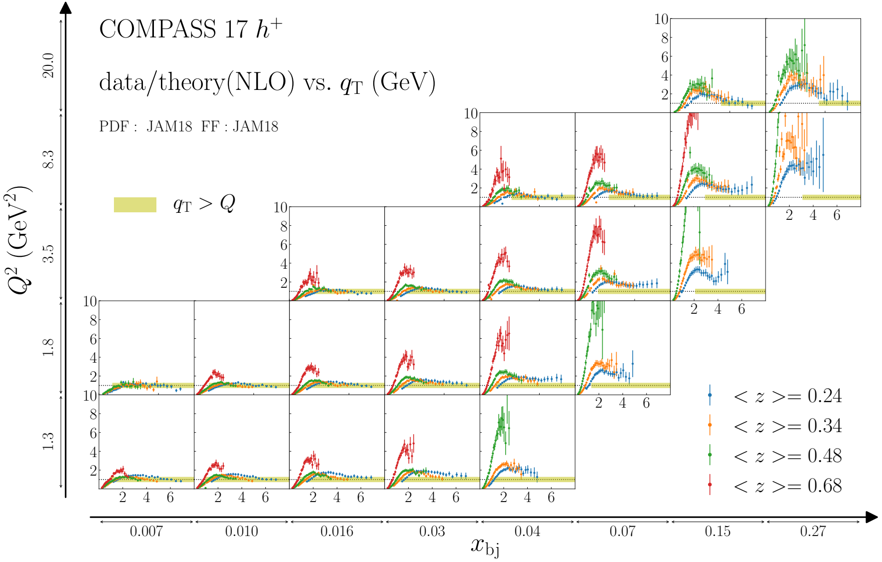
Real

- Dots indicates the fragmenting parton

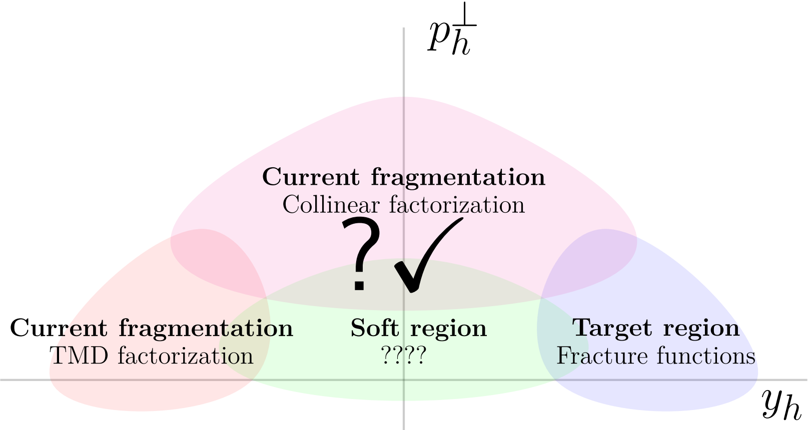
FO @ NLO benchmark



FO @ NLO (JAM FFs)



The large p_{T} puzzle



$$\underbrace{\frac{d\sigma}{dx\,dz\,dQ^2}}_{\checkmark} \stackrel{?}{=} \int dq_{\mathrm{T}} \underbrace{\left[\underbrace{\color{red}{\mathrm{W}}}_{\checkmark} + \underbrace{\color{blue}{\mathrm{FO}}}_{?\checkmark} - \underbrace{\color{teal}{\mathrm{ASY}}}_{?}\right]}_{?} + \mathcal{O}(m^2/Q^2)$$

Summary and outlook

$$\frac{d\sigma}{dx\ dy\ d\Psi\ dz\ d\phi_h\ dP_{hT}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\sum_{i=1}^{18} F_i(x,z,Q^2,P_{hT}^2)\beta_i$$

F_i	Standard label	β_i
F_1	$F_{UU,T}$	1
F_2	$F_{UU,L}$	ε
F_3	F_{LL}	$S_{\|\|}\lambda_e\sqrt{1-\varepsilon^2}$
F_4	$F_{UT}^{\sin(\phi_h+\phi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(\phi_h+\phi_S)$
F_5	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\sin(\phi_h-\phi_S)$
F_6	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(\phi_h-\phi_S)$
F_7	$F_{UU}^{\cos 2\phi_h}$	$\varepsilon\ \cos(2\phi_h)$
F_8	$F_{UT}^{\sin(3\phi_h-\psi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(3\phi_h-\phi_S)$
F_9	$F_{LT}^{\cos(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)$
F_{10}	$F_{UL}^{\sin\ 2\phi_h}$	$S_{\|\|}\varepsilon\sin(2\phi_h)$
F_{11}	$F_{LT}^{\cos\phi_S}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S$
F_{12}	$F_{LL}^{\cos\phi_h}$	$S_{\|\|}\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h$
F_{13}	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)$
F_{14}	$F_{UL}^{\sin\phi_h}$	$S_{\|\|}\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h$
F_{15}	$F_{LU}^{\sin\phi_h}$	$\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h$
F_{16}	$F_{UU}^{\cos\phi_h}$	$\sqrt{2\varepsilon(1+\varepsilon)}\ \cos\phi_h$
F_{17}	$F_{UT}^{\sin\phi_S}$	$\|\vec{S}_\perp\|\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S$
F_{18}	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)$

- The apparent disagreement between data and FO can be resolved by tunning FFs+NLO
- Maybe it might be possibility to describe F_{UU} in the full $\mathrm{W}+\mathrm{FO}-\mathrm{ASY}$
- This is important as all the structure functions that are typically provided in a form of asymmetries $A_i = F_i/F_{UU}$