Semi-Inclusive DIS at low to moderate Q

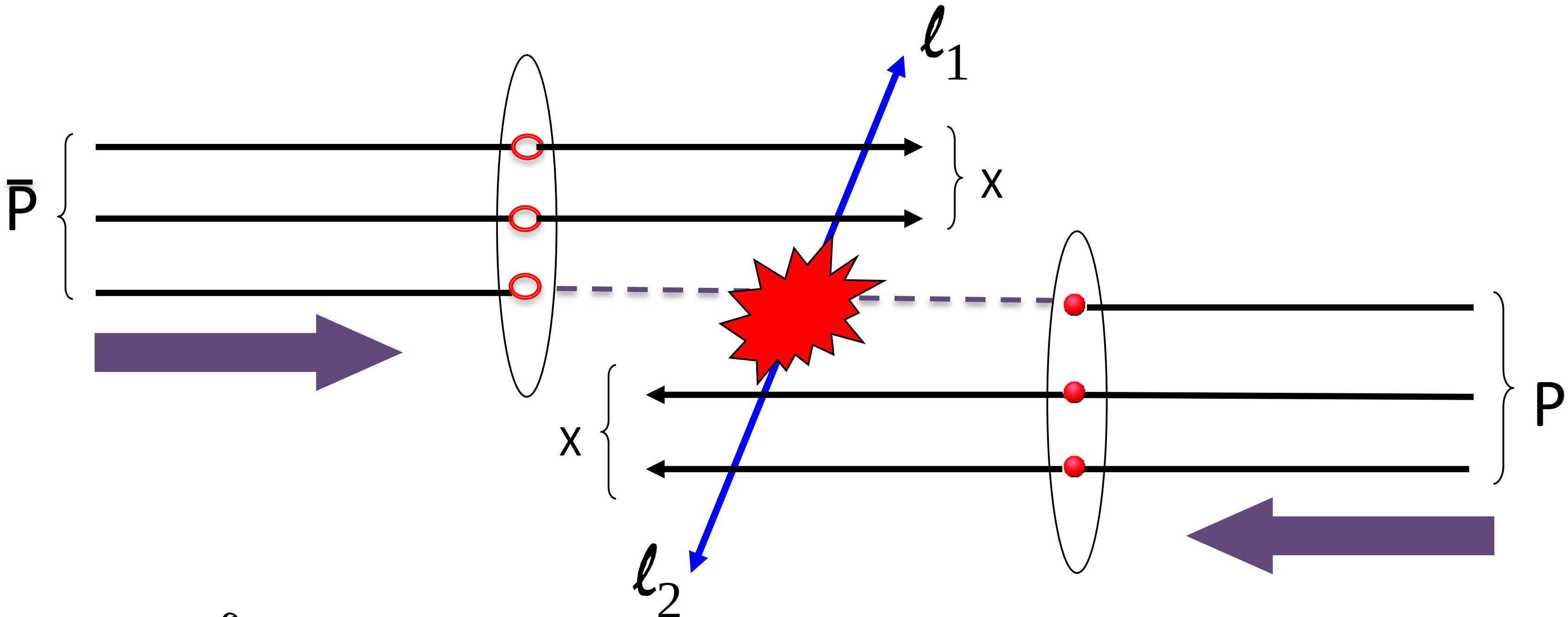
Ted Rogers

Jefferson Lab/Old Dominion University

- Overview of TMD factorization
- SIDIS
- Issues at small/moderate Q

INT workshop, October 8, 2018

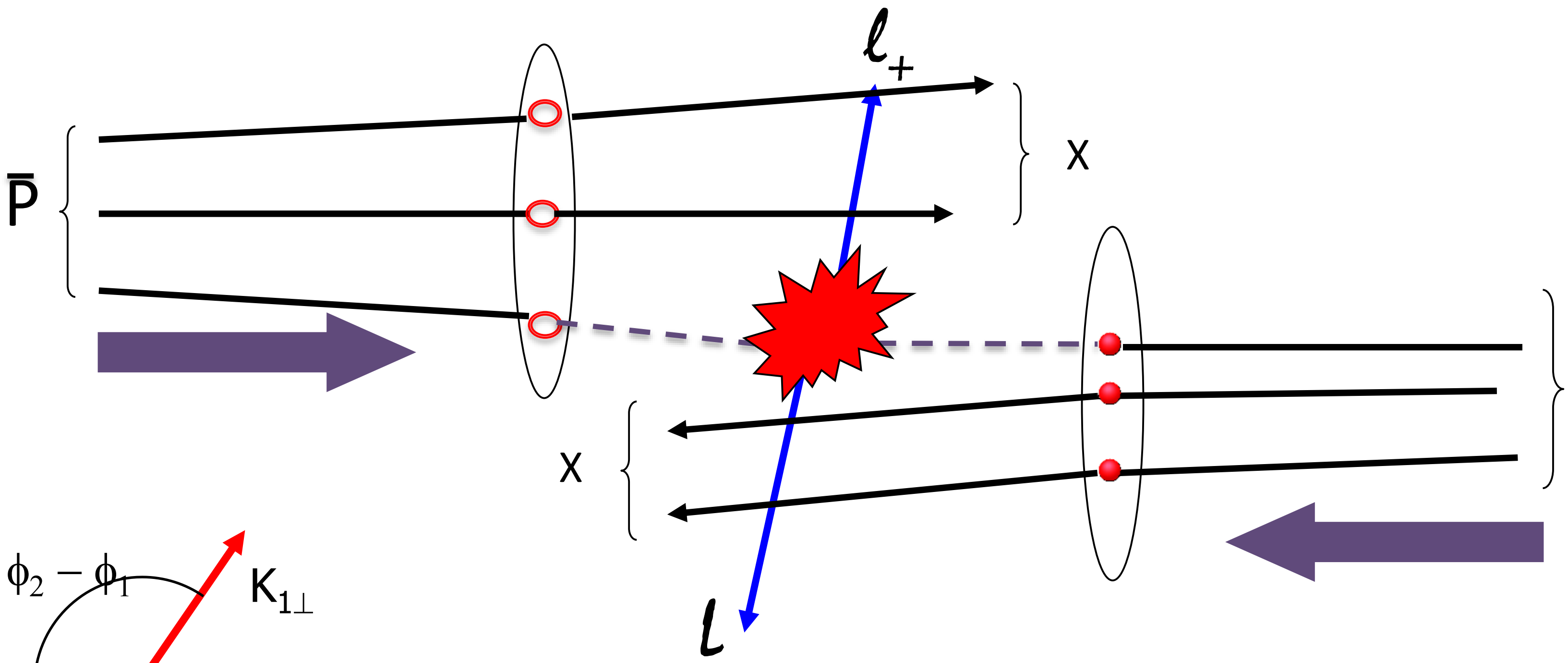
TMD Example: Drell-Yan

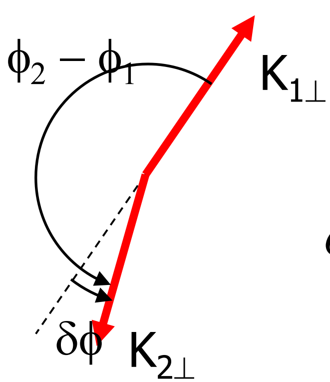


$$\sigma \sim \int \mathcal{H} \otimes f_{q/P}(x_1) \otimes f_{\bar{q}/\bar{P}}(x_2)$$

(Scale Dependence: DGLAP)

TMD Example: Drell-Yan

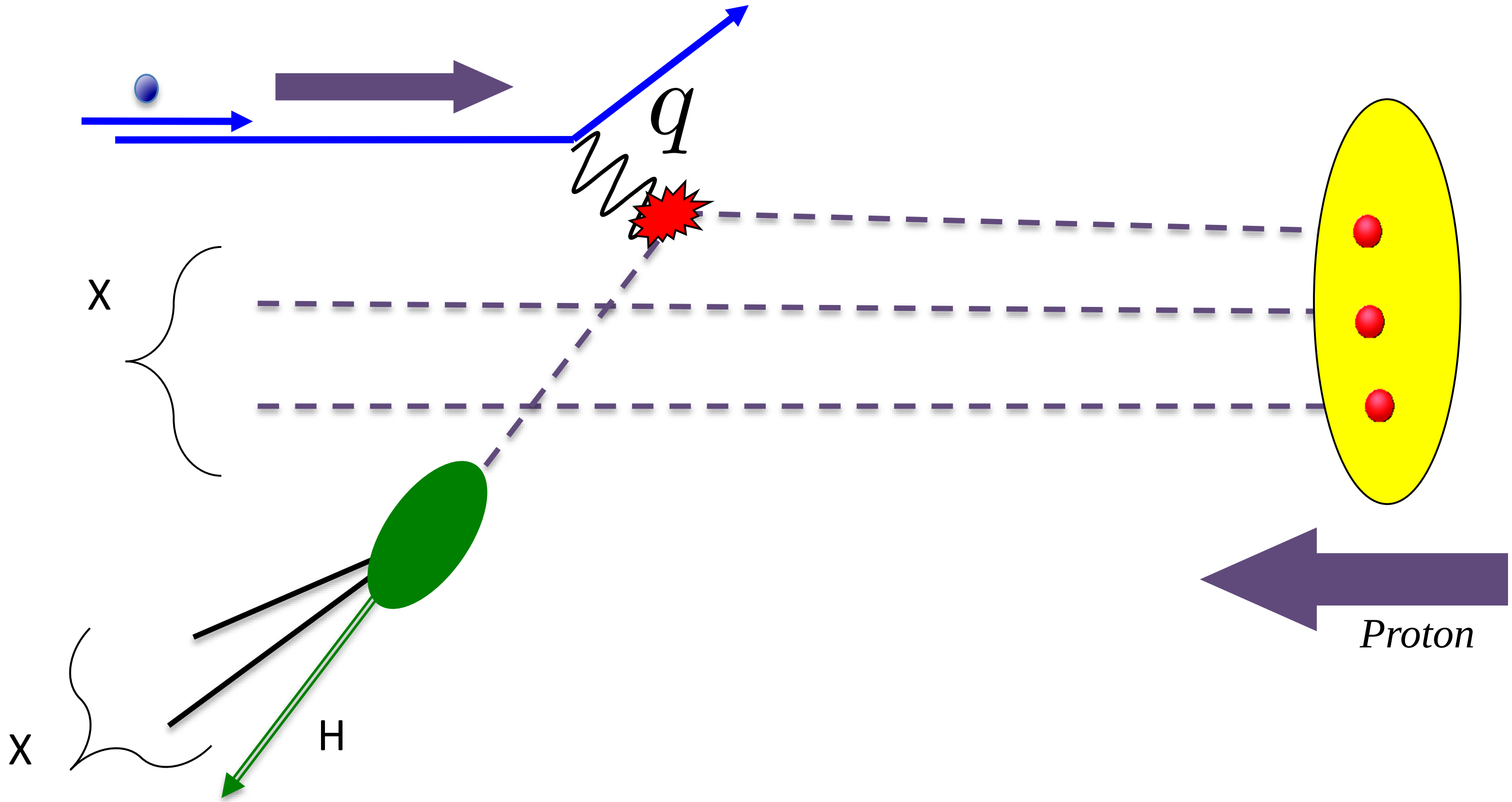




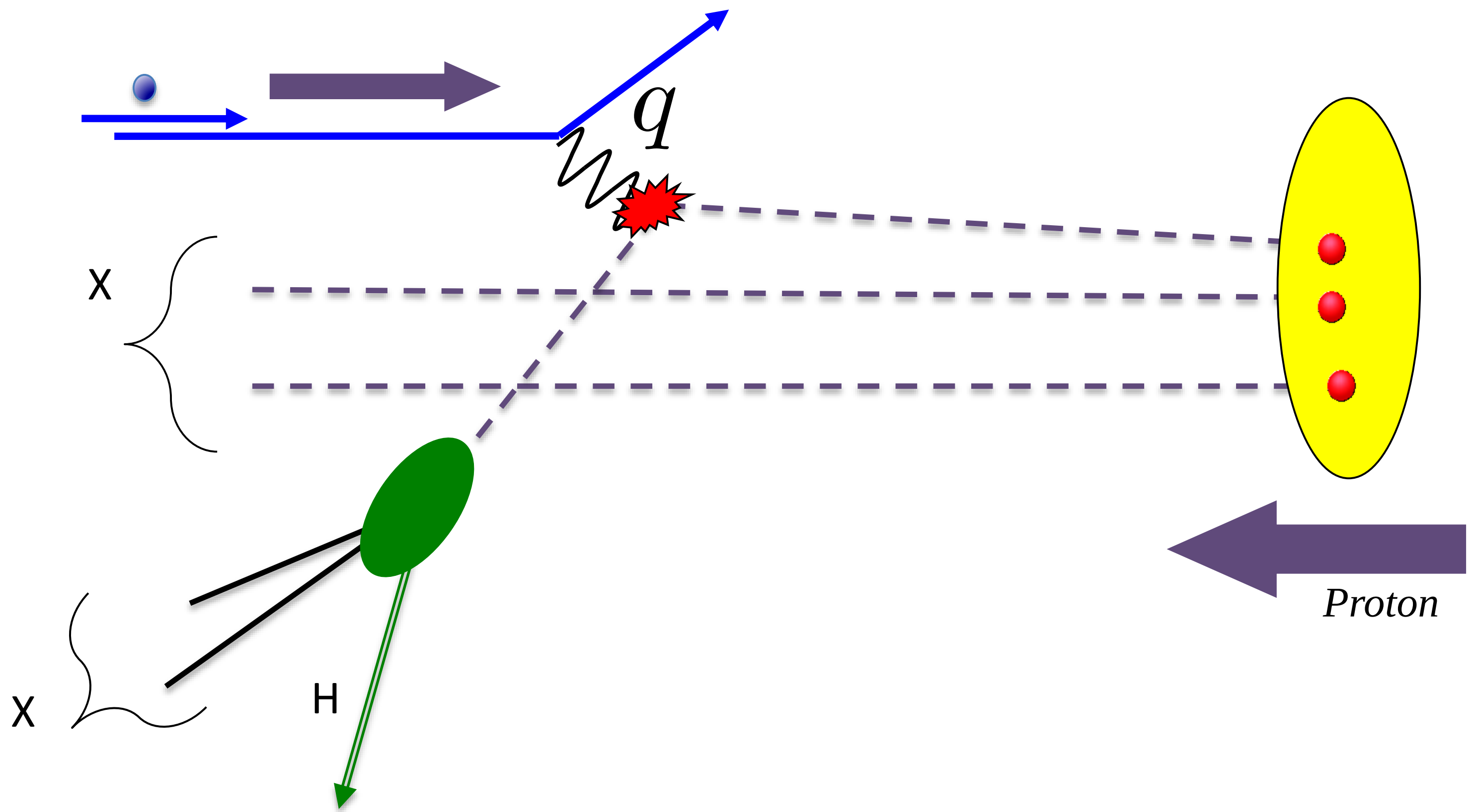
$$\sigma \sim \int \mathcal{H} \otimes F_{q/P}(x_1, \mathbf{k}_{1T}) \otimes \underbrace{F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T})}$$

(Scale Dependence: TMD Evolution)

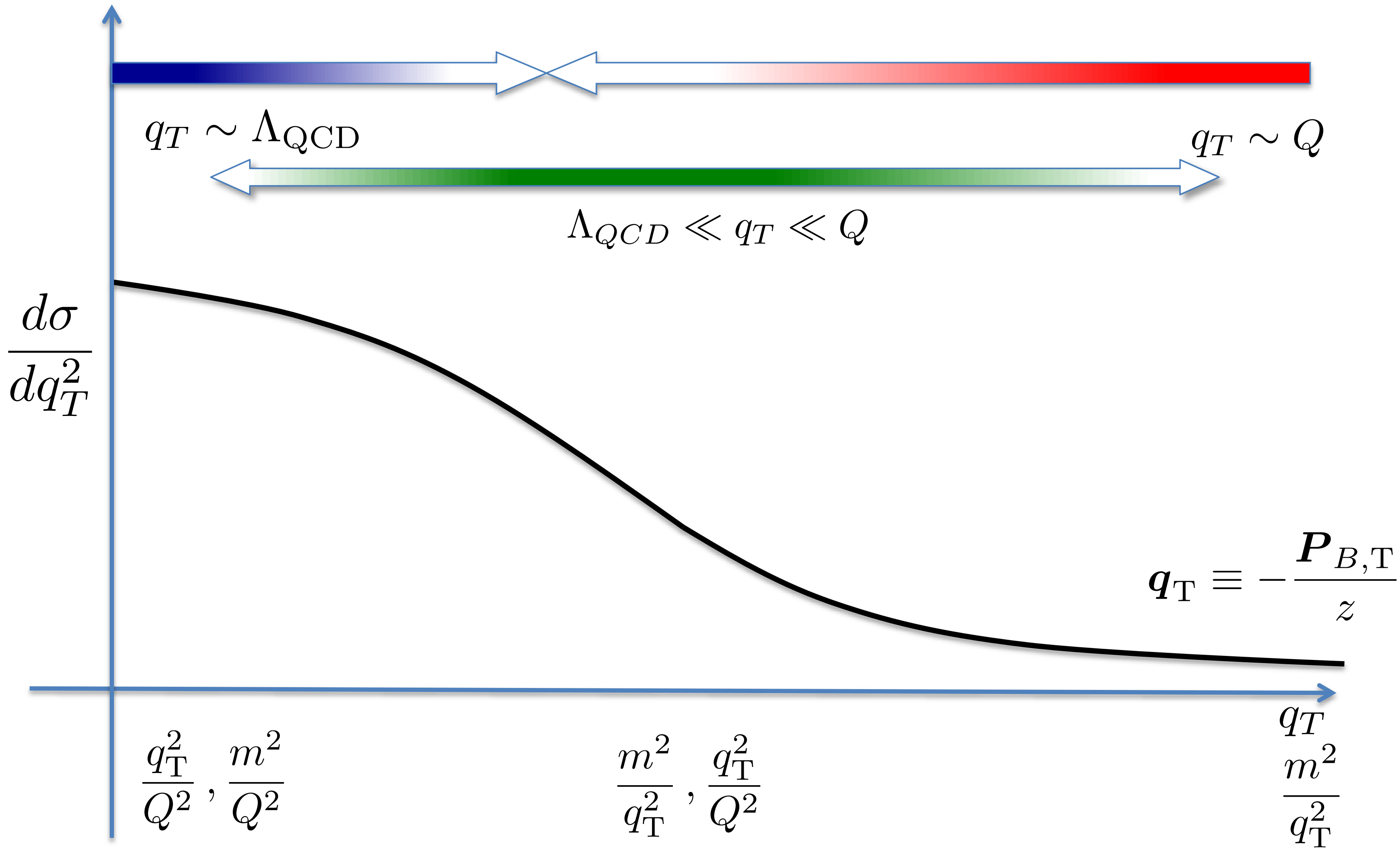
Example: SIDIS



Example: SIDIS



Large and Small Transverse Momentum



Taxonomy

Sivers

Quark \ Proton	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x, k_T)$	✖	$f_{1T}^{\perp}(x, k_T)$
Longitudinally polarized	✖	$g_{1L}(x, k_T)$	$g_{1T}(x, k_T)$
Transversely polarized	$h_1^{\perp}(x, k_T)$	$h_{1L}(x, k_T)$	$h_{1T}(x, k_T)$ $h_{1T}^{\perp}(x, k_T)$

Boer-Mulders

"Worm Gear"

"Pretzelocity"

$$\sigma \sim \int \mathcal{H}(Q) \otimes F_{q/P}(x_1, \mathbf{k}_{1T}, S_1) \otimes F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T}, S_2)$$

$$\sigma \sim \int \mathcal{H}(Q) \otimes F_{q/P}(x_1, \mathbf{k}_{1T}, S_1) \otimes D_{H/q}(z, \mathbf{q}_T + \mathbf{k}_{1T})$$

Quark \ Proton	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x, k_T)$	✖	$-f_{1T}^{\perp}(x, k_T)$ $+f_{1T}^{\perp}(x, k_T)$
Longitudinally polarized	✖	$g_{1L}(x, k_T)$	$g_{1T}(x, k_T)$
Transversely polarized	$h_1^{\perp}(x, k_T)$	$h_{1L}(x, k_T)$	$h_{1T}(x, k_T)$ $h_{1T}^{\perp}(x, k_T)$

Non-Zero!

Collins-Soper / Light-cone Renormalization

- Collinear PDFs:

Independent of hadron

$$f_{j/p}(\xi;\mu) = \sum_i \int \frac{dz}{z} Z_{ji}(z, \alpha_s(\mu)) \, f_{0,i/p}(\xi/z) \quad = \underbrace{Z_{ji}} \otimes f_{0,i/p}$$

- TMD PDFs, CS Equation:

$$\tilde{F}_{f/P}(x_1, \mathbf{b}_T; \mu, y_s) = \lim_{\text{WL Raps}\to\infty} \left(\tilde{F}^{\text{unsub.}}_{f/P}(x_1, \mathbf{b}_T; \mu) \times \underbrace{Z_{\text{CS}}(\mathbf{b}_T; y_s)} \right)$$

Independent of hadron

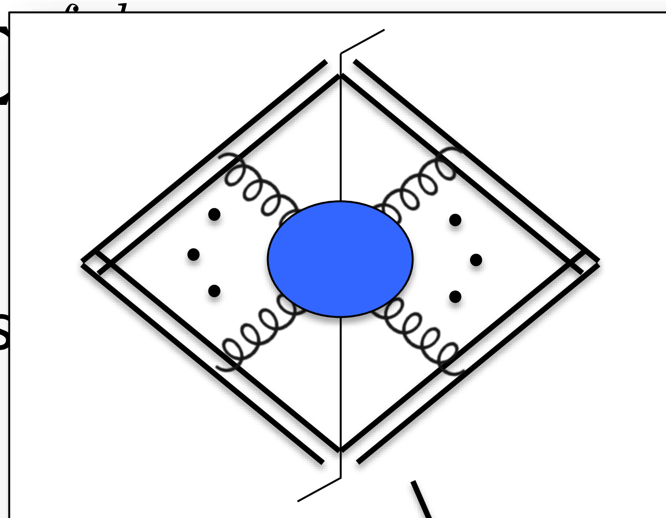
X UV renormalization

Collins-Soper / Light-cone Renormalization

- Collinear PDFs:

Independent of hadron

$$f_{j/p}(\xi;\mu) = \sum_i \quad /z) \quad = Z_{ji} \otimes f_{0,i/p}$$



- TMD PDFs

$$\tilde{F}_{f/P}(x_1, \mathbf{b}_T; \mu, y_s) = \lim \Big(F^{\text{unsub.}}_{f/P}(x_1, \mathbf{b}_T; \mu) \times Z_{\text{CS}}(\mathbf{b}_T; y_s)\Big)$$

$$\sqrt{\frac{\tilde{S}(b_{\text{T}}; +\infty, y_s)}{\tilde{S}(b_{\text{T}}; +\infty, -\infty)\, \tilde{S}(b_{\text{T}}; y_s, -\infty)}}$$

ndent of hadron

Transverse Momentum Dependent Evolution

- Collinear / DGLAP, Evolution with Scale:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} f_{j/P}(x;\mu) = 2\int P_{jj'}(x') \otimes f_{j'/P}(x/x';\mu)$$

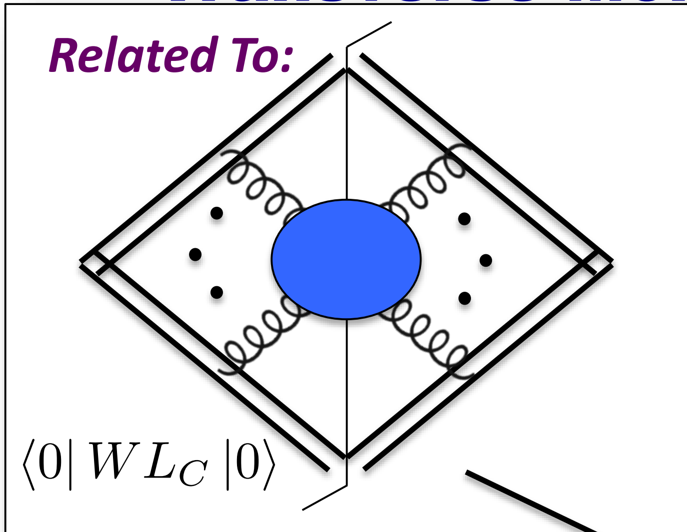
- TMD Case:

$$\frac{\partial \ln \tilde{F}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T;\mu)$$

$$\frac{\mathrm{d}\tilde{K}(b_T;\mu)}{\mathrm{d}\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{\mathrm{d}\ln \tilde{F}(x, b_T; \mu, \zeta)}{\mathrm{d}\ln\mu} = \gamma(g(\mu); \zeta/\mu^2)$$

Transverse Momentum Dependent Evolution



volution with Scale:

$$\mu) = 2 \int P_{jj'}(x') \otimes f_{j'/P}(x/x';\mu)$$

- TMD Case:

$$\frac{\partial \ln \tilde{F}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

$$\frac{\mathrm{d}\tilde{K}(b_T;\mu)}{\mathrm{d}\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{\mathrm{d}\ln \tilde{F}(x, b_T; \mu, \zeta)}{\mathrm{d}\ln\mu} = \gamma(g(\mu); \zeta/\mu^2)$$

One TMD PDF: Solution to Evolution

Ex: Cutoff Prescription:

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

$$\mu_b \equiv C_1/|\mathbf{b}_*(b_T)|$$

$$\tilde{F}_{f/P}(x, \mathbf{b}_\mathrm{T}; Q, Q^2) =$$

Collinear PDFs

$$\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \times$$

$$\times \exp\left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times$$

$$\times \exp\left\{ -g_{f/P}(x, b_T; b_{\max}) - g_K(b_T; b_{\max}) \ln \frac{Q}{Q_0} \right\}$$

Nonperturbative parts large b_T

Combining Results in TMD Factorization

Translation of results: Collins, TCR (2017)

Sudakov Form Factor: (Moch,Vermaseren (2005), Vogt, Gehrmann et al (2014))

α_s^2 Wilson Coefficients from Collinear Factorization: (Catani et al, (2012)), and SCET (Echevarria, Scimemi, Vladimirov (2016))

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_\mathrm{T}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} H^{\mathrm{DY}}_{j\bar{\jmath}}(Q,\mu_Q,a_s(\mu_Q)) \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}$$
$$\times\, e^{-g_{j/A}(x_A,b_\mathrm{T};b_\mathrm{max})} \int_{x_A}^1 \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$
$$\times\, e^{-g_{\bar{\jmath}/B}(x_B,b_\mathrm{T};b_\mathrm{max})} \int_{x_B}^1 \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{\bar{\jmath}/j_B}\left(\frac{x_B}{\xi_B}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$
$$\times\, \exp\left\{-g_K(b_\mathrm{T};b_\mathrm{max})\ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*})\ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$
$$+ \text{ suppressed corrections.}$$

Ex: Konychev, Nadolsky (2006)
ResBos extractions (and others)

Li, Zhu (2017)
Vladimirov (2017)

From
Sudakov Form Factor: (Moch,Vermaseren (2005), Vogt, Gehrmann et al (2014))

Low-to-Moderate Q SIDIS: Motivation

- Sensitivity to intrinsic non-perturbative effects.

- Many SIDIS measurements are at low/moderate Q.

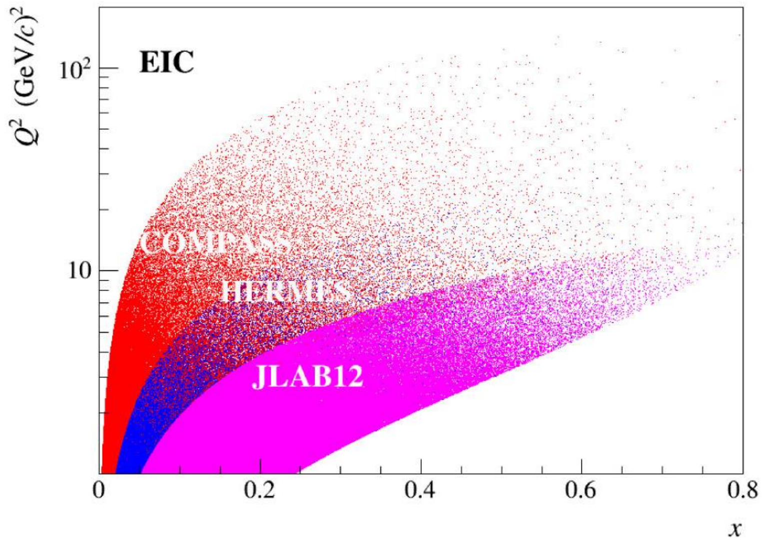
- Transition to partonic degrees of freedom.
  - E.g., quark-hadron duality

Low-to-moderate Q

$$\boldsymbol{q}_{\mathrm{T}} \equiv -\frac{\boldsymbol{P}_{B,\mathrm{T}}}{z}$$

Pion production

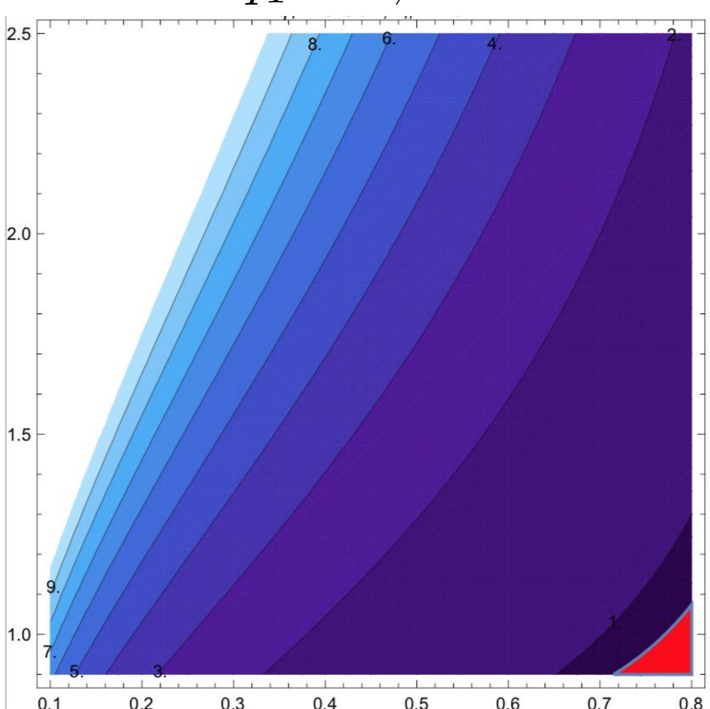
$$W^2_{\mathrm{SIDIS}} = (P + q - P_\pi)^2$$



H. Avakian, A. Bressan, and M. Contalbrigo, "Experimental results on TMDs" (2016)

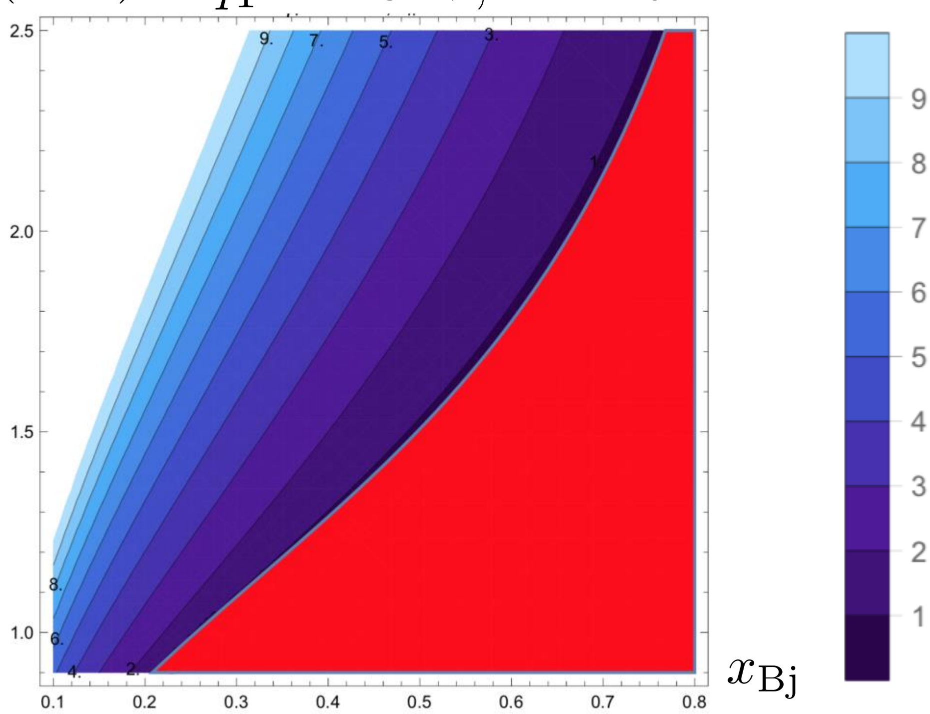
Help From :
Sterling
Gordon

Q (GeV) $\quad q_{\mathrm{T}} = 0,\ z = .25$



x_{Bj}

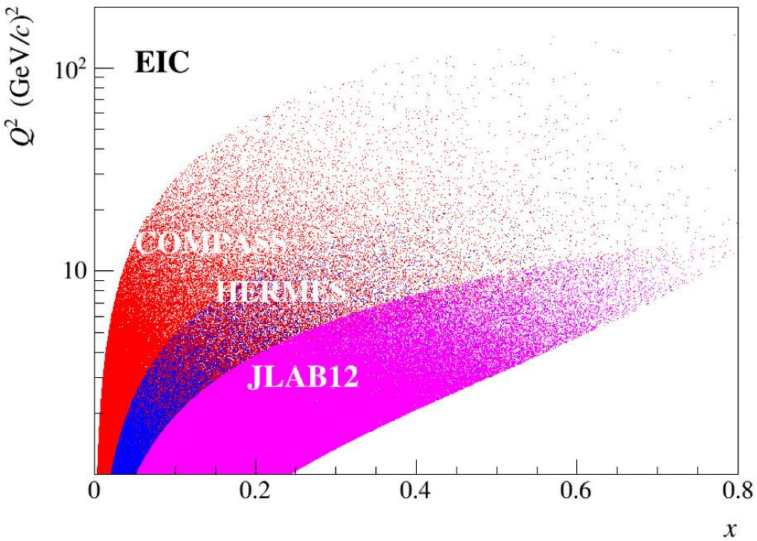
Q (GeV) $\quad q_{\mathrm{T}} = 2\,\mathrm{GeV},\ z = .25$



x_{Bj}

Low-to-moderate Q

$$\boldsymbol{q}_{\mathrm{T}} \equiv -\frac{\boldsymbol{P}_{B,\mathrm{T}}}{z}$$



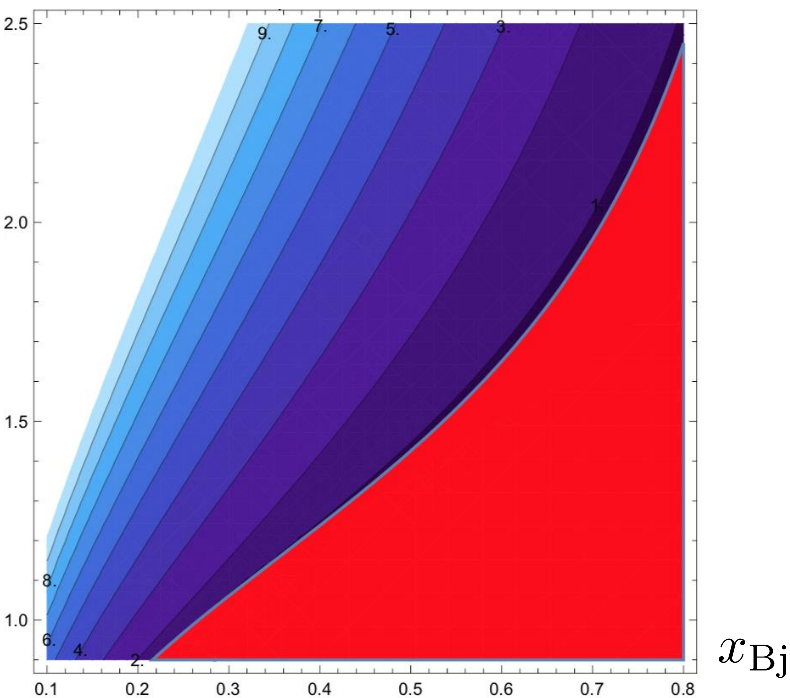
H. Avakian, A. Bressan, and M. Contalbrigo, “Experimental results on TMDs” (2016)

Kaon production

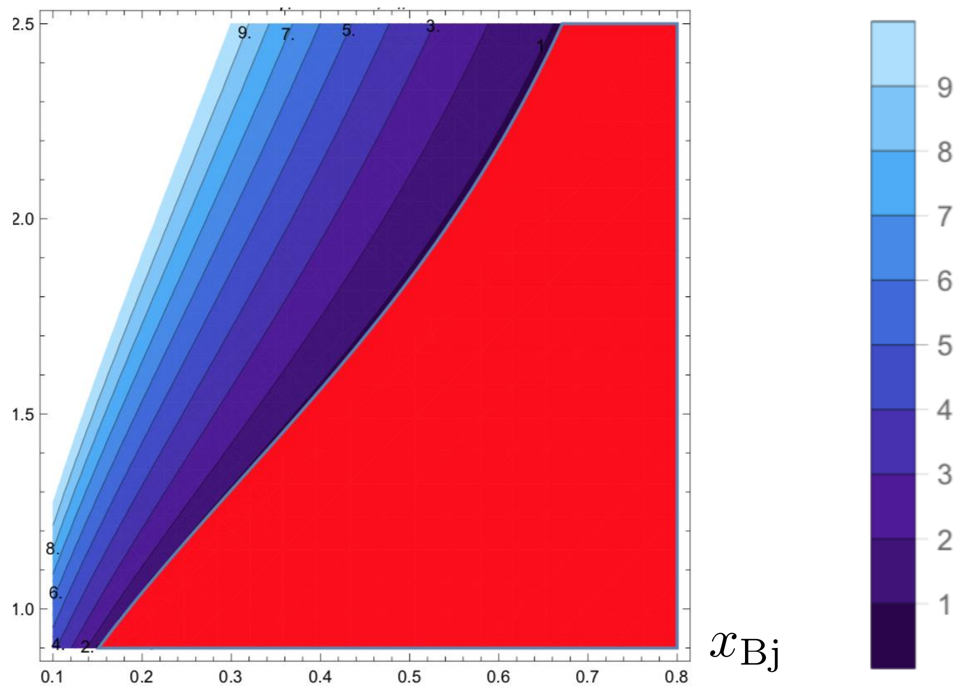
$$W^2_{\mathrm{SIDIS}} = (P + q - P_K)^2$$

Help From :
Sterling
Gordon

Q (GeV) $\quad q_{\mathrm{T}} = 0,\ z = .25$

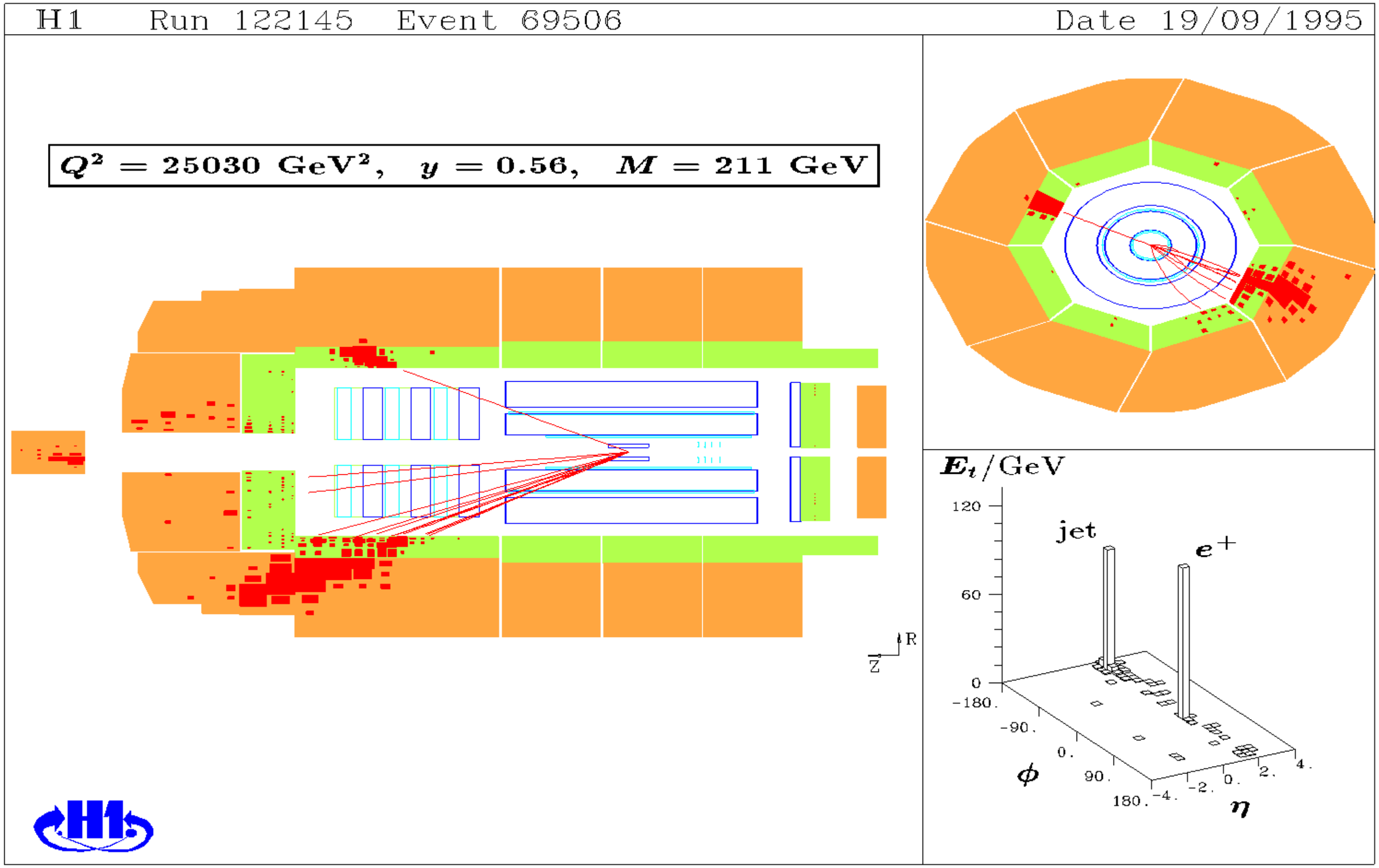


Q (GeV) $\quad q_{\mathrm{T}} = 2\,\mathrm{GeV},\ z = .25$

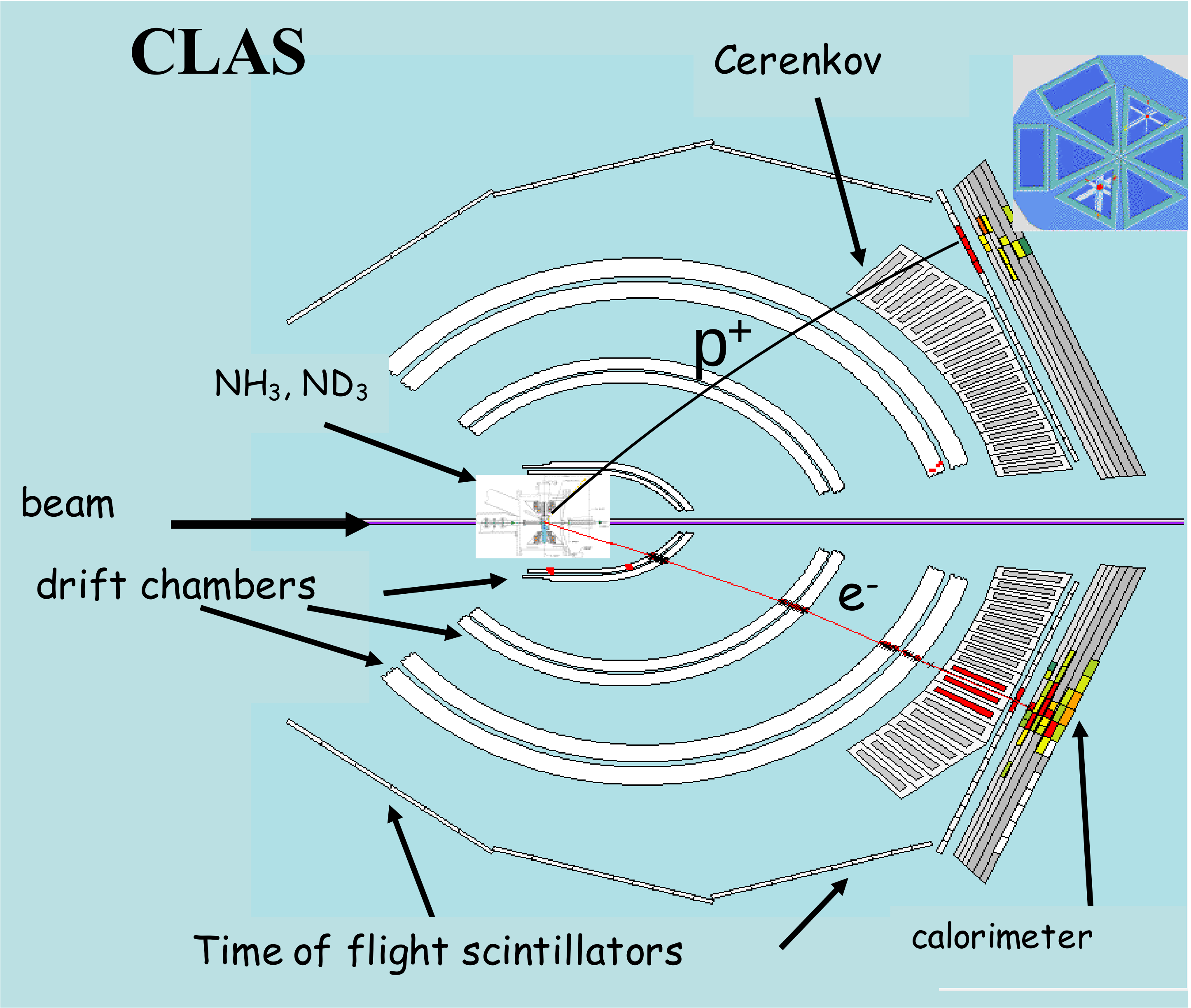


Large Q

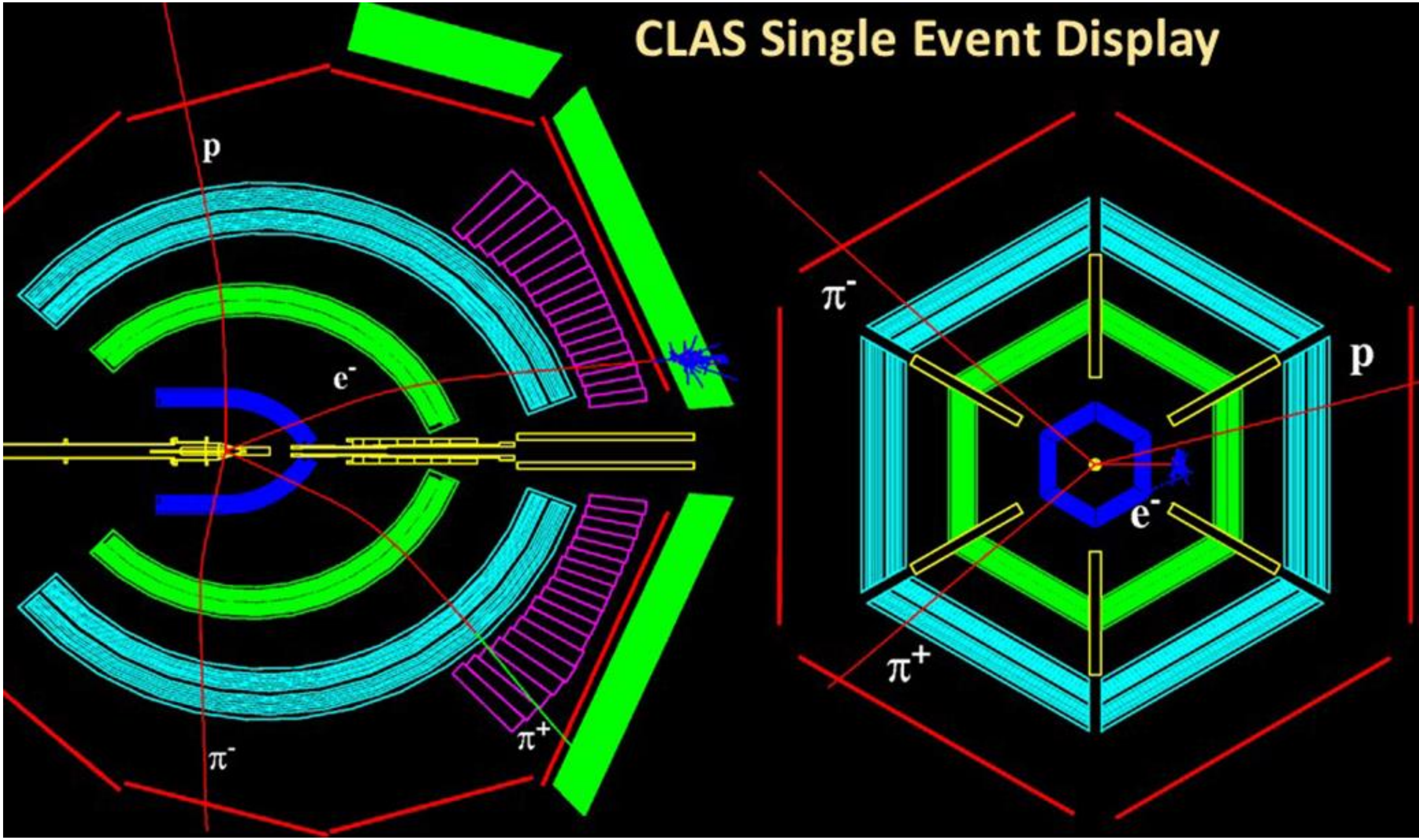
Candidate from NC sample



Low-to-moderate Q



Low-to-moderate Q



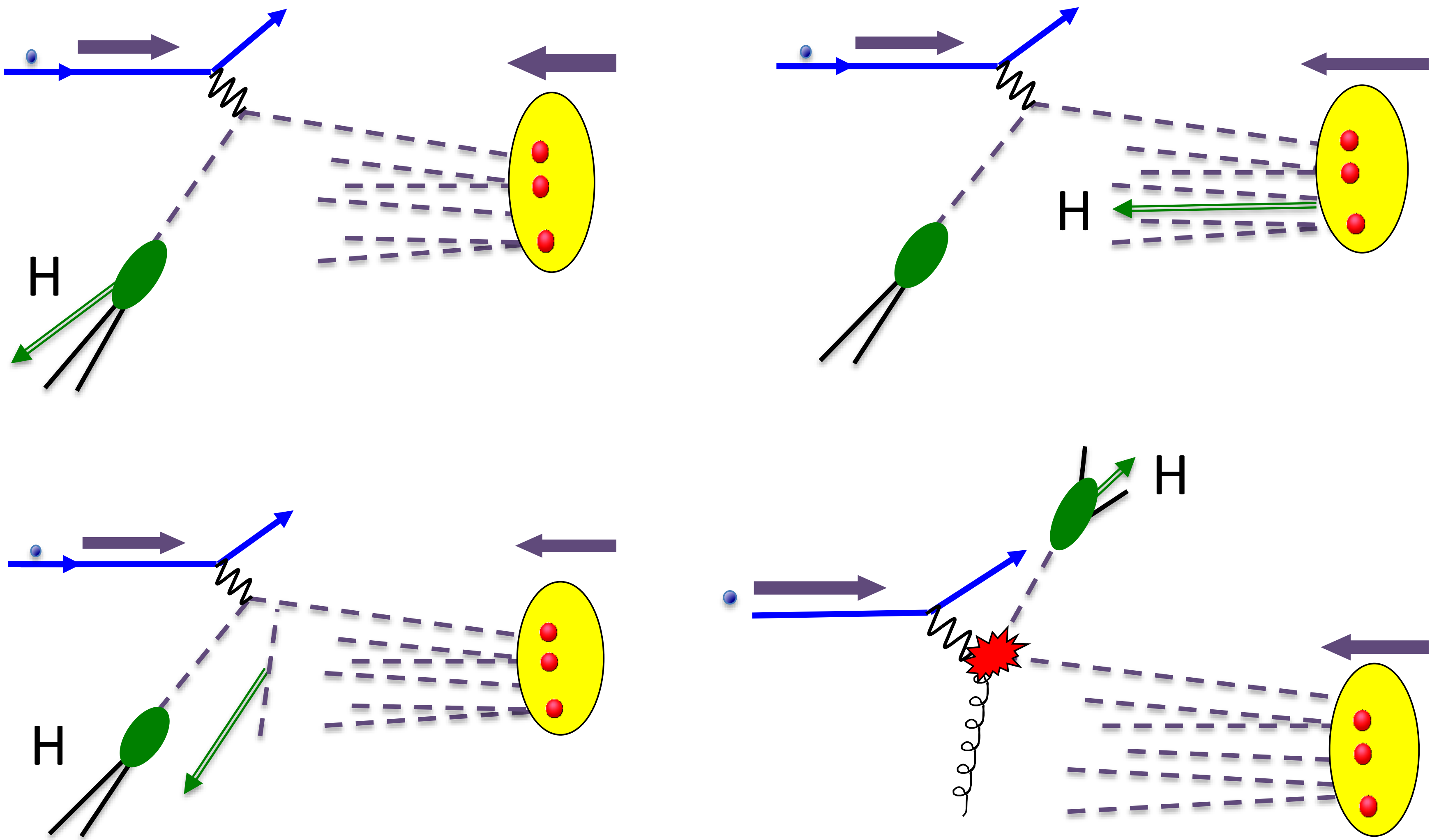
Challenges at moderate scales

- Non-zero hadron masses.

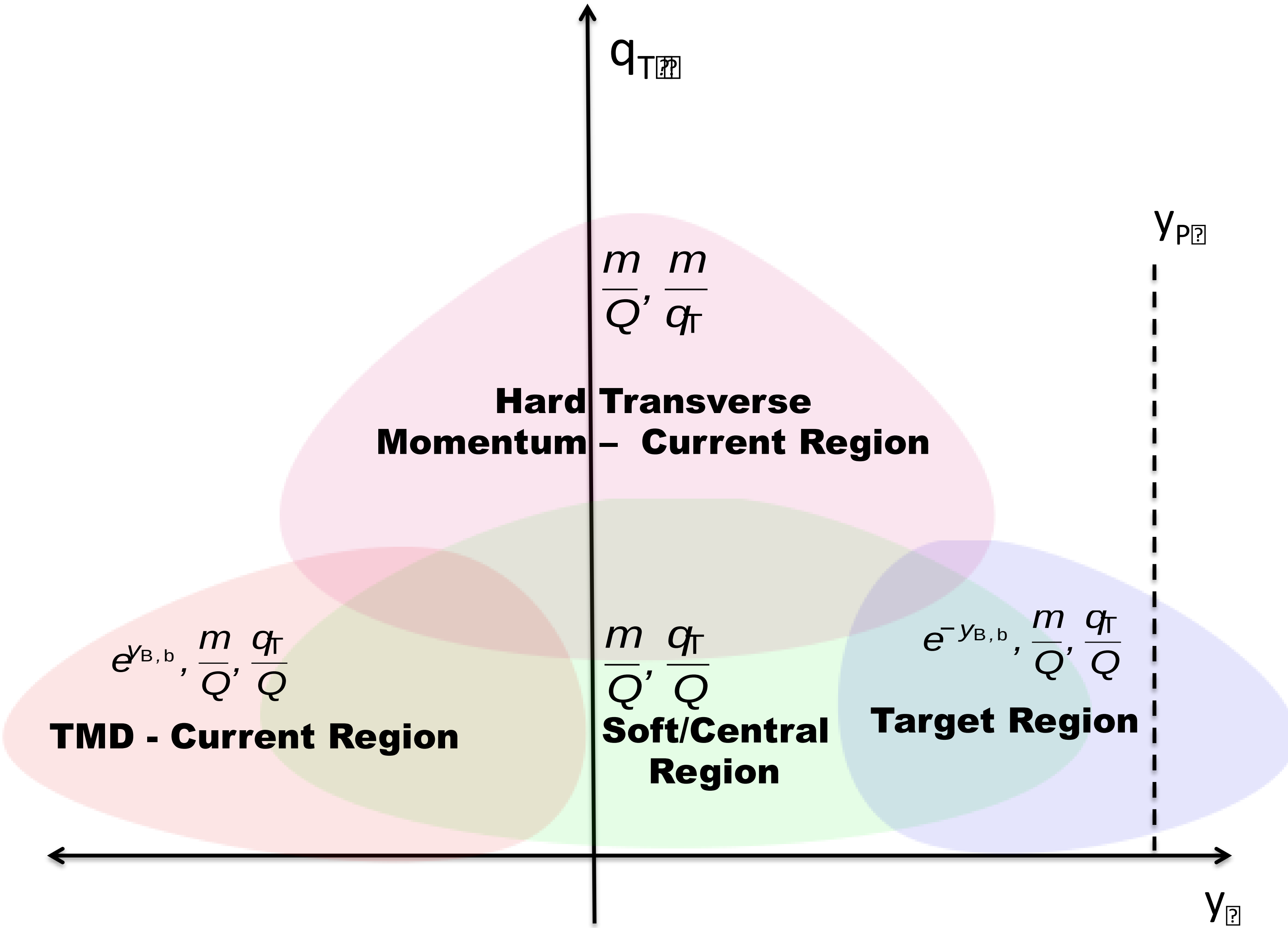
- Constituents have non-zero virtuality, mass, etc.

- The separation between regions gets squeezed.

Cartography of SIDIS



Cartography of SIDIS



Factorization: Inclusive Case

- Power expansion

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_{\mathrm{Bj}}\ \mathrm{d}Q^2} = \int \mathrm{d}\xi\, \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{x}_{\mathrm{Bj}}\ \mathrm{d}Q^2} f(\xi) + O\left(\frac{m^2}{Q^2}\right)$$

- m^2 = parton virtuality, transverse momentum, mass…

- What about hadron masses?

Massless Target Approximation (MTA)

- Exact:

$$P = \left(\sqrt{M^2 + P_z^2}, 0, 0, P_z\right) = \left(P^+, \frac{M^2}{2P^+}, \mathbf{0}_\mathrm{T}\right)$$

- The approximation:

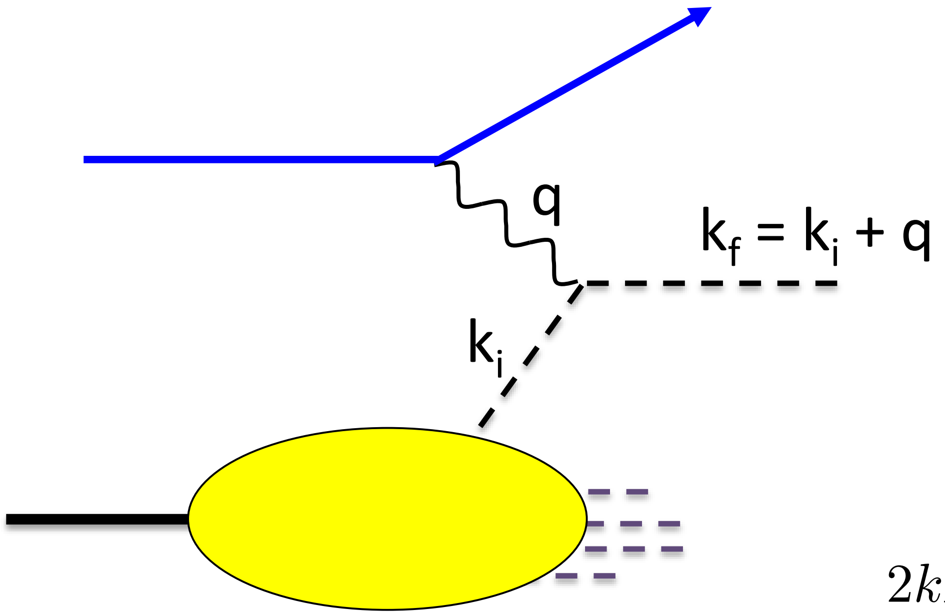
$$P \to \tilde{P} = (P_z, 0, 0, P_z) = \left(P^+, 0, \mathbf{0}_\mathrm{T}\right)$$

$$2P \cdot q \to 2\tilde{P} \cdot q \qquad M^2/Q^2 \to 0$$

MTA in Light-Cone Fractions

- Light-cone ratios:
  - No MTA: $$-\frac{q^+}{P^+} = x_{\mathrm{N}} \equiv \frac{2x_{\mathrm{Bj}}}{1+\sqrt{1+\frac{4x_{\mathrm{Bj}}^2 M^2}{Q^2}}}$$
  - MTA: $$-\frac{q^+}{P^+} = x_{\mathrm{Bj}} + O\left(\frac{x_{\mathrm{Bj}}^2 M^2}{Q^2}\right)$$

Factorization and Parton Approximations



$$q = \left(-x_{\rm N} P^+, \frac{Q^2}{2 x_{\rm N} P^+}, \mathbf{0}_{\rm T}\right)$$

$$k_{\rm i}^+ = O\left(Q\right)$$

$$k_{\rm i}^2 = O\left(m^2\right)$$

$$(k_{\rm i} + q)^2 = O\left(m^2\right)$$

$$2k_{\rm i}^+ q^- + 2k_{\rm i}^- q^+ - Q^2 + k_{\rm i}^2 = O\left(m^2\right)$$

$$2k_{\rm i}^+ q^- = Q^2 + O\left(m^2\right)$$

$$\Rightarrow \quad \xi \equiv \frac{k_{\rm i}^+}{P^+} = x_{\rm N} + O\left(\frac{m^2}{Q^2}\right)$$

$$\Rightarrow \quad = x_{\rm Bj} + O\left(\frac{x_{\rm Bj}^2 M^2}{Q^2}\right) + O\left(\frac{m^2}{Q^2}\right)$$

Aivazis, Olness, Tung (AOT)

Phys. Rev. D 50, 3085 (1994)

- Normal factorization, just keeping exact mass.

  - Target mass corrected (TMC)

$$W^{\mu\nu} = \int_{x_{\mathrm{N}}}^{1} \frac{\mathrm{d}\xi}{\xi} \hat{W}^{\mu\nu}(x_{\mathrm{N}}/\xi, q) f(\xi) + O\left(m^2/Q^2\right)$$

  - MTA

$$W^{\mu\nu} = \int_{x_{\mathrm{Bj}}}^{1} \frac{\mathrm{d}\xi}{\xi} \hat{W}^{\mu\nu}(x_{\mathrm{Bj}}/\xi, q) f(\xi) + O\left(m^2/Q^2\right) + O\left(x_{\mathrm{Bj}}^2 M^2/Q^2\right)$$

- Purely kinematical.

Extend AOT to SIDIS

- Light-cone fractions versus x and z:

$$x_{\mathrm{N}} = -\frac{q^+}{P^+} = \frac{2x_{\mathrm{Bj}}}{1+\sqrt{1+\frac{4x_{\mathrm{Bj}}^2 M^2}{Q^2}}} \qquad x_{\mathrm{Bj}} = \frac{Q^2}{2P\cdot q}$$

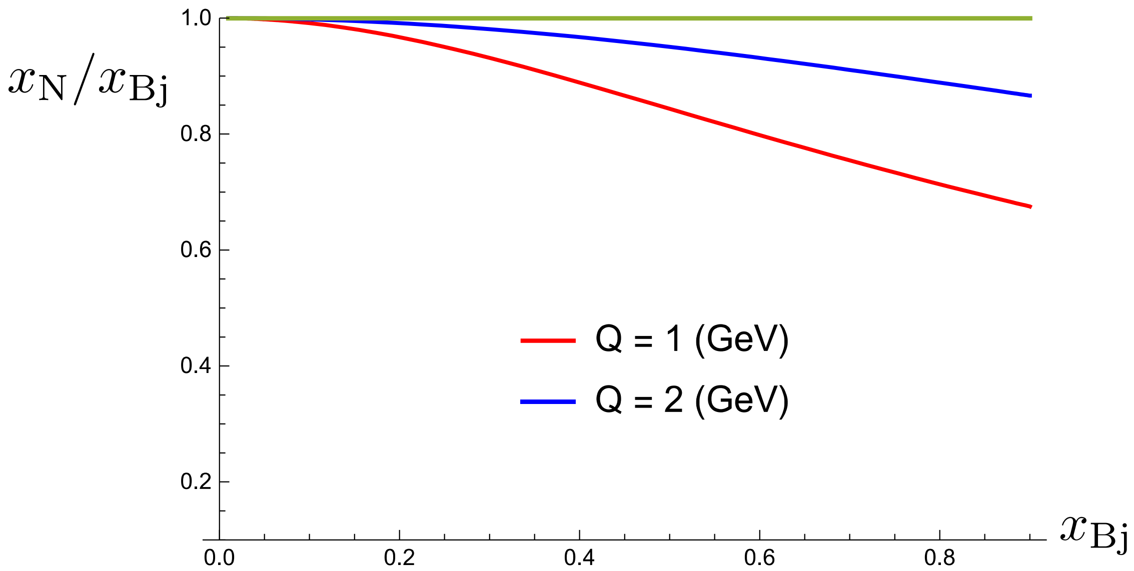
$$z_{\mathrm{N}} = \frac{P_{\mathrm{B}}^-}{q^-} \qquad z_{\mathrm{h}} = \frac{P\cdot P_{\mathrm{B}}}{P\cdot q} = 2x_{\mathrm{Bj}}\frac{P\cdot P_{\mathrm{B}}}{Q^2}$$

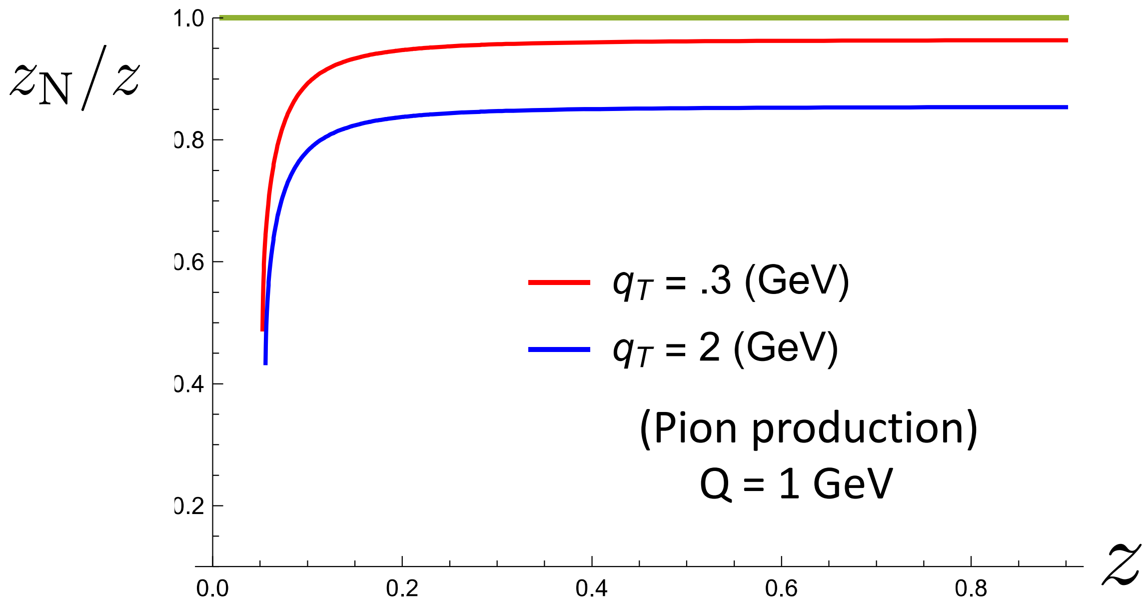
- Final state hadron mass (M_B) sensitivity:

$$z_{\mathrm{N}} = \frac{x_{\mathrm{N}} z_{\mathrm{h}}}{2x_{\mathrm{Bj}}}\left(1+\sqrt{1-\frac{4M^2 M_{\mathrm{B,T}}^2 x_{\mathrm{Bj}}^2}{Q^4 z_{\mathrm{h}}^2}}\right)$$

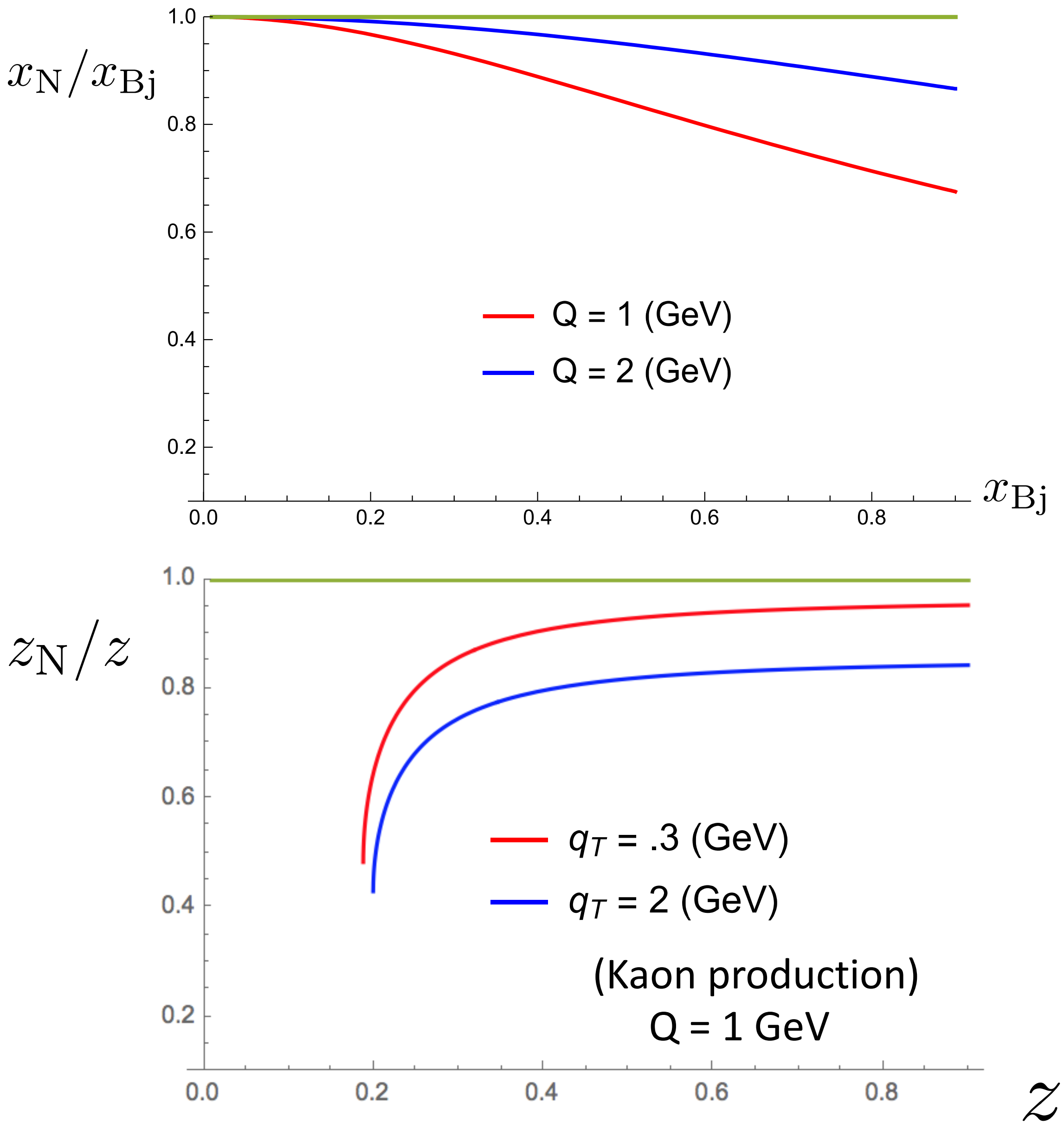
$$= z_{\mathrm{h}}\left(1-\frac{x_{\mathrm{Bj}}^2 M^2}{Q^2}\left(1+\frac{P_{\mathrm{B,T}}^2}{z_{\mathrm{h}}^2 Q^2}\right)+\left(\frac{x_{\mathrm{Bj}}^2 M^2}{Q^2}\right)^2\left(\frac{P_{\mathrm{B,T}}^2}{z_{\mathrm{h}}^2 Q^2}-\frac{P_{\mathrm{B,T}}^4}{z_{\mathrm{h}}^4 Q^4}+2-\frac{M_{\mathrm{B}}^2}{z_{\mathrm{h}}^2 M^2 x_{\mathrm{Bj}}^2}\right)+O\left(\left(\frac{x_{\mathrm{Bj}}^2 M^2}{Q^2}\right)^3\right)\right)$$

Light-cone fractions

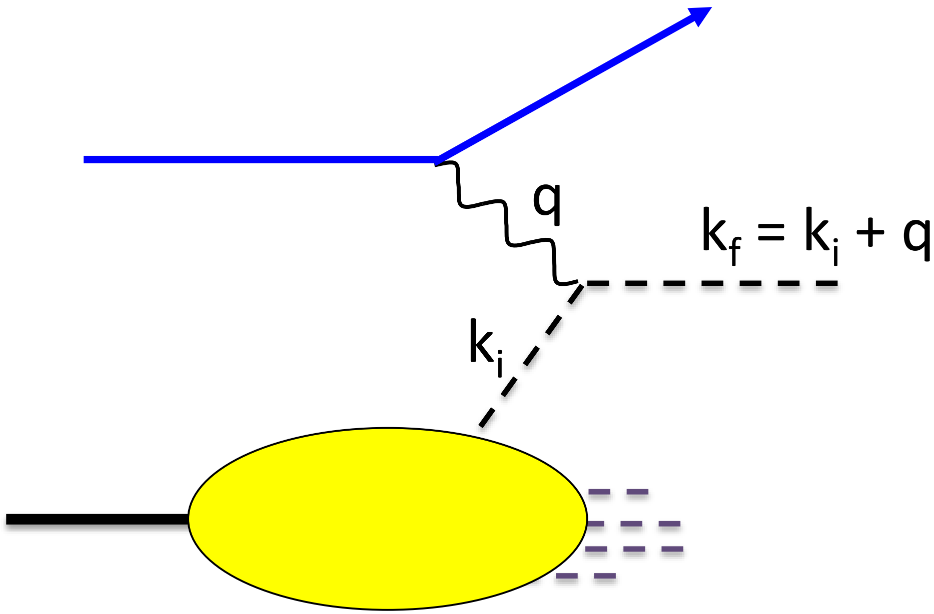




Light-cone fractions



Factorization and Parton Approximations



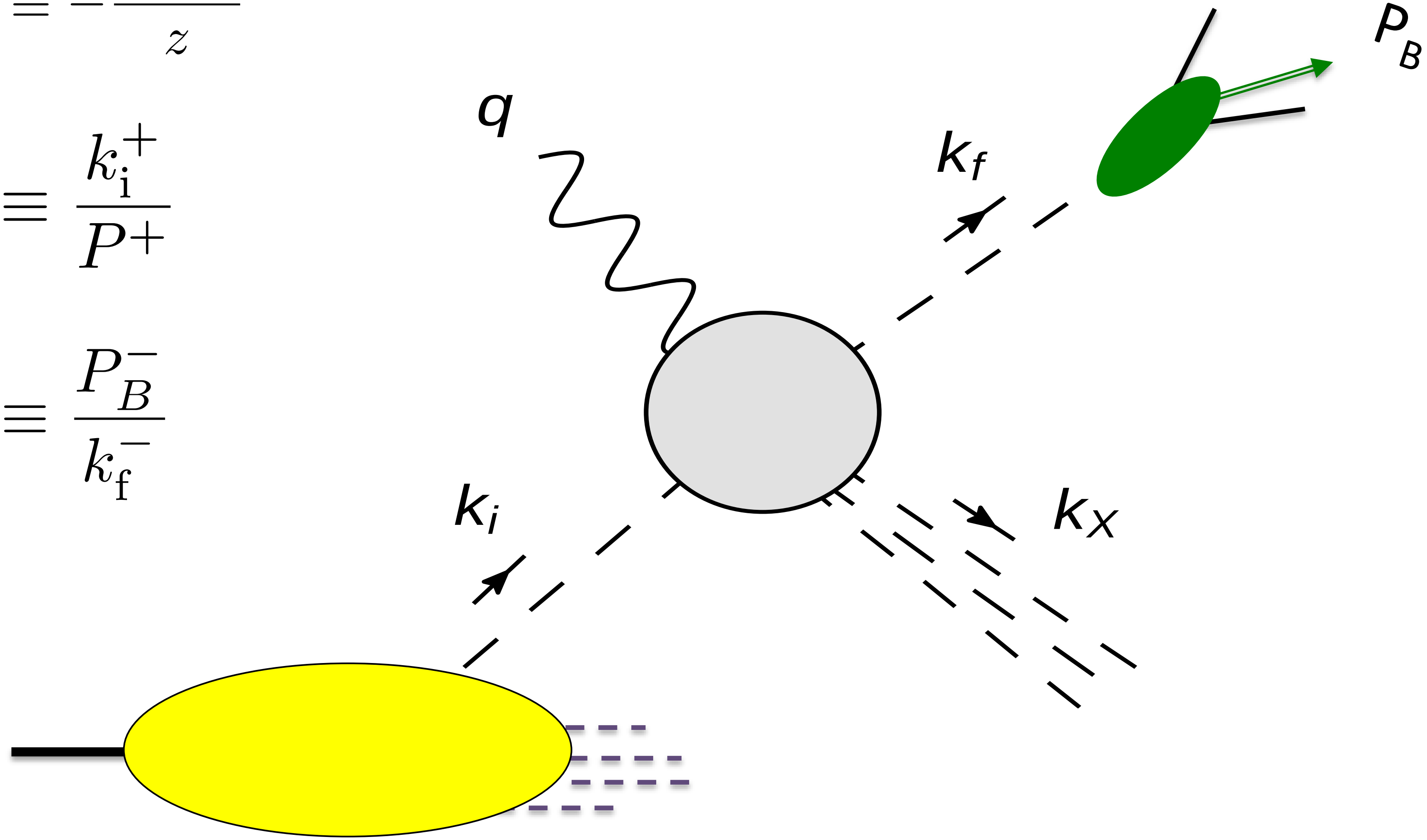
$$\xi = x_{\mathrm{N}} \left(1 + \frac{k_{\mathrm{f}}^2 + k_{\mathrm{T}}^2}{Q^2} + \cdots \right)$$

Current Fragmentation

$$q_T \equiv -\frac{\boldsymbol{P}_{B,T}}{z}$$

$$\xi \equiv \frac{k_i^+}{P^+}$$

$$\zeta \equiv \frac{P_B^-}{k_f^-}$$



Current fragmentation

$$R_1 \equiv \frac{P_{\rm B} \cdot k_{\rm f}}{P_{\rm B} \cdot k_{\rm i}} \overset{m^2/Q^2 \to 0}{=} e^{-\Delta y}$$

M. Boglione , J. Collins, L. Gamberg , J. O. Gonzalez-Hernandez , TCR , N. Sato (2017)

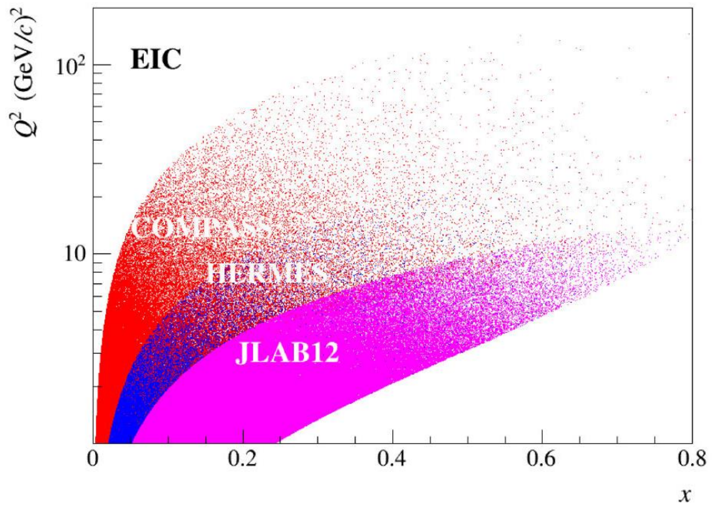
- Estimate of non-perturbative scales needed.

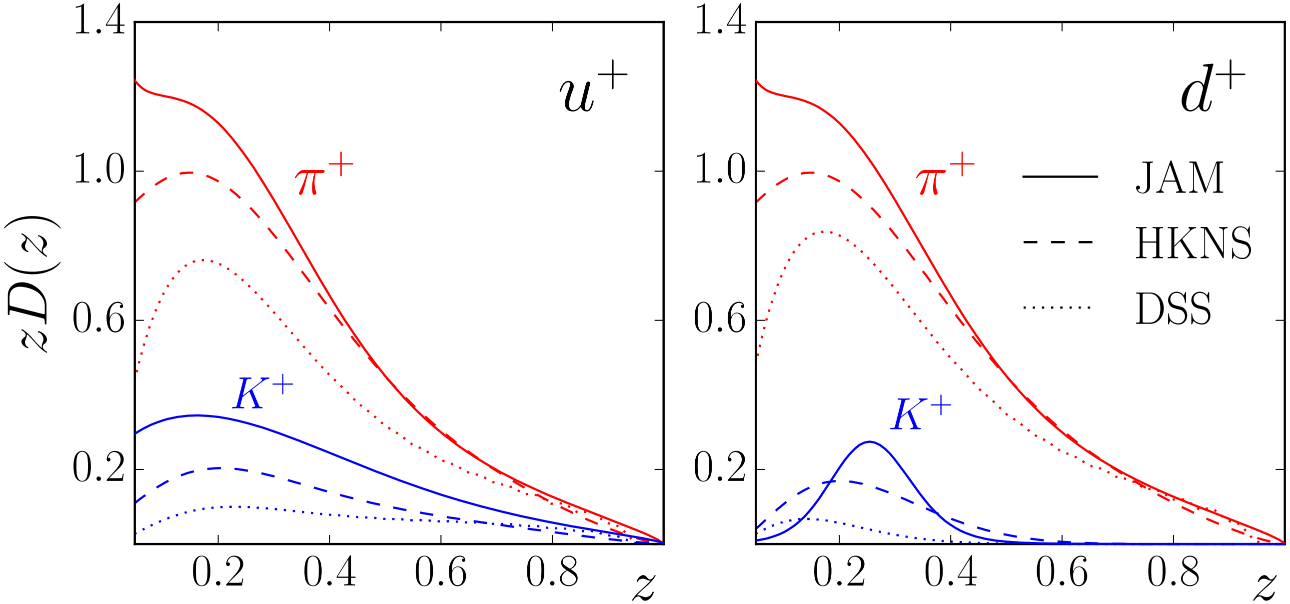
$$y_i = \ln \frac{Q}{M_{i,\rm T}} \,; \qquad y_f = -\ln \frac{Q}{M_{f,\rm T}}$$

Current fragmentation

“The overlap of kinematic coverage of COMPASS, HERMES and JLab (see fig. 1) would allow studies of Q^2-dependence in the range of Bjorken x ∼ 0.1–0.2, where the effects related to orbital motion of quarks are expected to be significant.”

-H. Avakian, A. Bressan, and M. Contalbrigo, “Experimental results on TMDs” (2016)





N. Sato et al, (20160

M. Hirai et al, (2007)

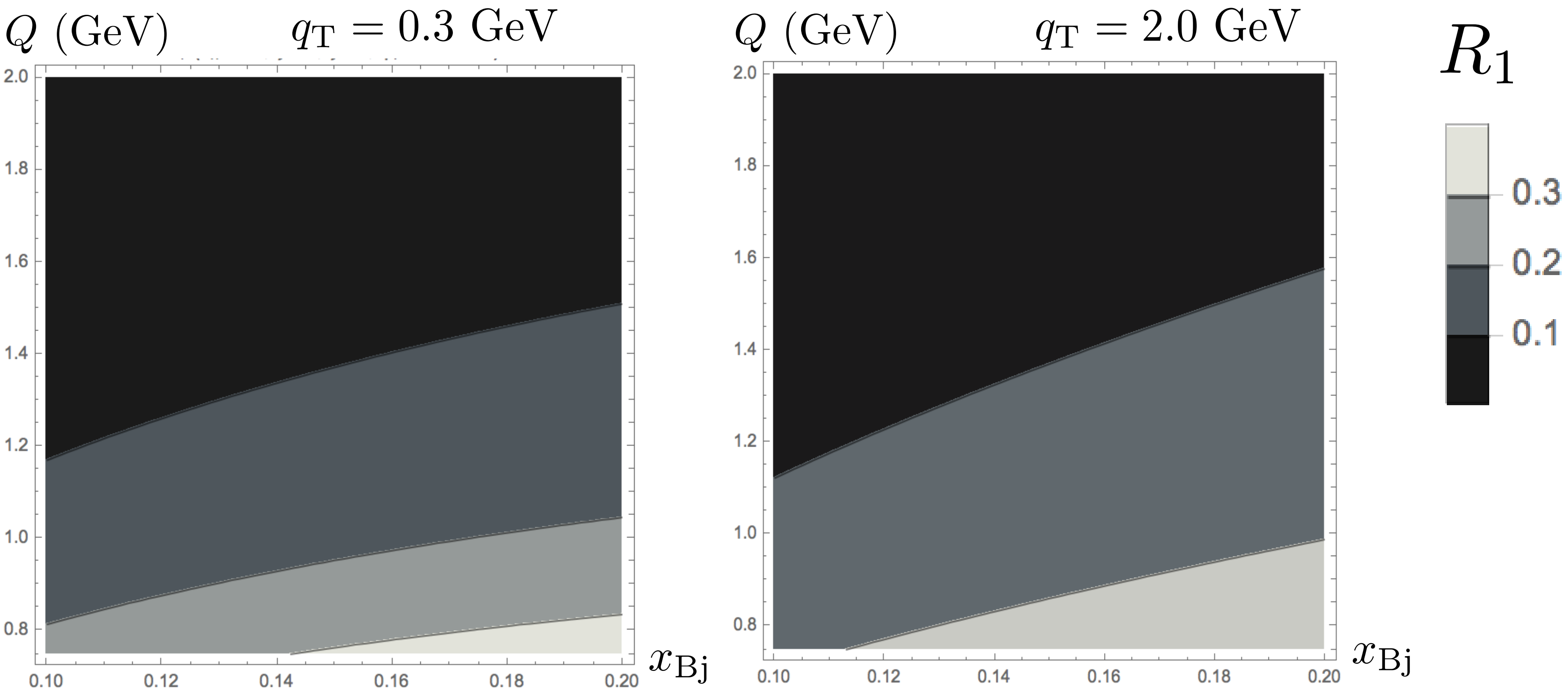
D. de Florian et al, (2007)

Current fragmentation

$$R_1 \equiv \frac{P_B \cdot k_f}{P_B \cdot k_i}$$

<u>Help From :</u>
Andrew Dotson
&
Sterling Gordon

Pion production



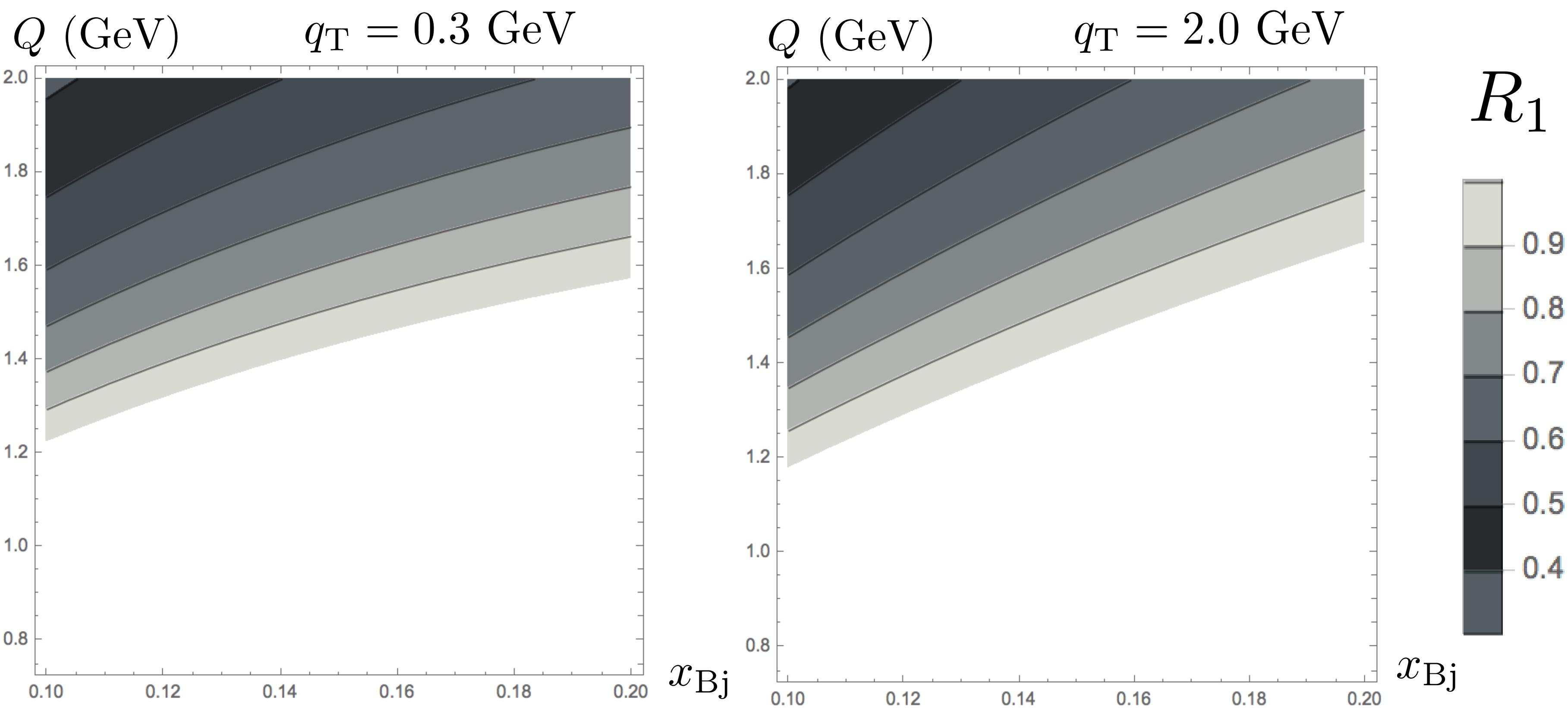
$z = .25,\ \zeta, = .3,\ \xi = .2,\ m = m_\pi$

Current fragmentation

$$R_1 \equiv \frac{P_B \cdot k_f}{P_B \cdot k_i}$$

Help From :
Andrew Dotson
&
Sterling Gordon

Kaon production

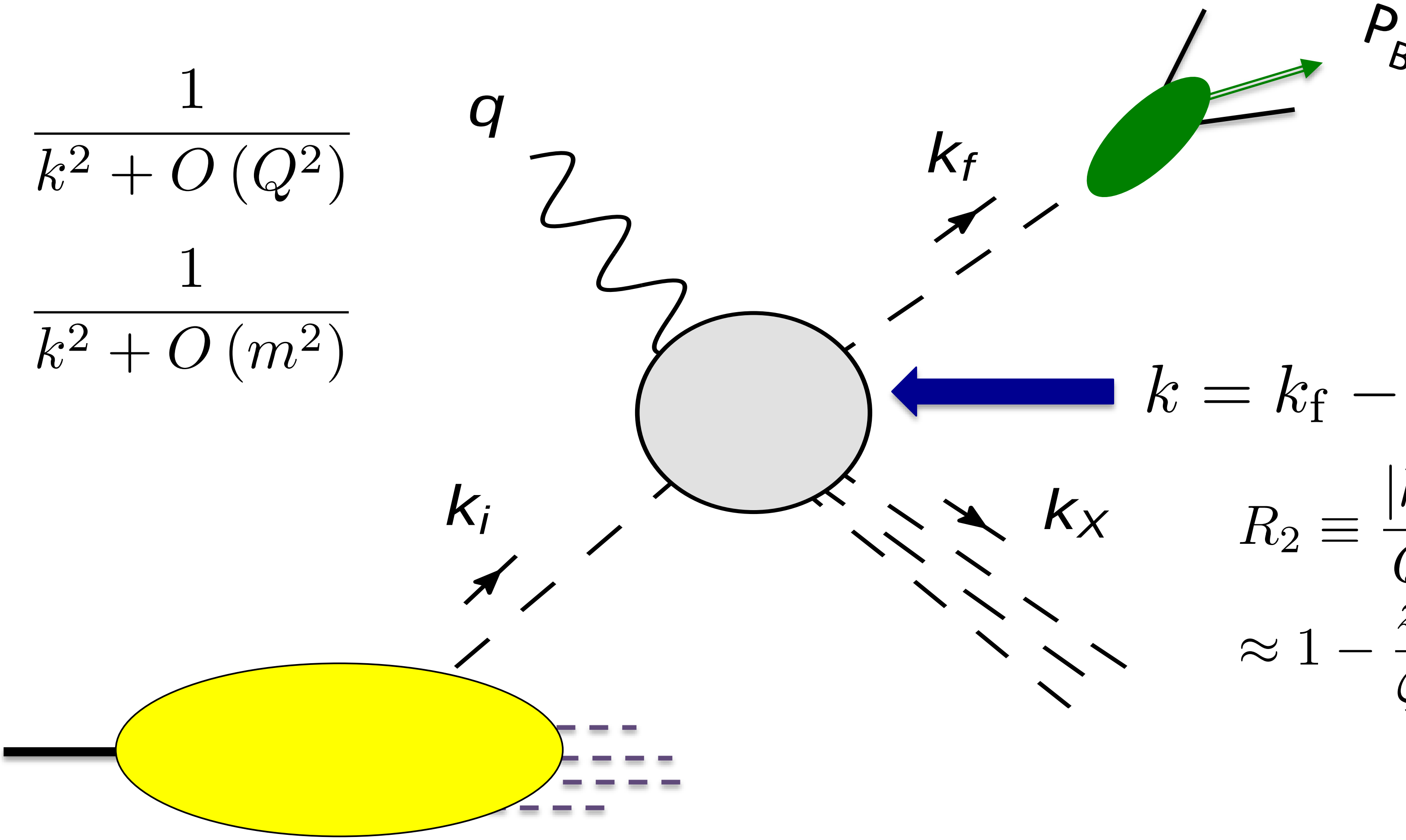


$z = .25,\ \zeta, = .3,\ x_{Bj} = .2,\ m = m_K$

Large and Small Transverse Momentum

$$\frac{1}{k^2 + O\left(Q^2\right)}$$

$$\frac{1}{k^2 + O\left(m^2\right)}$$



$$k = k_{\rm f} - q$$

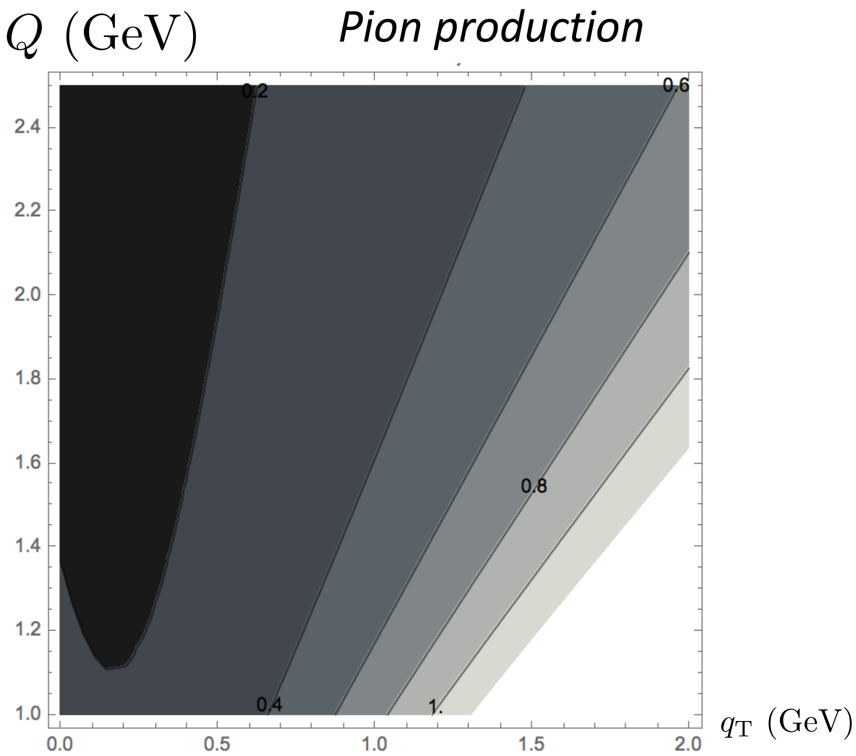
$$R_2 \equiv \frac{|k^2|}{Q^2}$$

$$\approx 1 - \frac{z}{\zeta} + \frac{z}{\zeta}\frac{q_{\rm T}^2}{Q^2}$$

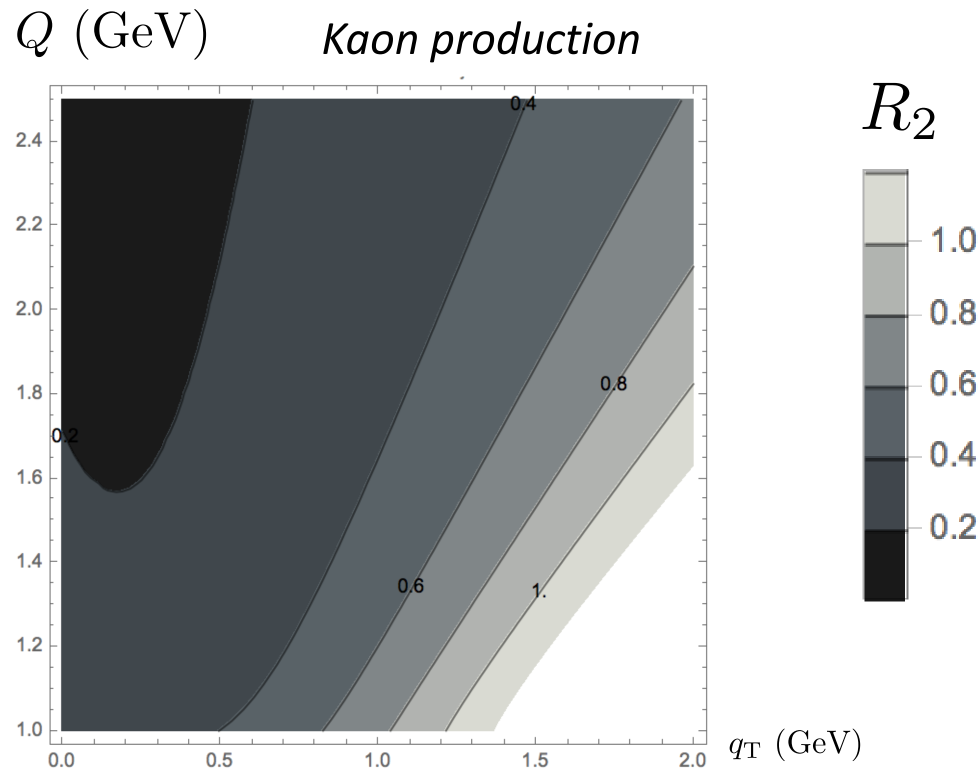
Large and Small Transverse Momentum

$$R_2 \equiv \frac{|k^2|}{Q^2}$$

Help From :
Andrew Dotson
&
Sterling Gordon

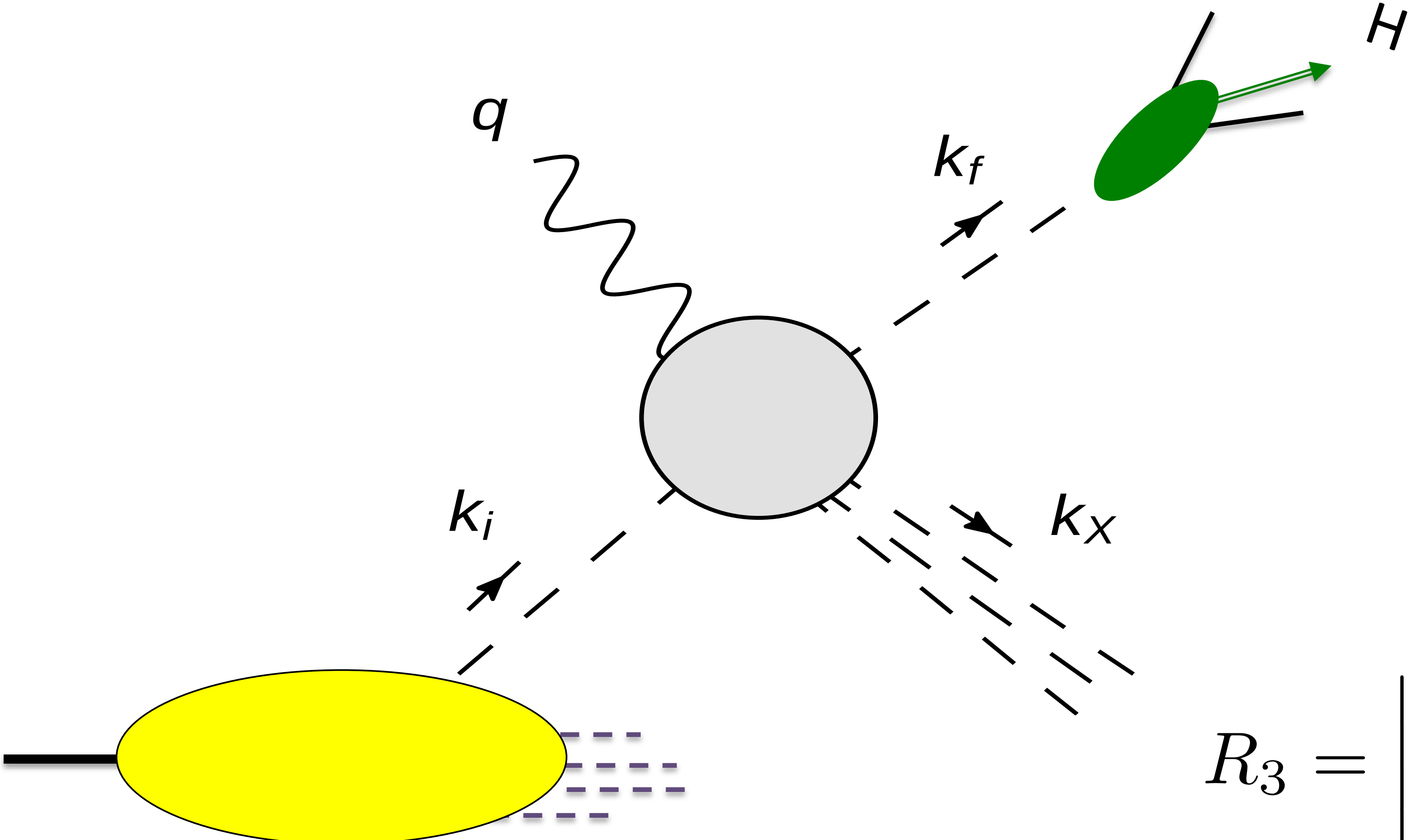


$z = .25,\ \zeta, = .3,\ x_{\rm Bj} = .2,\ m = m_\pi$



$z = .25,\ \zeta, = .3,\ x_{\rm Bj} = .2,\ m = m_K$

Extra Emissions



$$R_3 = \left| \frac{k_X^2}{Q^2} \right|$$

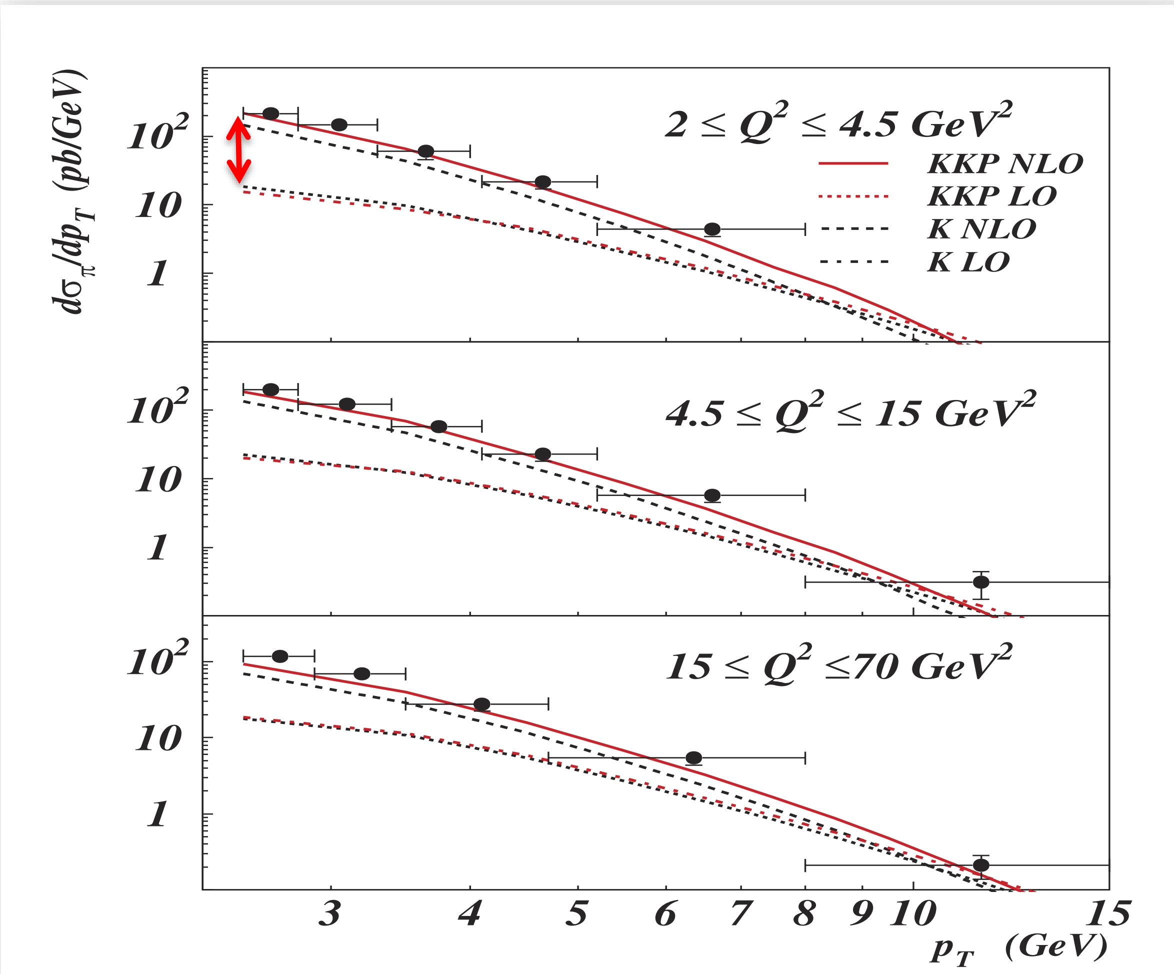
Region Diagnostics

- From model assumptions of underlying partonic picture, generate:
  - W^2_{SIDIS}
  - $x_{\mathrm{N}}/x_{\mathrm{Bj}}$, z_{N}/z
  - R_1
  - R_2
  - R_3
- Make a region map.
- Compare with measurements to constrain underlying picture.

Summary

- TMD factorization: Basics are well-established.
- SIDIS is important for TMD and related studies.
- Low-to-moderate Q opportunities: Access to interesting non-perturbative phenomena.
- Standard physical picture cannot be taken for granted.
  - Mass effects need to be accounted for.
  - Systematic diagnostic tools needed.

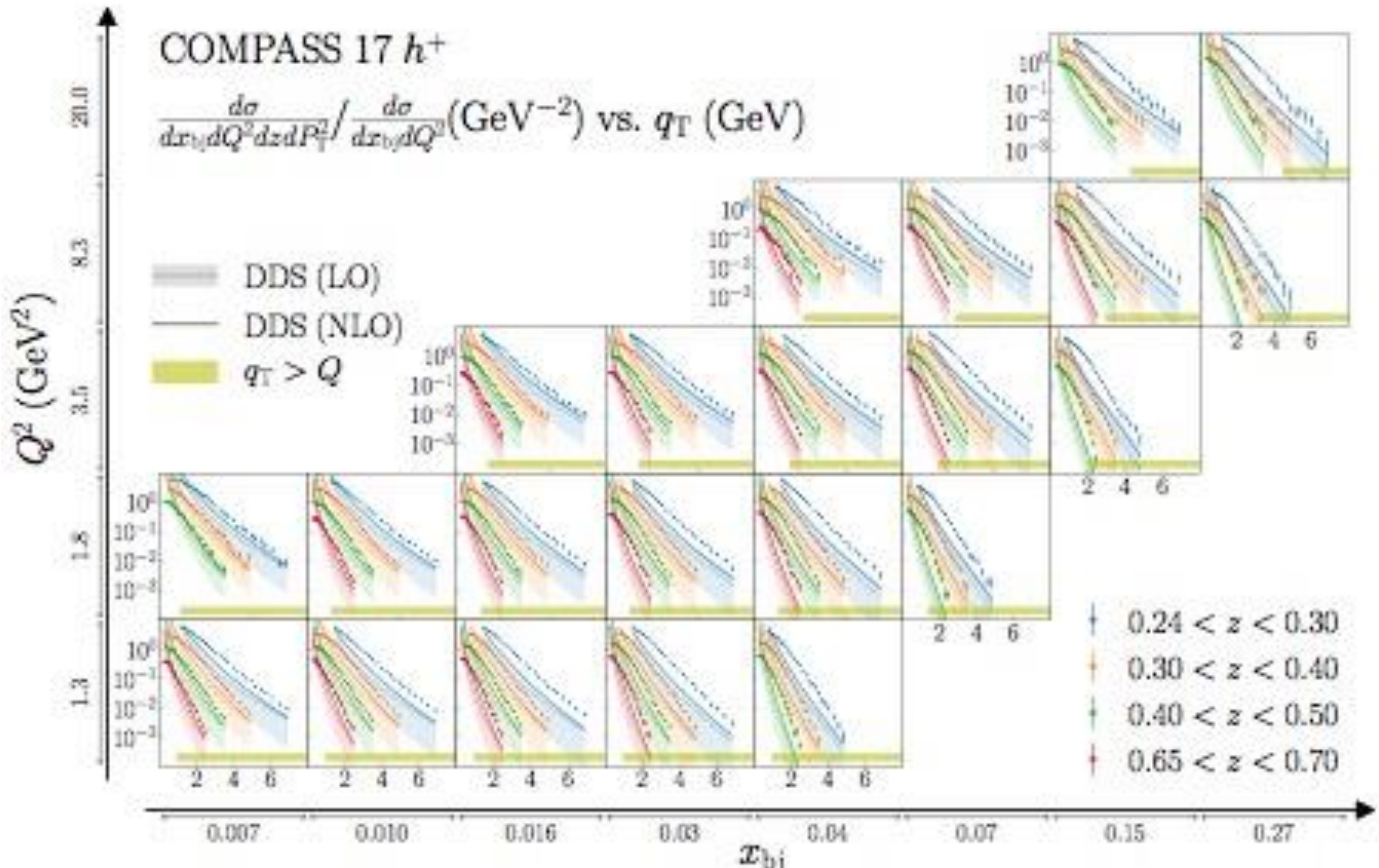
Small to large transverse momentum



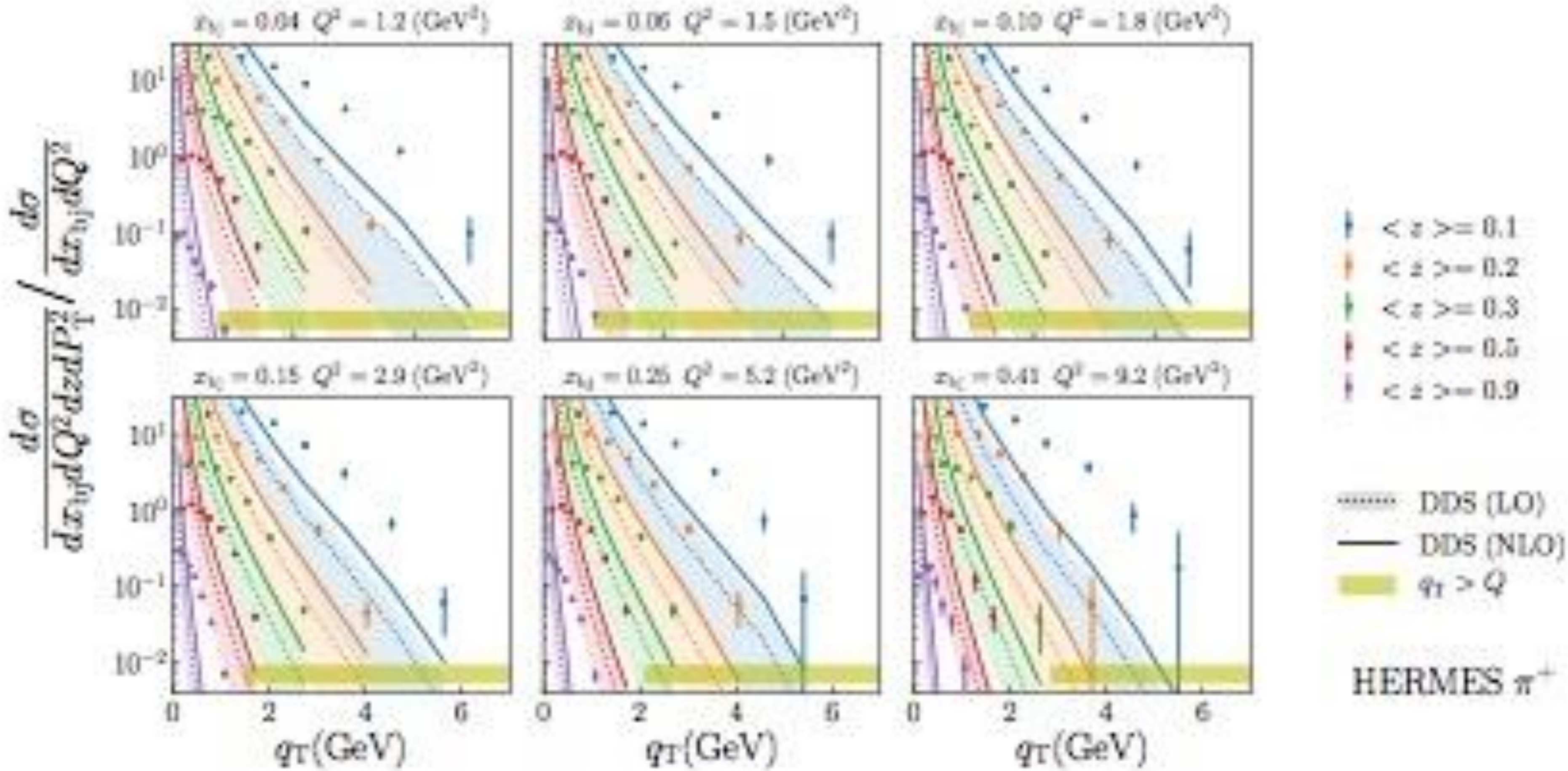
Daleo, de Florian, Sassot (2005)
Phys.Rev. D71 (2005) 034013

Data: H1 (2004)
Eur.Phys.J.C36:441-452,2004

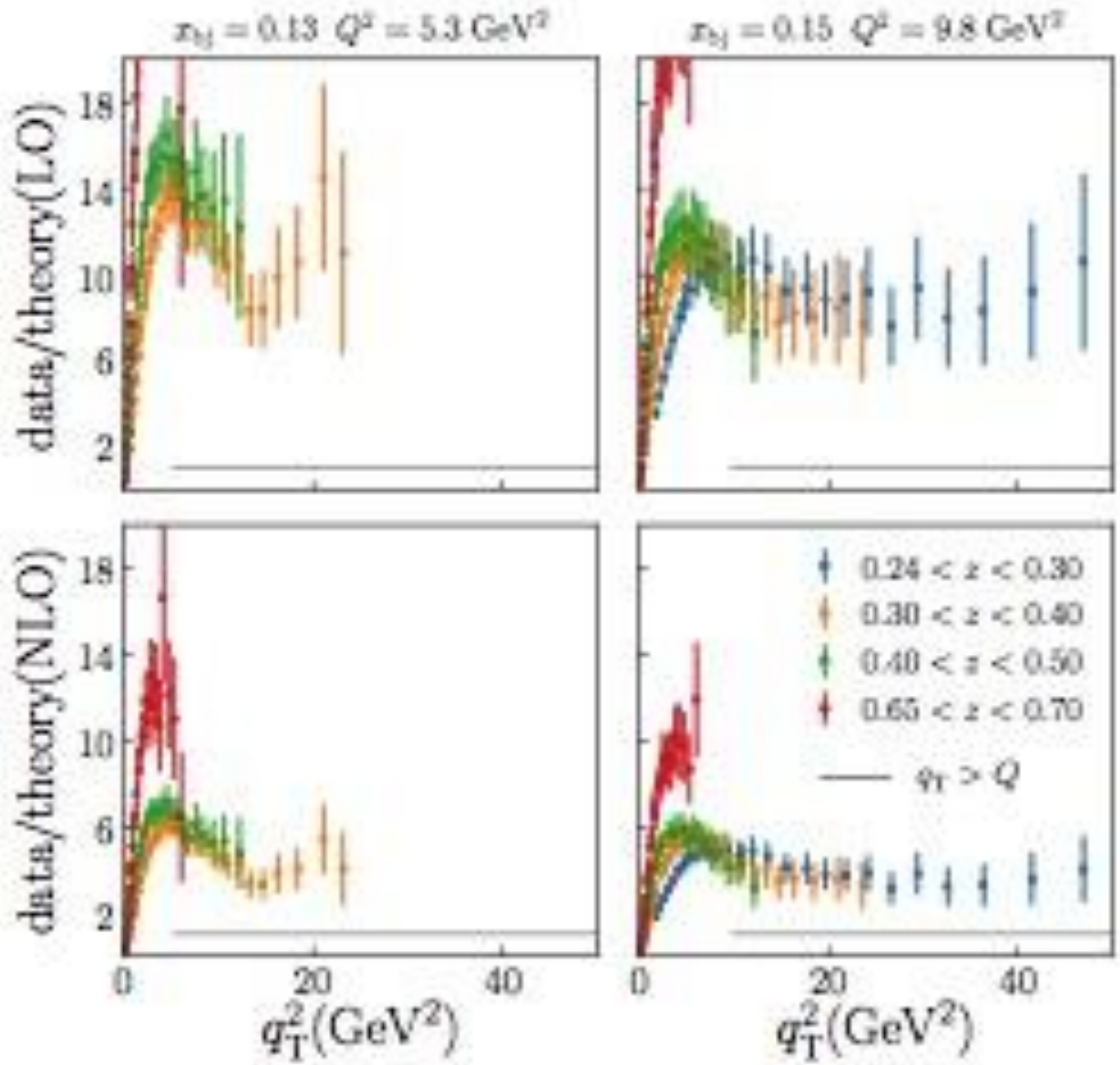
Small to large transverse momentum



Small to large transverse momentum



Small to large transverse momentum



Small to large transverse momentum