Global Analysis of Transverse-Spin Observables

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Workshop on Transverse Spin and TMDs
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Outline

- TMD and collinear twist-3 (CT3) functions
- Sivers and Collins effects & A_N in *pp* collisions
- Toward a global analysis of transverse spin observables
- Summary and outlook

TMD and Collinear Twist-3 Functions

TMD PDFs (x, k_T)

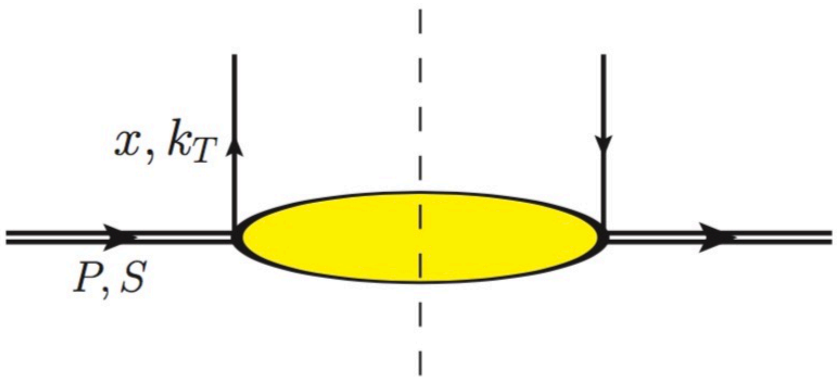
q pol. / H pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T} $h_{1T}^\perp$

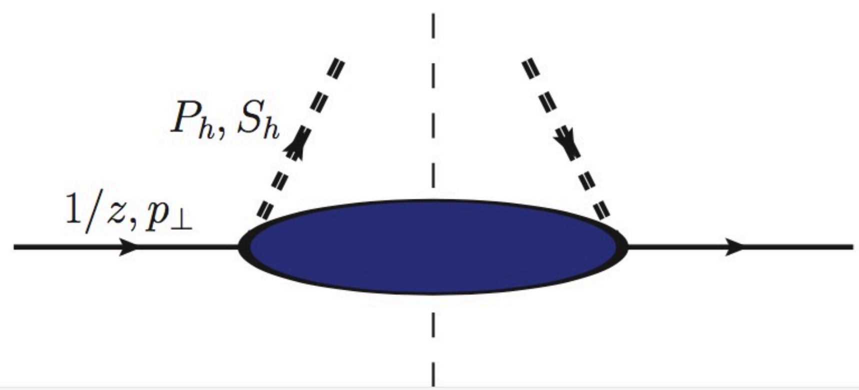
(Mulders, Tangerman (1996); Goeke, Metz, Schlegel (2005))

TMD FFs ($z, p_\perp$)

q pol. / H pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_{1T} $H_{1T}^\perp$

(Boer, Jakob, Mulders (1997))





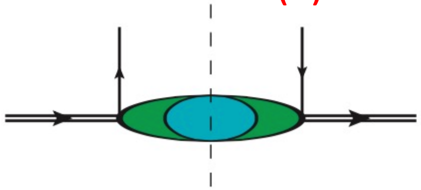
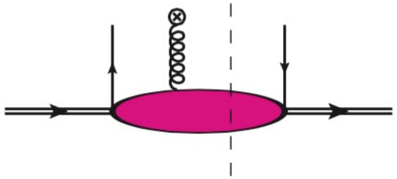
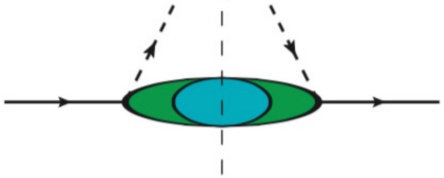
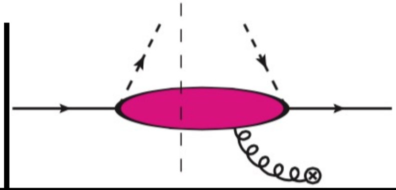
Collinear PDFs (x)

H pol. \ q pol.	U	L	T
U	f_1 unpolarized		
L		g_1 helicity	
T			h_1 transversity

Collinear FFs (z)

H pol. \ q pol.	U	L	T
U	D_1		
L		G_1	
T			H_1

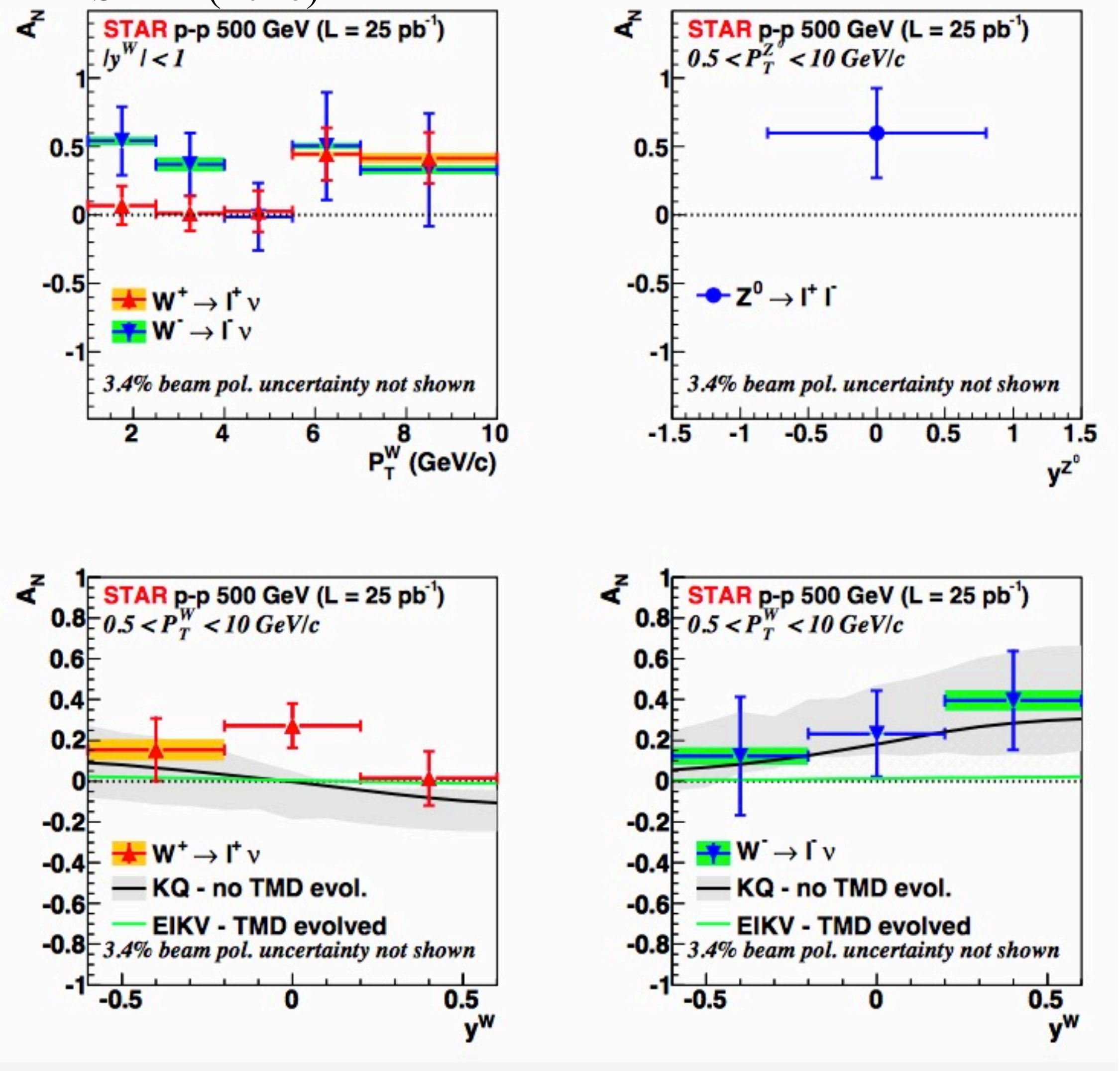
Integrate TMDs over k_T (or $p_\perp$) ➔ collinear PDFs and FFs

	CT3 PDF (x)		CT3 PDF (x, x_1)	CT3 FF (z)		CT3 FF (z, z_1)
Hadron Pol.						
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

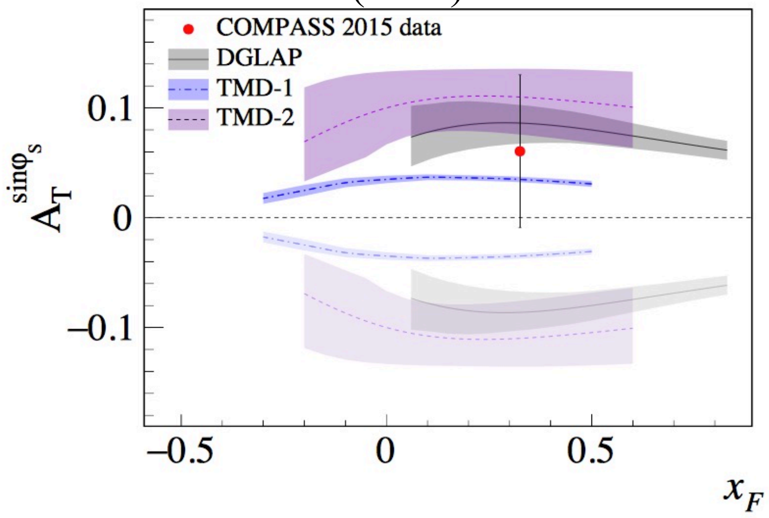
Sivers and Collins Effects & A_N in *pp* Collisions

Drell-Yan Sivers effect

STAR (2016)

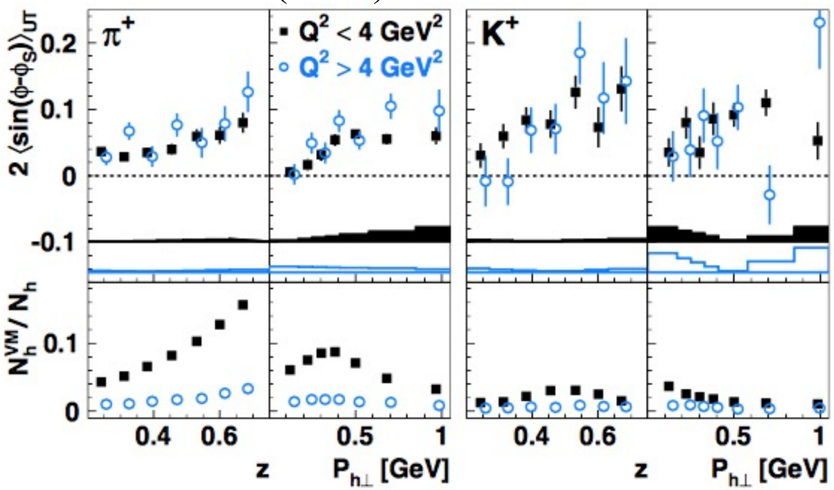


COMPASS (2017)

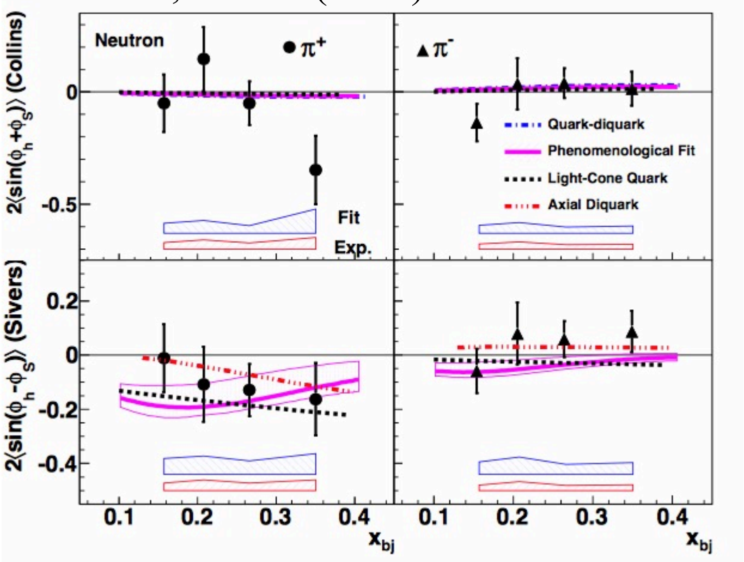


SIDIS Sivers effect (*sin(ϕ* $\phi_h - \phi_s$ *)*)

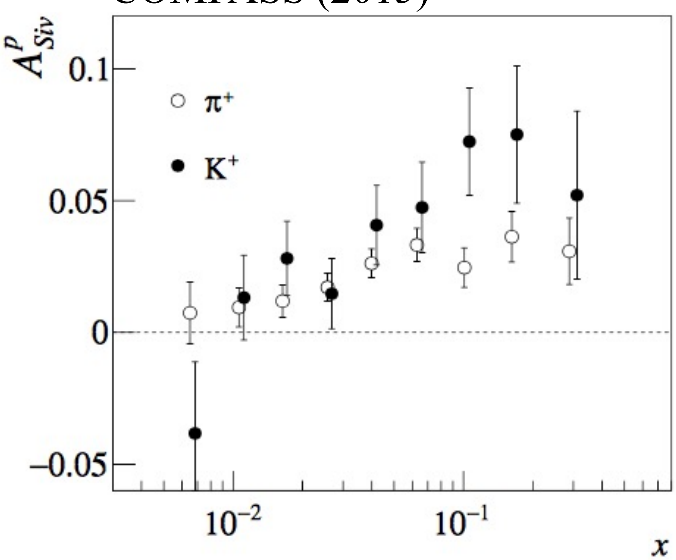
HERMES (2009)



JLab, Hall A (2011)

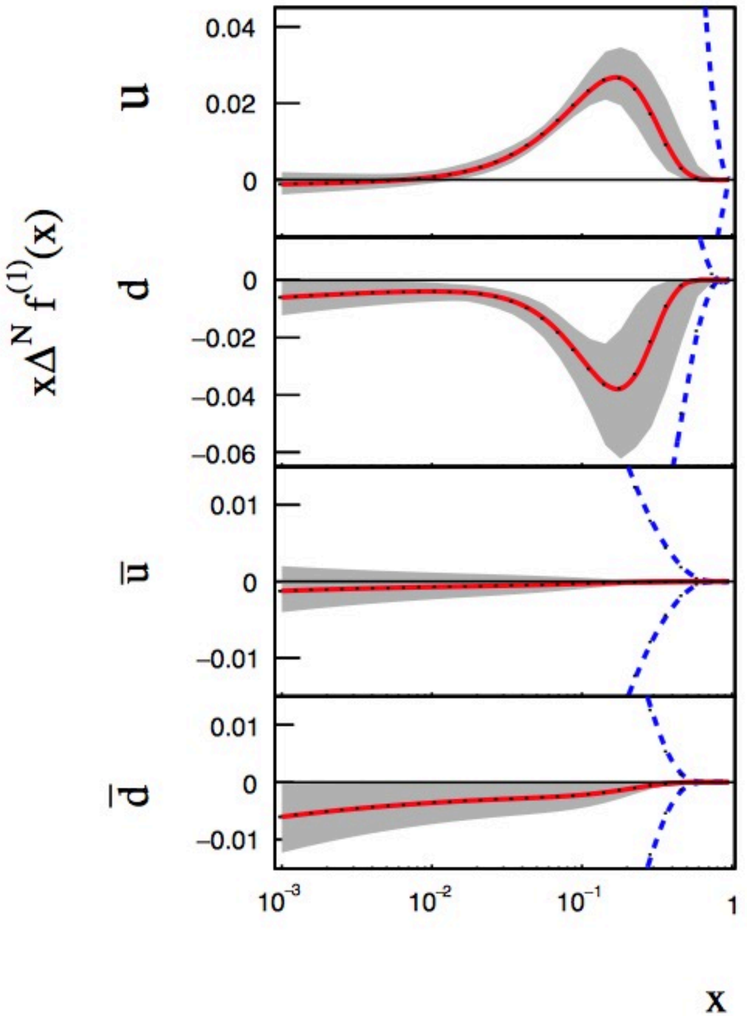


COMPASS (2015)

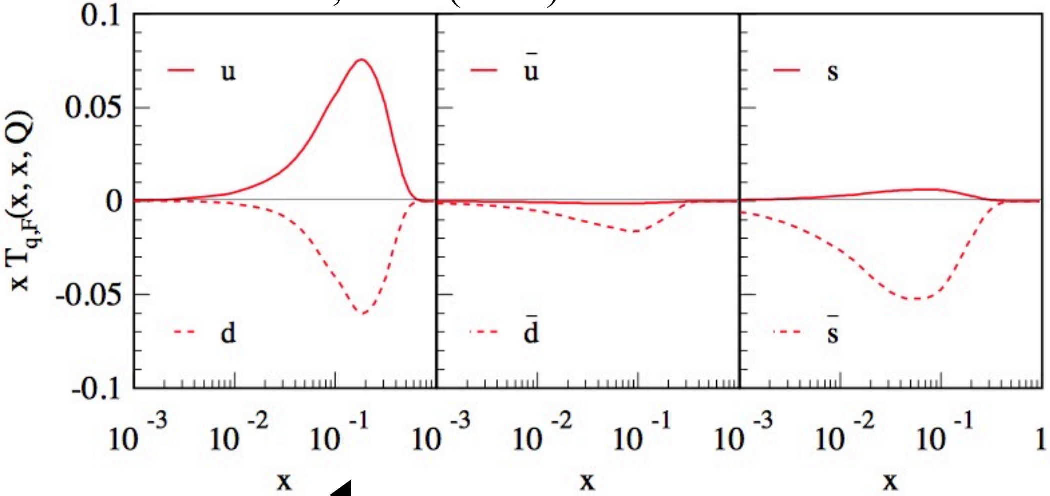


$$F_{UT}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_T}{M} f_{1T}^{\perp} D_1\right]$$

Anselmino, et al. (2017)



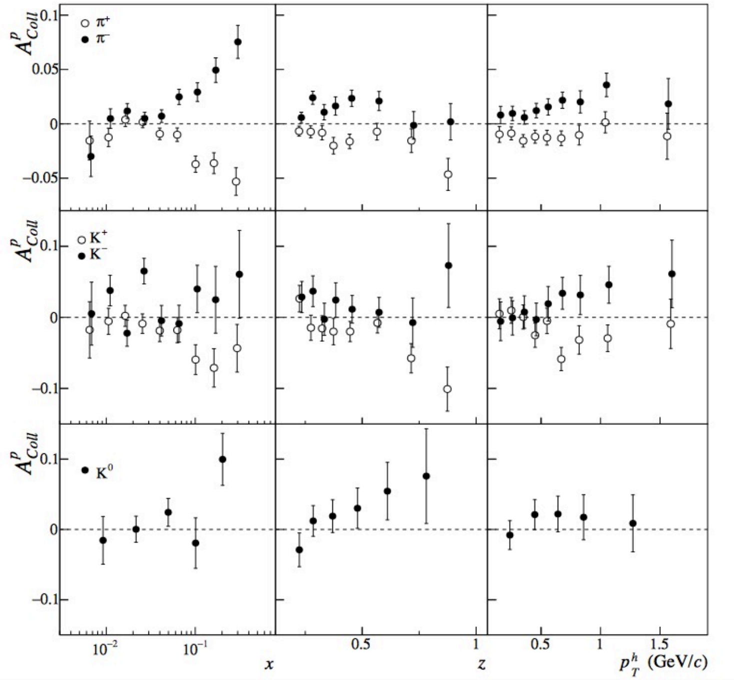
Echevarria, et al. (2014)



TMDs in Collins-Soper-Sterman (CSS) evolution formalism

SIDIS Collins effect ($sin(\phi_h + \phi_s)$)

COMPASS (2015)

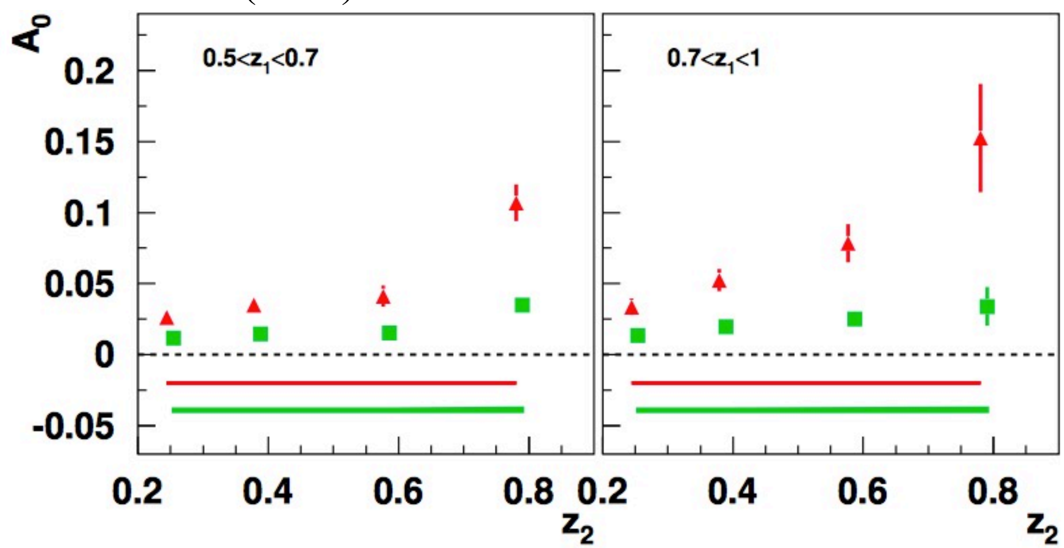


Also data from JLab Hall A (2011, 2014) and HERMES

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{p}_\perp}{M_h}\boldsymbol{h_1}\,\boldsymbol{H_1^\perp}\right]$$

e^+e^- Collins effect ($cos(2\phi_0)$)

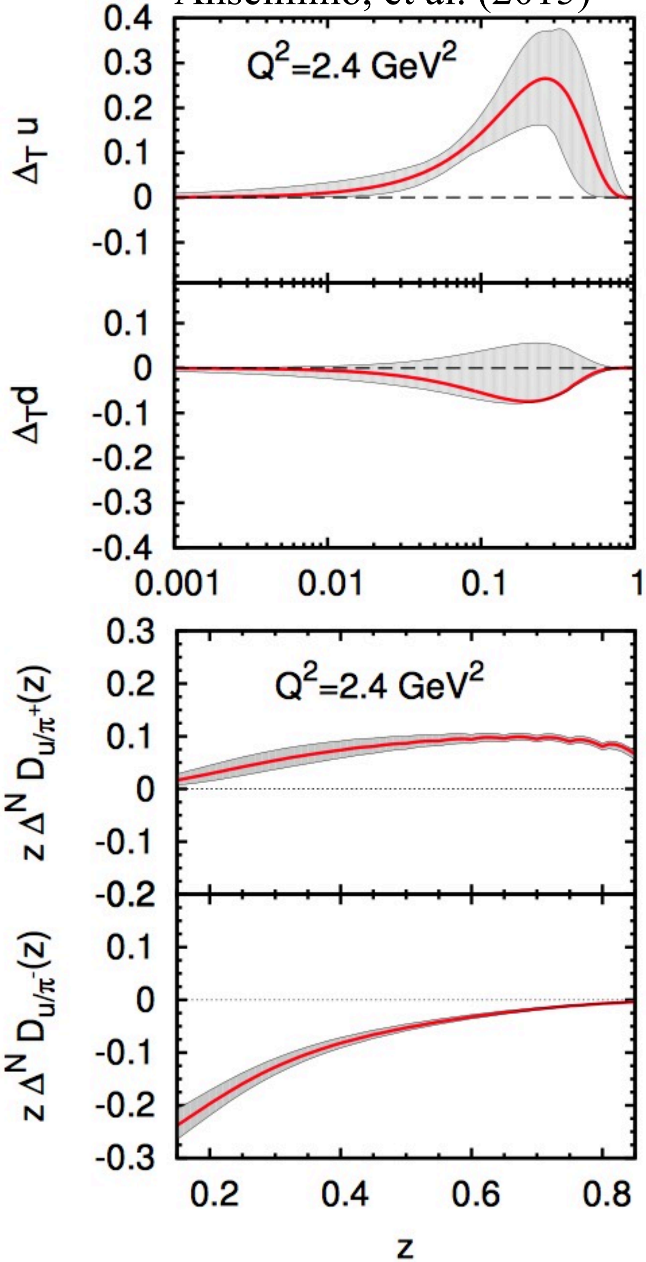
Belle (2008)



Also data from BaBar (2014) and BESIII (2016)

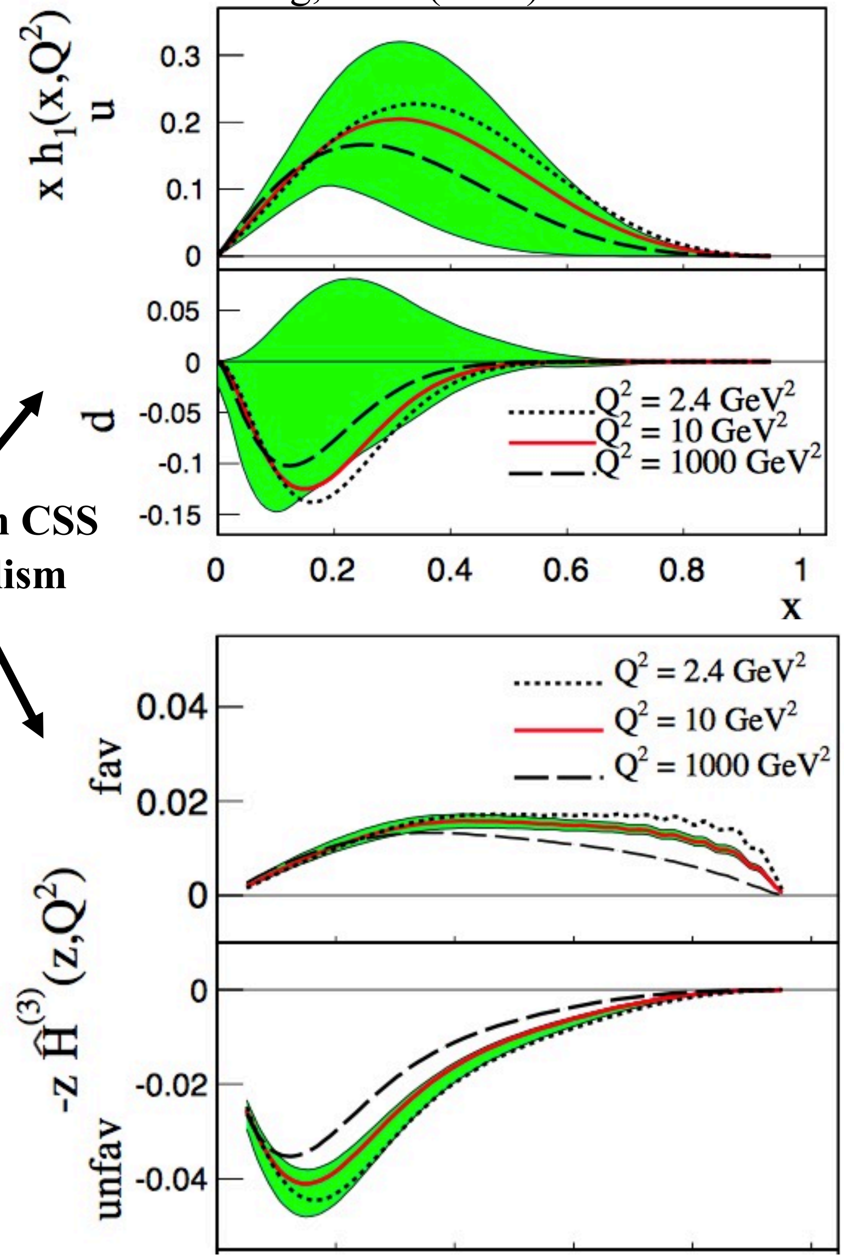
$$F_{UU}^{\cos(2\phi_0)} = \mathcal{C}\left[\frac{2\hat{h}\cdot\vec{p}_{a\perp}\,\hat{h}\cdot\vec{p}_{b\perp} - \vec{p}_{a\perp}\cdot\vec{p}_{b\perp}}{M_a M_b}\,\boldsymbol{H_1^\perp}\,\boldsymbol{\bar{H}_1^\perp}\right]$$

Anselmino, et al. (2015)



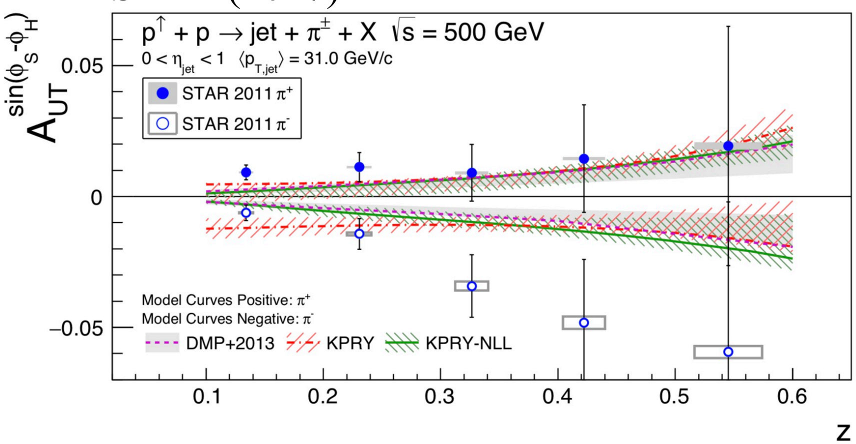
TMDs in CSS formalism

Kang, et al. (2016)



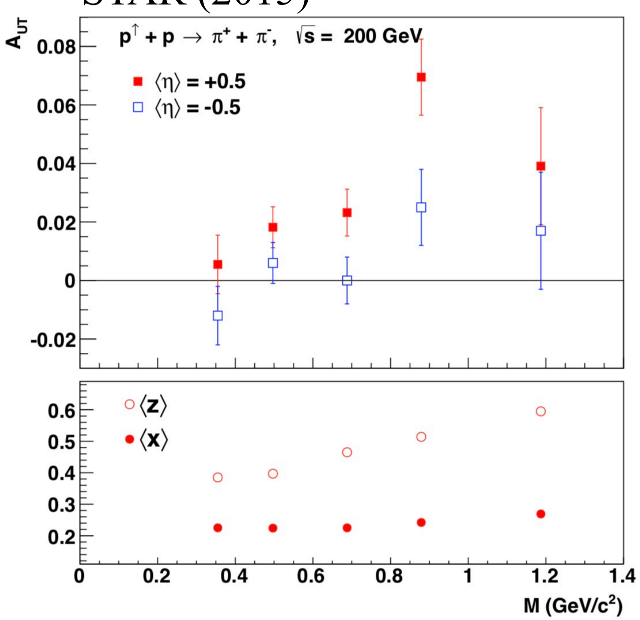
Hadron in a jet Collins effect

STAR (2017)

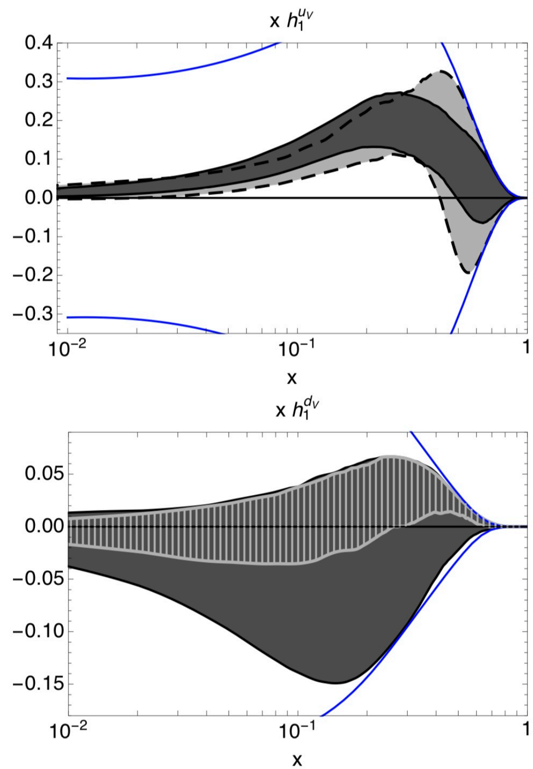


Theory curves from D'Alesio, et al. (2017) & Kang, et al. (2017)

STAR (2015)

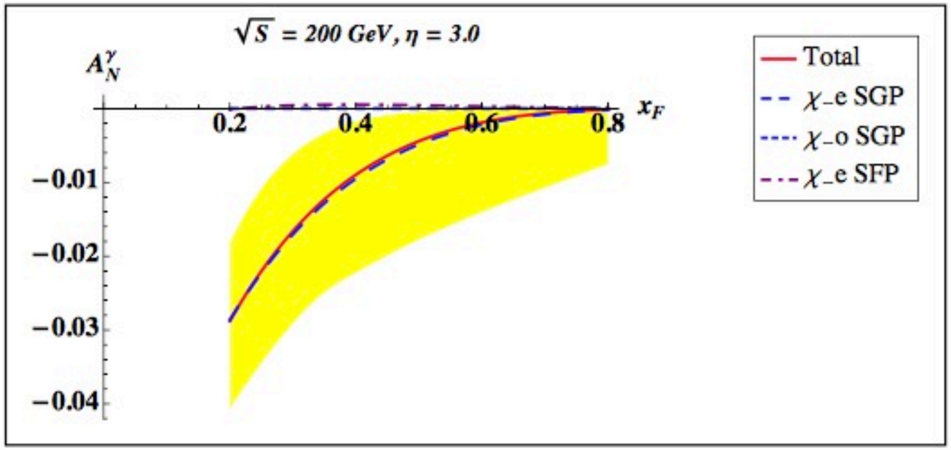


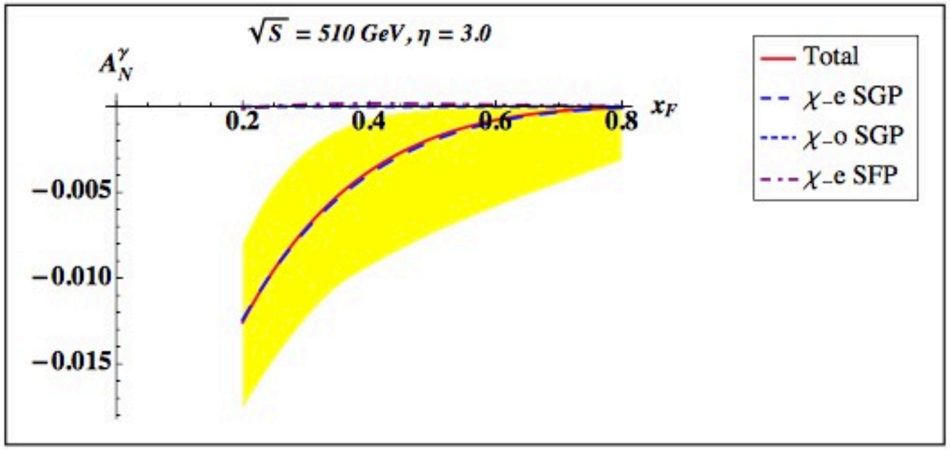
Transversity from dihadron FF



Bacchetta and Radici (2017)

A_N in $pp \to \gamma X$





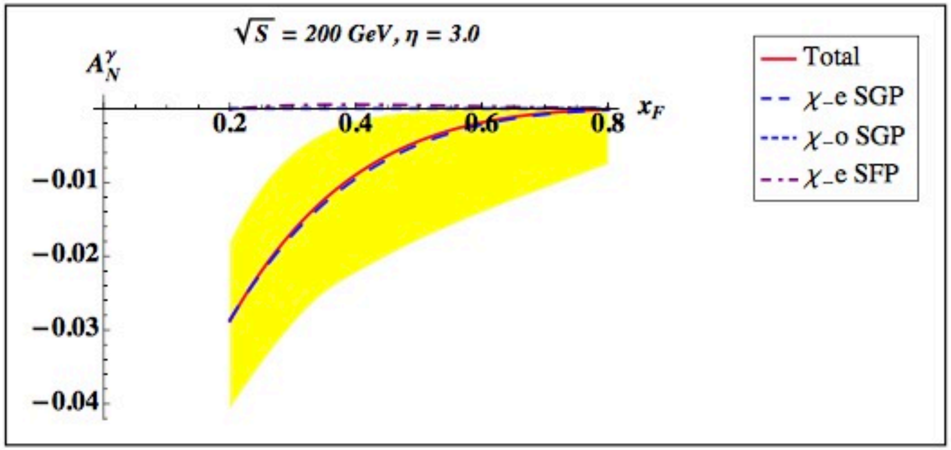
(Kanazawa, Koike, Metz, DP – PRD **91** (2015))
(See also Gamberg, Kang, Prokudin (2013))

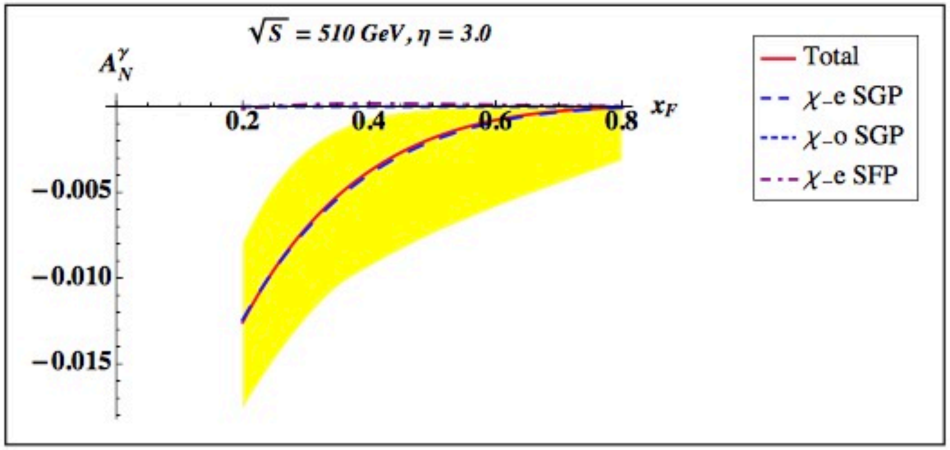
Qiu-Sterman term is the main cause of A_N in $pp \to \gamma X$ ➡ Test of the process dependence of the Sivers function

$$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

↘ Qiu-Sterman function

A_N in *pp* -> γ *X*





(Kanazawa, Koike, Metz, DP – PRD **91** (2015))
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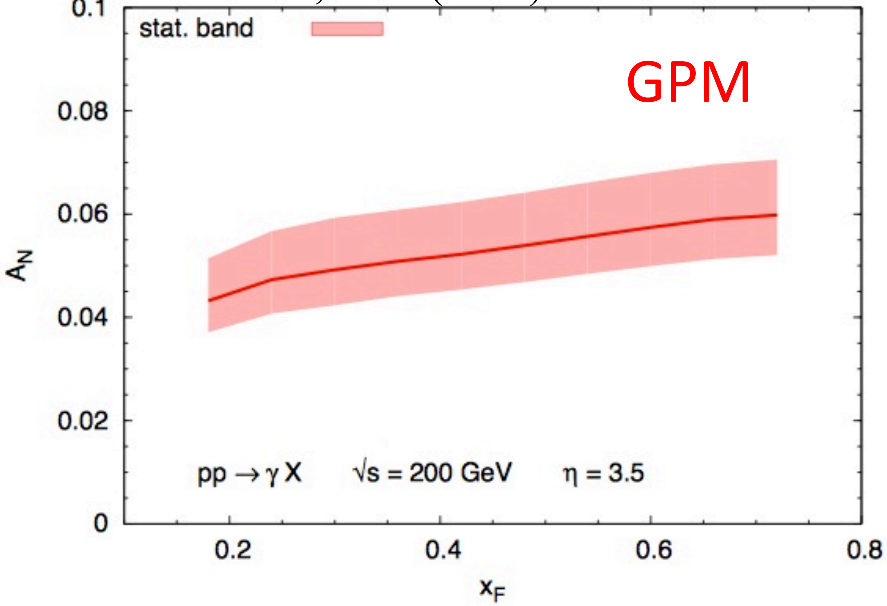
Qiu-Sterman term is the main cause of A_N in *pp* -> γ *X* ➡ Test of the process dependence of the Sivers function

$$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

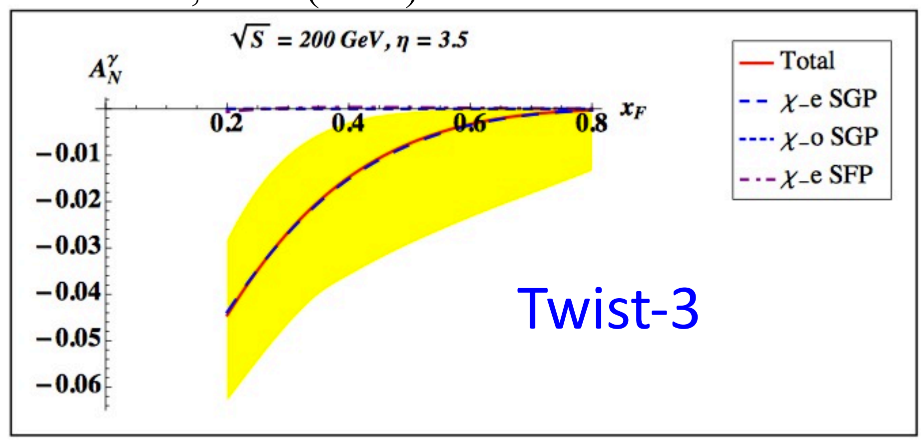
Can direct photon observables like A_N in *ep* -> γ *X* be measured at an EIC?

A_N in $pp \to \gamma X$

Anselmino, et al. (2013)

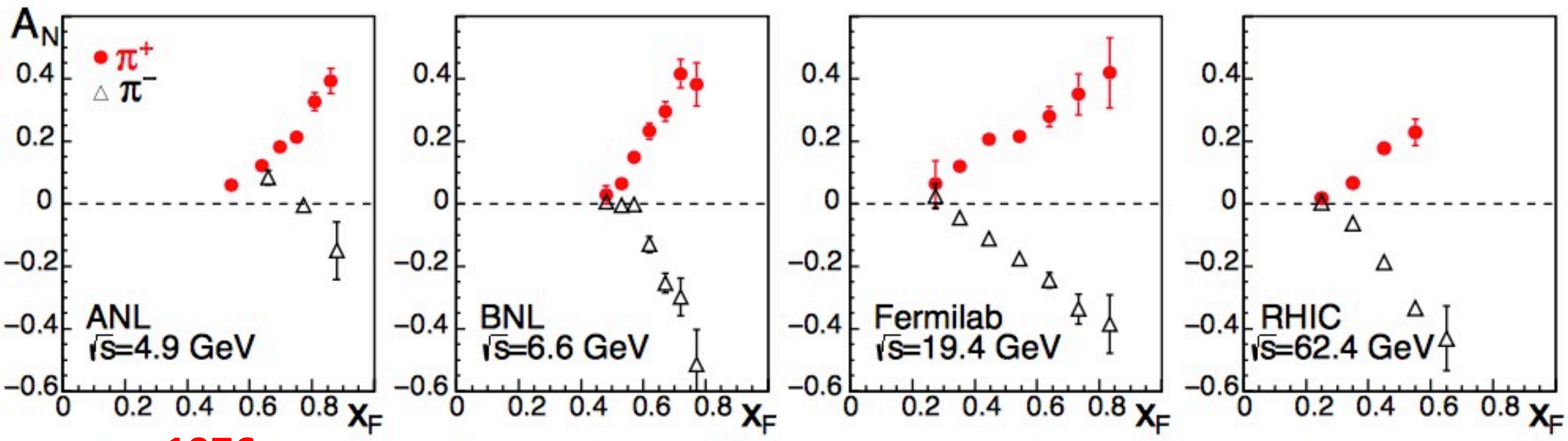


Kanazawa, et al. (2015)



GPM predicts a positive asymmetry while twist-3 predicts a negative one

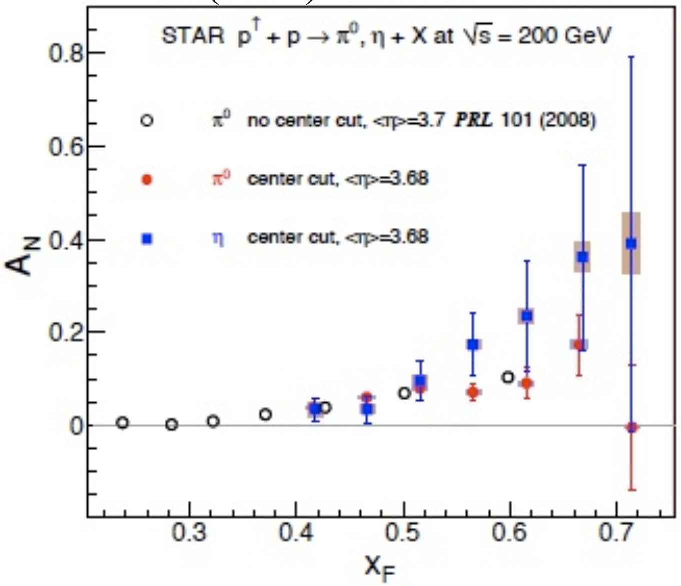
A_N in $pp \to \pi X$ – PUZZLE FOR 40+ YEARS!



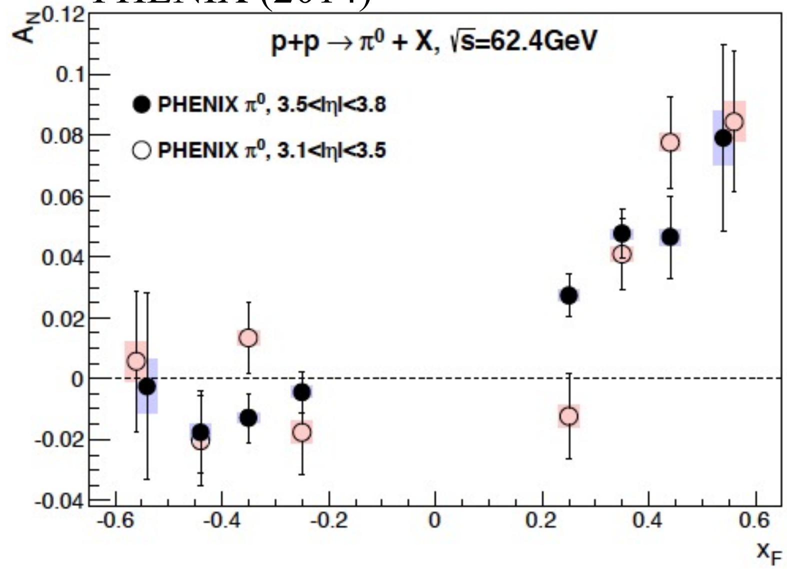
1976 →

A_N in *pp -> π X* – PUZZLE FOR 40+ YEARS!

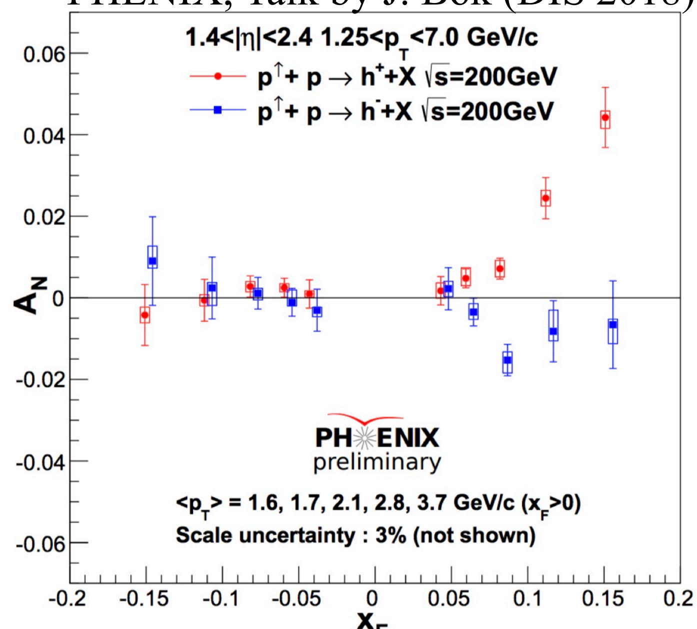
STAR (2012)



PHENIX (2014)



PHENIX, Talk by J. Bok (DIS 2018)



$$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

$$E_\ell \frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\min}}^{1} \frac{dz}{z^2} D_{c\to h}(z) \int_{x'_{\min}}^{1} \frac{dx'}{x'} \frac{1}{x'S + T/z} \phi_{b/B}(x') \times \sqrt{4\pi\alpha_s} \left(\frac{\epsilon^{\ell s_T n \bar{n}}}{z\hat{u}}\right) \frac{1}{x} \left[T_{a,F}(x,x) - x\left(\frac{d}{dx}T_{a,F}(x,x)\right)\right] H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

$$\boxed{F_{FT} \sim T_F}$$

(Qiu and Sterman (1999), Kouvaris, et al. (2006))

For many years the Qiu-Sterman/Sivers-type contribution was thought to be the dominant source of TSSAs in $\boldsymbol{p^{\uparrow}p \longrightarrow \pi\, X}$

$$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

(Kang, Qiu, Vogelsang, Yuan (2011); Kang and Prokudin (2012);
Metz, DP, Schäfer, Schlegel, Vogelsang, Zhou (2012))

~~$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$~~

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\, \epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\; \delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}\,(-x'\hat{t} - x\hat{u})}$$
$$\times\; h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz} \right] S^i_{H_1^\perp} + \frac{1}{z} H^c(z)\, S^i_H \right.$$
$$\left. + \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\left(\frac{1}{z} - \frac{1}{z_1}\right)^2} \hat{H}_{FU}^{c,\Im}(z,z_1)\, S^i_{\hat{H}_{FU}} \right\}$$

(Metz and DP - PLB **723** (2013))

We now believe the TSSAs in $\boldsymbol{p^{\uparrow}p \to \pi\, X}$ are due fragmentation effects as the partons form pions in the final state

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$H^q(z) = -2z\, H_1^{\perp(1),q}(z) + 2z \int_z^{\infty} \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{q,\Im}(z, z_1)$$

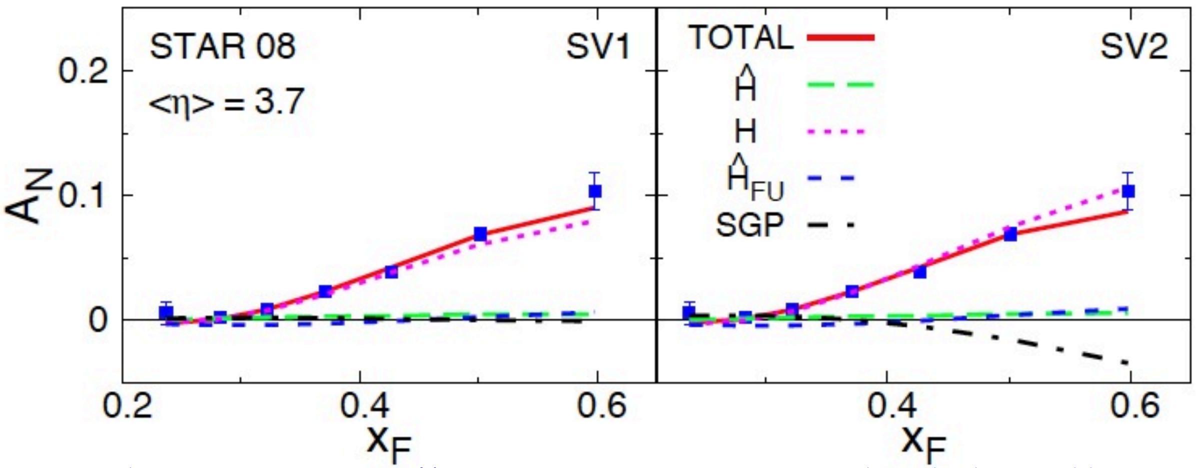
$$\equiv \tilde{H}^q(z)$$

QCD e.o.m. relation (EOMR)

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

Also included the Qiu-Sterman term $\pi \boldsymbol{F_{FT}(x,x)} = \boldsymbol{f_{1T}^{\perp(1)}(x)}$



(Kanazawa, Koike, Metz, DP, PRD **89**(RC) (2014))

Fragmentation term is the main cause of A_N in $pp \to \pi X$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^{\infty} \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z,z_1)}{(1/z - 1/z_1)^2}$$

Lorentz invariance relation (LIR)

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \tilde{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}} \right)$$

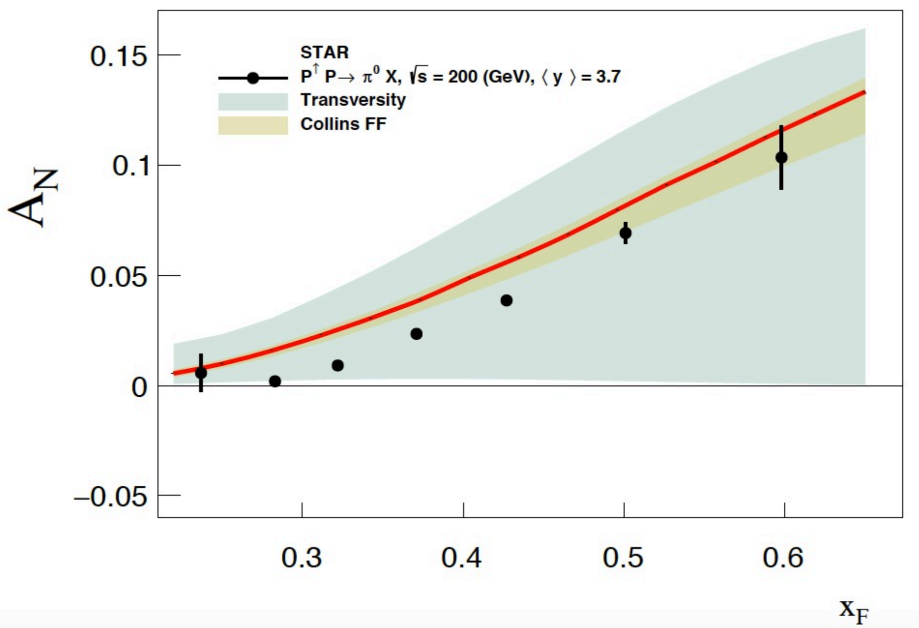
$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\, \epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\; \delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}}$$
$$\times\; h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz} \right] \tilde{S}^i_{H_1^\perp} + \left[-2H_1^{\perp(1),c}(z) + \frac{1}{z}\tilde{H}^c(z) \right] \tilde{S}^i_H \right\}$$

where $\tilde{S}^i_{H_1^\perp} \equiv \dfrac{S^i_{H_1^\perp} - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$ and $\tilde{S}^i_H \equiv \dfrac{S^i_H - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$

(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(H_1^{\perp(1)}, \tilde{H}, \int \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \tilde{S} \otimes \left(H_1^{\perp(1)}, \tilde{H} \right)$$



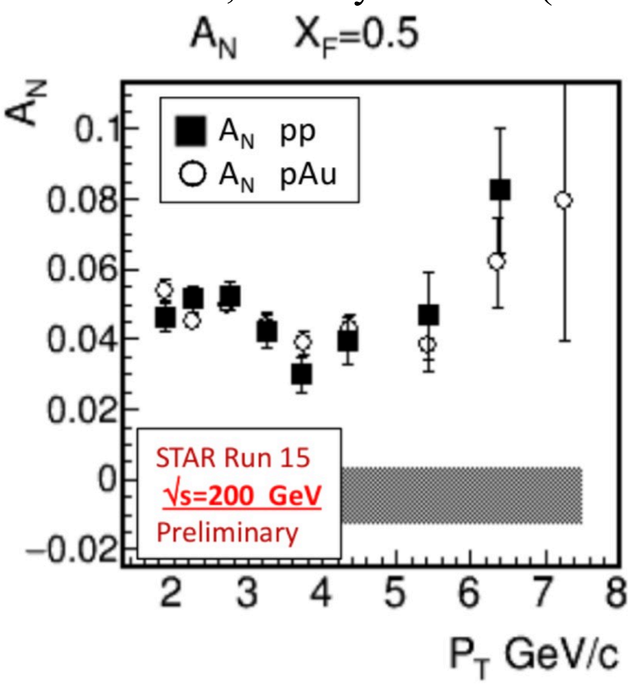
Fragmentation term is the main cause of A_N in *pp -> π X*

The ***A_N data from RHIC*** can be used along with ***measurements from SoLID at JLab*** to constrain ***transversity at large x!***

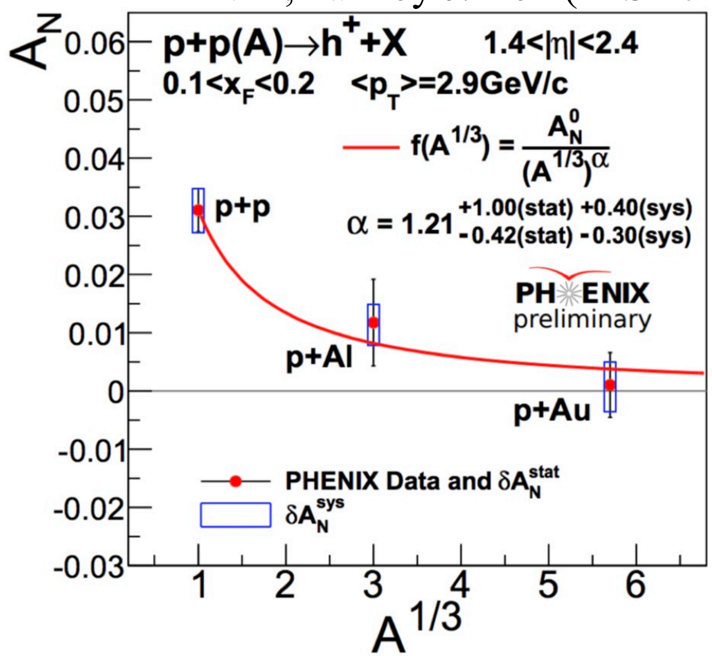
(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

A_N in $pA \to \pi X$

STAR, Talk by C. Dilks (DIS 2016)



PHENIX, Talk by J. Bok (DIS 2018)



STAR shows *no* suppression with *A*
(neutral pions, larger x_F region)

PHENIX shows $A^{1/3}$ suppression
(charged hadrons, smaller x_F region)

2013 expression from Metz and DP

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\,\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\,\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$

$$\times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\,\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H\right.$$

$$\left. + \,\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\,\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\,\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}\right\}$$

2013 expression from Metz and DP

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i \sum_{a,b,c}\int_0^1 \frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$

$$\times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz}\right] S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H + \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}\right\}$$

~ $A^{-1/3}$

~ $A^{-1/3}$

~ A^{0}

Include small *x* multiple-rescatterings to calculate *pA* TSSA (Hatta, Xiao, Yoshida, Yuan (2017))

2013 expression from Metz and DP

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$
$$\times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z)-z\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp}+\frac{1}{z}H^c(z)\,S^i_H\right.$$
$$\left.+\;\boxed{\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}}\right\}$$

~ A^0

EOMR + LIR →

$$\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{\pi/c,\Im}(z,z_1) = H_1^{\perp(1),c}(z)+z\frac{dH_1^{\perp(1),c}(z)}{dz}-\frac{1}{z}\tilde{H}^c(z)$$

2013 expression from Metz and DP

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$

$$\times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H\right.$$

$$\left. + \boxed{\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}}\right\}$$

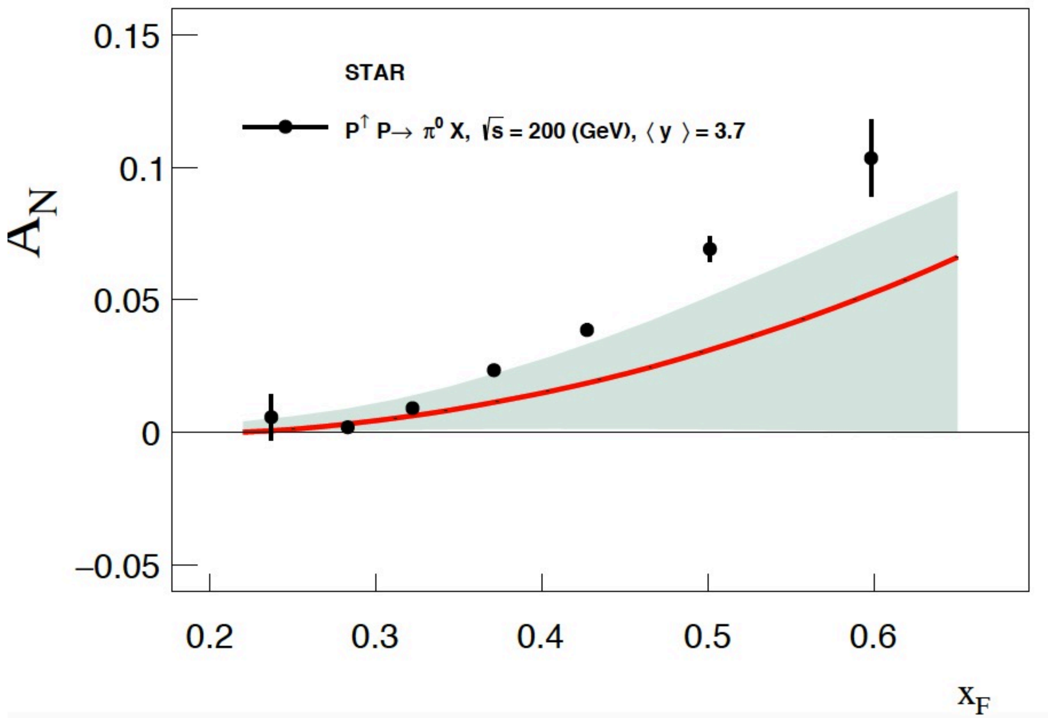
$\sim A^0$

EOMR + LIR ➔

$$\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{\pi/c,\Im}(z,z_1) = H_1^{\perp(1),c}(z) + z\frac{dH_1^{\perp(1),c}(z)}{dz} - \frac{1}{z}\tilde{H}^c(z)$$

Calculate pieces involving the (first k_T-moment of the) Collins function to get an updated estimate for the term in blue

D. Pitonyak

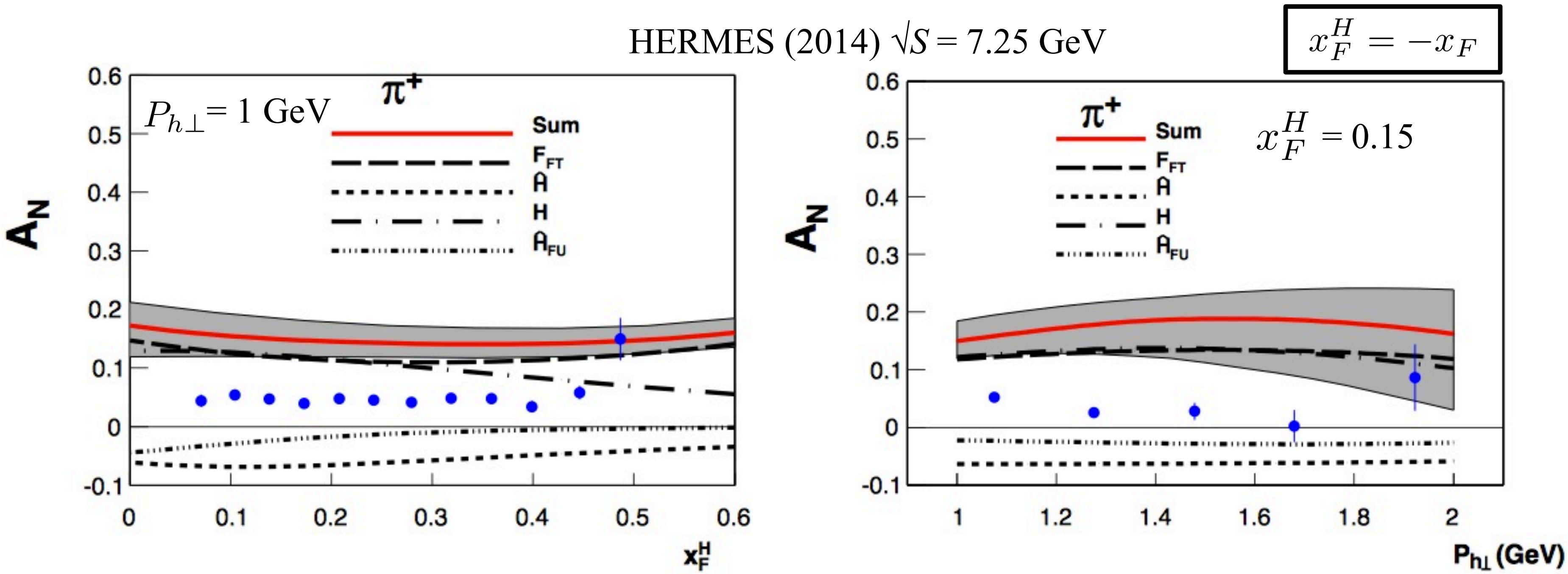


(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

Fragmentation term as the cause of A_N in *pp* collisions is *not* inconsistent with the RHIC *pA* A_N data.

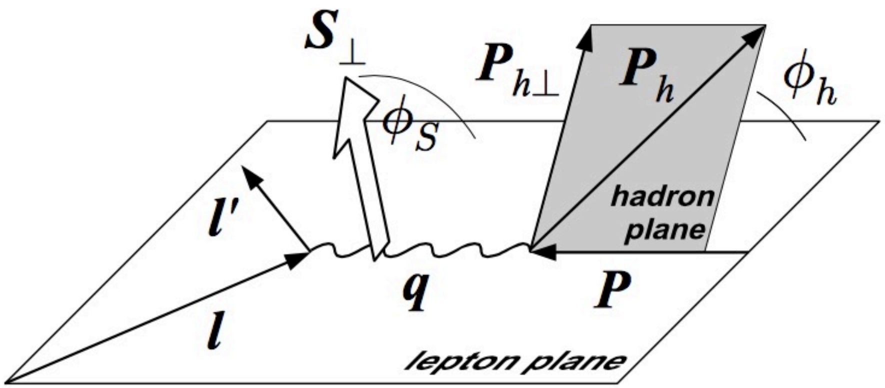
Reduced theoretical uncertainties are needed.

A_N in *ep -> π X*

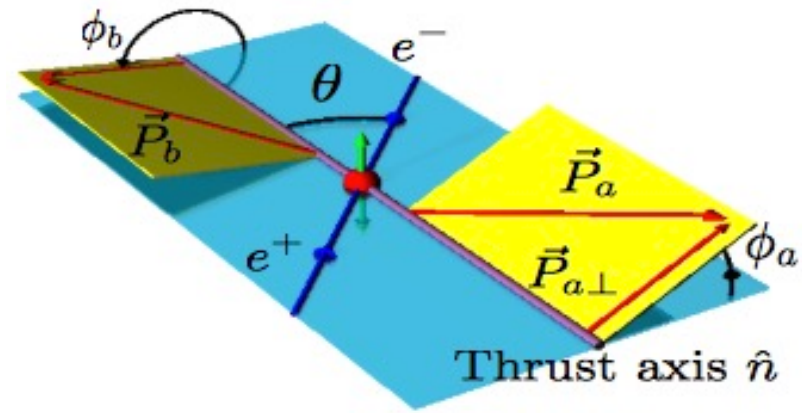


- JLab Hall A also has data for a neutron target, but P_T is too low
  - ➔ 12 GeV upgrade will give valuable data at higher P_T
- This process can help better constrain the 3-parton FFs that have been fit in *pp*
  - ➔ crucial to measure at at EIC – data in forward region!
  - ➔ need to update analysis to include constraints from LIRs

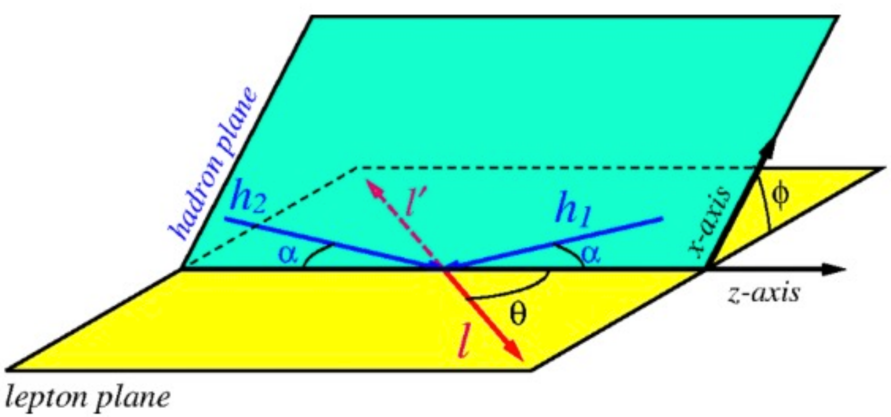
(Gamberg, Kang, Metz, DP, Prokudin - PRD **90** (2014))



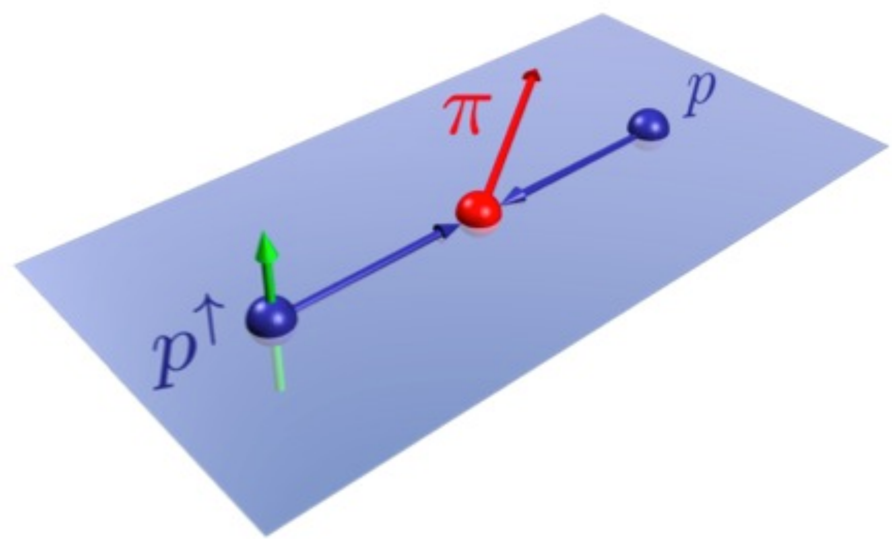
Sivers ~ $\sin(\phi_h - \phi_s)$, Collins ~ $\sin(\phi_h + \phi_s)$, …



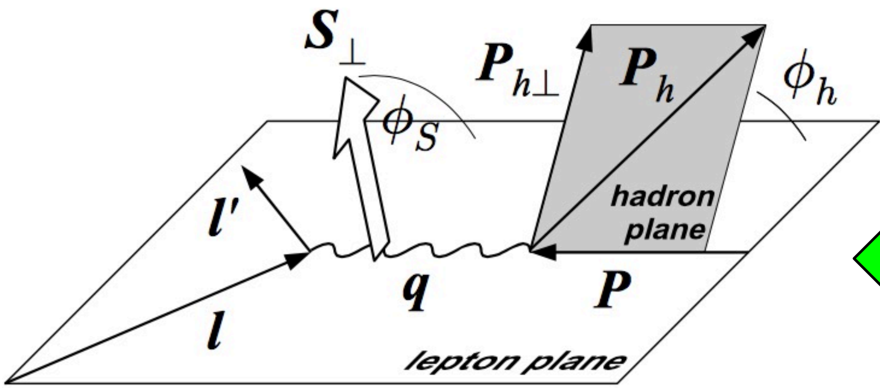
Collins ~ $\cos(\phi_a + \phi_b)$, …

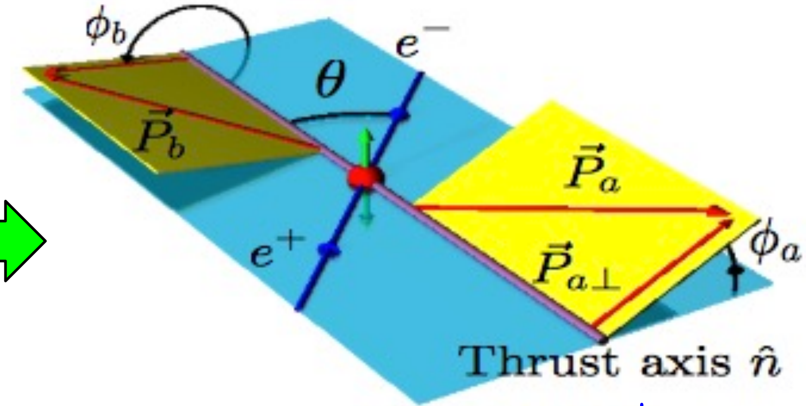


Sivers ~ $\sin(\phi_s)$ (lepton pair) / Sivers ~ $\cos(\phi_{W/Z})$ (boson)



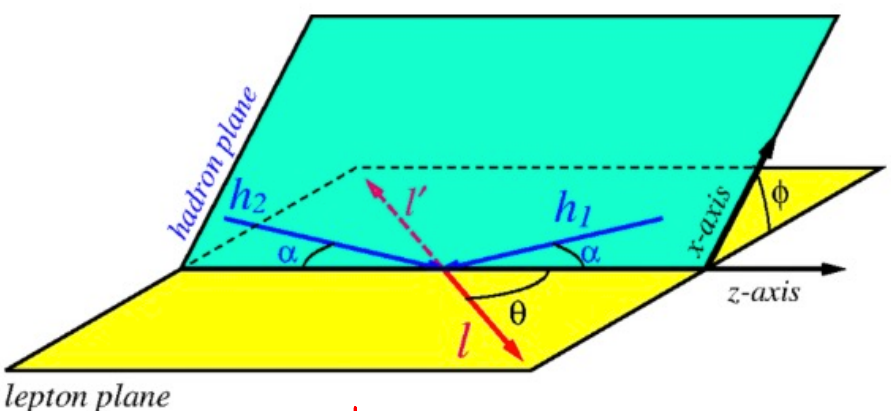
$A_N \sim d\sigma_L - d\sigma_R$

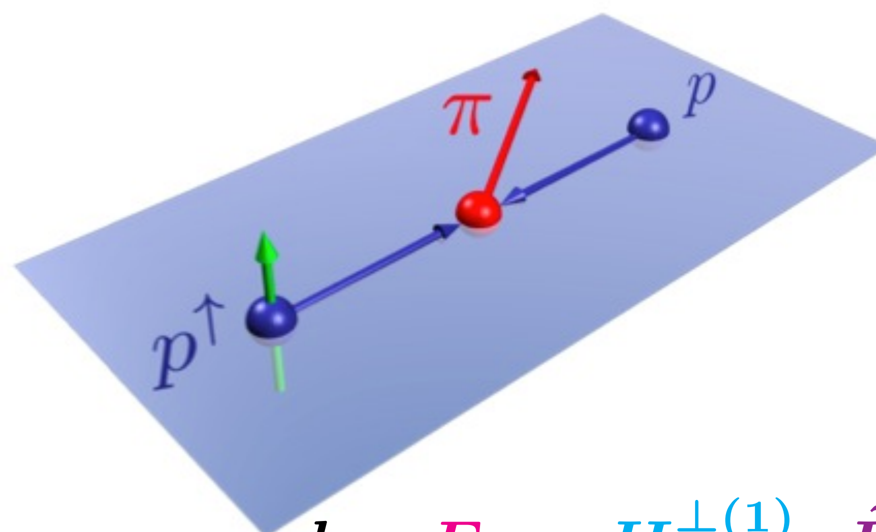




$h_1,\ f_{1T}^\perp,\ H_1^\perp$

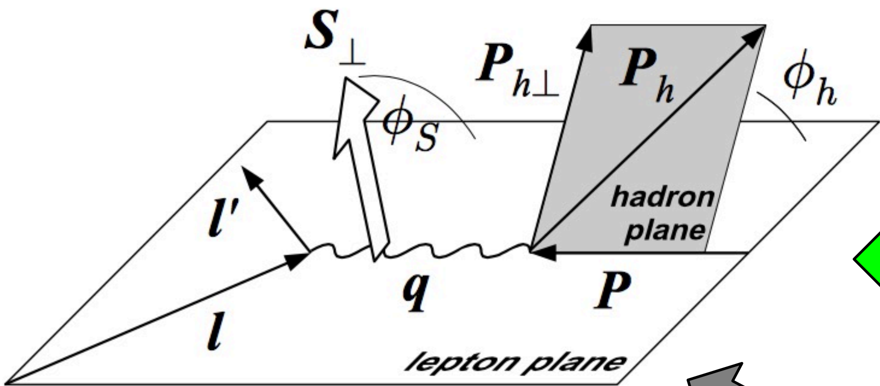
$H_1^\perp$



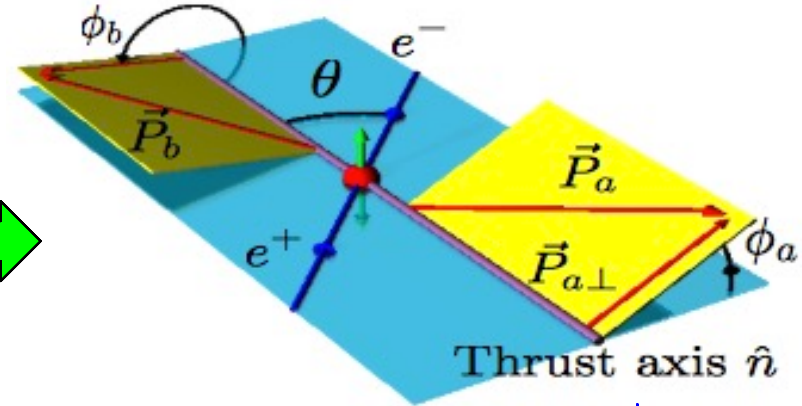


$f_{1T}^\perp$

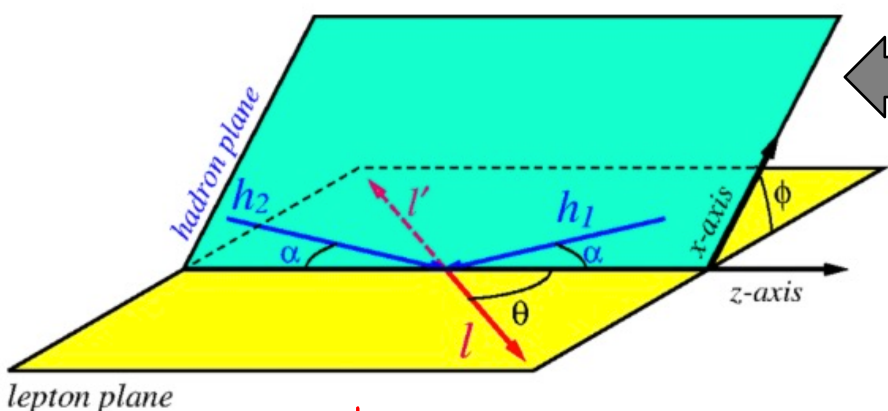
$h_1,\ F_{FT},\ H_1^{\perp(1)},\ \tilde{H}$



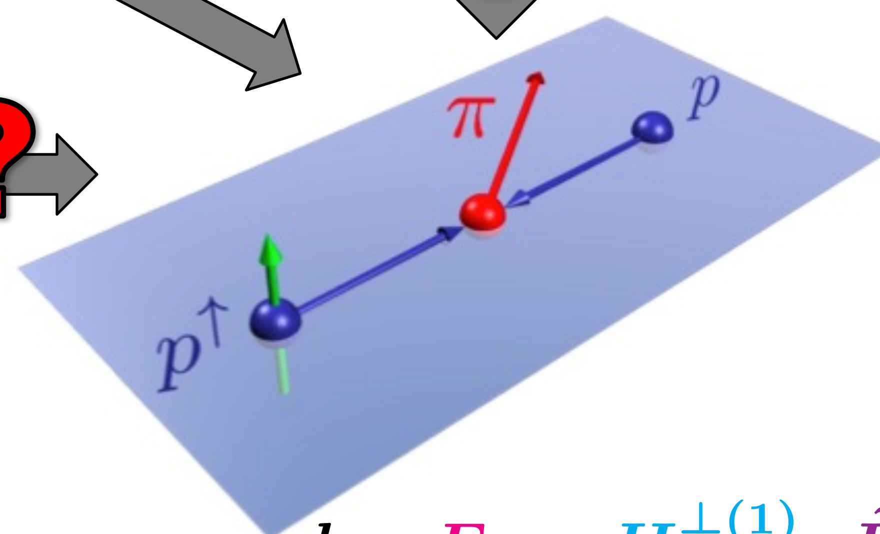
h_1, $f_{1T}^{\perp}$, $H_1^{\perp}$



$H_1^{\perp}$



$f_{1T}^{\perp}$



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

Toward a Global Analysis of Transverse Spin Observables

Recall the current phenomenology of TMD observables...

$$\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q) \sim \boxed{F_{FT}(x, x; \mu_{b_*})} \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^\perp}(b_T, Q)\right]$$

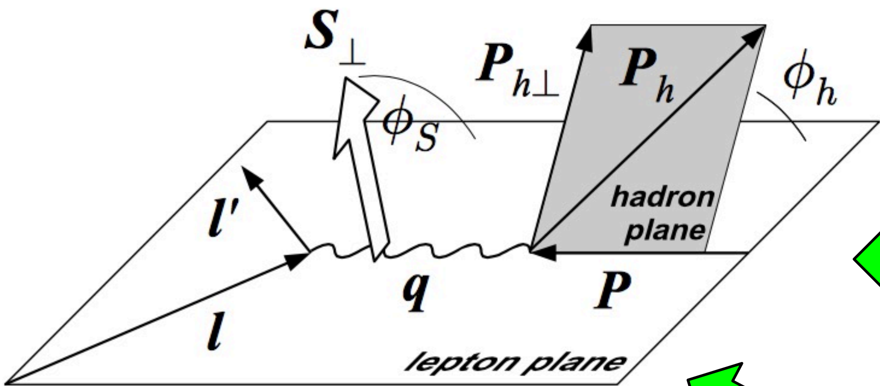
$$S_{NP}^{f_{1T}^\perp}(b_T, Q) \rightarrow \boxed{g_{f_{1T}^\perp}(x, b_T)} + g_K(b_T)\ln(Q/Q_0)$$

$$\tilde{H}_1^{\perp(1)}(z, b_T; Q^2, \mu_Q) \sim \boxed{H_1^{\perp(1)}(z; \mu_{b_*})} \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{H_1^\perp}(b_T, Q)\right]$$

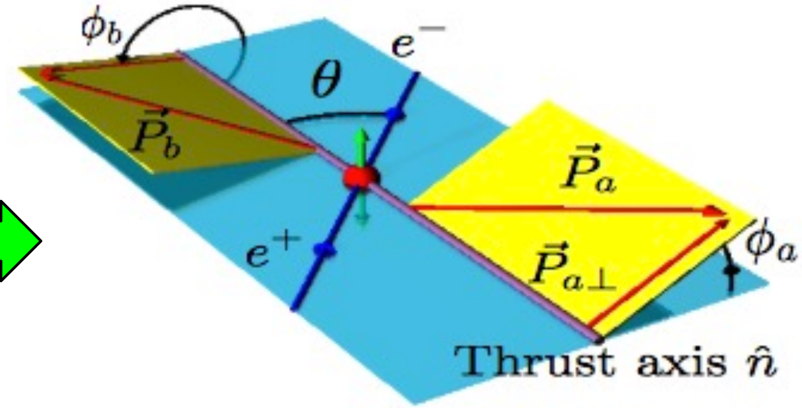
$$S_{NP}^{H_1^\perp}(b_T, Q) \rightarrow \boxed{g_{H_1^\perp}(z, b_T)} + g_K(b_T)\ln(Q/Q_0)$$

The **CT3 functions** (along with the NP *g*-functions) are what get extracted in analyses of TSSAs in ***TMD processes*** that use CSS evolution!

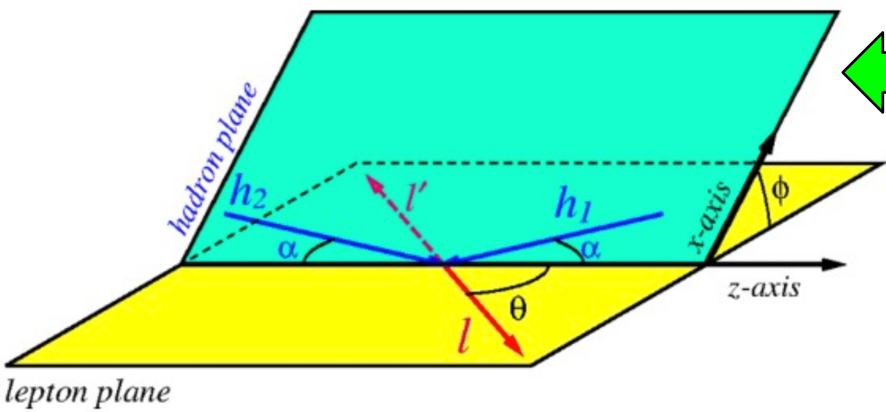
(Echevarria, Idilbi, Kang, Vitev (2014); Kang, Prokudin, Sun, Yuan (2016))



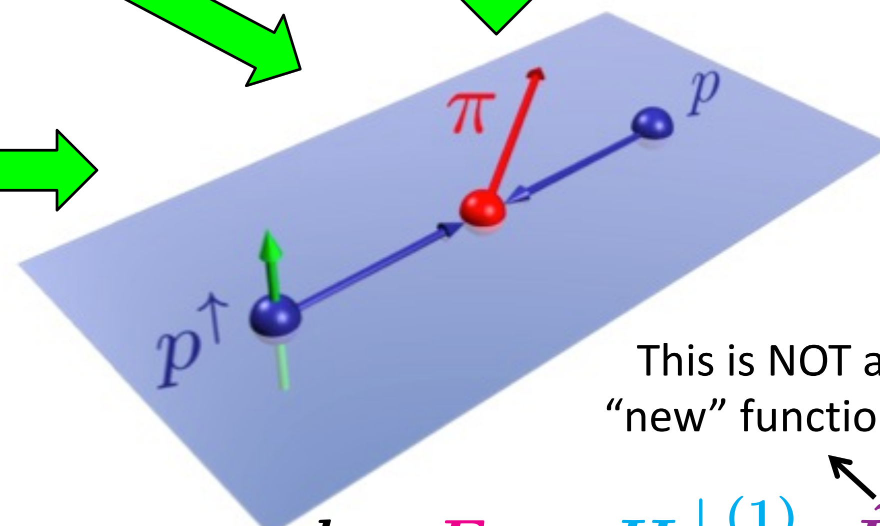
$h_1, F_{FT}, H_1^{\perp(1)}$



$H_1^{\perp(1)}$



F_{FT}

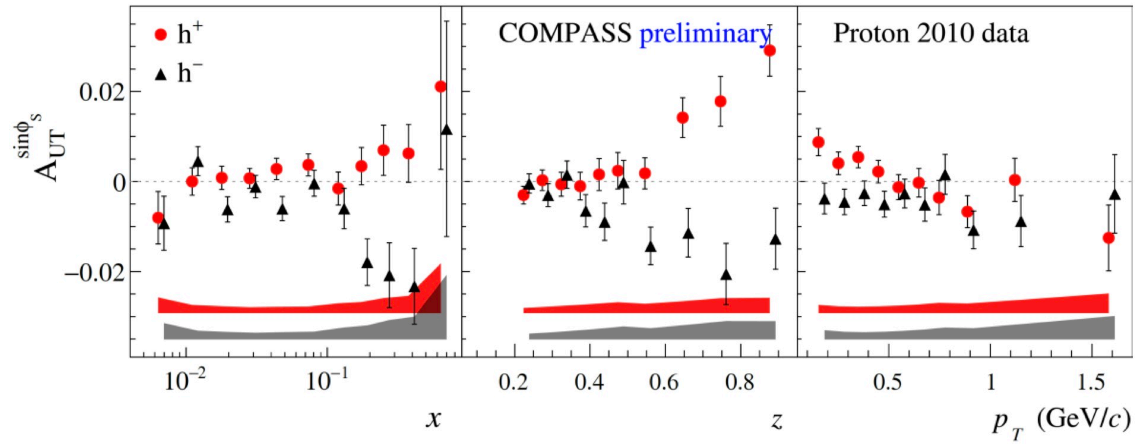


This is NOT a "new" function!

$h_1, F_{FT}, H_1^{\perp(1)}, \tilde{H}$

$A_{UT}^{\sin\phi_S}$ in SIDIS integrated over P_T (Mulders, Tangerman (1996); Bacchetta, et al. (2007))

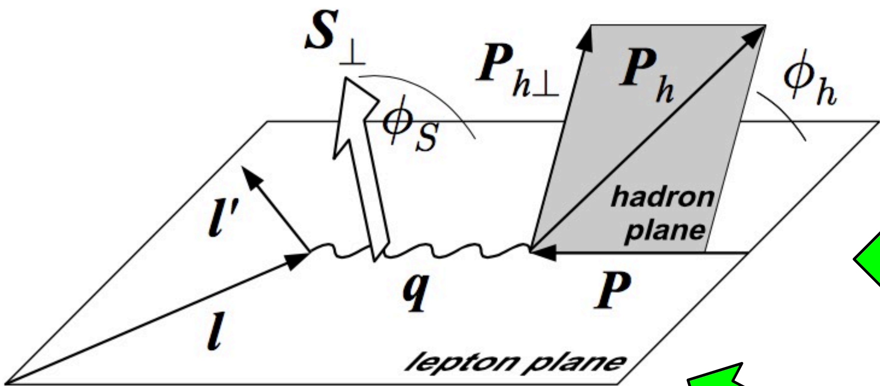
$$F_{UT}^{\sin\phi_S} \propto \sum_a e_a^2 \, \frac{2M_h}{Q} \, h_1^a(x) \, \frac{\tilde{H}^a(z)}{z}$$



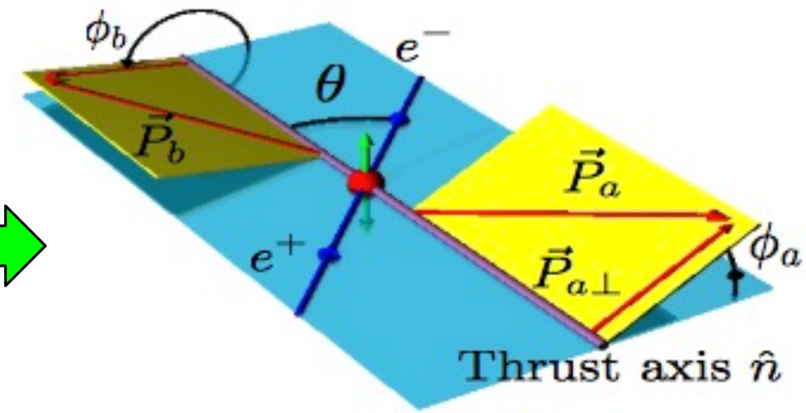
$A_{UT}^{\sin\phi_S}$ in $e^+e^- \rightarrow h_1 h_2 X$ integrated over q_T (Boer, Jakob, Mulders (1997))

$$F_{UT}^{\sin\phi_S} \propto \sum_{a,\bar{a}} e_a^2 \left(\frac{2M_2}{Q} \, D_1^a(z_1) \frac{D_T^{\bar{a}}(z_2)}{z_2} + \frac{2M_1}{Q} \, \frac{\tilde{H}(z_1)}{z_1} H_1^{\bar{a}}(z_2) \right)$$

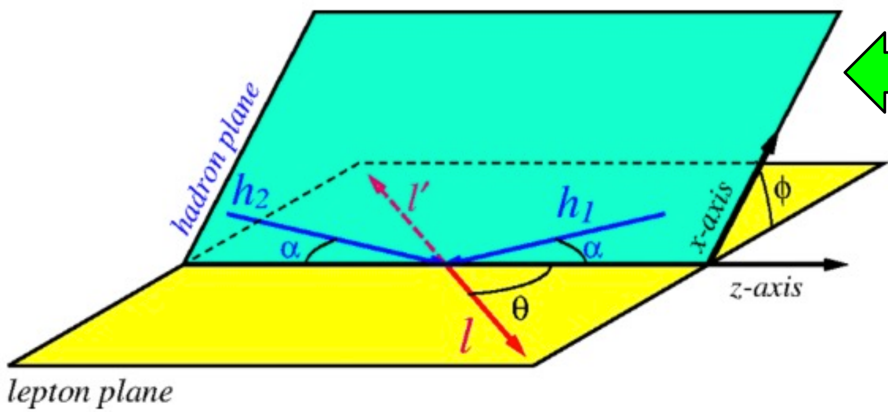
And also the TMD version of these (and other) observables (but with many more terms)



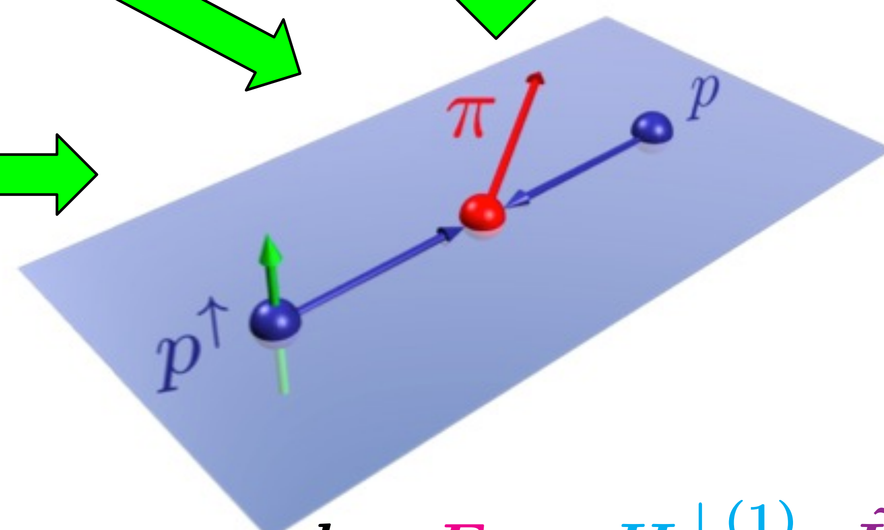
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$H_1^{\perp(1)}$, $\tilde{H}$



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

EOMR

$$H^q(z) = -2z\, H_1^{\perp(1),q}(z) + 2z \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{q,\Im}(z, z_1)$$

LIR

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z, z_1)}{(1/z - 1/z_1)^2}$$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

EOMR

$$H^q(z) = -2z\, H_1^{\perp(1),q}(z) + 2z \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{q,\Im}(z, z_1)$$

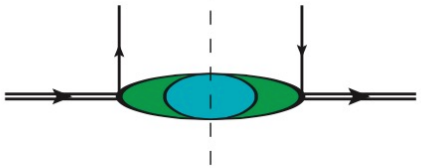
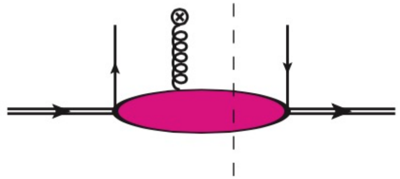
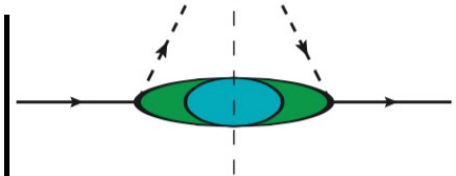
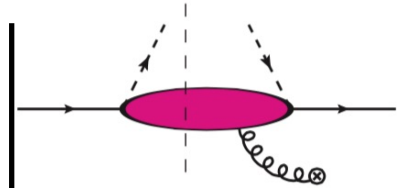
LIR

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z,z_1)}{(1/z - 1/z_1)^2}$$

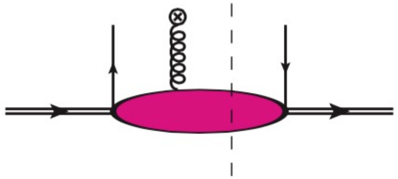
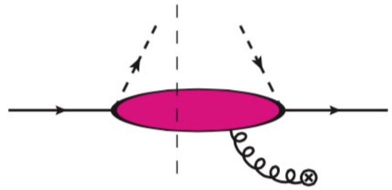
$$\boldsymbol{H(z)} = \int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2}\, 2\left[\frac{\left(2(\frac{2}{z_1} - \frac{1}{z_2}) + \frac{1}{z_1}\left(\frac{1}{z_1} - \frac{1}{z_2}\right)\delta\left(\frac{1}{z_1} - \frac{1}{z}\right)\right)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \boldsymbol{\hat{H}_{FU}^{\Im}(z_1, z_2)}\right]$$

$$\boldsymbol{H_1^{\perp(1)}(z)} = -\frac{2}{z}\int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2} \frac{\left(\frac{2}{z_1} - \frac{1}{z_2}\right)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \boldsymbol{\hat{H}_{FU}^{\Im}(z_1, z_2)}$$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

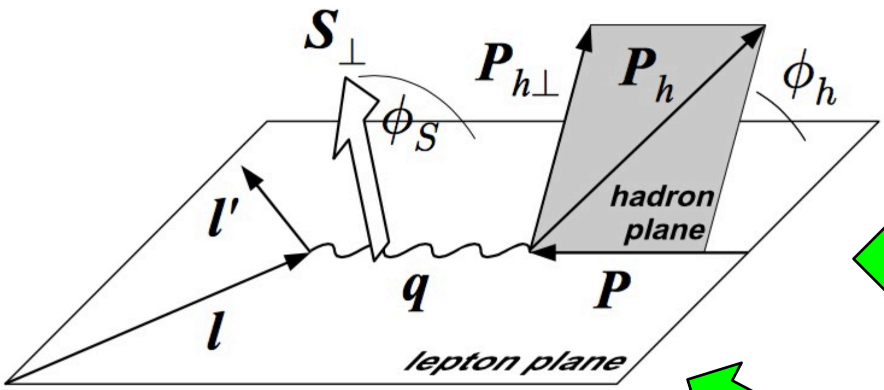
Hadron Pol.	PDF (x)		PDF (x, x_1)	FF (z)		FF (z, z_1)
						
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	~~e~~	~~$h_1^{\perp(1)}$~~	H_{FU}	~~E~~, ~~H~~	~~$H_1^{\perp(1)}$~~	$\hat{H}_{FU}^{\Re,\Im}$
L	~~h_L~~	~~$h_{1L}^{\perp(1)}$~~	H_{FL}	~~H_L~~, ~~E_L~~	~~$H_{1L}^{\perp(1)}$~~	$\hat{H}_{FL}^{\Re,\Im}$
T	~~g_T~~	~~$f_{1T}^{\perp(1)}$~~, ~~$g_{1T}^{\perp(1)}$~~	F_{FT}, G_{FT}	~~D_T~~, ~~G_T~~	~~$D_{1T}^{\perp(1)}$~~, ~~$G_{1T}^{\perp(1)}$~~	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

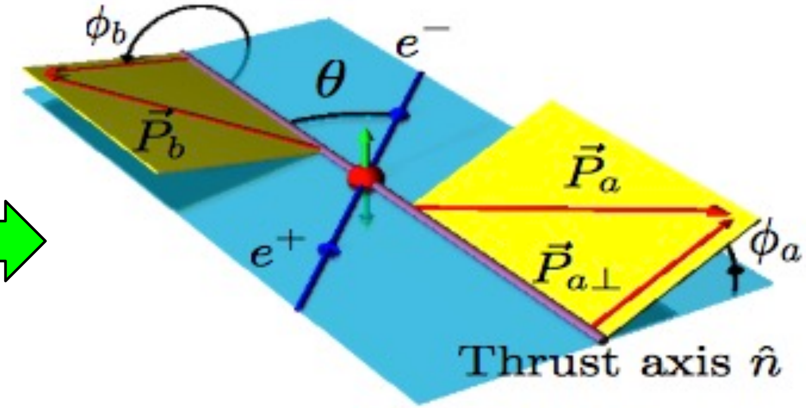
Hadron Pol.	PDF (x, x_1) 	FF (z, z_1) 
U	dynamical H_{FU}	dynamical $\hat{H}_{FU}^{\Re,\Im}$
L	H_{FL}	$\hat{H}_{FL}^{\Re,\Im}$
T	F_{FT}, G_{FT}	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

ALL *transverse spin observables* are driven by *multi-parton correlations*

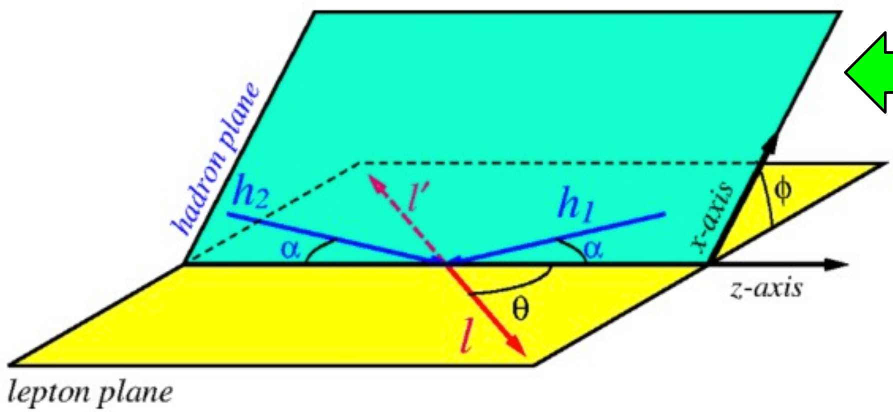
(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))



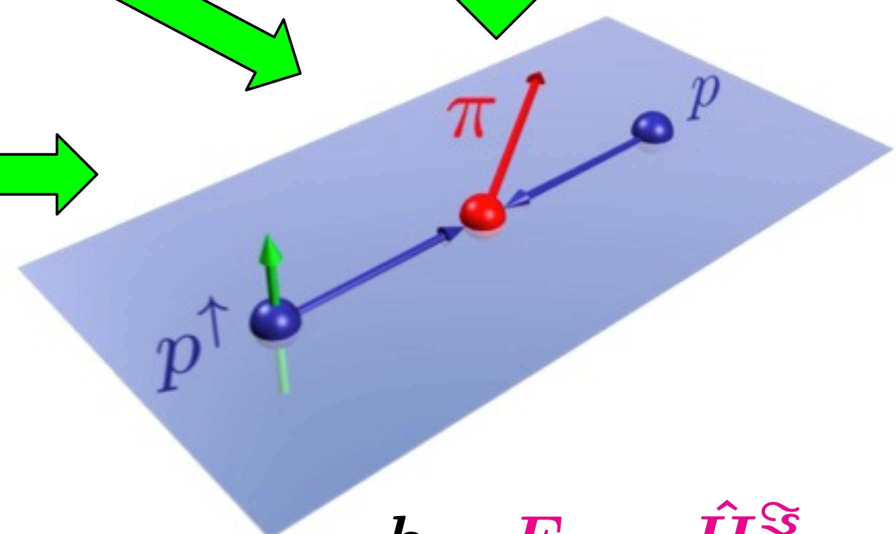
$h_1,\ F_{FT},\ \hat{H}_{FU}^{\Im}$



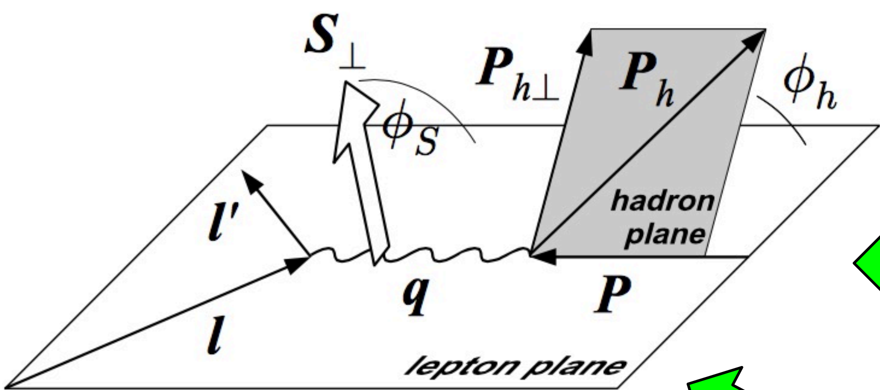
$\hat{H}_{FU}^{\Im}$



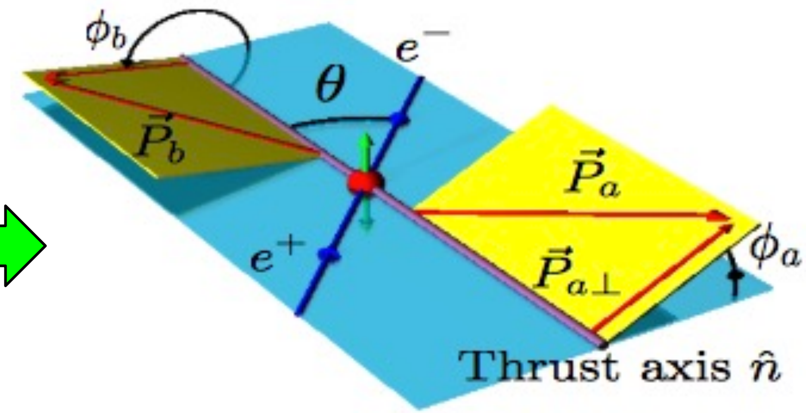
F_{FT}



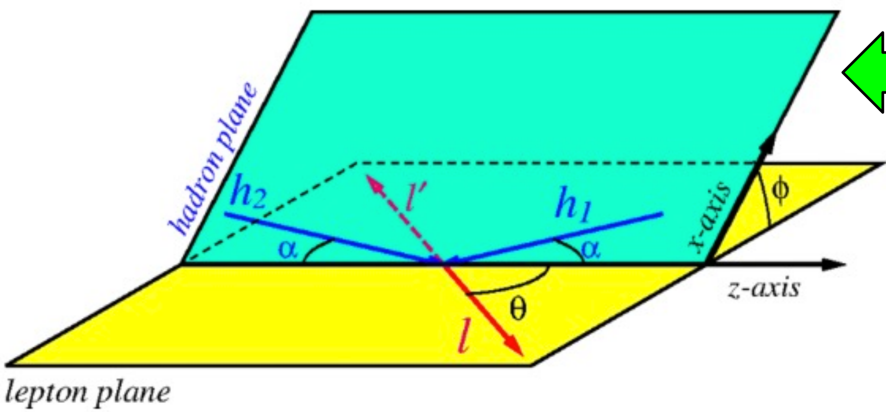
$h_1,\ F_{FT},\ \hat{H}_{FU}^{\Im}$



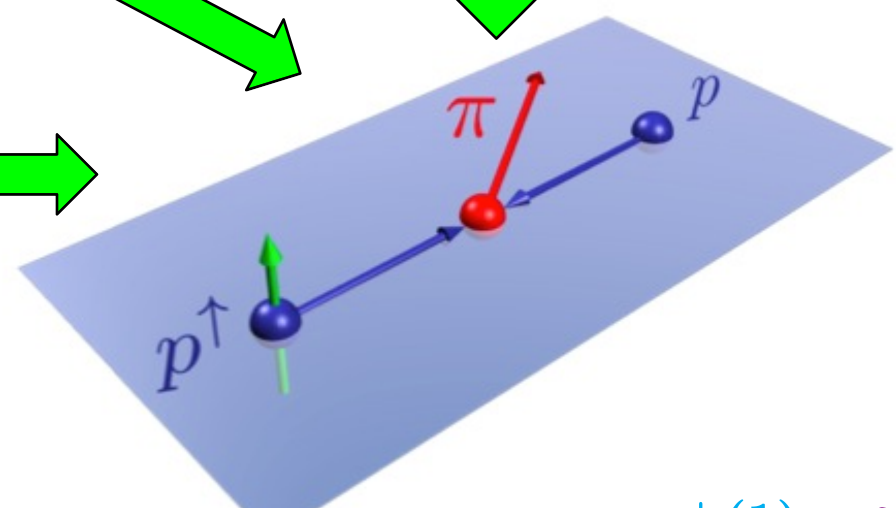
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$H_1^{\perp(1)}$, $\tilde{H}$



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

- What follows are *very preliminary* results of a global fit of
  1) Collins effect in e^+e^-
  2) Collins effect in SIDIS
  3) (Integrated) $A_{UT}^{\sin\phi_s}$ in SIDIS
  4) A_N in proton-proton collisions (fragmentation term)

  *Also will add Sivers and QS term of A_N to the analysis

- Monte Carlo (MC) sampling was used to determine error bands. For now, we use a simple Gaussian ansatz for TMDs.

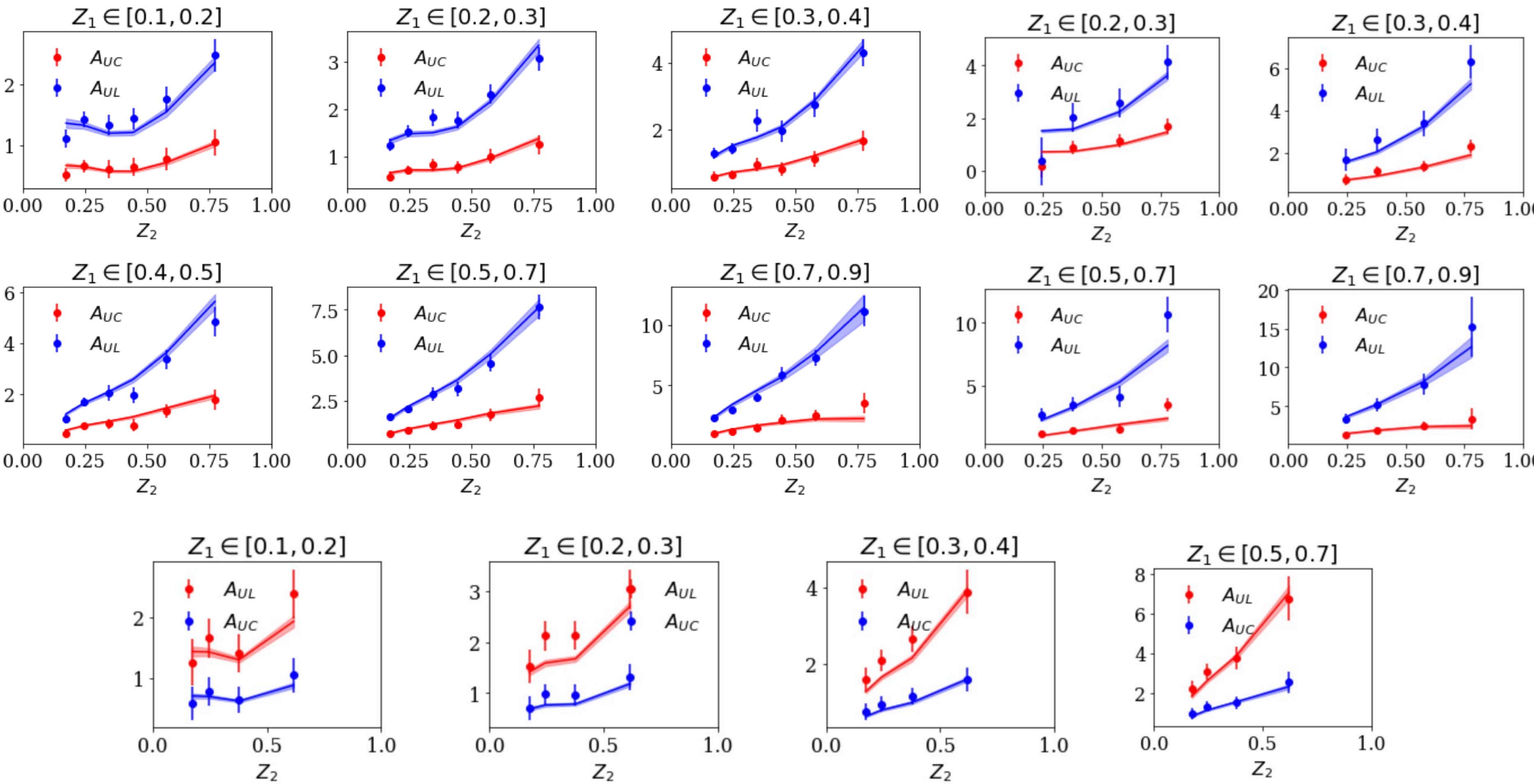
- We have found solutions for the relevant non-perturbative functions (including $\tilde{H}$!) that describe simultaneously a non-trivial amount of observables.

- Large errors in the (transversely polarized) deuteron SIDIS data make flavor separation subject to significant correlations which can only be estimated by MC – an EIC can hopefully deliver more accurate data.

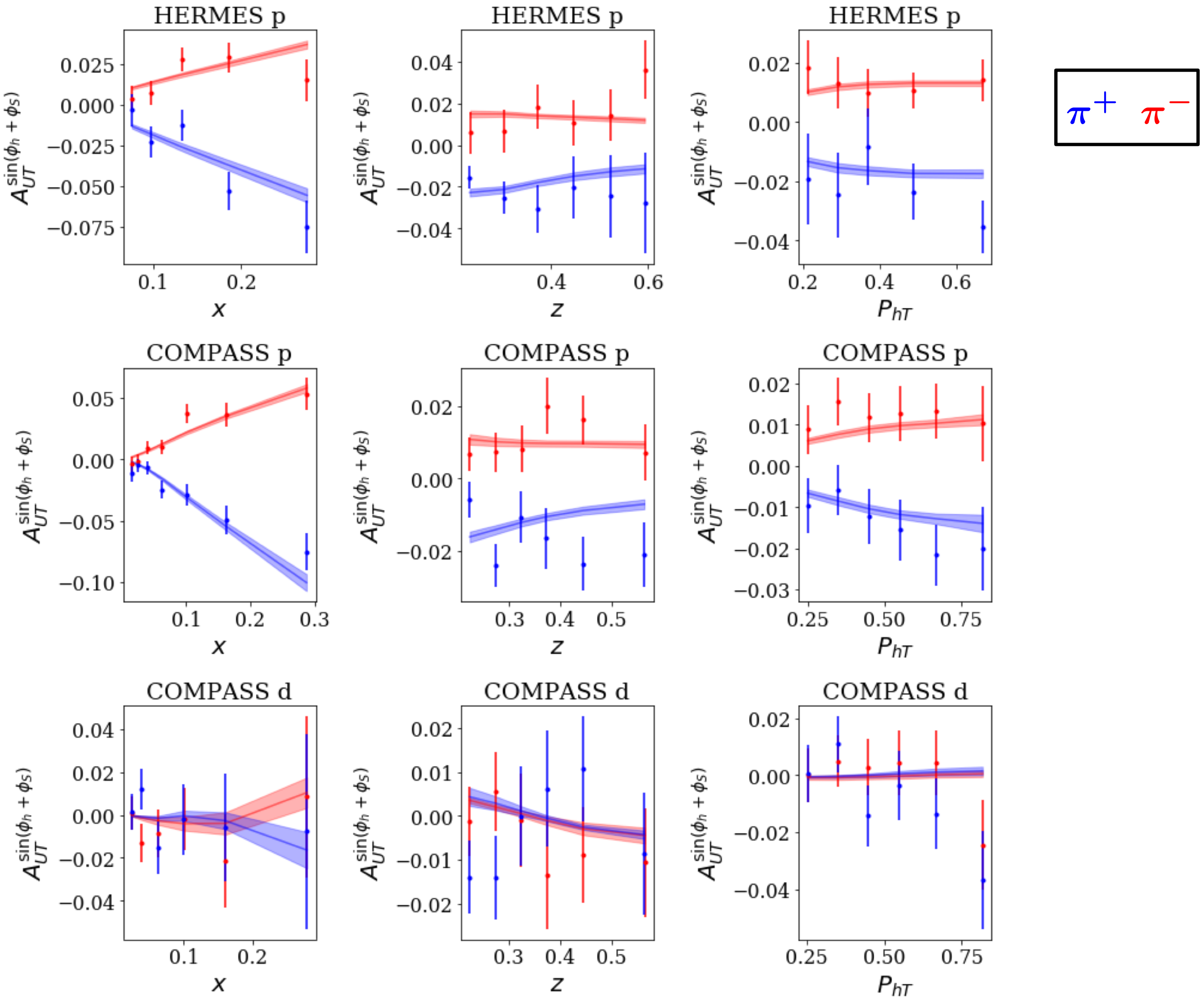
(Gamberg, Kang, DP, Prokudin, Sato, …, on-going work)

Collins effect e^+e^-

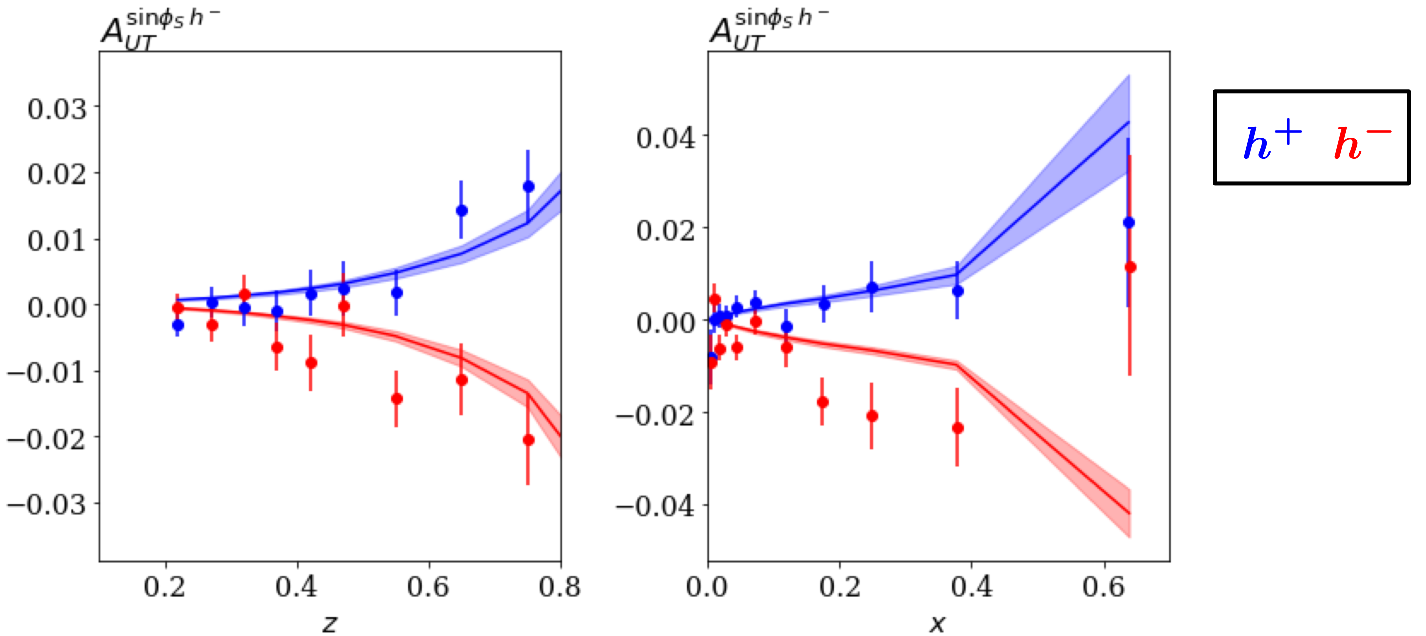
A_{UC} A_{UL}



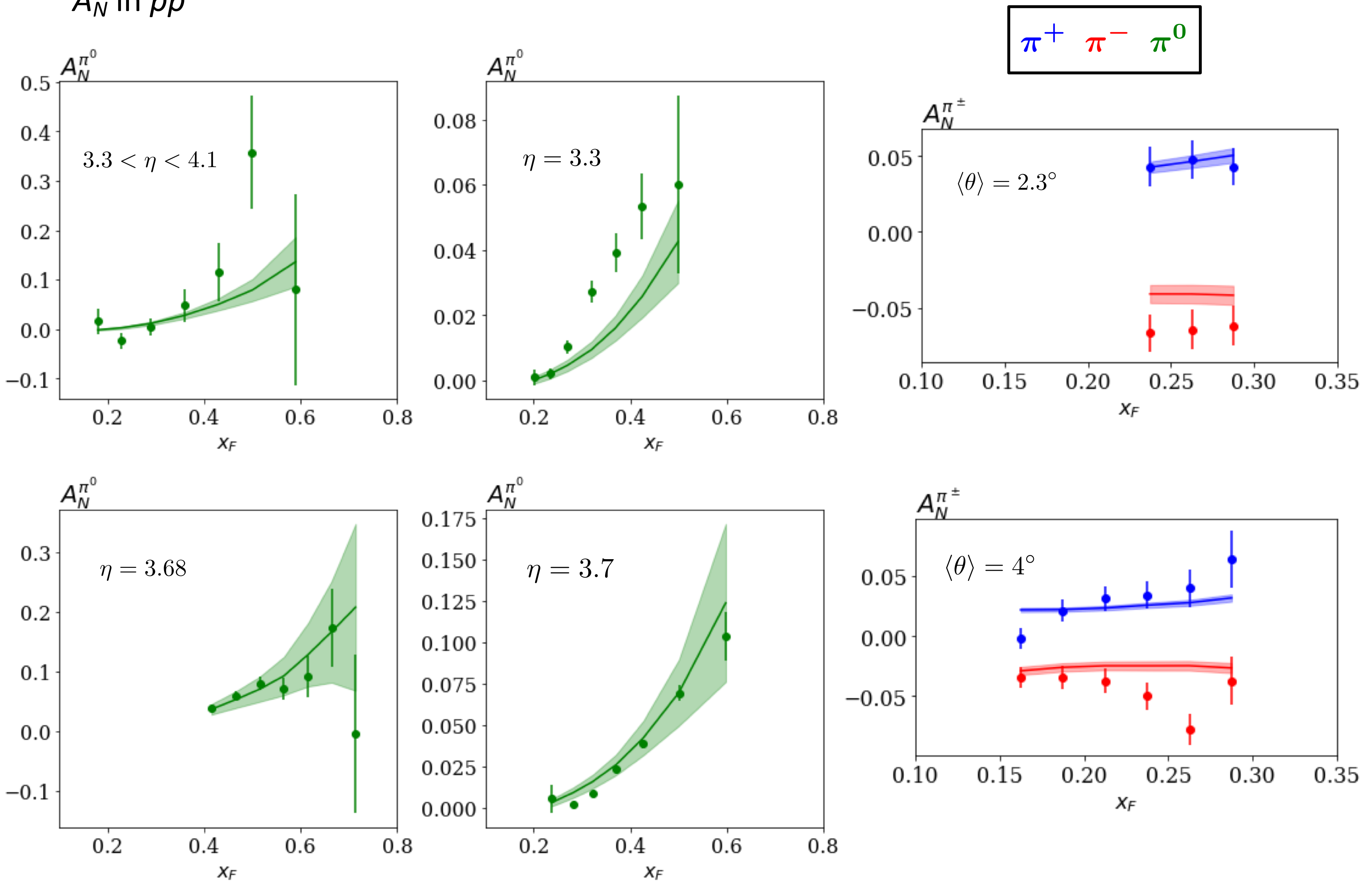
Collins effect SIDIS



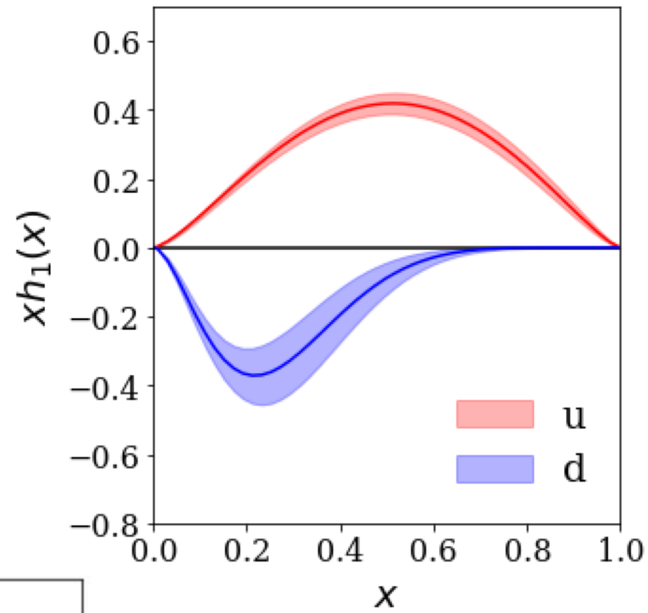
$A_{UT}^{\sin\phi_S}$ in SIDIS



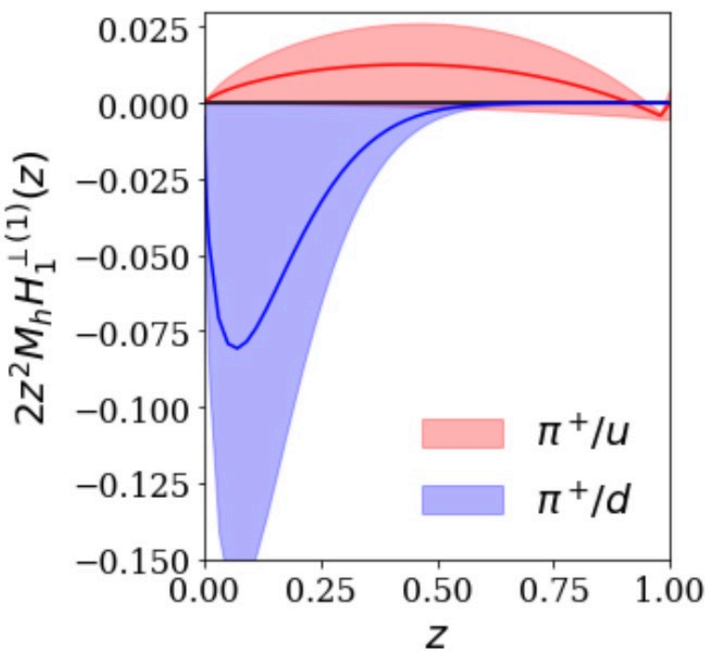
A_N in pp


π+ π− π0
$A_N^{\pi^0}$
$3.3 < \eta < 4.1$
$\eta = 3.3$
$\eta = 3.68$
$\eta = 3.7$
$A_N^{\pi^\pm}$
$\langle\theta\rangle = 2.3°$
$\langle\theta\rangle = 4°$
x_F

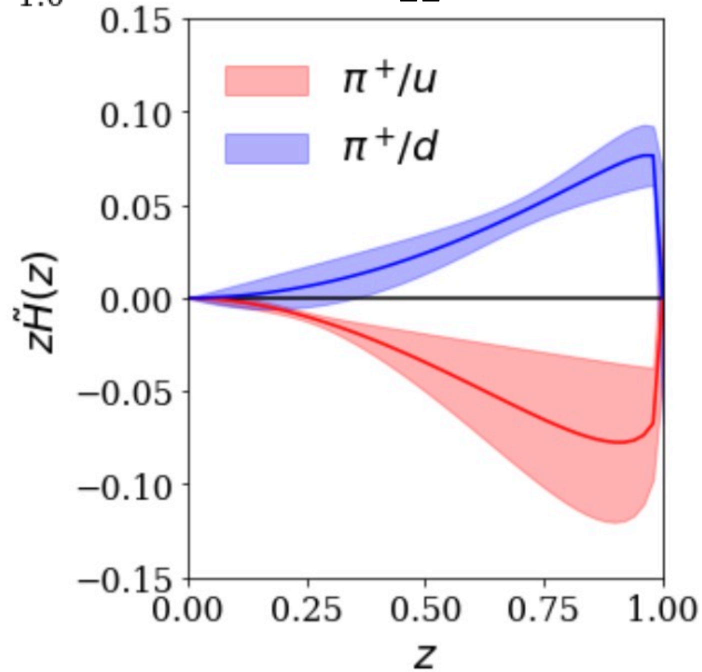
Transversity



Collins



$\tilde{H}$



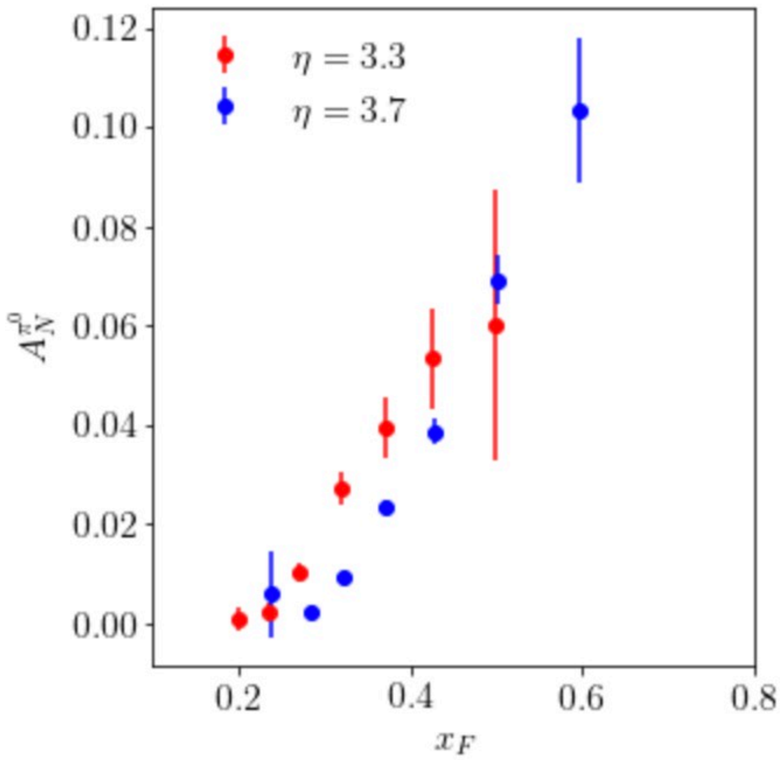
Summary and Outlook

- TMD and collinear functions are highly interconnected, especially for reactions involving transverse spin, and we should treat both types of observables on the same footing.

- A global analysis can be performed of TMD (Sivers and Collins effects) *AND* collinear twist-3 (A_N in *pp,* $A_{UT}^{\sin\phi_s}$ in SIDIS) transverse-spin observables.

- In addition to the Sivers and Collins effects that will be measured at a future EIC (with improved statistics needed for deuterium), we must also include measurements of A_N in electron-nucleon collisions.

Back-up Slides



There is an *increase* in A_N with P_T for P_T< 2 GeV. Need to see if evolution effects can account for this.