Gluon TMDs and Opportunities at EIC

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INT-18-3 Workshop
Transverse Spin and TMDs

October 8 - 12 2018
Seattle (USA)

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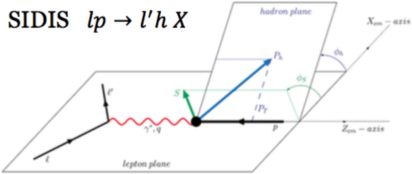
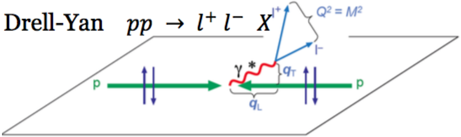
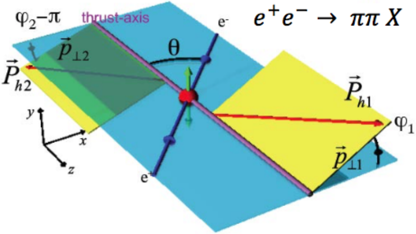
TMD factorization and color gauge invariance

Two scale processes $Q^2 \gg p_T^2$





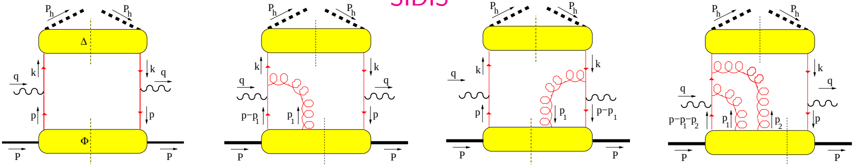


Factorization proven

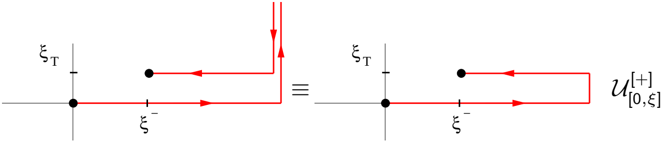
Gauge invariant definition of Φ (not unique)

$$\Phi^{[\mathcal{U}]} \propto \left\langle P,S \left| \overline{\psi}(0)\,\mathcal{U}^{\mathcal{C}}_{[0,\xi]}\,\psi(\xi) \right| P,S \right\rangle$$

SIDIS

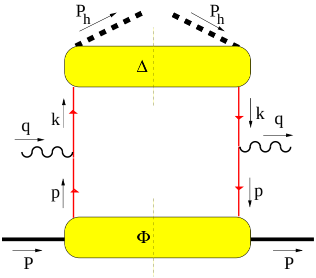


Belitsky, Ji, Yuan, NPB 656 (2003)
Boer, Mulders, Pijlman, NPB 667 (2003)

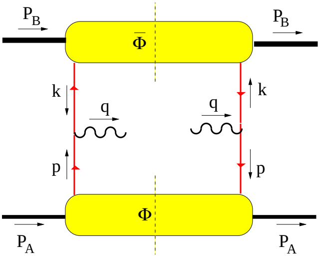


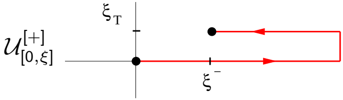
Possible effects in transverse momentum observables (ξ_T is conjugate to k_T)

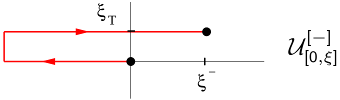
SIDIS



Drell-Yan

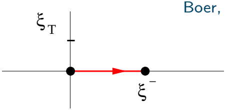






Belitsky, Ji, Yuan, NPB 656 (2003)
Boer, Mulders, Pijlman, NPB 667 (2003)
Boer, talk at RBRC Synergies workshop (2017)

$\int \mathrm{d}k_T \longrightarrow \xi_T = 0 \longrightarrow$

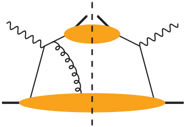


the same in both cases

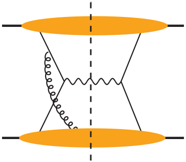
Fundamental test of TMD theory

$$f_{1T}^{\perp\,[DY]}(x, \boldsymbol{k}_\perp^2) = -f_{1T}^{\perp\,[SIDIS]}(x, \boldsymbol{k}_\perp^2) \qquad h_1^{\perp\,[DY]}(x, \boldsymbol{k}_\perp^2) = -h_1^{\perp\,[SIDIS]}(x, \boldsymbol{k}_\perp^2)$$

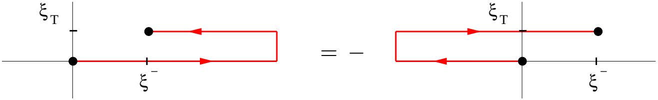
Collins, PLB 536 (2002)



FSI in SIDIS



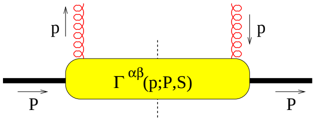
ISI in DY



ISI/FSI lead to process dependence of TMDs, could even break factorization

Collins, Qiu, PRD 75 (2007)
Collins, PRD 77 (2007)
Rogers, Mulders, PRD 81 (2010)

Process dependence of gluon TMDs



Gauge invariant definition of $\Gamma^{\mu\nu}$

$$\Gamma^{[\mathcal{U},\mathcal{U}']\mu\nu} \propto \langle P,S|\,\mathrm{Tr}_c\big[\,F^{+\nu}(0)\,\mathcal{U}^{\mathcal{C}}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}^{\mathcal{C}'}_{[\xi,0]}\,\big]\,|P,S\rangle$$

Mulders, Rodrigues, PRD 63 (2001)
Buffing, Mukherjee, Mulders, PRD 88 (2013)
Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)

The gluon correlator depends on two path-dependent gauge links

$ep \to e'\,Q\overline{Q}X$, $ep \to e'\,\mathrm{jet\,jet}\,X$ probe gluon TMDs with $[++]$ gauge links

$pp \to \gamma\gamma X$ (and/or other CS final state) probes gluon TMDs with $[--]$ gauge links

$pp \to \gamma\,\mathrm{jet}\,X$ probes an entirely independent gluon TMD: $[+-]$ links (dipole)

The gluon Sivers functions
Sign change test

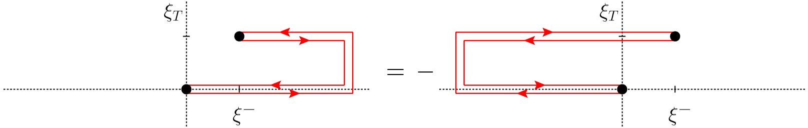
Related Processes

$ep^\uparrow \to e' Q\overline{Q}X$, $ep^\uparrow \to e'\ \mathrm{jet\,jet}\,X$ probe GSF with $[++]$ gauge links (WW)

$p^\uparrow p \to \gamma\gamma X$ (and/or other CS final state) probe GSF with $[--]$ gauge links

Analogue of the sign change of $f_{1T}^{\perp\, q}$ between SIDIS and DY (true also for h_1^g and $h_{1T}^{\perp\, g}$)

$$f_{1T}^{\perp\, g\, [e\, p^\uparrow \to e'\, Q\overline{Q}\, X]} = -f_{1T}^{\perp\, g\, [p^\uparrow\, p \to \gamma\, \gamma\, X]}$$



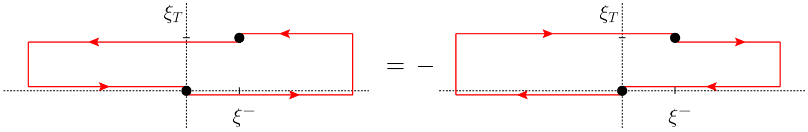
Boer, Mulders, CP, Zhou (2016)

Motivation to study gluon Sivers effects at both RHIC and the EIC

Complementary Processes

$ep^{\uparrow} \to e' Q\overline{Q}X$ probes a GSF with $[++]$ gauge links (WW)

$p^{\uparrow} p \to \gamma\,\mathrm{jet}\,X$ $(gq \to \gamma q)$ probes a gluon TMD with : $[+-]$ links (DP)



At small-x the WW Sivers function appears to be suppressed by a factor of x compared to the unpolarized gluon function, unlike the dipole one

The DP gluon Sivers function at small-x is the **spin dependent odderon** (single spin asymmetries from a single Wilson loop matrix element)

Boer, Echevarria, Mulders, Zhou, PRL 116 (2016)
Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)

Gluon TMDs

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)
Mulders, Rodrigues, PRD 63 (2001)
Meissner, Metz, Goeke, PRD 76 (2007)

- $h_1^{\perp\, g}$: T-even distribution of linearly polarized gluons inside an unp. hadron
- h_{1T}^g, $h_{1T}^{\perp\, g}$: helicity flip distributions like h_{1T}^q, $h_{1T}^{\perp\, q}$, but T-odd, chiral even!
- $h_1^g \equiv h_{1T}^g + \frac{\boldsymbol{p}_T^2}{2M_p^2}\, h_{1T}^{\perp\, g}$ does not survive under p_T integration, unlike transversity

In contrast to quark TMDs, gluon TMDs are almost unknown

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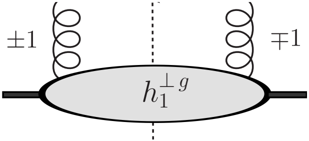
The distribution of linearly polarized gluons inside an unpolarized proton: $h_1^{\perp\, g}$

Gluons inside an unpolarized hadron can be linearly polarized

It requires nonzero transverse momentum



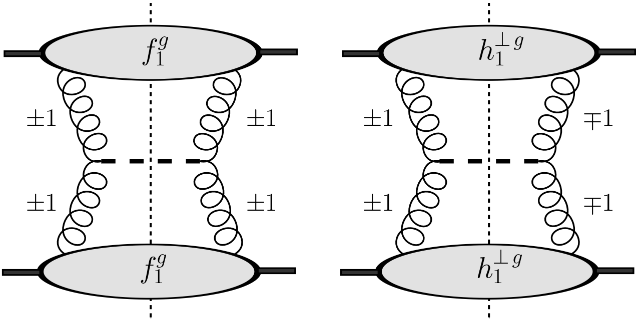
Interference between ±1 gluon helicity states

It does not need ISI/FSI to be nonzero, unlike the Sivers function. However it is affected by them $\Longrightarrow$ process dependence

Higgs boson production happens mainly via $gg \to H$

Pol. gluons affect the Higgs transverse spectrum at NNLO pQCD

Catani, Grazzini, NPB 845 (2011)



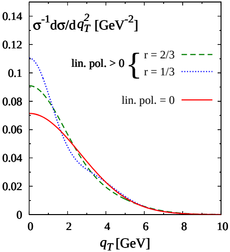
The nonperturbative distribution can be present at tree level and would contribute to Higgs production at low q_T

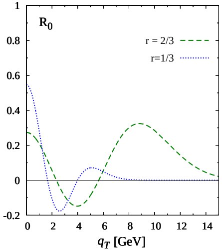
Sun, Xiao, Yuan, PRD 84 (2011)
Boer, den Dunnen, CP, Schlegel, Vogelsang, PRL 108 (2012)

q_T-distribution of the Higgs boson

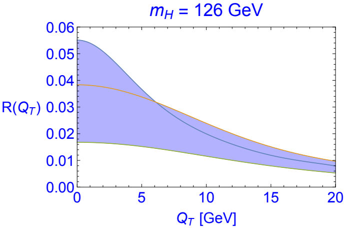
$$\frac{1}{\sigma}\frac{d\sigma}{d\boldsymbol{q}_T^2} \propto 1 + R(\boldsymbol{q}_T^2) \qquad R = \frac{h_1^{\perp\,g} \otimes h_1^{\perp\,g}}{f_1^g \otimes f_1^g} \qquad |h_1^{\perp\,g}(x,\boldsymbol{p}_T^2)| \le \frac{2M_p^2}{\boldsymbol{p}_T^2} f_1^g(x,\boldsymbol{p}_T^2)$$

Gaussian Model





TMD evolution



Echevarria, Kasemets, Mulders, CP, JHEP 1507 (2015) 158

Study of $H \to \gamma\gamma$ and interference with $gg \to \gamma\gamma$

Boer, den Dunnen, CP, Schlegel, PRL 111 (2013)

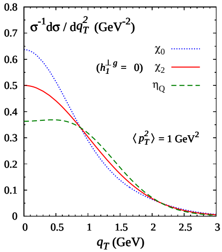
q_T-distribution of η_Q and χ_{QJ} $(Q = c, b)$ in the kinematic region $q_T \ll 2M_Q$

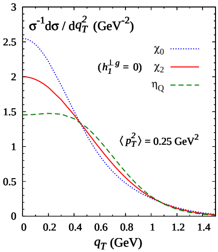
$$\frac{1}{\sigma(\eta_Q)} \frac{d\sigma(\eta_Q)}{d\boldsymbol{q}_T^2} \propto f_1^g \otimes f_1^g \, [1 - R(\boldsymbol{q}_T^2)] \qquad \text{[pseudoscalar]}$$

$$\frac{1}{\sigma(\chi_{Q0})} \frac{d\sigma(\chi_{Q0})}{d\boldsymbol{q}_T^2} \propto f_1^g \otimes f_1^g \, [1 + R(\boldsymbol{q}_T^2)] \qquad \text{[scalar]}$$

$$\frac{1}{\sigma(\chi_{Q2})} \frac{d\sigma(\chi_{Q2})}{d\boldsymbol{q}_T^2} \propto f_1^g \otimes f_1^g$$

Boer, CP, PRD 86 (2012) 094007





Proof of factorization at NLO for $p\,p \to \eta_Q\,X$ in the Color Singlet Model (CSM)

Ma, Wang, Zhao, PRD 88 (2013), 014027; PLB 737 (2014) 103

J/ψ-pair production at the LHC

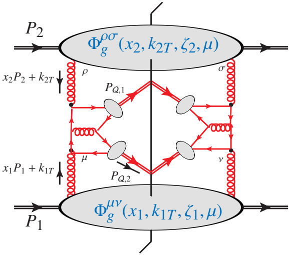
J/ψ-pair production at the LHC

J/ψ's are relatively easy to detect. Accessible at the LHC: already studied by LHCb, CMS & ATLAS

LHCb PLB 707 (2012)
CMS JHEP 1409 (2014)
ATLAS EPJC 77 (2017)

gg fusion dominant, negligible $q\bar{q}$ contributions even at AFTER@LHC energies

Lansberg, Shao, NPB 900 (2015)



No final state gluon needed for the Born contribution in the Color Singlet Model. Pure colorless final state, hence simple color structure because one has only ISI

Lansberg, Shao, PRL 111 (2013)

Negligible Color Octet contributions, in particular at low $P_T^{\Psi\Psi}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y\,\mathrm{d}^2 q_T\,\mathrm{d}\Omega} \approx A\, f_1^g \otimes f_1^g + B\, f_1^g \otimes h_1^{\perp g} \cos(2\phi_{CS}) + C\, h_1^{\perp g} \otimes h_1^{\perp g} \cos(4\phi_{CS})$$

Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)

- valid up to corrections $\mathcal{O}(q_T/Q)$
- Y: rapidity of the J/ψ-pair, along the beam in the hadronic c.m. frame
- $\mathrm{d}\Omega = \mathrm{d}\cos\theta_{CS}\,\mathrm{d}\phi_{CS}$: solid angle for J/ψ-pair in the Collins-Soper frame

Analysis similar to the one for $pp \to \gamma\gamma X$, $pp \to J\psi\, \gamma^{(*)} X$, $pp \to H\,\mathrm{jet}\, X$

Qiu, Schlegel, Vogelsang, PRL 107 (2011)
den Dunnen, Lansberg, CP, Schlegel, PRL 112 (2014)
Lansberg, CP, Schlegel, NPB 920 (2017)
Boer, CP, PRD 91 (2015)

The three contributions can be disentangled by defining the transverse moments

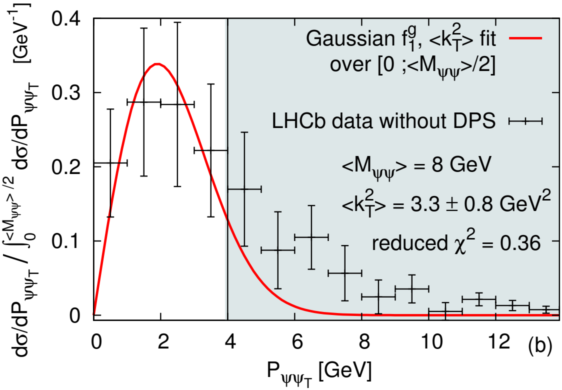
$$\langle \cos n\phi_{CS} \rangle \equiv \frac{\int_0^{2\pi} \mathrm{d}\phi_{CS}\, \cos(n\,\phi_{CS})\, \frac{\mathrm{d}\sigma}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}^2 q_T \mathrm{d}\Omega}}{\int_0^{2\pi} \mathrm{d}\phi_{CS}\, \frac{\mathrm{d}\sigma}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}^2 q_T \mathrm{d}\Omega}} \qquad (n = 2, 4)$$

$$\int \mathrm{d}\phi_{CS}\, \mathrm{d}\sigma \Longrightarrow f_1^g \otimes f_1^g$$

$$\langle \cos 2\phi_{CS} \rangle \Longrightarrow f_1^g \otimes h_1^{\perp g}$$

$$\langle \cos 4\phi_{CS} \rangle \Longrightarrow h_1^{\perp g} \otimes h_1^{\perp g}$$

We consider $q_T = P_T^{\Psi\Psi} \leq M_{\Psi\Psi}/2$ in order to have two different scales



Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)
LHCb Coll., JHEP 06 (2017)

Gaussian model:

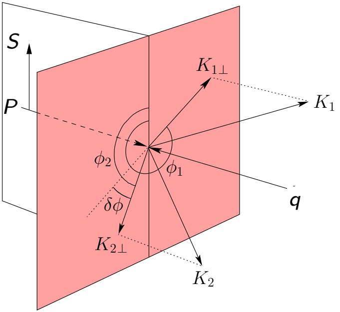
$$f_1^g(x, \boldsymbol{k}_T^2) = \frac{f_1^g(x)}{\pi \langle k_T^2 \rangle} \exp\left(-\frac{\boldsymbol{k}_T^2}{\langle k_T^2 \rangle}\right)$$

Heavy quark pair production at an EIC

Gluon TMDs probed directly in $e(\ell) + p(P,S) \to e(\ell') + Q(K_1) + \overline{Q}(K_2) + X$

Boer, Mulders, CP, Zhou, JHEP 1608 (2016)

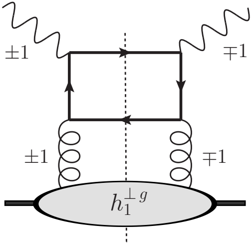
- the $Q\overline{Q}$ pair is almost back to back in the plane $\perp$ to q and P
- $q \equiv \ell - \ell'$: four-momentum of the exchanged virtual photon γ^*



$$\boldsymbol{q}_T \equiv \boldsymbol{K}_{1\perp} + \boldsymbol{K}_{2\perp}$$

$$\boldsymbol{K}_\perp \equiv (\boldsymbol{K}_{1\perp} - \boldsymbol{K}_{2\perp})/2$$

$\Longrightarrow$ Correlation limit: $|\boldsymbol{q}_T| \ll |\boldsymbol{K}_\perp|$, $\quad |\boldsymbol{K}_\perp| \approx |\boldsymbol{K}_{1\perp}| \approx |\boldsymbol{K}_{2\perp}|$



$\phi_T, \phi_\perp, \phi_S$ azimuthal angles of $q_T, K_\perp, S_T$

At LO in pQCD: only $\gamma^* g \to Q\overline{Q}$ contributes

$$\mathrm{d}\sigma(\phi_S, \phi_T, \phi_\perp) = \mathrm{d}\sigma^U(\phi_T, \phi_\perp) + \mathrm{d}\sigma^T(\phi_S, \phi_T, \phi_\perp)$$

Angular structure of the unpolarized cross section for $ep \to e' Q\overline{Q}X$, $\quad |\boldsymbol{q}_T| \ll |\boldsymbol{K}_\perp|$

$$\frac{\mathrm{d}\sigma^U}{\mathrm{d}^2\boldsymbol{q}_T\,\mathrm{d}^2\boldsymbol{K}_\perp} \propto \left\{ A_0^U + A_1^U \cos\phi_\perp + A_2^U \cos 2\phi_\perp \right\} f_1^g(x, \boldsymbol{q}_T^2) + \frac{\boldsymbol{q}_T^2}{M_p^2}\, h_1^{\perp\, g}(x, \boldsymbol{q}_T^2)$$
$$\times \left\{ B_0^U \cos 2\phi_T + B_1^U \cos(2\phi_T - \phi_\perp) + B_2^U \cos 2(\phi_T - \phi_\perp) + B_3^U \cos(2\phi_T - 3\phi_\perp) + B_4^U \cos 2(\phi_T - 2\phi_\perp) \right\}$$

The different contributions can be isolated by defining

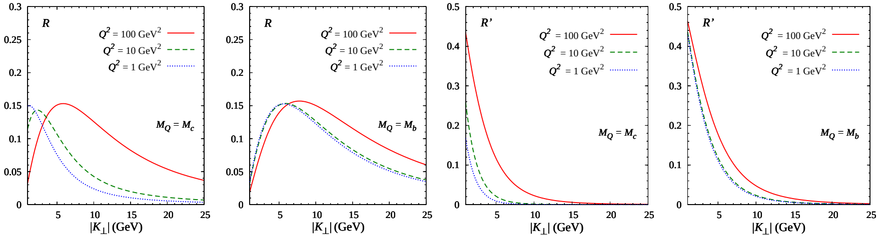
$$\langle W(\phi_\perp, \phi_T)\rangle = \frac{\int \mathrm{d}\phi_\perp \mathrm{d}\phi_T\, W(\phi_\perp, \phi_T)\, \mathrm{d}\sigma}{\int \mathrm{d}\phi_\perp \mathrm{d}\phi_T\, \mathrm{d}\sigma}, \quad W = \cos 2\phi_T,\ \cos 2(\phi_\perp - \phi_T), \ldots$$

Positivity bound for $h_1^{\perp g}$: $\quad |h_1^{\perp g}(x, \boldsymbol{p}_T^2)| \leq \frac{2M_p^2}{\boldsymbol{p}_T^2} f_1^g(x, \boldsymbol{p}_T^2)$

It can be used to estimate maximal values of the asymmetries

Asymmetries usually larger when Q and $\overline{Q}$ have same rapidities

Upper bounds on $R \equiv |\langle \cos 2(\phi_T - \phi_\perp)\rangle|$ and $R' \equiv |\langle \cos 2\phi_T \rangle|$ at $y = 0.01$



CP, Boer, Brodsky, Buffing, Mulders, JHEP 1310 (2013)
Boer, Brodsky, Mulders, CP, PRL 106 (2011)

Spin asymmetries in $ep^\uparrow \to e' Q\overline{Q} X$

Angular structure of the single polarized cross section for $ep^\uparrow \to e' Q\overline{Q}X$, $|\boldsymbol{q}_T| \ll |\boldsymbol{K}_\perp|$

$$
\begin{aligned}
\mathrm{d}\sigma^T \propto\ & \sin(\phi_S - \phi_T)\Big[A_0^T + A_1^T \cos\phi_\perp + A_2^T \cos 2\phi_\perp\Big] f_{1T}^{\perp\, g} + \cos(\phi_S - \phi_T)\Big[B_0^T \sin 2\phi_T \\
& + B_1^T \sin(2\phi_T - \phi_\perp) + B_2^T \sin 2(\phi_T - \phi_\perp) + B_3^T \sin(2\phi_T - 3\phi_\perp) + B_4^T \sin(2\phi_T - 4\phi_\perp)\Big] h_{1T}^{\perp\, g} \\
& + \Big[B_0'^{\,T} \sin(\phi_S + \phi_T) + B_1'^{\,T} \sin(\phi_S + \phi_T - \phi_\perp) + B_2'^{\,T} \sin(\phi_S + \phi_T - 2\phi_\perp) \\
& + B_3'^{\,T} \sin(\phi_S + \phi_T - 3\phi_\perp) + B_4'^{\,T} \sin(\phi_S + \phi_T - 4\phi_\perp)\Big] h_{1T}^{g}
\end{aligned}
$$

Boer, Mulders, CP, Zhou, JHEP 1608 (2016)

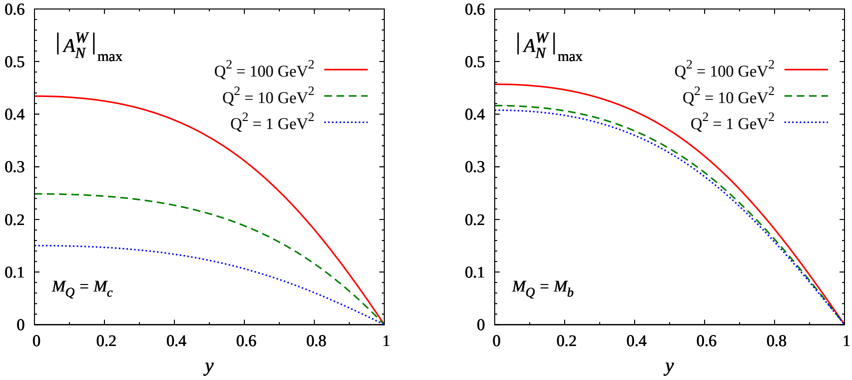
The ϕ_S dependent terms can be singled out by means of azimuthal moments A_N^W

$$
A_N^{W(\phi_S,\phi_T)} \equiv 2\,\frac{\int \mathrm{d}\phi_T\, \mathrm{d}\phi_\perp\, W(\phi_S,\phi_T)\, \mathrm{d}\sigma_T(\phi_S,\, \phi_T,\, \phi_\perp)}{\int \mathrm{d}\phi_T\, \mathrm{d}\phi_\perp\, \mathrm{d}\sigma_U(\phi_T,\phi_\perp)}
$$

$$
A_N^{\sin(\phi_S-\phi_T)} \propto \frac{f_{1T}^{\perp\, g}}{f_1^g} \qquad A_N^{\sin(\phi_S+\phi_T)} \propto \frac{h_1^g}{f_1^g} \qquad A_N^{\sin(\phi_S-3\phi_T)} \propto \frac{h_{1T}^{\perp\, g}}{f_1^g}
$$

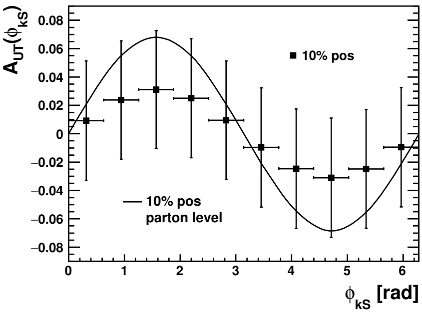
Same modulations as in SIDIS for quark TMDs ($\phi_T \to \phi_h$)

Maximal values for $|A_N^W|$, $W = \sin(\phi_S + \phi_T)$, $\sin(\phi_S - 3\phi_T)$ $\quad (|\boldsymbol{K}_\perp| = 1$ GeV$)$



The Sivers asymmetry in open heavy quark production is bounded by 1

$$A^{\sin(\phi_S-\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p} \frac{f_{1T}^{\perp g}(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)}$$



$\mathcal{L}_{\rm int} = 10 \text{ fb}^{-1}$

Zheng, Aschenauer, Lee, Xiao, Yin, PRD 98 (2018)

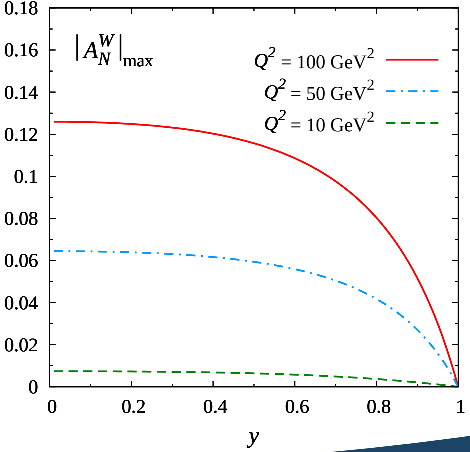
If $f_{1T}^{\perp g}$ is 10% of the positivity bound, then the measurement of $A_N^{\sin(\phi_S-\phi_T)}$ is problematic because of statistics

The situation for dijets is more promising, but theoretically less clean

Contribution to the denominator also from $\gamma^* q \to gq$, negligible at small-x

Asymmetries much smaller than in $c\bar{c}$ case for $Q^2 \leq 10$ GeV^2

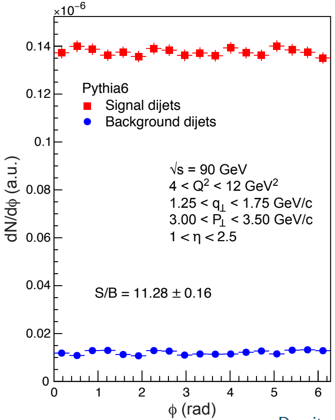
Upper bounds for A_N^W for $K_\perp \geq 4$ GeV



Also in a $e\,A$ collisions polarization shows itself through a $\cos 2\phi$ distribution

Dumitru, Lappi, Skokov, PRL 115 (2015)

$\langle \cos 2\phi \rangle$ has opposite signs for L and T γ^*-polarization, large effects



Dumitru, Skokov, Ullrich, arXiv:1809.02615

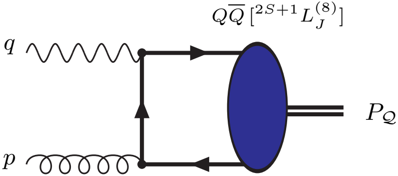
Monte-Carlo Generator: measurement feasible at the EIC

Quarkonium production at the EIC

Bacchetta, Boer, CP, Taels, arXiv:1809.02056

$e\, p^{\uparrow} \to e'\, \mathcal{Q}\, X$ with $\mathcal{Q}$ either a J/ψ or a Υ meson, with $P^2_{\mathcal{Q}T} \ll M^2_{\mathcal{Q}} \sim Q^2$

Color octect (CO) production dominates

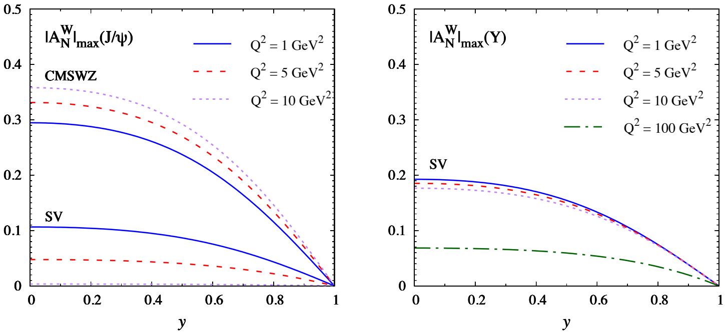


Theoretically described by Color Evaporation Model or NRQCD

Godbole, Misra, Mukherjee, Rawoot, PRD 85 (2012)
Godbole, Kaushik, Misra, Rawoot, PRD 91 (2015)
Mukherjee, Rajesh, EPJC 77 (2017)
Rajesh, Kishore, Mukherjee, PRD 98 (2018)

Results depend on the quite uncertain CO $^1S_0$ and ^3P_J LDMEs

Upper bounds for $\langle \cos 2\phi_T \rangle$ and A_N^W with $W = \sin(\phi_S + \phi_T)$, $\sin(\phi_S - 3\phi_T)$



$\langle \cos 2\phi_T \rangle$ still sizeable at small x, using the MV model as a starting input for $h_1^{\perp g}$ and f_1^g at $x = 10^{-2}$ and evolve them with the JIMWLK equations

Bacchetta, Boer, CP, Taels, arXiv:1809.02056
Marquet, Rosnel, Taels, PRD 97 (2018)

The Sivers asymmetry does not depend on the CO NRQCD LDMEs

$$A^{\sin(\phi_S-\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p}\,\frac{f_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

The other asymmetries depend on them, but one can consider ratios of asymmetries to cancel them out

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S+\phi_T)}} = \frac{\boldsymbol{q}_T^2}{M_p^2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_1^g(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S-3\phi_T)}} = -\frac{1}{2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\sin(\phi_S-3\phi_T)}}{A^{\sin(\phi_S+\phi_T)}} = -\frac{\boldsymbol{q}_T^2}{2M_p^2}\,\frac{h_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_1^g(x,\boldsymbol{q}_T^2)}$$

Same relations hold for $e\,p \to e'\,Q\,\bar{Q}\,X$

One can consider ratios where the TMDs cancel out and one can obtain new experimental information on the CO NRQCD LDMEs

It requires comparison with $e\, p^{\uparrow} \to e'\, Q\, \overline{Q}\, X$

$$\mathcal{R}^{\cos 2\phi} = \frac{\int \mathrm{d}\phi_T \cos 2\phi_T \, \mathrm{d}\sigma^{\mathcal{Q}}(\phi_S, \phi_T)}{\int \mathrm{d}\phi_T \, \mathrm{d}\phi_\perp \cos 2\phi_T \, \mathrm{d}\sigma^{Q\overline{Q}}(\phi_S, \phi_T, \phi_\perp)}$$

$$\mathcal{R} = \frac{\int \mathrm{d}\phi_T \, \mathrm{d}\sigma^{\mathcal{Q}}(\phi_S, \phi_T)}{\int \mathrm{d}\phi_T \, \mathrm{d}\phi_\perp \, \mathrm{d}\sigma^{Q\overline{Q}}(\phi_S, \phi_T, \phi_\perp)}$$

Two observables depending on two unknowns: $\mathcal{O}_8^S \equiv \langle 0|\mathcal{O}_8^{\mathcal{Q}}({}^1S_0)|0\rangle$, $\mathcal{O}_8^P \equiv \langle 0|\mathcal{O}_8^{\mathcal{Q}}({}^3P_0)|0\rangle$

$$\mathcal{R}^{\cos 2\phi} = \frac{27\pi^2}{4} \frac{1}{M_Q} \left[\mathcal{O}_8^S - \frac{1}{M_Q^2} \mathcal{O}_8^P \right]$$

$$\mathcal{R} = \frac{27\,\pi^2}{4} \frac{1}{M_Q} \frac{[1 + (1-y)^2]\, \mathcal{O}_8^S + (10 - 10y + 3y^2)\, \mathcal{O}_8^P / M_Q^2}{26 - 26y + 9y^2}$$

Plus similar (but different) equations for polarized quarkonium production

Conclusions

- Azimuthal asymmetries in heavy quark pair and dijet production in DIS could probe WW-type gluon TMDs (similar to SIDIS for quark TMDs)
- Quarkonia are also good probes for gluon TMDs: first extraction of unpolarized gluon TMD from LHC data on di-J/Ψ production
- Asymmetries maximally allowed by positivity bounds of gluon TMDs can be sizeable in specific kinematic region
- Different behavior of WW and dipole gluon TMDs accessible at RHIC, AFTER@LHC and at EIC, overlap of both *spin* and *small-x* programs