

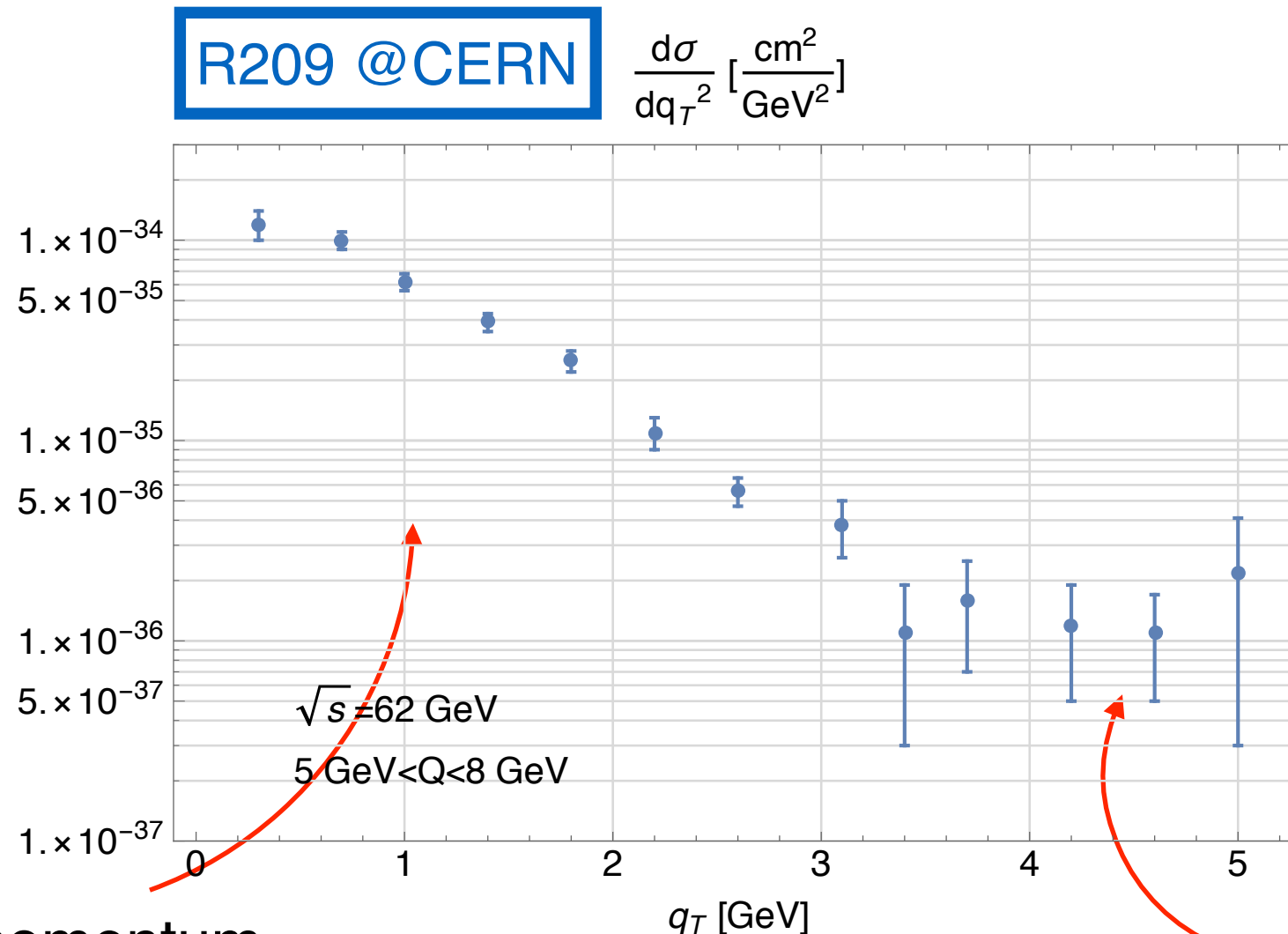
Drell-Yan at low energy and high transverse momentum

Fulvio Piacenza

in collaboration with A.Bacchetta, G.Bozzi, W.Vogelsang, M.Lambertsen



Transverse momentum of Drell-Yan pairs



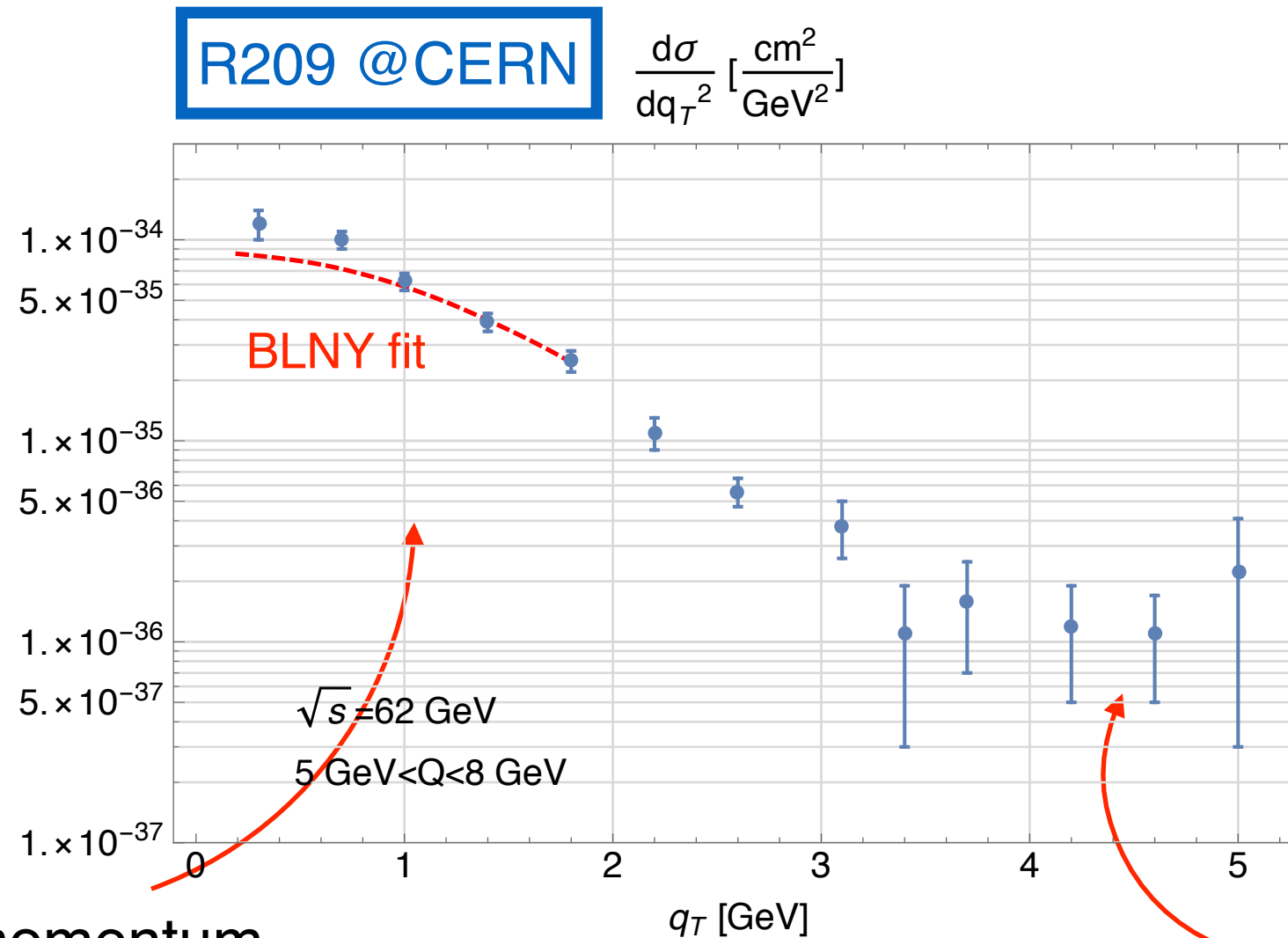
Transverse momentum
resummation

$$(q_T\text{-logs}) \quad \alpha_s^n \ln^m \frac{q_T^2}{Q^2}$$

+ non perturbative
effects

Fixed order
collinear factorization

Transverse momentum of Drell-Yan pairs



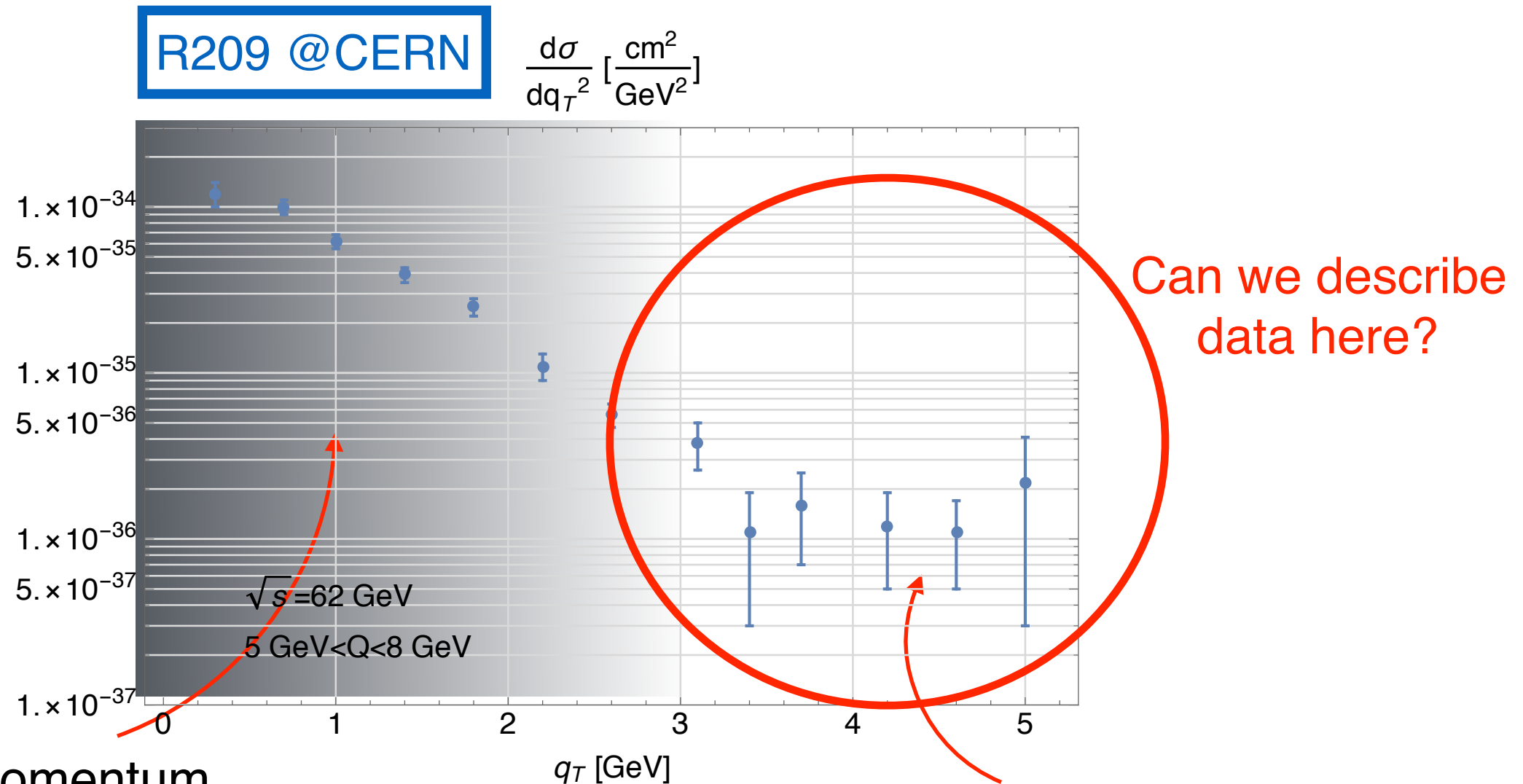
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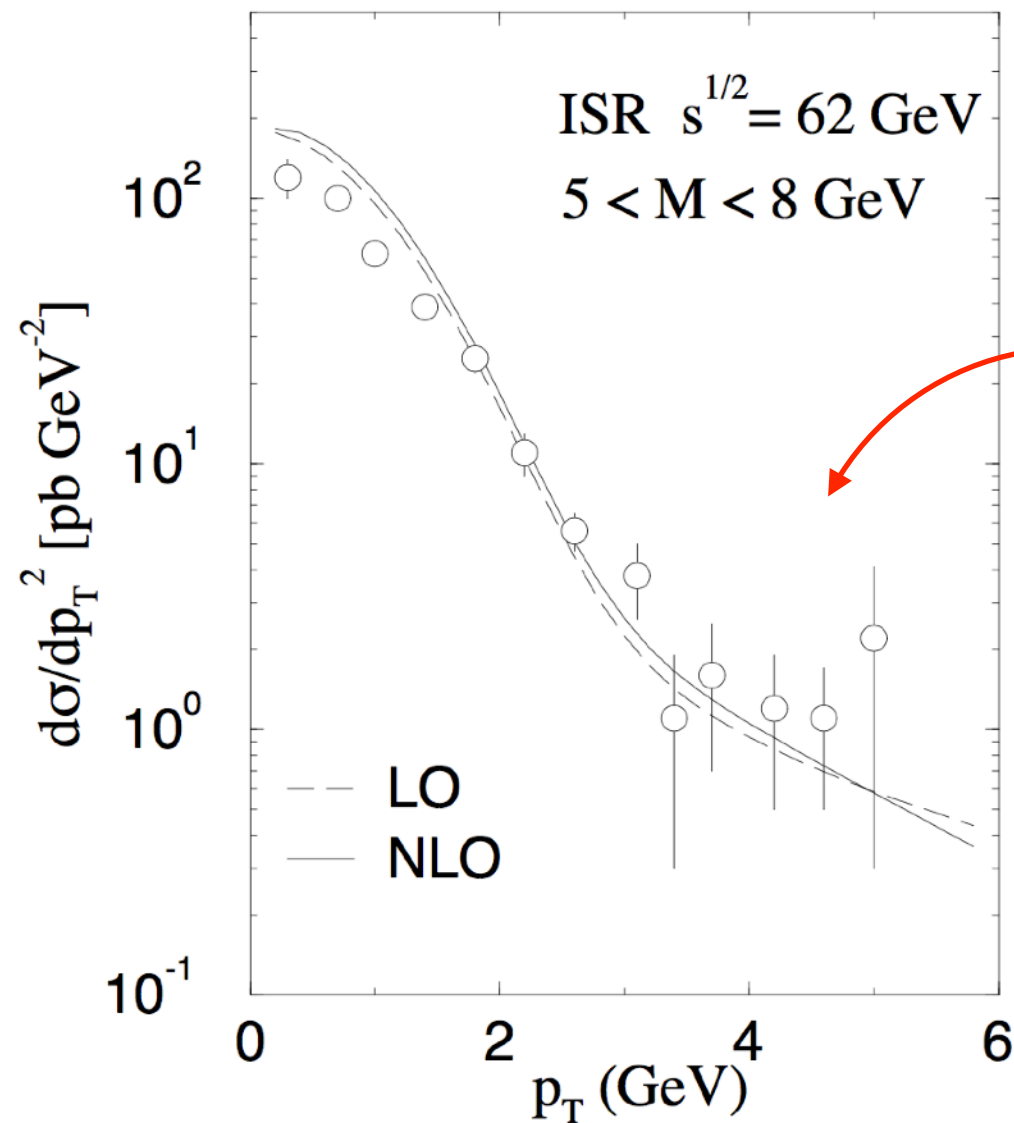
Transverse momentum
resummation

(q_T -logs) $\alpha_s^n \ln^m \frac{q_T^2}{Q^2}$

+ non perturbative
effects

Matching resummation and fixed order

$$\frac{d\sigma}{dq_T^2}(\text{matched}) = \underbrace{\frac{d\sigma}{dq_T^2}(\text{resum})_{NLL}}_W - \underbrace{\frac{d\sigma}{dq_T^2}(\text{expanded})}_{\mathcal{O}(\alpha_s)} + \frac{d\sigma}{dq_T^2}(\text{LO})_Y$$



W + Y

Gavin et al
 I.J.Mod.Phys. A10 (1995)

Arnold & Kauffman's rule

$$\frac{d\sigma}{dq_T^2}(\text{matched}) = \underbrace{\frac{d\sigma}{dq_T^2}(\text{resum})_{NLL}}_W - \underbrace{\frac{d\sigma}{dq_T^2}(\text{expanded})}_{\mathcal{O}(\alpha_s)} + \frac{d\sigma}{dq_T^2}(\text{LO})$$

When $W < 0$ switch to fixed order!

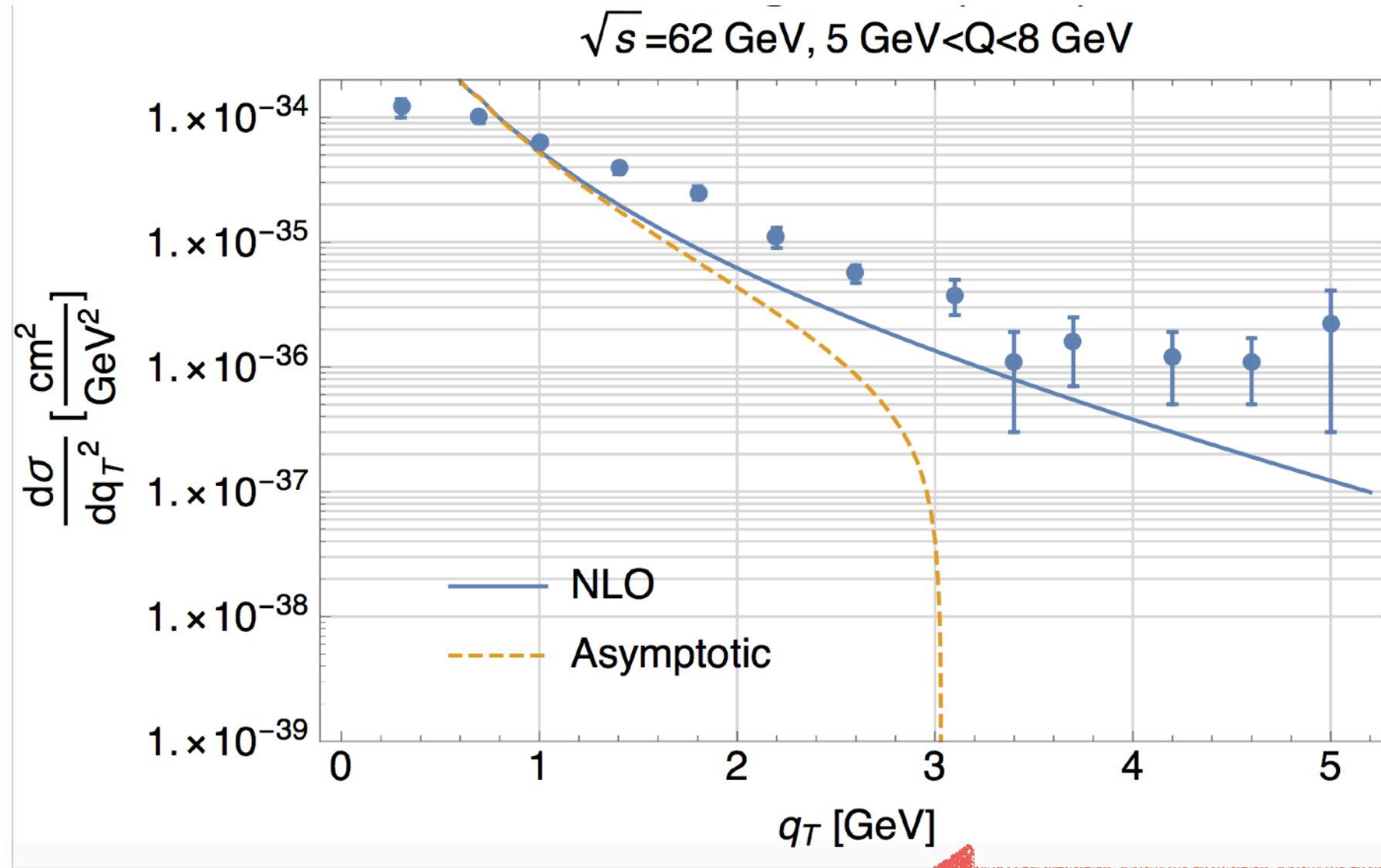
(usually happens at $q_T = Q/2$)

Arnold & Kauffman NP B349 381

Collins et al PRD94 034014

Transverse momentum of Drell-Yan pairs

NLO collinear factorization: $\mathcal{O}(\alpha_s^2)$



Asymptotic =
expansion of resummed result
at fixed order

here collinear factorization
should be reliable

low energy DY data

Experiment	Reaction	Year	TMD fits	PDF fits
R209	p-p	1981	✓	✗
E288	p-Cu(Pt)	1981	✓	✗
E605	p-Cu	1991	✓	✓

$$20 \text{ GeV} \lesssim \sqrt{s} \lesssim 60 \text{ GeV}$$

low energy DY data

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E605	p-Cu	1991	✓	✓
E866	p-p(d)	2003	✗	✓
E615	π -W	1989	✗	✓

$$20 \text{ GeV} \lesssim \sqrt{s} \lesssim 60 \text{ GeV}$$

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E615	π -W	1989	✗	✓
E772	p-d	1994		
E537	pbar-W	1988		

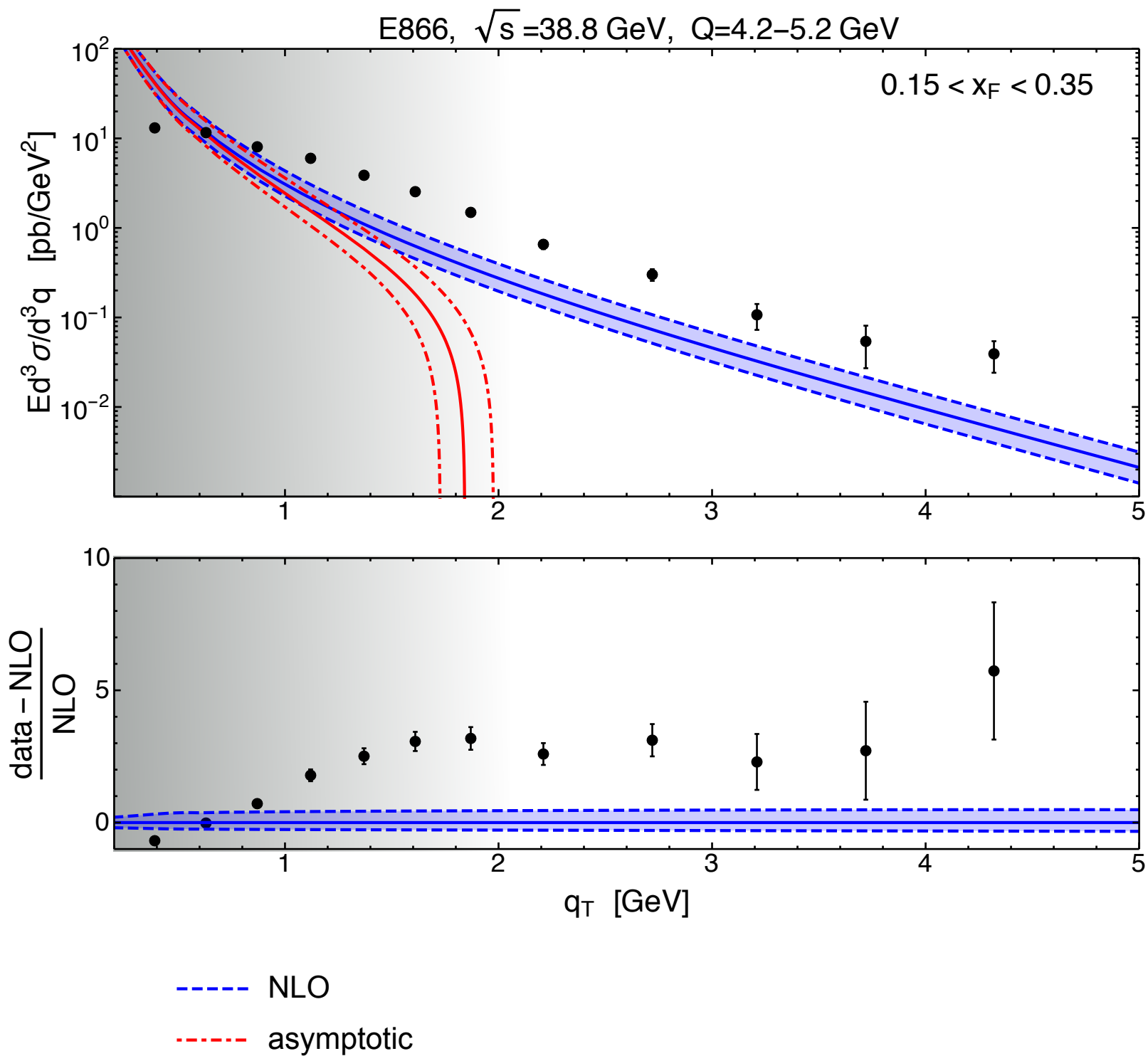


$$20 \text{ GeV} \lesssim \sqrt{s} \lesssim 60 \text{ GeV}$$

E866/NuSea

$$pp \rightarrow \mu^+ \mu^- X$$

$$\sqrt{s} = 38.8 \text{ GeV}$$

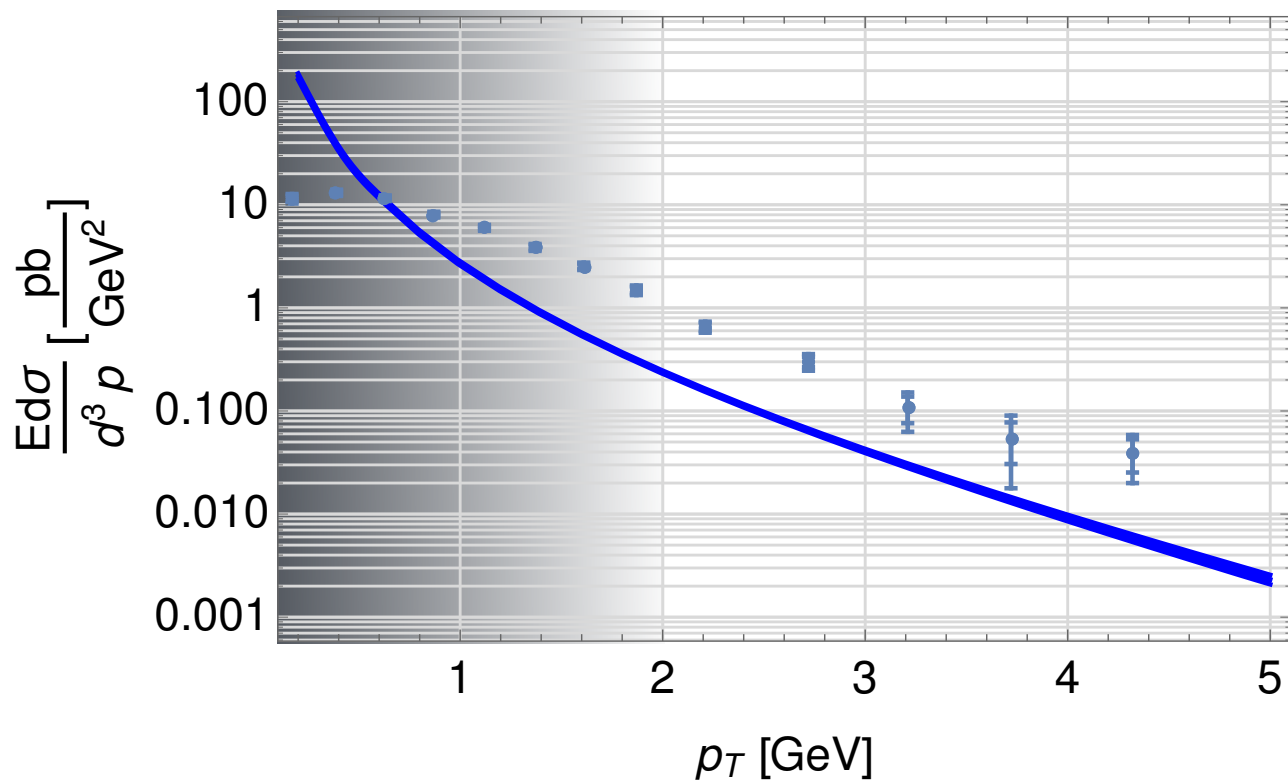


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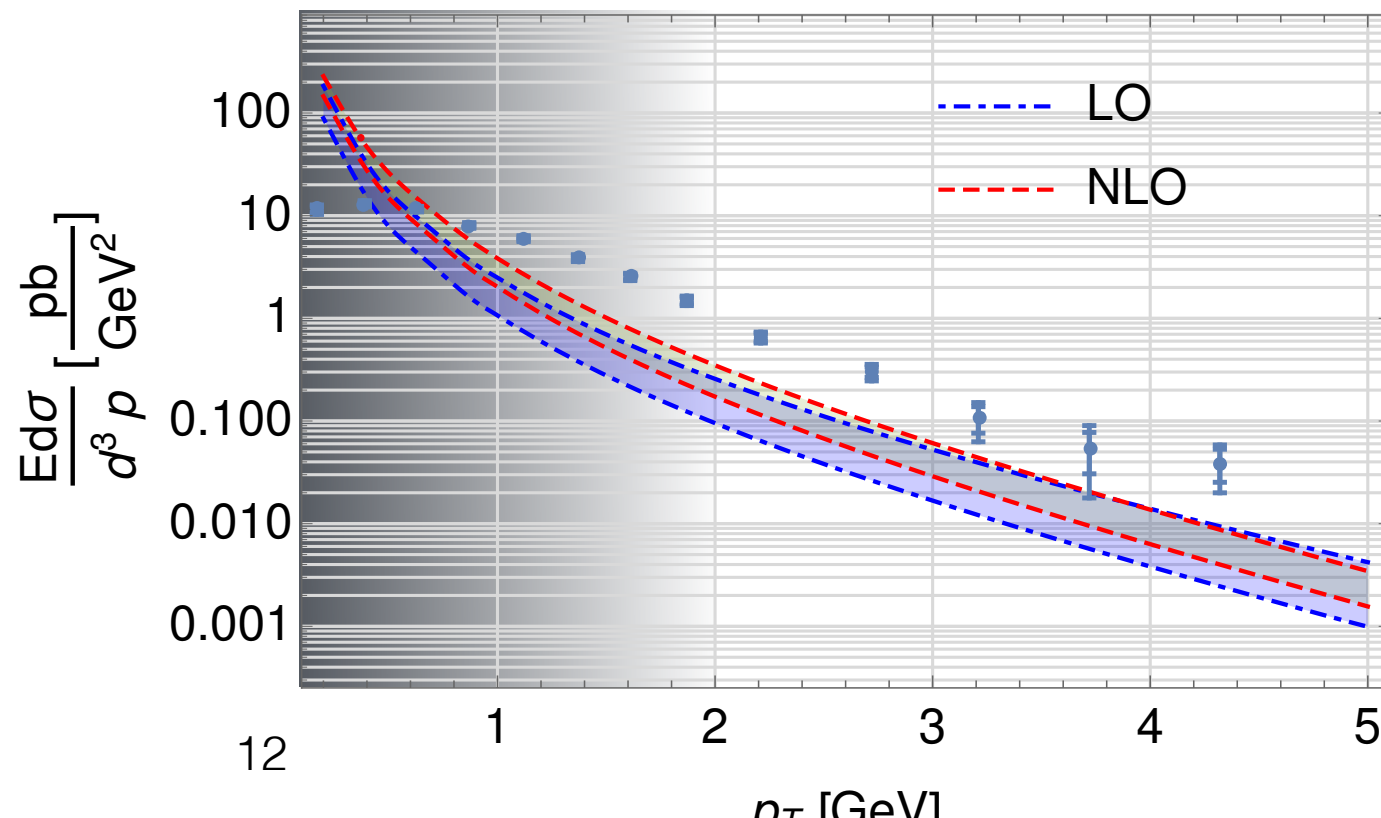
$Q=4.2-5.2 \text{ GeV}, x_F=0.15-0.35$



NLO $\mathcal{O}(\alpha_s^2)$
+PDF uncertainty

scale variations

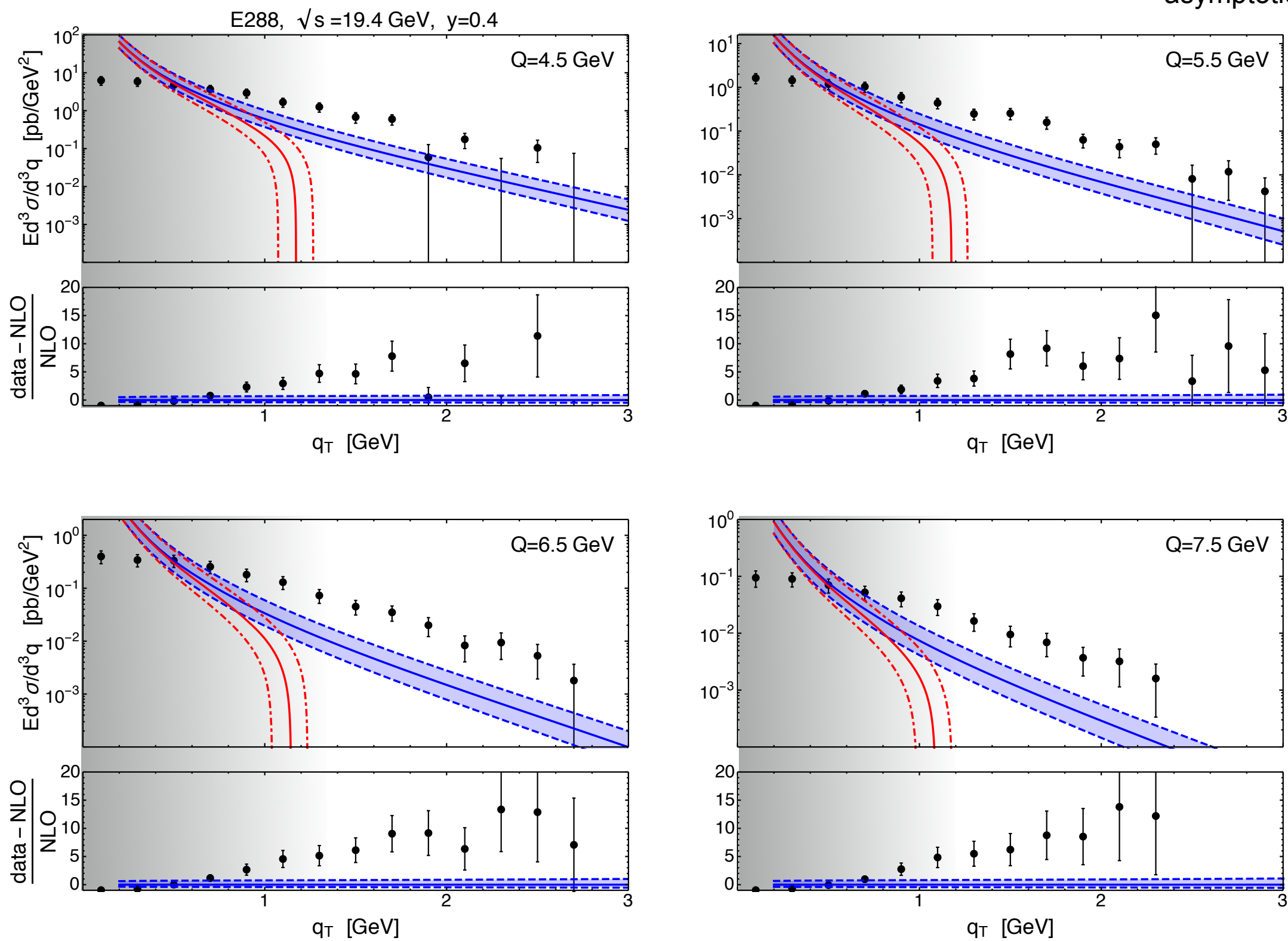
$Q=4.7 \text{ GeV}, x_F=\{0.15,0.35\}, \text{target}=p$



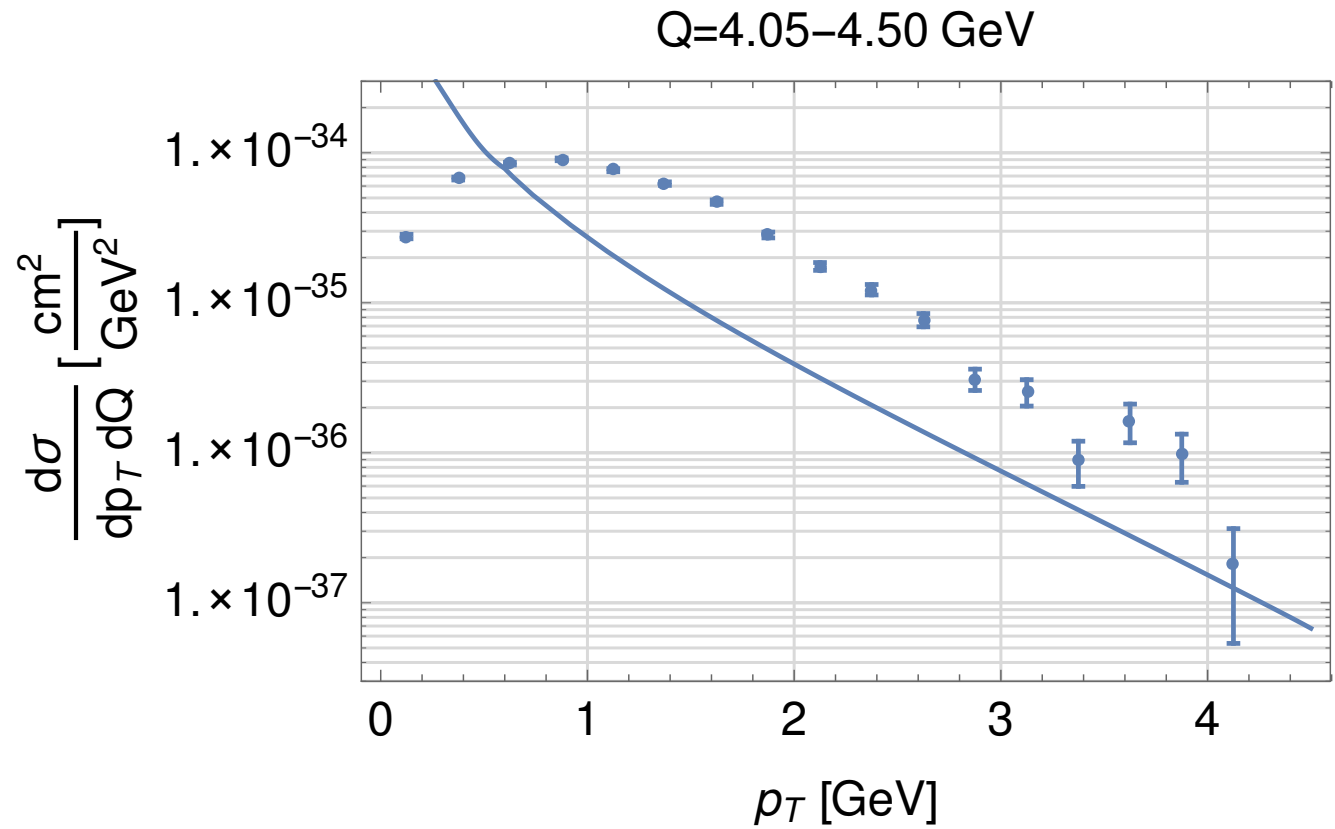
E288

$p\text{Cu}, p\text{Pt} \rightarrow \mu^+ \mu^- X$

--- NLO
- - - asymptotic



pion-nucleus Drell-Yan

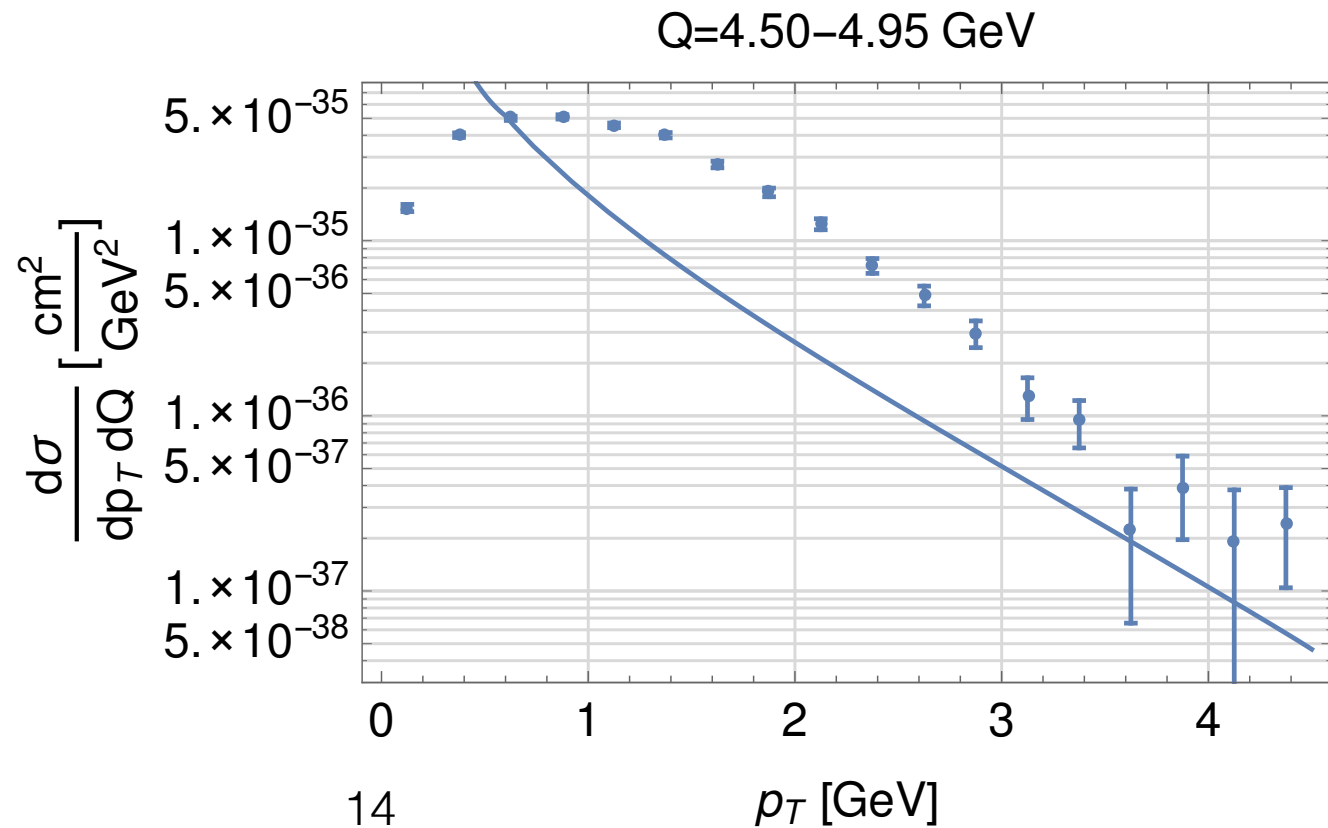


NLO $\mathcal{O}(\alpha_s^2)$

E615

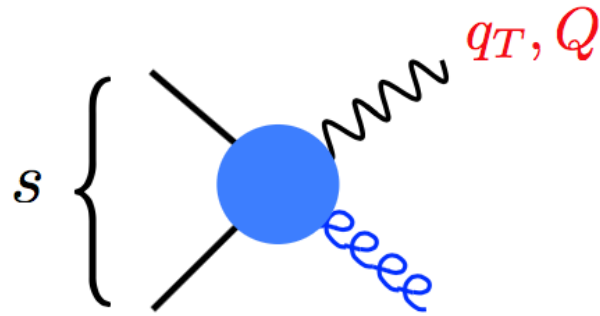
$\pi W \rightarrow \mu^+ \mu^- X$

$\sqrt{s} = 21.8 \text{ GeV}$



threshold resummation

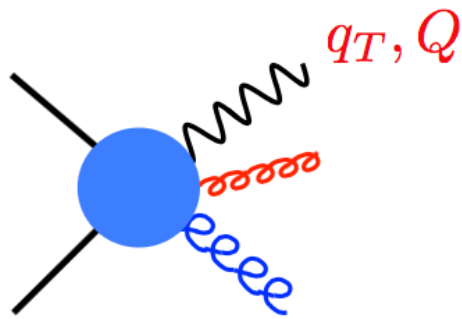
• LO :



$$\sqrt{s} \geq q_T + \sqrt{Q^2 + q_T^2}$$

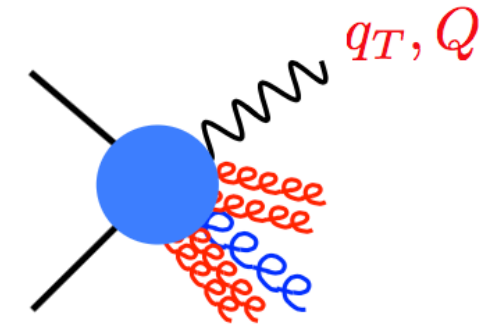
$$y_T \equiv \frac{q_T + \sqrt{q_T^2 + Q^2}}{\sqrt{s}} \leq 1$$

• NLO :



$$\frac{d\hat{\sigma}^{\text{NLO}}}{dq_T} \propto \alpha_s [\mathcal{A} \log^2(1 - y_T^2) + \mathcal{B} \log(1 - y_T^2) + \mathcal{C}]$$

• N^kLO :

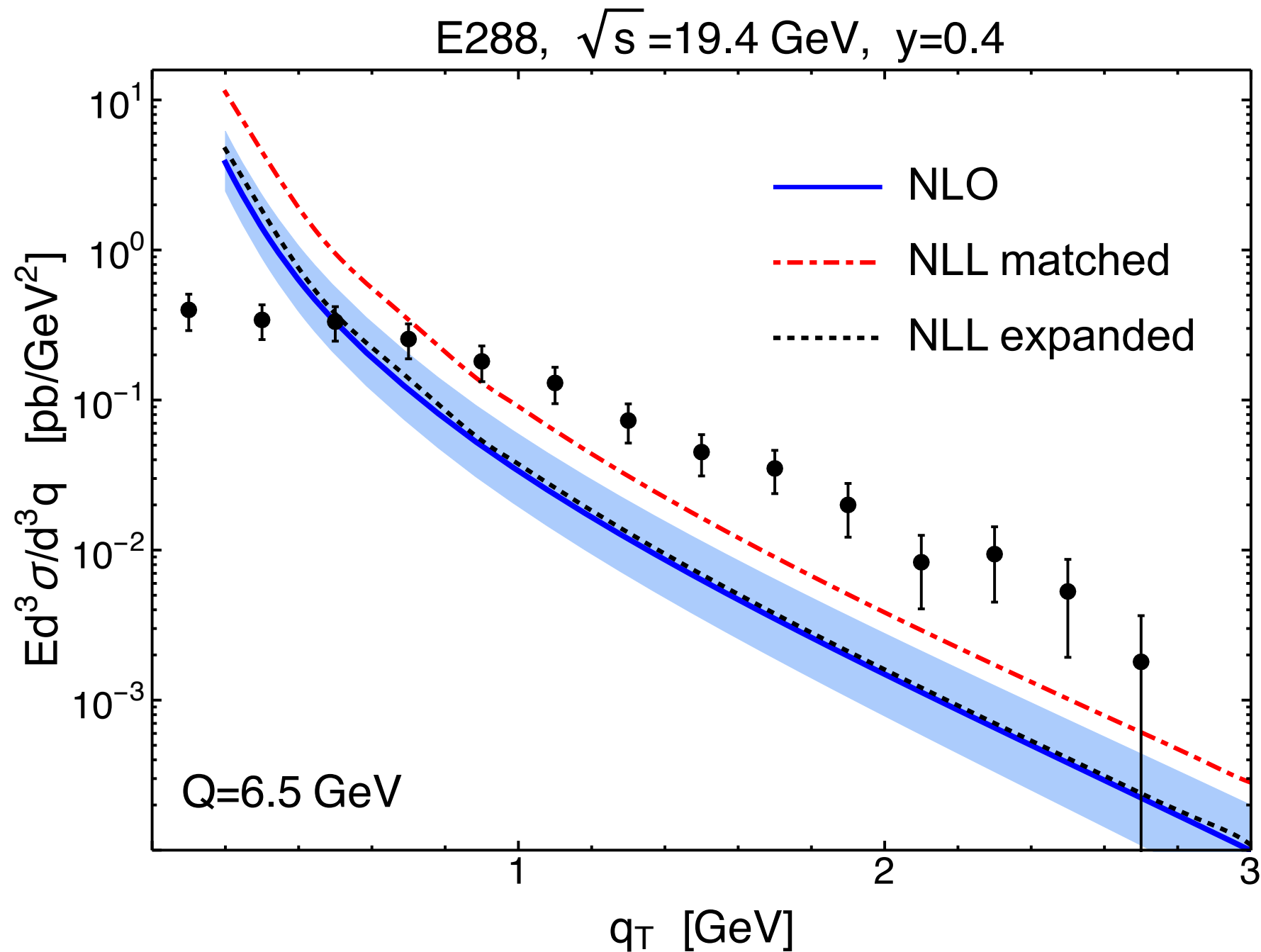


$$\frac{d\hat{\sigma}^{\text{N}^k\text{LO}}}{dq_T} \propto \alpha_s^k \log^{2k}(1 - y_T^2) + \dots$$

• threshold logarithms

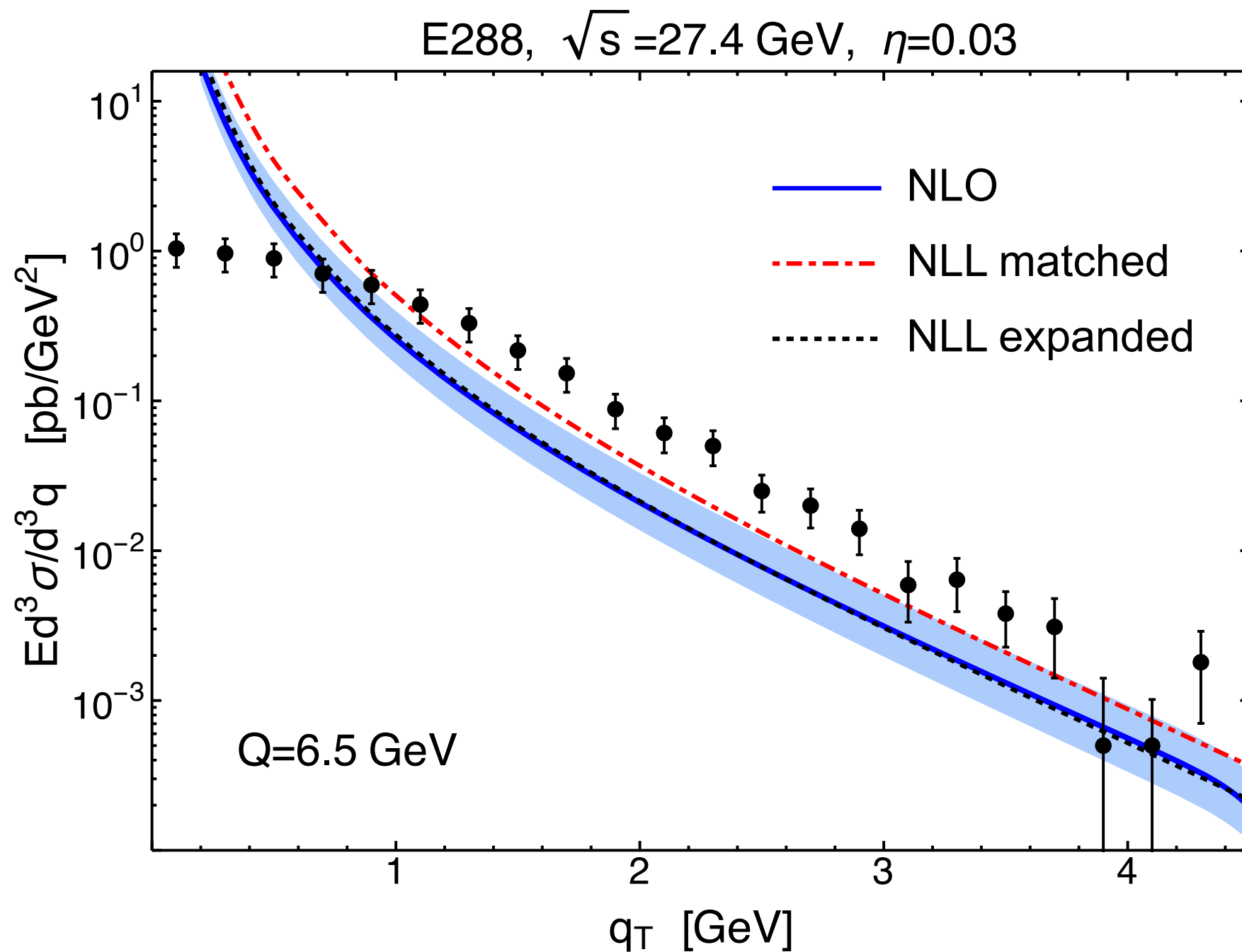
W. Vogelsang

threshold resummation



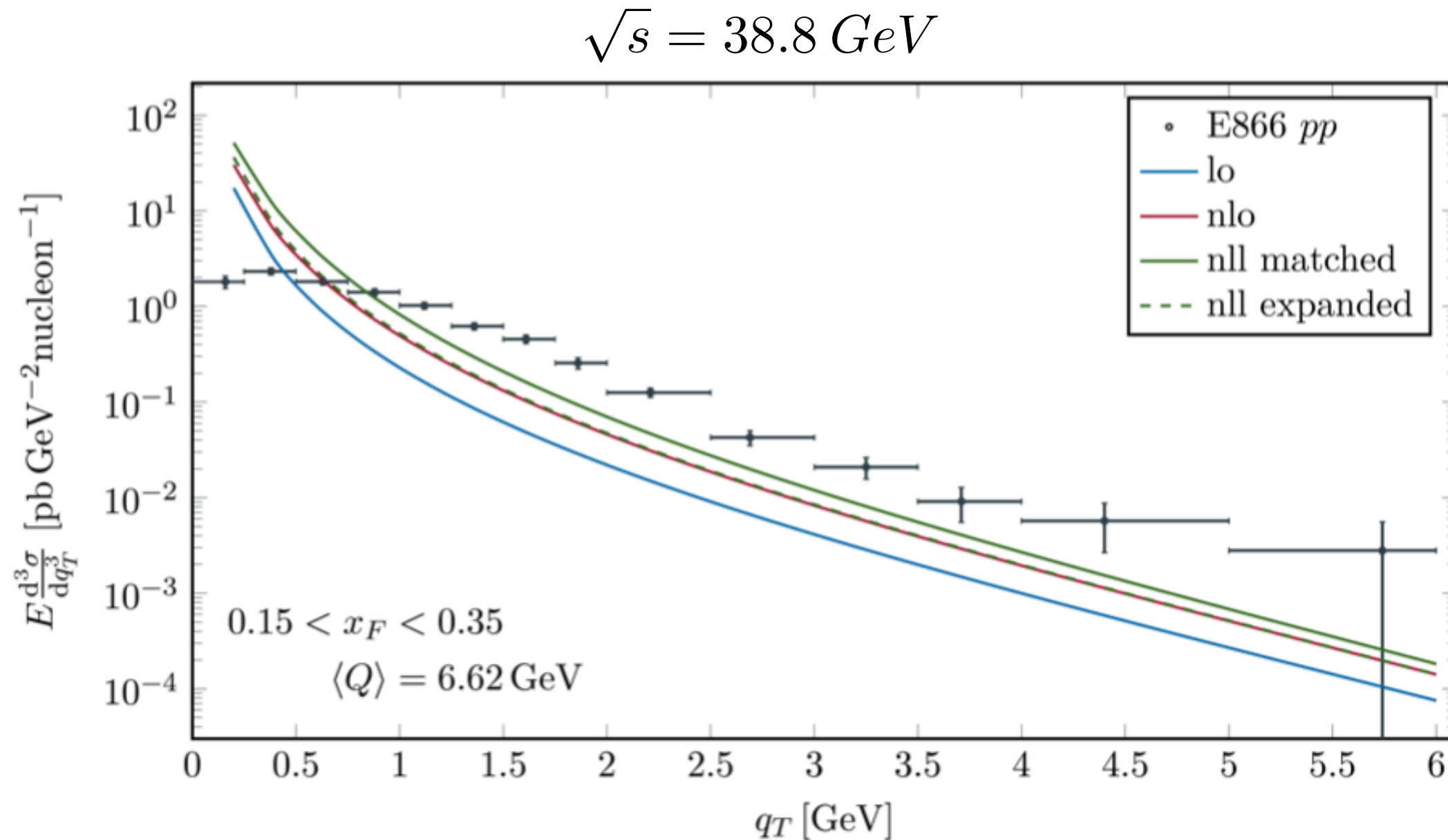
Vogelsang, Lambertsen, Steiglechner

threshold resummation



Vogelsang, Lambertsen, Steiglechner

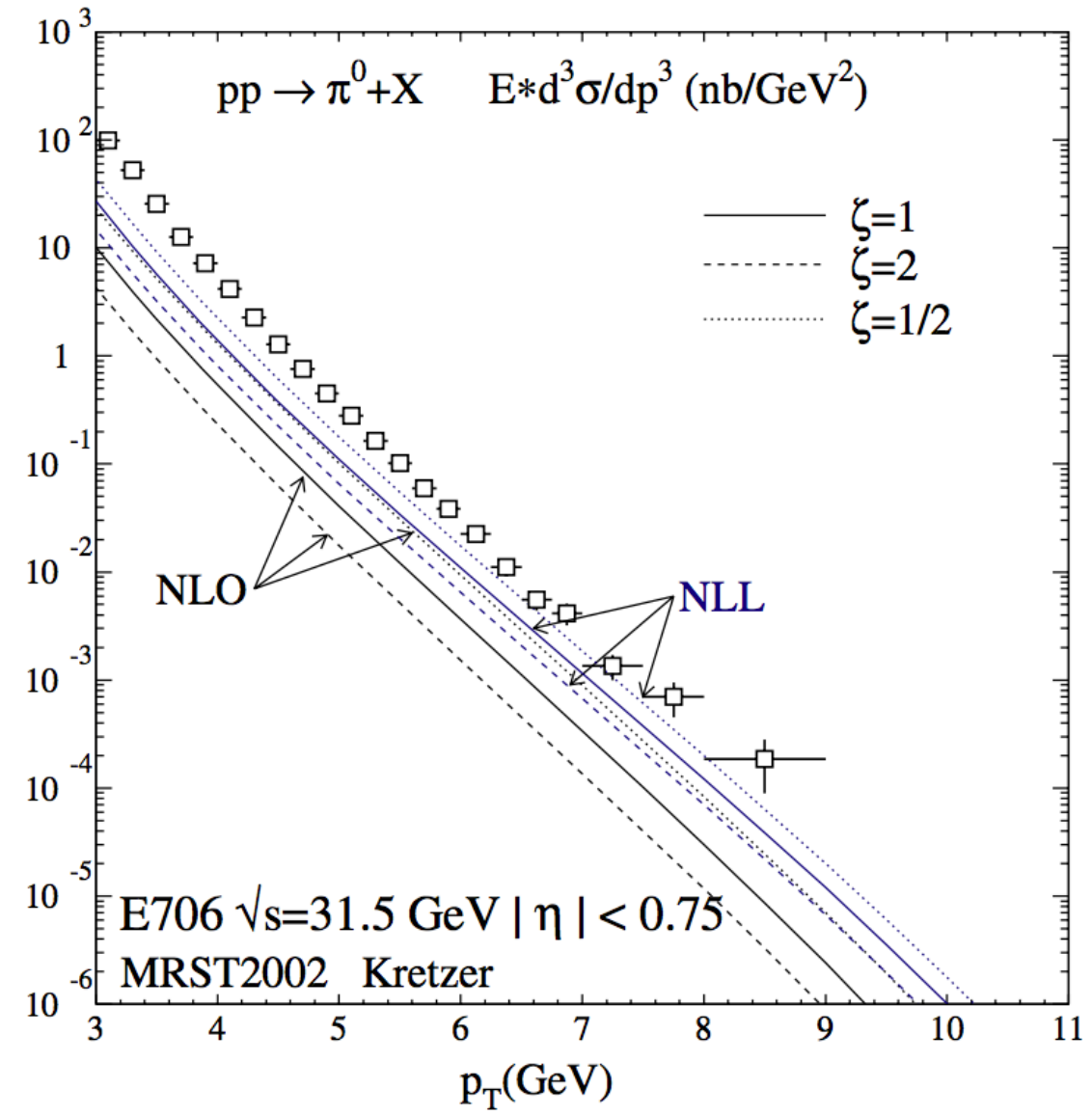
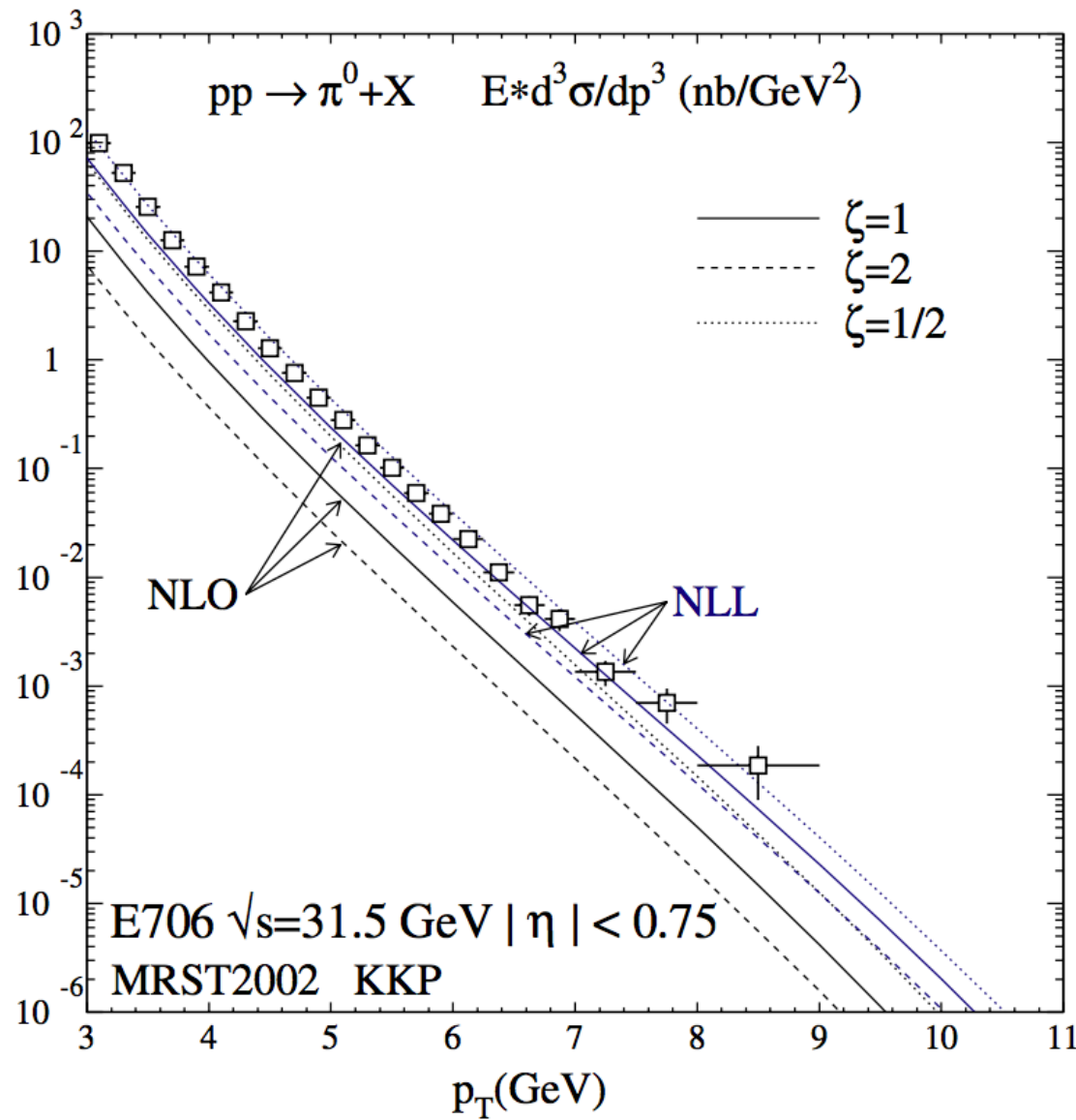
threshold resummation



Vogelsang, Lambertsen, Steiglechner

known similar cases

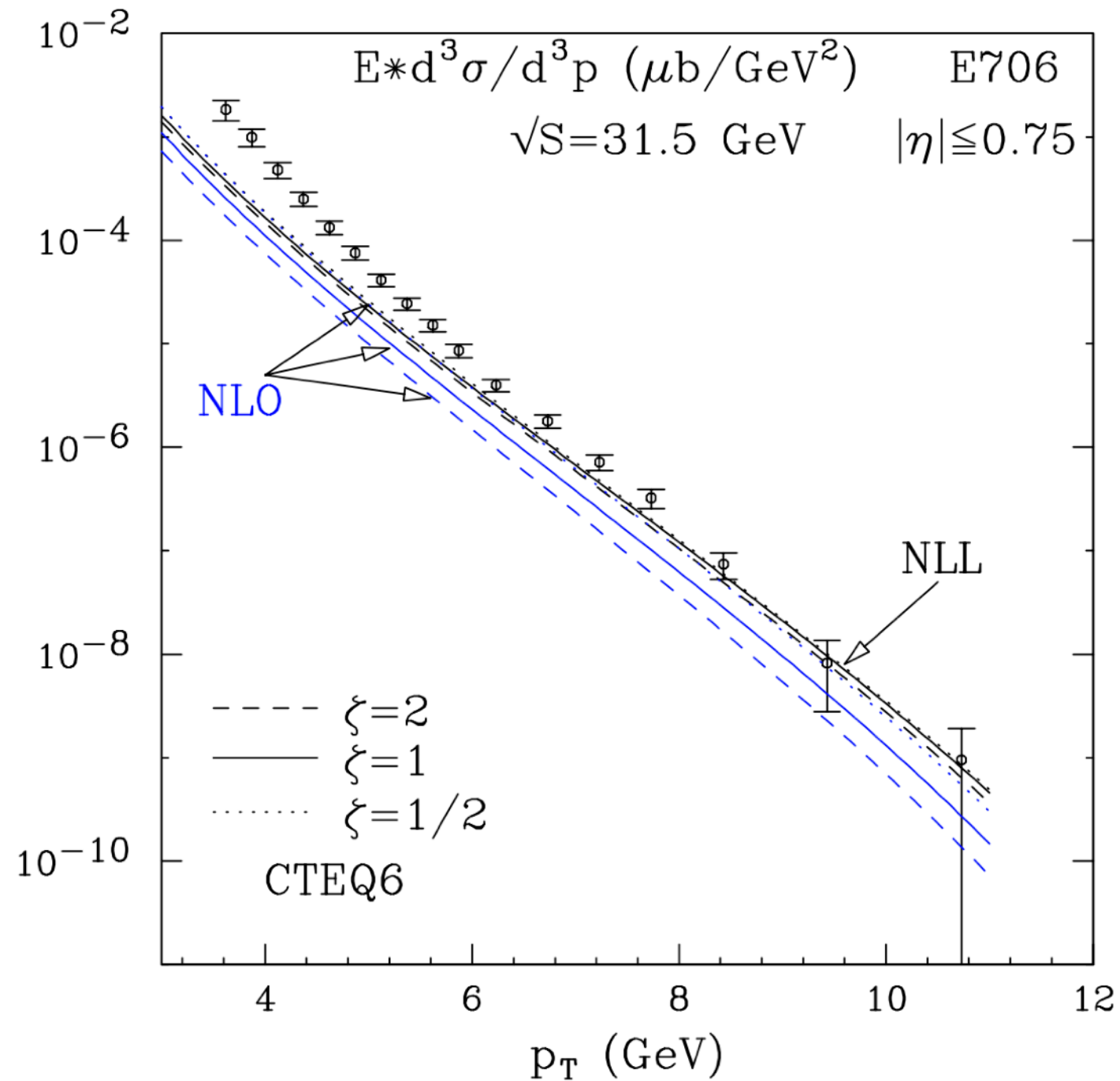
pion production



de Florian Vogelsang PRD71 114004 (2005)

known similar cases

prompt photon



de Florian Vogelsang PRD72 014014 (2005)

intrinsic k_T smearing

- take collinear factorization formula

$$d\sigma = \sum_{ab} \int dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) d\hat{\sigma}^{ab \rightarrow l^+ l^-}$$

- give the incoming partons a small k_T

$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb} \\ \times f_{a/A}(x, \mathbf{k}_{Ta}) f_{b/B}(x, \mathbf{k}_{Tb}) \frac{\hat{s}}{x_a x_b s} d\sigma^{ab \rightarrow l^+ l^-}$$

intrinsic k_T smearing

intrinsic k_T has a long history...
(for prompt photon and pion production)

Owens RMP59, 465 (1987)

Sivers PRD41, 83 (1990)

D'Alesio Murgia PRD70, 074009 (2004)

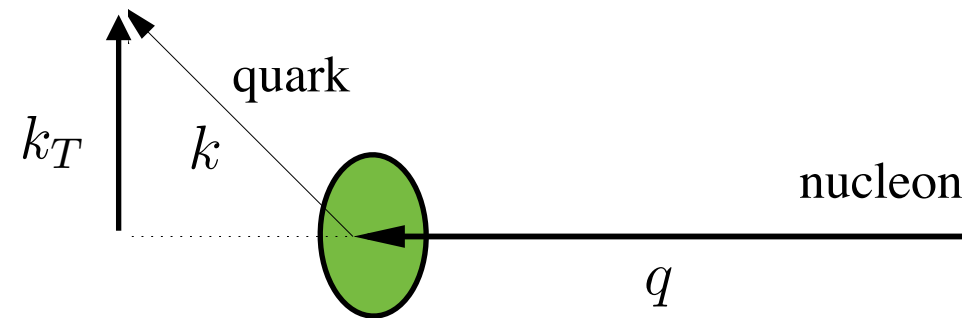
...

- give the incoming partons a small k_T

$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb} \\ \times f_{a/A}(x, \mathbf{k}_{Ta}) f_{b/B}(x, \mathbf{k}_{Tb}) \frac{\hat{s}}{x_a x_b s} d\sigma^{ab \rightarrow l^+ l^-}$$

intrinsic k_T smearing

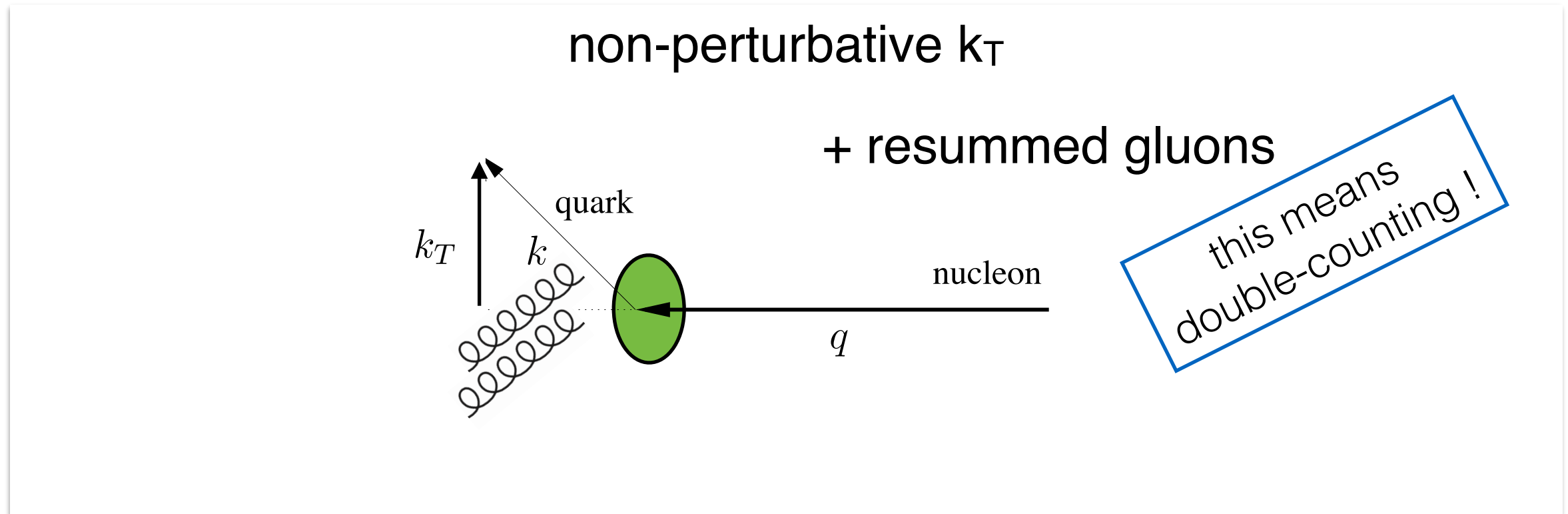
non-perturbative k_T



- give the incoming partons a small k_T

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intrinsic k_T smearing



- give the incoming partons a small k_T

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parton momentum:

$$p_a^\mu \doteq (p_a^0, \mathbf{p}_a^T, p_a^3) = \left(x_a P_A + \frac{k_{Ta}^2}{4x_a P_A}, \mathbf{k}_{Ta}, x_a P_A - \frac{k_{Ta}^2}{4x_a P_A} \right)$$

this enforces: $p_a^\mu p_{a\mu} = 0$ $x_a = p_a^+ / P_A^+$

intrinsic k_T smearing

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this enforces:

$$p_a^\mu p_{a\mu} = 0 \quad x_a = p_a^+ / P_A^+$$

must make sure

- quark energy < proton energy
- quark direction = proton direction

$$k_{Ta} < \sqrt{x_a(1-x_a)}\sqrt{s}$$

$$k_{Ta} < x_a \sqrt{s}$$

better to stay far from the bounds!

intrinsic k_T smearing

$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb} \\ \times f_{a/A}(x, \mathbf{k}_{Ta}) f_{b/B}(x, \mathbf{k}_{Tb}) \frac{\hat{s}}{x_a x_b S} d\sigma^{ab \rightarrow l^+ l^-}$$

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$$\hat{t} = (q_\gamma - p_a)^2 = Q^2 - 2q_\gamma^- p_a^+ - 2q_\gamma^+ p_a^- + 2\mathbf{q}_T \cdot \mathbf{p}_{Ta}$$


make sure

\hat{t}, \hat{u}

not too small

Fixed Order must be valid!

intrinsic k_T smearing

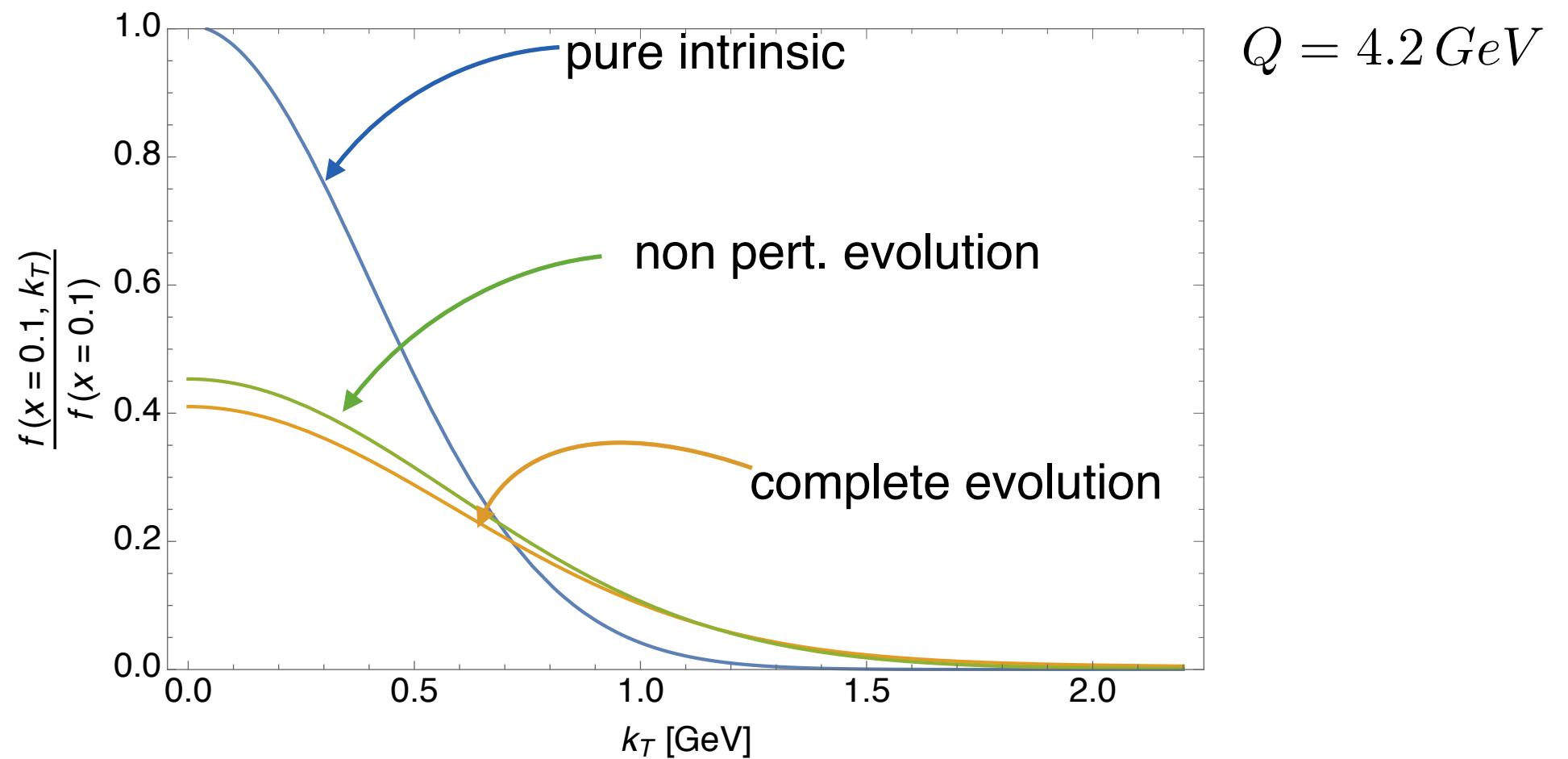
$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb}$$
$$\times f_{a/A}(x, \mathbf{k}_{Ta}) f_{b/B}(x, \mathbf{k}_{Tb}) \frac{\hat{s}}{x_a x_b S} d\sigma^{ab \rightarrow l^+ l^-}$$

$$\int_0^{k_{Tmax}^2} \pi dk_T^2$$

check independence from cutoff!

intrinsic k_T smearing

$$\frac{f_{a/A}(x_a, \mathbf{k}_{T a})}{f_{a/A}(x_a)}$$

Q evolution from TMD extraction:

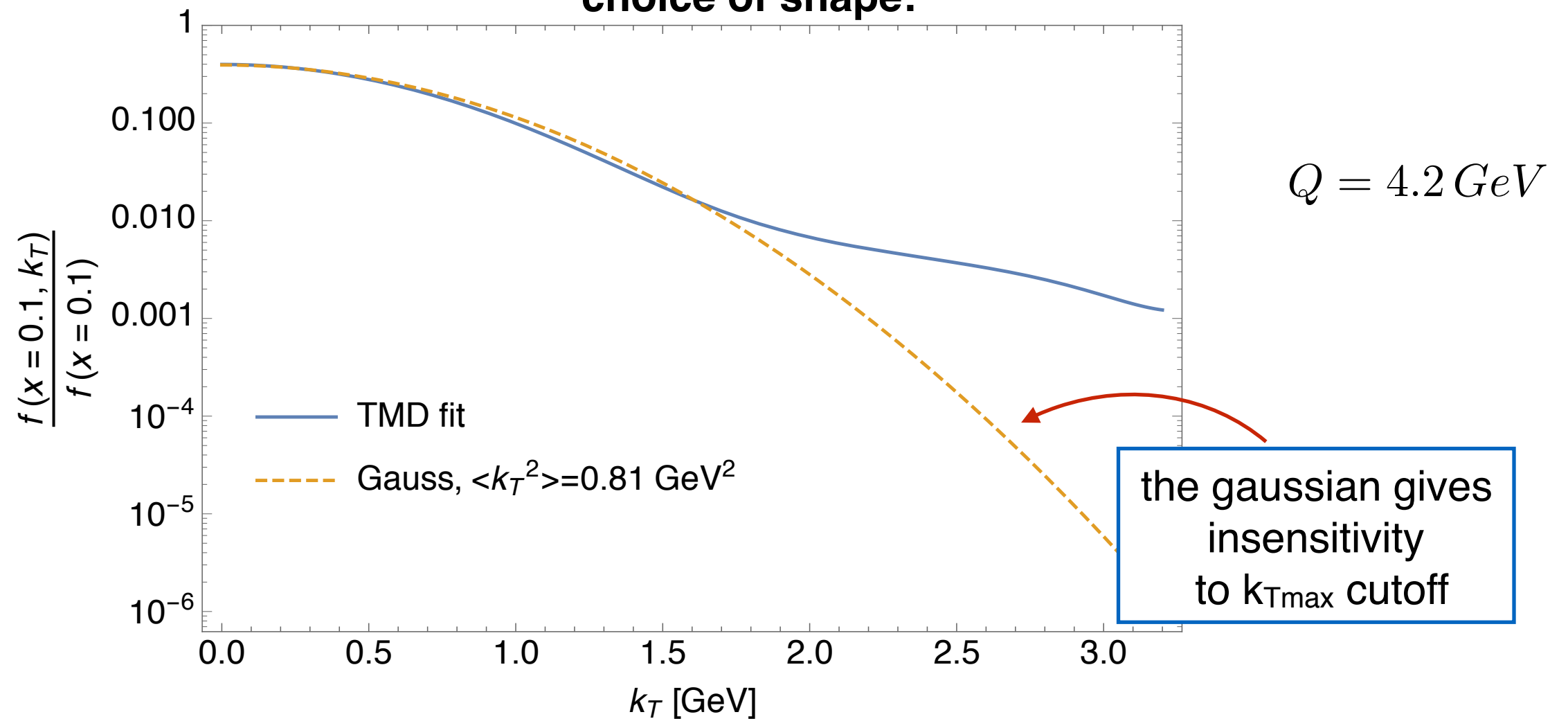


Bacchetta et al JHEP 1706 081

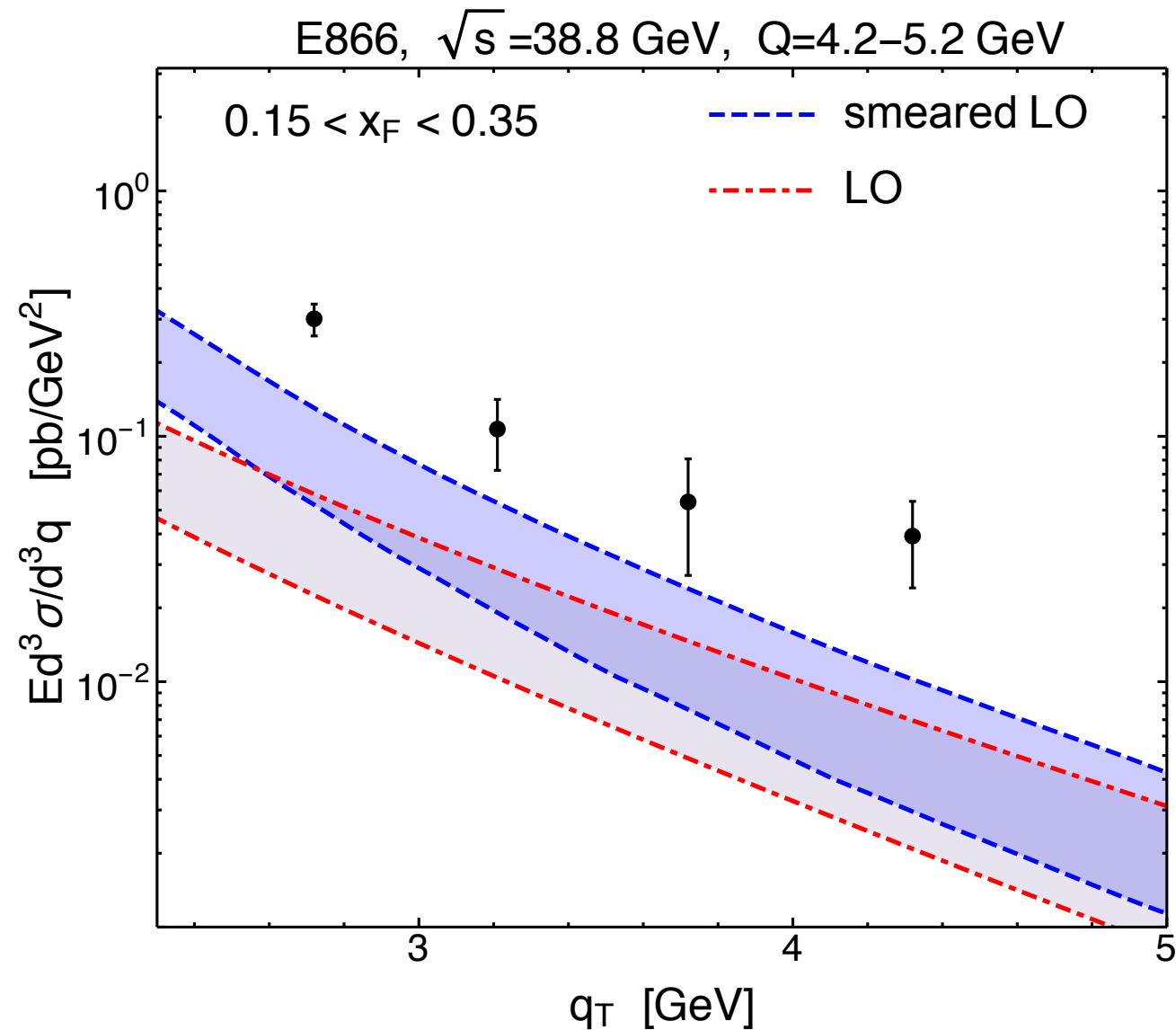
intrinsic k_T smearing

$$\frac{f_{a/A}(x_a, \mathbf{k}_{T a})}{f_{a/A}(x_a)}$$

choice of shape:



intrinsic k_T smearing



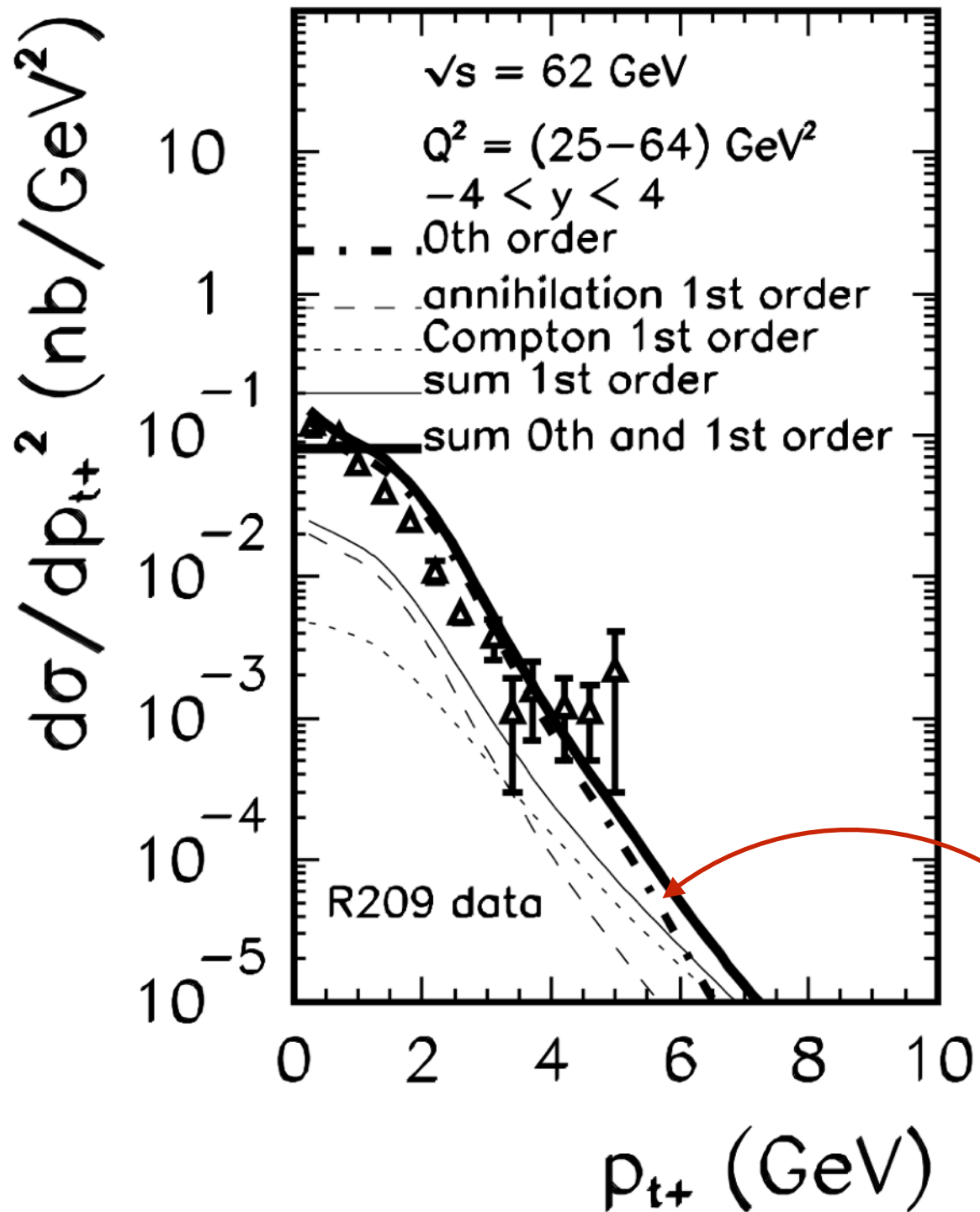
$$f_q(x, \mathbf{k}_T) = f(x) \frac{1}{\pi \langle k_T^2 \rangle} e^{-\frac{k_T^2}{\langle k_T^2 \rangle}}$$

$$\langle k_T^2 \rangle = 0.81 \text{ GeV}^2 \quad \text{same for any } x \text{ and flavor} \\ \dots \text{and for gluons!!}$$

Summary

- fixed-order pQCD largely underestimates low-energy DY data at high q_T
- only partial improvement from threshold resummation, and only for some kinematics
- intrinsic- k_T model does not seem to help
- more low Q and high q_T data needed

... "k_T-factorization" formalism?

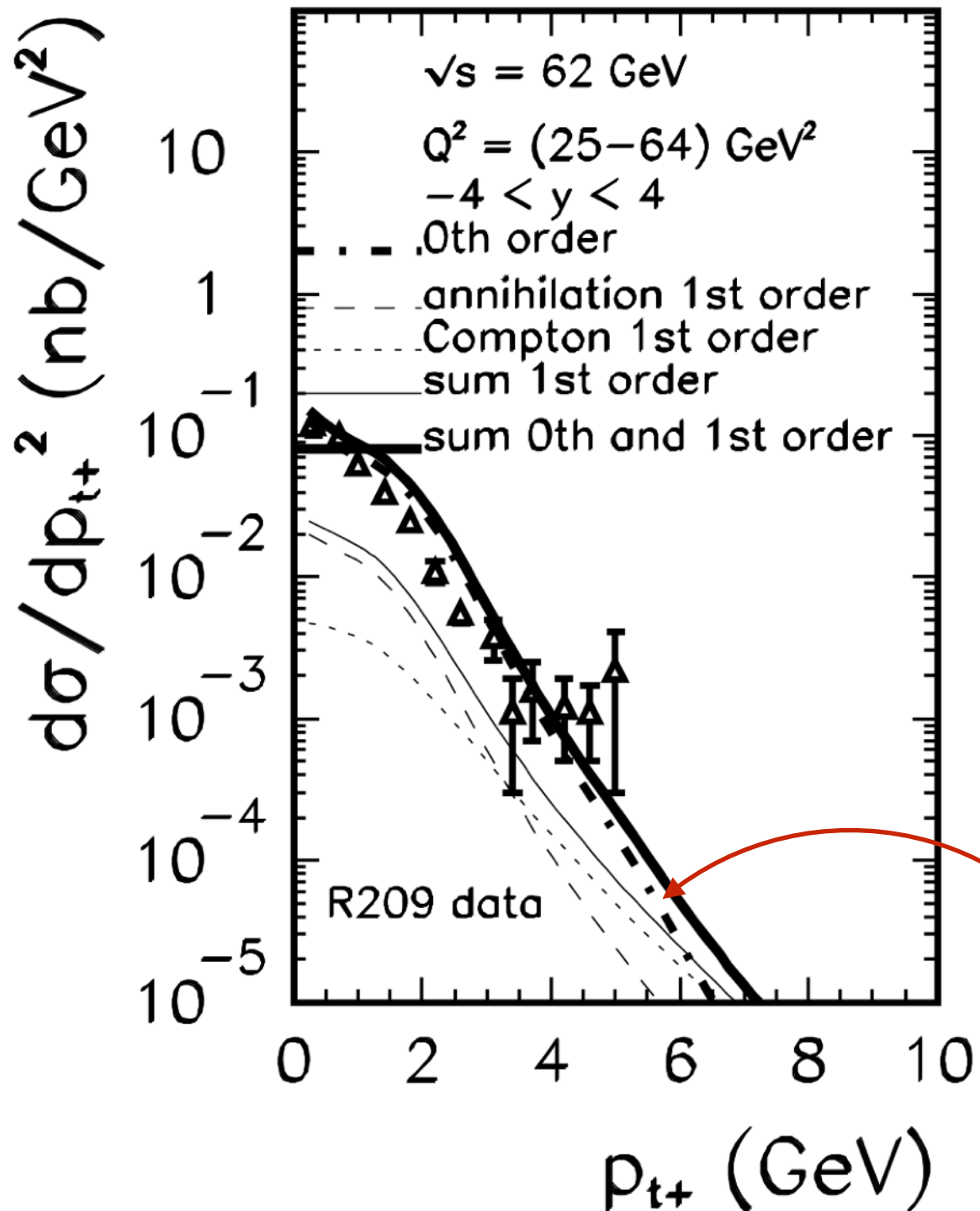


$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb} \times F_{a/A}^u(x_a, \mathbf{k}_{Ta}, \mu_F) F_{b/B}^u(x_b, \mathbf{k}_{Tb}, \mu_F) \frac{\hat{s}}{x_a x_b} d\hat{\sigma}^{ab \rightarrow l^+ l^-}$$

“unintegrated parton distributions”
(from extension of CCFM equations)

almost all of the cross-section
given by soft gluons +
intrinsic k_T

... "k_T-factorization" formalism?



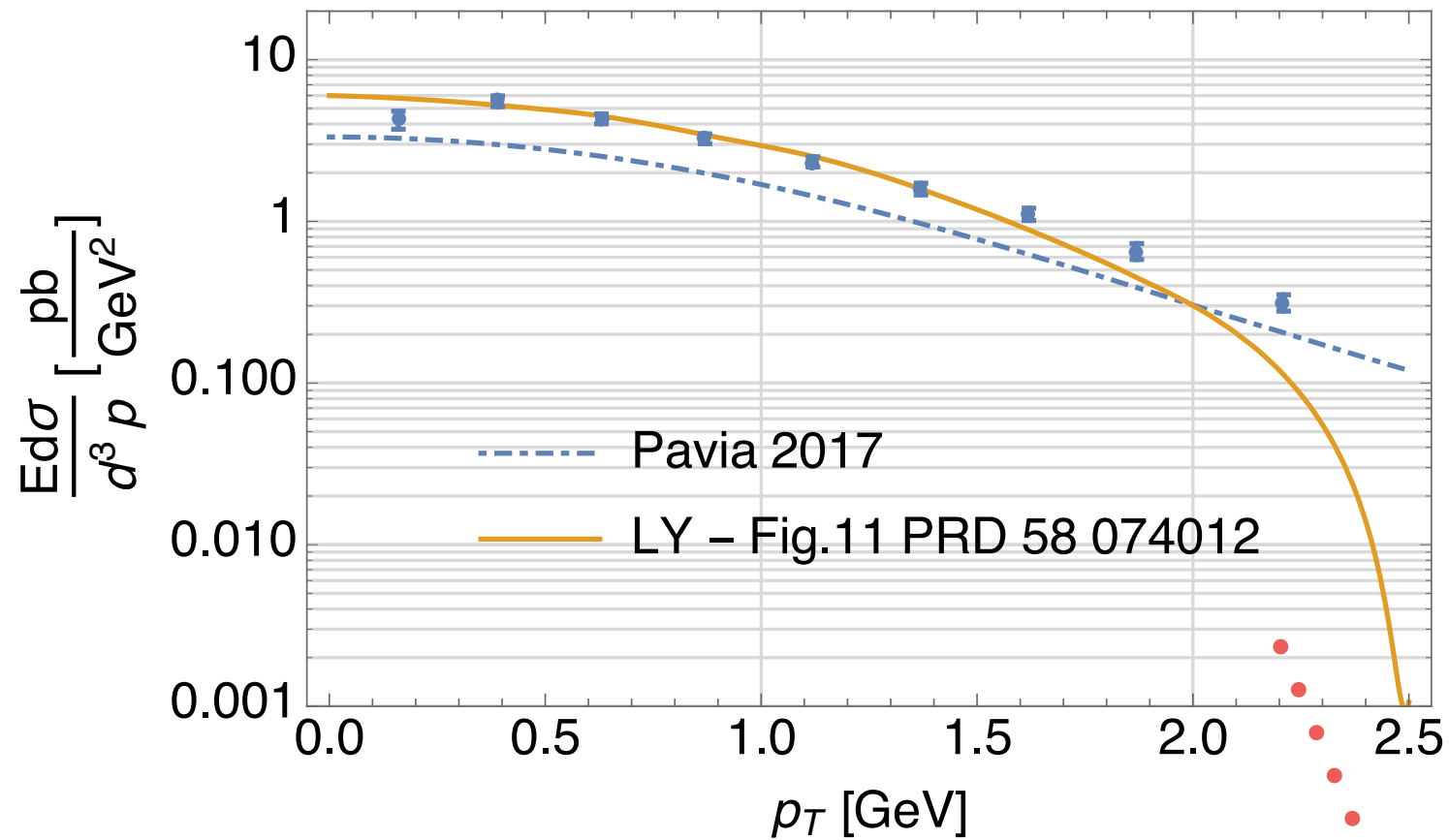
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**apparently difficult to find
 a formal proof**
Avsar, Collins arXiv:1209.1675

**not clear why
 it should differ from "traditional"
 formalism at this kinematics
 (not small-x)**

almost all of the cross-section
 given by soft gluons +
 intrinsic k_T

Q=5.2–6.2 GeV, $x_F=0.15-0.35$

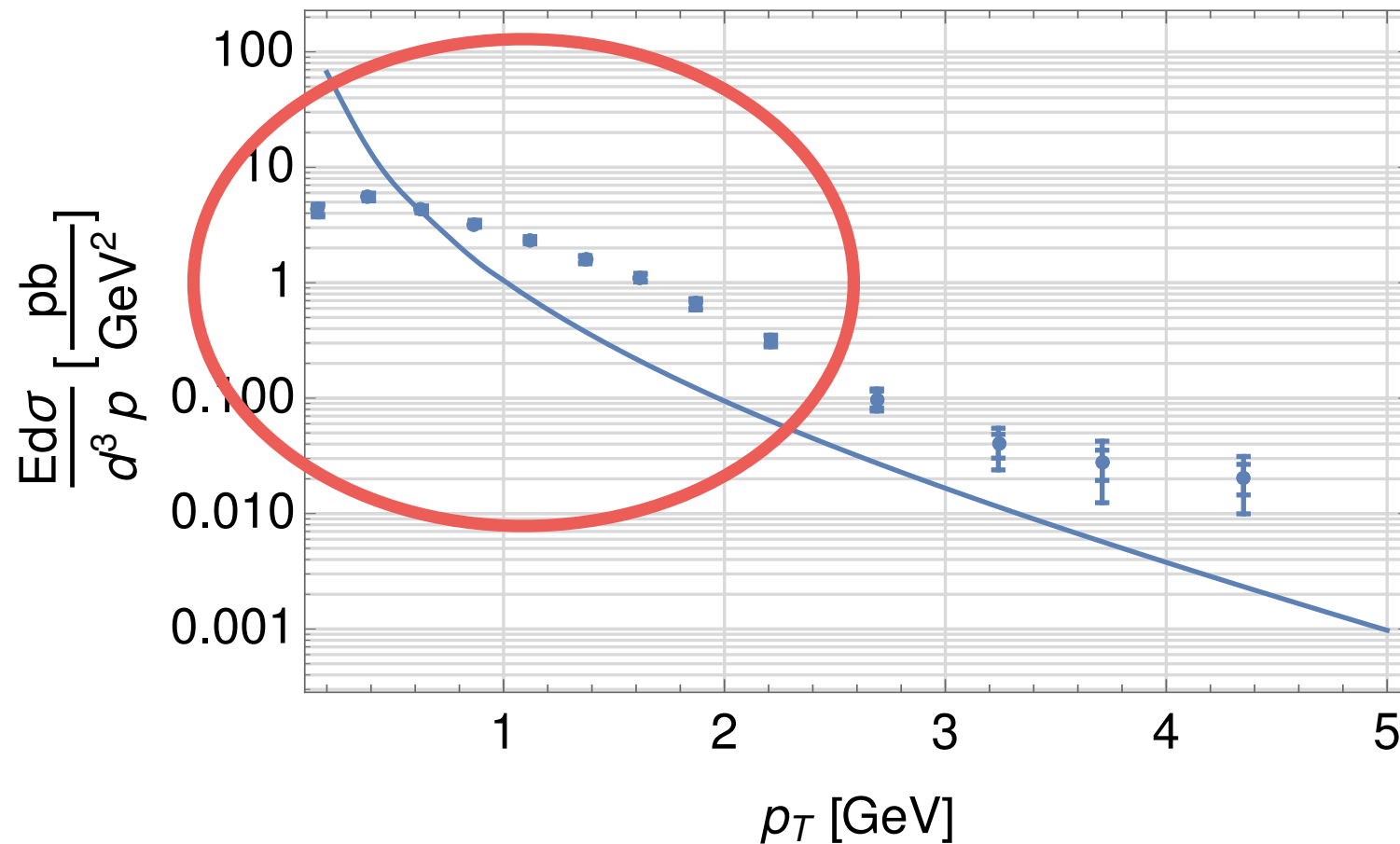


TMD
parametrizations

Q=5.2–6.2 GeV, $x_F=0.15-0.35$

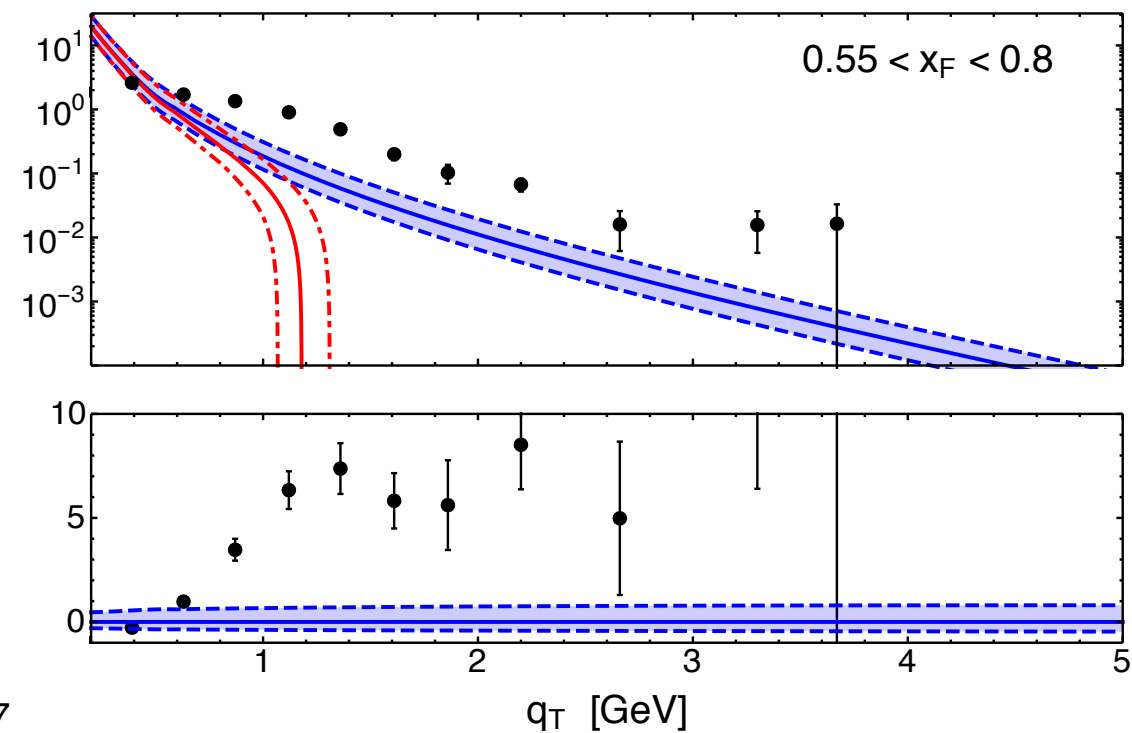
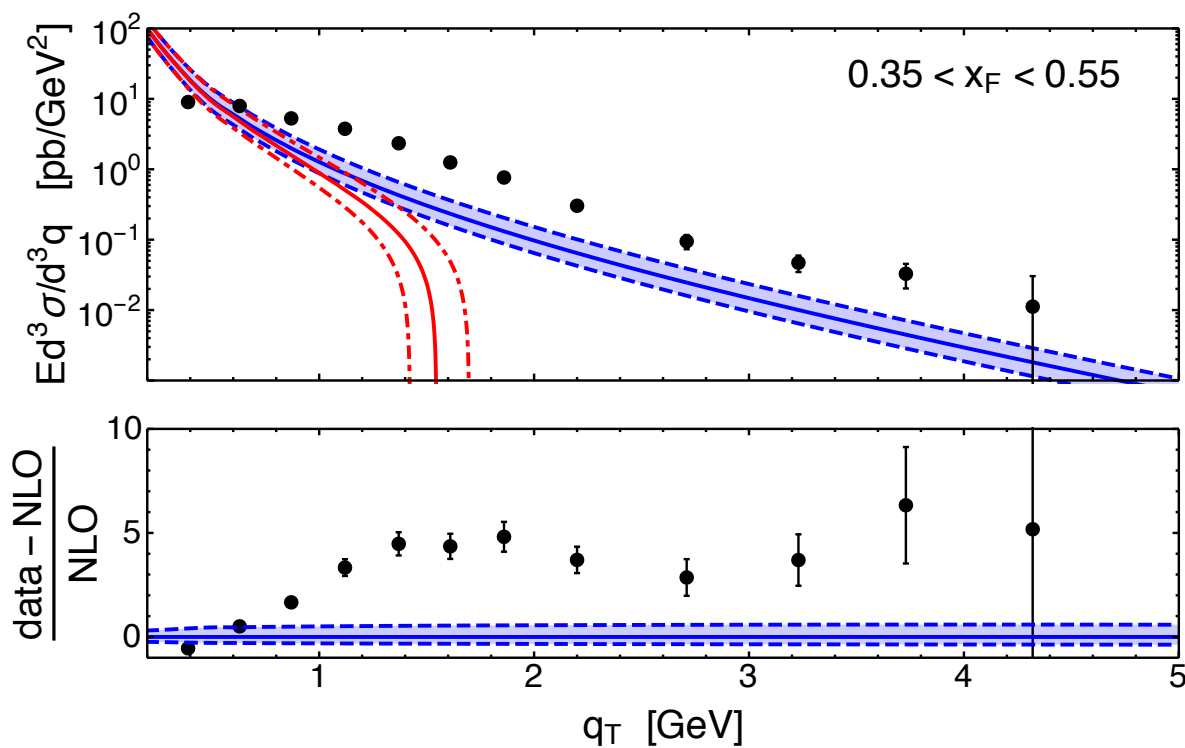
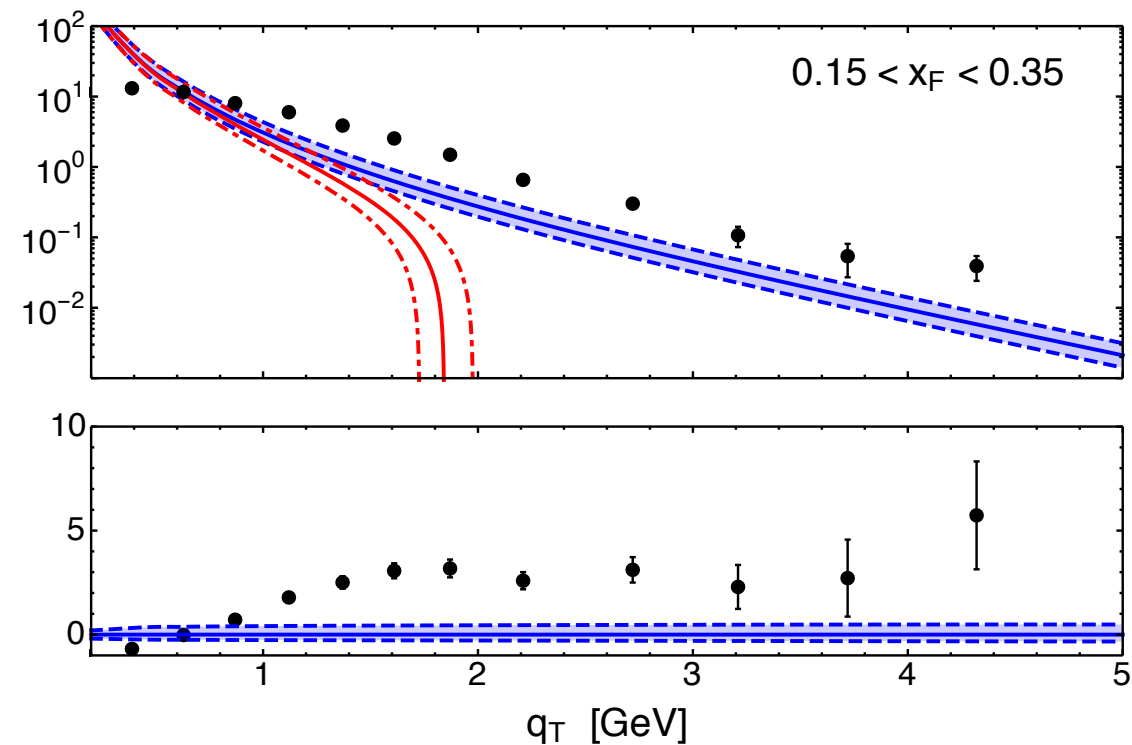
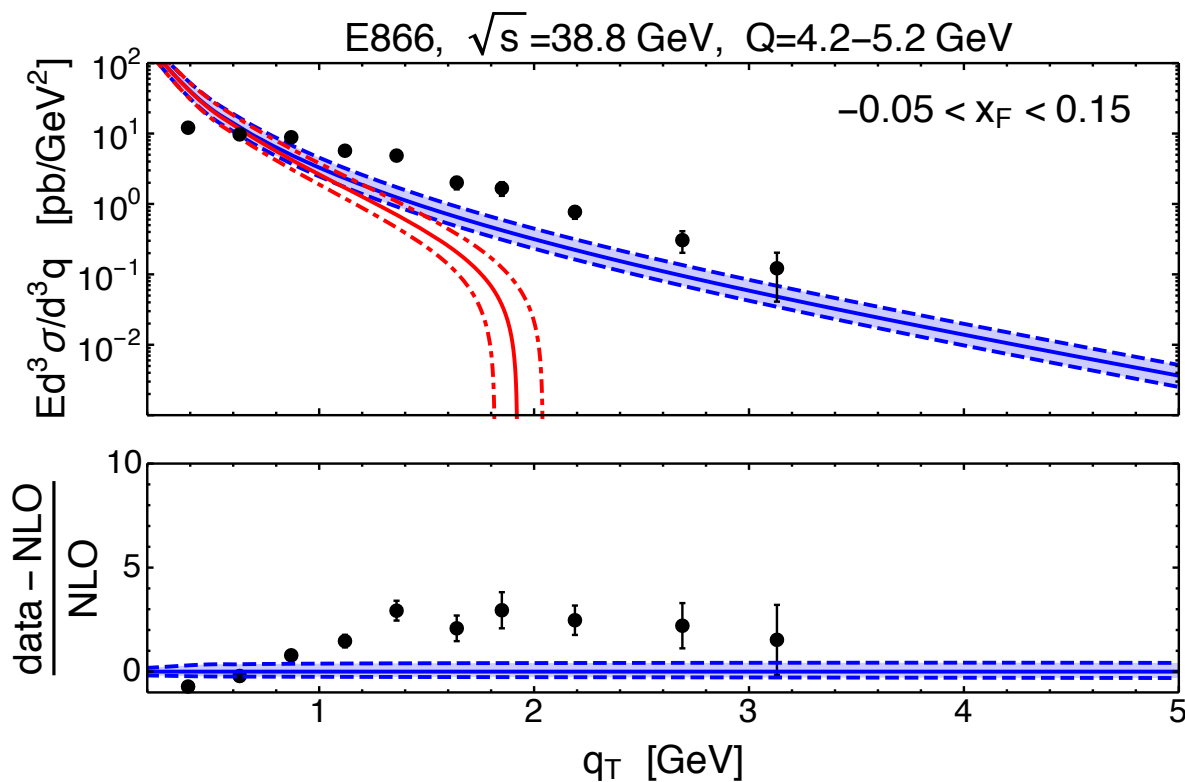
E866/NuSea

$pp \rightarrow \mu^+ \mu^- X$
 $\sqrt{s} = 38.8 \text{ GeV}$



E866/NuSea

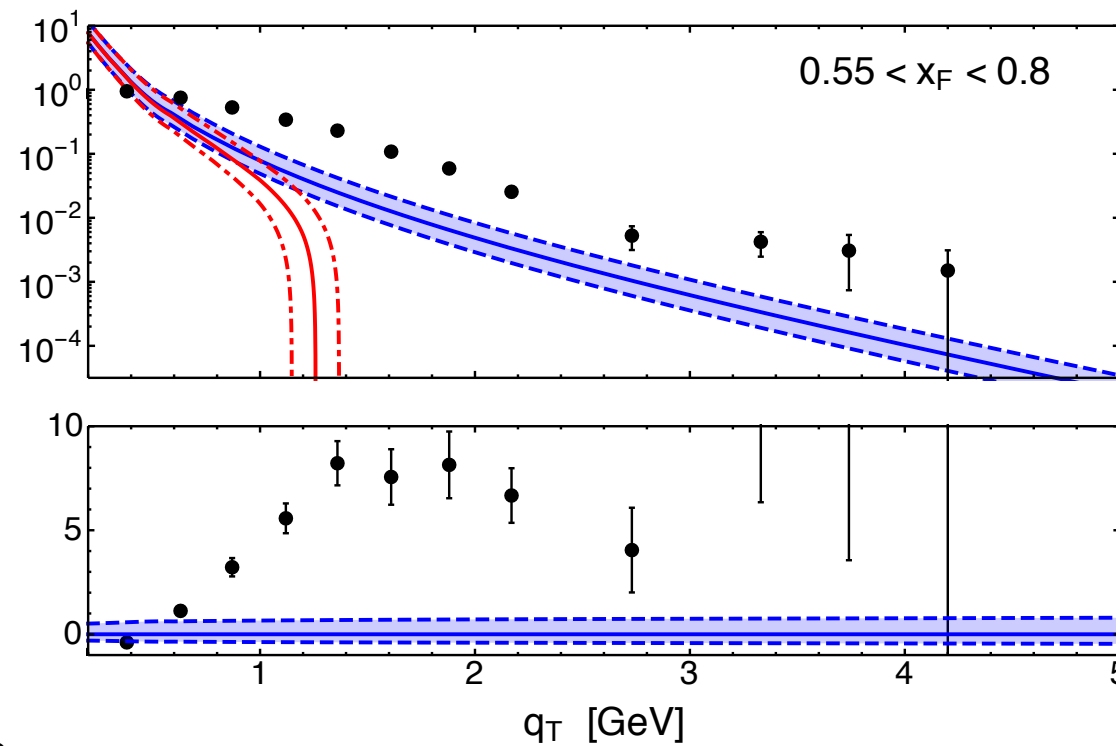
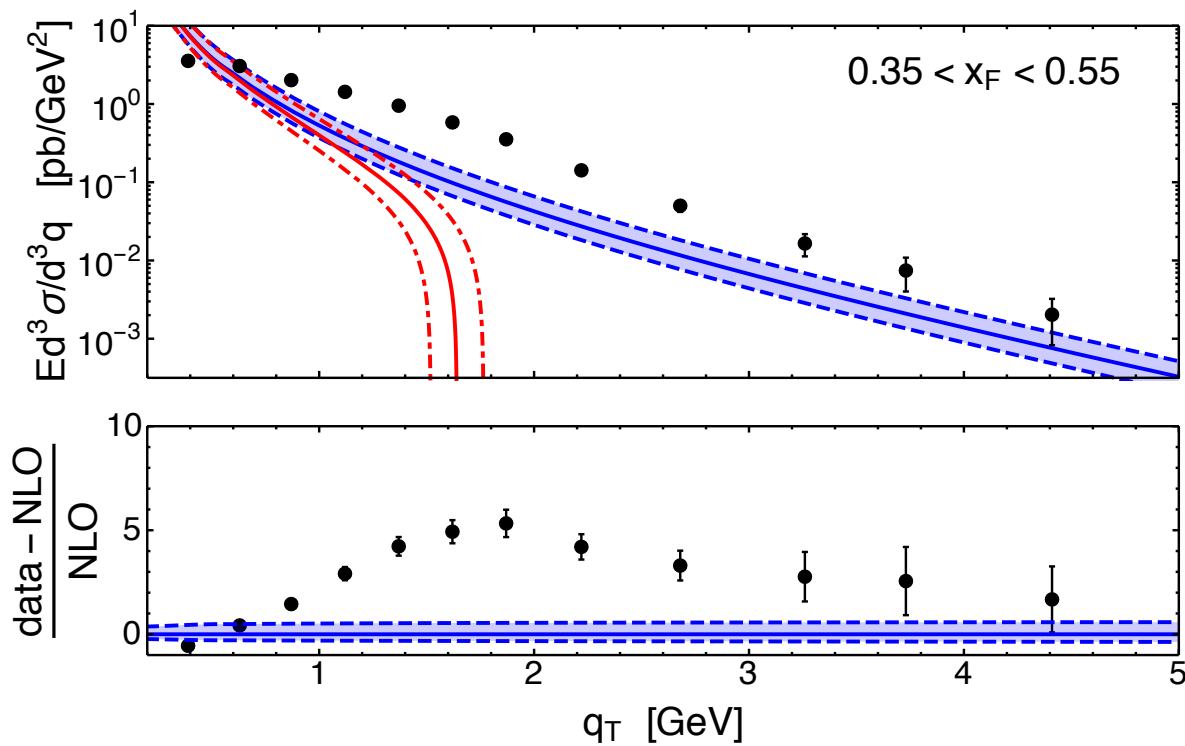
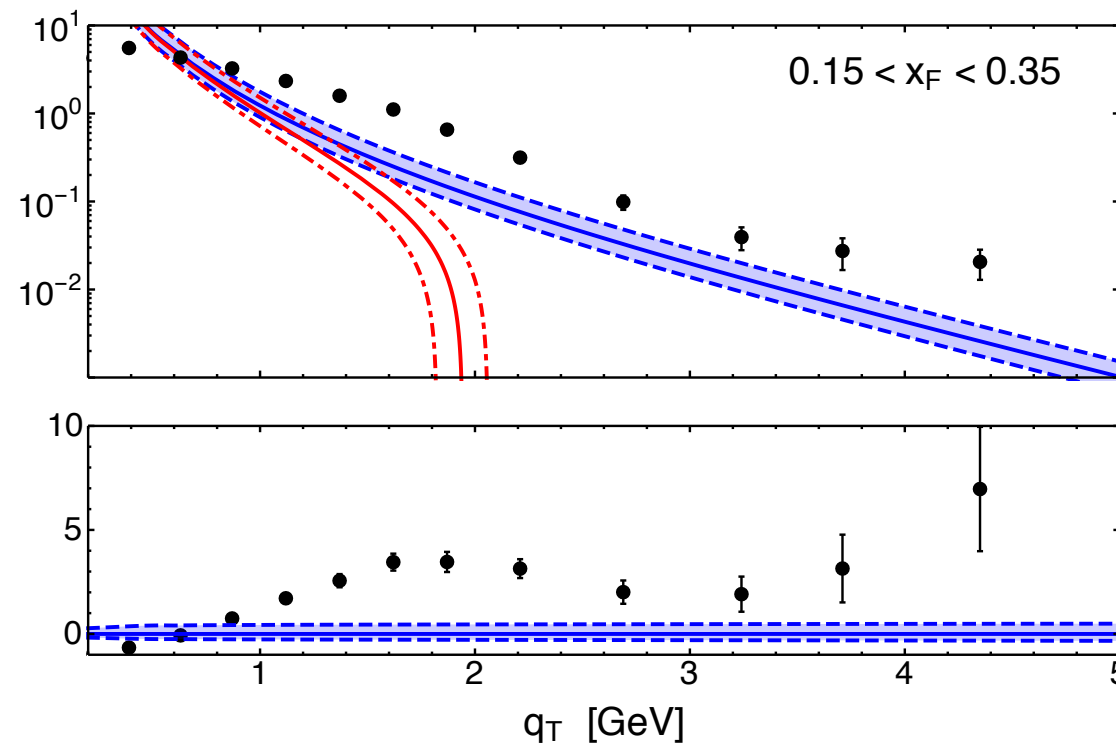
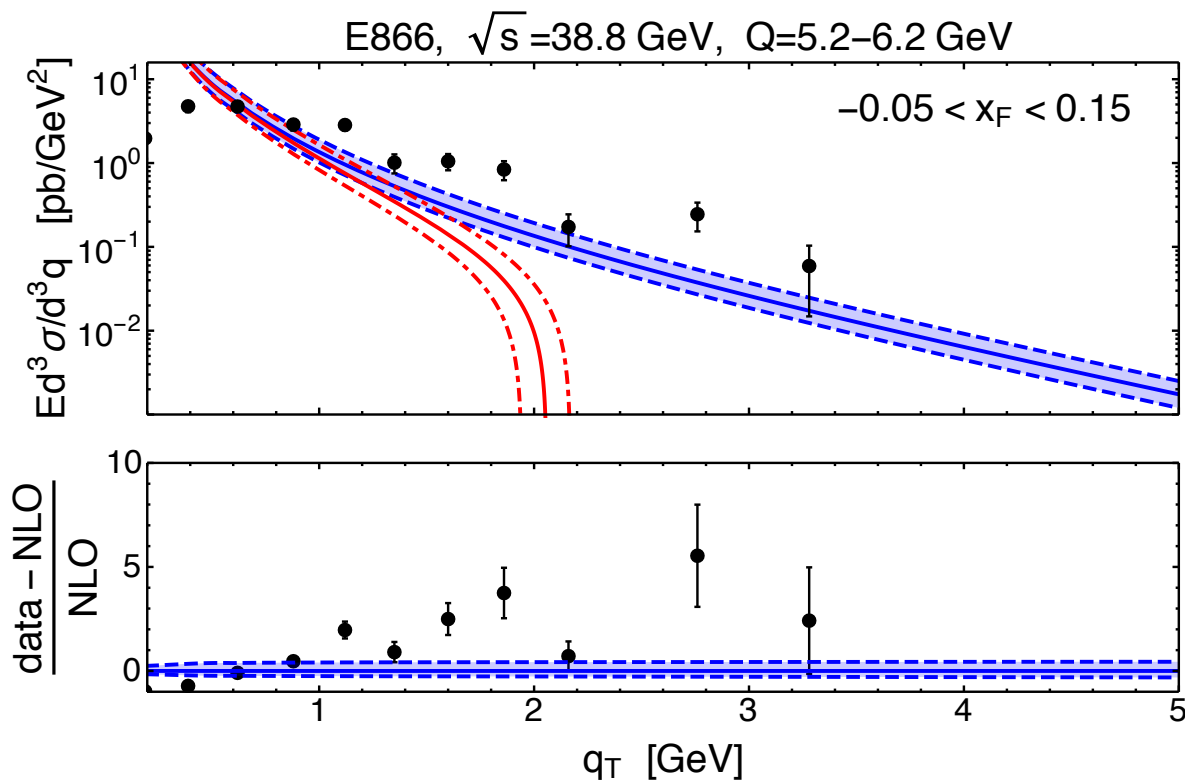
$$pp \rightarrow \mu^+ \mu^- X \quad \sqrt{s} = 38.8 \text{ GeV}$$



--- NLO

E866/NuSea

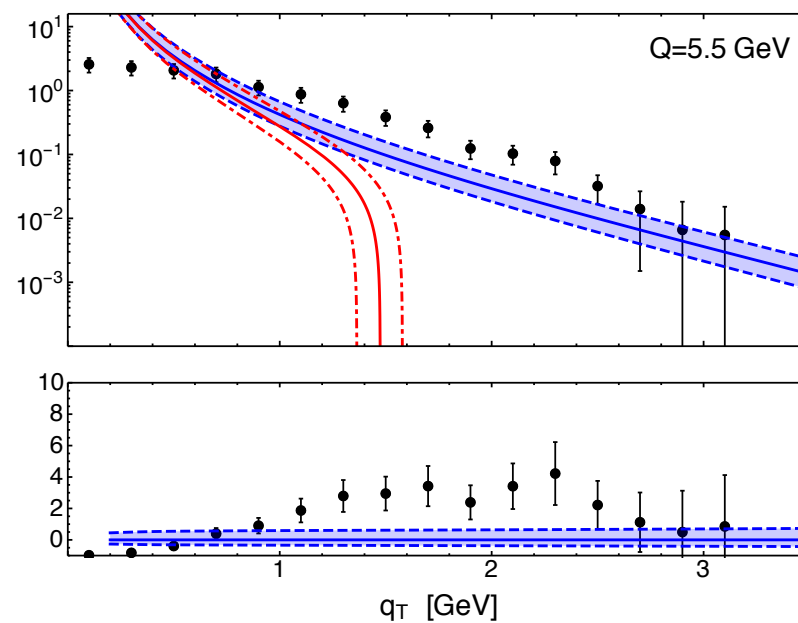
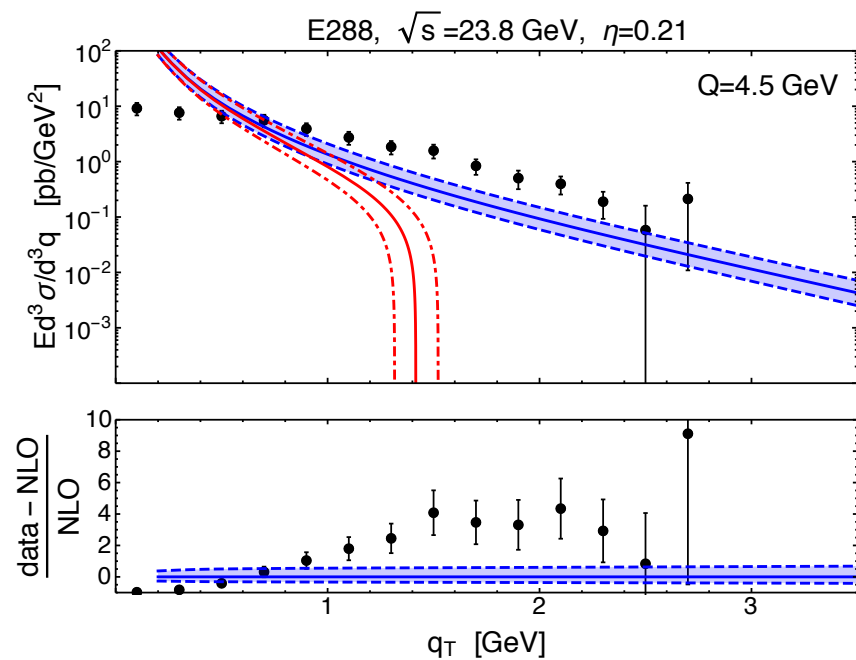
$$pp \rightarrow \mu^+ \mu^- X \quad \sqrt{s} = 38.8 \text{ GeV}$$



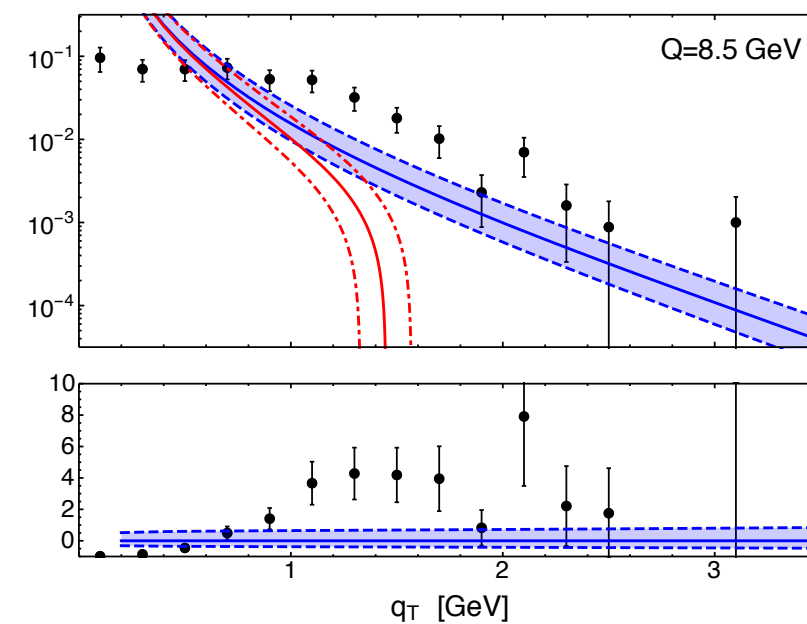
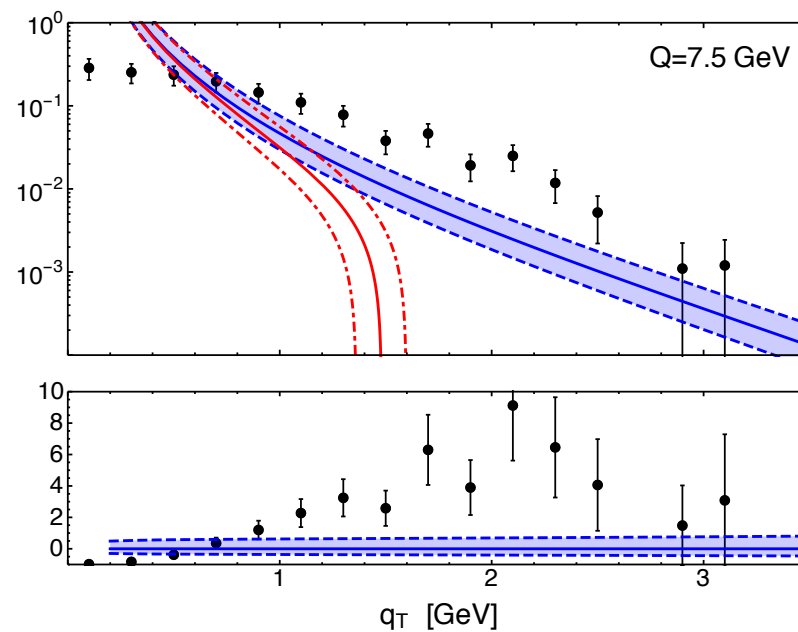
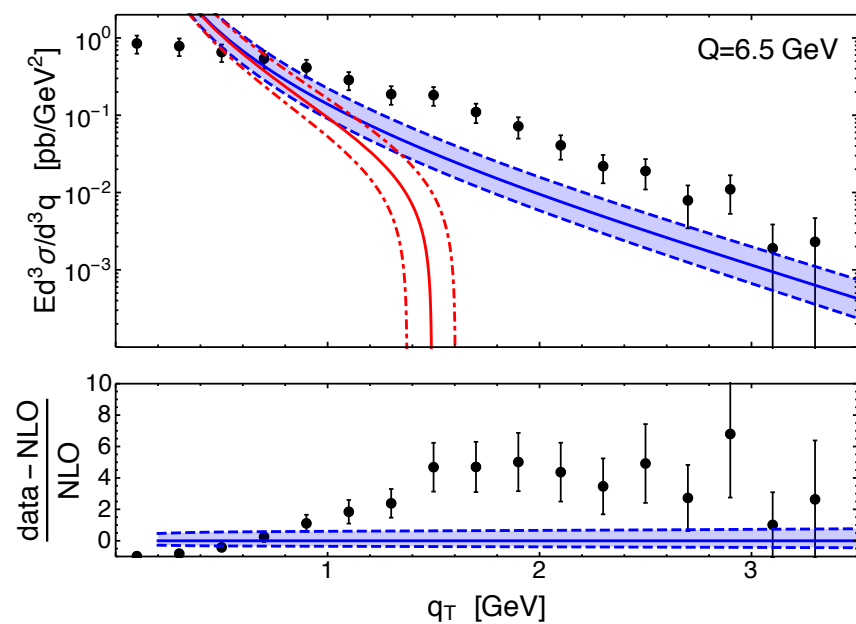
--- NLO

E288

$$p \text{ Cu}, p \text{ Pt} \rightarrow \mu^+ \mu^- X$$

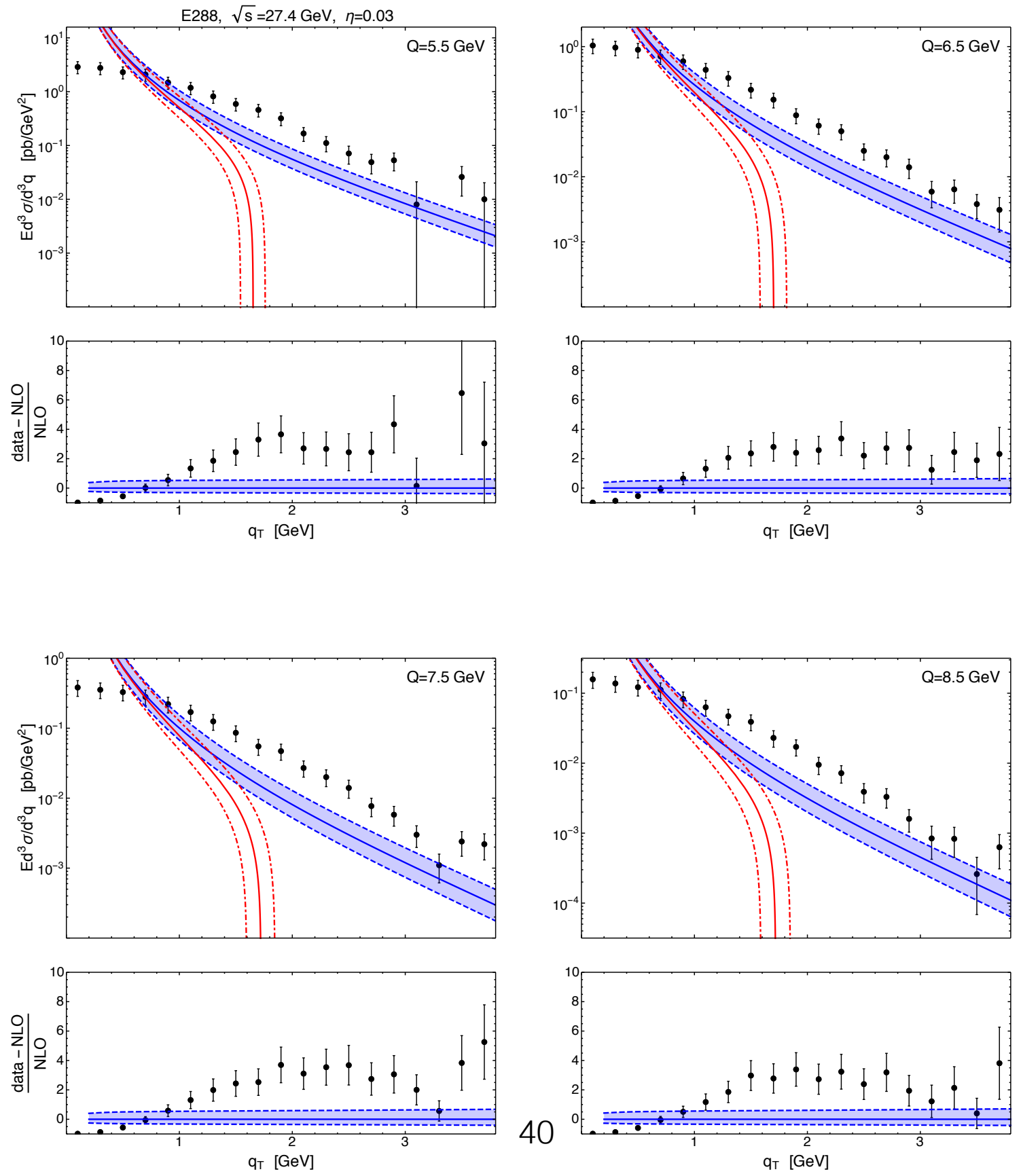


--- NLO
--- asymptotic



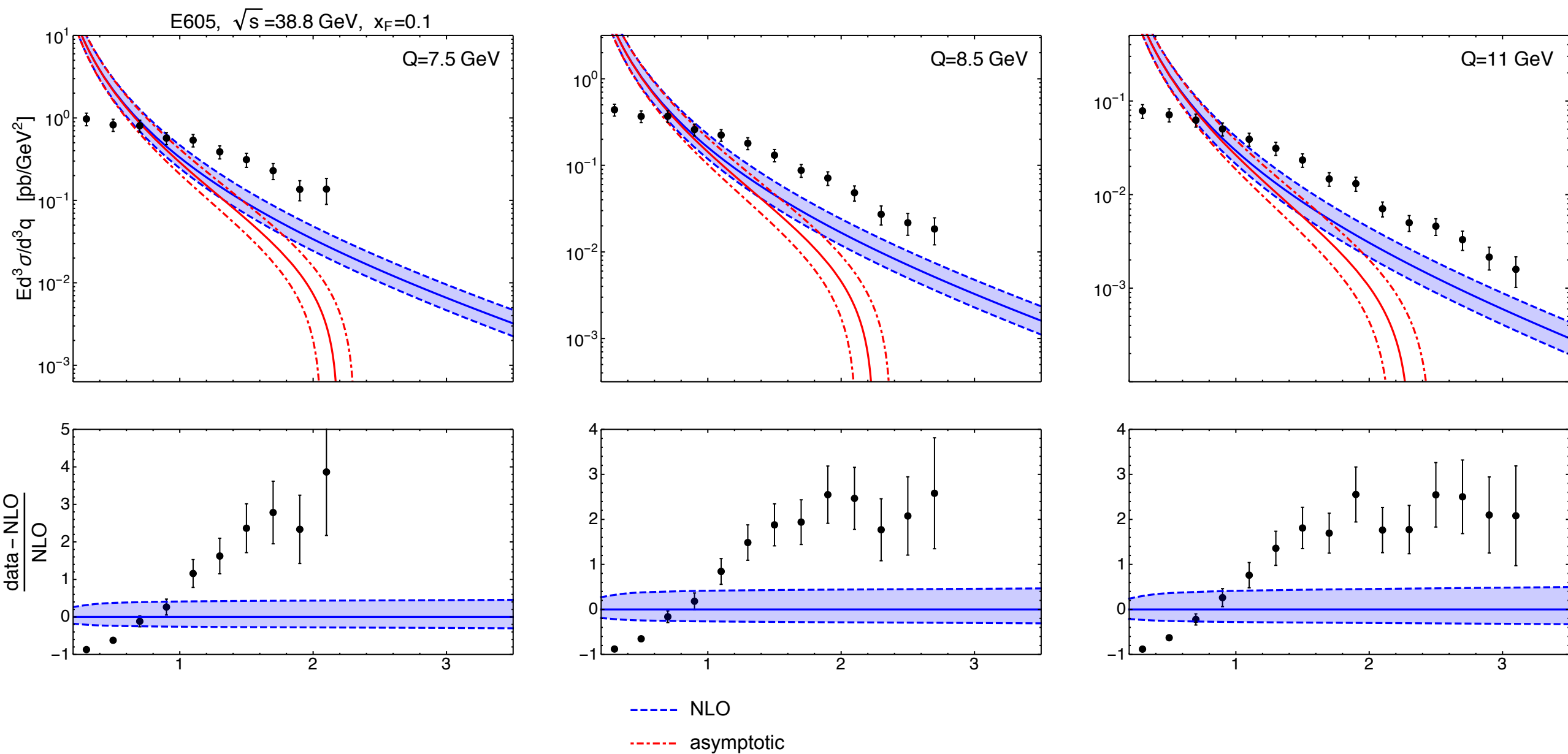
E288

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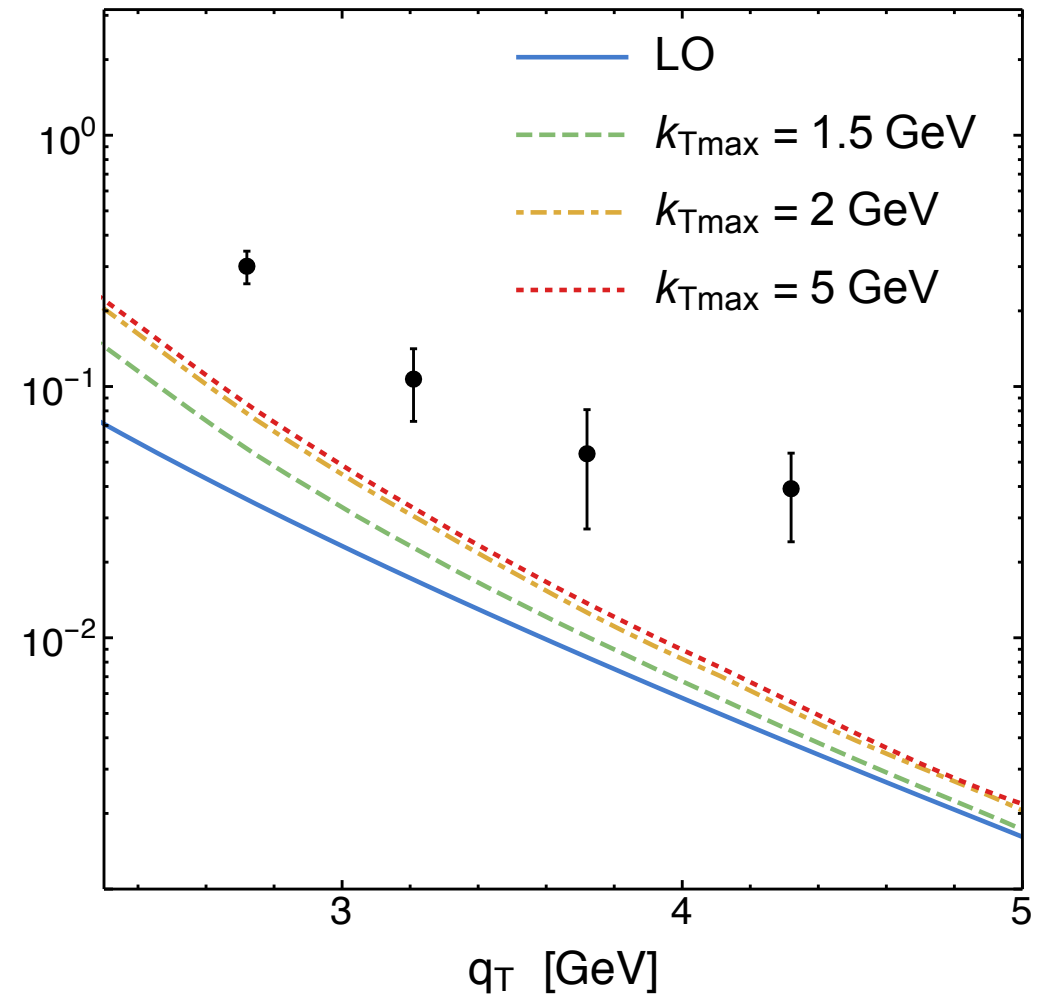
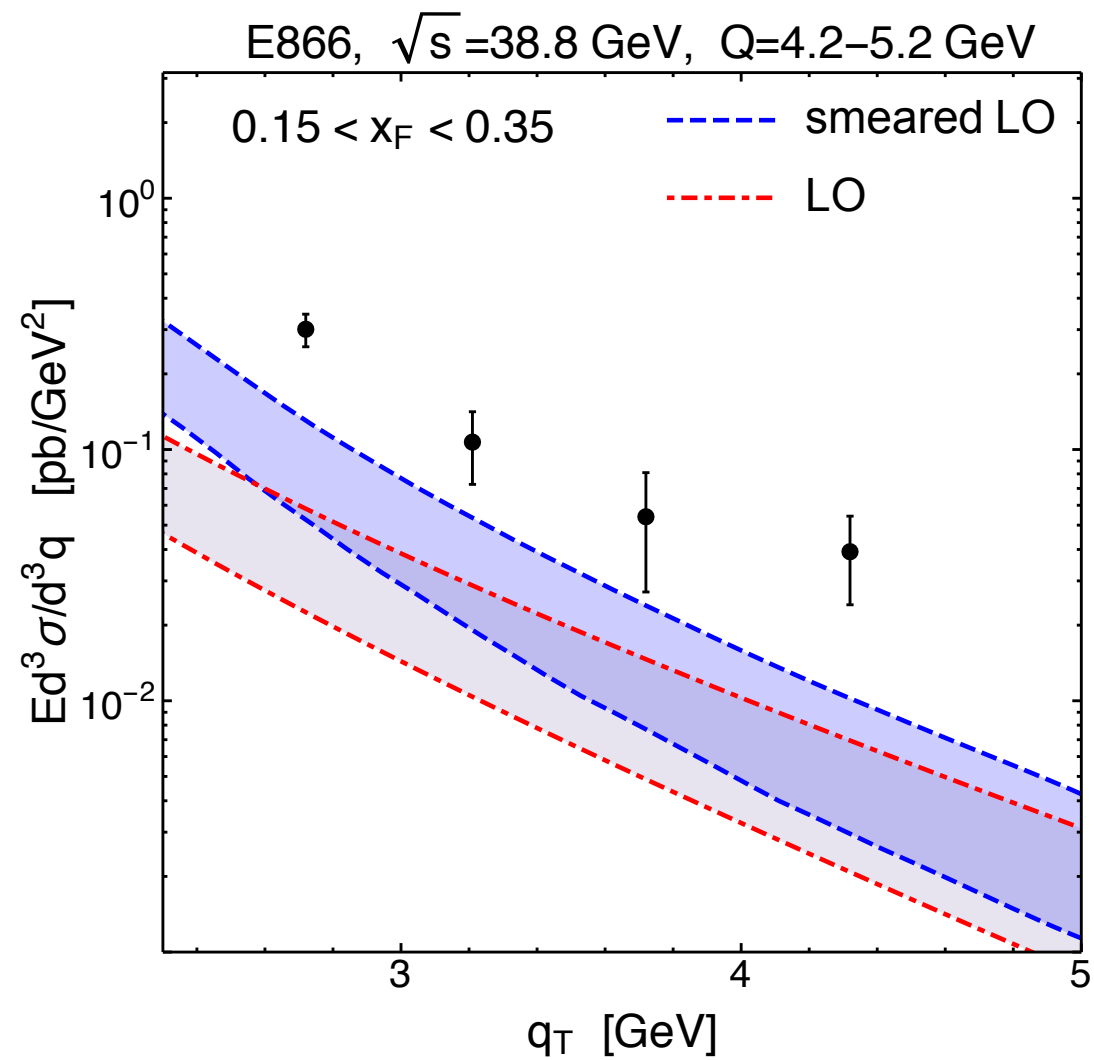


E605

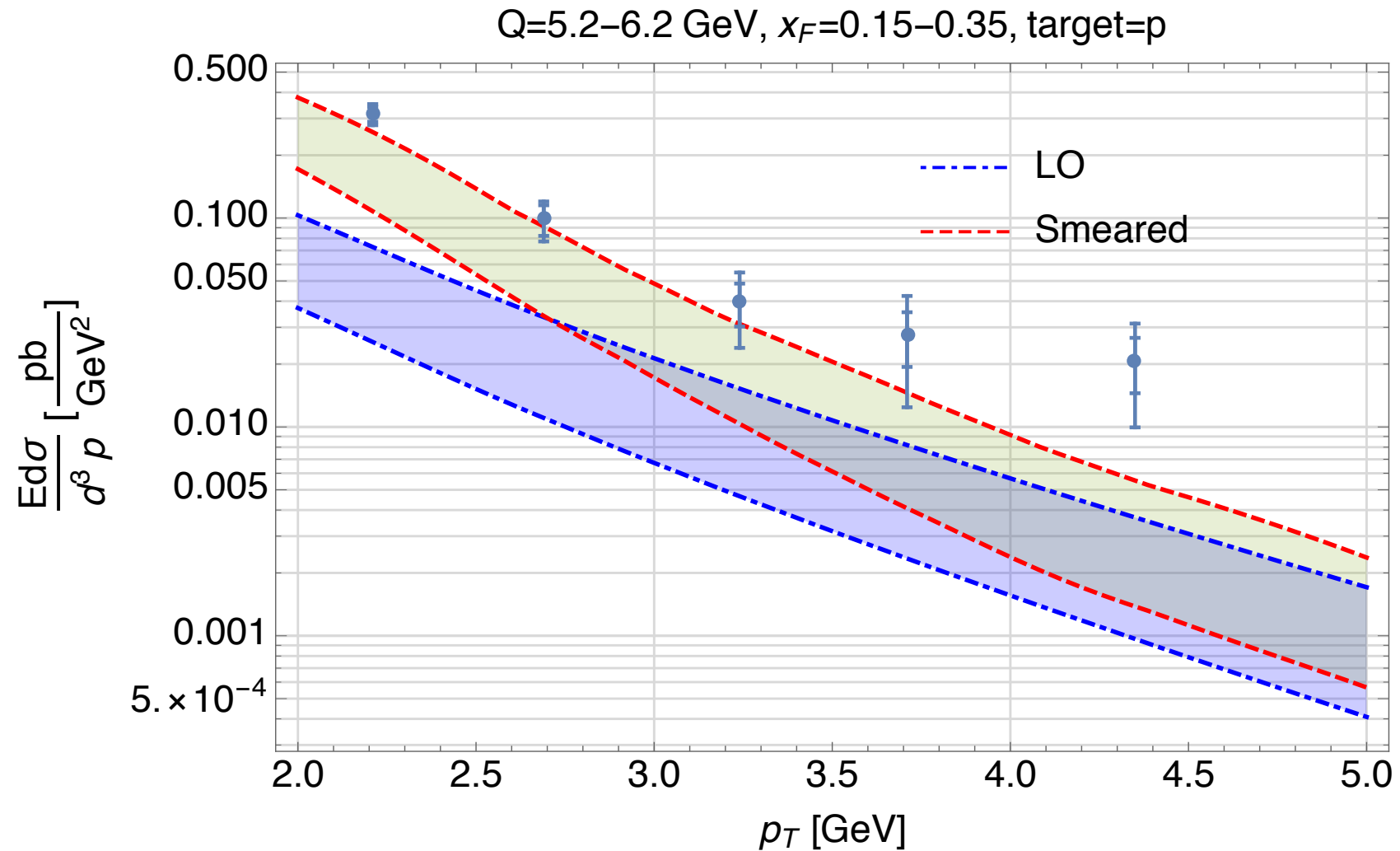
$$p Cu \rightarrow \mu^+ \mu^- X$$



intrinsic k_T smearing



intrinsic k_T smearing



$$f_q(x, \mathbf{k}_T) = f(x) \frac{1}{\pi \langle k_T^2 \rangle} e^{-\frac{k_T^2}{\langle k_T^2 \rangle}}$$

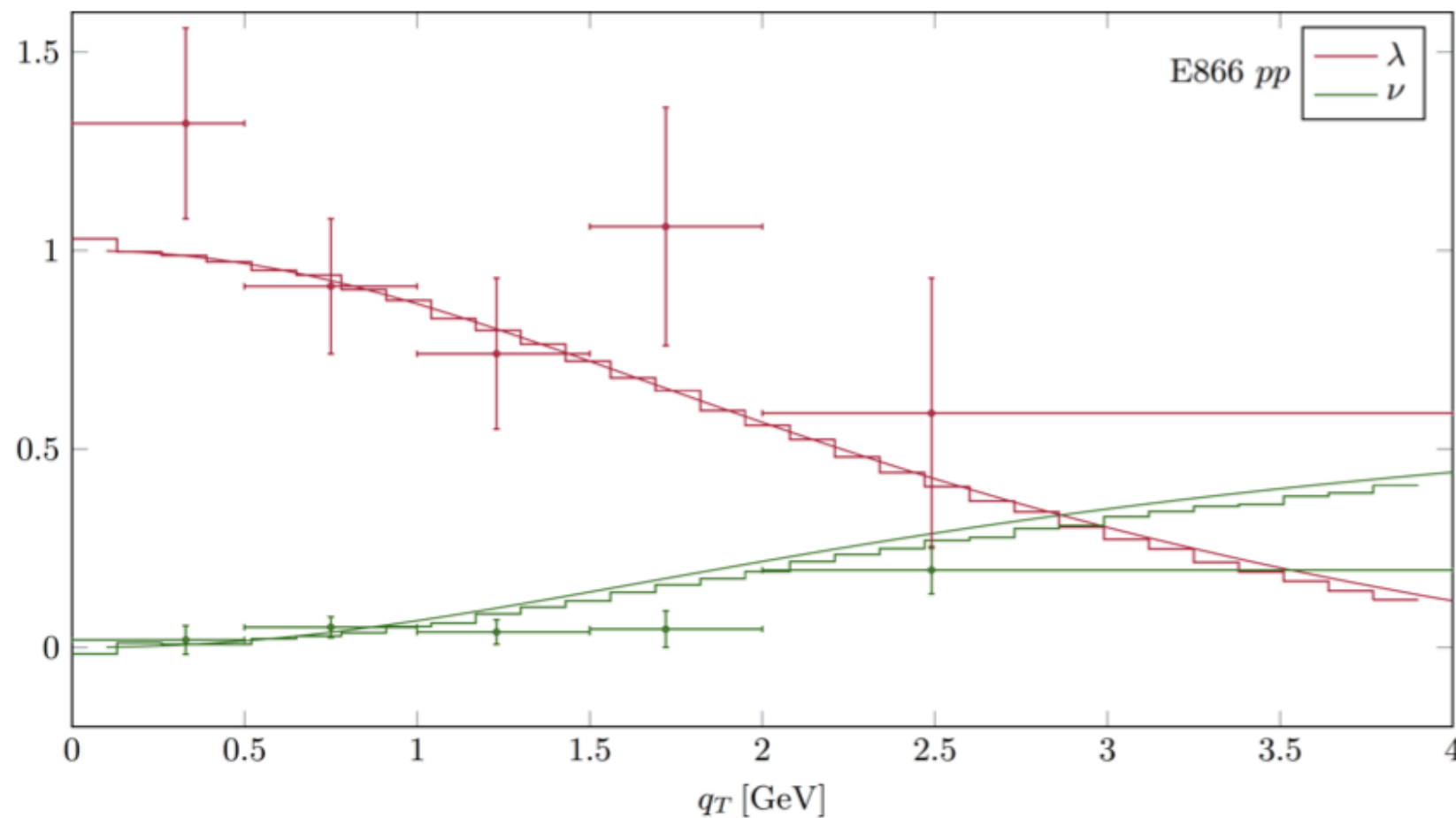
$$\langle k_T^2 \rangle = 1 \text{ GeV}^2$$

angular coefficients

Lambertsen, WV '16

$pp, E = 800 \text{ GeV}$

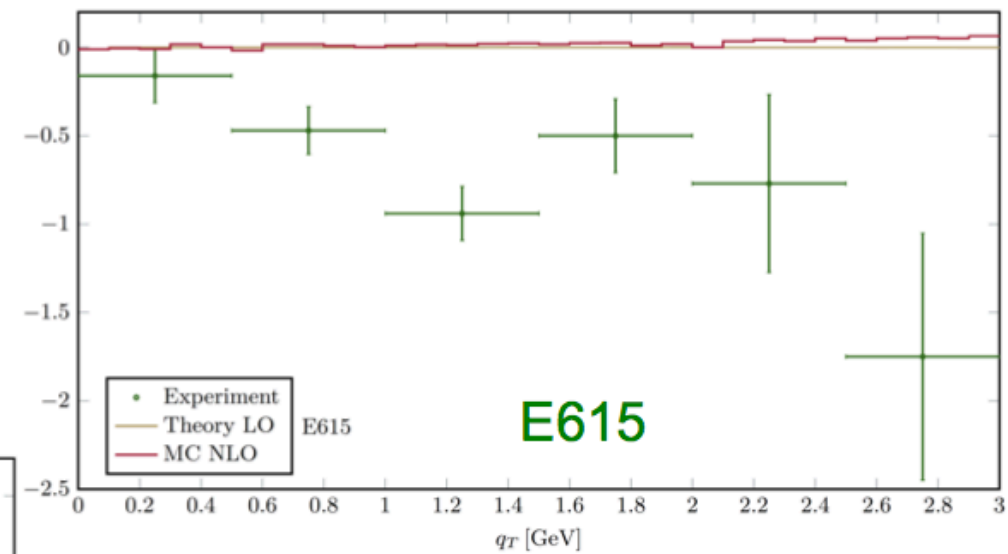
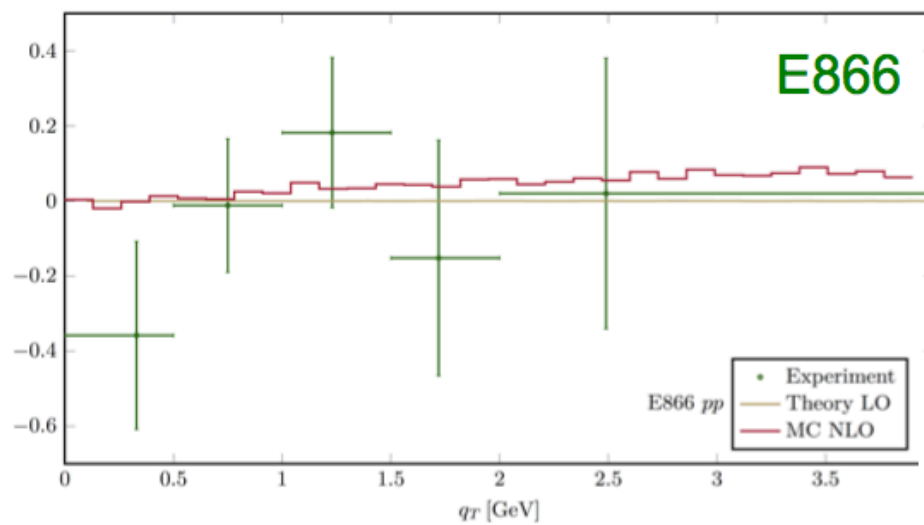
E866



W. Vogelsang @Transversity 2017

angular coefficients

Lam-Tung $1 - \lambda - 2\nu$



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