Wigner functions and nucleon structure

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3DSPIN MAPPING THE PROTON IN 3D
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Outline

- Introduction to Wigner distributions
- Model independent decomposition in phase-space transverse modes
- OAM from Wigner distributions
- Examples in a light-front quark model

Phase-Space Distributions in Quantum-Mechanics

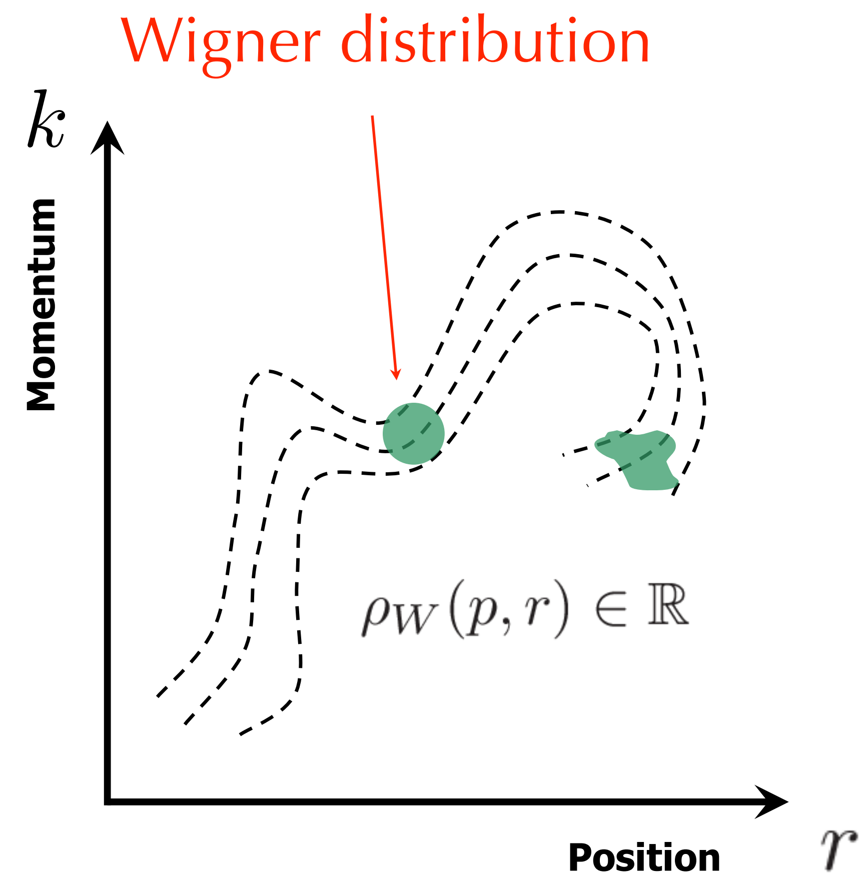
Wigner (1932); Moyal (1949)

$$\rho_W(r,k) = \int \frac{\mathrm{d}z}{2\pi} e^{-ikz} \psi^*(r-\frac{z}{2})\psi(r+\frac{z}{2})$$
$$= \int \frac{\mathrm{d}\Delta}{2\pi} e^{-i\Delta r} \phi^*(k+\frac{\Delta}{2})\phi(k-\frac{\Delta}{2})$$

Position-space density $|\psi(r)|^2 = \int \mathrm{d}k\, \rho_W(r,k)$

Momentum-space density $|\phi(k)|^2 = 2\pi \int \mathrm{d}r\, \rho_W(r,k)$

Quantum average $\langle \hat{O} \rangle = \int \mathrm{d}r\, \mathrm{d}k\, O(r,k)\, \rho_W(r,k)$



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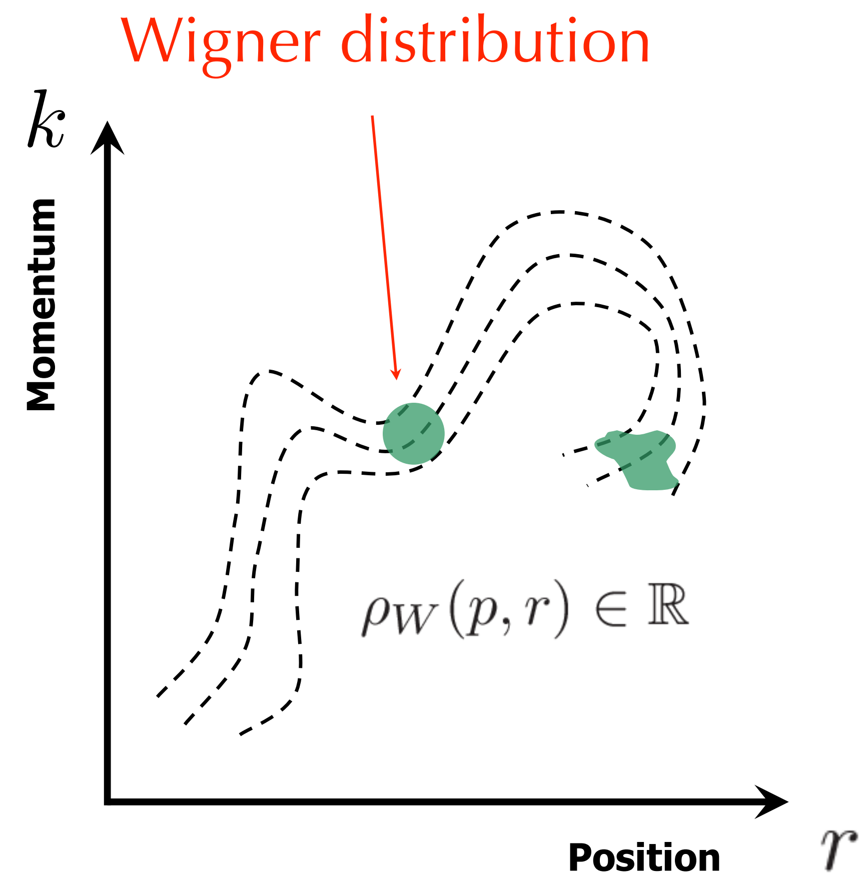


Position-space density $|\psi(r)|^2 = \int dk\, \rho_W(r,k)$

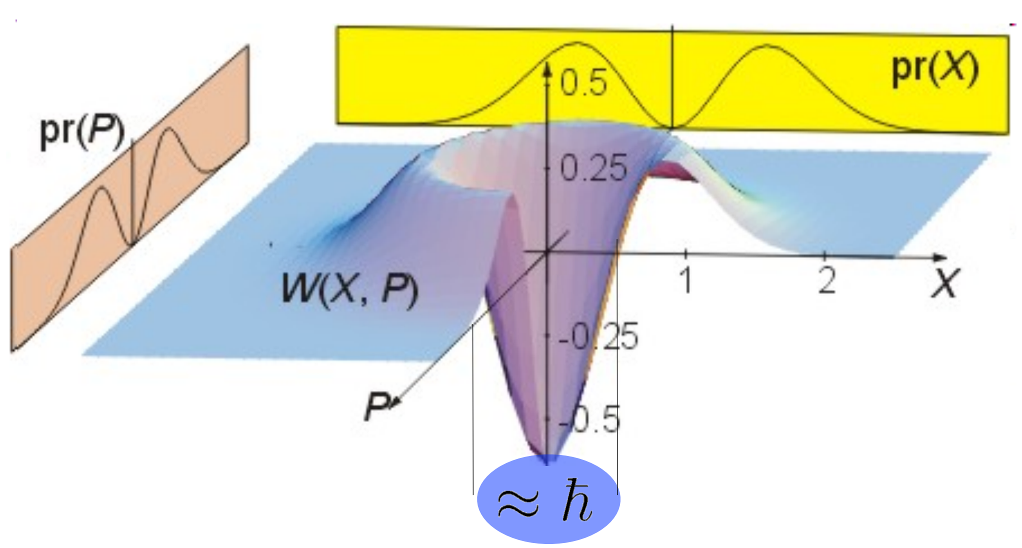
Momentum-space density $|\phi(k)|^2 = 2\pi \int dr\, \rho_W(r,k)$

Quantum average $\langle \hat{O} \rangle = \int dr\, dk\, O(r,k)\, \rho_W(r,k)$

Heisenberg's uncertainty relation ⚠ $\rho(r,k) \not\geq 0$

Quasi-probabilistic interpretation

$\xrightarrow{\hbar \to 0}$ classical density



Wigner Distributions in QFT

Quark Wigner operator

$$\widehat{W}^{[\Gamma]}(\vec{r}, k) = \int \frac{d^4 z}{(2\pi)^4} e^{ik\cdot z} \bar{\psi}(\vec{r} - \frac{z}{2})\, \Gamma\, \mathcal{W}\, \psi(\vec{r} + \frac{z}{2})$$

Dirac matrix
~ quark polarization

Wilson line

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Fixed light-front time

$$z^+ = 0 \quad \Longleftrightarrow \quad \int \mathrm{d}k^-$$

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Wigner distributions
in the Breit frame

$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{r},k^+,\vec{k}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \langle \frac{\vec{\Delta}}{2},\Lambda'|\widehat{W}^{[\Gamma]}(0,k^+,\vec{k}_\perp)|-\frac{\vec{\Delta}}{2},\Lambda\rangle$$

3+3 D

no semi-classical interpretation

Ji (2003)
Belitsky, Ji, Yuan (2004)

Wigner Distributions in QFT

Dirac matrix
$\sim$ quark polarization

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Wigner distributions in the Drell-Yan frame ($\Delta^+ = 0$)
$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{b}_\perp, k^+, \vec{k}_\perp) = \frac{1}{2} \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \langle p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda' | \widehat{W}^{[\Gamma]}(0, k^+, \vec{k}_\perp) | p^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda \rangle$$

2+3 D

semi-classical interpretation

GTMDs

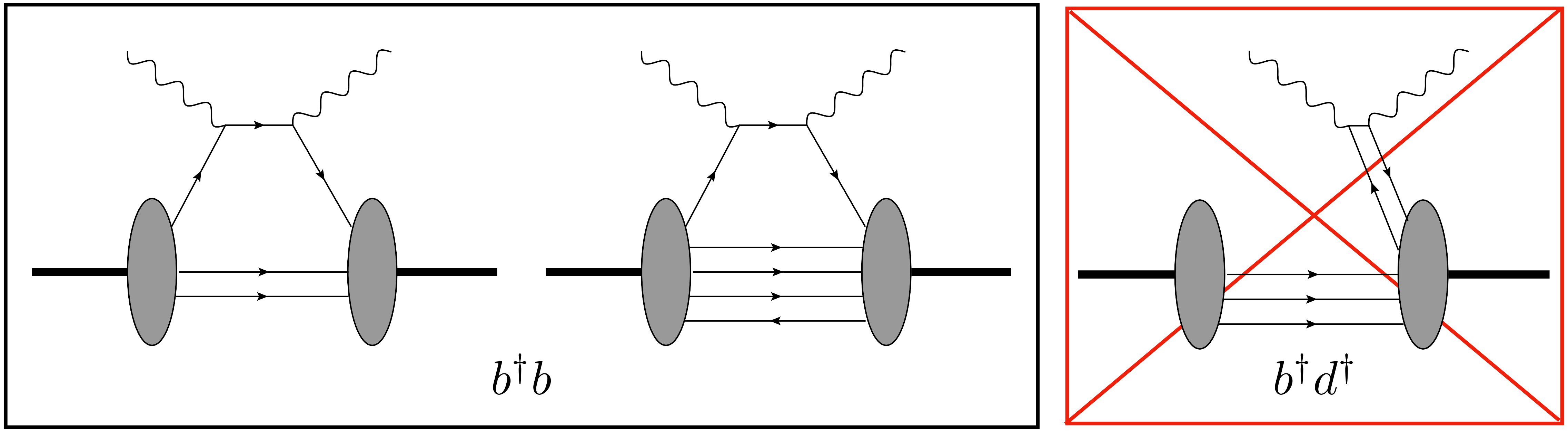
Lorcè, BP (2011)
Lorcè, BP, Xiong, Yuan (2012)

Quasi-probabilistic interpretation

✓ $\int \mathrm{d}r^- \sim \Delta^+ = 0$ ⟶ no sensitivity to longitudinal Lorentz contraction

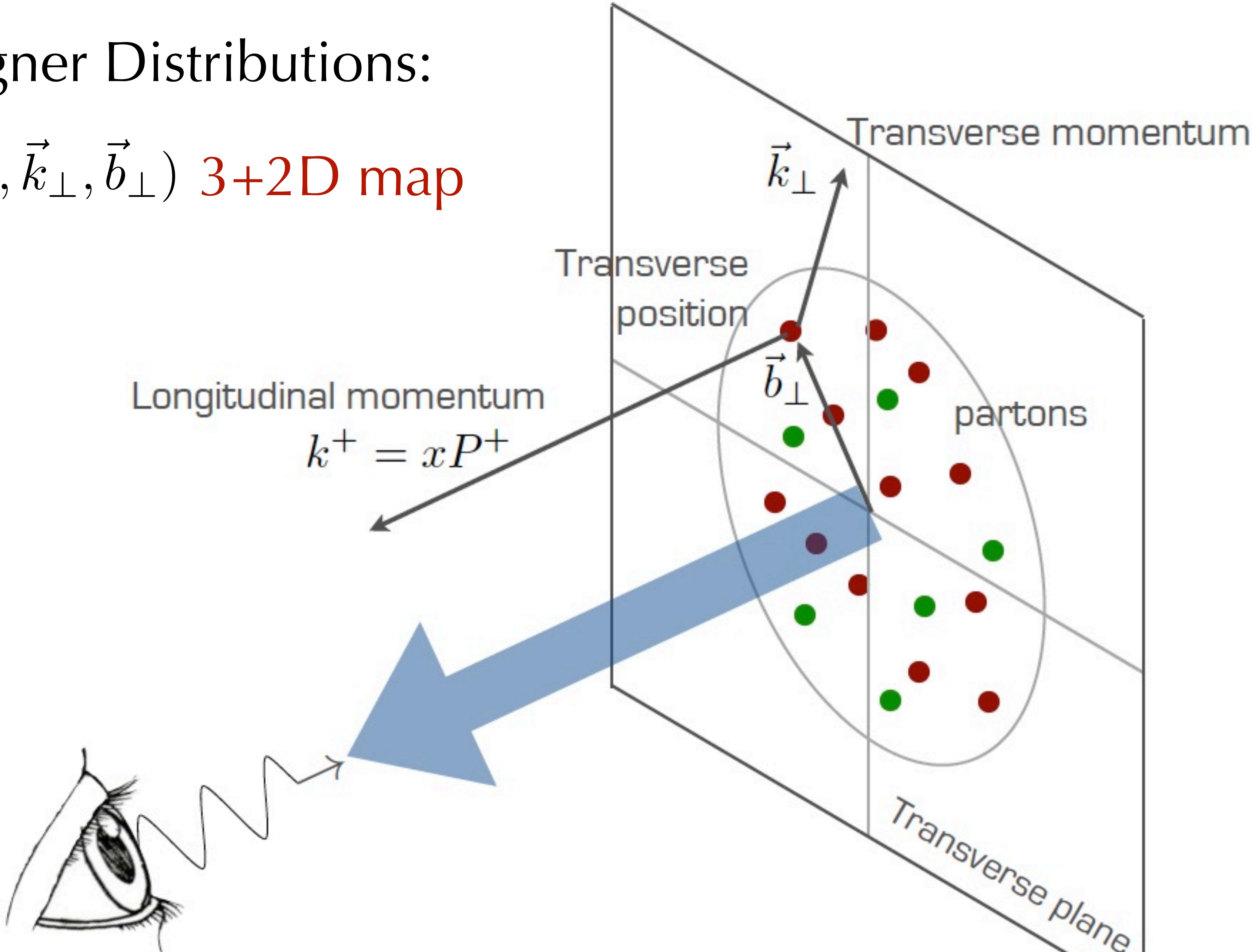
✓ $\Delta_\perp \neq 0$: Transverse boosts ⟶ no transverse Lorentz contraction

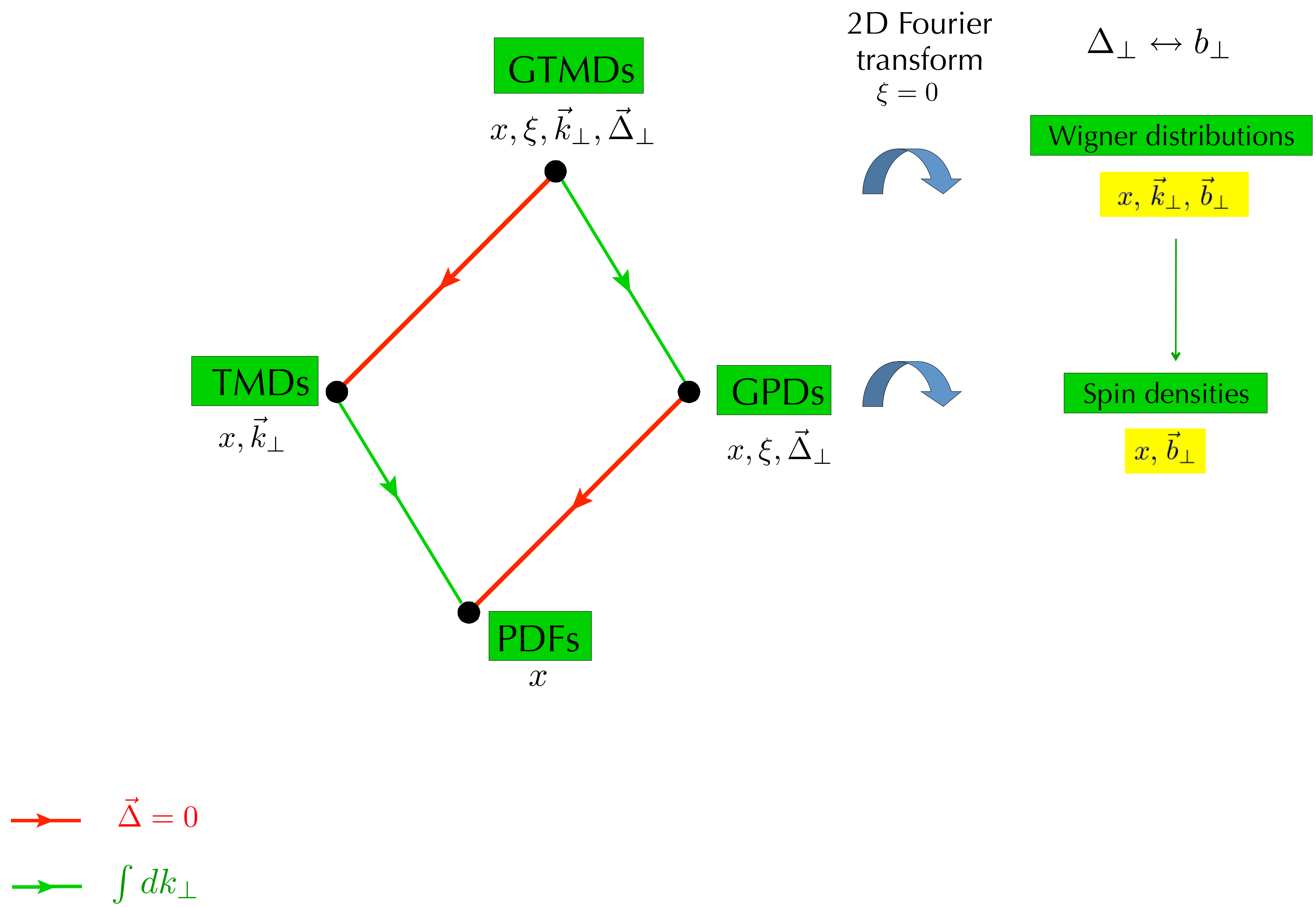
✓ Particle number is conserved in Drell-Yan frame $\Delta^+ = 0$



Wigner Distributions:

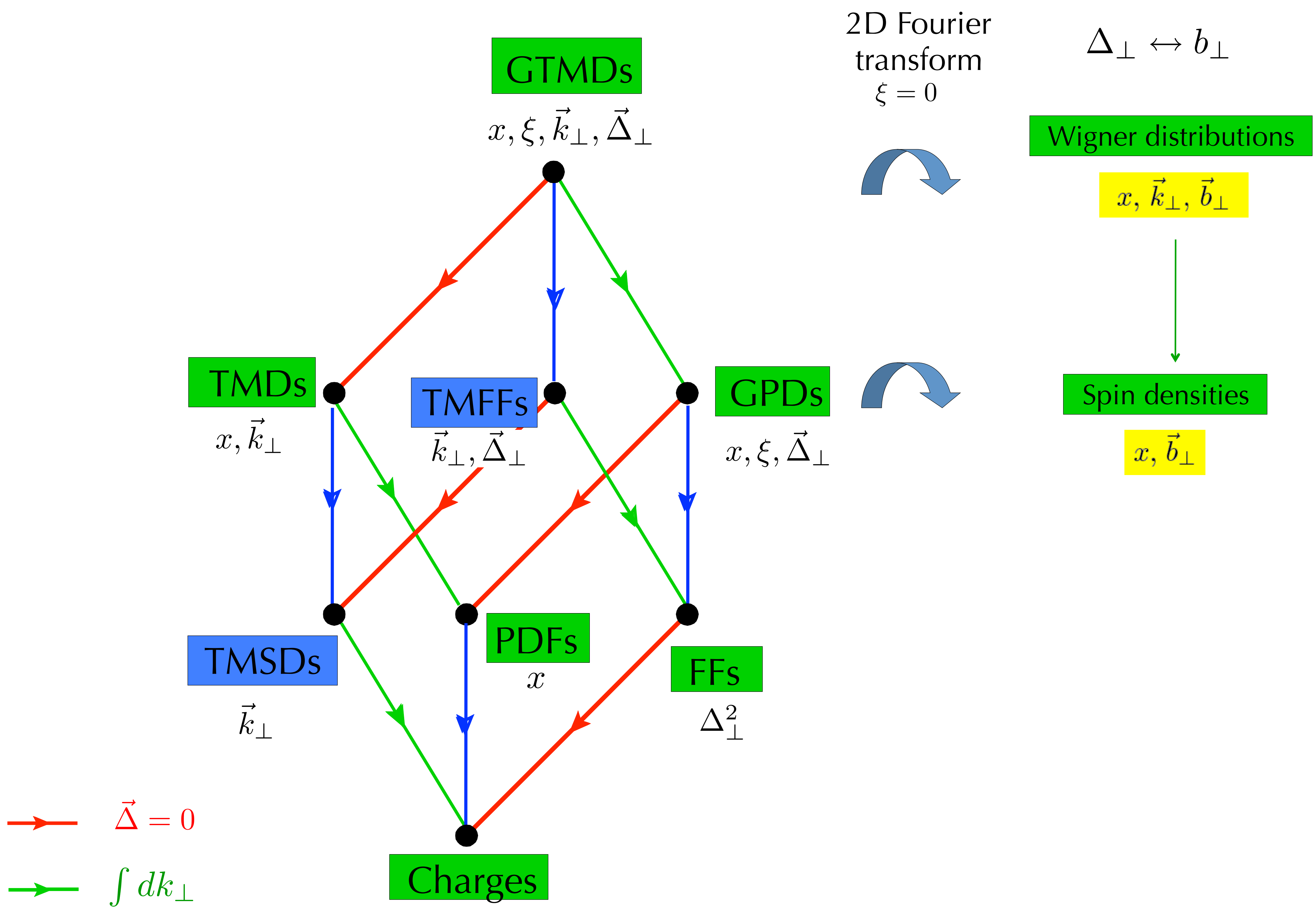
$\rho(x, \vec{k}_\perp, \vec{b}_\perp)$ 3+2D map





Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

Burkardt, BP, EPJA 52 (2016) 161



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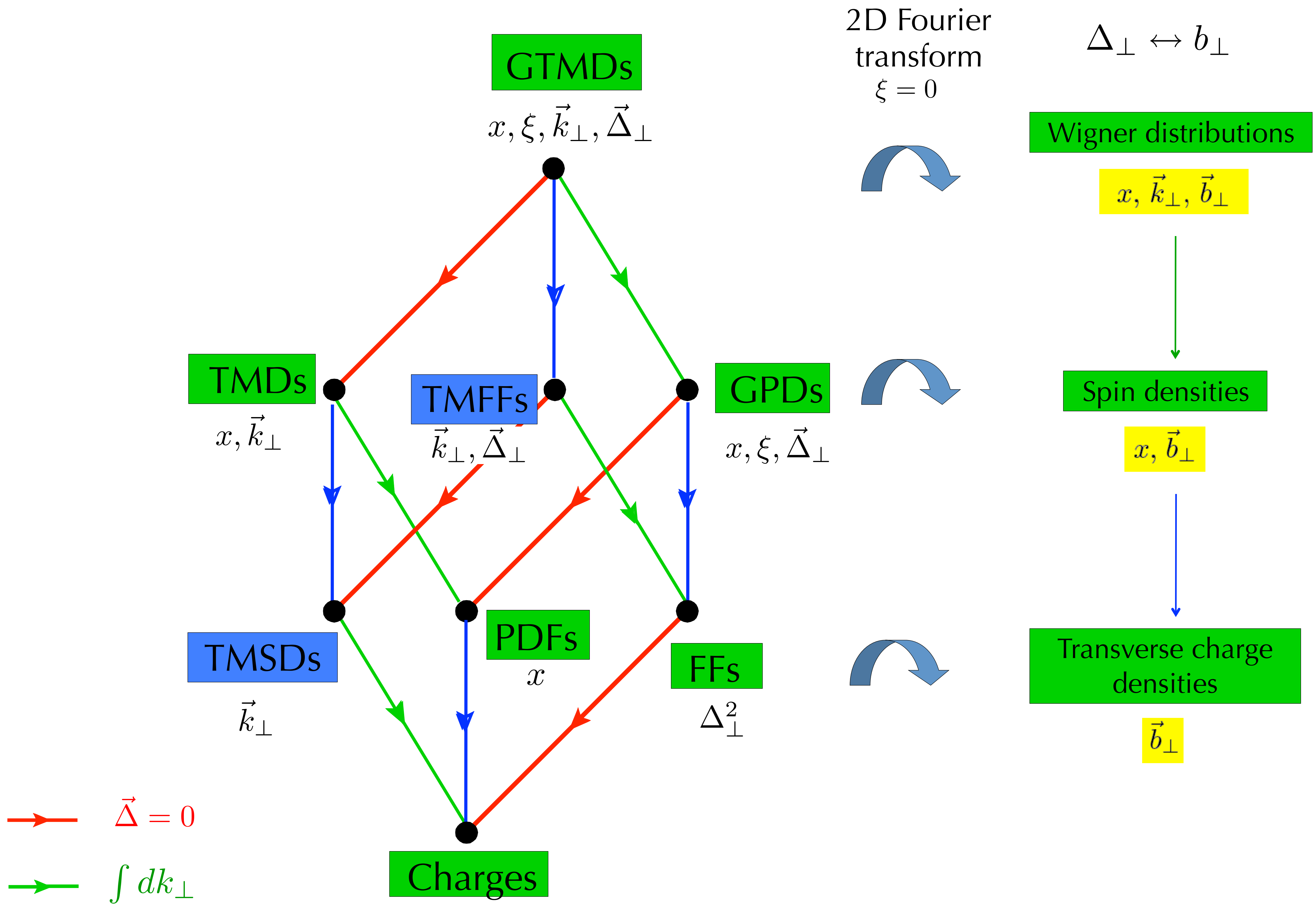

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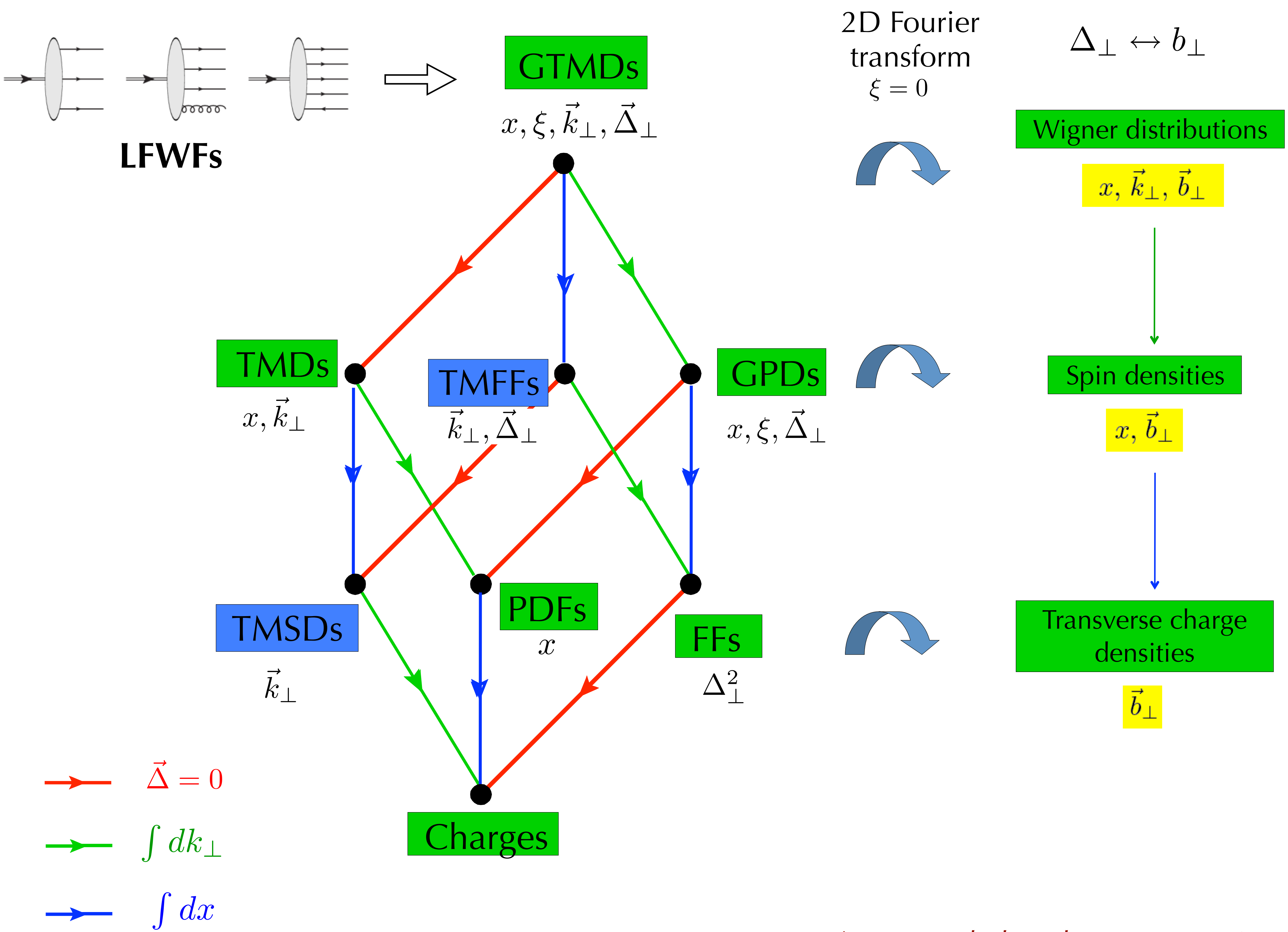

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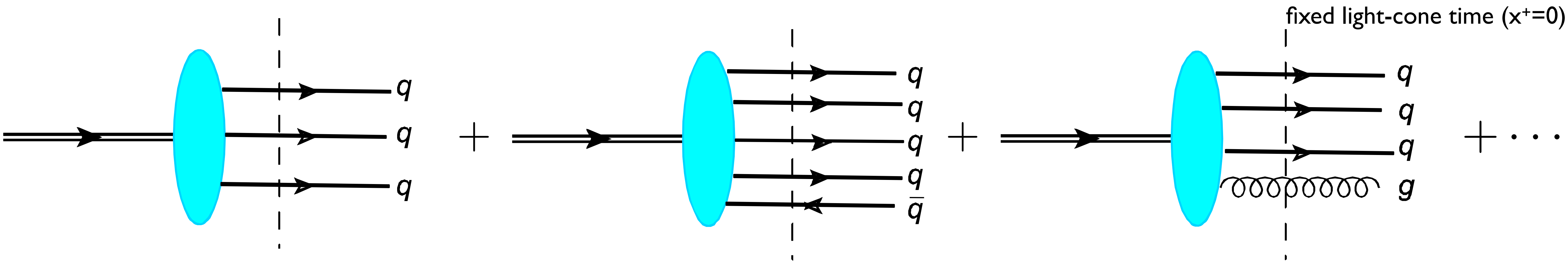
Light-Front Wave Functions (LFWFs)

- Fock expansion of Nucleon state:

$$|N\rangle = \Psi_{3q}|qqq\rangle + \Psi_{3q\,q\bar{q}}|3q\,q\bar{q}\rangle + \Psi_{3q\,g}|qqqg\rangle + \cdots$$



- Probability to find N partons in the nucleon $\rho^{\Lambda}_{N,\beta} = \int [dx]_N [d^2k_\perp]_N |\Psi^{\Lambda}_{\lambda_1 \cdots \lambda_N}|^2$

  normalization $\sum_{N,\beta} \rho^{\Lambda}_{N,\beta} = 1$

- Invariant under boost $\longrightarrow$ independent on P^μ

- Linear and angular momentum conservation

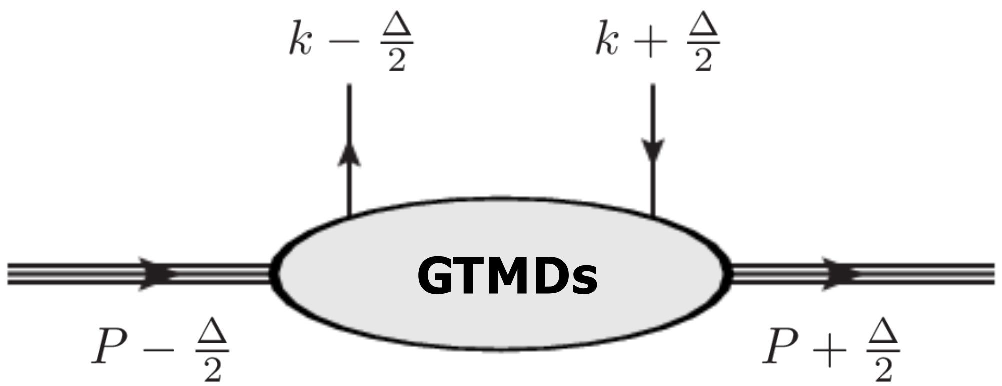
$$P^+ = \sum_{i=1}^{N} k_i^+$$

$$\vec{P}_\perp = \sum_{i=1}^{N} \vec{k}_{i\,\perp} = \vec{0}_\perp$$

$$\Lambda = \sum_{i=1}^{N} \lambda_i + l_z$$

⚠ $A^+ = 0$ gauge

LFWF overlap representation



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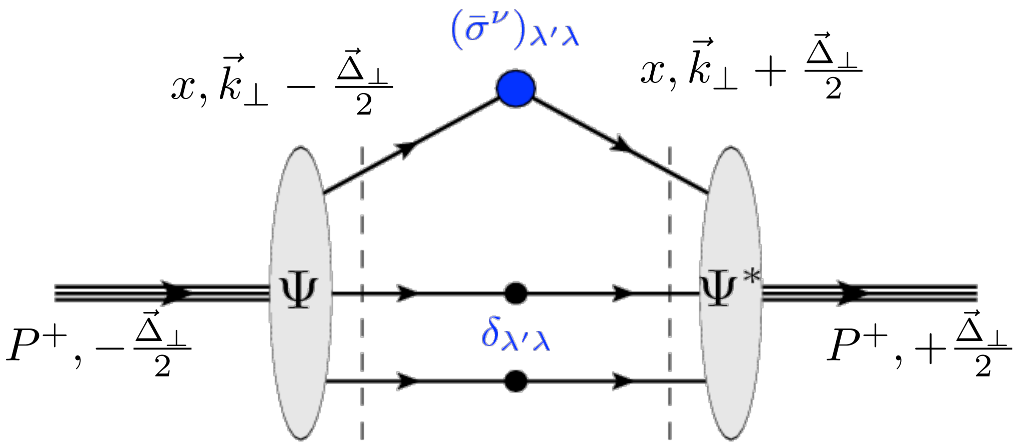

$$W^{[\Gamma]}_{\Lambda'\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp) = \frac{1}{\sqrt{1-\xi^2}} \sum_{\beta,\beta'} \int [\mathrm{d}x]_3 [\mathrm{d}^2k_\perp]_3 \bar\delta(\vec{k}) \psi^*_{\Lambda'\beta'}(r')\psi_{\Lambda\beta}(r) M^{[\Gamma]\beta'\beta}$$

Momentum ↑↓ Polarization

$$\bar\delta(\vec{k}) \equiv \textstyle\sum_{i=1}^3 \Theta(x)\delta(x-x_i)\delta^{(2)}(\vec{k}_\perp - \vec{k}_{i\perp})$$

$$M^{[\Gamma]\beta'\beta} = M^{[\Gamma]\lambda'_1\lambda_1}\delta^{\lambda'_2\lambda_2}\delta^{\lambda'_3\lambda_3}$$

$$[\mathrm{d}x]_3 = \left[\prod_{i=1}^3 \mathrm{d}x_i\right]\delta\left(1-\sum_{i=1}^3 x_i\right)$$

$$M^{[\Gamma]\lambda'\lambda} = \frac{\bar u(p',\lambda')\Gamma u(p,\lambda)}{2P^+\sqrt{1-\xi^2}}$$

$$[\mathrm{d}^2k_\perp]_3 = \left[\prod_{i=1}^3 \frac{\mathrm{d}^2k_{i\perp}}{2(2\pi)^3}\right] 2(2\pi)^3\delta^{(2)}\left(\sum_{i=1}^3 \vec{k}_{i\perp}\right)$$

Lorcé, BP, Vanderhaeghen, JHEP05 (2011)

Light-Front Constituent Quark Model

- momentum-space wf
  Schlumpf, Ph.D. Thesis, hep-ph/921155

$$\Psi(k_i) = \frac{N}{(M_0^2 + \beta^2)^\gamma} \qquad M_0 = \sum_i^3 \sqrt{m_i^2 + \vec{k}_i^2}$$

N: normalization constant

β, γ parameters fitted to anomalous magnetic moments of the nucleon

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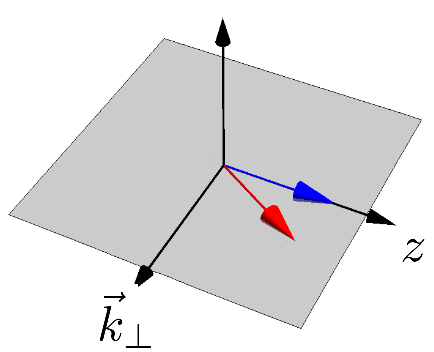
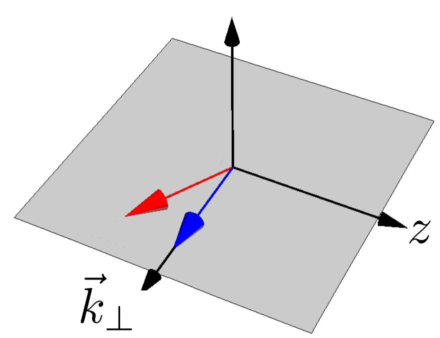
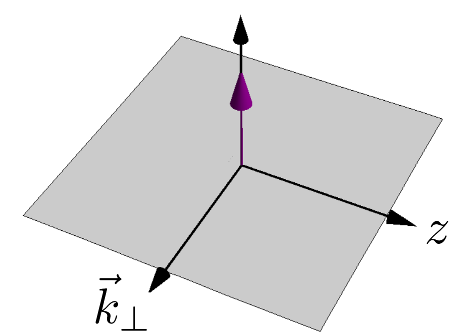
- spin-structure:

$$q_\lambda^{LC}(k) = D_{\lambda s}^{(1/2)*}\, q_s^C(k) \qquad D_{\lambda s}^{(1/2)*}(k) = \frac{1}{|\vec{K}|}\begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}$$







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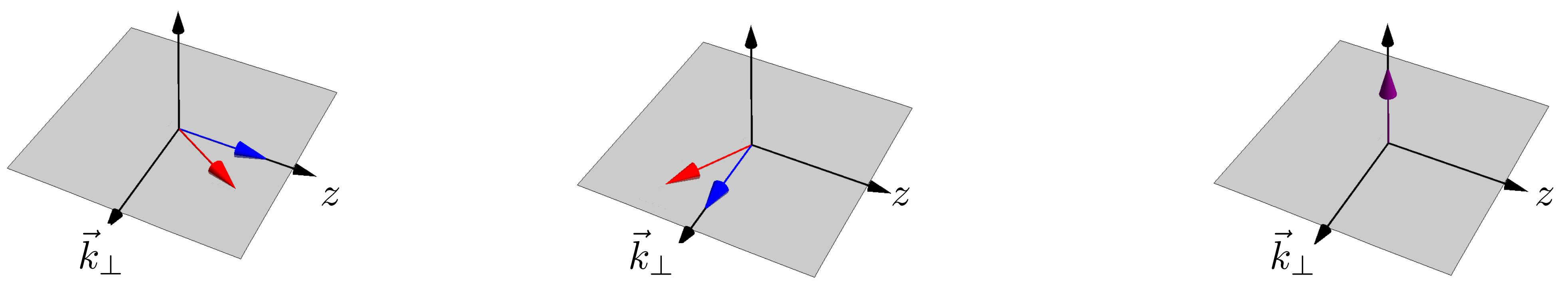
free quarks → $K_z = m + x\mathcal{M}_0$ $\quad \vec{K}_\perp = \vec{k}_\perp$ $\quad$ (Melosh rotation)



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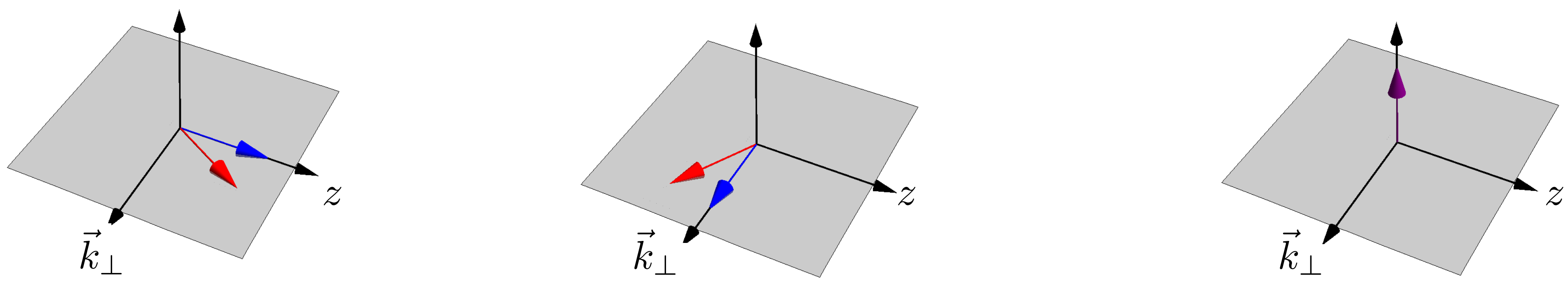

- SU(6) symmetry

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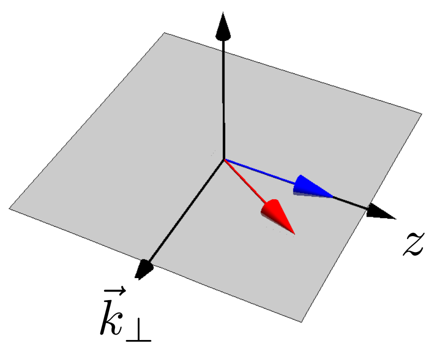
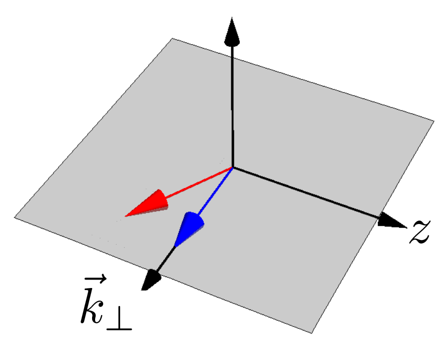
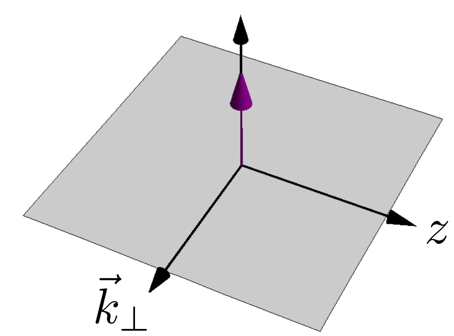






- SU(6) symmetry

Applications to:

GPDs and Form Factors: *BP, Boffi, Traini (2003)-(2007);*
TMDs: *BP, Cazzaniga, Boffi (2008); BP, Yuan (2010); BP, Schweitzer (2011)-(2016)*
Azimuthal Asymmetries: *Schweitzer, BP, Boffi, Efremov (2009)*
Wigner distributions: *Lorcé, BP (2011)-(2017)*

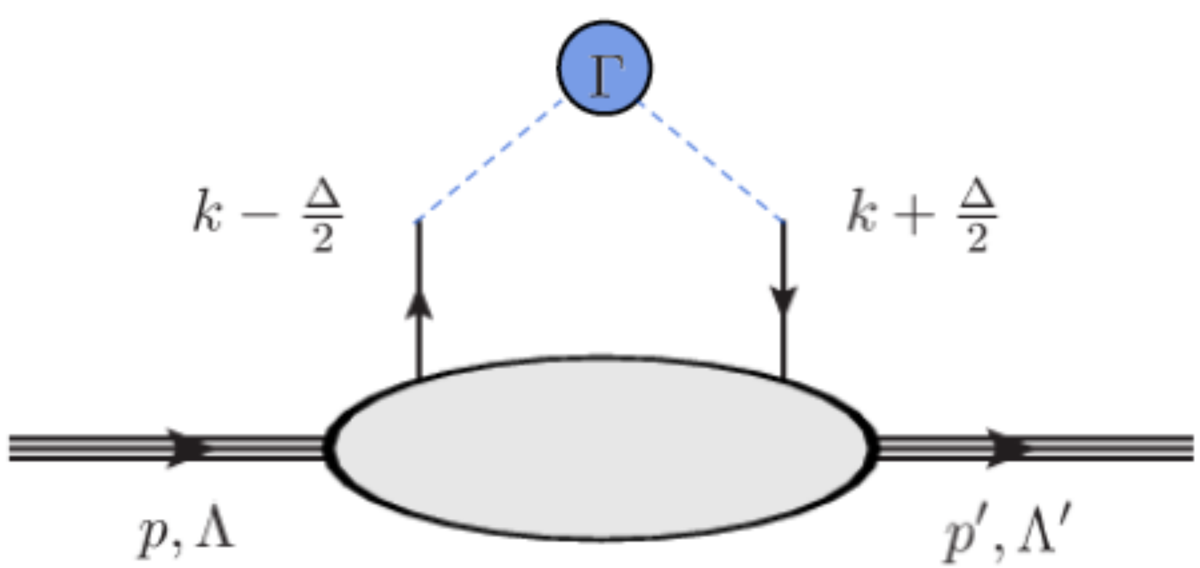
What can we learn from the Wigner distributions ❓

Transverse phase-space distributions

★ Twist-2: $\Gamma_{\text{twist}-2} = \gamma^+,\, \gamma^+\gamma_5,\, i\sigma^{j+}\gamma_5$

quark polarization: **U** **L** **T**

★ Nucleon polarization: **U** **L** **T**



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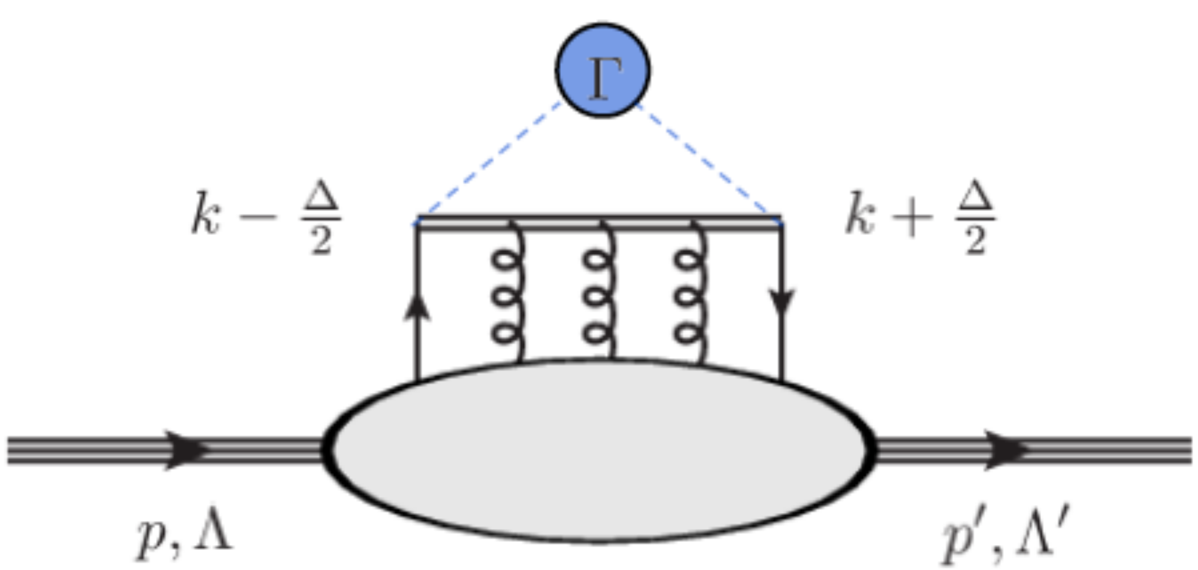
★Gauge link: T-even and T-odd functions



Transverse phase-space distributions

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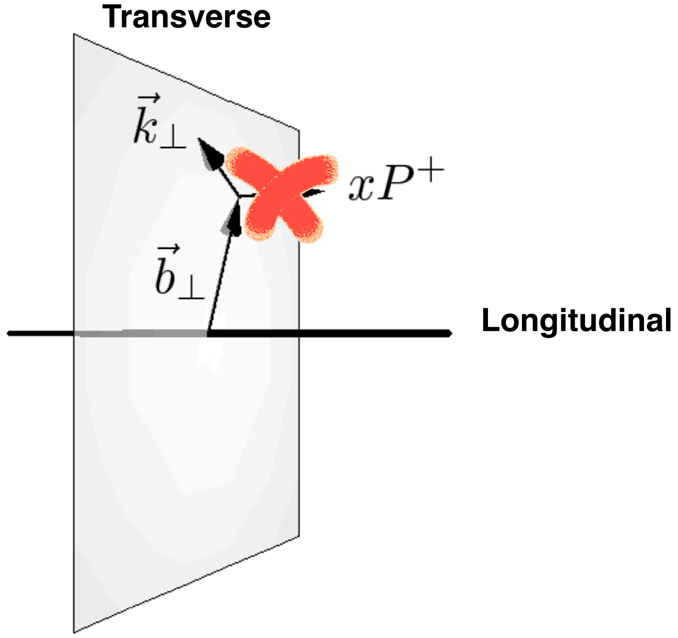
★ Nucleon polarization: **U** **L** **T** → 16 complex GTMDs → 32 real Wigner Distributions

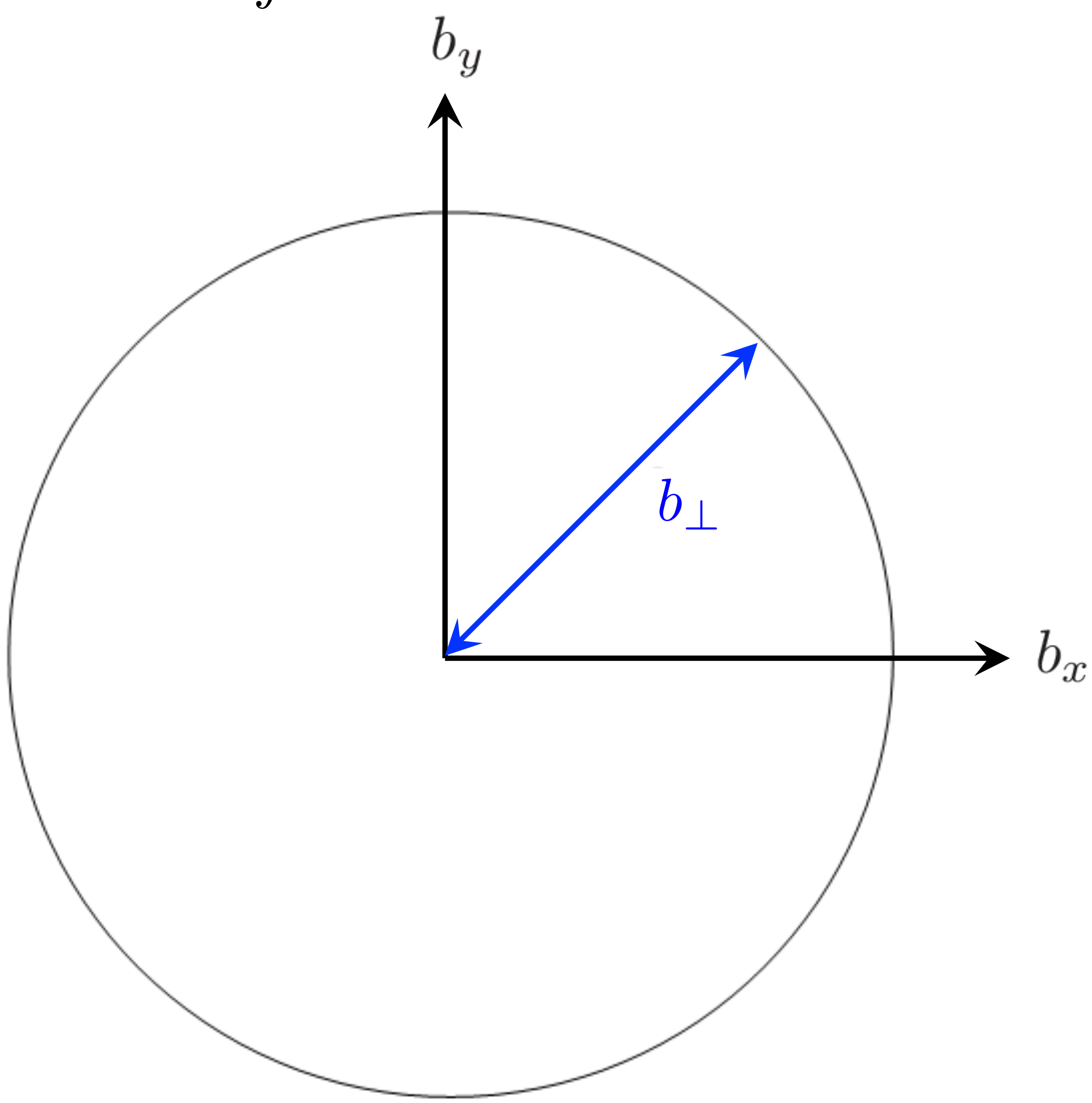


Transverse Phase-Space distributions

$$\rho_X(\vec{k}_\perp, \vec{b}_\perp) = \int {\rm d}x\, \rho_X(x, \vec{k}_\perp, \vec{b}_\perp) \quad X = UU,\ UL, UT,\ LU,\ \ldots$$

Phase-Space Transverse Modes

$$\rho_X(\vec{k}_\perp|\,\vec{b}_\perp) = \int \mathrm{d}x\,\rho_X(x,\vec{k}_\perp,\vec{b}_\perp;\hat{P}=\vec{e}_z,\eta=+1)\big|_{\vec{b}_\perp\ \text{fixed}}$$



Lorcé, BP, PRD 96, 2016

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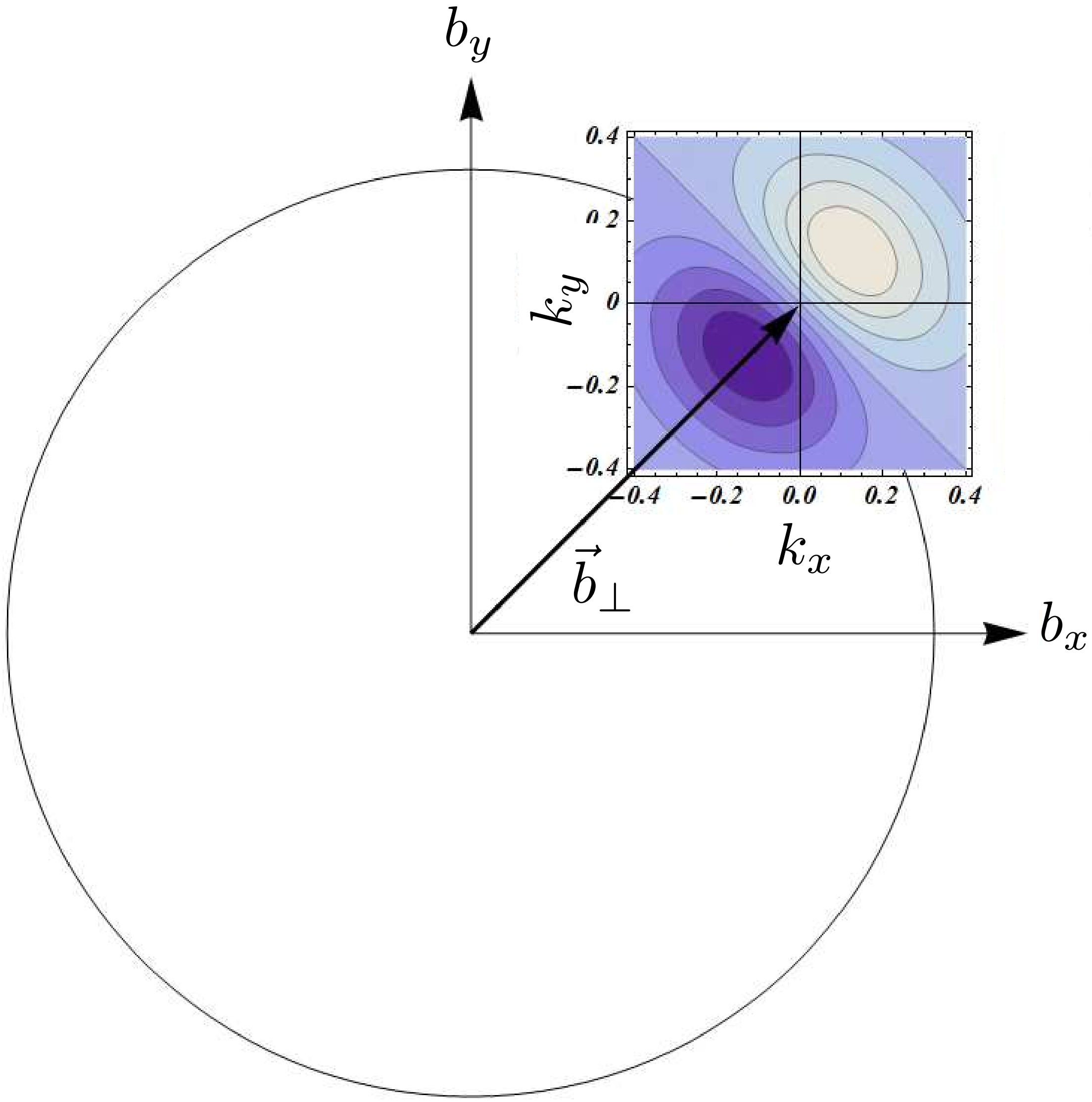

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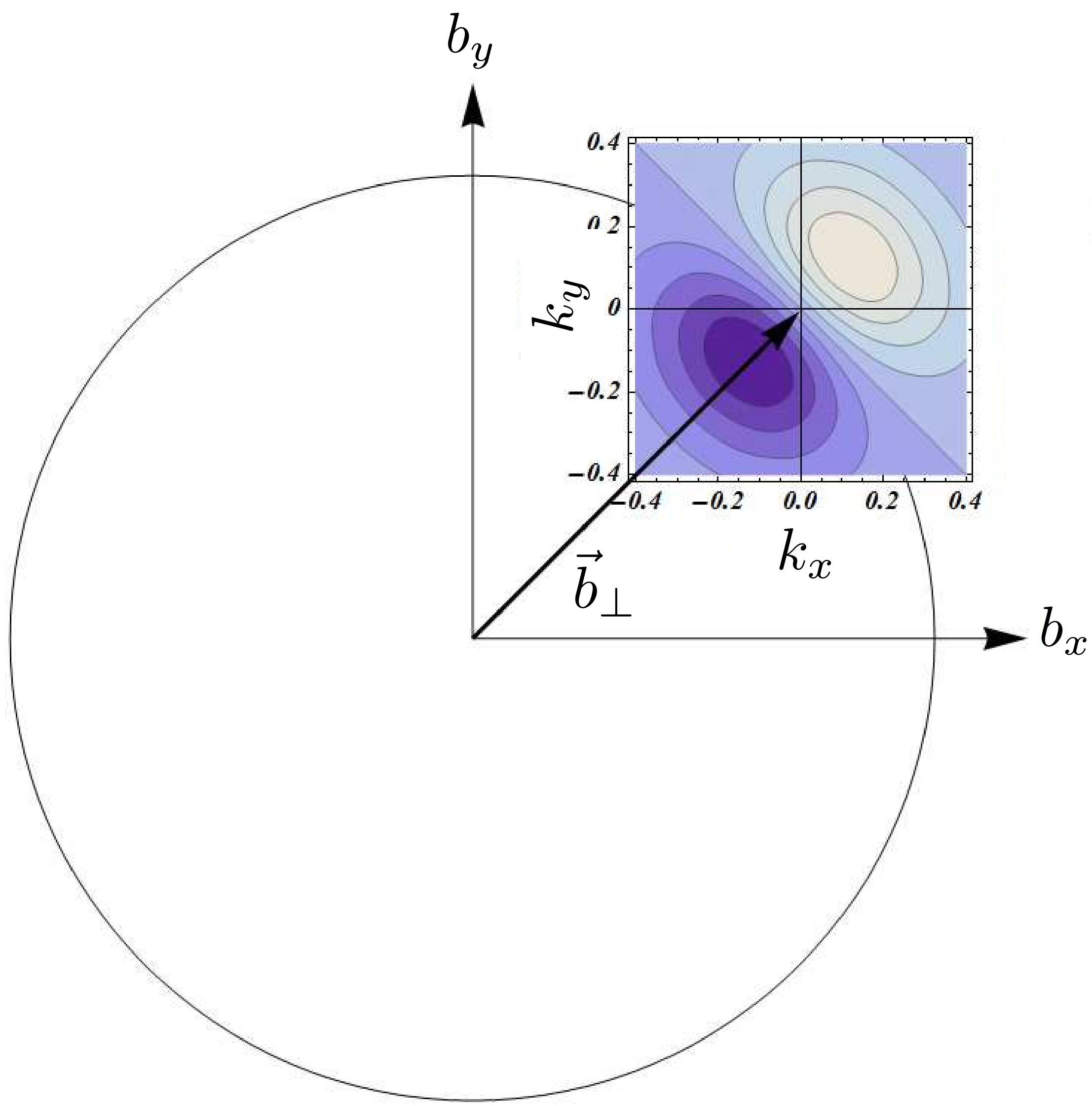
Phase-Space Transverse Modes

$$\rho_X(\vec{k}_\perp|\vec{b}_\perp) = \int \mathrm{d}x\, \rho_X(x,\vec{k}_\perp,\vec{b}_\perp;\hat{P}=\vec{e}_z,\eta=+1)\big|_{\vec{b}_\perp \text{ fixed}}$$



Multipole decomposition

$$\rho_X = \sum_{m_k,m_b} \rho_X^{(m_k,m_b)}$$

from parity and time-reversal properties

$$\vec{a}_\mathsf{P} = -c_\mathsf{P}\vec{a} \qquad \times_\mathsf{P} = c_\mathsf{P}\times$$
$$\vec{a}_\mathsf{T} = c_\mathsf{T}\vec{a} \qquad \times_\mathsf{T} = c_\mathsf{T}\times$$

	$\vec{b}_\perp$	$\vec{k}_\perp$	$\vec{e}_z \equiv \frac{\vec{P}}{P}$	$\vec{S}$	$\times$
c_P	+	+	+	-	-
c_T	+	-	-	-	+

Lorcé, BP, PRD 96, 2016

Phase-Space Transverse Modes

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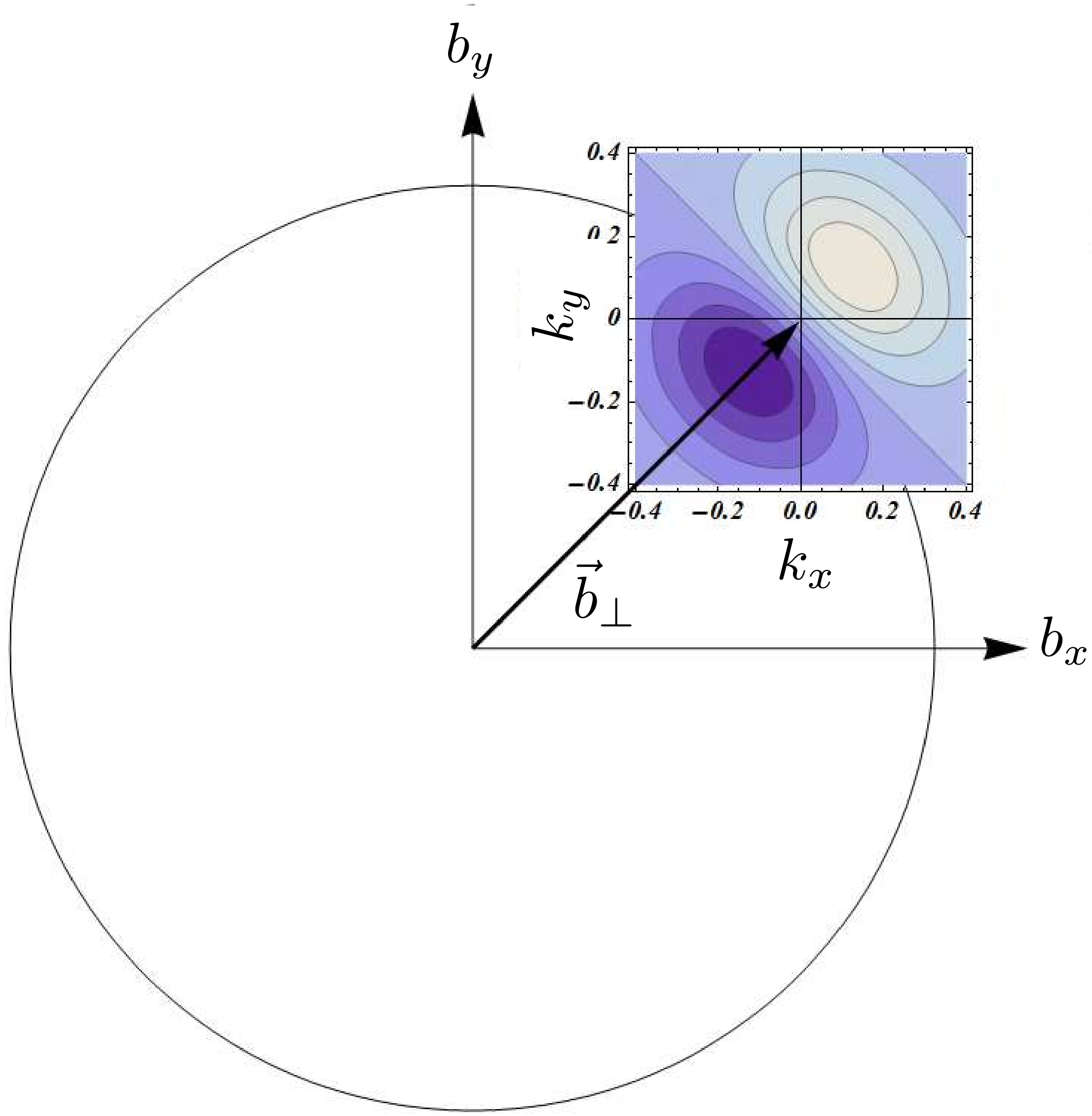

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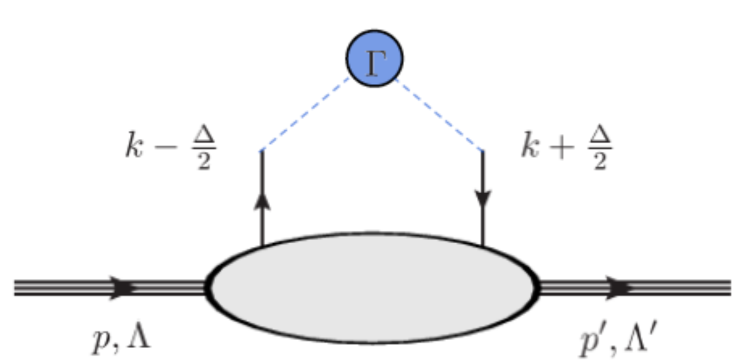
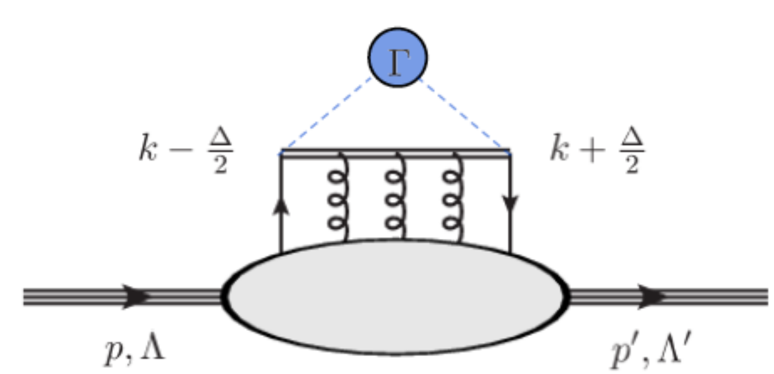
	$\vec{b}_\perp$	$\vec{k}_\perp$	$\vec{e}_z \equiv \frac{\vec{P}}{P}$	$\vec{S}$	$\times$
c_P	+	+	+	-	-
c_T	+	-	-	-	+

ρ_X^e T-even



ρ_X^o T-odd



Lorcé, BP, PRD 96, 2016

Angular Correlations

$$\rho_{\vec{S}\vec{S}^q} = \rho_{UU} + S_L\, \rho_{LU} + S_L^q\, \rho_{UL} + S_L S_L^q\, \rho_{LL} + S_T^i\, (\rho_{T^iU} + S_L^q\, \rho_{T^iL}) + S_T^{qi}\, (\rho_{UT^i} + S_L\, \rho_{LT^i}) + S_T^i S_T^{qj}\, \rho_{T^iT^j}$$

quark polarization (columns); nucleon polarization (rows)

ρ_X	U	L	T_x	T_y
U	$\langle 1 \rangle$	$\langle S_L^q \ell_L^q \rangle$	$\langle S_x^q \ell_x^q \rangle$	$\langle S_y^q \ell_y^q \rangle$
L	$\langle S_L \ell_L^q \rangle$	$\langle S_L S_L^q \rangle$	$\langle S_L \ell_L^q S_x^q \ell_x^q \rangle$	$\langle S_L \ell_L^q S_y^q \ell_y^q \rangle$
T_x	$\langle S_x \ell_x^q \rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q \rangle$	$\langle S_x S_x^q \rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q \rangle$
T_y	$\langle S_y \ell_y^q \rangle$	$\langle S_y \ell_y^q S_L^q \ell_L^q \rangle$	$\langle S_y \ell_y^q S_x^q \ell_x^q \rangle$	$\langle S_y S_y^q \rangle$

$\xi = 0$

GPD	U	L	T
U	H		$\mathcal{E}_T$
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	$H_T,\ \tilde{H}_T$

TMD	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

each distribution contains unique information

the distributions in **red** vanish if there is no quark orbital angular momentum

the distributions in **black** survive in the collinear limit

Angular Correlations

$$\rho_{\vec{S}\vec{S}^q} = \rho_{UU} + S_L\,\rho_{LU} + S_L^q\,\rho_{UL} + S_L S_L^q\,\rho_{LL} + S_T^i\,(\rho_{T^iU} + S_L^q\,\rho_{T^iL}) + S_T^{qi}\,(\rho_{UT^i} + S_L\,\rho_{LT^i}) + S_T^i S_T^{qj}\,\rho_{T^iT^j}$$

quark polarization (columns) / nucleon polarization (rows)

ρ_X	U	L	T_x	T_y
U	$\langle 1\rangle$	$\langle S_L^q \ell_L^q\rangle$	$\langle S_x^q \ell_x^q\rangle$	$\langle S_y^q \ell_y^q\rangle$
L	$\langle S_L \ell_L^q\rangle$	$\langle S_L S_L^q\rangle$	$\langle S_L \ell_L^q S_x^q \ell_x^q\rangle$	$\langle S_L \ell_L^q S_y^q \ell_y^q\rangle$
T_x	$\langle S_x \ell_x^q\rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q\rangle$	$\langle S_x S_x^q\rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q\rangle$
T_y	$\langle S_y \ell_y^q\rangle$	$\langle S_y \ell_y^q S_L^q \ell_L^q\rangle$	$\langle S_y \ell_y^q S_x^q \ell_x^q\rangle$	$\langle S_y S_y^q\rangle$

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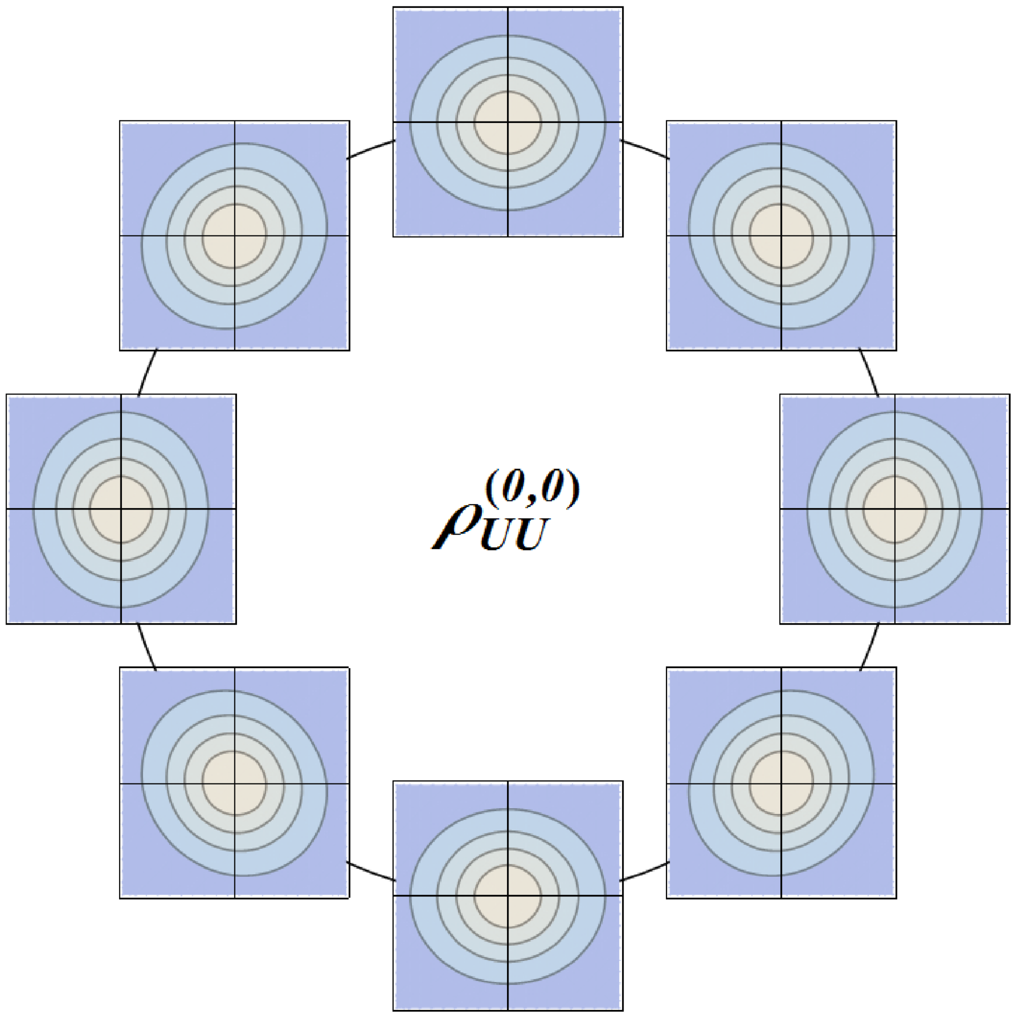
Unpolarized quarks in unpolarized proton

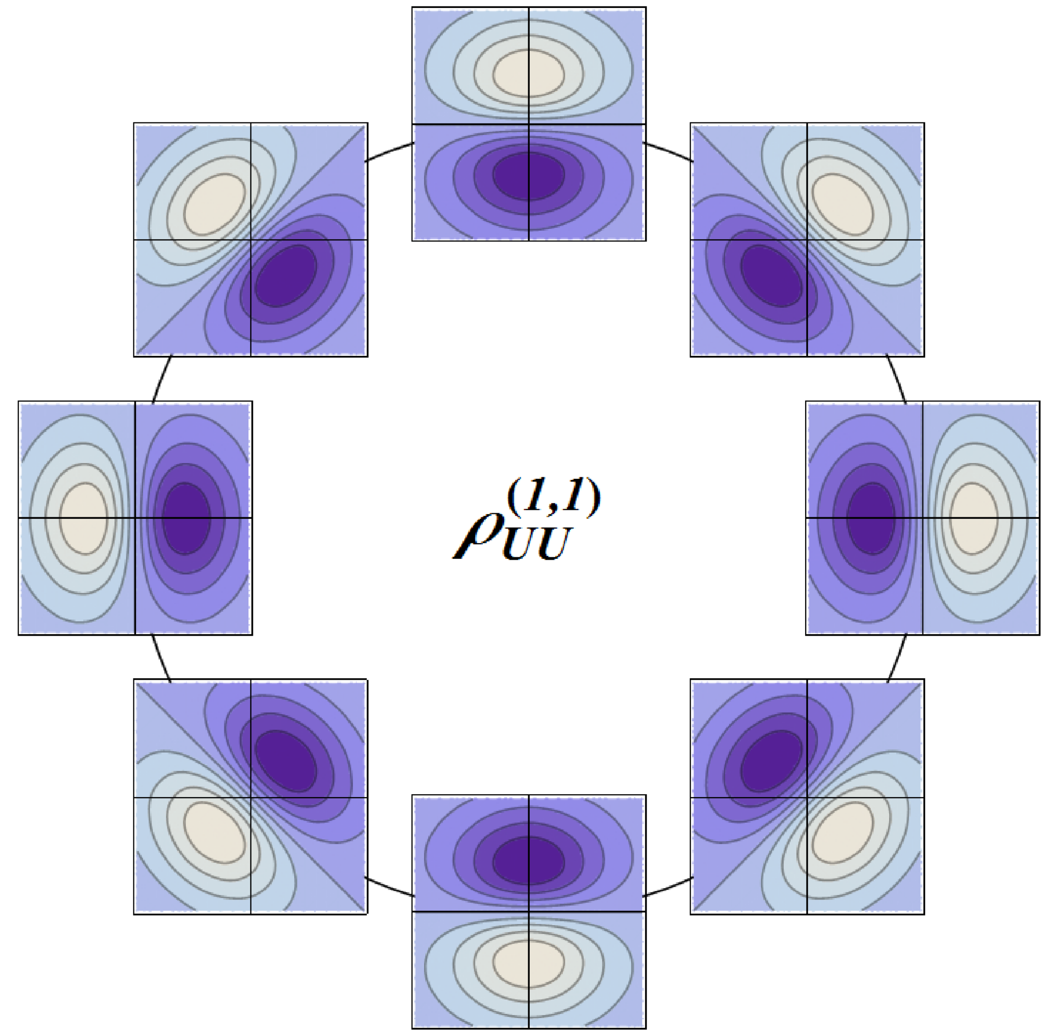
UU

$\Re e[F_{11}]$



naive time-reversal even

$\Im m[F_{11}]$



naive time-reversal odd

Integral over $k_\perp$ ➔ GPD (monopole)

Integral over $b_\perp$ ➔ TMD (monopole)

no counterpart in the GPD and TMD cases

polar flow $(\vec{k}_\perp \perp \vec{b}_\perp)$ preferred over radial flow $(\vec{k}_\perp \parallel \vec{b}_\perp)$

bottom-up symmetry ➔ no net OAM

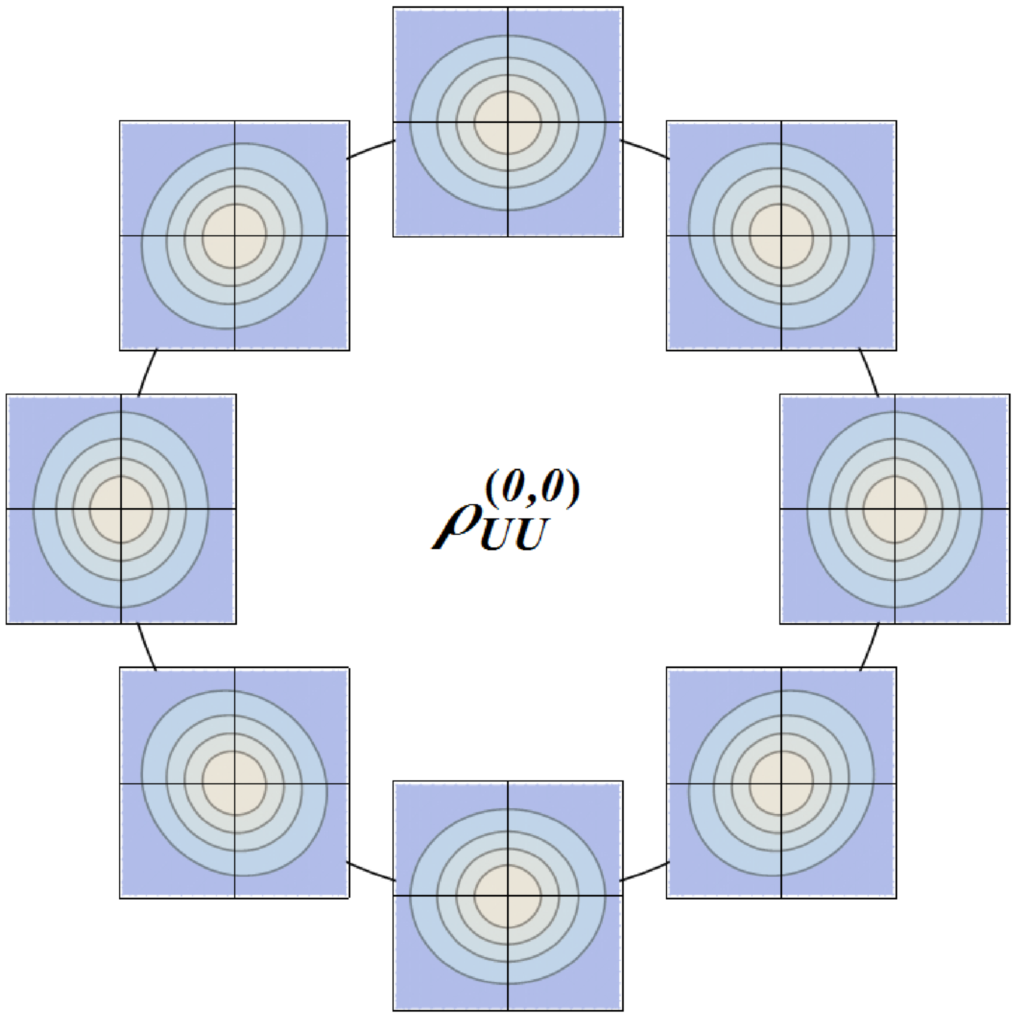
net radial flow $(\vec{k}_\perp \parallel \vec{b}_\perp)$

due to initial/final state interactions

UU

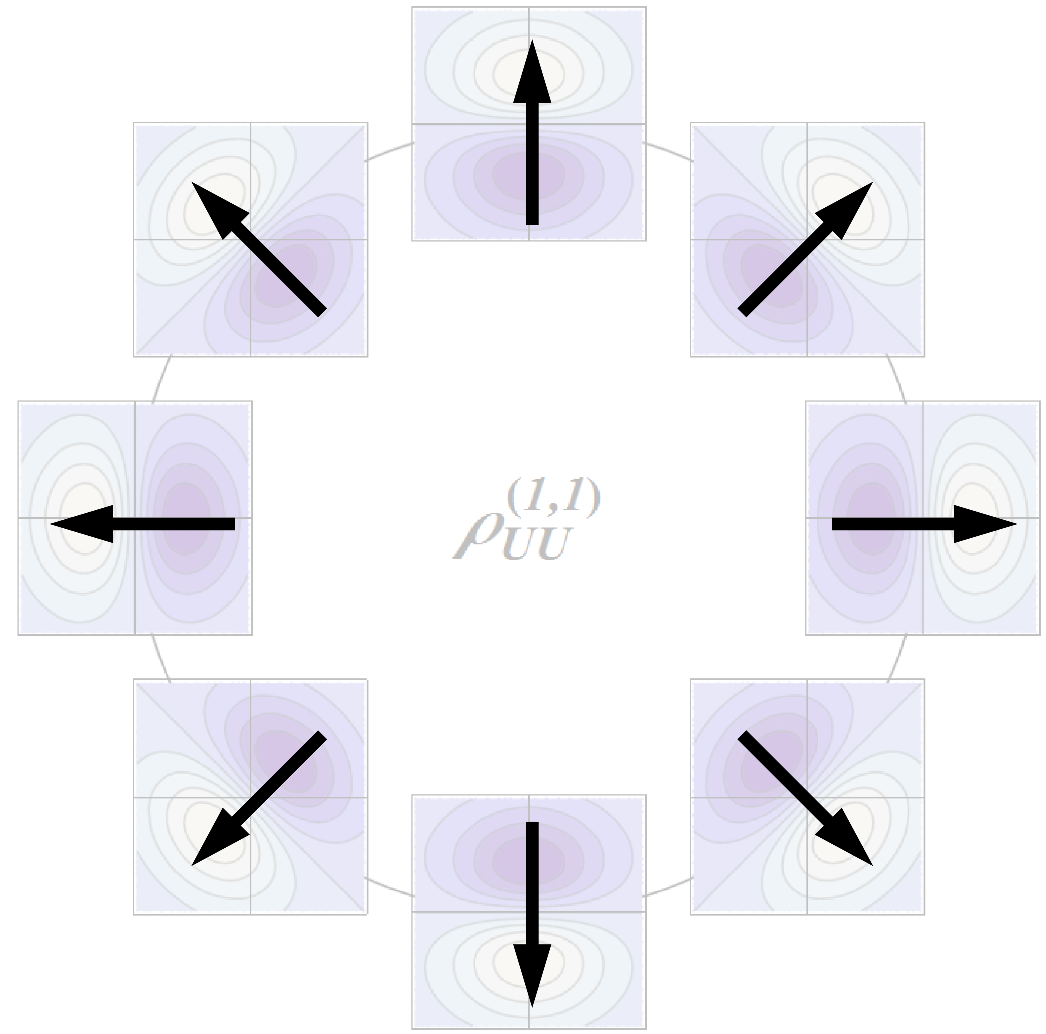
Unpolarized quarks in unpolarized proton

$\Re e[F_{11}]$



naive time-reversal even

$\Im m[F_{11}]$



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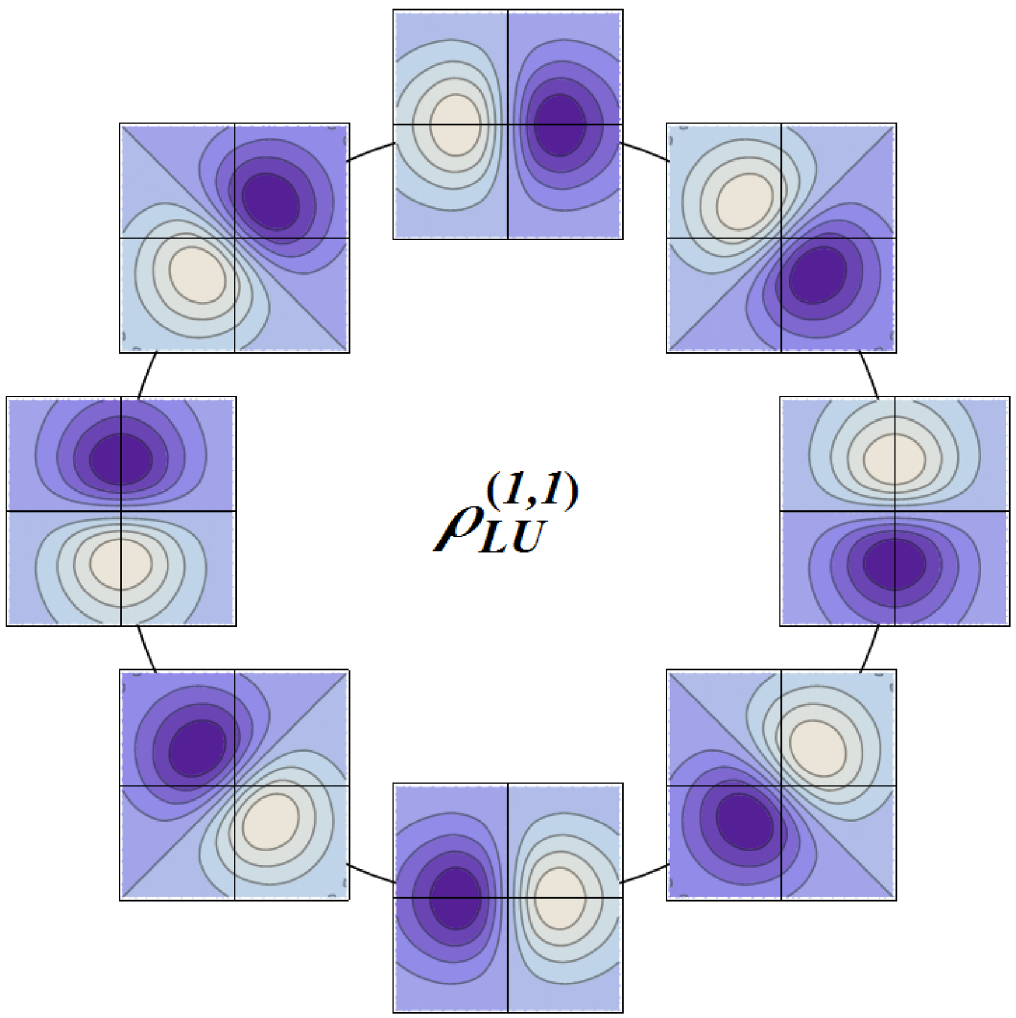
due to initial/final state interactions

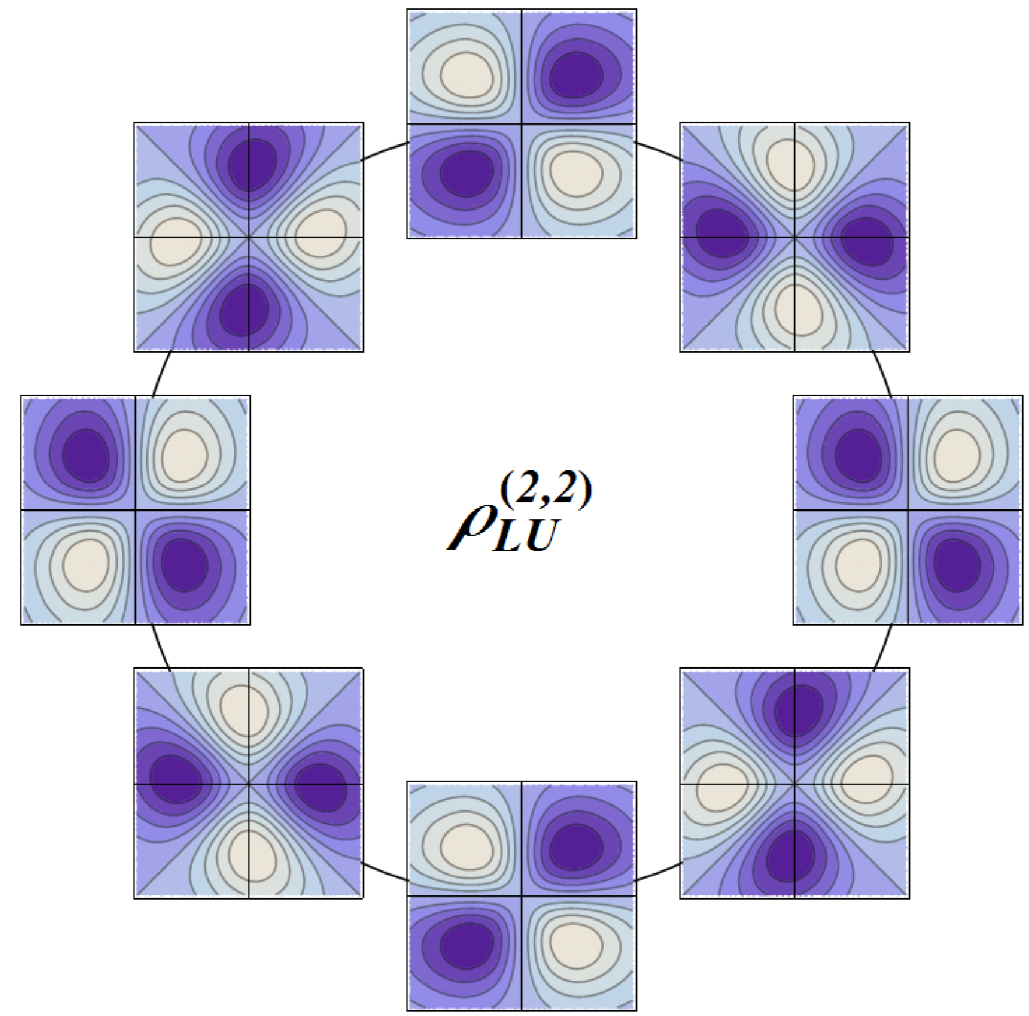
Unpolarized quarks in Longitudinally pol. proton

unique information from GTMDs

$\Re e[F_{14}]$



$\Im m[F_{14}]$



naive time-reversal even

$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z$

orbital flow → net OAM correlated S_z with

naive time-reversal odd

$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z\,(\vec{b}_\perp \cdot \vec{k}_\perp)$

spiral flow correlated with S_z

with no-net quark flow

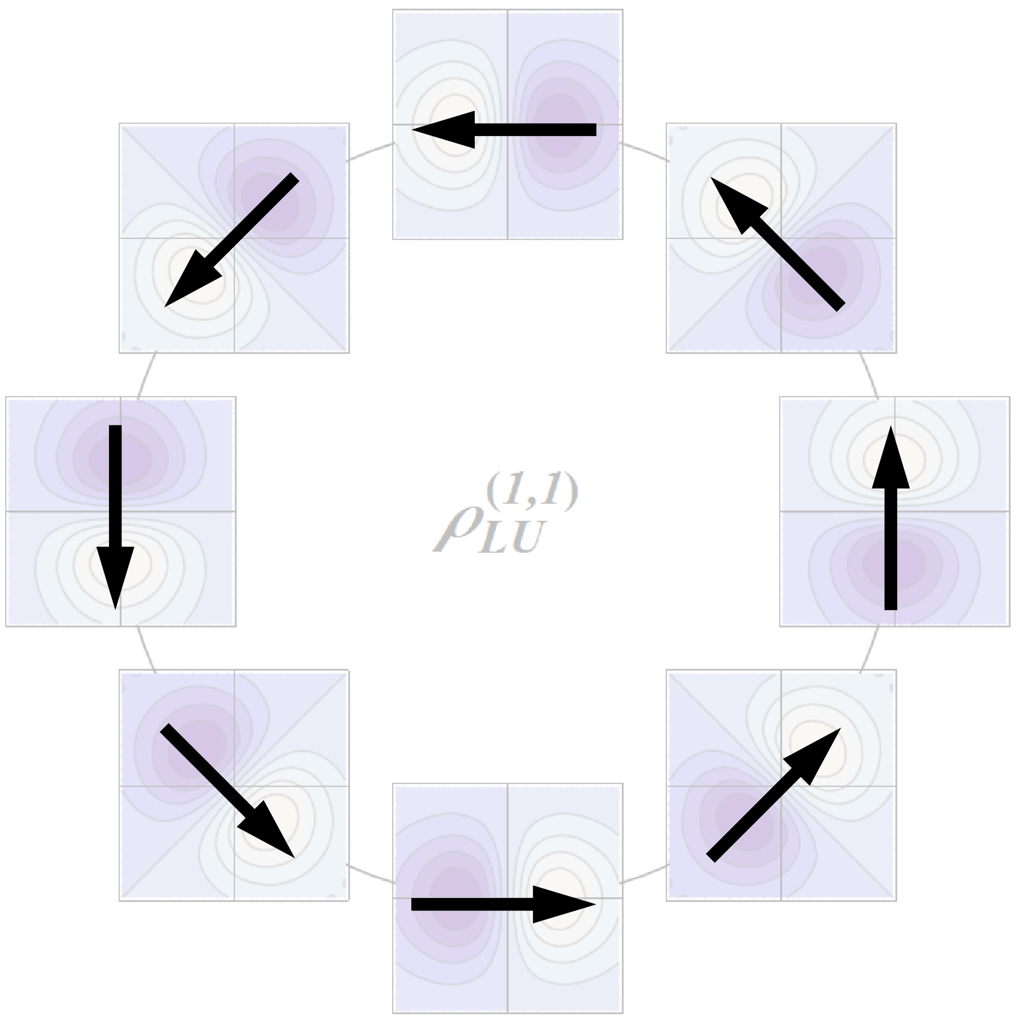
Lorcé, BP, PRD 96, 2016; PRD 84, 2011

Unpolarized quarks in Longitudinally pol. proton

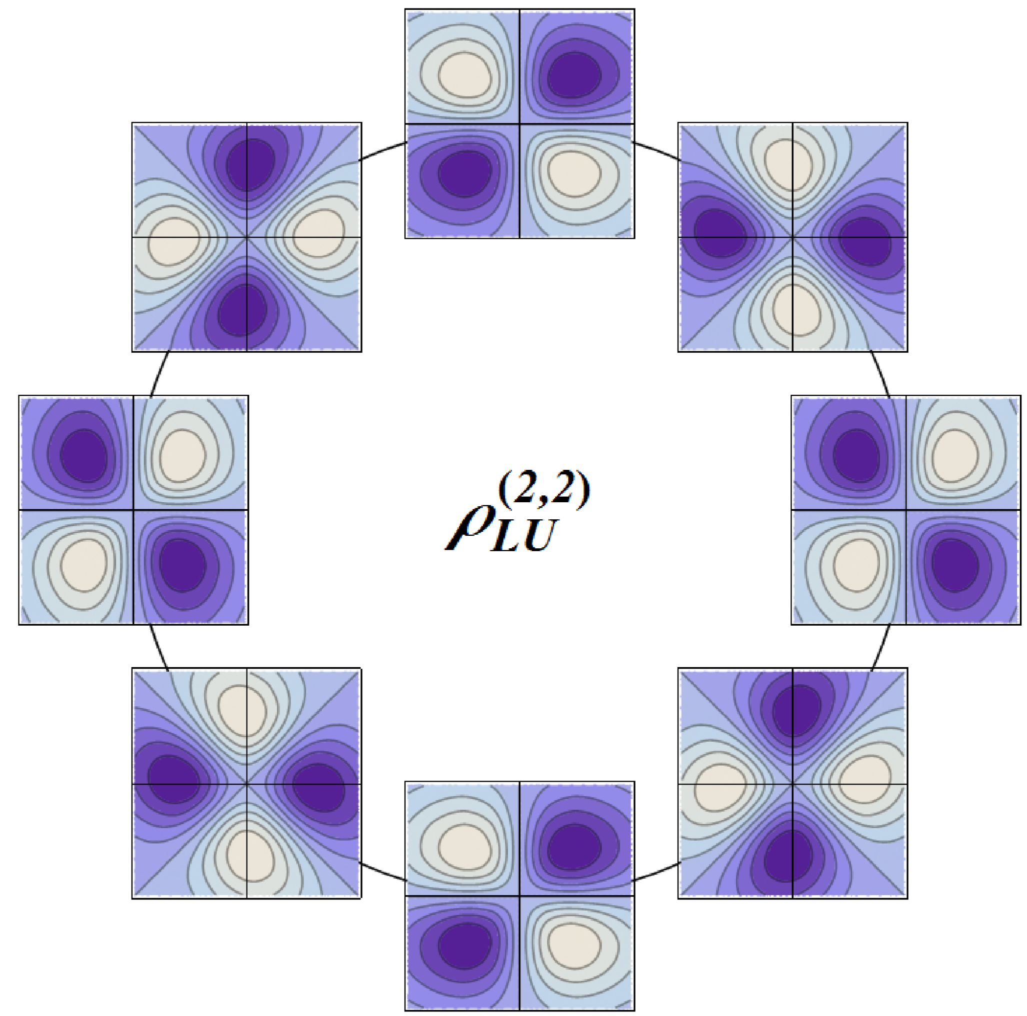
LU

unique information from GTMDs

$\Re e[F_{14}]$



$\Im m[F_{14}]$



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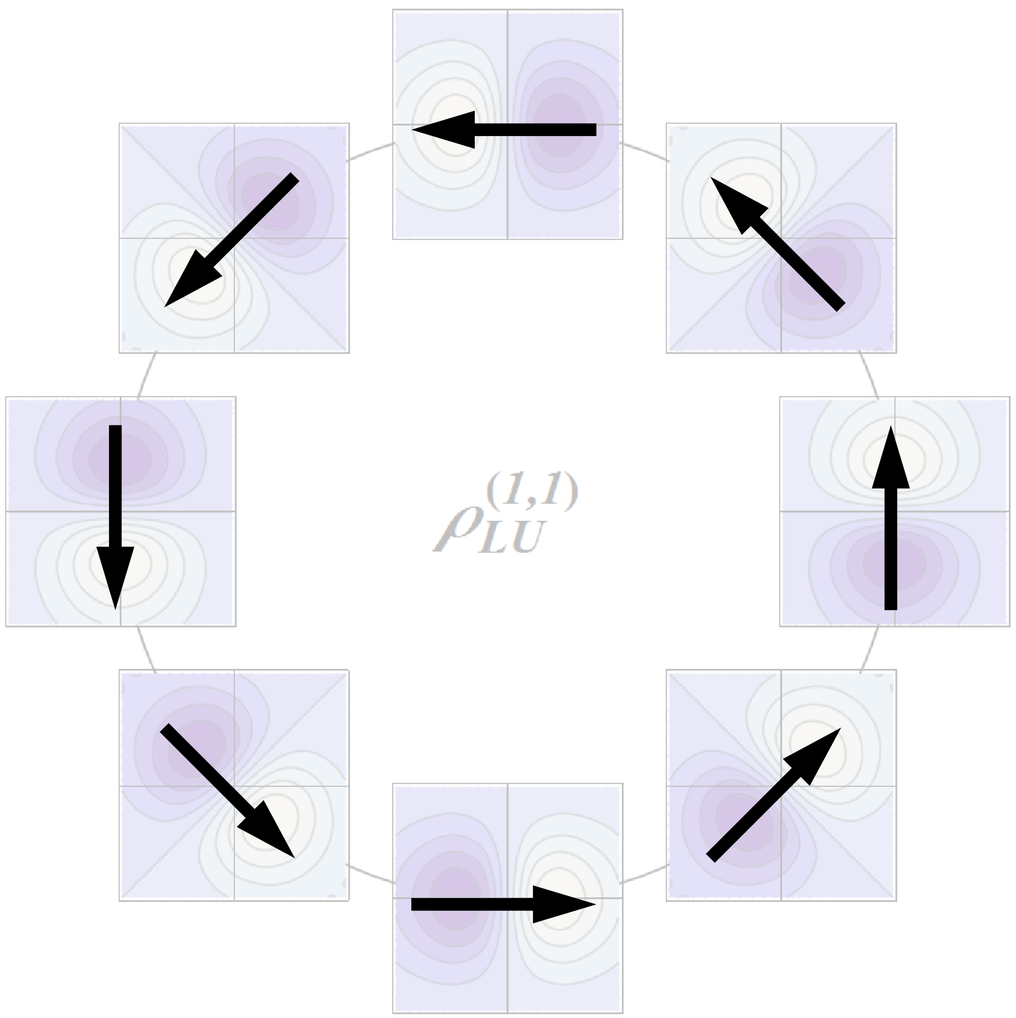
Lorcé, BP, PRD 96, 2016; PRD 84, 2011

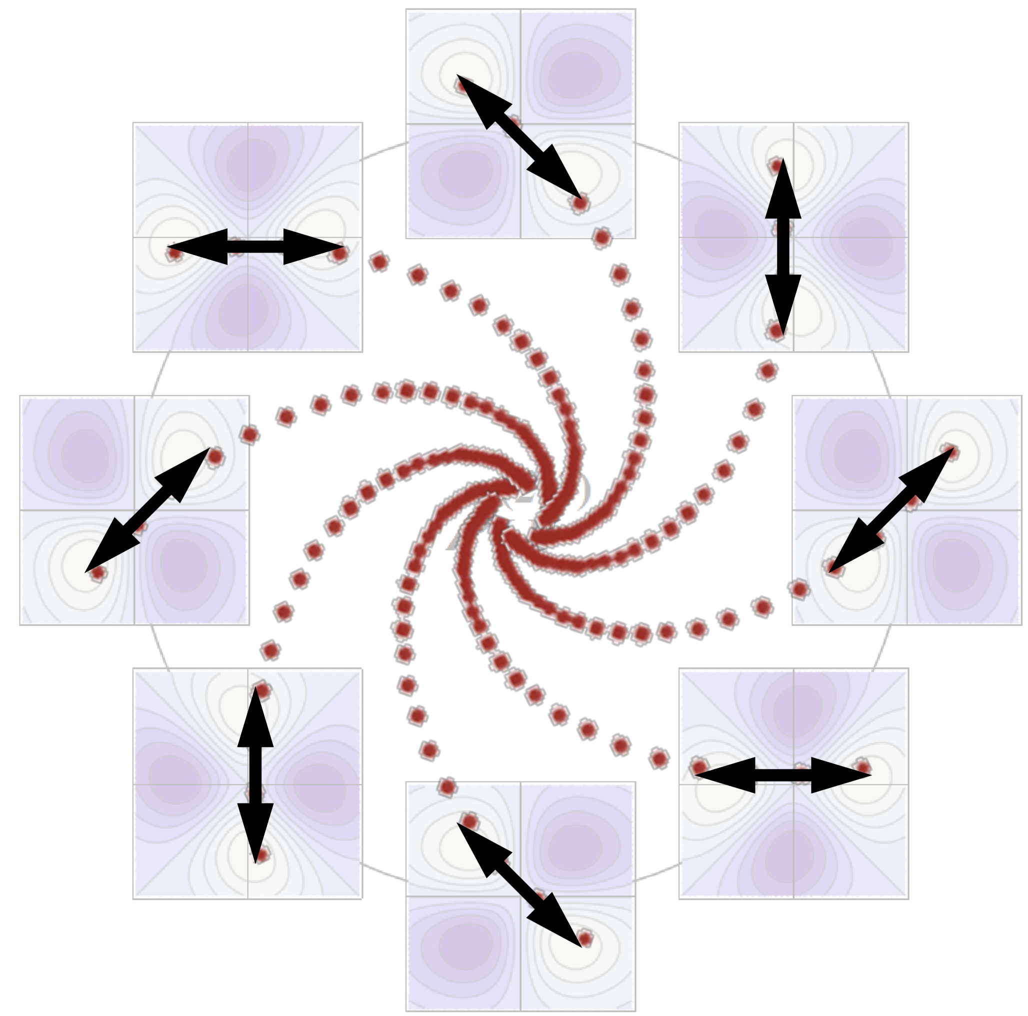
Unpolarized quarks in Longitudinally pol. proton

unique information from GTMDs

$\Re e[F_{14}]$



$\Im m[F_{14}]$



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$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z$

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naive time-reversal odd

$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z\,(\vec{b}_\perp \cdot \vec{k}_\perp)$

spiral flow correlated with S_z

with no-net quark flow

Lorcé, BP, PRD 96, 2016; PRD 84, 2011

Quark Orbital Angular Momentum

$$\ell_z^q = \int \mathrm{d}x\, \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x) = -\int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F^q_{1,4}(x, 0, \vec{k}_\perp^2, 0, 0)$$

Lorcé, BP, PRD 84 (2011) 014015

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

Quark Orbital Angular Momentum

$$\ell_z^q = \int \mathrm{d}x\, \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho_{LU}^q(\vec{b}_\perp, \vec{k}_\perp, x) = -\int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^q(x, 0, \vec{k}_\perp^2, 0, 0)$$

- mutually orthogonal components of quark position and momentum

  ⟶ no conflict with uncertainty principle

- the integrand $\ell_z^q(x)$ represents the OAM density

Lorcé, BP, PRD 84 (2011) 014015

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

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$$\ell_z^q = \int \mathrm{d}^2\vec{b}_\perp\, \vec{b}_\perp \times \langle \vec{k}^q_\perp \rangle \longrightarrow \langle \vec{k}_\perp(\vec{b}_\perp) \rangle = \int \mathrm{d}x \mathrm{d}\vec{k}_\perp\, \vec{k}_\perp\, \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

Lorcé, BP, PRD 84 (2011) 014015

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

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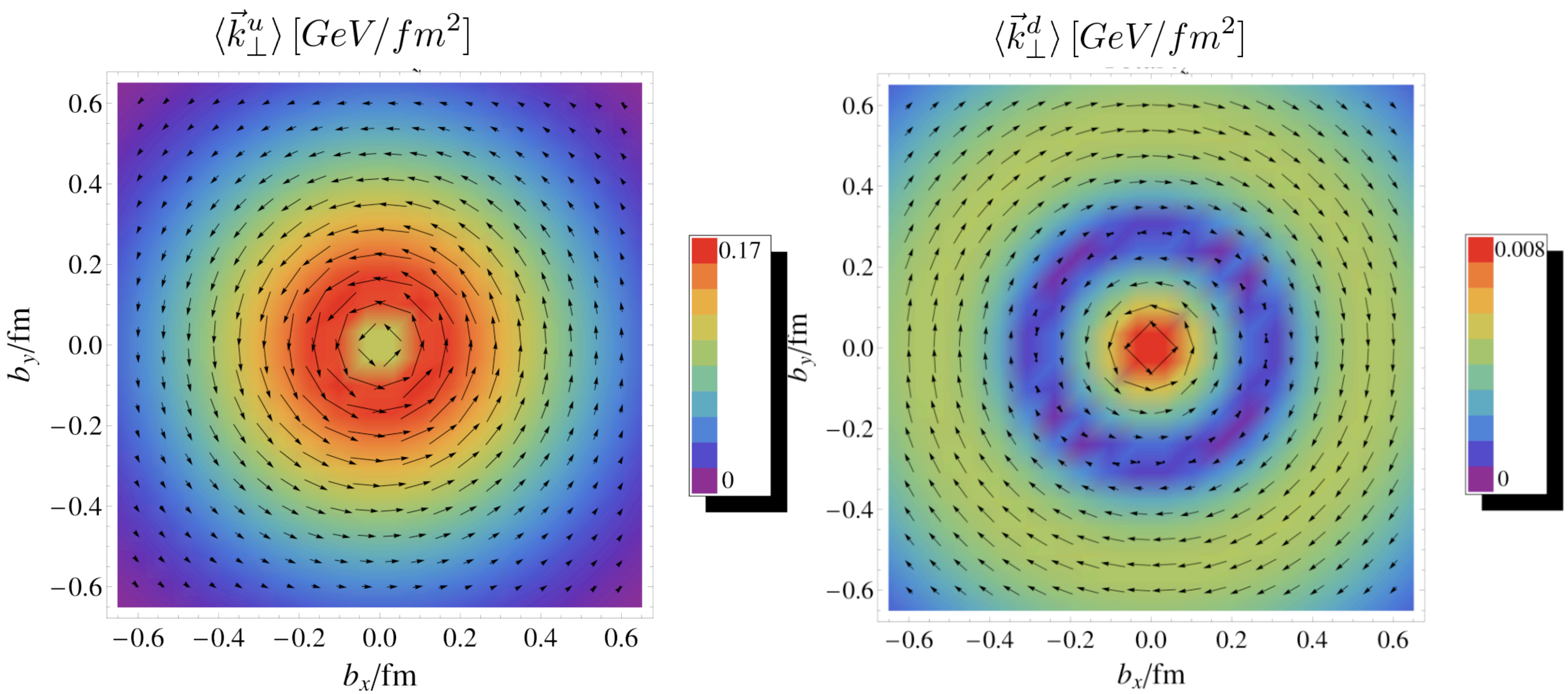

Lorcé, BP, PRD 84 (2011) 014015

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

Quark Orbital Angular Momentum

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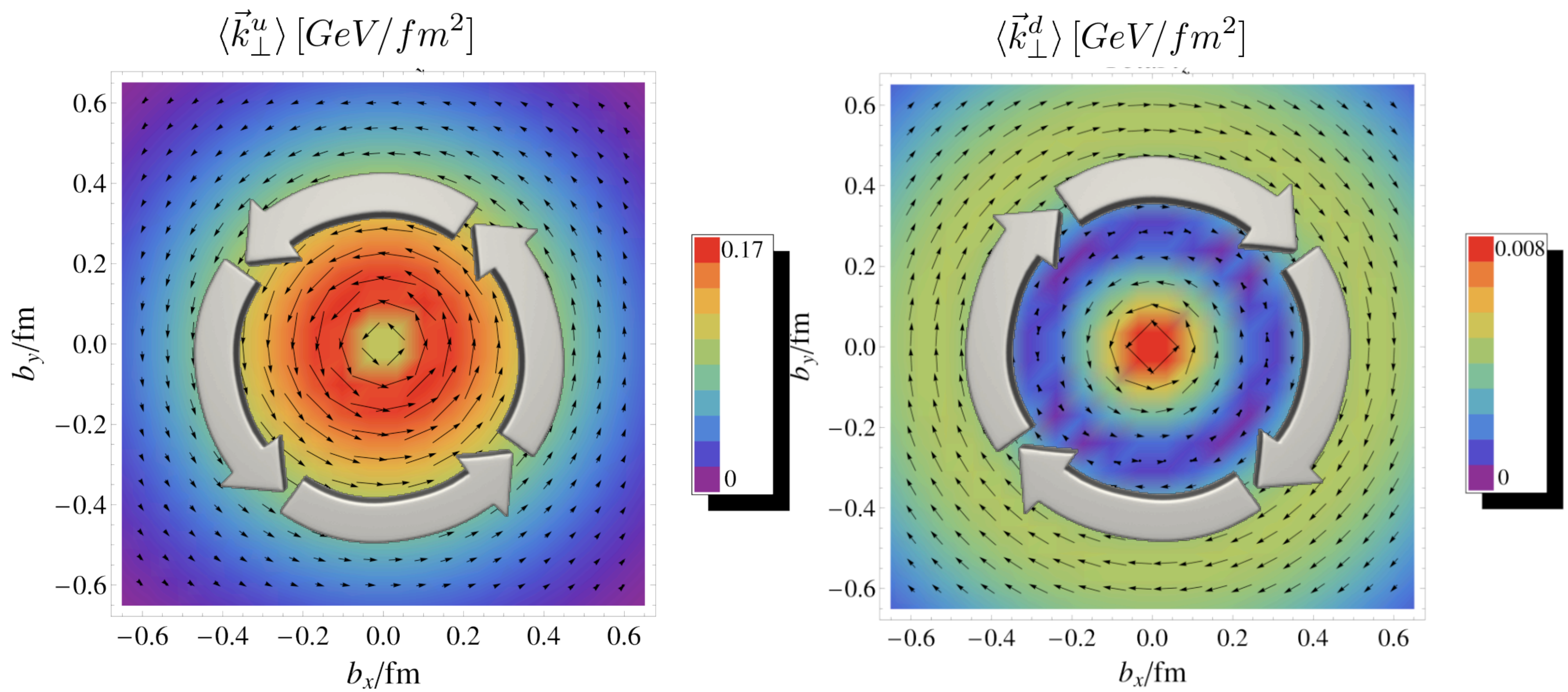


Proton spin

u-quark OAM

d-quark OAM

Lorcé, BP, PRD 84 (2011) 014015

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

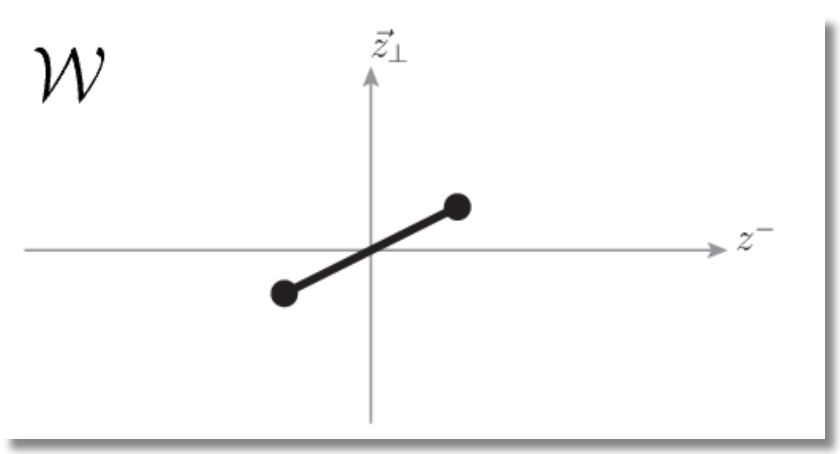
$$\ell_z^q = \int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

[Lorcé, BP (2011)]
[Lorcé, BP, Xiong, Yuan(2011)]

Light-cone gauge $A^+ = 0$
not gauge invariant, but with simple partonic interpretation

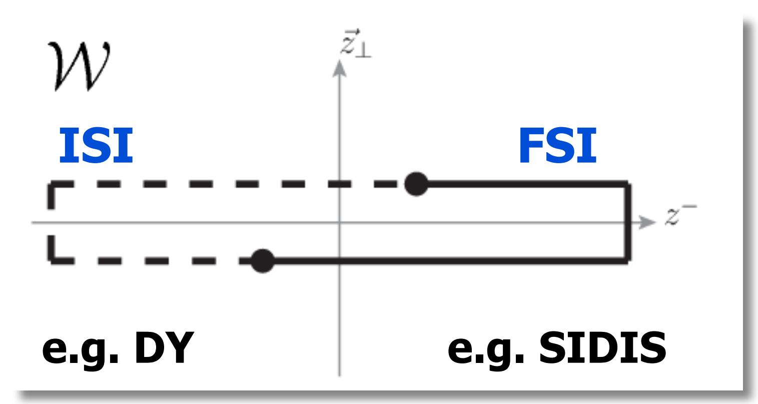
Gauge-invariant extension
$\rho_{LU} \to \rho_{LU}^{\mathcal{W}}$

Kinetic *[Ji (1997)]*



[Ji, Xiong, Yuan (2012)]
[Burkardt (2012)]

Canonical *[Jaffe, Manohar (1990)]*

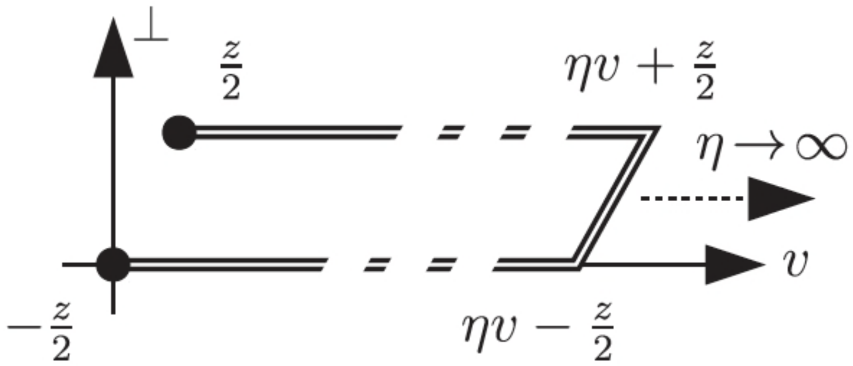


[Hatta (2012)]

difference between the two definitions can be interpreted as
the change in the quark OAM as the quark leaves the target in a DIS experiment
[M. Burkardt (2013)]

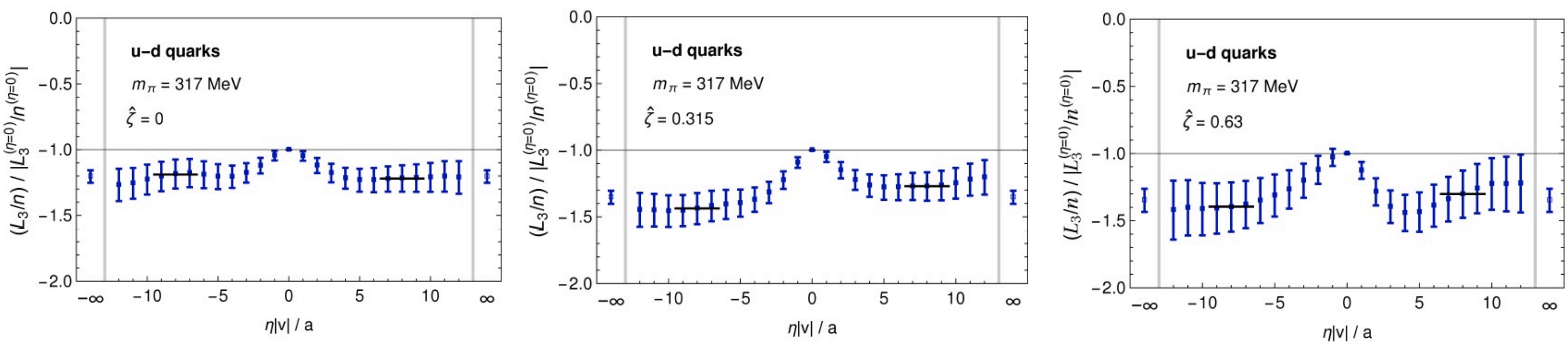
Lattice calculation

M. Engelhardt, Phys. Rev. D95, 094505 (2017)
and talk at SPIN2018



Continuous interpolation between the Ji limit $\eta = 0$ and the Jaffe-Manohar limit $\eta \to \infty$

Jaffe-Manohar OAM ← **Ji OAM** ↓ → **Jaffe-Manohar OAM**



nucleon rapidity →

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M. Engelhardt, Phys. Rev. D95, 094505 (2017)
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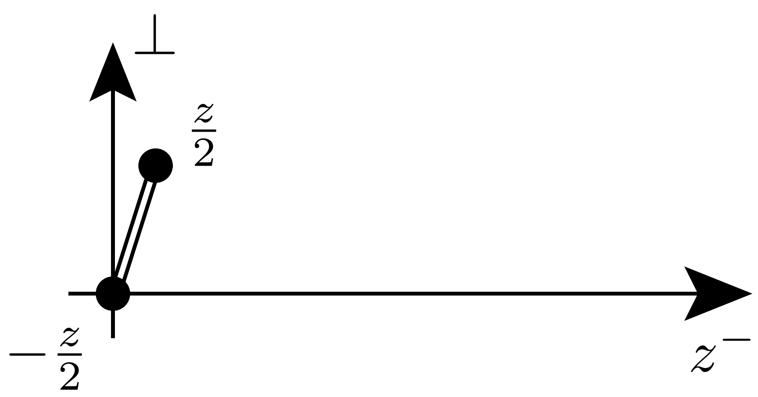

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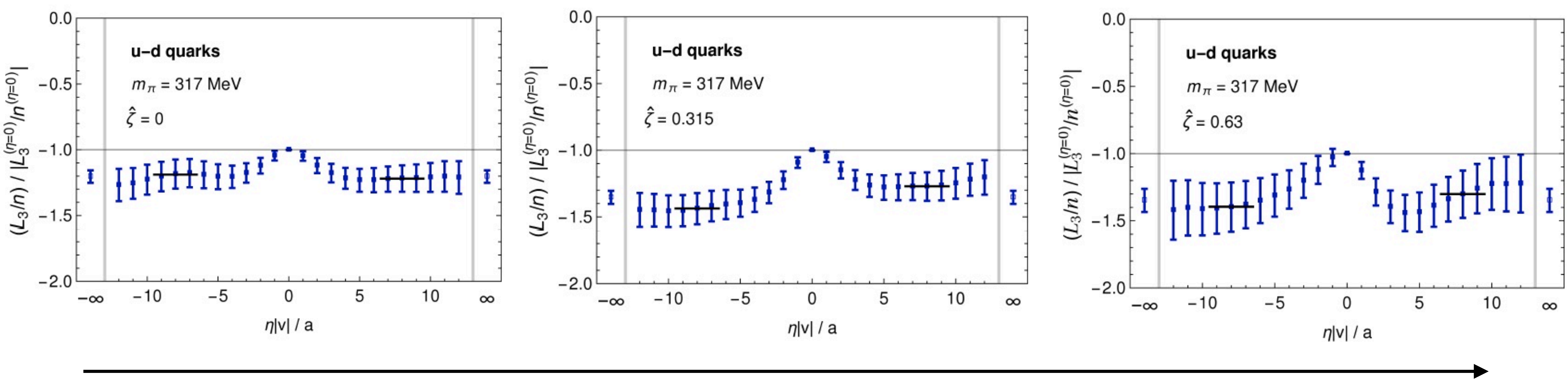

nucleon rapidity

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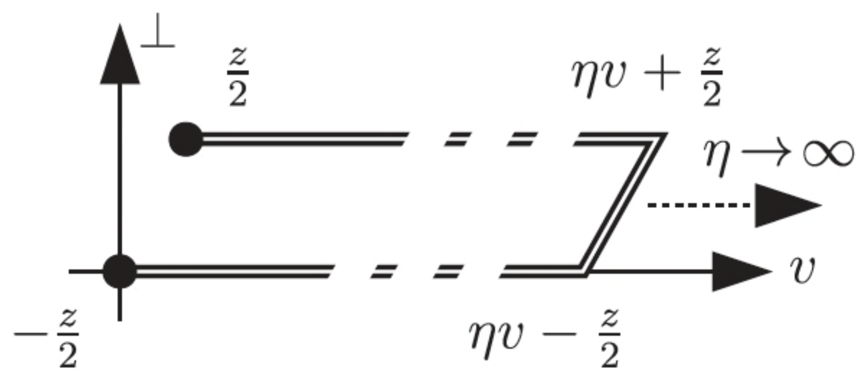

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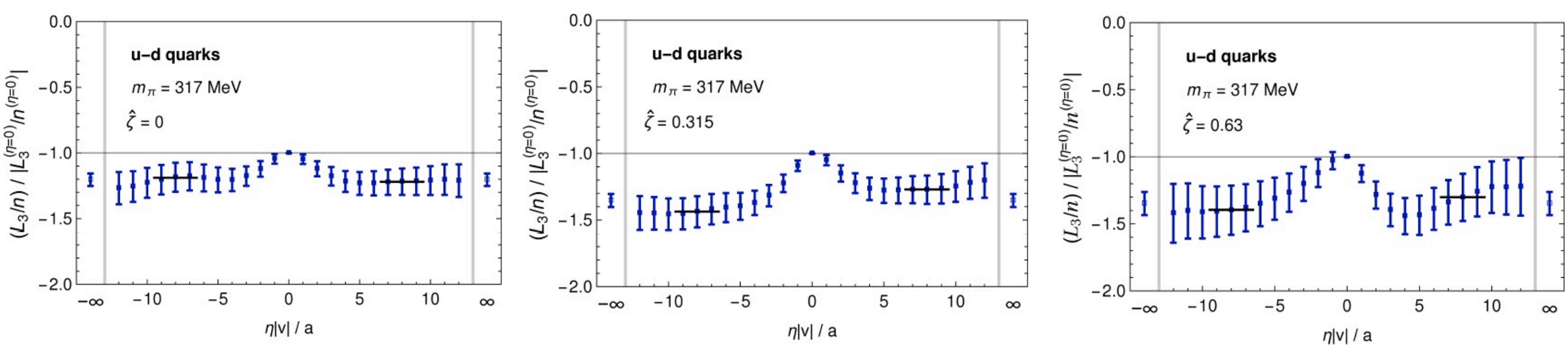

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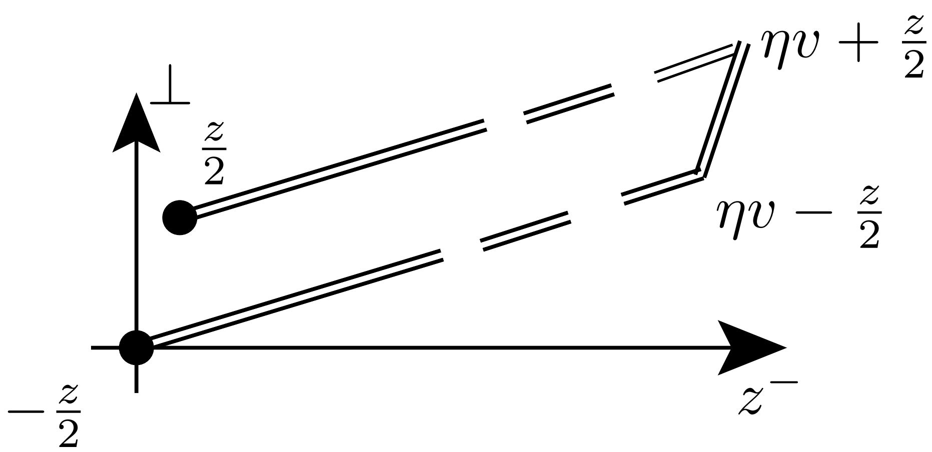
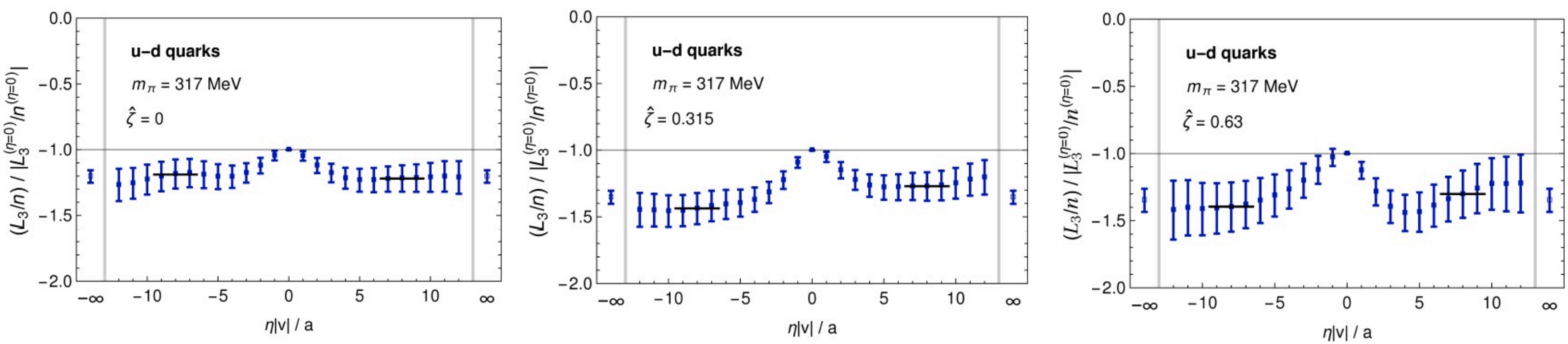


Continuous interpolation between the Ji limit $\eta = 0$ and the Jaffe-Manohar limit $\eta \to \infty$

Staple direction off the light-cone

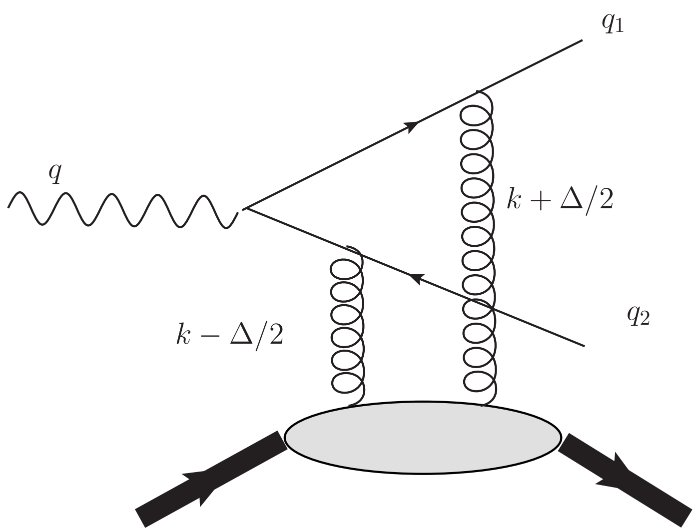
light-cone limit for $\hat{\zeta} = \frac{v \cdot P}{\sqrt{|v^2|}\sqrt{|P^2|}} \to \infty$

Jaffe-Manohar OAM ← **Ji OAM** ↓ **Jaffe-Manohar OAM** →



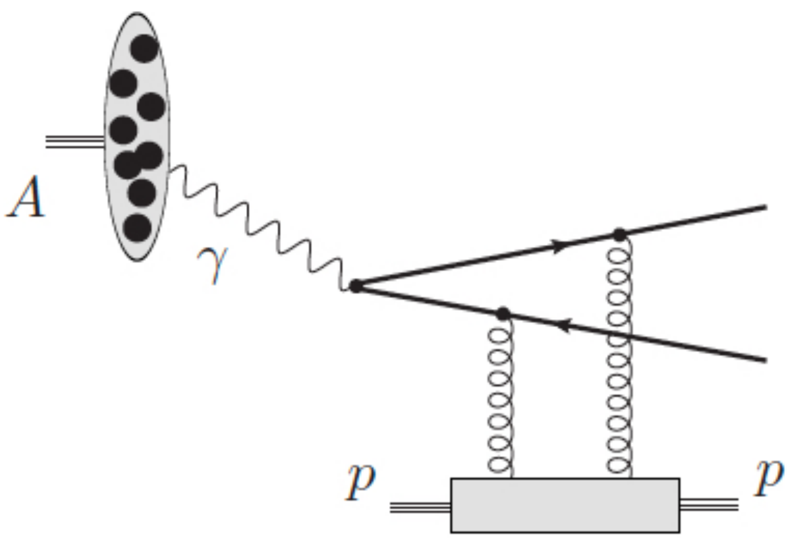
nucleon rapidity →

Wigner Distributions (WD) and GTMDs from



Exclusive dijet production in ep DIS (gluon GTMDs)

Hatta, Xiao, Yuan, PRL 116 (2016) 202301
Hatta, Nakagawa, Xiao, Yuan, Zhao, PRD 95 (2017) 114032
Ji, Yuan, Zhao, PRL 118 (2017) 192004



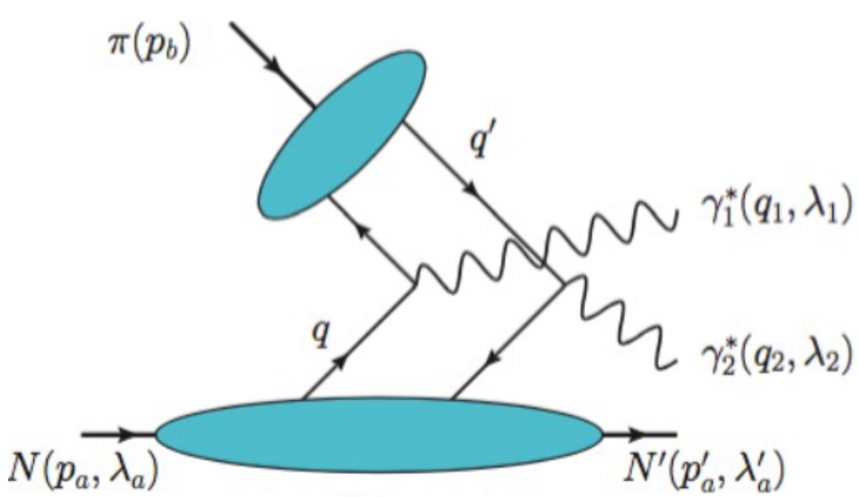
Exclusive dijet production in pA UPC (gluon GTMDs)

Hagiwara, Hatta, Pasechnik, Tasevsky, Teryaev, PRD 96 (2016) 034009

Exclusive double quarkonia production
in nucleon-nucleon collisions (gluon GTMDs)

Bhattacharya, Metz, Ojha, Tsai, Zhou, arXiv:1802.10550

Boussarie, Hatta, Xiao, Yuan, arXiv: 1807.08697



Exclusive pion-nucleon double Drell-Yan
(quark GTMDs)

Bhattacharya, Metz, Zhou, PLB 771 (2017) 396

→ Talk of F. Yuan

Conclusions

- Wigner distributions provide a new multidimensional imaging of the nucleon
- Decomposition in phase-space transverse modes helps to understand the physical content of parton distributions
- Complete characterization of spin structure requires spin-orbit correlation
- OAM from Wigner distributions $\longrightarrow$ calculable in the lattice
- First processes identified to access gluon and quark GTMDs, but still far away from Wigner distributions