Color correlations in the proton

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Based on 1808.02501
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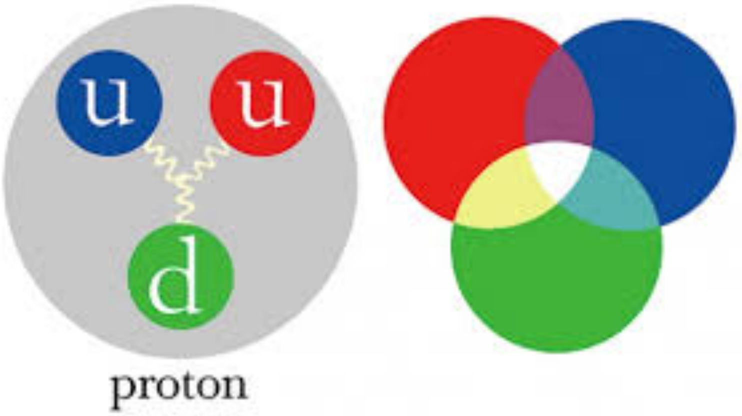


Image courtesy of Carole Kliger, Dept. of Physics, University of Maryland

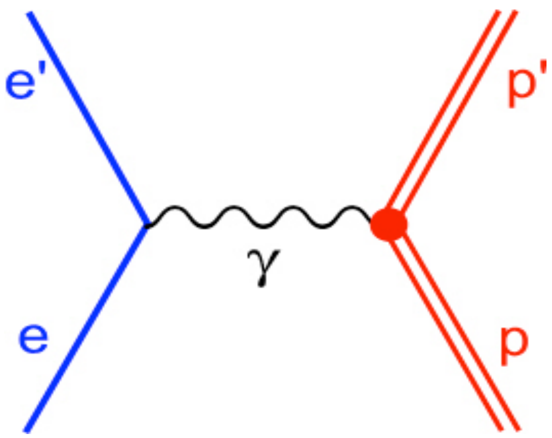
Where is the color charge located in a nucleon, nucleus ?

Is the color charge distribution the same as the charge distribution?
Can't be -integral =0
Located on quarks and gluons need to deal with moments ρ^2, ρ^3
This talk introduces a new way of looking at nucleon and nuclear structure:
Moments of charge density operators as in
The Mclerran-Venugopalan model for RHIC physics

Start with the electric charge

Electron-nucleon scattering

$j_\mu = \langle e'|\gamma_\mu|e\rangle$



$$\Gamma_\mu(p',p) = \langle p'|J_\mu|p\rangle,\ J_\mu = \sum_q e_q \bar{q}\gamma_\mu q$$

Nucleon vertex: $\Gamma_\mu(p',p) = \gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q_\nu}{2M} F_2(Q^2)$ **sandwich in spinors**

Dirac **Pauli**

Light front dynamics:

'time' $x^+ = (x^0 + x^3)/\sqrt{2} = 0,\ \text{space}: (x^-, r_\perp) = (x^-, \mathbf{b})$

Transverse boosts are kinematic

Model independent transverse charge density

Charge Density

b is impact parameter

$$\rho_\infty(x^-,\mathbf{b}) = \langle p^+,\mathbf{R}=\mathbf{0},\lambda|\sum_q e_q q_+^\dagger(x^-,b)q_+(x^-,b)|p^+,\mathbf{R}=\mathbf{0},\lambda\rangle$$

+-component of electromagnetic current operator

$$\rho(b) \equiv \int dx^- \rho_\infty(x^-,\mathbf{b}) = \int \frac{QdQ}{2\pi} F_1(Q^2) J_0(Qb)$$

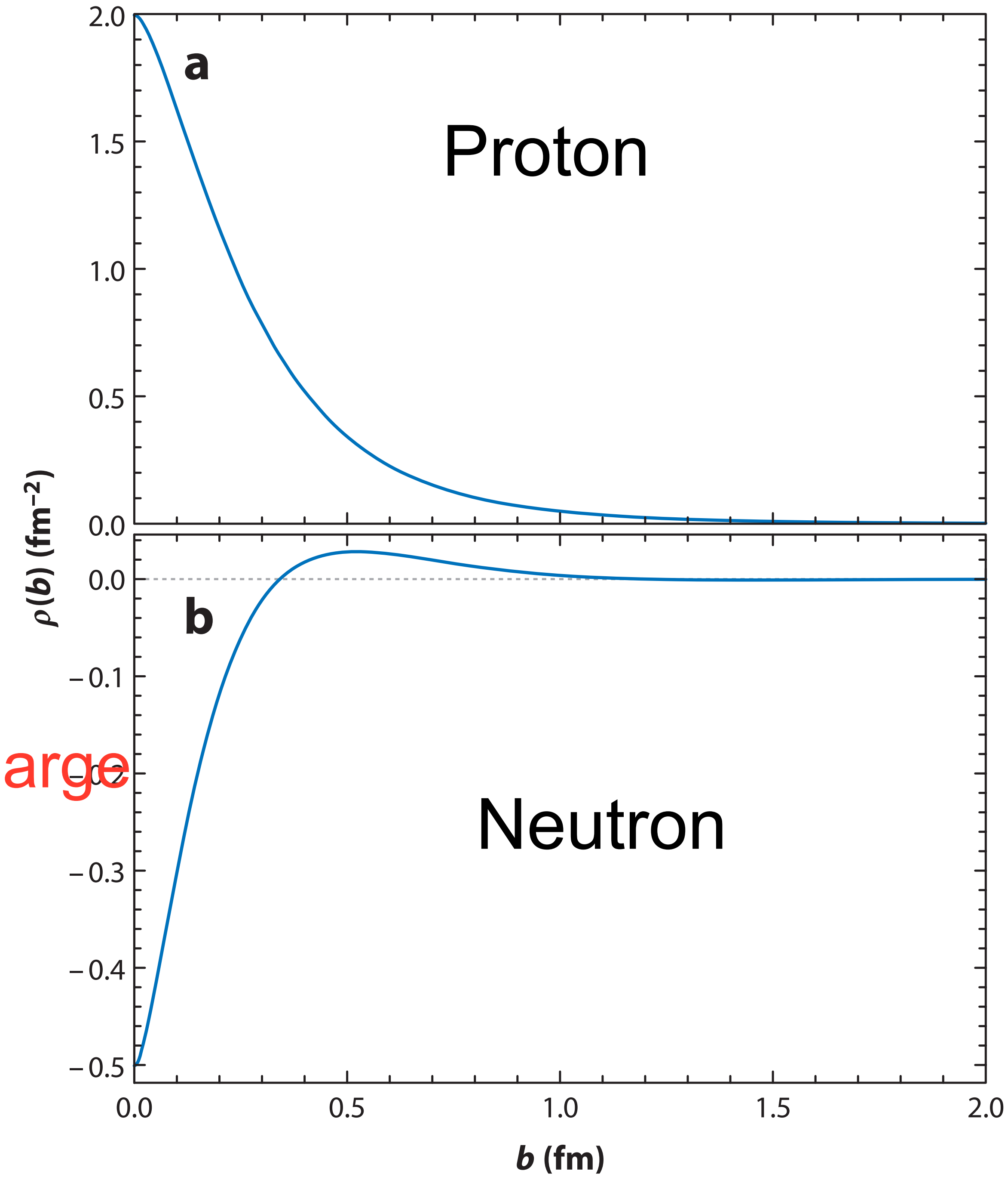
Soper '77

$$\text{Density is } u-\bar{u},\ d-\bar{d}$$

$$F_1(Q^2) = \int d^2b\rho(b)e^{i\mathbf{q}\cdot\mathbf{b}}$$

Two dimensional FT

Transverse charge densities from parameterizations (Alberico)



Negative central charge density

Negative central density - GAM PRL ’07

PRL99,112001

0705.2409

Neutron case - integral of density=0, similar to color charge density

Can we get a picture like this for the color charge density with moments of density?

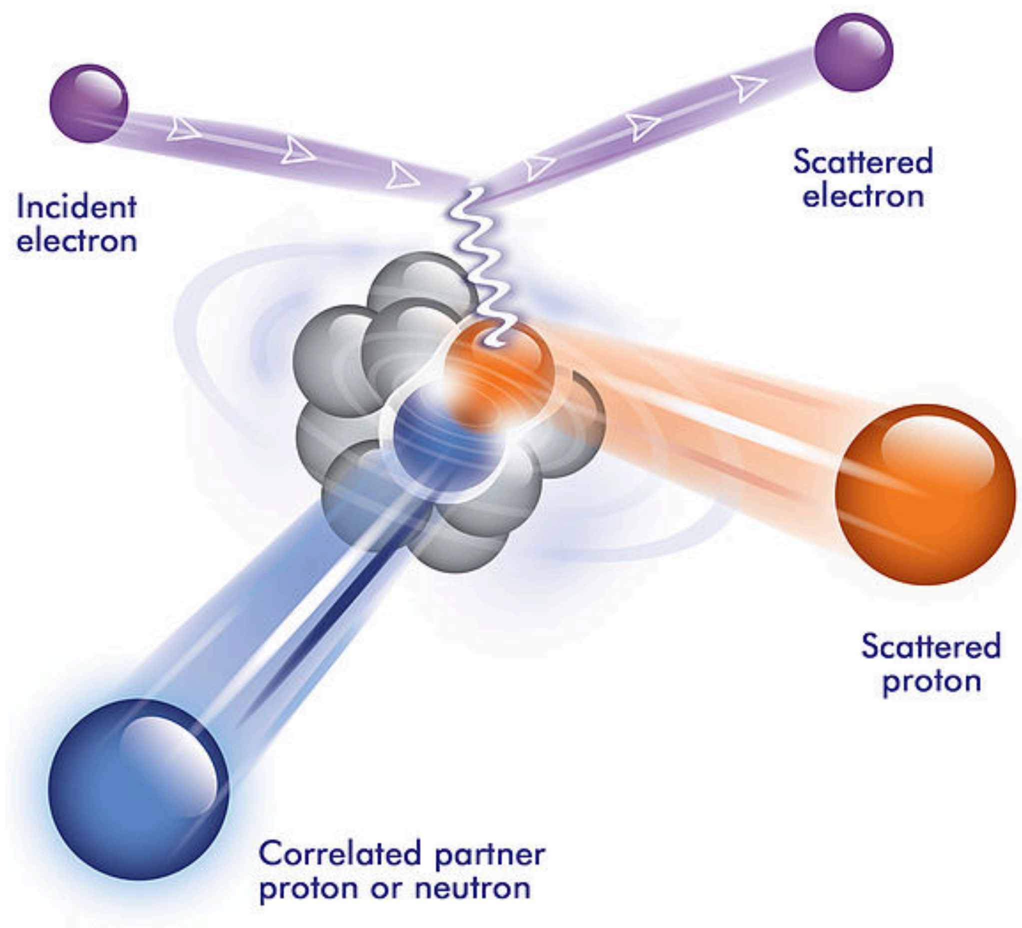
Given a u quark at given position, what is the probability that a d is a distance $(\Delta x^-, \Delta r_\perp)$ away?

$$-\frac{2}{9}\langle p^+\mathbf{R}=0,\lambda|\int dx^- d^2r_\perp \rho_u(x^-,r_\perp)\rho_d(x^-+\Delta x^-, r_\perp+\Delta r_\perp)|p^+\mathbf{R}=0,\lambda\rangle$$

$$\rho_u(x^-,r_\perp)\equiv u_+^\dagger(x^-,r_\perp)u_+(x^-,r_\perp),\ \text{etc.}$$

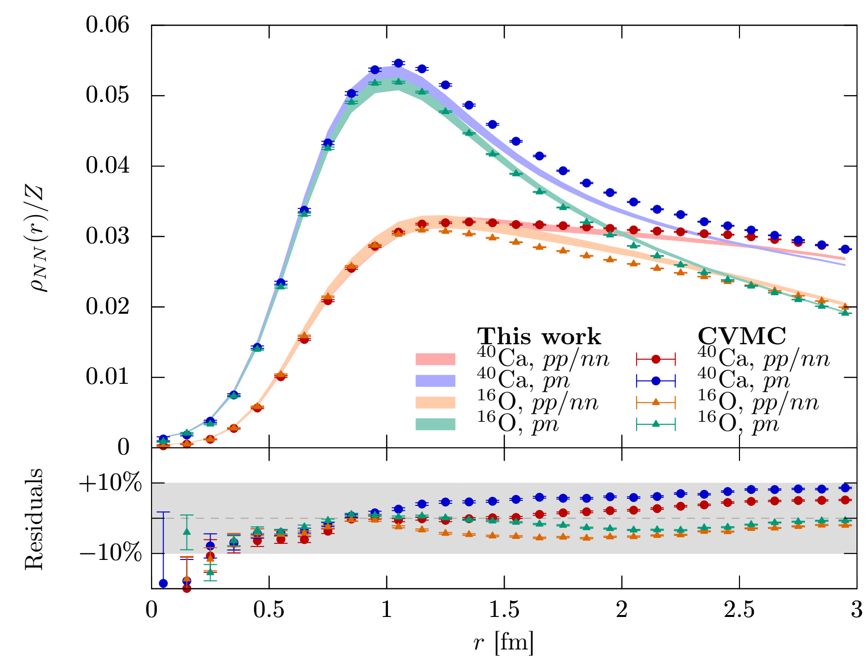
- needs higher order electromagnetic, seen in two photon exchange
- Maybe better with gluons?

Nucleon-nucleon correlations in nuclei



Basic part of nuclear binding

$$\rho_{NN,s}(\vec{r}) \equiv \sum_{\substack{i,j\in NN \\ i<j}} \langle\psi|\delta(\vec{r}-\vec{r}_{ij})P_s|\psi\rangle,$$



Cruz-Torres et al PLB785, 304 (2018)
1710.07966

After 50 years we are finally getting information about two nucleon correlations - see Hen et al RMP 89 (2017) 045002 and subsequent references

Color charge density - one-body, two-body etc

$$\rho^a(x) \equiv \bar{\psi}_{i,f}(x)\,\gamma^+\,\psi_{j,f}(x)(t^a)_{ij} + \text{ gluon terms}$$

What are:

$$\langle\,\text{proton}|\rho^a(x)|\text{proton}\rangle, \langle\,\text{proton}|\rho^a(x)\rho^b(y)|\text{proton}\rangle \text{ etc}$$

Idea comes from Mclerran-Venugopalan model

Observables are computed in terms of moments of ρ^a:

quarks at higher x emit gluons of lower x

Color charge operator

Light-front dynamics- expand quark-field ops at $x^+ = 0 \;\; r = (x^-, \vec{x}_\perp)$ as creation destruction ops. (ignore anti-quarks)

$$\rho^a(r) = 2P^+ \sum_{\lambda,\lambda'} \int \frac{dx_q d^2q}{16\pi^3\sqrt{x_q}}\, b^\dagger_{q,i,\lambda} e^{iq\cdot r} \int \frac{dx_p d^2p}{16\pi^3\sqrt{x_p}}\, b_{p,j,\lambda'} e^{-ip\cdot r}\, (t^a)_{ij}\, \delta_{\lambda\lambda'}$$

$\lim_{P^+\to\infty} P^+ e^{i(x_q-x_p)P^+r^-} \to 2\pi\delta(x_p - x_q)\delta(r^-)$

Pancake shape

Color charge per unit area $\rho^a(\vec{x}_\perp) = \int \frac{d^2k}{(2\pi)^2}\, e^{i\vec{k}\cdot\vec{x}_\perp} \int_0^\infty \frac{dq^+}{q^+} \int \frac{d^2q}{16\pi^3} \sum_\lambda b^\dagger_{x_q,\vec{q}-\vec{k},i,\lambda}\, b_{x_q,\vec{q},j,\lambda}\, (t^a)_{ij}$

Fourier Transform to momentum space $\tilde{\rho}^a(\vec{k}) = \int_0^\infty \frac{dq^+}{q^+} \int \frac{d^2q}{16\pi^3} \sum_\lambda b^\dagger_{x_q,\vec{q}-\vec{k},i,\lambda}\, b_{x_q,\vec{q},j,\lambda}\, (t^a)_{ij}$

$$\langle \mathbf{O} \rangle_{K_\perp} = \frac{\left\langle P^+,\vec{K}_\perp \middle| \mathbf{O} \middle| P^+,\vec{P}_\perp{=}0 \right\rangle}{\langle K|P\rangle}\,,$$

Evaluate matrix elements

Using light front wave function:

Brodsky, LePage 1989, Brodsky et al NP B593,311, Kovchechov Levin book

Only way to have a wave function in QFT , coordinates are $x_i, p_{\perp,i}$

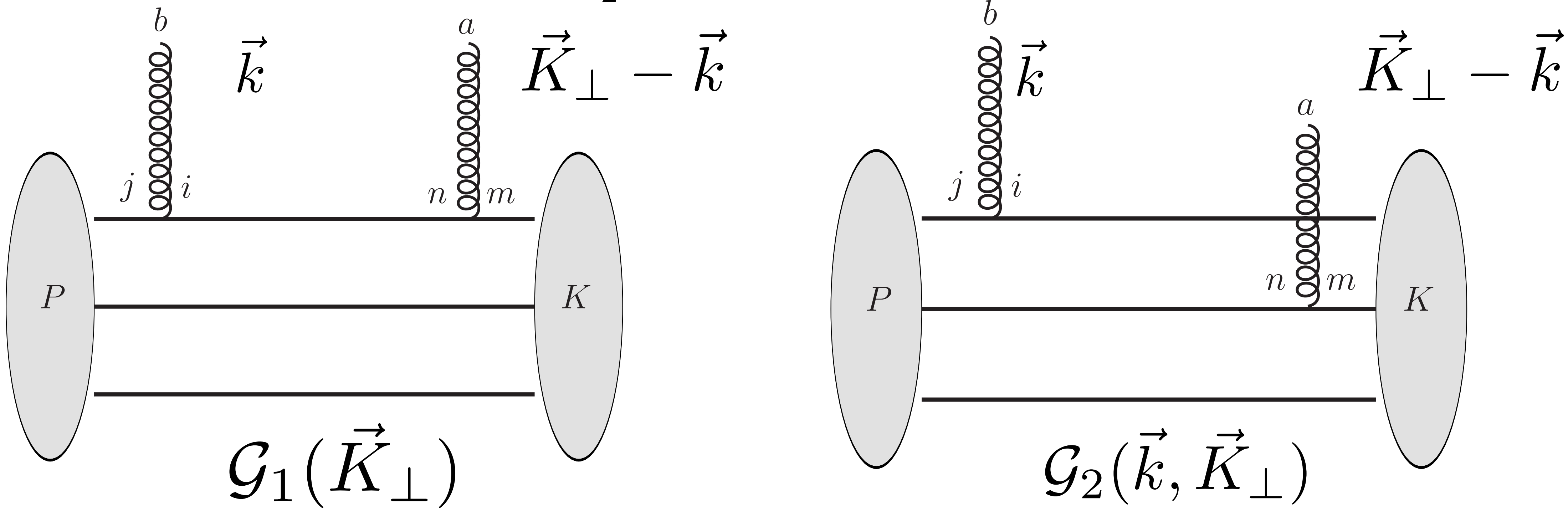
First calculations in 1808.02501—3 quark Fock space:

$\langle \rho^a(k_\perp) \rangle_{K_\perp} = 0$

Two-quark color charge density

Use proton light front 3-quark wave function

$$\langle \tilde\rho^a(q)\, \tilde\rho^b(k) \rangle_{K_\perp} = \tfrac{1}{2}\, \delta^{ab}(\mathcal{G}_1(\vec K_\perp) - \mathcal{G}_2(\vec k, \vec K_\perp))$$



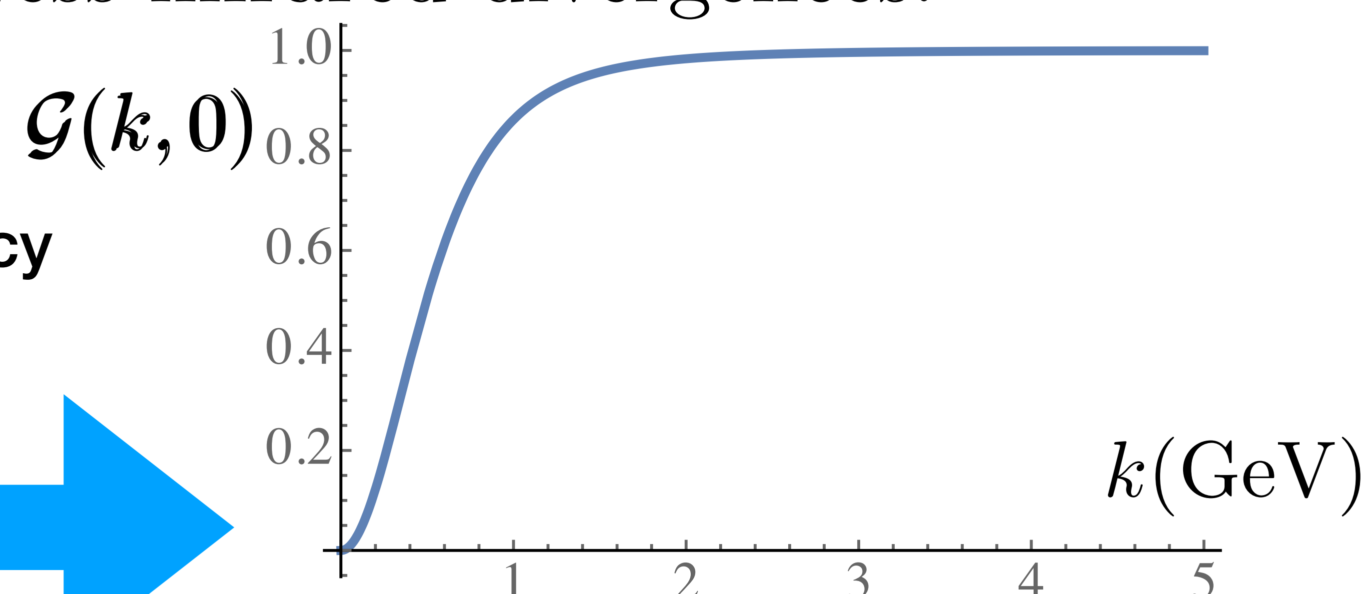
forward scattering $\vec K_\perp = 0$, $\mathcal{G}(\vec k, 0) = 1 - \mathcal{G}_2(\vec k, 0)$

$\mathcal{G}(0,0) = 0$

Color neutrality, $\to$ suppress infrared divergences.

Cancellation is basis of color transparency AND factorization theorems

Cancellation



Gluon distribution in a proton

lowest order YM in light cone gauge

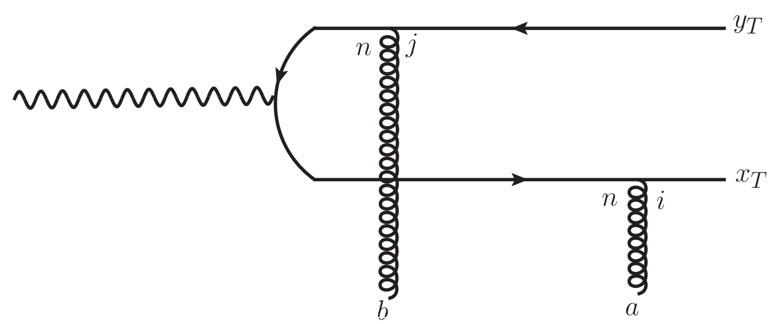
Poisson Eq. $F_a^{+i}(\mathrm{k}_\perp) \simeq ig\frac{k^i}{k_\perp^2}\,\rho^a(\mathrm{k}_\perp)$

$$xG(x,Q^2) \simeq \frac{g^2}{4\pi^2}\,\frac{(N_c^2-1)}{2}\int_0^{Q^2}\frac{dk_\perp^2}{k_\perp^2}(1-\mathcal{G}_2(k_\perp,0))\,.$$

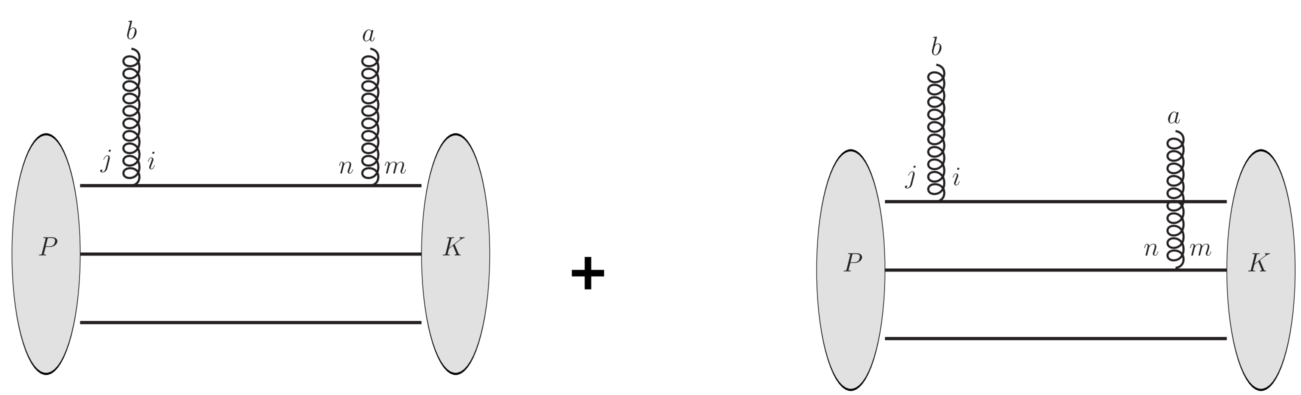
NO infrared divergence due to cancellation

Large Q^2: $xG(x,Q^2) \simeq \frac{N_c\alpha_S}{\pi}C_F\ln(Q^2/\Lambda^2)$

Two gluon exchange in J/psi production



$$i\int d^2r\int_0^1 \frac{dz}{4\pi}\left(\Psi_{\gamma^*}\Psi^*_{Q\bar{Q}}\right)(r,z,Q^2)\; e^{-i\frac{(1-2z)}{2}r\cdot\vec{K}_\perp}\;\times$$



$$\times\int d^2b_\perp\, e^{i\vec{b}_\perp\cdot\vec{K}_\perp}\;\mathcal{T}(r,b_\perp;K_\perp)\,.$$

$$\mathcal{T}(r,b_\perp,K_\perp) = -\frac{g^4}{2N_c}\,\delta_{ab}\int_{q_1}\int_{q_2}\frac{e^{i\vec{b}_\perp\cdot(\vec{q}_1+\vec{q}_2)}}{q_1^2q_2^2}\left[e^{i\frac{\vec{r}}{2}\cdot(\vec{q_1}-\vec{q_2})}-\frac{1}{2}e^{i(q_1+q_2)\cdot\frac{\vec{r}}{2}}-\frac{1}{2}e^{-i(q_1+q_2)\cdot\frac{\vec{r}}{2}}\right]\mathcal{G}_2(q_1,K_\perp)$$

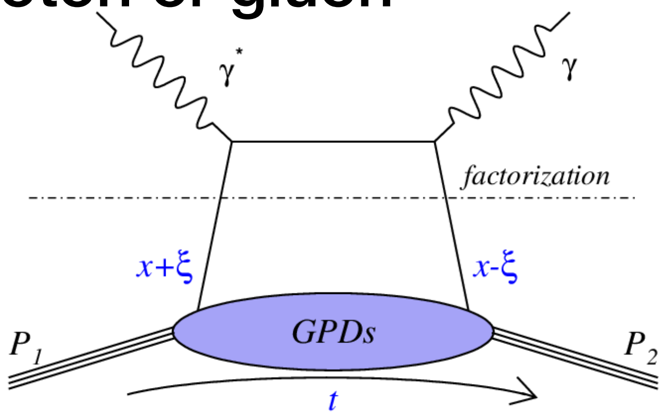
No singularity if either q_1 or q_2 =0 due to cancellation needs both diagrams

This amplitude is called the pomeron

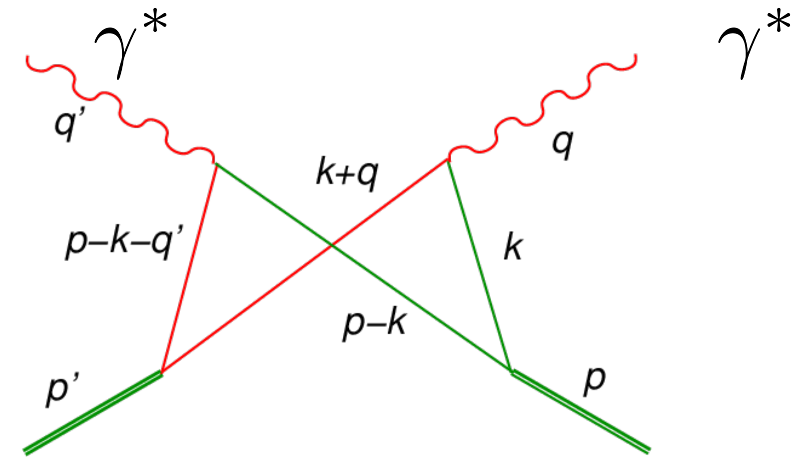
Moments of color charge density vs GPDs

photon or gluon

GPDs:



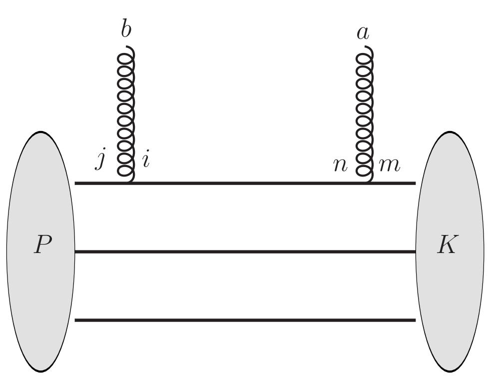
Handbag HB



Cat's Ears (CE)

For current conservation need HB +CE, with hard photon HB>CE

Moments of color charge density:
soft gluons not hard gluons, photons



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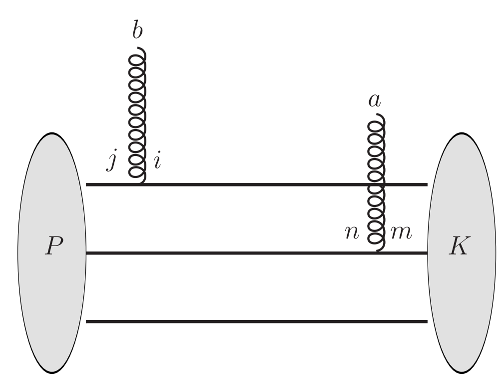


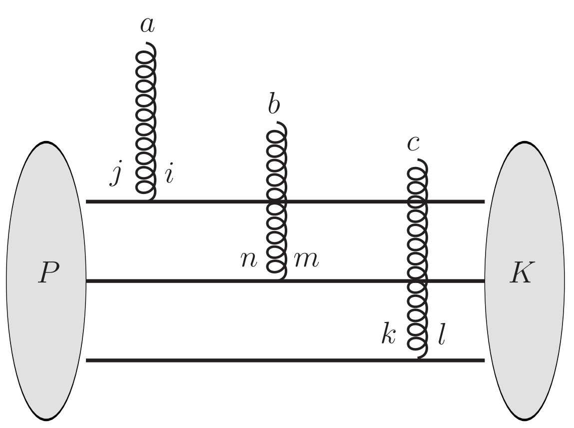
HB

CE

Both needed for gauge invariance and IR safety

Three-quark color charge density

$\langle \tilde{\rho}^a \tilde{\rho}^b \tilde{\rho}^c \rangle_{K_\perp}$



There are three types of terms:

1. All three gluons from same quark
2. Two gluons from same quark one from another (3)
3. Each gluon is emitted from a different quark

$$\langle [\tilde{\rho}^a \tilde{\rho}^b \tilde{\rho}^c]^1_S \rangle_{K_\perp} = \frac{d^{abc}}{N_c} \mathcal{G}_O(q_1, q_2, q_3)$$

Expressions in 1808.02501

- Three gluons can be odderon have C=-1, otherwise vacuum quantum numbers,
- odderon: coupling to quark=-coupling to anti-quark
- Three-gluon exchange contributes to $\gamma^* + p \to p + J/\Psi$
- Three-gluon exchange contributes to $\gamma^* + p \to p + \eta_c$
- Potential to discover the odderon!

Summary-1808.02501
Dumitru, Miller, Venugopalan

- New way of looking at proton structure
- Use moments of color charge density operator
- Quadratic and cubic correlates in the proton have been constructed for 3-quark light-front wave function
- Can do same for more complicated wave functions
- Quadratic correlator corresponds to Pomeron, cubic to Odderon
- Complementary to GPD formalism