Generalized TMDs and Wigner Functions

(A. Metz, Temple University)

Related dedicated talks (Week 1)

- Marc Schlegel: *Definition of generalized TMDs*
- Barbara Pasquini: *Wigner functions and nucleon structure*
- Feng Yuan: *Observables for partonic Wigner functions*

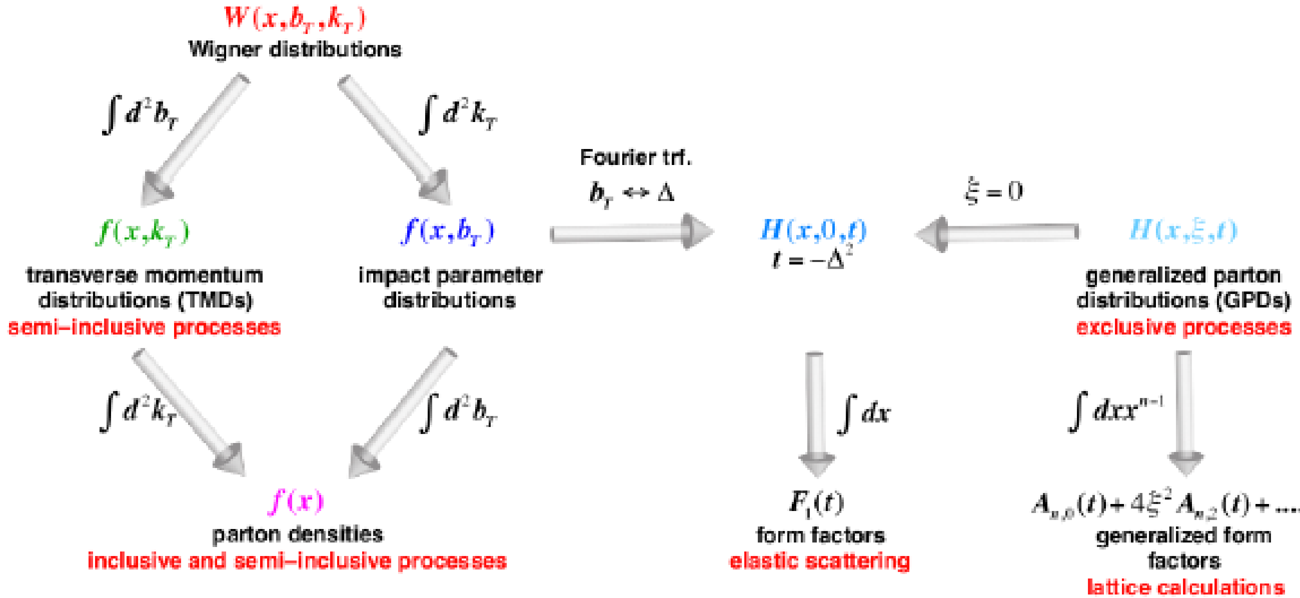
Some other talks have overlap as well

supported by the NSF

Outline

- Introduction (brief)
- Generalized TMDs
- Wigner Functions
- Applications of GTMDs and Wigner Functions
- Observables for GTMDs and Wigner Functions
- Summary

Introduction



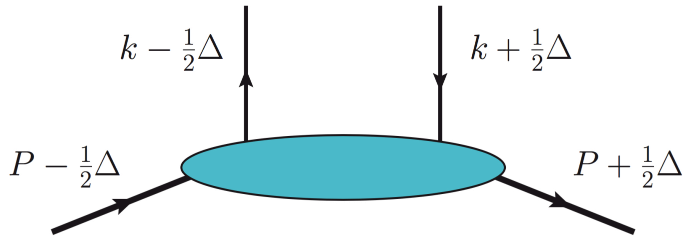
(from EIC White Paper, arXiv:1212.1701)

"... $f(x, b_T)$ and $f(x, k_T)$ give complementary information about partons, and both types of quantities can be thought of as descendants of Wigner distributions $W(x, b_T, k_T)$, which are used extensively in other branches of physics. Although there is no known way to measure Wigner distributions for quarks and gluons, they provide a unifying theoretical framework for the different aspects of hadron structure we have discussed."

- In the meantime, some interesting developments in this field
- Can Wigner functions (GTMDs) play important role for EIC science case — are they sufficiently interesting and can they be measured ?

Generalized TMDs

- Graphical representation of GTMD correlator for quarks; kinematics in symmetric frame



$$P = \frac{p + p'}{2} \qquad \Delta = p' - p$$

- GTMD correlator: definition through traces (can appear in observables)

$$W^{q\,[\Gamma]} = \int \frac{dz^-\, d^2\vec{z}_\perp}{2\,(2\pi)^3}\, e^{ik\cdot z}\, \langle p' \,|\, \bar{\psi}^q(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}_{\rm TMD}[-\tfrac{z}{2}, \tfrac{z}{2}]\, \psi^q(\tfrac{z}{2}) \,|\, p\rangle\Big|_{z^+=0}$$

  – $W^{q\,[\Gamma]}$ parameterized through GTMDs $X^q(x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp)$

$$x = \frac{k^+}{P^+} \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+} \qquad \vec{k}_\perp \qquad \vec{\Delta}_\perp = \vec{p}\,'_\perp - \vec{p}_\perp$$

  – two auxiliary scales (for which evolution equations exist) omitted

  – issue of light-cone singularities in definition of GTMDs can be dealt with
  (Echevarria, Idilbi, Kanazawa, Lorcé, Kanazawa, Metz, Pasquini, Schlegel, 1602.06953)

- Leading-twist chiral-even quark GTMDs (Meissner, Metz, Schlegel, 0906.5323)

$$W^{[\gamma^+]} = \frac{1}{2M}\,\bar{u}(p')\Big[F_{1,1} + \frac{i\sigma^{i+}k_\perp^i}{P^+}F_{1,2} + \frac{i\sigma^{i+}\Delta_\perp^i}{P^+}F_{1,3} + \frac{i\sigma^{ij}k_\perp^i\Delta_\perp^j}{M^2}F_{1,4}\Big]u(p)$$

$$W^{[\gamma^+\gamma_5]} = \frac{1}{2M}\,\bar{u}(p')\Big[-\frac{i\varepsilon_\perp^{ij}k_\perp^i\Delta_\perp^j}{M^2}G_{1,1} + \frac{i\sigma^{i+}\gamma_5 k_\perp^i}{P^+}G_{1,2} + \frac{i\sigma^{i+}\gamma_5\Delta_\perp^i}{P^+}G_{1,3} + i\sigma^{+-}\gamma_5\,G_{1,4}\Big]u(p)$$

- General results
  - 16 leading-twist GTMDs for quarks (Meissner, Metz, Schlegel, 0906.5323)
  - 16 leading-twist GTMDs for gluons (Lorcé, Pasquini, 1307.4497)
  - GTMDs have real and imaginary part

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- “Philosophical” remark
  - community should not be afraid of “collecting stamps” (GTMDs, GPDs, TMDs)
  - many “stamps” in other areas (e.g., periodic table of elements)
  - we find “stamps” in nature (which is never boring)

Generalized TMDs as "Mother Functions"

- GTMD-correlator

$$W^{q\,[\Gamma]} = \int \frac{dz^-\, d^2\vec{z}_\perp}{2\,(2\pi)^3}\, e^{ik\cdot z}\, \langle p' \,|\, \bar{\psi}^q(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}_{\rm TMD}[-\tfrac{z}{2},\tfrac{z}{2}]\, \psi^q(\tfrac{z}{2}) \,|\, p\rangle\Big|_{z^+=0}$$

- Projection onto TMDs and GPDs

$$\Phi^{q\,[\Gamma]} = \int \frac{dz^-\, d^2\vec{z}_\perp}{2\,(2\pi)^3}\, e^{ik\cdot z}\, \langle p \,|\, \bar{\psi}^q(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}_{\rm TMD}[-\tfrac{z}{2},\tfrac{z}{2}]\, \psi^q(\tfrac{z}{2}) \,|\, p\rangle\Big|_{z^+=0}$$

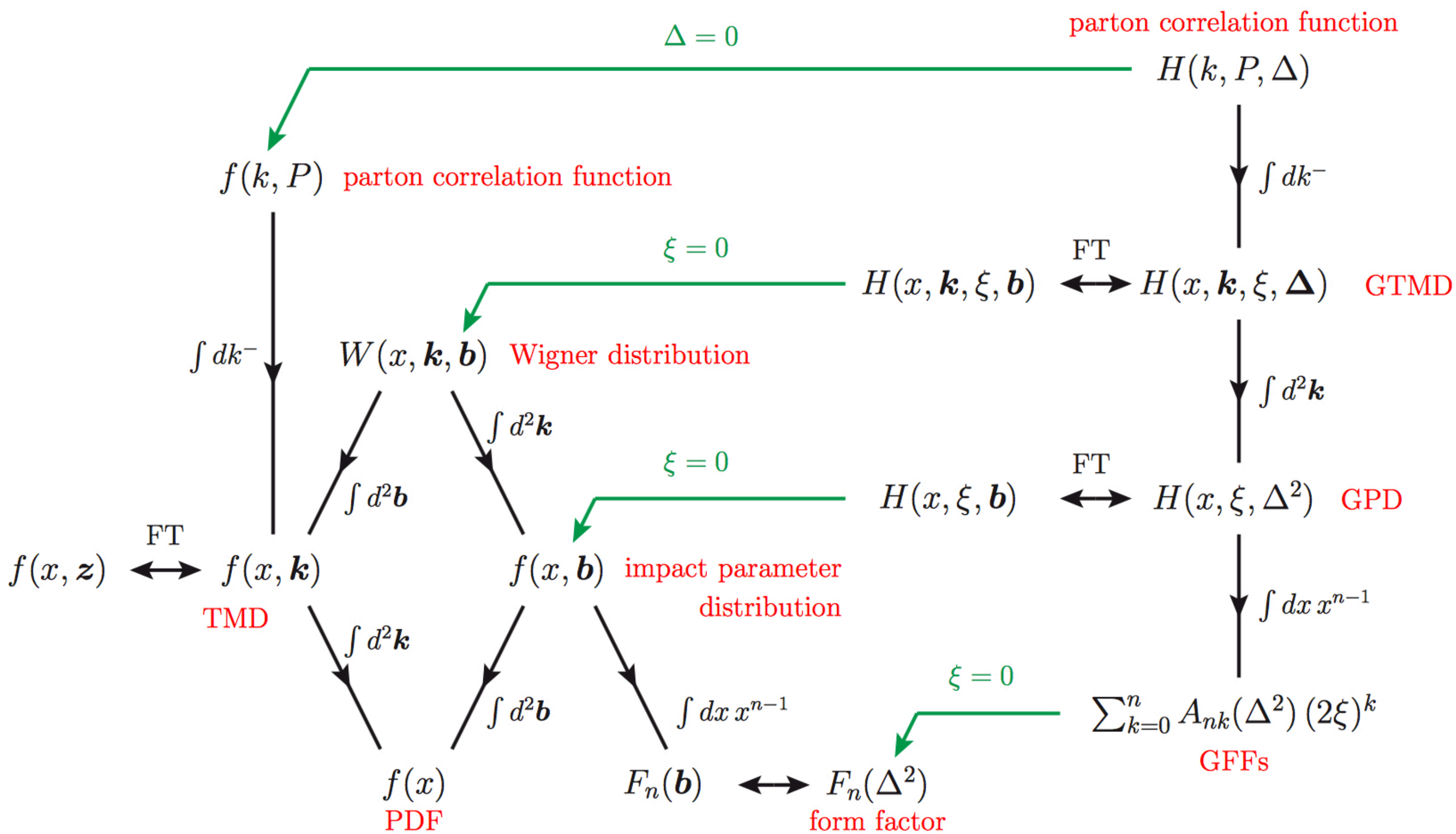
$$= W^{q\,[\Gamma]}\Big|_{\Delta=0}$$

$$F^{q\,[\Gamma]} = \int \frac{dz^-}{4\pi}\, e^{ik\cdot z}\, \langle p' \,|\, \bar{\psi}^q(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}_{\rm PDF}[-\tfrac{z}{2},\tfrac{z}{2}]\, \psi^q(\tfrac{z}{2}) \,|\, p\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

$$= \int d^2\vec{k}_\perp\, W^{q\,[\Gamma]}$$

  - all TMDs and GPDs are projections of GTMDs
  - first application: no model-independent non-trivial relation btw GPDs and TMDs
    $\rightarrow$ affects also relation between GPD E and Sivers function $f_{1T}^{\perp}$
    (Meissner, Metz, Schlegel, Goeke, 0805.3165, 0906.5323)

- Overview of quantities characterizing parton structure of hadrons



(figure from Diehl, arXiv:1512.01328)

  - not many studies on k^- dependent correlators (parton correlation functions)
  - GTMDs describe most general two-parton structure of hadrons
  - GTMDs contain genuine new physics (beyond GPDs and TMDs)
  - modelling/measuring GTMDs may be an important goal of parton structure studies

Wigner Functions

- Wigner functions in non-relativistic QM (Wigner, 1932)
  - calculable from wave function

$$\mathcal{W}(x,k) = \int \frac{dx'}{2\pi}\, e^{-i\,k\,x'} \psi^*\left(x - \frac{x'}{2}\right) \psi\left(x + \frac{x'}{2}\right)$$

$$= \int \frac{dk'}{2\pi}\, e^{+i\,k'x}\, \tilde{\psi}^*\left(k - \frac{k'}{2}\right) \tilde{\psi}\left(k + \frac{k'}{2}\right)$$

  - relation to densities and observables

$$|\psi(x)|^2 = \int dk\, \mathcal{W}(x,k)$$

$$|\tilde{\psi}(k)|^2 = \int dx\, \mathcal{W}(x,k)$$

$$\langle O(x,k)\rangle = \int dx\, dk\, O(x,k)\, \mathcal{W}(x,k)$$

  - $\mathcal{W}(x,k)$ can become negative (quantum effect) $\rightarrow$ quasi-probabilty distribution
  - $\mathcal{W}(x,k)$ may help to study transition btw quantum and classical mechanics

- Partonic Wigner functions
  (Belitsky, Ji, Yuan, hep-ph/0307383 / Lorcé, Pasquini, 1106.0139)
  - Wigner operator

$$\widehat{W}^{q\,[\Gamma]}(x,\vec{k}_\perp,\vec{b}_\perp) = \int \frac{dz^-\,d^2\vec{z}_\perp}{2\,(2\pi)^3}\, e^{ik\cdot z}\, \bar{\psi}^q\Big(\vec{b}_\perp - \frac{z}{2}\Big)\,\Gamma\,\mathcal{W}_{\text{TMD}}\,\psi^q\Big(\vec{b}_\perp + \frac{z}{2}\Big)\Big|_{z^+=0}$$

  - correlator for Wigner functions ($\xi = 0$) $\rightarrow$ Fourier transform of GTMD correlator

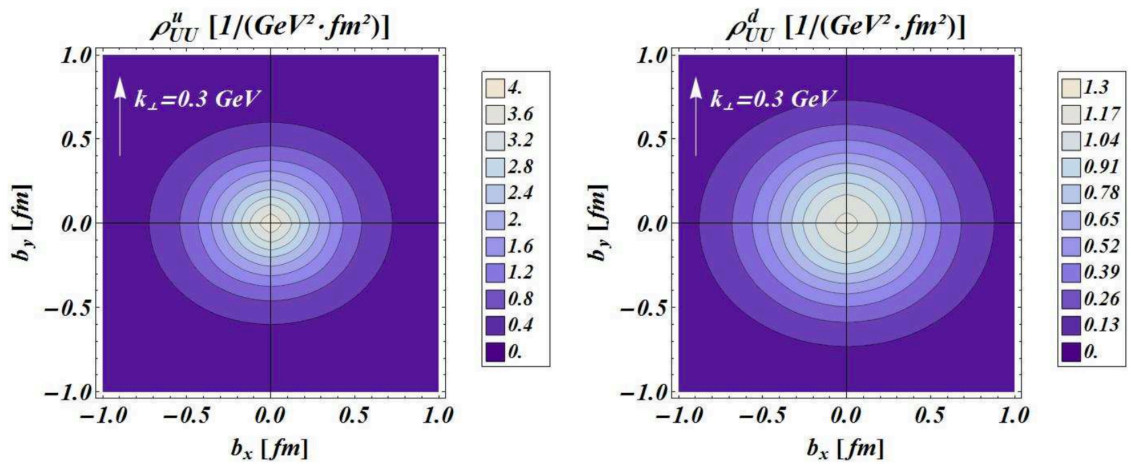
$$\begin{aligned}\mathcal{W}^{q\,[\Gamma]}(x,\vec{k}_\perp,\vec{b}_\perp) &= \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\Big\langle p^+, \frac{\vec{\Delta}_\perp}{2}\Big|\,\widehat{W}^{q\,[\Gamma]}(x,\vec{k}_\perp,\vec{b}_\perp)\,\Big|p^+, -\frac{\vec{\Delta}_\perp}{2}\Big\rangle \\ &= \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-i\,\vec{\Delta}_\perp\cdot\vec{b}_\perp}\, W^{q\,[\Gamma]}(x,\vec{k}_\perp,\vec{\Delta}_\perp)\Big|_{\xi=0}\end{aligned}$$

  - relation to densities and observables (note analogy to non-relativistic QM)

$$\begin{aligned}\mathcal{F}^{q\,[\Gamma]}(x,\vec{b}_\perp) &= \int d^2\vec{k}_\perp\, \mathcal{W}^{q\,[\Gamma]}(x,\vec{k}_\perp,\vec{b}_\perp) \\ \Phi^{q\,[\Gamma]}(x,\vec{k}_\perp) &= \int d^2\vec{b}_\perp\, \mathcal{W}^{q\,[\Gamma]}(x,\vec{k}_\perp,\vec{b}_\perp) \\ \langle O(x,\vec{k}_\perp,\vec{b}_\perp)\rangle &= \int dx\, d^2\vec{k}_\perp\, d^2\vec{b}_\perp\, O(x,\vec{k}_\perp,\vec{b}_\perp)\, \mathcal{W}^{q[\Gamma]}(x,\vec{k}_\perp,\vec{b}_\perp)\end{aligned}$$

Applications of GTMDs and Wigner Functions

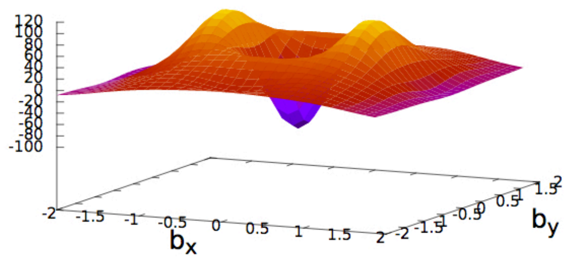
1. 5-D Imaging of Hadrons
   - Numerical example of Wigner distributions (Lorcé, Pasquini, 1106.0139)



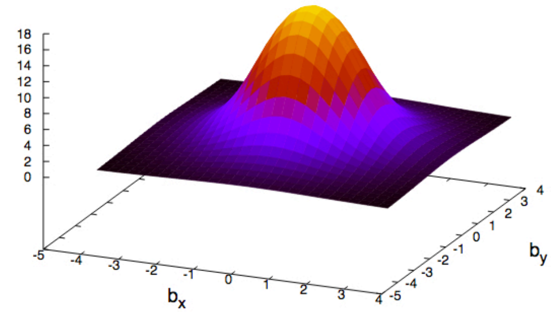
     - figures show $\mathcal{W}_U^{q\,[\gamma^+]}(x, \vec{k}_\perp, \vec{b}_\perp)$ (unpolarized quarks and unpolarized nucleon), integrated upon x, for fixed $\vec{k}_\perp$
     - results in light-cone constituent quark model
     - wider distribution for down quarks (known also from form factor studies)
     - distortion due to dependence on $\vec{k}_\perp \cdot \vec{b}_\perp$
     - top-bottom symmetry since $\mathcal{W}_U^{q\,[\gamma^+]}$ is even function of $\vec{k}_\perp \cdot \vec{b}_\perp$
     - interpreting results as prob. densities agrees with intuition from confinement
     - similar examples including polarization available (Lorcé, Pasquini, 1106.0139 / ...)

- But Wigner functions are quasi-probability distributions that can become negative
  - example: quark-target model at one loop (Hagiwara, Hatta, 1412.4591)

Wigner distribution



Husimi distribution



$x = 0.5$ $\qquad$ $k_\perp = 0.5\,\mathrm{GeV}$ in x-direction

  - Husimi distribution involves Gaussian smearing for $b_\perp$ and $k_\perp$ (Husimi, 1940)
    $\rightarrow$ positive-definite in non-relativistic QM
  - Husimi distributions may be considered for hadrons (Hagiwara, Hatta, 1412.4591)
  - Husimi distributions expected to be positive-definite in field theory (Hagiwara, Hatta, 1412.4591)
  - connection btw Husimi distributions and GPDs/TMDs ?

- Some questions
  - is there "optimal" tool for 5-D imaging of hadrons ?
  - interpretation of Wigner functions (if they are negative) ?
    $\rightarrow$ input from other fields ?

2. Orbital Angular Momentum of Partons

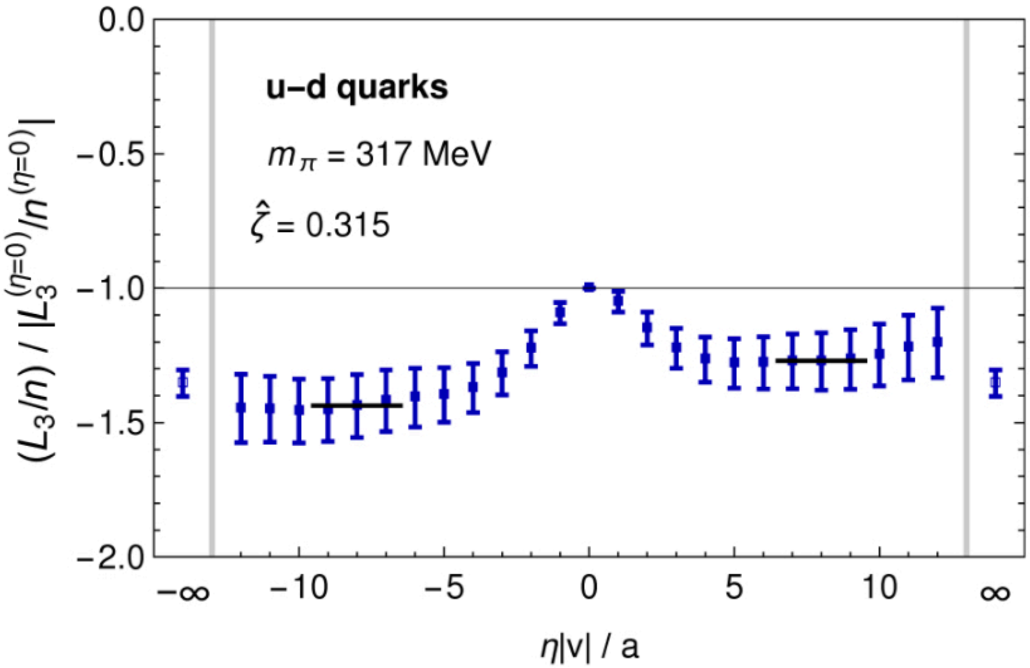
- Parton OAM in longitudinally polarized nucleon
  (Lorcé, Pasquini, 1106.0139 / Hatta, 1111.3547 / Hägler, Mukherjee, Schäfer, hep-ph/0310136)

$$\begin{aligned} L_z^q &= \int dx\, d^2\vec{k}_\perp\, d^2\vec{b}_\perp\, (\vec{b}_\perp \times \vec{k}_\perp)_z\, \mathcal{W}_L^{q\,[\gamma^+]}(x, \vec{k}_\perp, \vec{b}_\perp) \\ &= -\int dx\, d^2\vec{k}_\perp\, \frac{\vec{k}_\perp^{\,2}}{M^2}\, F_{1,4}^q(x, \vec{k}_\perp^{\,2})\Big|_{\Delta=0} \end{aligned}$$

  - intuitive definition of OAM
  - same equation for both L_{JM} (staple-like link) and L_{Ji} (straight link)
    (Ji, Xiong, Yuan, 1202.2843)
  - this definition of OAM allows for intuitive interpretation of $L_{\mathrm{JM}} - L_{\mathrm{Ji}}$
    (Burkardt, 1205.2916, talk in Week 3)
  - equation holds for gluons as well
    (Hatta, 1111.3547 / Ji, Xiong, Yuan, 1207.5221 / Hatta, Yoshida, 1207.5332)

- By using Wigner functions one can also define OAM density
  (Lorcé, Pasquini, 1106.0139 / Hatta, 1111.3547 / Ji, Xiong, Yuan, 1207.5221 /
  Hatta, Yoshida, 1207.5332 / Lorcé, 1210.2581 / Rajan, Courtoy, Engelhardt, Liuti, 1601.06117 / ...)

- Exploratory calculation of $L_{\rm JM}^{u-d}$ in lattice QCD
  (Engelhardt, 1701.01536, talk at SPIN 2018)



  - figure shows essentially $L_{\rm JM}^{u-d}/|L_{\rm Ji}^{u-d}|$ (for large $\eta|v|/a$)
  - considerable numerical difference btw $L_{\rm JM}^{u-d}$ and $L_{\rm Ji}^{u-d}$

- Developments in this area can be considered milestone in spin physics

3. Spin-Orbit Correlations

(Lorcé, Pasquini, 1106.0139 / Lorcé, 1401.7784 / Lorcé, Pasquini, 1512.06744 / ...)

quark polarization (columns) / nucleon polarization (rows)

ρ_X	U	L	T_x	T_y
U	$\langle 1 \rangle$	$\langle S_L^q \ell_L^q \rangle$	$\langle S_x^q \ell_x^q \rangle$	$\langle S_y^q \ell_y^q \rangle$
L	$\langle S_L \ell_L^q \rangle$	$\langle S_L S_L^q \rangle$	$\langle S_L \ell_L^q S_x^q \ell_x^q \rangle$	$\langle S_L \ell_L^q S_y^q \ell_y^q \rangle$
T_x	$\langle S_x \ell_x^q \rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q \rangle$	$\langle S_x S_x^q \rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q \rangle$
T_y	$\langle S_y \ell_y^q \rangle$	$\langle S_y \ell_y^q S_L^q \ell_L^q \rangle$	$\langle S_y \ell_y^q S_x^q \ell_x^q \rangle$	$\langle S_y S_y^q \rangle$

$\xi = 0$

GPD	U	L	T
U	H		$\mathcal{E}_T$
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	H_T, $\tilde{H}_T$

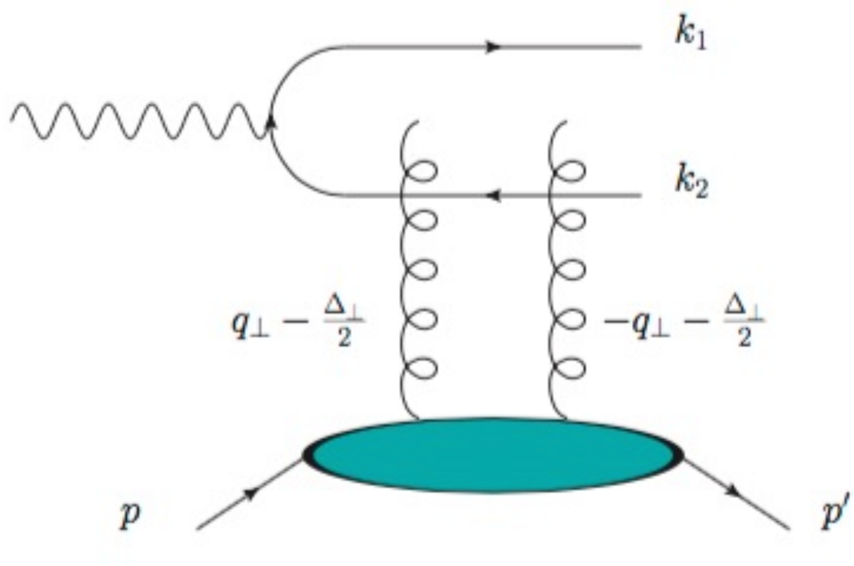
TMD	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

(from talk of B. Pasquini in Week 1)

- Number 16 of leading-twist Wigner functions (GTMDs) appears "naturally"
- Dependence on both $b_\perp$ and $k_\perp$ allows one to talk about spin-orbit correlations (similar to AMO physics)

Observables for GTMDs and Wigner Functions

1. Diffractive Exclusive Back-To-Back Dijet Production in $\ell N/\ell A$ Collisions
(Hatta, Xiao, Yuan, 1601.01585 / Altinoluk, Armesto, Beuf, Rezaeian, 1511.07452)



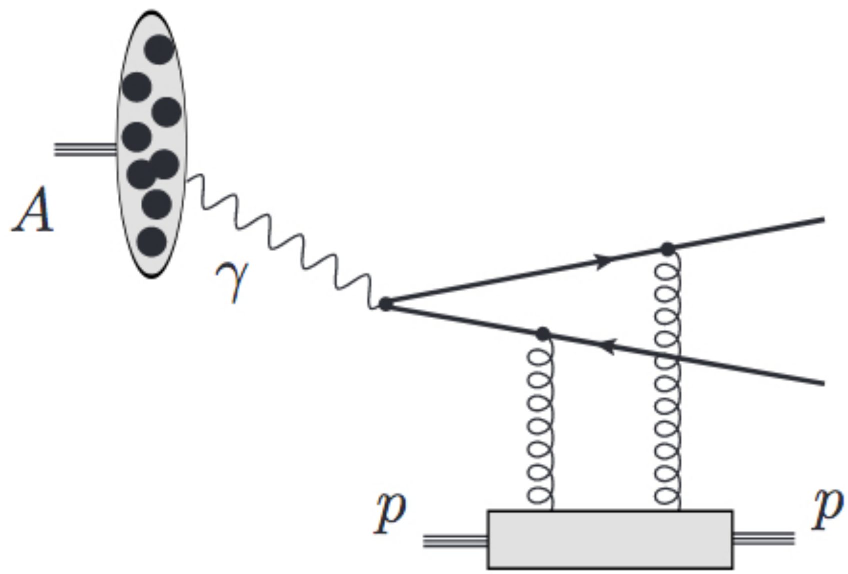
- Direct sensitivity to longitudinal and transverse parton momenta
$\rightarrow$ (gluon) GTMDs

- How precisely can one measure the (transverse) jet momenta ?

- With target polarization one may address GTMD $F_{1,4}^g \rightarrow$ gluon OAM $L_{\rm JM}^g$
(Ji, Yuan, Zhao, 1612.02438 / Hatta, Nakagawa, Xiao, Yuan, Zhao, 1612.02445)

2. Diffractive Dijet Production in Ultra-Peripheral pA/AA Collisions

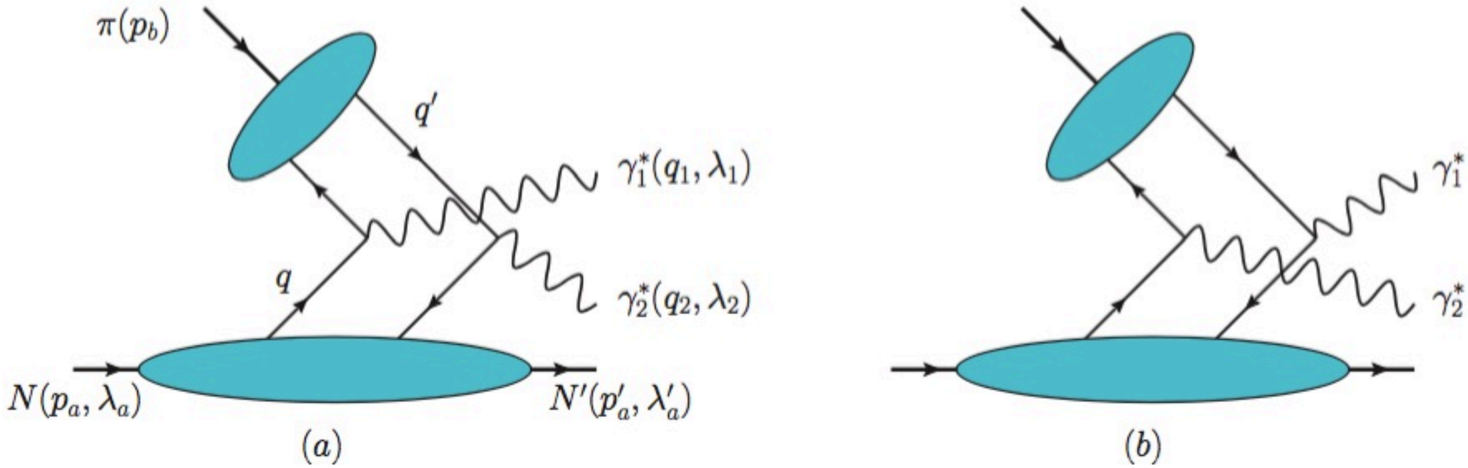
(Hagiwara, Hatta, Pasechnik, Tasevsky, Teryaev, 1706.01765)



- Use photon-flux provided by (heavy) nucleus
- Could be explored at LHC and RHIC (→ see also talk by E. Aschenauer)

3. Exclusive Double Drell-Yan Process: $\pi\, N \rightarrow (\ell_1^- \ell_1^+)\, (\ell_2^- \ell_2^+)\, N'$

(Bhattacharya, Metz, Zhou, 1702.04387)

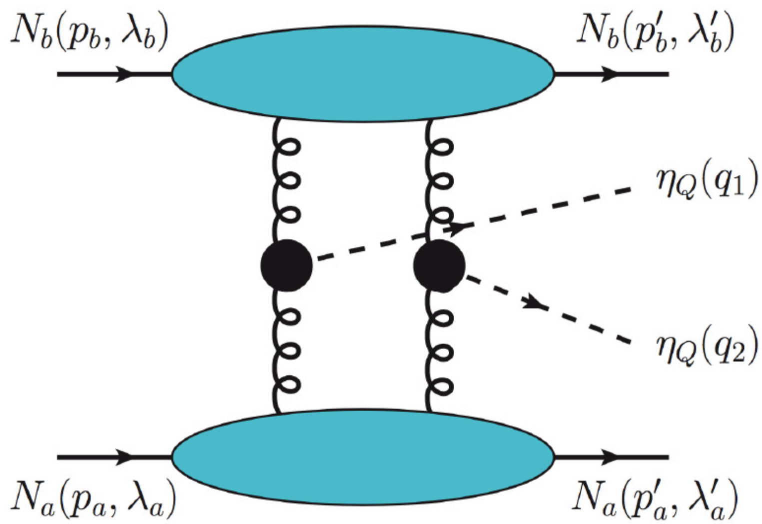


- At present, only known process that is sensitive to quark GTMDs
- In leading-order one is sensitive to ERBL region only
- Low count rate (amplitude $T \sim \alpha_{\rm em}^2$, like double-DVCS)
- Can one measure quark GTMDs in $\ell N / \ell A$ collisions ?

4. Exclusive Double η_Q-Production in Hadronic Collisions

- Double-exclusive NN scattering

  (Bhattacharya, Metz, Ojha, Tsai, Zhou, 1802.10550)



  - general topology like for double Drell-Yan, but larger count rate
  - sensitivity to gluon GTMDs
  - can the process be measured ?

- Single-exclusive NN scattering, exploiting double-parton scattering

  (Boussarie, Hatta, Xiao, Yuan, 1807.08697)

  - one nucleon breaks up
  - should have larger count rate than double-exclusive process

Summary

- GTMDs and partonic Wigner functions have attracted considerable interest
- Already several interesting applications (5-D imaging, OAM, spin-orbit correlations)
- Wigner functions can bring parton structure studies of hadrons to next level (more information than GPDs and TMDs, less information than wave function)
- GTMDs appear "naturally" in QCD description of certain exclusive processes
- Can one learn even more about "physics content" of Wigner functions ?
- Can one measure (at all) quark GTMDs in $\ell N/\ell A$ collisions (at an EIC) ?
- Dijet production may be key process for studying gluon GTMDs at an EIC
- Other processes in $\ell N/\ell A$ collisions that are sensitive to gluon GTMDs ?