Model Calculations of Euclidean Correlators

(A. Metz, Temple University)

- Introduction: quasi-distributions
- Model calculations: some results
- Quasi-GPDs in diquark spectator model
  (Bhattacharya, Cocuzza, AM, arXiv:1808.01437)
  - definition of quasi-GPDs
  - analytical results
  - numerical results
- Summary

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Quasi-PDFs

- Standard (light-cone) unpolarized quark PDF (support: $-1 \leq x \leq 1$)

$$f_1(x) = \frac{1}{2} \int \frac{dz^-}{2\pi} \, e^{ik \cdot z} \, \langle P | \bar{\psi}(-\tfrac{z}{2}) \, \gamma^+ \, \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) \, \psi(\tfrac{z}{2}) | P \rangle \Big|_{z^+ = 0, \vec{z}_\perp = \vec{0}_\perp}$$

  - nonlocal correlator depending on time $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}} \, z^-$
  - cannot be computed on Euclidean lattice

- Suggestion: consider quasi-PDF instead (Ji, 2013) (support: $-\infty < x < \infty$)

$$f_{1,\mathrm{Q}(3)}(x, P^3) = \frac{1}{2} \int \frac{dz^3}{2\pi} \, e^{ik \cdot z} \, \langle P | \bar{\psi}(-\tfrac{z}{2}) \, \gamma^3 \, \mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2}, \tfrac{z}{2}) \, \psi(\tfrac{z}{2}) | P \rangle \Big|_{z^0 = 0, \vec{z}_\perp = \vec{0}_\perp}$$

  - nonlocal correlator depending on position z^3
  - can be computed on Euclidean lattice
  - quasi-PDF depends on $x = k^3 / P^3$, and on hadron momentum P^3
  - for $P^3 \to \infty$, quasi-PDF and standard PDF contain same IR physics
  - LQCD calculations at finite $P^3 \to$ power corrections
  - difference in UV behavior is dealt with via perturbative matching
    (e.g., Xiong, Ji, Zhang, Zhao, 2013 / Stewart, Zhao, 2017 / Izubuchi, Ji, Jin, Stewart, Zhao, 2018)

- Generic structure of matching formula (scale-dependence omitted) ($\rightarrow$ talk by Y. Zhao)

$$f_{1,\mathrm{Q}(3)}(x, P^3) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}\right) f_1(y) + \mathcal{O}\left(\frac{M^2}{(P^3)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(P^3)^2}\right)$$

  - C is matching coefficient
  - matching presently known to one-loop order
  - several works on power corrections available

- Choosing γ^0 (instead of γ^3) for unpolarized quasi-PDF (Radyushkin, 2016)

$$f_{1,\mathrm{Q}(0)}(x, P^3) = \frac{1}{2} \int \frac{dz^3}{2\pi}\, e^{ik\cdot z}\, \langle P|\bar{\psi}(-\tfrac{z}{2})\, \gamma^0\, \mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2}, \tfrac{z}{2})\, \psi(\tfrac{z}{2})|P\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

  - better behaved w.r.t. power corrections (Radyushkin, 2016)
  - better behaved w.r.t. renormalization (Constantinou, Panagopoulos, 2017)

- Several other suggestions for computing PDFs and related quantities;
  some of them were proposed before quasi-PDFs and/or are related to quasi-PDFs
  $\rightarrow$ talk by V. Braun
  (Braun, Müller, 2008 / Ma, Qiu, 2014 / Radyushkin, 2017 / ...)

Spectator Model Calculations

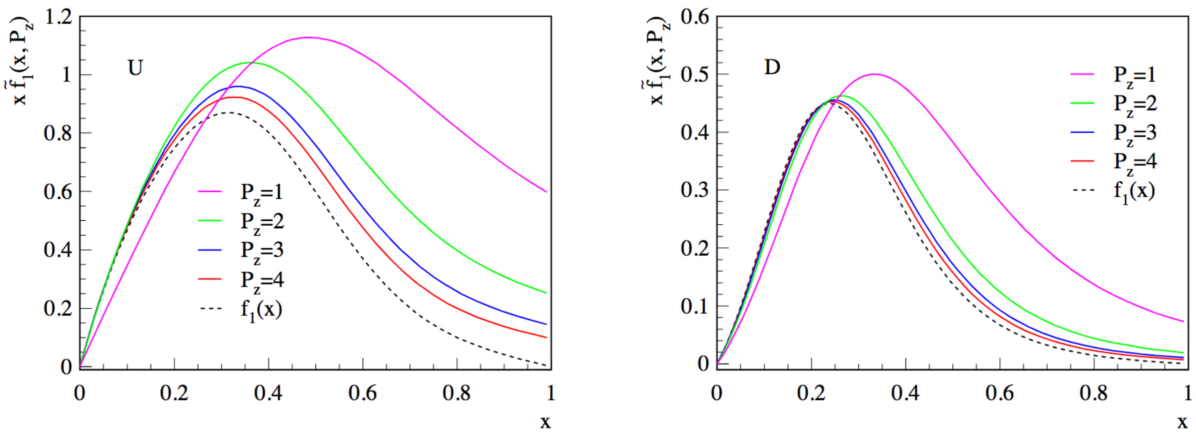
- Ingredients of spectator models for nucleon (e.g., Jakob, Mulders, Rodrigues, 1997)
  - idea: describe spectator partons as diquark (of spin-0 or spin-1)
  - graphical representation of two-quark correlator

  - Lagrange density for scalar diquark model

$$\mathcal{L}_{\rm SDM} = \bar{\Psi}\big(i\,\partial\!\!\!/ - M\big)\Psi + \bar{\psi}\big(i\,\partial\!\!\!/ - m_q\big)\psi + \frac{1}{2}\big(\partial_\mu\varphi\,\partial^\mu\varphi - m_s^2\,\varphi^2\big)$$
$$+\, g\big(\bar{\Psi}\,\psi\,\varphi + \bar{\psi}\,\Psi\,\varphi\big)$$

  - often phenomenological nucleon-quark-diquark vertex with form factor used
  - various studies available in literature
  - cut-graph (diquark on-shell) can be used to compute PDFs, but care has to be taken for quasi-PDFs (Bhattacharya, Cocuzza, AM, 2018)

- Spectator model calculation of quasi-PDFs (Gamberg, Kang, Vitev, Xing, 2014)
  - use scalar and vector diquark, and form factors at nucleon-quark-diquark vertices
  - parameters taken from previous work (Bacchetta, Conti, Radici, 2008)
  - results for up and down quarks in proton

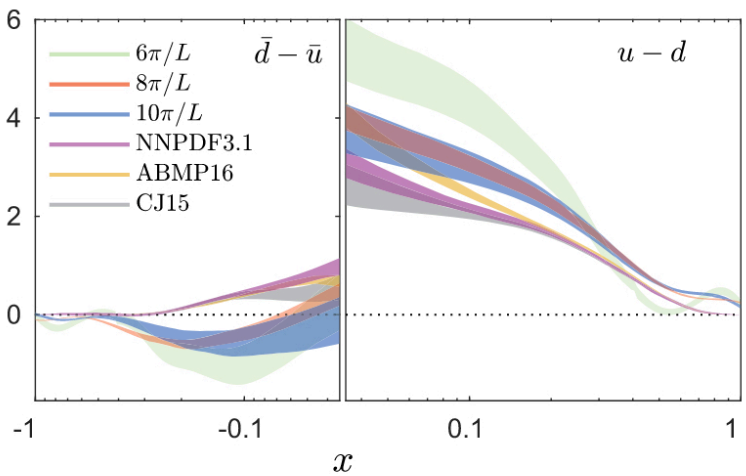


$$x\tilde{f}_1(x, P_z) = x f_{1,\mathrm{Q}(3)}(x, P^3)$$

  - for large P^3, quasi-PDFs are close to f_1 in wide x range
  - considerable discrepancies between quasi-PDFs and f_1 at large x
  - qualitatively, very similar findings for g_1 and h_1
  - related study proposes method to improve situation at large x (Bacchetta, Radici, Pasquini, Xiong, 2016)
  - similar study/findings for PDF of π and ρ mesons (Hobbs, 2017)

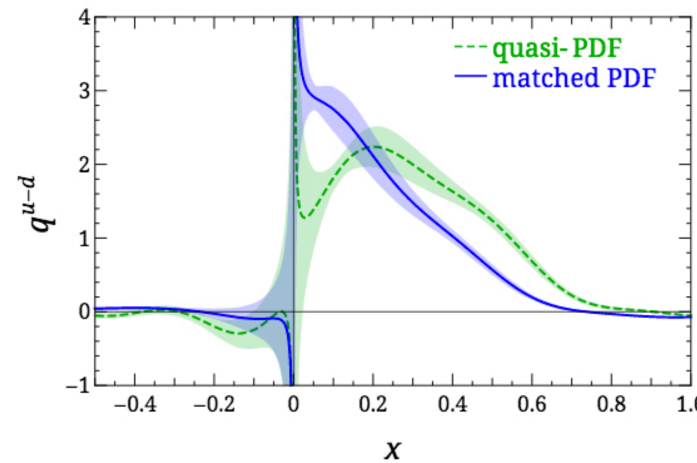
- Some results from LQCD ($f_{1,\mathrm{Q}(0)}$, at physical m_π)

(Alexandrou et al, 2018)



largest $P^3 = 1.38\,\mathrm{GeV}$ ($\widehat{=} 10\pi/L$)

(Chen et al, 2018)



$P^3 = 2.2\,\mathrm{GeV}$

  – overall, encouraging results

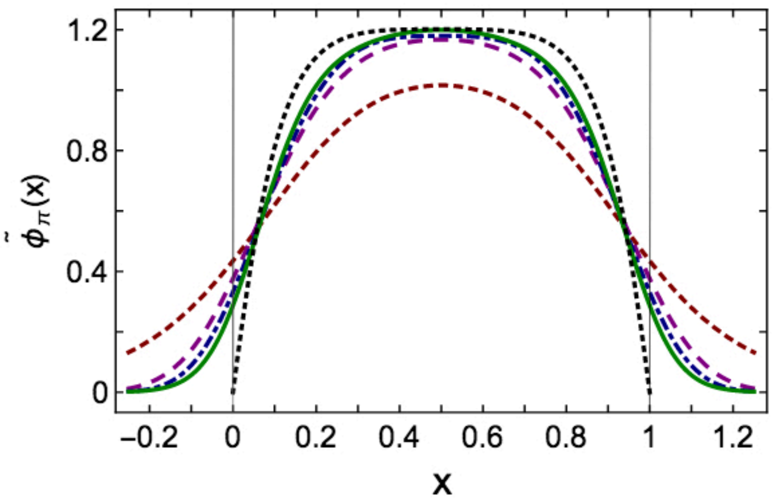
  – at large x, discrepancies btw quasi-PDFs and standard PDFs clearly seen in LQCD calculations

  – matching may reduce discrepancies at large x

Further Model Results

- Model for π and K using Bethe-Salpeter wave functions (Xu, Zhang, Roberts, Zong, 2018)

Quasi-DA for π



  - dotted: standard DA
    full: quasi-DA for $P^3 = 3.0\,\mathrm{GeV}$
  - good/decent agreement btw quasi-DA and standard DA in wide x range
  - discrepancies at end points
  - similar results for ϕ_K, and PDFs of π and K

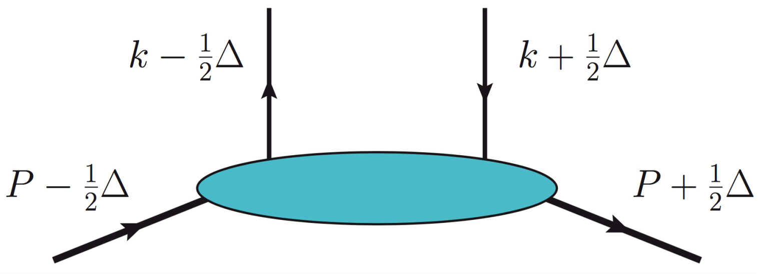
- Related study of quasi-DAs for π and K in nonlocal chiral quark model (Nam, 2017)

- Calculation of quasi-DA and quasi-PDF for π in NJL model and spectral quark model; calculation of pseudo-PDF; etc (Broniowski, Ruiz Arriola, 2017)

- Calculations of quasi-DA and quasi-PDF in 2D-QCD (in large N_c limit: t'Hooft model) (Jia, Liang, Xiong, Yu, 2018 / Ji, Liu, Zahed, 2018)
  - no UV divergence $\rightarrow$ no nontrivial matching
  - calculations were motivated as check that IR physics of quasi and standard distributions identical for large P^3

Definition of (Quasi-) GPDs

- GPD correlator: graphical representation



$$P = \frac{p + p'}{2} \qquad \Delta = p' - p$$

- (Light cone) correlator for standard GPDs of quarks

$$F^{[\Gamma]}(x, \Delta) = \frac{1}{2} \int \frac{dz^-}{2\pi} \, e^{ik \cdot z} \, \langle p' | \bar{\psi}(-\tfrac{z}{2}) \, \Gamma \, \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) \, \psi(\tfrac{z}{2}) | p \rangle \Big|_{z^+ = 0, \vec{z}_\perp = \vec{0}_\perp}$$

correlator parameterized through GPDs $X(x, \xi, t)$

$$x = \frac{k^+}{P^+} \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+} \qquad t = \Delta^2$$

- Kinematic relation

$$t = -\frac{1}{1 - \xi^2} \left(4\xi^2 M^2 + \vec{\Delta}_\perp^2 \right)$$

- (Spatial) correlator for quasi-GPDs of quarks (Ji, 2013)

$$F_{\rm Q}^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi}\, e^{ik\cdot z}\,\langle p'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\rm Q}(-\tfrac{z}{2},\tfrac{z}{2})\,\psi(\tfrac{z}{2})|p\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

- Definition of twist-2 vector quasi-GPDs $H_{\rm Q}$ and $E_{\rm Q}$

$$F^{[\gamma^0]}(x,\Delta;P^3) \;=\; \frac{1}{2P^0}\,\bar{u}(p')\Big[\gamma^0\,H_{\rm Q(0)} + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}\,E_{\rm Q(0)}\Big]u(p)$$

$$F^{[\gamma^3]}(x,\Delta;P^3) \;=\; \frac{1}{2P^3}\,\bar{u}(p')\Big[\gamma^3\,H_{\rm Q(3)} + \frac{i\sigma^{3\mu}\Delta_\mu}{2M}\,E_{\rm Q(3)}\Big]u(p)$$

  - we have explored both definitions of quasi-GPDs
  - quasi-GPDs depend on

$$x = \frac{k^3}{P^3} \neq \frac{k^+}{P^+} \qquad\qquad \xi \qquad\qquad t = \Delta^2 \qquad\qquad P^3$$

  - other skewness variable, such as $\tilde{\xi}_3 = -\Delta^3/(2P^3)$, could be used, but numerical difference btw ξ and $\tilde{\xi}_3$ can be significant for finite P^3

- Previous work: matching calculations for quasi-GPDs (Ji, Schäfer, Xiong, Zhang, 2015 / Xiong, Zhang, 2015)

Analytical Results in Scalar Diquark Model

- Correlator for quasi-GPDs

$$F_{\mathrm{Q}}^{[\Gamma]}(x,\Delta;P^3) = \frac{i\,g^2}{2(2\pi)^4}\int dk^0\,d^2\vec{k}_\perp\,\frac{\bar{u}(p')\left(\not{k}+\frac{\not{\Delta}}{2}+m_q\right)\Gamma\left(\not{k}-\frac{\not{\Delta}}{2}+m_q\right)u(p)}{D_{\mathrm{GPD}}}$$

$$D_{\mathrm{GPD}} = \left[\left(k+\frac{\Delta}{2}\right)^2 - m_q^2 + i\varepsilon\right]\left[\left(k-\frac{\Delta}{2}\right)^2 - m_q^2 + i\varepsilon\right]\left[(P-k)^2 - m_s^2 + i\varepsilon\right]$$

- Quasi-GPDs: example

$$H_{\mathrm{Q}(0)}(x,\xi,t;P^3) = \frac{i\,g^2 P^3}{(2\pi)^4}\int dk^0\,d^2\vec{k}_\perp\,\frac{N_{H(0)}}{D_{\mathrm{GPD}}}$$

$$\begin{aligned} N_{H(0)} \quad = \quad & \delta(k^0)^2 - \frac{2}{P^3}\left[x(P^3)^2 - m_q M - x\,\frac{t}{4} - \frac{1}{2}\,\delta\xi t\,\frac{\vec{k}_\perp\cdot\vec{\Delta}_\perp}{\vec{\Delta}_\perp^2}\right]k^0 \\ & + \delta\left[x^2(P^3)^2 + \vec{k}_\perp^2 + m_q^2 + (1-2x)\,\frac{t}{4} - \delta\xi t\,\frac{\vec{k}_\perp\cdot\vec{\Delta}_\perp}{\vec{\Delta}_\perp^2}\right] \end{aligned}$$

  - kinematic variable $\delta \to 1$ for $P^3 \to \infty$
  - quasi-GPDs are continuous at $x = \pm\,\xi$ (even beyond leading twist)

- For $P^3 \to \infty$, we recover standard GPDs for all cases

Numerical Results in Scalar Diquark Model

- Parameter choice
  - coupling (exact value of g irrelevant for our purpose)

$$g = 1$$

  - masses must satisfy $M < m_s + m_q$; we mostly use

$$m_s = 0.7\,\mathrm{GeV} \qquad m_q = 0.35\,\mathrm{GeV}$$

    values similar to previous work (Gamberg, Kang, Vitev, Xing, 2014)
    "optimal choice" for minimizing difference btw quasi and standard distributions

  - momentum transfer

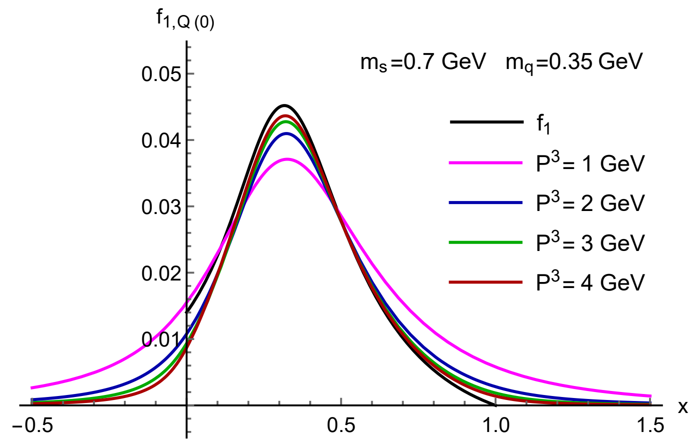
$$|\vec{\Delta}_\perp| = 0$$

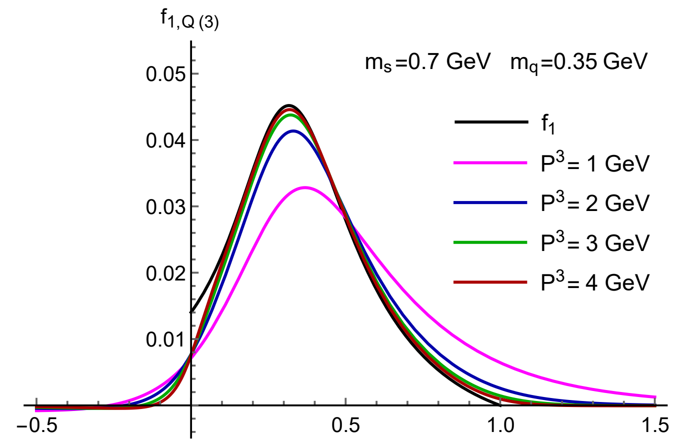
  - cutoff for $k_\perp$ integration

$$\Lambda = 1\,\mathrm{GeV}$$

  - variations of $|\vec{\Delta}_\perp|$ and Λ do not affect general results
  - using form factor (rather than $k_\perp$ cutoff) does not affect general results

- Quasi-PDFs





  – for larger P^3 ($\gtrsim 2\,\mathrm{GeV}$), quasi-PDFs are close to f_1 in wide x range

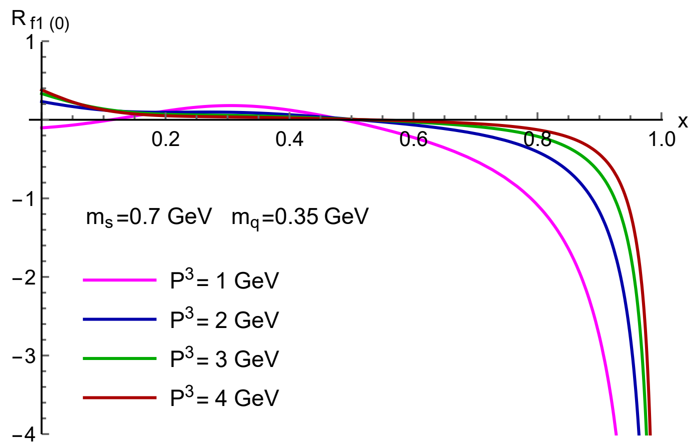
  – for larger P^3, not much difference between $f_{1,\mathrm{Q}(0)}$ and $f_{1,\mathrm{Q}(3)}$;
    this is general feature for all cases

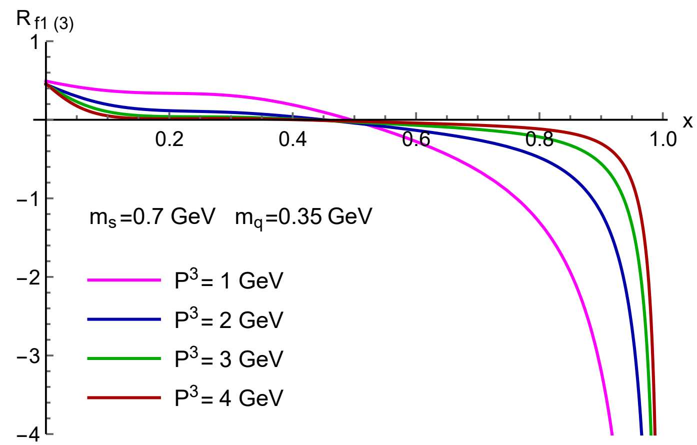
  – considerable discrepancies between quasi-PDFs and f_1 at large x
    (compare Gamberg, Kang, Vitev, Xing, 2014)

  – considerable discrepancies between quasi-PDFs and f_1 at small x
    f_1 is discontinuous at $x = 0$ ($f_1(x < 0) = 0$)
    quasi-PDFs are continuous at $x = 0$ and must change rapidly around $x = 0$
    discontinuity is probably not just a model artifact $(f_1^q(x < 0) = -f_1^{\bar{q}}(x > 0))$

- Relative difference for quasi-PDFs

$$R_{f1(0/3)}(x;P^3) = \frac{f_1(x) - f_{1,\mathrm{Q}(0/3)}(x;P^3)}{f_1(x)}$$





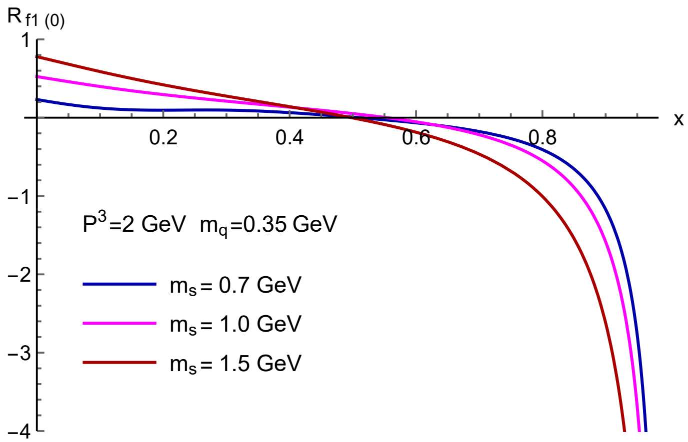
– relative difference makes discrepancies at large x very explicit

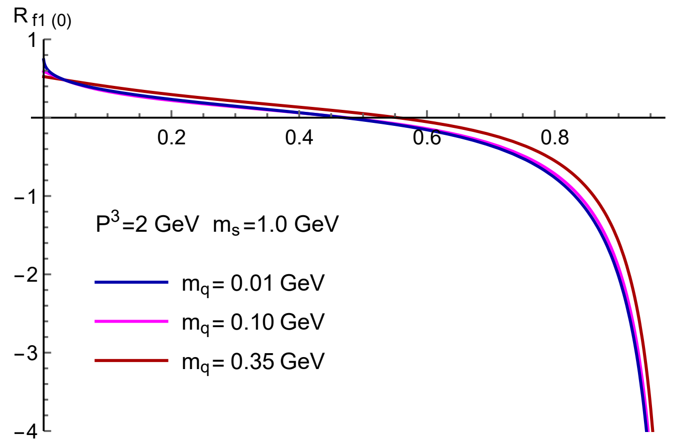
– for $P^3 = 2\,\mathrm{GeV}$, one can hardly go beyond $x = 0.8$ for decent results

– main cause of problem is large mismatch in this region btw k^+/P^+ and k^3/P^3 for finite P^3

– calculations of quasi-PDFs in LQCD also lead to discrepancies at large x, but situation may improve after matching

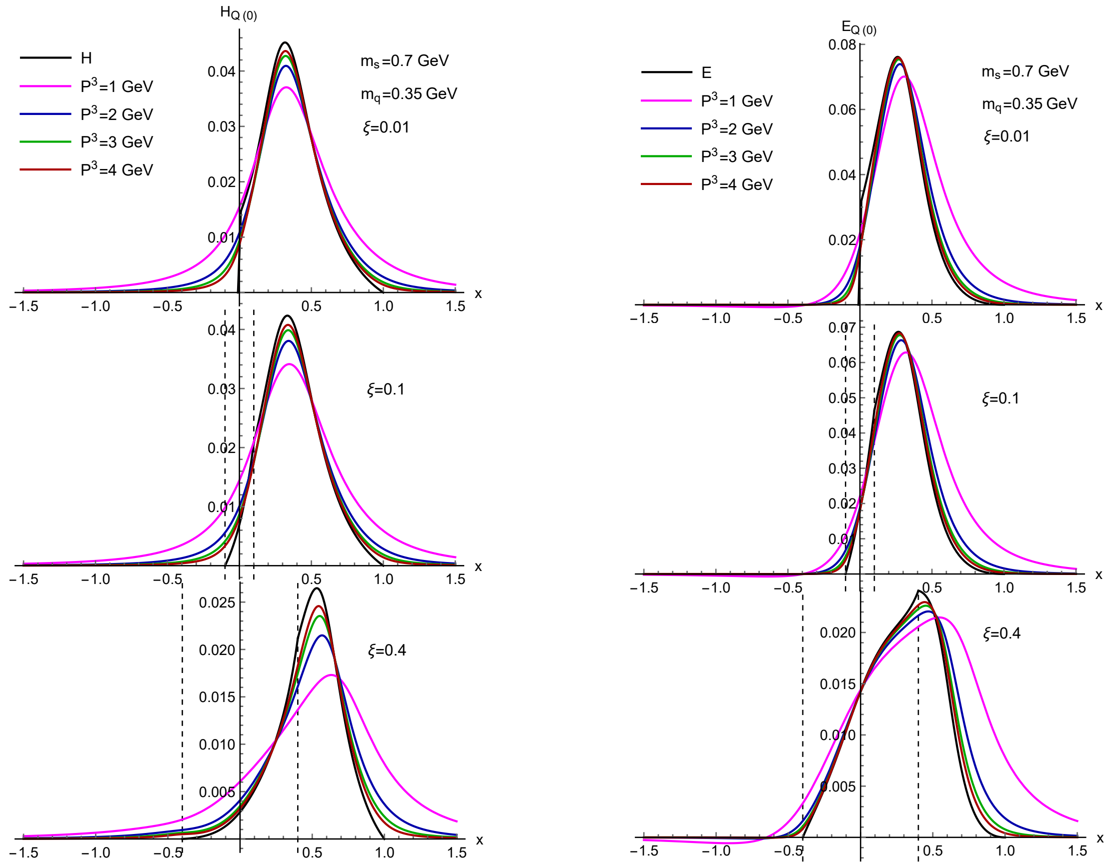
- Parameter dependence for PDFs
  - variation of m_s changes f_1 and quasi-PDFs in entire x range
  - variation of m_q changes f_1 and quasi-PDFs mostly for smaller x
  - parameter dependence of relative difference





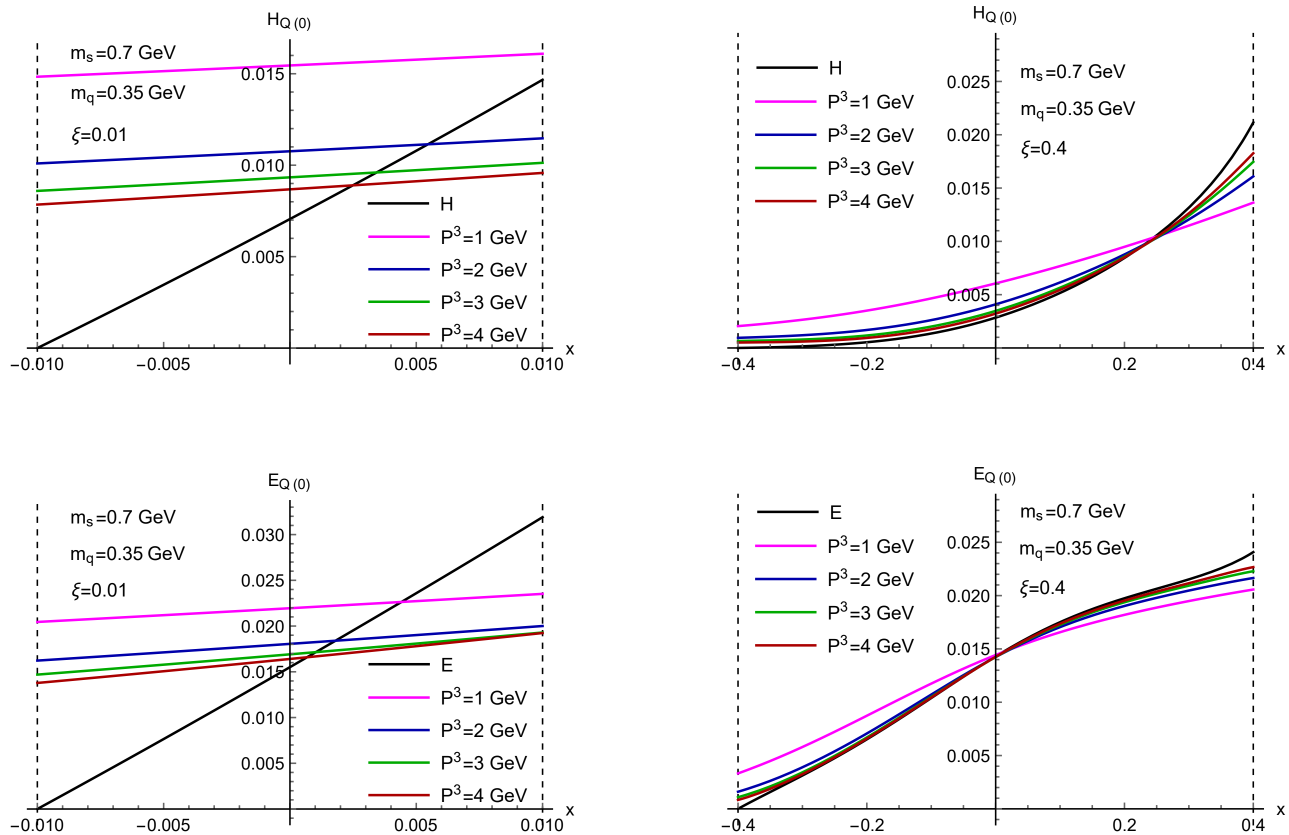
  - variation of m_s and m_q has milder impact on relative difference
  - general finding: the smaller m_s the better agreement btw f_1 and quasi-PDFs

- Quasi-GPDs



  - standard (twist-2) GPDs are continuous in entire x range
  - for larger P^3 ($\gtrsim 2\,\mathrm{GeV}$), quasi-GPDs are close to standard GPDs in wide x range
  - considerable discrepancies between quasi and standard GPDs for large x;
    issue becomes more severe as ξ increases
  - no non-trivial matching for E (Ji, Schäfer, Xiong, Zhang, 2015)
    $\rightarrow$ will issue at large x persist for LQCD calculations ?

- Quasi-GPDs in ERBL region



- large discrepancies between quasi and standard GPDs in ERBL region for small ξ (compare region around $x = 0$ for PDFs)

- good agreement between quasi and standard GPDs in large part of ERBL region for large ξ $\rightarrow$ nice opportunity for LQCD calculations ?

Summary

- Quasi-PDFs have attracted enormous interest;
  they allow to compute x-dependence of PDFs and related quantities in LQCD
- First encouraging LQCD results for quasi-PDFs
- Model results show large discrepancies between quasi and standard PDFs at large x;
  discrepancies have been seen in LQCD calculations as well
- Quasi-GPDs in diquark spectator model
  - for $P^3 \to \infty$, $H_{\rm Q}$ and $E_{\rm Q}$ exactly agree with the respective standard GPDs
  - for finite P^3, large discrepancies btw quasi and standard GPDs at large x
    for H and E; issue becomes more severe as ξ increases
  - for finite P^3 and large ξ, good agreement btw quasi and standard GPDs
    in ERBL region
- Outlook
  - LQCD may provide new important results for GPDs
  - combination of experimental data (also from EIC) and input from LQCD may
    be ideal to pin down GPDs