TMDs through jets and quarkonium

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TMDs through jets and quarkonium

Part 1:
TMD and groomed Jets

Part 2:
EFT approach

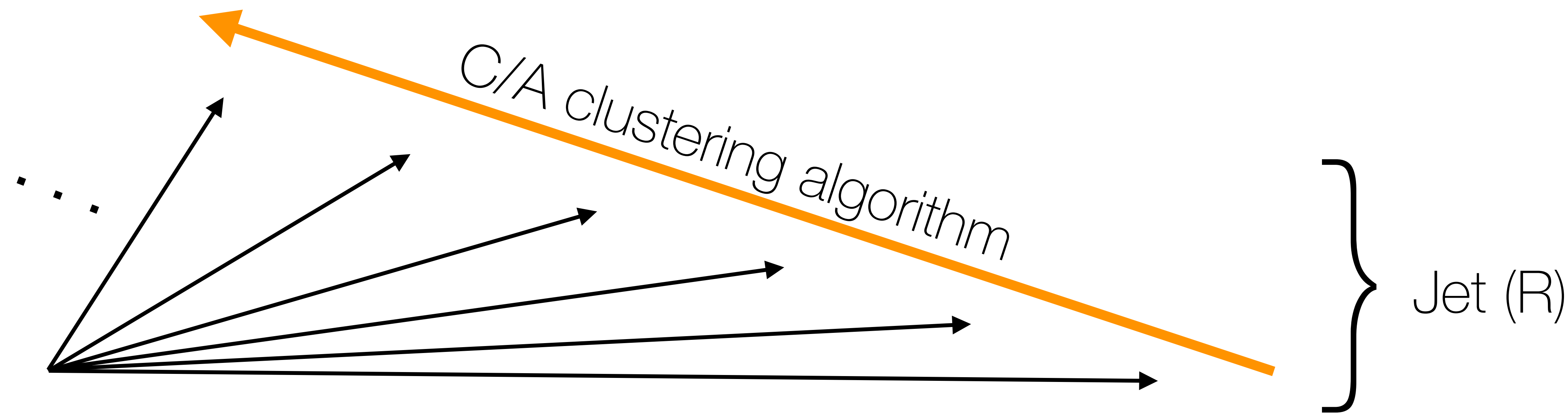
Part 1:TMD fragmentation within groomed Jets

Light: **arXiv:1712.07653:** In collaboration with Duff Neill and Varun Vaidya   This Talk

Heavy: **arXiv:1807.09805:** In collaboration with Varun Vaidya

- grooming procedure (soft-drop)
- factorization of groomed jets with SCET
- resummation in momentum space
- resummation in impact parameter (b-)space
- probing TMD evolution

Grooming algorithm: mMDT/soft-drop ($\beta = 0$)

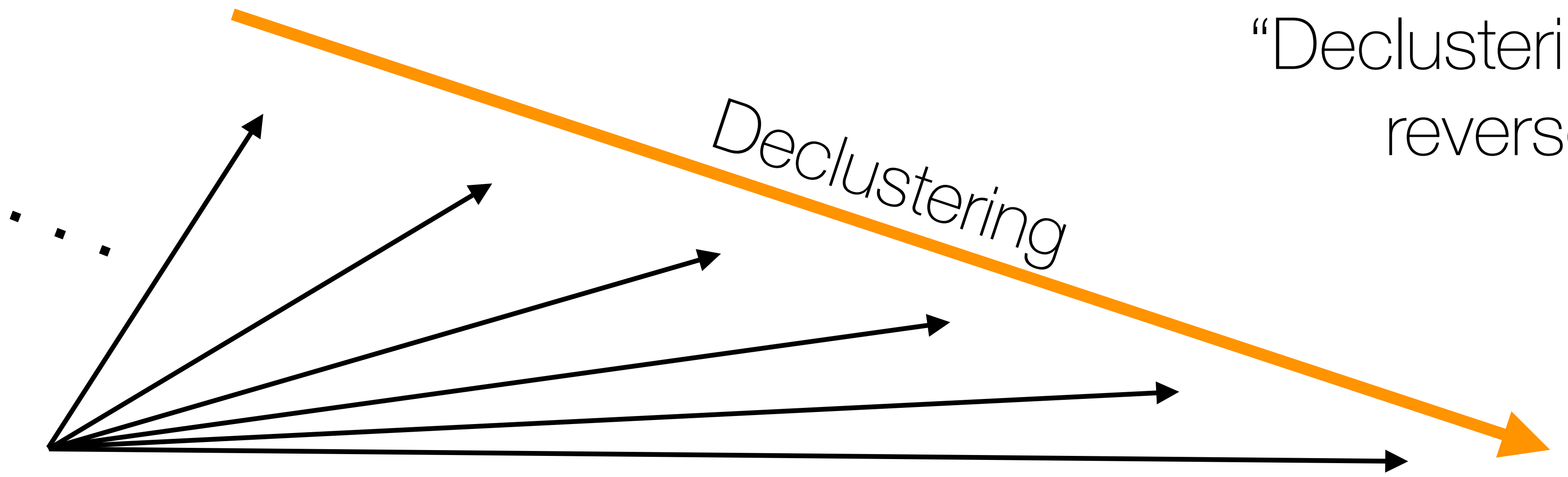


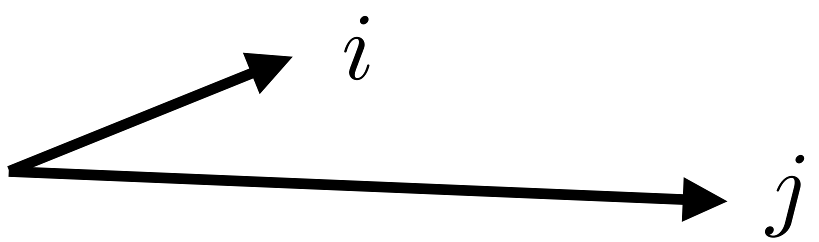
- The algorithm is imposed **only** on the jet constituents
- Record clustering history in each step
- Particles closer in angle get clustered first

For details on soft-drop see: **arXiv:1402.2657** A. J. Larkoski, S. Marzani, G. Soyez, and J. Thaler

Grooming algorithm: mMDT/soft-drop ($\beta = 0$)

"Declustering" with the reverse order:



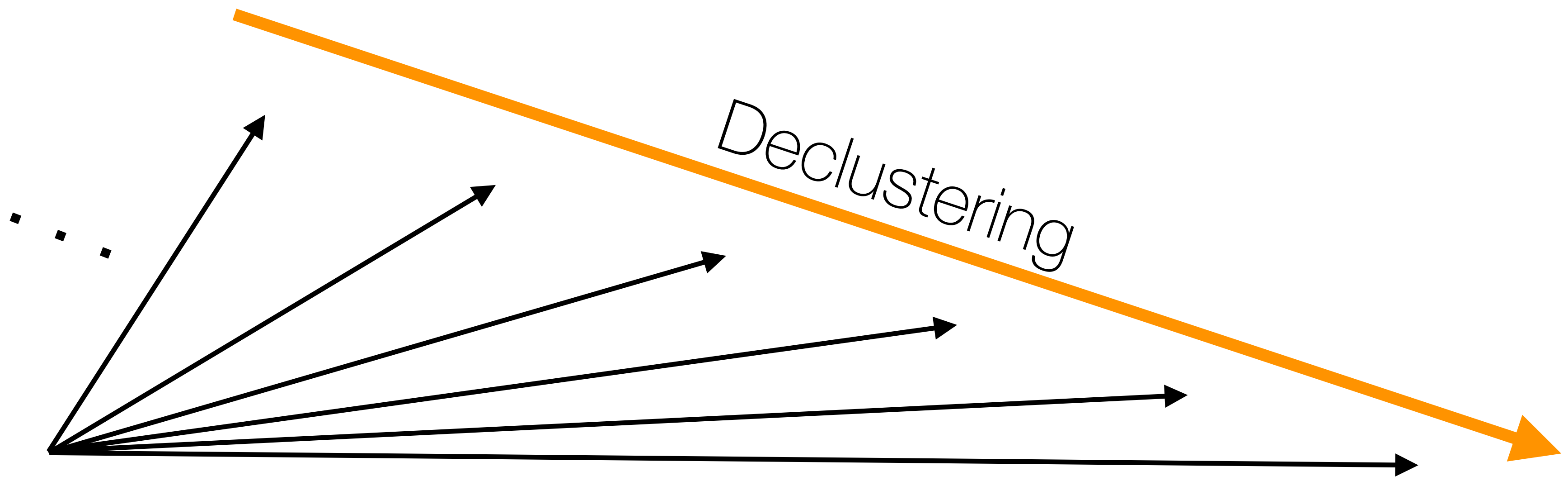


$$\frac{\min\{E_i, E_j\}}{E_i + E_j} > z_{\rm cut}$$

True
Stop. Remaining particles consist groomed jet

False
Drop the softer of the two branchings

Grooming algorithm: mMDT/soft-drop ($\beta = 0$)



- Removes soft wide angle radiation sensitive to the cone/boundary and non-global effects (NGLs)
- Isolates collinear-energetic radiation near the center of the jet
- Smaller sensitivity to underlying event

Factorization with groomed jets

$$\frac{d\sigma}{d\vec{p}_J\, d\mathcal{M}} = \sum_{i=g,q} F_i(Q, R, z_{\rm cut}, \vec{p}_J, \mathcal{C})\, J_i(\mathcal{M}, z_{\rm cut}, R, E_J)$$

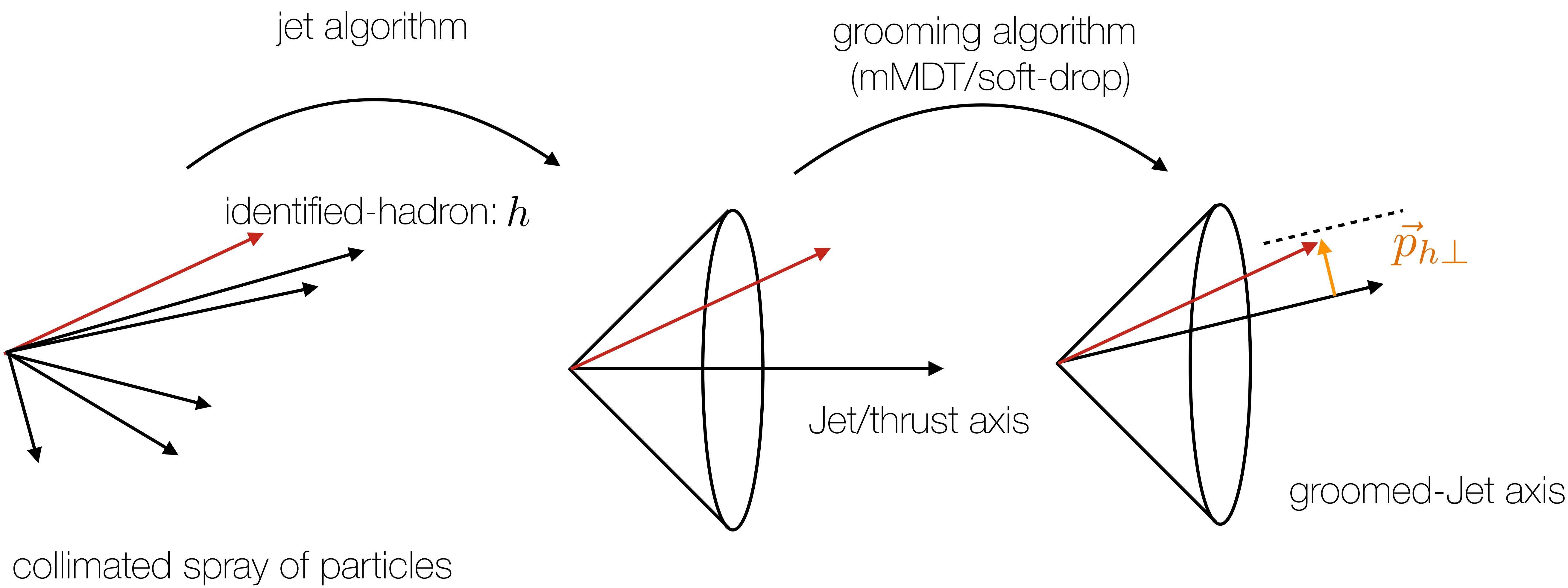
Fraction of quark and gluon-initiated jets Independent of the measurement within the jet

Collinear function describing radiation within groomed jet

$\mathcal{C}$: Collider specific + Initial conditions

$\mathcal{M}$: Measurements on the jet constituents (e.g. jets invariant mass)

Measurement



Measurements: $\vec{p}_{h\perp}, \ z = \dfrac{E_h}{E_J}$

Only the particles that pass the grooming process will determine the direction of the groomed-jet axis

Factorization with groomed jet(h)

$$\frac{d\sigma}{d\vec{p}_J\, d\vec{k}_\perp dz_h} = \sum_{i=g,q} F_i(Q, R, z_{\rm cut}, \vec{p}_J, \mathcal{C})\, \mathcal{G}_{i/h}(z_h, \vec{k}_\perp, z_{\rm cut}, R, E_J)$$

Fraction of quark and gluon-initiated jets Independent of the measurement within the jet

groomed TMD Fragmenting Jet Function (TMDFJF)

$\vec{k}_\perp$ is the transverse momentum of jet with respect to hadron

See also:

$$\vec{k}_\perp = -\frac{\vec{p}_{h\perp}}{z_h}$$

- TMDFJF (measurement along the jet axis)

arXiv:1610.06508 (Reggie Bain, YM, Thomas Mehen)

- JTMDFF (measurement along the winner-take-all axis)

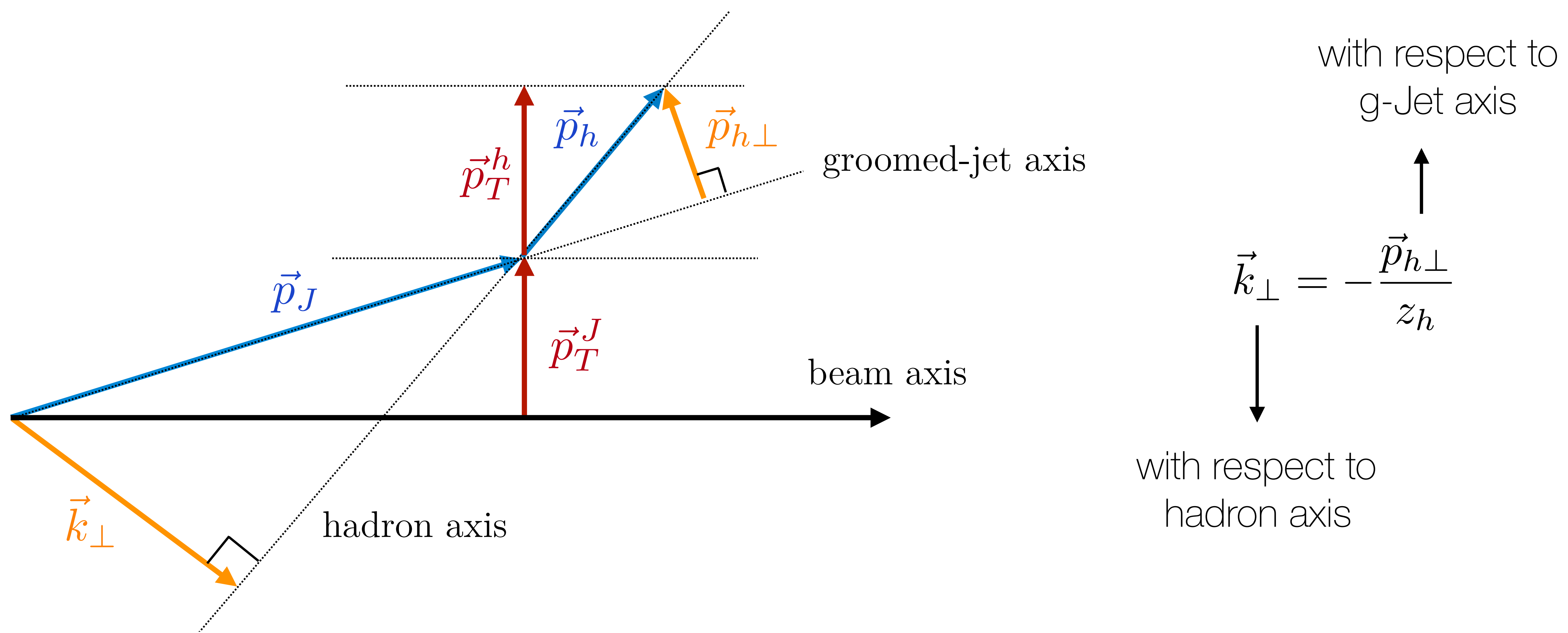
arXiv:1612.04817 (Duff Neill, Ignazio Scimemi, Wouter J. Waalewijn)

- siTMDFJF (semi-inclusive)

arXiv:1705.08443 (Zhong-Bo Kang, Xiaohui Liu, Felix Ringer, Hongxi Xing)

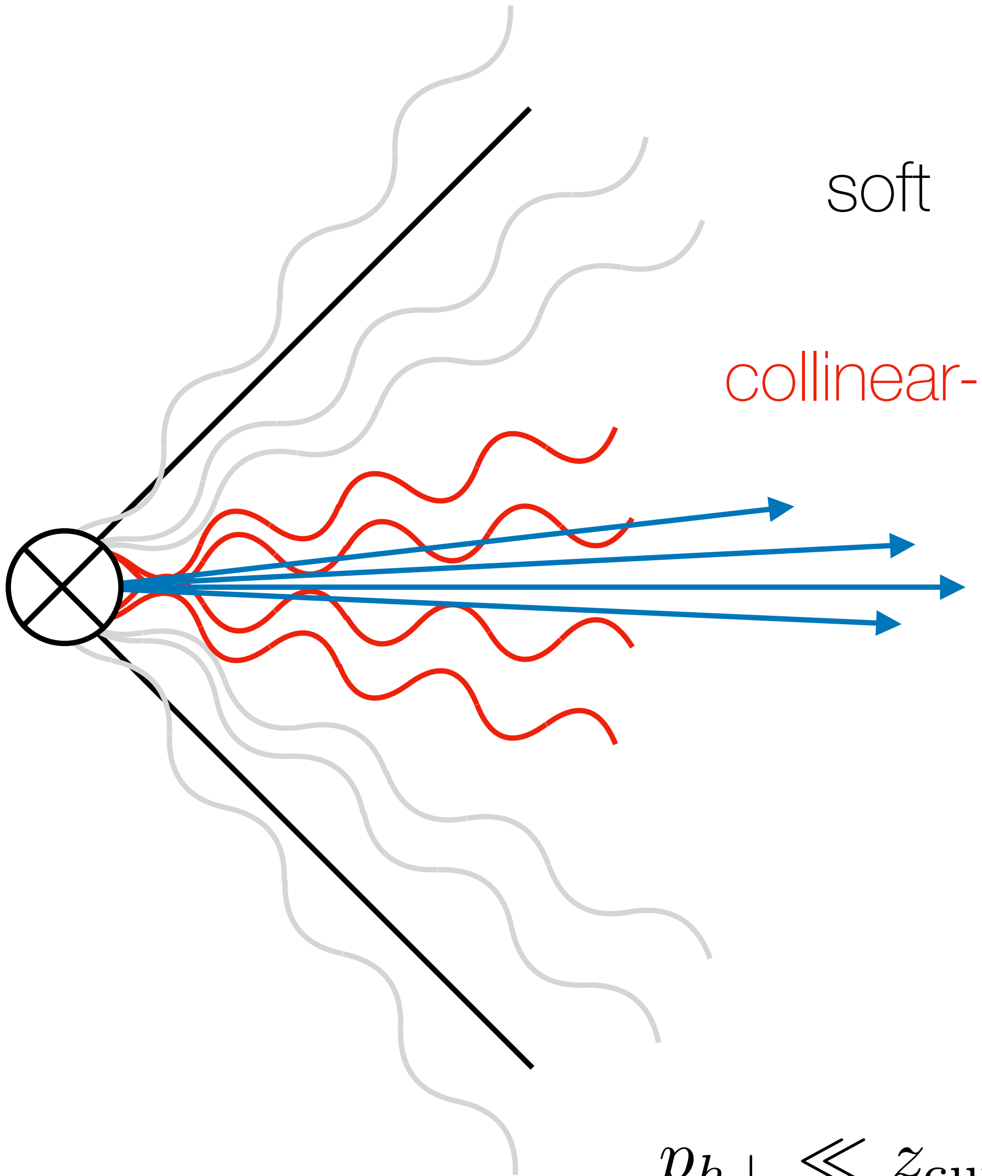
Groomed TMD fragmenting jet function

$$\mathcal{G}_{q/h}\Big(z_h, \vec{k}_\perp, z_{\rm cut}, R, E_J\Big)$$
$$= z_h \sum_{X\in {\rm Jet}(R)} \frac{1}{2N_c}\delta(2E_J - p_X^- - p_h^-){\rm tr}\left[\frac{\bar{n}\!\!\!/}{2}\langle 0|\delta^{(2)}(\vec{k}_\perp + \vec{\mathcal{P}}_\perp^{SD})\chi_n(0)|Xh\rangle\langle Xh|\bar{\chi}_n(0)|0\rangle\right]_{\vec{p}_{h\perp}=\vec{0}}$$



Factorization of the TMD (FJF) in SCET

$$\mathcal{G}_{i/h}(z_h, \vec{k}_\perp, E_J, z_{\rm cut}; \mu_L) = \int d^2\vec{k}_{c\perp} \int d^2\vec{k}_{s\perp} \delta^2\left(\vec{k}_\perp + \vec{k}_{c\perp} + \vec{k}_{s\perp}\right) S_i^\perp\left(\vec{k}_{s\perp}, z_{cut}\right) \mathcal{D}_{i/h}^\perp\left(z_h, \vec{k}_{c\perp}\right)$$



soft $p_s^\mu \sim z_{\rm cut} Q(1,1,1)$

collinear-soft $p_{sc}^\mu \sim z_{\rm cut} Q(\lambda_{sc}^2, 1, \lambda_{sc})$ $\lambda_{sc} = \frac{p_{h\perp}}{z_{\rm cut} Q}$

collinear $p_c^\mu \sim Q(\lambda_c^2, 1, \lambda_c)$ $\lambda_c = \frac{p_{h\perp}}{Q}$

$$p_{h\perp} \ll z_{\rm cut} Q, \; z_{\rm cut} \ll R \sim 1$$

Factorization of the TMD (FJF) in SCET

$$\mathcal{G}_{i/h}(z_h, \vec{k}_\perp, E_J, z_{\rm cut}; \mu_L) = \int d^2\vec{k}_{c\perp} \int d^2\vec{k}_{s\perp} \delta^2\left(\vec{k}_\perp + \vec{k}_{c\perp} + \vec{k}_{s\perp}\right) S_i^\perp\left(\vec{k}_{s\perp}, z_{cut}\right) \mathcal{D}_{i/h}^\perp\left(z_h, \vec{k}_{c\perp}\right)$$

$$\mathcal{D}_{q/h}^\perp(z_h, \vec{k}_{c\perp}, E_J) = \sum_X \frac{z_h}{2N_c} \delta(2E_J - p^-_{Xh}) \mathrm{tr}\left[\frac{\bar{\not n}}{2} \langle 0|\delta^{(2)}(\vec{k}_{c\perp} - \vec{\mathcal{P}}_\perp)\chi_n(0)|Xh\rangle\langle Xh|\bar{\chi}_n(0)|0\rangle\right]_{\vec{p}_{h\perp}=0}$$

- collinear modes are energetic and always pass the grooming constraint

→ independent of the cutoff parameter $(z_{\rm cut})$

- contains the non-perturbative information of the fragmentation process

$$S_i^\perp(\vec{k}_{s\perp}, E_J, z_{\rm cut}) = \frac{1}{N_i} \mathrm{tr}\left[\langle 0|T\{S_n^i S_{\bar{n}}^i\}(0)\delta^{(2)}\left(\vec{k}_{s\perp} - \vec{\mathcal{P}}_\perp^{SD}\right)\bar{T}\{S_n^i S_{\bar{n}}^i\}(0)|0\rangle\right]$$

- describes collinear-soft radiation that can pass the grooming constraint

- universal to all light hadrons → independent of hadron's energy fraction (z_h)

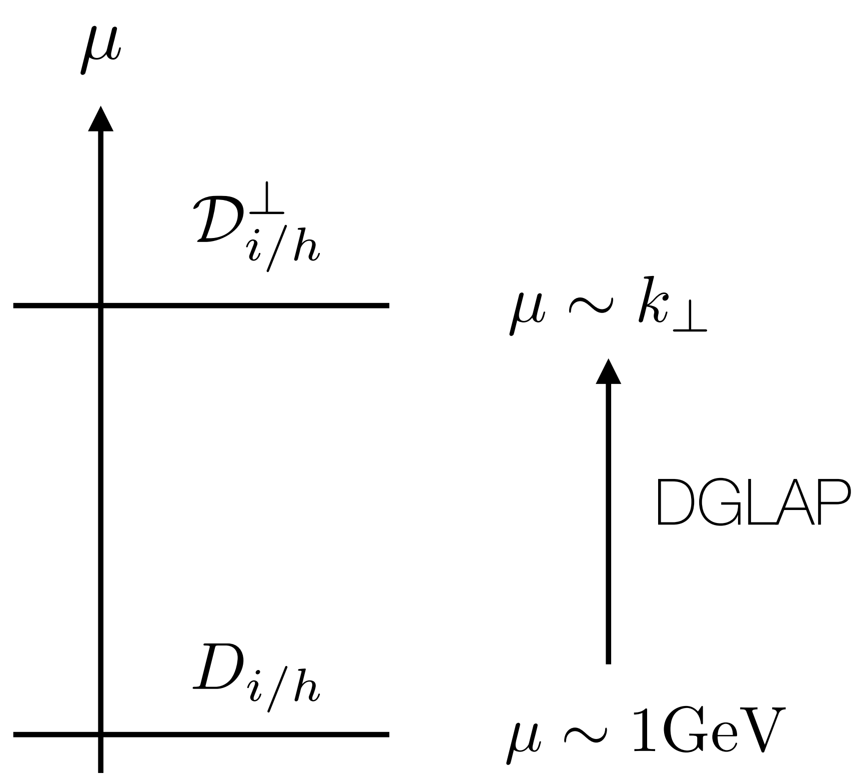
Matching onto collinear Fragmentation Functions

Although $\mathcal{D}^{\perp}_{q/h}(z_h, \vec{k}_{c\perp}, E_J)$ is a fundamentally non-perturbative object, for $k_\perp \gg \Lambda_{\rm QCD}$ can be matched onto the collinear Fragmentation Functions:

$$\mathcal{D}^{\perp}_{i/h}\left(z_h, \vec{k}_{c\perp}, E_J\right) = \int_{z_h}^{1} \frac{dx}{x} \mathcal{J}^{\perp}_{ij}\left(x, \vec{k}_{c\perp}, E_J\right) D_{j/h}\left(\frac{z_h}{x}\right)$$

short distance matching coefficients
and collinear-soft
calculable in perturbation theory
→ rapidity divergences

collinear
Fragmentation
Functions



Renormalization Group and Resummation

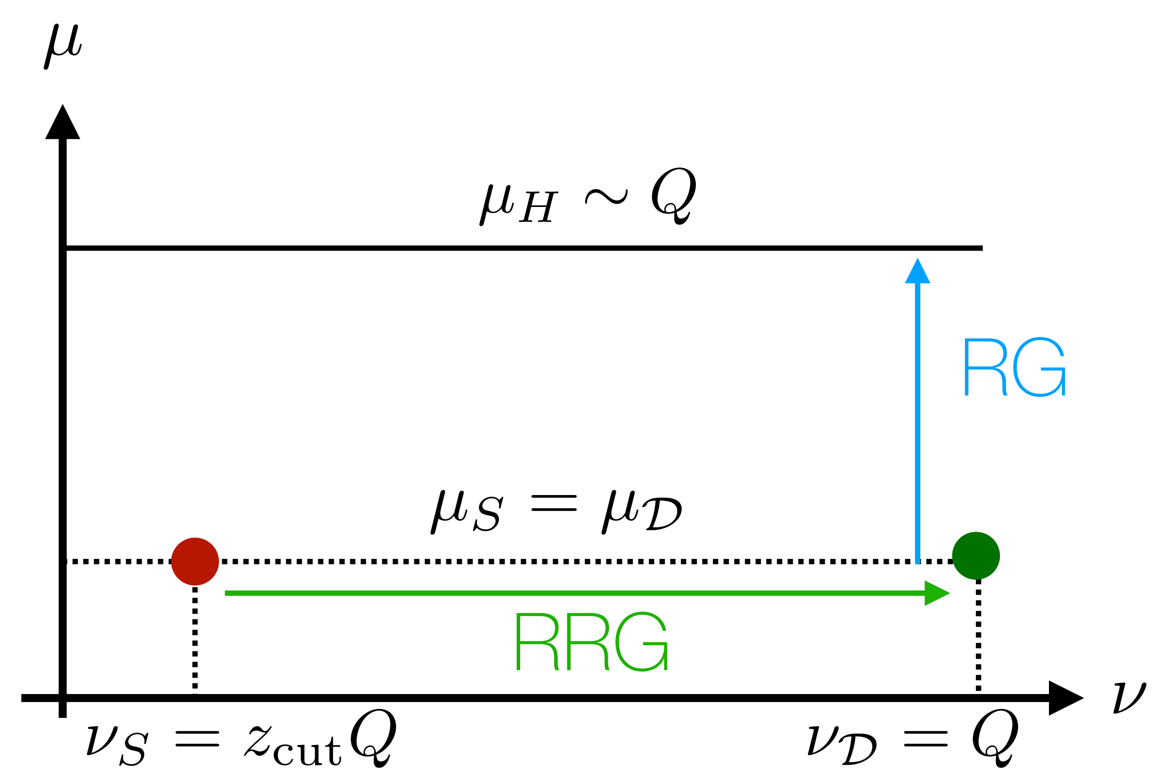
Rapidity regulator and Rapidity Renormalization Group (RRG):

$$\frac{d}{d\ln\nu}G(\mu,\nu) = \gamma_\nu^G G(\mu,\nu)$$

J.-Y. Chiu, A. Jain, D. Neill and I. Z. Rothstein **arXiv:1202.0814**

$$\gamma_\nu^S + \gamma_\nu^{\mathcal{D}} = 0$$

The combined object soft+collinear does not evolve in rapidity



Virtuality Renormalization Group (RG):

$$\frac{d}{d\ln\mu}G(\mu,\nu) = \gamma_\mu^G G(\mu,\nu)$$

Talk by Ignazio S. for ambiguities regarding the path choice

NLL-Resummation in momentum space

Fourier Transform ⟶ Solve RGE ⟶ Inverse Fourier Tranform ⟶ Fix Scales

$$\mathcal{G}_{j/h}^{\mathrm{NLL}}(z_h, \vec{k}_\perp, z_{\mathrm{cut}}; \mu) = \mathcal{V}(\vec{k}_\perp, z_{\mathrm{cut}}, \mu_0)\mathcal{U}(\mu, \mu_0) D_{j/h}(z_h, \mu_0)\Big|_{\mu_0 = k_\perp}$$

$$\mathcal{U}(\mu, \mu_0) = \exp\left[2\pi \frac{\gamma^{D\otimes S(\mu, z_{\mathrm{cut}})}}{\beta_0\ \alpha_s(\mu)} \ln(\alpha_s(\mu_0)/\alpha_s(\mu))\right]$$

$$\mathcal{V}(\vec{k}_\perp, z_{\mathrm{cut}}, \mu) = \frac{\exp(-2\gamma_E \omega_S)}{\pi} \frac{\Gamma(1-\omega_S)}{\Gamma(\omega_S)} \frac{1}{\mu^2} \left(\frac{\mu^2}{k_\perp^2}\right)^{1-\omega_S}$$

$$\omega_S = \frac{\alpha(\mu) C_i}{\pi} \ln\left(\frac{\nu_{\mathcal{D}}}{\nu_S}\right)$$

traditional TMDs

$$-\frac{\alpha(k_\perp) C_i}{\pi} \ln\left(\frac{k_\perp}{Q}\right)$$

groomed TMDFJF

$$-\frac{\alpha(k_\perp) C_i}{\pi} \ln(z_{\mathrm{cut}})$$

NLL-Resummation in momentum space

$$\omega_S = \frac{\alpha(\mu)C_i}{\pi}\ln\left(\frac{\nu_{\mathcal{D}}}{\nu_S}\right)$$

traditional TMDs

$$-\frac{\alpha(k_\perp)C_i}{\pi}\ln\left(\frac{k_\perp}{Q}\right)$$

Solution:

Fix scales in coordinate space
and take Fourier transform numerically

groomed TMDFJF

$$-\frac{\alpha(k_\perp)C_i}{\pi}\ln(z_{\rm cut})$$

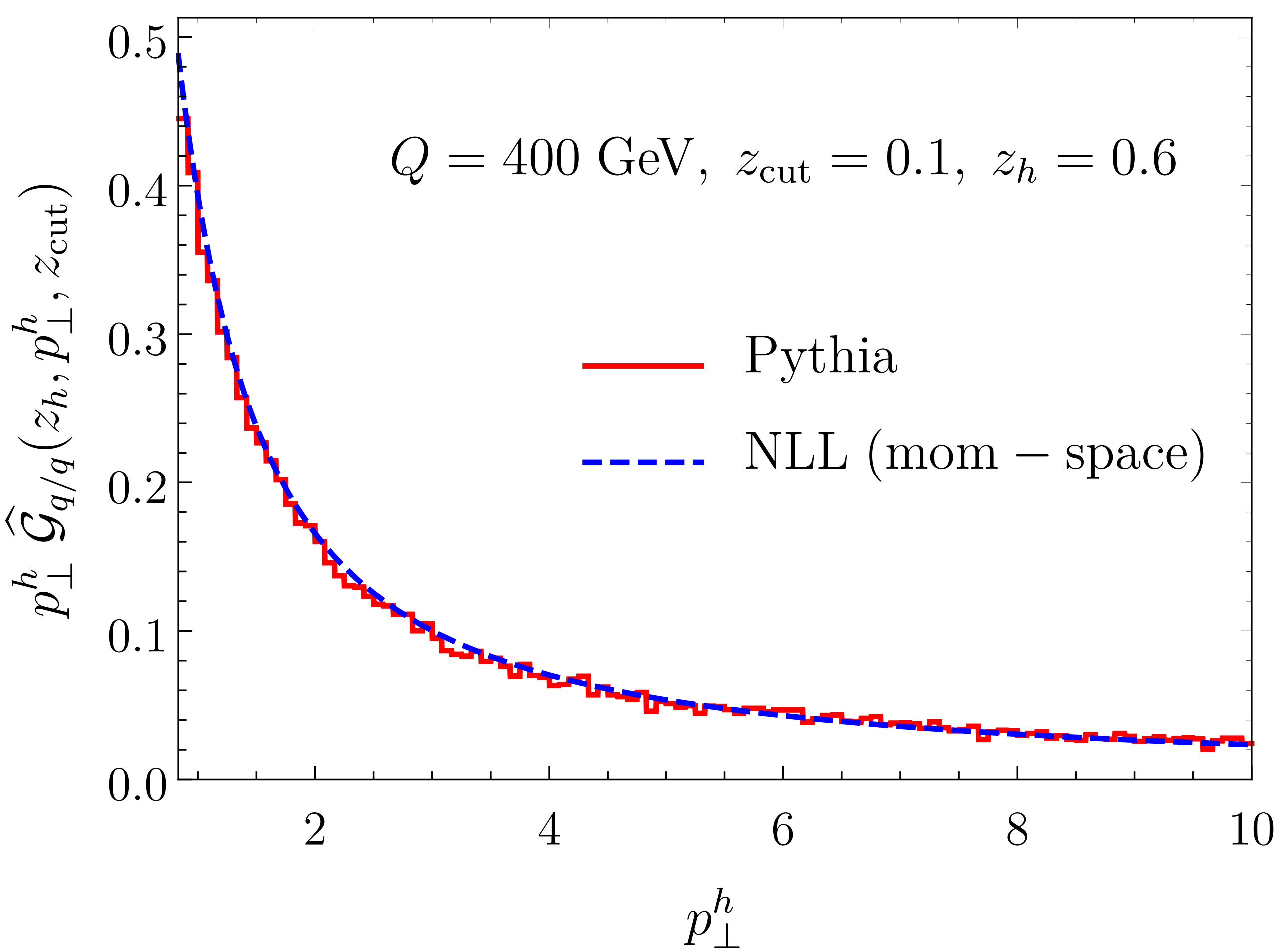
In the perturbative region ω_S is small, therefore we can fix the scales momentum space directly.

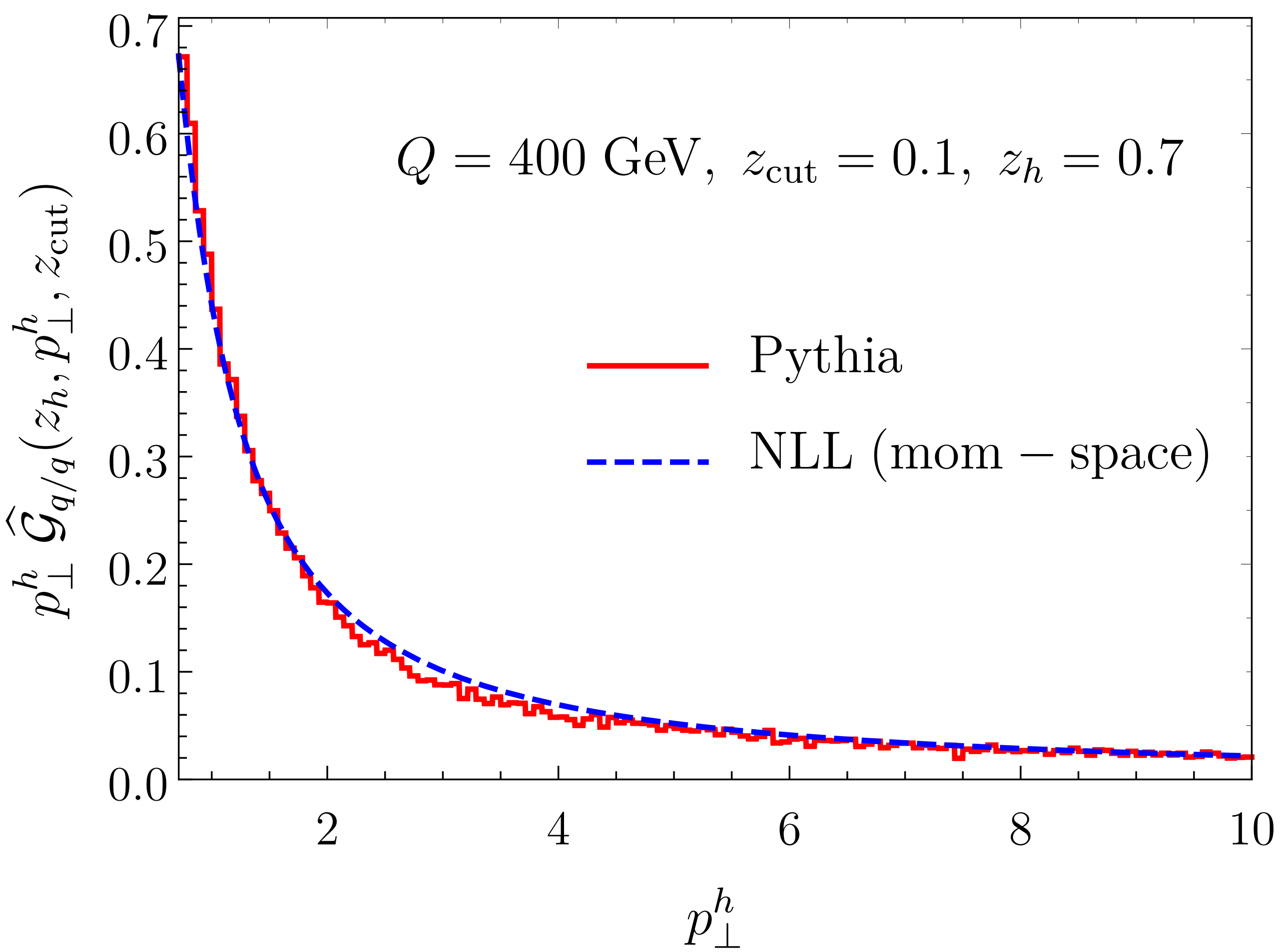
Common choice: $z_{\rm cut} = 0.1$

$\omega_S \sim 1$: Only in the non-perturbative regime

NLL-Resummation in momentum space

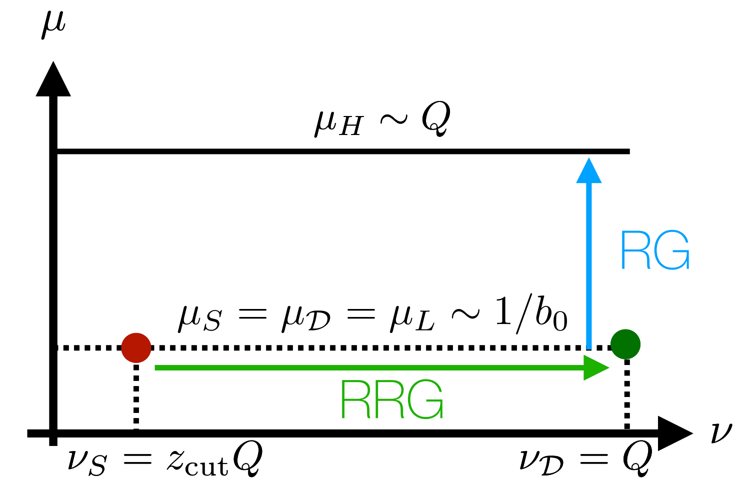
test against Pythia partonic shower: quark-to-quark case





Resummation in b-space

Fourier Transform ⟶ Solve RGE ⟶ Fix Scales ⟶ Inverse Fourier Tranform



$$\mathcal{D}^{\perp}_{i/h}(\mu_H, \nu = 2E_J) S^{\perp}_i(\mu_H, \nu = 2E_J) = U_i(\mu_L, \mu_H) \times \Big[\mathcal{D}^{\perp}_{i/h}(\mu_L, \nu = 2E_J) S^{\perp}_i(\mu_L, \nu = 2E_J z_{\rm cut})\Big]$$

$$U_i(\mu_L, \mu_H) \equiv \mathrm{Exp}\Big[-\int_{\mu_L}^{\mu_H} d\ln\mu\, \gamma^F_i[\alpha_s(\mu)] + 2\ln(z_{\rm cut})\Big(\int_{1/b_0}^{\mu_H} d\ln\mu\, \Gamma^i_{\rm cusp}[\alpha_s(\mu)] + \gamma^r(1/b_0)\Big)\Big]$$

Rapidity anomalous dimension

$$\gamma^S_{\nu,i}(\mu) = -2\int_{1/b_0}^{\mu} d\ln\mu'\, \Gamma^i_{\rm cusp}[\alpha_s(\mu')] + \gamma^r(1/b_0)$$

Resummation in b-space

$$\gamma^S_{\nu,i}(\mu) \to \gamma^S_{\nu,i}(\mu)\Big|_{b\to b_*} - g_K(b;b_{\max}) \qquad b_* = \frac{b}{\sqrt{1+(b/b_{\max})^2}} \qquad g_K(b;b_{\max}) \xrightarrow[b\to 0]{} 0$$

Universal component of TMD observables:

$$U_i(\mu_L,\mu_H) \equiv \text{Exp}\Big[-\int_{\mu_L}^{\mu_H} d\ln\mu\,\gamma_i^F[\alpha_s(\mu)] + 2\ln(z_{\text{cut}})\Big(\int_{1/b_0}^{\mu_H} d\ln\mu\,\Gamma^i_{\text{cusp}}[\alpha_s(\mu)] + \gamma^r(1/b_0)\Big)\Big]$$

Rapidity anomalous dimension

Non-perturbative TMD evolution

$$\gamma^S_{\nu,i}(\mu) \to \gamma^S_{\nu,i}(\mu)\Big|_{b\to b_*} - g_K(b;b_{\max}) \qquad b_* = \frac{b}{\sqrt{1+(b/b_{\max})^2}} \qquad g_K(b;b_{\max}) \xrightarrow[b\to 0]{} 0$$

Model:Fits	g_2	$b_{\max}$ [GeV^{-1}]	$b_{\rm NP}$ [GeV^{-1}]
CSS:BNLY 2003	0.68	0.5	n.a.
CSS:KN 2006	0.18	1.5	n.a.
CSS:Pavia 2016	0.12	1.123	n.a.
AFGR: n.a.	0.10	0.5	2.0

CSS:

$$g_K(b;b_{\max}) = \frac{1}{2} g_2(b_{\max}) b^2$$

AFGR:

$$g_K(b;b_{\max}) = \frac{g_2(b_{\max}) b_{\rm NP}^2}{2} \ln\left(1 + \frac{b^2}{b_{\rm NP}^2}\right)$$

BNLY: **arXiv:0212159** F. Landry, R. Brock, P.M. Nadolsky, C.-P. Yuan

KN: **arXiv:0506225** A. V. Konychev, P. M. Nadolsky

Pavia: **arXiv:1703.10157** A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A Signori

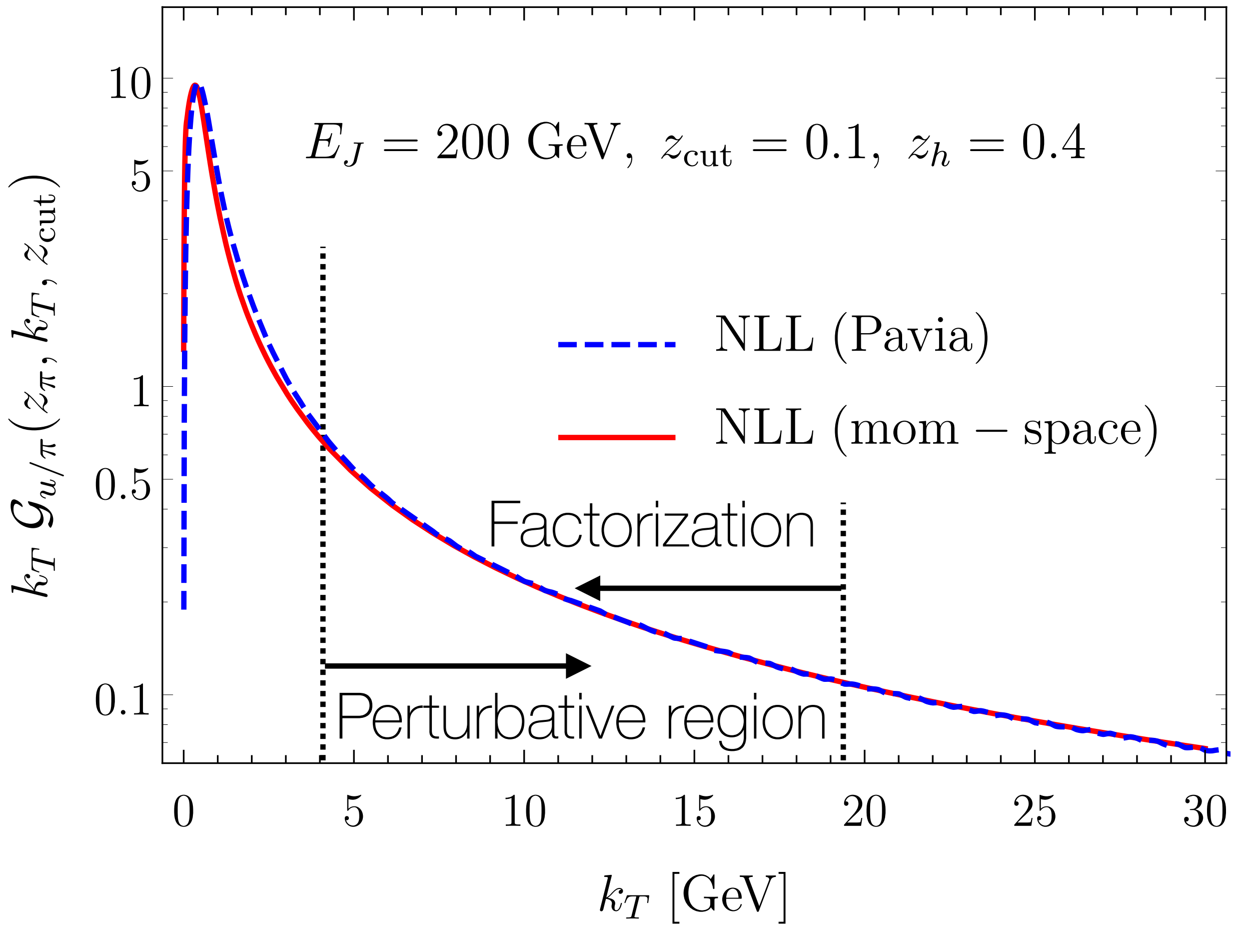
AFGR: **arXiv:1401.2654** C. A. Aidala, B. Field, L. P. Gamberg, T. C. Rogers

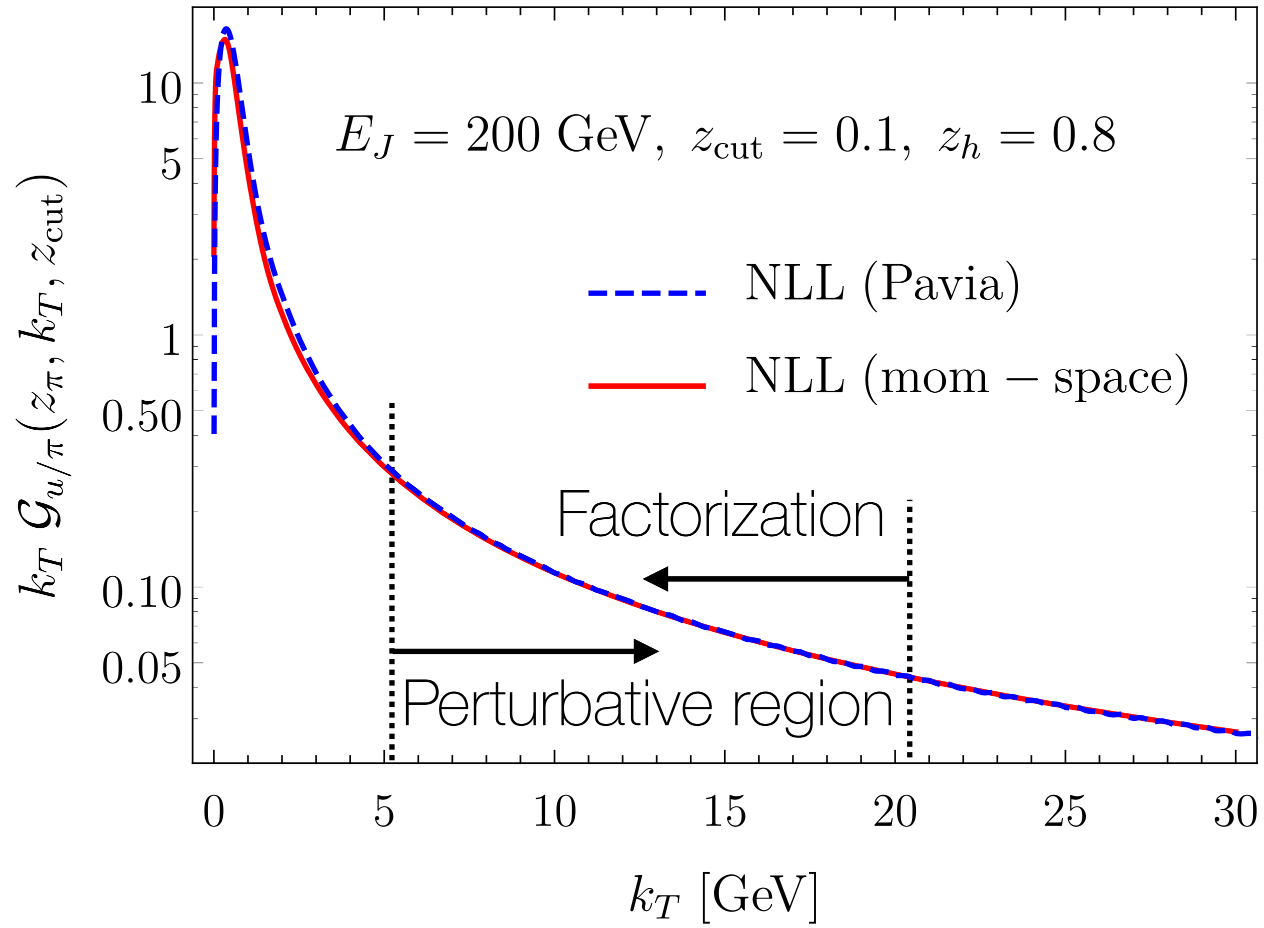
NLL: momentum space vs b-space

$$\mathcal{N}\frac{d\sigma}{dk_\perp}(e^+e^- \to jet + jet(\pi))$$

CSS:

$$g_K(b; b_{\max}) = \frac{1}{2} g_2(b_{\max}) b^2$$





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Resummation in b-space

$$\gamma^S_{\nu,i}(\mu) \to \gamma^S_{\nu,i}(\mu)\Big|_{b\to b_*} - g_K(b;b_{\max}) \qquad b_* = \frac{b}{\sqrt{1+(b/b_{\max})^2}} \qquad g_K(b;b_{\max}) \xrightarrow[b\to 0]{} 0$$

Universal component of TMD observables:

$$U_i(\mu_L,\mu_H) \equiv \mathrm{Exp}\Big[-\int_{\mu_L}^{\mu_H} d\ln\mu\,\gamma_i^F[\alpha_s(\mu)] + 2\ln(z_{\rm cut})\Big(\int_{1/b_0}^{\mu_H} d\ln\mu\,\Gamma^i_{\rm cusp}[\alpha_s(\mu)] + \gamma^r(1/b_0)\Big)\Big]$$

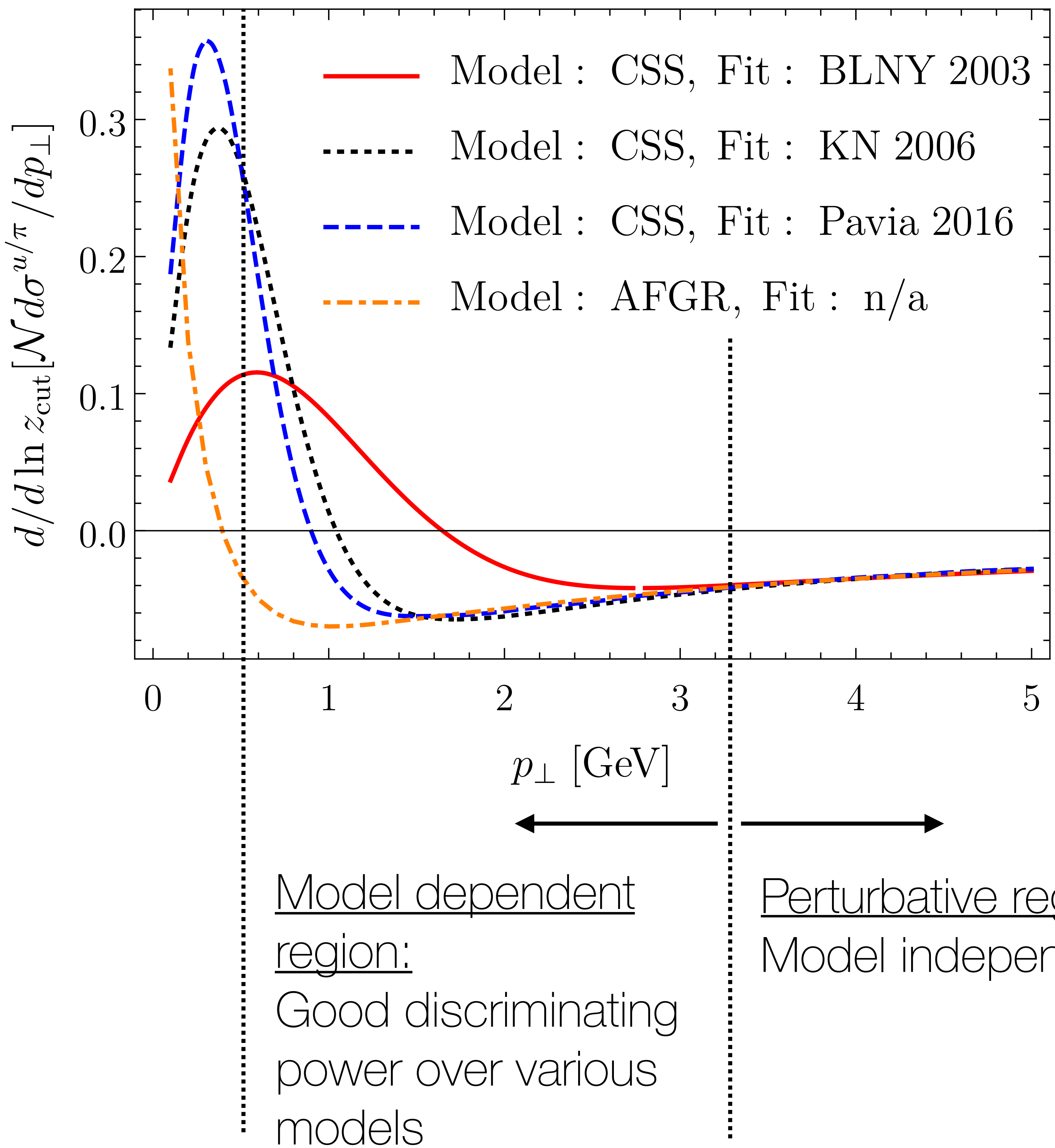
Rapidity anomalous dimension

variations of the cutoff parameter give as direct access to the rapidity anomalous dimension:

$$\frac{d}{d\ln z_{\rm cut}}\Big[\mathcal{N}(z_{\rm cut})\frac{d\sigma}{dp_{h\perp}}\Big]$$

Normalized cross section

Proposed observable



$$\frac{d}{d\ln z_{\rm cut}}\left[\mathcal{N}(z_{\rm cut})\frac{d\sigma}{dp_{h\perp}}\right]$$

<u>Model dependent region:</u>
Good discriminating power over various models

<u>Perturbative region:</u>
Model independent

Summary

Study fragmentation within groomed jets:

- Use EFT (SCET) for factorization and resummation of large logarithms
- No logarithmic enhancements from boundary effects (NGLs)
- Can easily extended for hadrons + jet substructure (e.g. jet mass and angularities in preparation)
- Observable easy to relate between e+ e- and EIC
- This study was expanded for heavy quarks + threshold resummation

In the perturbative regime:

- Groomed TMD fragmentation can be studied directly in momentum space

In the non-perturbative regime:

- Good discriminating observable for extracting non-perturbative TMD evolution

Part 2: Effective Field Theory Approach for Quarkonium at Low pT

In collaboration with: Sean Fleming, and Thomas C. Mehen

<u>1-slide review</u>

Quarkonium production at moderate pT (standard NRQCD): $p_T^{\mathcal{Q}} \sim m_{\mathcal{Q}}$

<u>Preliminary</u>

Quarkonium production at low pT (TMD region of NRQCD): $p_T^{\mathcal{Q}} \ll m_{\mathcal{Q}}$

NRQCD Factorization

LDME: Long Distance Matrix Elements

$$d\sigma(a+b\to \mathcal{Q}+X) = \sum_n d\sigma(a+b\to Q\overline{Q}(n)+X)\langle \mathcal{O}_n^{\mathcal{Q}}\rangle$$

Perturbative expansion in the strong coupling.

NRQCD Scaling Rules

$$d\sigma_0(1+\alpha_s C_1 + \alpha_s^2 C_2 + \ldots)$$

$$\langle \mathcal{O}({}^{2S+1}L_J^{[1,8]})\rangle \sim v^{3+2L+2E+4M}$$

$$Q\bar{Q}(n) \xrightarrow{\langle \mathcal{O}_n^{\mathcal{Q}}\rangle} \mathcal{Q}$$

$$\mathcal{O}_n^{\mathcal{Q}} = \mathcal{O}_2^{n\,\dagger}\left(\sum_X |X+\mathcal{Q}\rangle\langle X+\mathcal{Q}|\right)\mathcal{O}_2^n$$

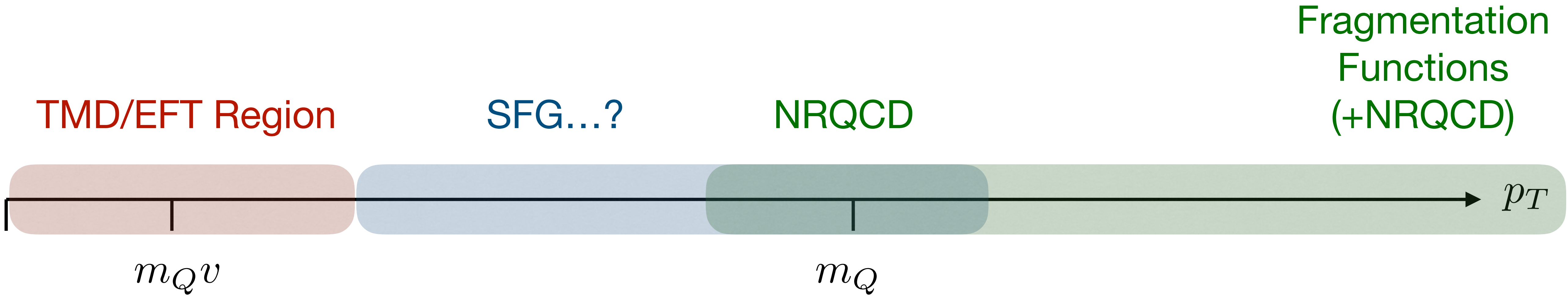
ultra-soft + soft

$$\mathcal{O}_2^n = \psi^\dagger \mathcal{K}^n \chi$$

$$n = {}^{2S+1}L_J^{[c]}$$

NRQCD Factorization

Upsilon spectrum vs EFT regions



Quarkonium at low pT (previous attempts)

Many attempts that approach the problem in CEM and CSM

Cannot be improved the same way EFTs can.

CEM and CSM fail in other regions/aspects of quarkonium production

CGC methods

arXiv:1408.4075, Yan-Qing Ma, Raju Venugopalan

arXiv:1503.07772, Yan-Qing Ma, Raju Venugopalan, and Hong-Fei Zhang

small-x resummation/ NO pT/M resummation: OK for charmonium at LHC.

NRQCD attempt (including evolution)

arXiv:1210.3432, Peng Sun, C.-P. Yuan, and Feng Yuan

Assumption: NRQCD factorization holds down to low pT?

NRQCD+SCET scales and Lagrangian

$\mathcal{L} =$"$\mathcal{L}_{\mathrm{SCET}} + \mathcal{L}_{\mathrm{NRQCD}}$"

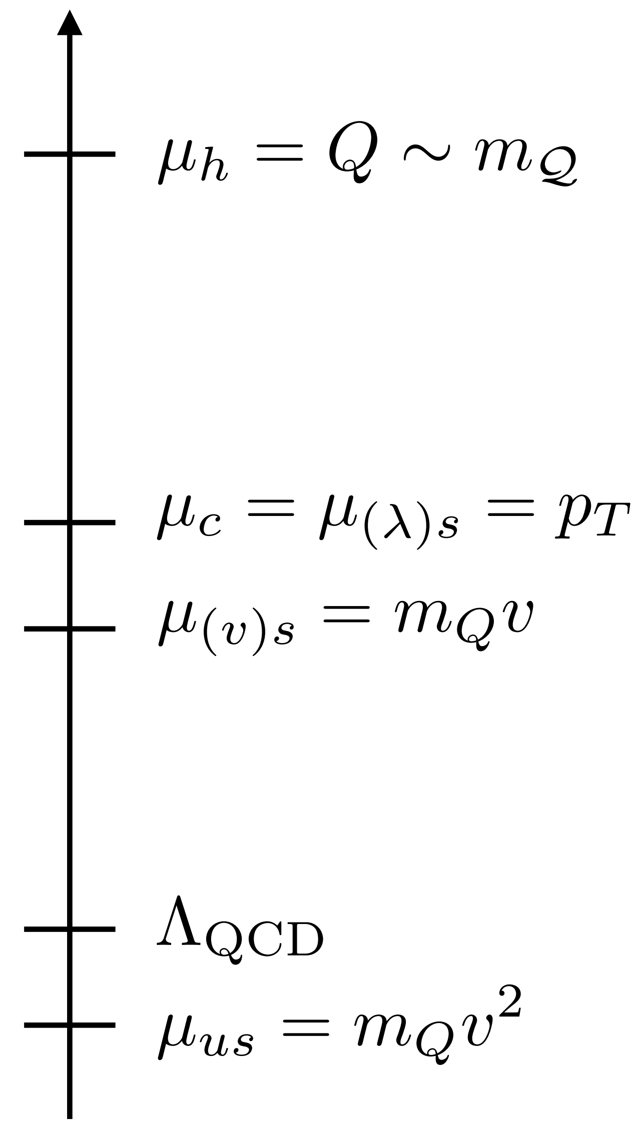
- $p_c^\mu = (p_c^+, p_c^-, \vec{p}_c^{\,\perp}) \sim Q(\lambda^2, 1, \lambda)$ $\qquad \lambda = \frac{p_T}{Q}$

- $p_{(\lambda)s}^\mu = (p_{(\lambda)s}^+, p_{(\lambda)s}^-, \vec{p}_{(\lambda)s}^{\,\perp}) \sim Q(\lambda, \lambda, \lambda)$

- $p_{(v)s}^\mu = (p_{(v)s}^+, p_{(v)s}^-, \vec{p}_{(v)s}^{\,\perp}) \sim Q(v, v, v)$

+ heavy quarks

+ ultra-soft



NRQCD at low pT - Factorization

h+h (central rapidity) / e+ e- (threshold)

$$\mathcal{O}^{\rm QCD}_{2+2} = (\bar{q}_i \Gamma q_i)(\bar{Q}\Gamma' Q)$$

$$\mathcal{O}^{n}_{2} = \psi^\dagger \mathcal{K}^n \chi$$

$$\mathcal{O}^{\rm SCET}_{2} = \bar{\xi}_{n_{\bar{B}}} \Gamma \xi_{n_B}$$

$$\mathcal{O}^{\rm QCD}(2+2) \to \sum_n C^n_{(2+2)} \Big(\mathcal{O}^{\rm SCET}_2 \times \mathcal{O}^n_2 \Big)$$

Factorize the cross section using BPS field redefinition for decoupling collinear and heavy from ultra-soft modes.

$$\frac{d\sigma}{dy\, d^2\mathbf{p}_T} = \sum_n \left(\sigma_0(n)\frac{m^2}{s}\right) \times H^n_{ij} \times \mathcal{B}_i(x_1) \otimes \mathcal{B}_j(x_2) \otimes S^{\mathcal{Q}}_n \times \Big(1 + \mathcal{O}(\lambda)\Big)$$

NRQCD at low pT - Hard function

$$\mathcal{O}^{\rm QCD}_{2+2} = (\bar{q}_i \Gamma q_i)(\bar{Q}\Gamma' Q)$$

$$\mathcal{O}^{n}_{2} = \psi^\dagger \mathcal{K}^n \chi$$

$$\mathcal{O}^{\rm SCET}_{2} = \bar{\xi}_{n_{\bar{B}}} \Gamma \xi_{n_B}$$

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Hard function: $\quad H^n_{ij} \sim C^n_{(2+2)} (C^n_{(2+2)})^*$

NRQCD at low pT - Beam function

$$\mathcal{O}^{\rm QCD}_{2+2} = (\bar{q}_i \Gamma q_i)(\bar{Q}\Gamma' Q)$$

$$\mathcal{O}^{n}_{2} = \psi^\dagger \mathcal{K}^n \chi$$

$$\mathcal{O}^{\rm SCET}_{2} = \bar{\xi}_{n_{\bar{B}}} \Gamma \xi_{n_B}$$

$$\mathcal{O}^{\rm QCD}(2+2) \to \sum_n C^n_{(2+2)} \Big(\mathcal{O}^{\rm SCET}_2 \times \mathcal{O}^n_2 \Big)$$

Factorize the cross section using BPS field redefinition for decoupling collinear and heavy from ultra-soft modes.

$$\frac{d\sigma}{dy\, d^2\mathbf{p}_T} = \sum_n \left(\sigma_0(n) \frac{m^2}{s} \right) \times H^n_{ij} \times \mathcal{B}_i(x_1) \otimes \mathcal{B}_j(x_2) \otimes S^{\mathcal{Q}}_n \times \Big(1 + \mathcal{O}(\lambda) \Big)$$

Beam Function: $$\mathcal{B}_{i/h}(x_i, p_T) = \int_{x_i}^1 \frac{dx}{x} C_{i/j}(x, p_T) f_{j/h}(x_i/x)$$ + model ?

short distance matching coefficients ($C_{i/j}(x, p_T)$)

collinear PDFs/FFs ($f_{j/h}(x_i/x)$)

NRQCD at low pT - Shape function

$$\mathcal{O}_{2+2}^{\rm QCD} = (\bar{q}_i \Gamma q_i)(\bar{Q}\Gamma' Q)$$

$$\mathcal{O}_2^n = \psi^\dagger \mathcal{K}^n \chi$$

$$\mathcal{O}_2^{\rm SCET} = \bar{\xi}_{n_{\bar{B}}} \Gamma \xi_{n_B}$$

$$\mathcal{O}^{\rm QCD}(2+2) \to \sum_n C^n_{(2+2)} \Big(\mathcal{O}_2^{\rm SCET} \times \mathcal{O}_2^n \Big)$$

Factorize the cross section using BPS field redefinition for decoupling collinear and heavy from ultra-soft modes.

$$\frac{d\sigma}{dy\, d^2\mathbf{p}_T} = \sum_n \left(\sigma_0(n) \frac{m^2}{s} \right) \times H^n_{ij} \times \mathcal{B}_i(x_1) \otimes \mathcal{B}_j(x_2) \otimes S_n^{\mathcal{Q}} \times \Big(1 + \mathcal{O}(\lambda) \Big)$$

Quarkonium TMD shape function:

$$S_n^{\mathcal{Q}} = \frac{1}{\mathcal{N}} \mathrm{Tr}\Big[\sum_{X_s} \Big\langle 0 \Big| (\chi^\dagger \mathcal{K}_n^\dagger \psi)\, \mathcal{Y}^\dagger_{(\lambda)s,n_{\bar{B}}} \mathcal{Y}^\dagger_{(\lambda)s,n_B} \delta^{(2)}(\mathbf{p}_T - \vec{\mathcal{P}}_\perp) \Big| X_s + \mathcal{Q} \Big\rangle$$
$$\times \Big\langle X_s + \mathcal{Q} \Big| \mathcal{Y}_{(\lambda)s,n_B} \mathcal{Y}_{(\lambda)s,n_{\bar{B}}}\, (\psi^\dagger \mathcal{K}_n \chi) \Big| 0 \Big\rangle \Big]$$

NRQCD at low pT - Shape function

$$\mathcal{O}^{\rm QCD}_{2+2} = (\bar{q}_i \Gamma q_i)(\bar{Q}\Gamma' Q)$$

$$\mathcal{O}_2^n = \psi^\dagger \mathcal{K}^n \chi$$

$$\mathcal{O}_2^{\rm SCET} = \bar{\xi}_{n_{\bar{B}}} \Gamma \xi_{n_B}$$

$$\mathcal{O}^{\rm QCD}(2+2) \to \sum_n C^n_{(2+2)} \left(\mathcal{O}_2^{\rm SCET} \times \mathcal{O}_2^n\right)$$

Factorize the cross section using BPS field redefinition for decoupling collinear and heavy from ultra-soft modes.

$$\frac{d\sigma}{dy\, d^2\mathbf{p}_T} = \sum_n \left(\sigma_0(n) \frac{m^2}{s}\right) \times H^n_{ij} \times \mathcal{B}_i(x_1) \otimes \mathcal{B}_j(x_2) \otimes S_n^{\mathcal{Q}} \times \Big(1 + \mathcal{O}(\lambda)\Big)$$

Quarkonium TMD shape function: S-waves: octet case, not IR finite cross section !

$$S_n^{\mathcal{Q}} = \frac{1}{\mathcal{N}} \mathrm{Tr}\Big[\sum_{X_s} \Big\langle 0 \Big| (\chi^\dagger \mathcal{K}_n^\dagger \psi)\, \mathcal{Y}^\dagger_{(\lambda)s,n_{\bar{B}}} \mathcal{Y}^\dagger_{(\lambda)s,n_B} \delta^{(2)}(\mathbf{p}_T - \vec{\mathcal{P}}_\perp) \Big| X_s + \mathcal{Q} \Big\rangle$$

$$\times \Big\langle X_s + \mathcal{Q} \Big| \mathcal{Y}_{(\lambda)s,n_B} \mathcal{Y}_{(\lambda)s,n_{\bar{B}}} (\psi^\dagger \mathcal{K}_n \chi) \Big| 0 \Big\rangle \Big]$$

NRQCD at low pT - Shape function

The correct operator definition of the shape function is the same as

before but with : $\mathcal{O}_2^n \to (\psi Y_u)^\dagger \mathcal{K}_n (Y_u \chi)$

$$\psi \to Y_u \psi$$

$$Y_u(x) = \mathrm{T}\Big[\exp\Big(ig\int_{-\infty}^{0} dt'\; u\cdot A_s^a(\vec{x}, t+t')T^a\Big)\Big]$$

$$u^\mu = (1,0,0,0)$$

Singlet	Octet
$\mathcal{K}_1 = \delta_{ab}\,\Gamma({}^{2S+1}L_J)$	$\mathcal{K}_8 = T^A_{ab}\,\Gamma({}^{2S+1}L_J)$
$Y_u^\dagger Y_u = 1$	$Y_u^\dagger T^A Y_u = \mathcal{Y}_u^{AB} T^B$
$\mathcal{O}_2^{[1]} \to \mathcal{O}_2^{[1]}$	$\mathcal{O}_2^{[8]} \to \mathcal{O}_2^{[8]}\mathcal{Y}_u$

NRQCD at low pT - Soft Wilson lines

G. C. Nayak, J.-W. Qiu, G. F. Sterman: arXiv:hep-ph/0501235, hep-ph/0509021

light-like wilson-lines for color octet LDMEs for gauge invariance and factorization of the fragmentation processes

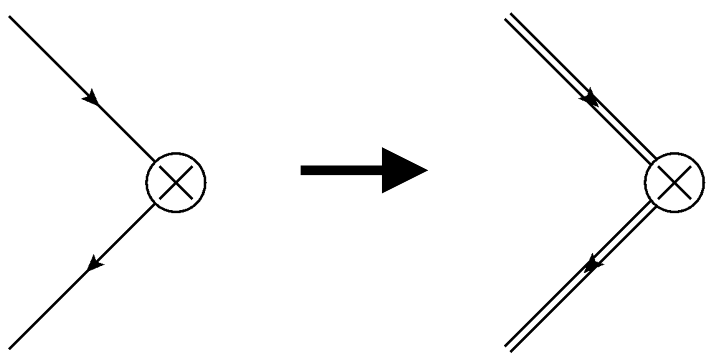
$$Y_u(x) = \mathrm{P}\Big[\exp\Big(ig\int_{-\infty}^{0} dt' u \cdot A_s^a(x^\mu + u^\mu t')\Big)\Big] \qquad u^\mu = (1, \vec{u})$$

Manifestly Soft Gauge Invariant Formulation of vNRQCD
(Ira Z. Rothstein, Prashant Shrivastava, and Iain W. Stewart: arXiv:1806.07398)

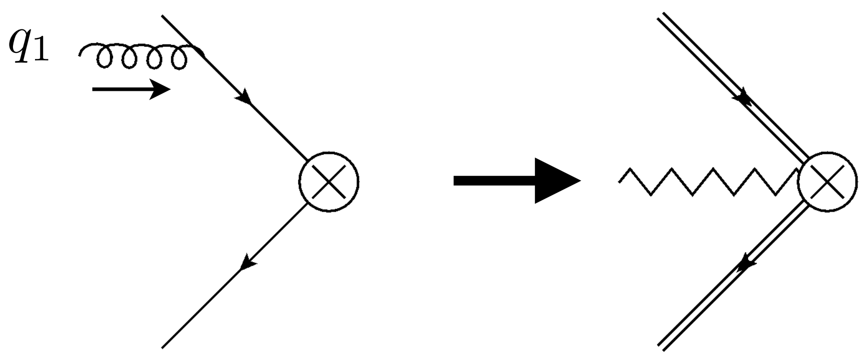
soft Wilson-lines for gauge invariance of the Lagrangian

NRQCD at low pT - Soft Wilson lines

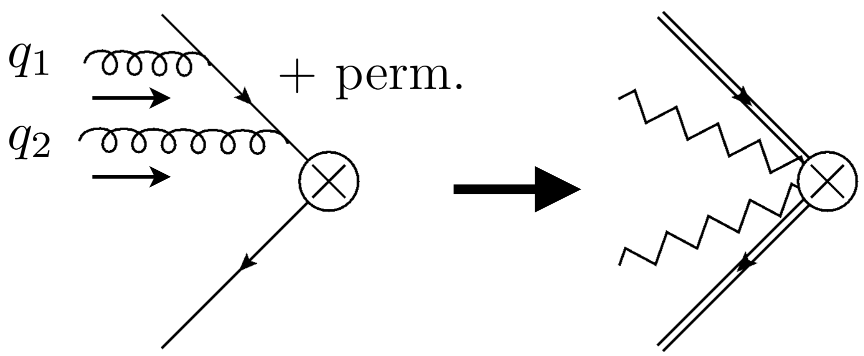
QCD | Non-relativistic limit



$$= \chi^\dagger \, \mathcal{K}_n \, \psi$$



$$= \chi^\dagger \, \mathcal{K}_n \Big(\frac{-g \; u \cdot A_s(q_1)}{u \cdot q_1} \Big) \psi \qquad u^\mu = (1,0,0,0)$$



$$= \chi^\dagger \, \mathcal{K}_n \Big(\frac{g^2 \; [u \cdot A_s(q_1)][u \cdot A_s(q_2)]}{2 \; [u \cdot q_1][u \cdot (q_1 + q_2)]} \Big) \psi \quad + \text{perm.}$$

$$\rightarrow \quad Y_u = \sum_k \sum_{\text{perm}} \frac{(-g)^k [u \cdot A_s(q_1)] \cdots [u \cdot A_s(q_k)]}{(k!)[u \cdot q_1] \cdots [u \cdot (q_1 + q_2 + \cdots q_k)]}$$

NRQCD at low pT - Soft Wilson lines

(1) This soft Wilson line is it a “v-soft” or a “λ-soft”?

(2) What are the different hierarchies between the two soft scales?

(3) Can we do a perturbative calculation of the shape function to check consistency of the factorization?

(4) How this factorization is related with the standard NRQCD factorization if the small pT limit is taken?

NRQCD at low pT - Soft Wilson lines

(1) This soft Wilson line is it a "v-soft" or a "λ-soft"?

There is **no** hierarchy

No two modes to choose from.

There is hierarchy

Question 2

NRQCD at low pT - Soft Wilson lines

(2) What are the different hierarchies between the two soft scales?

Region (1)	Region (2)	Region (3)
$p_T \gg m_Q v \gtrsim \Lambda_{\text{QCD}}$	$p_T \sim m_Q v \sim \Lambda_{\text{QCD}}$	$p_T \sim m_Q v \gg \Lambda_{\text{QCD}}$
Two separated soft modes	No hierarchy of scales	Hadronization effects happen at much lower scale but no re-factorization is possible.
$A_s \to A_{(\lambda)s} + A_{(v)s}$ $Y_{s,u} \to Y_{(\lambda)s,u} \times Y_{(v)s,u}$ $\|X_s\rangle \to \|X_{(\lambda)s}\rangle \times \|X_{(v)s}\rangle$	Shape function introduces **new** non-perturbative effects	Can be matched onto a **new** set of LDMEs
$S_n^{\mathcal{Q}}(p_T) \to S_c(p_T) \times \langle \mathcal{O}_n^{\mathcal{Q}} \rangle$	No further re-factorization	$S_n^{\mathcal{Q}}(p_T) \to C_n(p_T) \times \langle \overline{\mathcal{O}}_n^{\mathcal{Q}} \rangle$

NRQCD at low pT - Soft Wilson lines

(3) Can we do a perturbative calculation of the shape function to check consistency of out factorization?

Region (1)

$p_T \gg m_Q v \gtrsim \Lambda_{\text{QCD}}$

Two separated soft modes

$$A_s \to A_{(\lambda)s} + A_{(v)s}$$
$$Y_{s,u} \to Y_{(\lambda)s,u} \times Y_{(v)s,u}$$
$$|X_s\rangle \to |X_{(\lambda)s}\rangle \times |X_{(v)s}\rangle$$

$$S_n^{\mathcal{Q}}(p_T) \to S_c(p_T) \times \langle \mathcal{O}_n^{\mathcal{Q}} \rangle$$

Region (2) ✓

$p_T \sim m_Q v \sim \Lambda_{\text{QCD}}$

$$S_n^{[Q\bar{Q}]}\Big|_{\text{NLO}} = S_{[c]}\Big|_{\text{NLO}} + \langle \mathcal{O}_n^{Q\bar{Q}} \rangle\Big|_{\text{NLO}}$$

$$\gamma_\mu^H + \gamma_\mu^S + 2\gamma_\mu^{\mathcal{B}} = 0$$

$$\gamma_\nu^S + 2\gamma_\nu^{\mathcal{B}} = 0$$

Region (3)

$p_T \sim m_Q v \gg \Lambda_{\text{QCD}}$

Hadronization effects happen at much lower scale but no re-factorization is possible.

Can be matched onto a **new** set of LDMEs

$$S_n^{\mathcal{Q}}(p_T) \to C_n(p_T) \times \langle \overline{\mathcal{O}}_n^{\mathcal{Q}} \rangle$$

Rapidity regulator and rapidity-RG:
arXiv:1202.0814, J.-Y. Chiu, A. Jain, D. Neill and I. Z. Rothstein

NRQCD at low pT - Soft Wilson lines

(4) How this factorization is related with the standard NRQCD factorization if the small pT limit is taken?

Region (1) ✓

$p_T \gg m_Q v \gtrsim \Lambda_{\mathrm{QCD}}$

Two separated soft modes

$$A_s \to A_{(\lambda)s} + A_{(v)s}$$
$$Y_{s,u} \to Y_{(\lambda)s,u} \times Y_{(v)s,u}$$
$$|X_s\rangle \to |X_{(\lambda)s}\rangle \times |X_{(v)s}\rangle$$

$$S_n^{\mathcal{Q}}(p_T) \to S_c(p_T) \times \langle \mathcal{O}_n^{\mathcal{Q}} \rangle$$

arXiv:1210.3432, Peng Sun, C.-P. Yuan, and Feng Yuan

Region (2) ✓

$p_T \sim m_Q v \sim \Lambda_{\mathrm{QCD}}$

$$S_n^{[Q\bar{Q}]}\Big|_{\mathrm{NLO}} = S_{[c]}\Big|_{\mathrm{NLO}} + \langle \mathcal{O}_n^{Q\bar{Q}} \rangle\Big|_{\mathrm{NLO}}$$

$$\gamma_\mu^H + \gamma_\mu^S + 2\gamma_\mu^{\mathcal{B}} = 0$$

$$\gamma_\nu^S + 2\gamma_\nu^{\mathcal{B}} = 0$$

Rapidity regulator and rapidity-RG:
arXiv:1202.0814, J.-Y. Chiu, A. Jain, D. Neill and I. Z. Rothstein

Region (3)

$p_T \sim m_Q v \gg \Lambda_{\mathrm{QCD}}$

Hadronization effects happen at much lower scale but no re-factorization is possible.

Can be matched onto a **new** set of LDMEs

$$S_n^{\mathcal{Q}}(p_T) \to C_n(p_T) \times \langle \overline{\mathcal{O}}_n^{\mathcal{Q}} \rangle$$

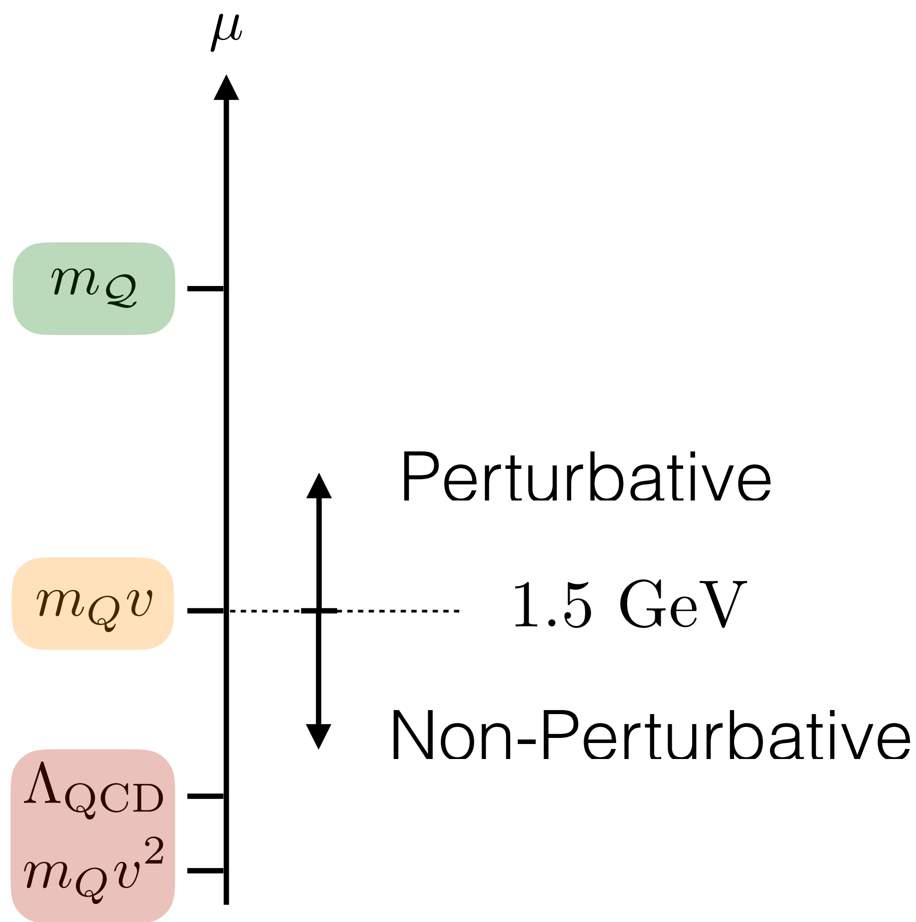
Summary

- Time-like Wilson line is necessary for consistency in the case of color octet mechanisms
- Quarkonium production in low pT standard LDMEs are “promoted” to shape/soft functions
- Small pT limit of NRQCD is recovered only in the limit: $p_T \gg m_Q v$
- Shape function introduces new non-perturbative effects (universal ?).
- Extension to:
  - polarized cross sections
  - TMD-fragmentation
  - In-jet Quarkonia

Back Up

NRQCD scales

NRQCD = Non-Relativistic QCD



$b\bar{b}: \quad v^2 \sim 0.1$ bottomonium

$c\bar{c}: \quad v^2 \sim 0.3$ charmonium

→ Relative velocity of the heavy quark and antiquark in the quarkonium

typical momentum of heavy quark: $|\mathbf{p}_Q| \sim m_Q v$ (soft)

typical kinetic energy of heavy quark: $K_Q \sim m_Q v^2$ (ultra-soft)

(v)NRQCD Lagrangian

NRQCD = Non-Relativistic QCD

soft: $p_s^\mu \sim m_Q v(1,1,1,1)$

ultra-soft: $p_{us}^\mu \sim m_Q v^2(1,1,1,1)$

ultra-soft

subheading

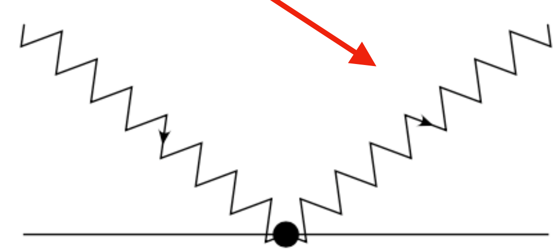
$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_p \left|p^\mu A_p^\nu - p^\nu A_p^\mu\right|^2 + \sum_{\mathbf{p}} \psi_{\mathbf{p}}^\dagger \left\{ iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m}\right\}\psi_{\mathbf{p}}$$

$$-4\pi\alpha_s \sum_{q,q'\mathbf{p},\mathbf{p}'} \left\{ \frac{1}{q^0}\psi^\dagger_{\mathbf{p}'}\left[A^0_{q'},A^0_q\right]\psi_{\mathbf{p}} + \frac{g^{\nu 0}(q'-p+p')^\mu - g^{\mu 0}(q-p+p')^\nu + g^{\mu\nu}(q-q')^0}{(\mathbf{p}'-\mathbf{p})^2}\psi^\dagger_{\mathbf{p}'}\left[A^\nu_{q'},A^\mu_q\right]\psi_{\mathbf{p}}\right\}$$

$$+\psi\leftrightarrow\chi,\ T\leftrightarrow\bar{T}$$

$$+\sum_{\mathbf{p},\mathbf{q}} \frac{4\pi\alpha_s}{(\mathbf{p}-\mathbf{q})^2}\psi^\dagger_{\mathbf{q}}T^A\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{q}}\bar{T}^A\chi_{-\mathbf{p}}+\dots$$

soft



arXiv:hep-ph/9910209 M. E. Luke, A. V. Manohar, I. Z. Rothstein

NRQCD Factorization

LDME: Long Distance Matrix Elements

$$d\sigma(a+b\to \mathcal{Q}+X) = \sum_n d\sigma(a+b\to Q\overline{Q}(n)+X)\langle \mathcal{O}_n^{\mathcal{Q}}\rangle$$

Perturbative expansion in the strong coupling.

NRQCD Scaling Rules

$$d\sigma_0(1+\alpha_s C_1+\alpha_s^2 C_2+\ldots) \qquad \langle \mathcal{O}(^{2S+1}L_J^{[1,8]})\rangle \sim v^{3+2L+2E+4M}$$

$$Q\bar{Q}(n) \xrightarrow{\langle \mathcal{O}_n^{\mathcal{Q}}\rangle} \mathcal{Q}$$

$$\mathcal{O}_n^{\mathcal{Q}} = \mathcal{O}_2^{n\,\dagger}\left(\sum_X |X+\mathcal{Q}\rangle\langle X+\mathcal{Q}|\right)\mathcal{O}_2^n$$

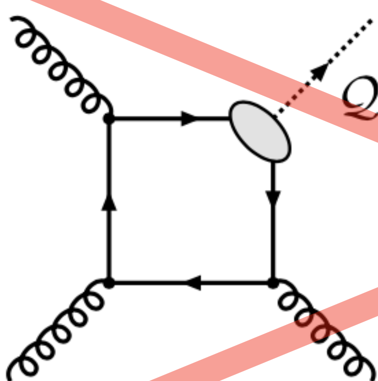
$$\mathcal{O}_2^n = \psi^\dagger \mathcal{K}^n \chi$$

$$n = {}^{2S+1}L_J^{[c]}$$

ultra-soft + soft

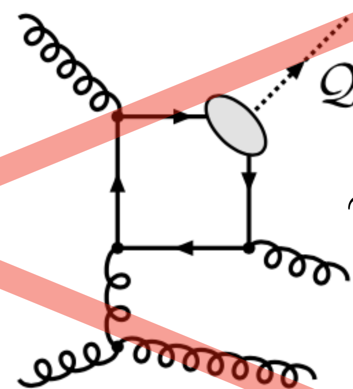
NRQCD at large pT

(Hadron colliders)



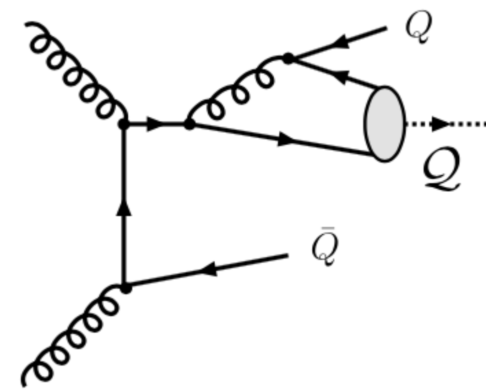
$\sim \frac{1}{p_\perp^8}$

LO



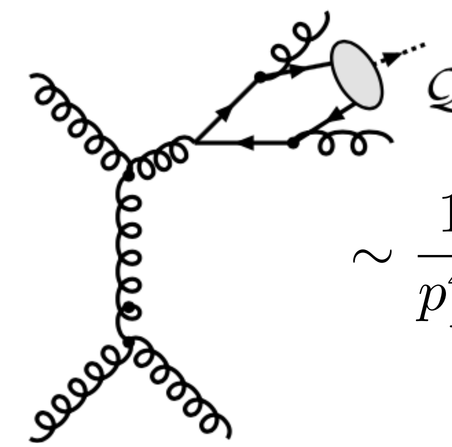
$\sim \frac{1}{p_\perp^6}$

NLO



$\sim \frac{1}{p_\perp^4}$

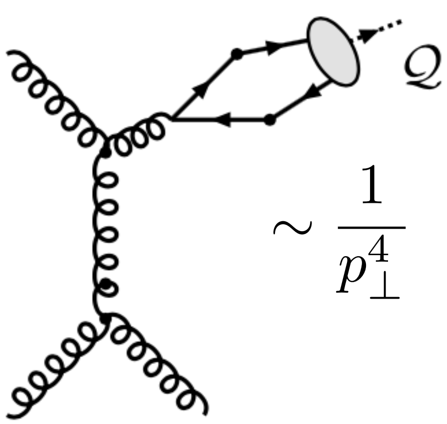
NLO



$\sim \frac{1}{p_\perp^4}$

NNLO

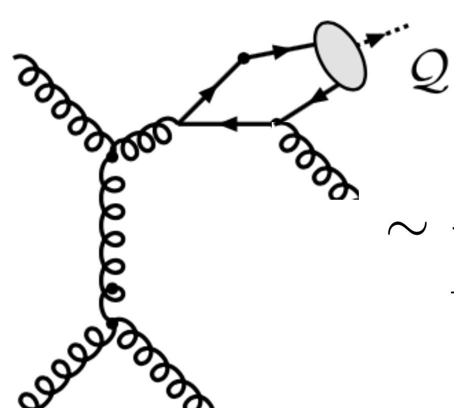
${}^3S_1^{[8]}$



$\sim \frac{1}{p_\perp^4}$

LO

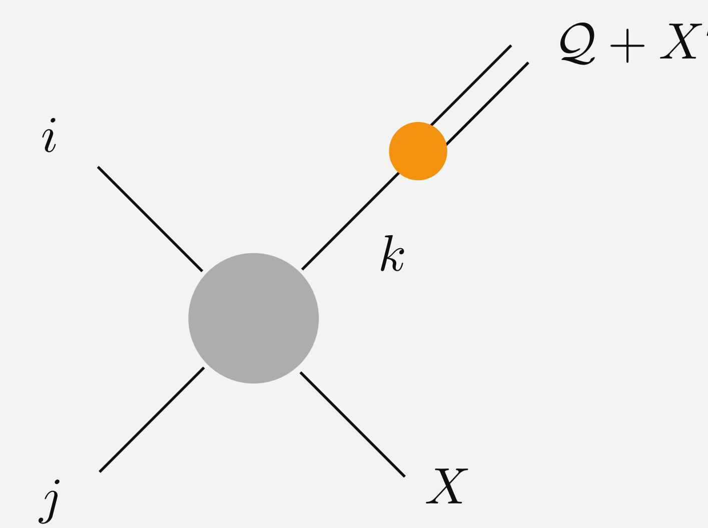
${}^1S_0^{[8]}, \; {}^3P_J^{[8]}$



$\sim \frac{1}{p_\perp^4}$

NLO

(single) Parton fragmentation process



NRQCD at large pT

<u>Leading Power (LP) Factorization</u>

$$\frac{d\sigma_h}{dp_\perp}(p_\perp) = \sum_i \int_z^1 \frac{dx}{x} \frac{d\sigma_i}{dp_\perp}\left(\frac{p_\perp}{x}, \mu\right) D_{i/h}(x,\mu) + \mathcal{O}\left(\frac{m_h^2}{p_\perp^2}\right)$$

Expansion in: $\frac{m_{\mathcal{Q}}}{p_\perp}$

At sufficiently large pT the fragmentation processes will dominate the cross section:

Only few simple diagrams for each mechanism

Large Logarithms

$\ln(p_T/m_{\mathcal{Q}})$

NRQCD at large pT

Leading Power (LP) Factorization

$$\frac{d\sigma_h}{dp_\perp}(p_\perp) = \sum_i \int_z^1 \frac{dx}{x} \frac{d\sigma_i}{dp_\perp}\left(\frac{p_\perp}{x}, \mu\right) D_{i/h}(x,\mu) + \mathcal{O}\left(\frac{m_h^2}{p_\perp^2}\right)$$

$$\ln\left(\frac{\mu}{p_T}\right) - \ln\left(\frac{\mu}{2m_Q}\right)$$

Expansion in: $\frac{m_{\mathcal{Q}}}{p_\perp}$

At sufficiently large pT the fragmentation processes will dominate the cross section:

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NRQCD at large pT

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$$\frac{d\sigma_h}{dp_\perp}(p_\perp) = \sum_i \int_z^1 \frac{dx}{x}\, \frac{d\sigma_i}{dp_\perp}\left(\frac{p_\perp}{x}, \mu\right) D_{i/h}(x,\mu) + \mathcal{O}\left(\frac{m_h^2}{p_\perp^2}\right)$$

Expansion in: $\frac{m_{\mathcal{Q}}}{p_\perp}$

$$\ln\left(\frac{\mu}{p_T}\right) - \ln\left(\frac{\mu}{2m_Q}\right)$$

DGLAP Evolution

$$\mu\frac{d}{d\mu}D_{i/h}(z,\mu) = \sum_i \int_z^1 \frac{dx}{x}\, P_{ij}(x) D_{i/h}\left(\frac{z}{x},\mu\right)$$

Resummation: $\ln(p_T/m_{\mathcal{Q}})$

NRQCD at large pT

Leading Power (LP) Factorization

$$\frac{d\sigma_h}{dp_\perp}(p_\perp) = \sum_i \int_z^1 \frac{dx}{x} \frac{d\sigma_i}{dp_\perp}\left(\frac{p_\perp}{x}, \mu\right) D_{i/h}(x,\mu) + \mathcal{O}\left(\frac{m_h^2}{p_\perp^2}\right)$$

Expansion in: $\frac{m_Q}{p_\perp}$

$$\ln\left(\frac{\mu}{p_T}\right) - \ln\left(\frac{\mu}{2m_Q}\right) \qquad d_{i/n}(x,\mu)\langle \mathcal{O}_n^h \rangle$$

DGLAP Evolution

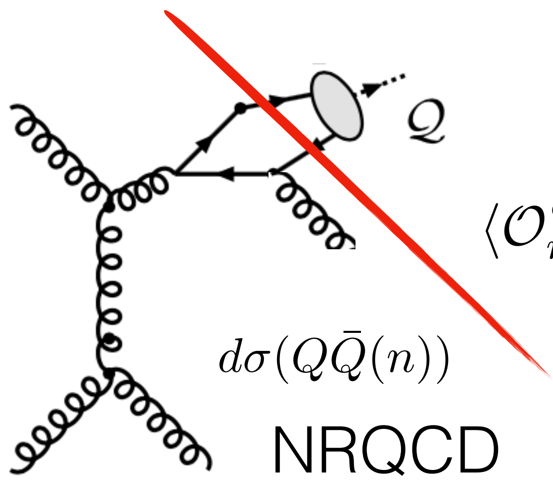
$$\mu \frac{d}{d\mu} D_{i/h}(z,\mu) = \sum_i \int_z^1 \frac{dx}{x} P_{ij}(x) D_{i/h}\left(\frac{z}{x}, \mu\right)$$

Resummation: $\ln(p_T/m_Q)$

Quarkonium production until now

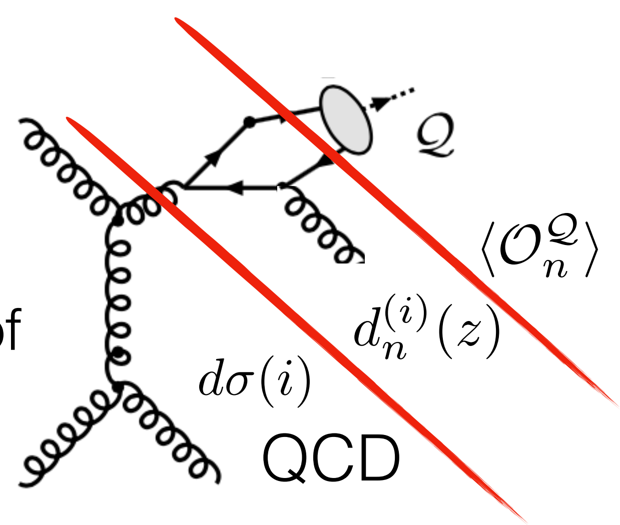
Fixed order NRQCD

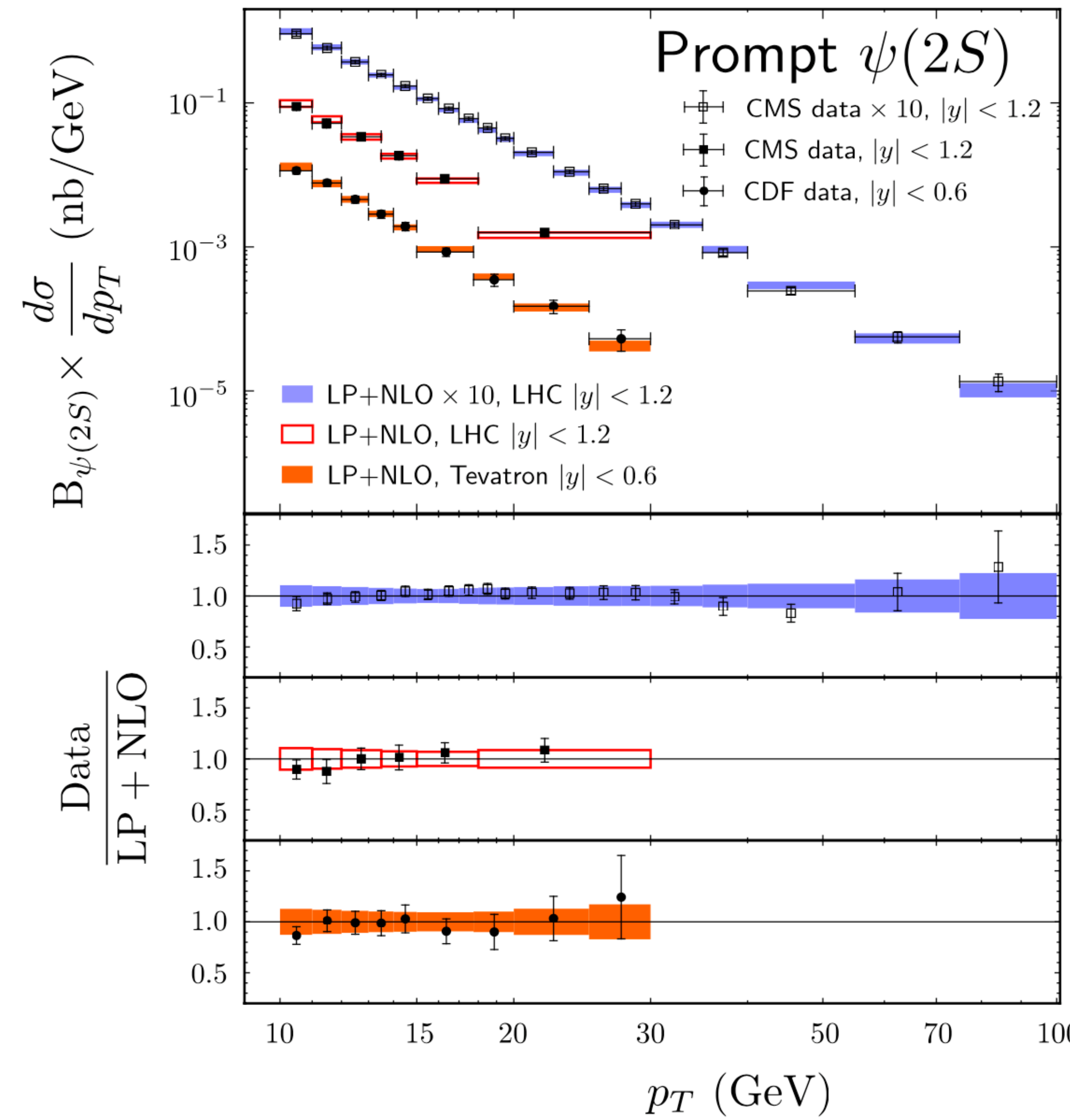
Preferred at
$p_T \sim m_Q$
corrections of
p_T/m_Q



Leading Power NRQCD

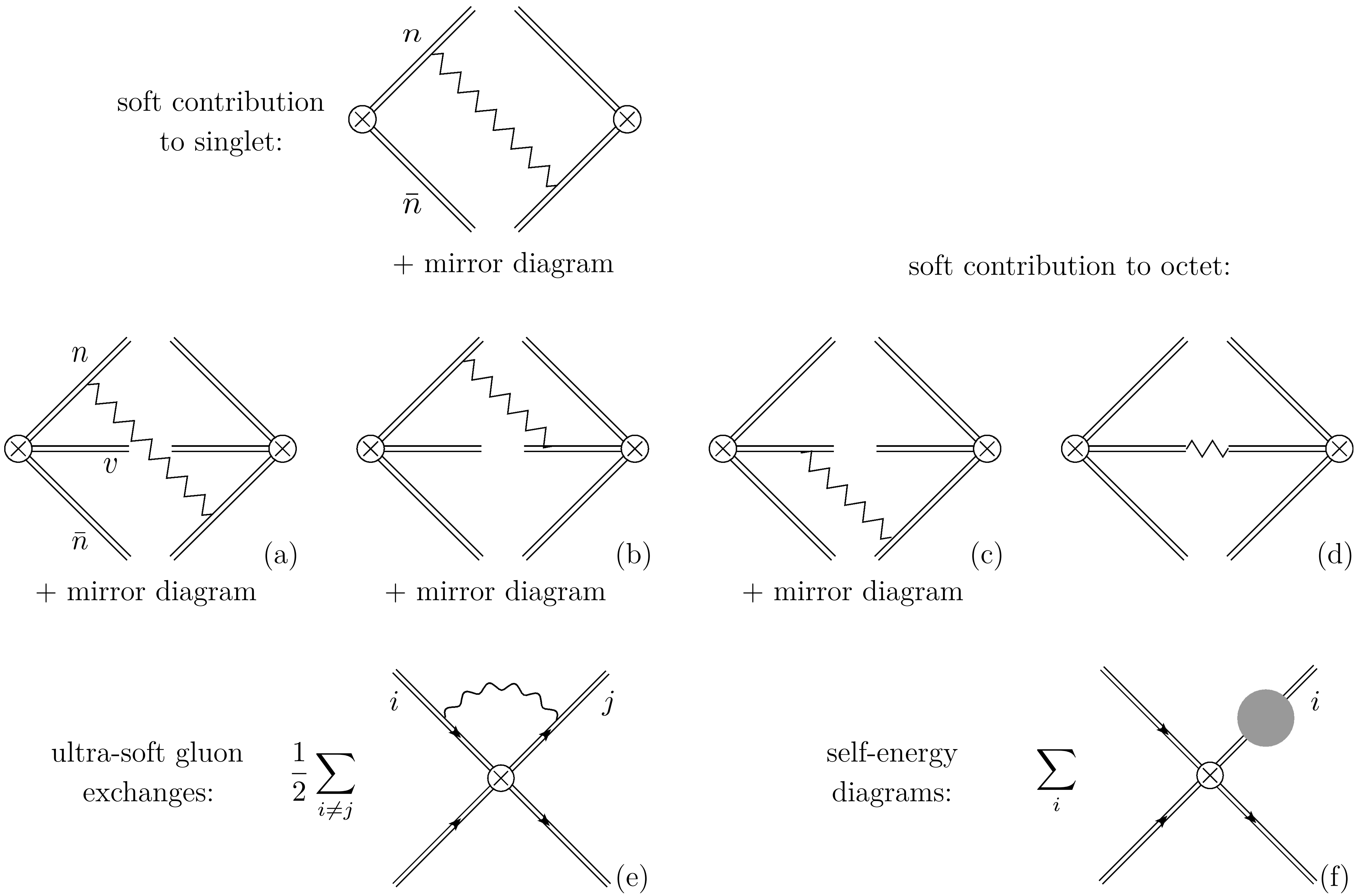
Preferred at
$p_T \gg m_Q$
Resummation of
$\ln(p_T/m_Q)$





G. T. Bodwin, K-T. Chao, H. S. Chung, U-R. Kim,J. Lee, and Y-Q. Ma (PRD) 2016

Quarkonium at low pT (shape function perturbative)



Quarkonium at low pT (shape function perturbative)

$$S_{ij}^{[8]} = S_{ij}^{[1]} + \frac{\alpha_s C_A}{2\pi}\Big\{\frac{1}{\epsilon}\delta^{(2)}(\mathbf{p}_T) - 2\mathcal{L}_0(\mathbf{p}_T^2,\mu^2)\Big\}$$

$$S_{ij}^{[1]}(\mathbf{p}_T) = \delta^{(2)}(\mathbf{p}_T) + \frac{\alpha_s C_{ij}}{2\pi}\Big\{\frac{4}{\eta}\Big[2\mathcal{L}_0(\mathbf{p}_T^2,\mu^2) - \frac{1}{\epsilon}\delta^{(2)}(\mathbf{p}_T)\Big] + \frac{2}{\epsilon}\Big[\frac{1}{\epsilon} - \ln\Big(\frac{\nu^2}{\mu^2}\Big)\Big]\delta^{(2)}(\mathbf{p}_T) - \frac{\pi^2}{6}\delta(\mathbf{p}_T)$$

$$- 4\mathcal{L}_1(\mathbf{p}_T^2,\mu^2) + 4\mathcal{L}_0(\mathbf{p}_T^2,\mu^2)\ln\Big(\frac{\nu^2}{\mu^2}\Big)\Big\}$$

<u>NLO hard function:</u>

$$H_{ij}^{[n],\mathrm{b}} = 1 + \frac{\alpha_s C_{ij}}{2\pi}\Big\{\frac{2}{\epsilon}\Big[\ln\Big(\frac{M^2}{\mu^2}\Big) - \bar{\gamma}_i - \frac{1}{\epsilon}\Big] + 2B(n = {}^{2S+1}L_J^{[c]}) - \ln^2\Big(\frac{M^2}{\mu^2}\Big) - \frac{\pi^2}{6} + 2\bar{\gamma}_i\ln\Big(\frac{M^2}{\mu^2}\Big)\Big\}$$

$$+ \delta_{c8}\frac{\alpha_s C_A}{2\pi}\Big\{ - \frac{1}{\epsilon} + \ln\Big(\frac{M^2}{\mu^2}\Big)\Big\}$$

NRQCD at low pT - Soft Wilson lines

(4) How this factorization is related with the standard NRQCD factorization if the small pT limit is taken?

Region (1)

$p_T \gg m_Q v \gtrsim \Lambda_{\rm QCD}$

Two separated soft modes

$$A_s \to A_{(\lambda)s} + A_{(v)s}$$

$$Y_{s,u} \to Y_{(\lambda)s,u} \times Y_{(v)s,u}$$

$$|X_s\rangle \to |X_{(\lambda)s}\rangle \times |X_{(v)s}\rangle$$

$$S_n^{\mathcal{Q}}(p_T) \to S_c(p_T) \times \langle \mathcal{O}_n^{\mathcal{Q}} \rangle$$

Region (2)

$p_T \sim m_Q v \sim \Lambda_{\rm QCD}$

No hierarchy of scales

Shape function introduces **new** non-perturbative effects

No further re-factorization

Region (3)

$p_T \sim m_Q v \gg \Lambda_{\rm QCD}$

Hadronization effects happen at much lower scale but no re-factorization is possible.

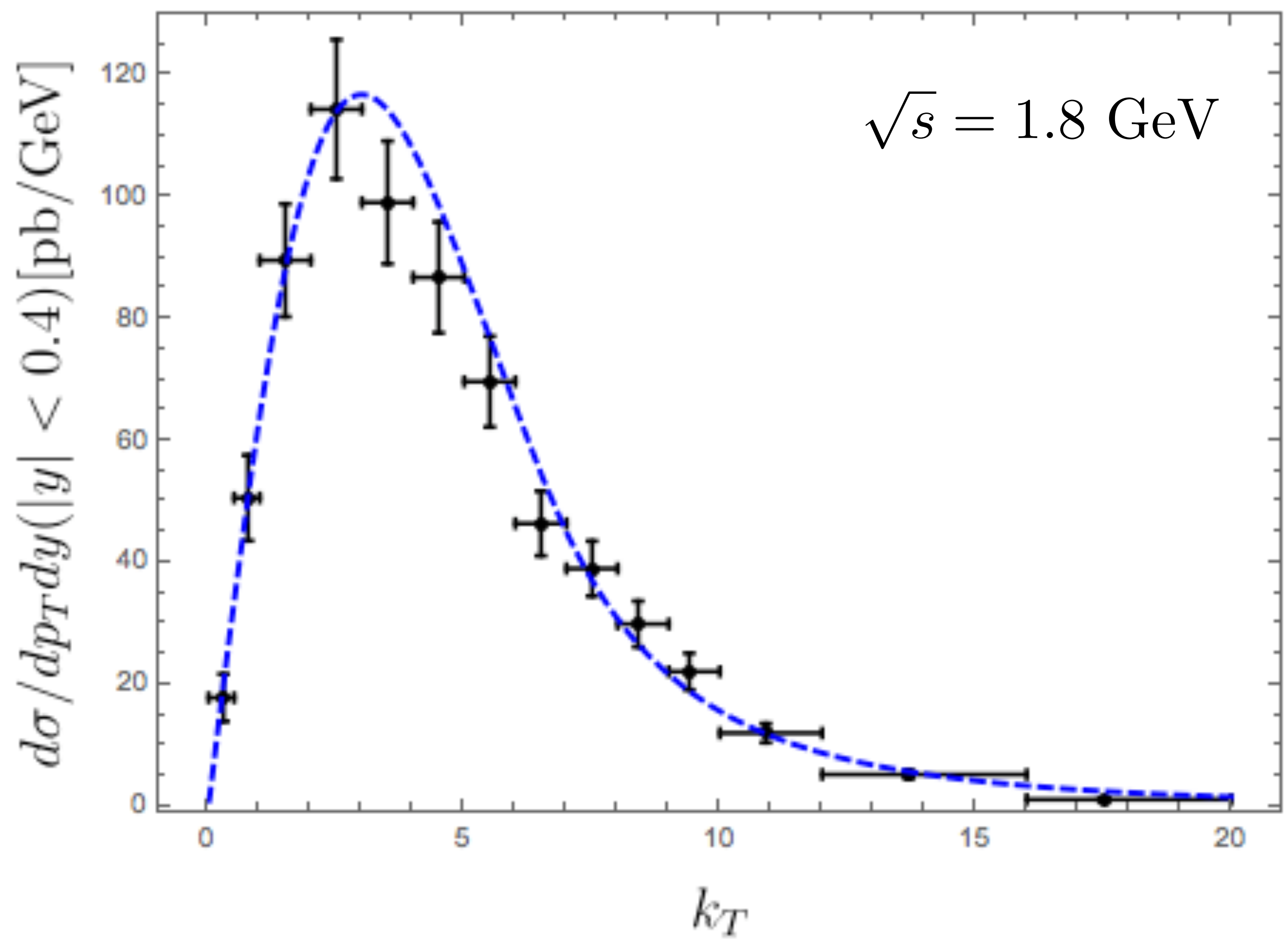
Can be matched onto a **new** set of LDMEs

$$S_n^{\mathcal{Q}}(p_T) \to C_n(p_T) \times \langle \overline{\mathcal{O}}_n^{\mathcal{Q}} \rangle$$

$$\overline{\mathcal{O}}_n^{\mathcal{Q}}(c=1) = \mathcal{O}_n^{(2)\dagger} \mathcal{Y}_{n,s}^{a\dagger} \mathcal{Y}_{\bar{n},s}^{b\dagger} \left(a_Q^\dagger a_Q \right) \mathcal{Y}_{\bar{n},s}^{b} \mathcal{Y}_{n,s}^{a} \mathcal{O}_n^{(2)}$$

$$\overline{\mathcal{O}}_n^{\mathcal{Q}}(c=8) = \mathcal{O}_n^{(2)\dagger} \mathcal{Y}_{v,s}^{c\dagger} \mathcal{Y}_{n,s}^{a\dagger} \mathcal{Y}_{\bar{n},s}^{b\dagger} \left(a_Q^\dagger a_Q \right) \mathcal{Y}_{\bar{n},s}^{b} \mathcal{Y}_{n,s}^{a} \mathcal{Y}_{v,s}^{c} \mathcal{O}_n^{(2)}$$

NRQCD at low pT - Numerics (preliminary)



- Region (1) + (2) factorization

- All leading and subheading channels.

- Octet LDMEs are fitted.