OCTOBER 1 – NOVEMBER 16, 2018 • SEATTLE, WASHINGTON

PROBING NUCLEONS AND NUCLEI IN HIGH ENERGY COLLISIONS

Dedicated to the Physics of the Electron Ion Collider

Program held at the Institute for Nuclear Theory, supported by the US Department of Energy

http://www.int.washington.edu/PROGRAMS/18-3

Mass structure and pressure forces inside the nucleon

Based on **[C.L., Eur. Phys. J. C78 (2018)]**
[C.L., Moutarde, Trawinski, arXiv:1810.xxxxx]

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October 23, INT, Seattle, USA

Outline

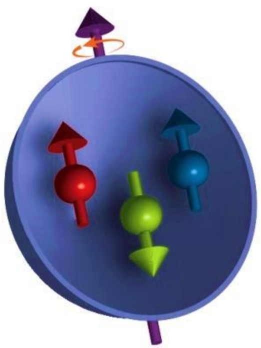
1. **Nucleon structure**
2. **Energy-momentum tensor**
3. **Mass decompositions**
4. **3D distributions in Breit frame**
5. **Comparison with neutron stars**
6. **Summary**

Origin of mass and spin?

Non-relativistic picture
dominated by constituents

Until ~ 1980

Spectroscopy



Mass

$$M_N \sim \sum_Q M_Q + E_{\mathrm{binding}}$$

~ 102 % **~ - 2 %**

Spin

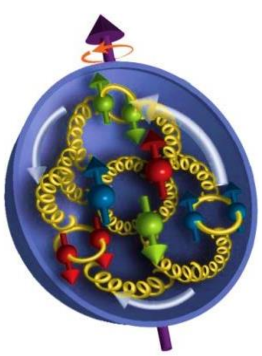
$$J_z^N \sim \sum_Q S_z^Q$$

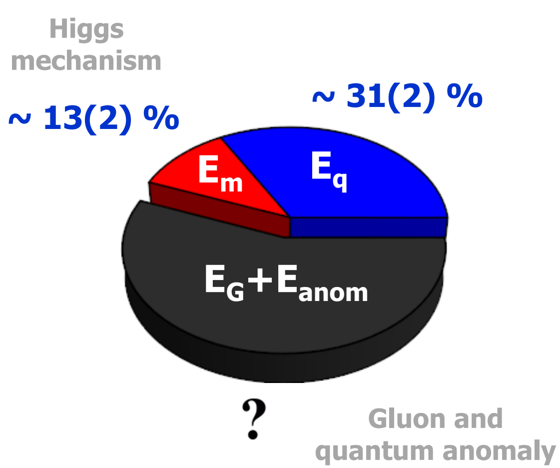
~ 100 %

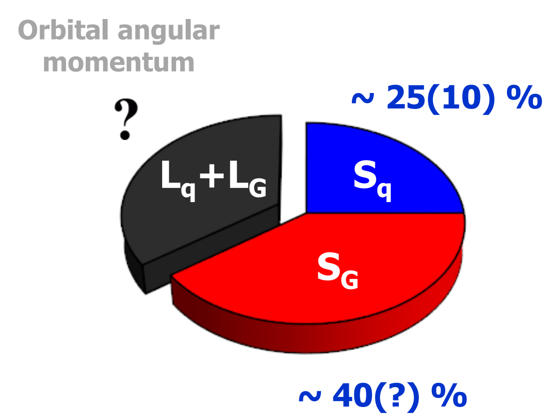
Relativistic picture
dominated by dynamics

Now

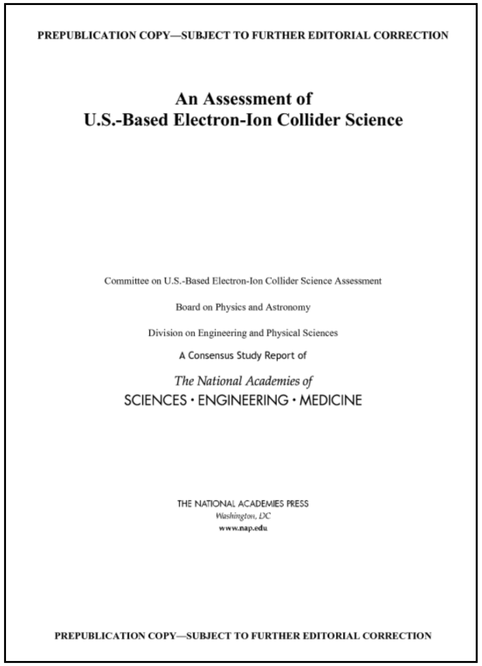
High-energy scattering







Hot news



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An Assessment of
U.S.-Based Electron-Ion Collider Science

Committee on U.S.-Based Electron-Ion Collider Science Assessment

Board on Physics and Astronomy

Division on Engineering and Physical Sciences

A Consensus Study Report of

The National Academies of
SCIENCES · ENGINEERING · MEDICINE

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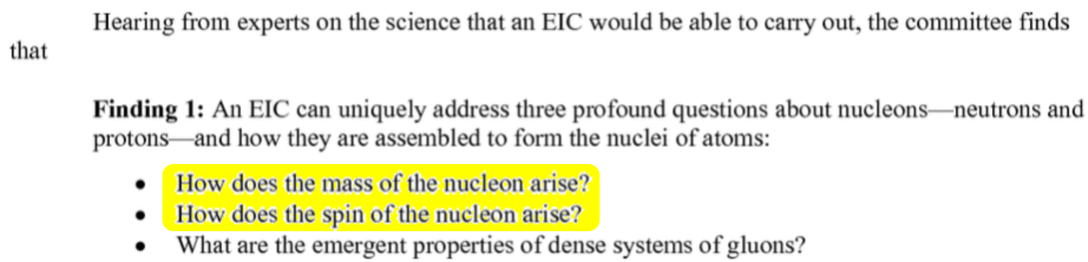


Hearing from experts on the science that an EIC would be able to carry out, the committee finds that

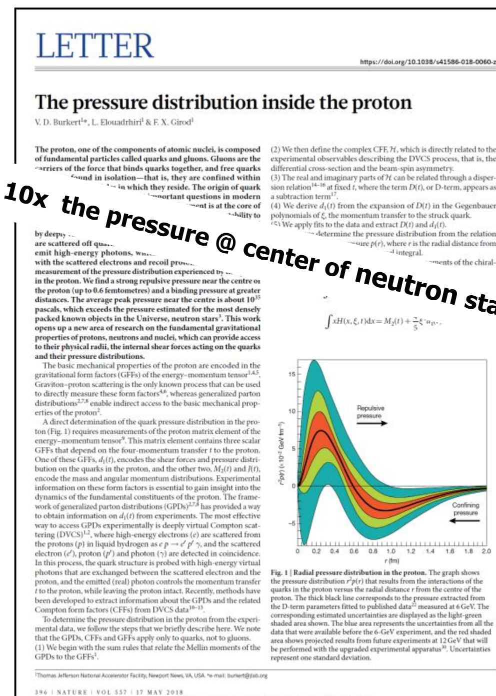
Finding 1: An EIC can uniquely address three profound questions about nucleons—neutrons and protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

Summary extract from a report by the U.S. National Academy of Sciences on the science of U.S.-based Electron-Ion Collider (July 24, 2018)

First experimental extraction of the pressure distribution inside the proton

[Burkert, Elouadrhiri, Girod, *Nature 557* (2018)]



LETTER

https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1*], L. Elouadrhiri[1] & F. X. Girod[1]

Fig. 1 | Radial pressure distribution in the proton.

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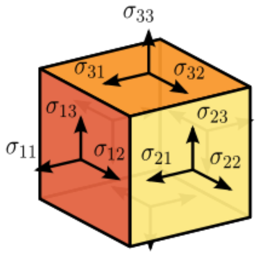
10x the pressure @ center of neutron stars!

Soon brand new data from JLab 12GeV and COMPASS II!

Energy-momentum tensor (EMT)

Mass, **spin** and **pressure** all encoded in



Energy density | **Momentum density**

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

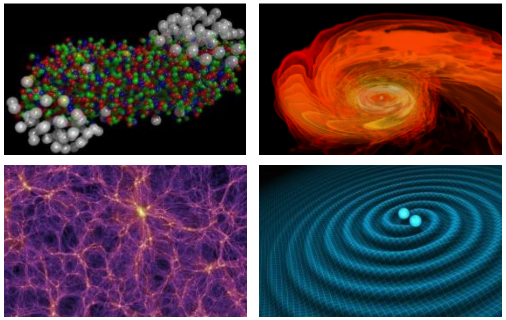
Shear stress

Normal stress **(pressure)**

Energy flux | **Momentum flux**

Key concept for

- **Nucleon mechanical properties**
- **Quark-gluon plasma**
- **Relativistic hydrodynamics**
- **Stellar structure and dynamics**
- **Cosmology**
- **Gravitational waves**
- **Modified theories of gravitation**
- **...**



Gravitational form factors (GFFs)

[Kobzarev, Okun (1962)]
[Pagels (1966)]
[Ji (1996)]
[Bakker, Leader, Trueman (2004)]
[Leader, C.L. (2014)]

Matrix elements $P = \frac{p' + p}{2}, \qquad \Delta = p' - p, \qquad t = \Delta^2$

$$\langle p', s'|T^{\mu\nu}(0)|p, s\rangle =$$

$$\overline{u}(p', s')\left[\frac{P^\mu P^\nu}{M}A(t) + \frac{P^{\{\mu}i\sigma^{\nu\}\lambda}\Delta_\lambda}{4M}(A+B)(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}C(t)\right.$$

$$\left. + \boxed{Mg^{\mu\nu}\bar{C}(t)} + \boxed{\frac{P^{[\mu}i\sigma^{\nu]\lambda}\Delta_\lambda}{4M}D(t)}\right]u(p, s)$$

Non-conservation

Intrinsic spin

Mass-shell conditions

$$p'^2 = p^2 = M^2 \quad \Longrightarrow \quad P^2 = M^2 - \frac{t}{4}, \qquad P\cdot\Delta = 0$$

Timelike ⟹ « frame »

Spacelike ⟹ « position »

Quantum chromodynamics (QCD)

Classical QCD energy-momentum tensor

$$T^{\mu\nu} = \overline{\psi}\gamma^{\mu}\,\tfrac{i}{2}\overleftrightarrow{D}^{\nu}\psi - G^{a\mu\alpha}G^{a\nu}{}_{\alpha} + \tfrac{1}{4}\,\eta^{\mu\nu}G^{2}$$

Renormalized trace of the QCD EMT

[Crewther (1972)]
[Chanowitz, Ellis (1972)]
[Nielsen (1975)]
[Adler, Collins, Duncan (1977)]
[Collins, Duncan, Joglekar (1977)]
[Nielsen (1977)]

$$T^{\mu}{}_{\mu} = \underbrace{\frac{\beta(g)}{2g}\,G^{2}}_{\text{Trace anomaly}} + (1+\gamma_m)\,\overline{\psi}m\psi$$

Quark mass matrix

Poincaré invariance

[Kobzarev, Okun (1962)]
[Teryaev (1999)]
[Brodsky, Hwang, Ma, Schmidt (2001)]
[Leader, C.L. (2014)]
[Teryaev (2016)]
[Lowdon, Chiu, Brodsky (2017)]

$$\partial_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \sum_{i=q,g} A_i(0) = 1, \qquad \sum_{i=q,g} \bar{C}_i(t) = 0$$

$$\partial_\mu \underbrace{J^{\mu\alpha\beta}}_{x^{\alpha}T^{\mu\beta} - x^{\beta}T^{\mu\alpha} + S^{\mu\alpha\beta}} = 0 \quad \Rightarrow \quad \sum_{i=q,g} B_i(0) = 0, \qquad D_q(t) = -G_A^q(t)$$

Textbook decomposition

Forward matrix element

$$\langle P|T^{\mu\nu}(0)|P\rangle = 2P^\mu P^\nu$$

$$\langle P'|P\rangle = 2P^0\,(2\pi)^3\,\delta^{(3)}(\vec{P}'-\vec{P})$$

Trace decomposition

$$2M^2 = \langle P|T^\mu{}_\mu(0)|P\rangle$$
$$= \underbrace{\langle P|\frac{\beta(g)}{2g}\,G^2|P\rangle}_{\sim 89\%} + \underbrace{\langle P|(1+\gamma_m)\,\overline{\psi}m\psi|P\rangle}_{\sim 11\%}$$

[Shifman, Vainshtein, Zakharov (1978)]
[Luke, Manohar, Savage (1992)]
[Donoghue, Golowich, Holstein (1992)]
[Kharzeev (1996)]
[Bressani, Wiedner, Filippi (2005)]
[Roberts (2017)]
[Krein, Thomas, Tsushima (2017)]

- ✓ **Manifestly covariant**
- ✓ **Compatible with Gell-Mann–Oakes–Renner formula for pion**

- ✗ **Depends on state normalization**
- ✗ **No spatial extension**
- ✗ **No clear relation to energy**

Ji's decomposition

[Ji (1995)]

Separation of quark and gluon contributions

$$T^{\mu\nu} = \underbrace{\bar{T}^{\mu\nu}}_{\text{Traceless}} + \underbrace{\hat{T}^{\mu\nu}}_{\text{Pure trace}} \qquad \bar{T}^{\mu\nu} = \bar{T}^{\mu\nu}_q + \bar{T}^{\mu\nu}_g \qquad \hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_m + \hat{T}^{\mu\nu}_a$$

Forward matrix elements

$$\langle P|\bar{T}^{\mu\nu}_i(0)|P\rangle = 2\left(P^\mu P^\nu - \tfrac{1}{4}\,\eta^{\mu\nu}M^2\right) A_i(0)$$
$$\langle P|\hat{T}^{\mu\nu}_i(0)|P\rangle = \tfrac{1}{2}\,\eta^{\mu\nu}M^2\left[A_i(0) + 4\bar{C}_i(0)\right]$$

$$\langle O\rangle = \frac{\langle P|\int \mathrm{d}^3r\, O(r)|P\rangle}{\langle P|P\rangle}$$

Ji's decomposition

[Gao *et al.* (2015)]

$$M = \underbrace{M_q}_{\sim 31\%} + \underbrace{M_g}_{\sim 34\%} + \underbrace{M_m}_{\sim 13\%} + \underbrace{M_a}_{\sim 22\%}$$

$\mu = 2\,\mathrm{GeV}$

$$M_q = \langle \bar{T}^{00}_q\rangle|_{\vec{P}=\vec{0}} - \frac{3}{1+\gamma_m}\,\langle \hat{T}^{00}_m\rangle|_{\vec{P}=\vec{0}}$$
$$M_g = \langle \bar{T}^{00}_g\rangle|_{\vec{P}=\vec{0}}$$
$$M_m = \frac{4+\gamma_m}{1+\gamma_m}\,\langle \hat{T}^{00}_m\rangle|_{\vec{P}=\vec{0}}$$
$$M_a = \langle \hat{T}^{00}_a\rangle|_{\vec{P}=\vec{0}}$$

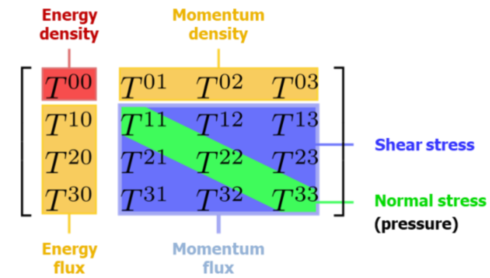
- ✓ **Proper normalization**
- ✓ **Clear relation to energy distribution**
- ! **Scale-dependent interpretation in the rest frame**
- ✗ **Pressure effects not taken into account**

New decomposition

[C.L. (2018)]

Forward matrix element

$$\langle P|T_i^{\mu\nu}(0)|P\rangle = 2P^\mu P^\nu A_i(0) + 2M^2\eta^{\mu\nu}\bar{C}_i(0)$$



Analogy with relativistic hydrodynamics

Perfect fluid element

$$\Theta_i^{\mu\nu} = (\varepsilon_i + p_i)u^\mu u^\nu - p_i\,\eta^{\mu\nu}$$

Four-velocity

$$u^\mu = P^\mu/M$$

Energy density

$$\varepsilon_i = [A_i(0) + \bar{C}_i(0)]\,\frac{M}{V}$$

Isotropic pressure

$$p_i = -\bar{C}_i(0)\,\frac{M}{V}$$

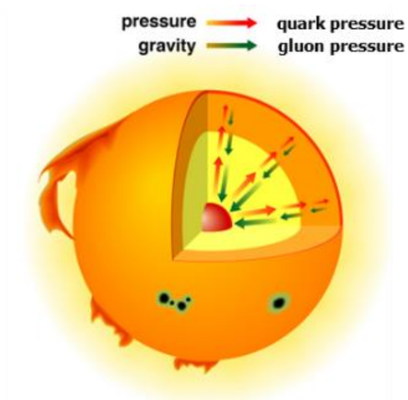
Nucleon mass decomposition $U_i = \varepsilon_i V$

$$M = \underbrace{U_q}_{\sim 44\%} + \underbrace{U_g}_{\sim 56\%}$$

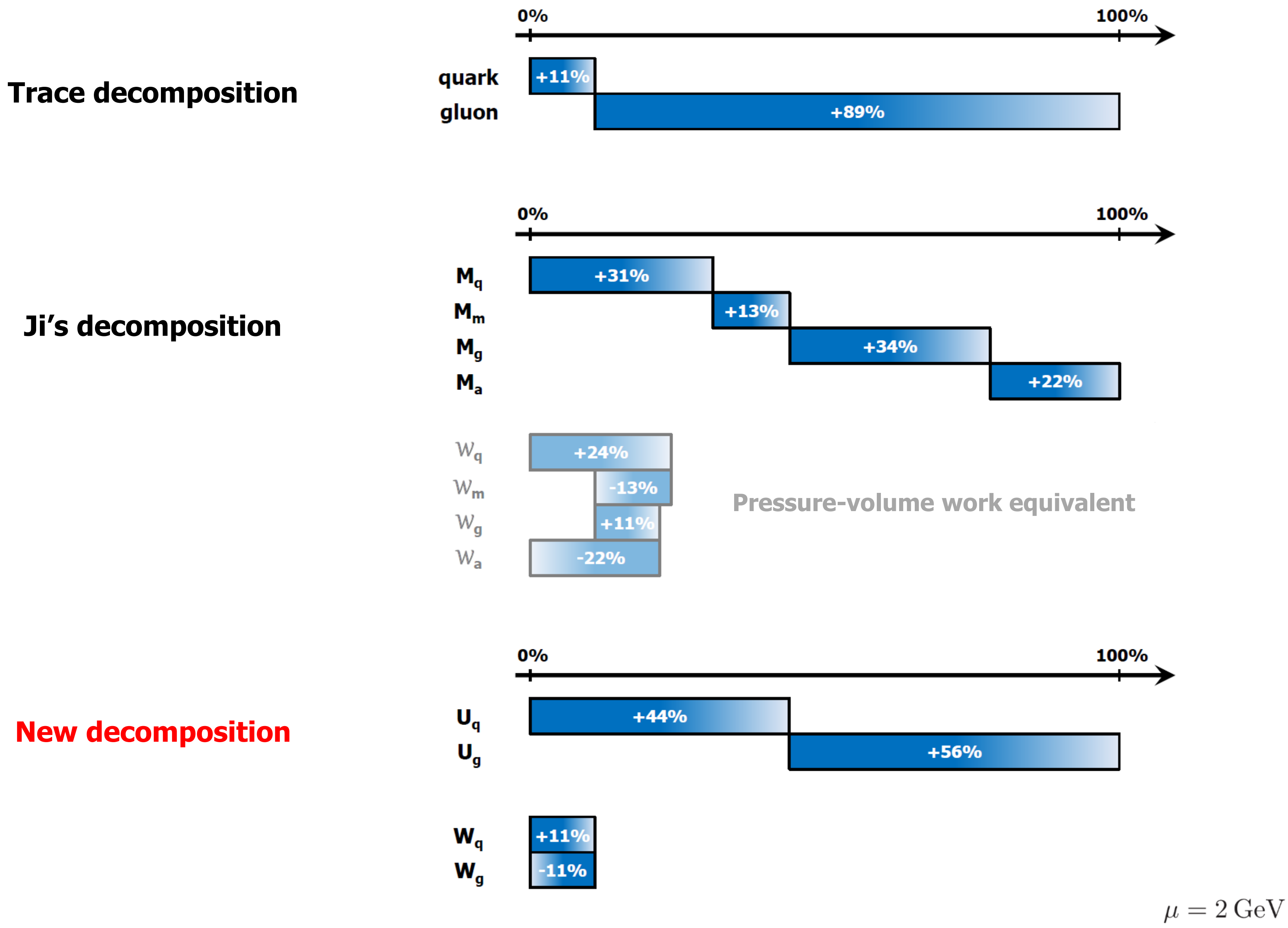
$$p_q = -p_g$$

~ 11%

$\mu = 2\,\mathrm{GeV}$

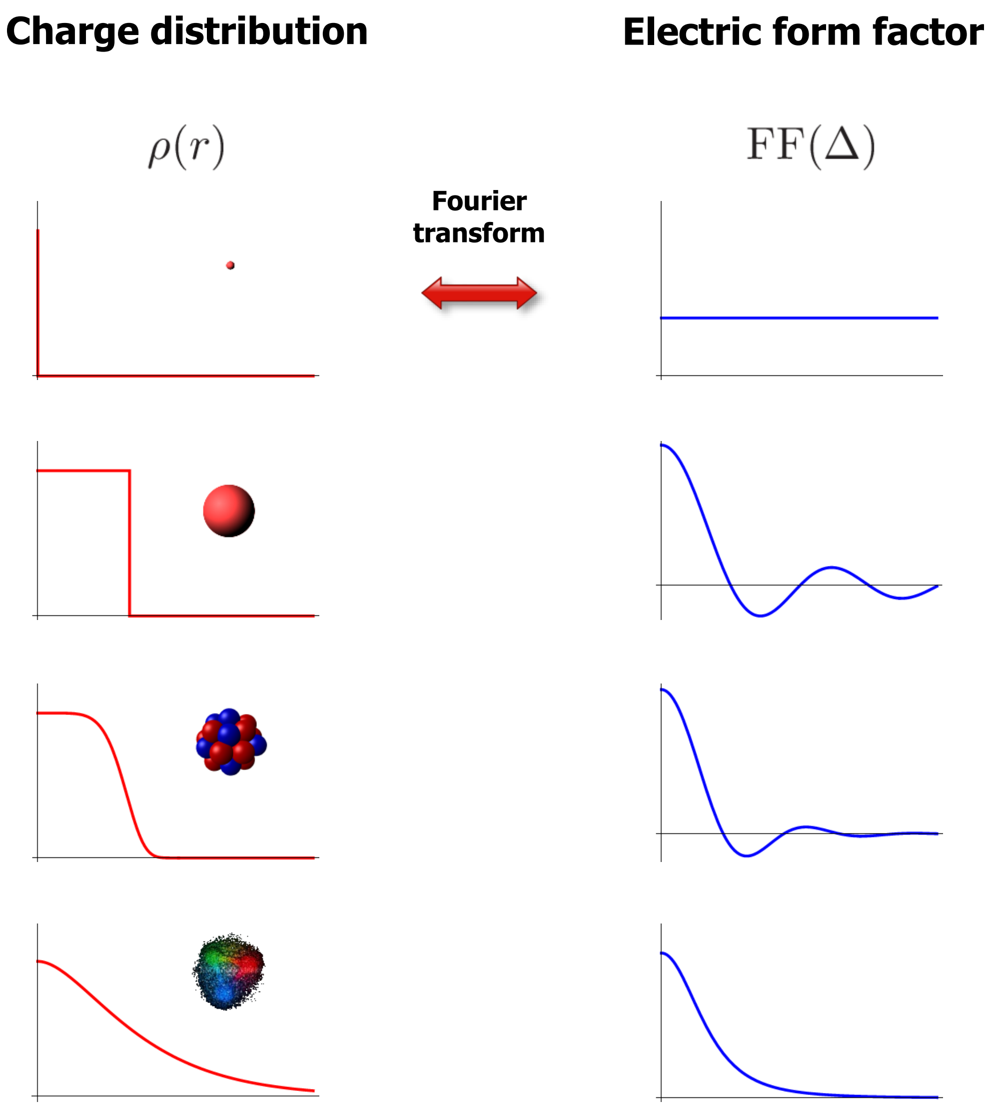


In short



Trace decomposition
Ji's decomposition
New decomposition
0%
100%
quark
gluon
+11%
+89%
M_q
M_m
M_g
M_a
+31%
+13%
+34%
+22%
W_q
W_m
W_g
W_a
+24%
-13%
+11%
-22%
Pressure-volume work equivalent
U_q
U_g
+44%
+56%
W_q
W_g
+11%
-11%
$\mu = 2\,\mathrm{GeV}$

Spatial information

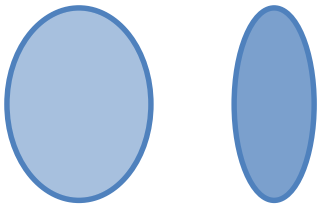


3D distribution in Breit frame

Lorentz factors

$$p^0 = \gamma M, \qquad p'^0 = \gamma' M$$

Initial ψ **Final** ψ'^*

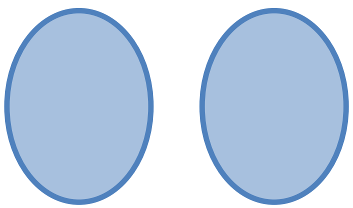


$$\Delta^0 \neq 0$$

Breit frame

$$\xrightarrow{\vec{p}\,'}$$
$$\xleftarrow[\vec{p}]{}$$

$$\vec{P} = \vec{0} \quad \Rightarrow \quad \Delta^0 = \frac{\vec{P}\cdot\vec{\Delta}}{P^0} = 0$$



$$\Delta^0 = 0$$

$$\rho \sim \psi^* \psi$$

3D distribution

$$T_i^{\mu\nu}(\vec{r}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3 2P^0}\, e^{-i\vec{\Delta}\cdot\vec{r}}\, \langle \tfrac{\vec{\Delta}}{2} | T_i^{\mu\nu}(0) | -\tfrac{\vec{\Delta}}{2} \rangle$$

Anisotropic medium

[Polyakov (2003)]
[Goeke *et al.* (2007)]
[Polyakov, Schweitzer (2018)]
[C.L., Moutarde, Trawinski, arXiv:1810.xxxxx]

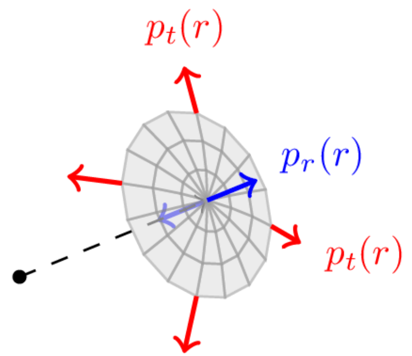
Breit frame amplitude $t = -\vec{\Delta}^2$

$$\frac{\langle \frac{\vec{\Delta}}{2}|T_i^{\mu\nu}(0)|-\frac{\vec{\Delta}}{2}\rangle}{2P^0} = M\left\{\eta^{\mu 0}\eta^{\nu 0}\left[A_i(t) + \frac{t}{4M^2}\,B_i(t)\right] + \eta^{\mu\nu}\left[\bar{C}_i(t) - \frac{t}{M^2}\,C_i(t)\right] + \frac{\Delta^\mu\Delta^\nu}{M^2}\,C_i(t)\right\}$$

Analogy with relativistic hydrodynamics $r = |\vec{r}|$

Anisotropic fluid

$$\Theta_i^{\mu\nu}(\vec{r}) = [\varepsilon_i(r) + p_{t,i}(r)]\,u^\mu u^\nu - p_{t,i}(r)\eta^{\mu\nu} + [p_{r,i}(r) - p_{t,i}(r)]\,\frac{r^\mu r^\nu}{r^2}$$



Isotropic pressure

$$p_i(r) = \frac{p_{r,i}(r) + 2p_{t,i}(r)}{3}$$

Pressure anisotropy

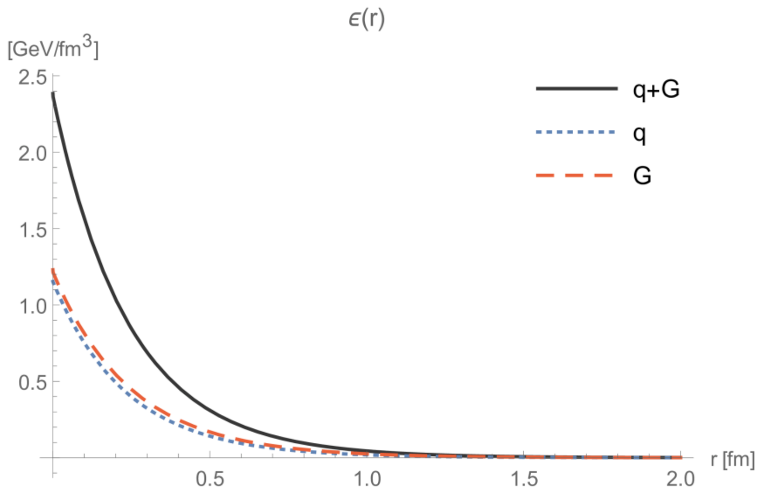
$$s_i(r) = p_{r,i}(r) - p_{t,i}(r)$$

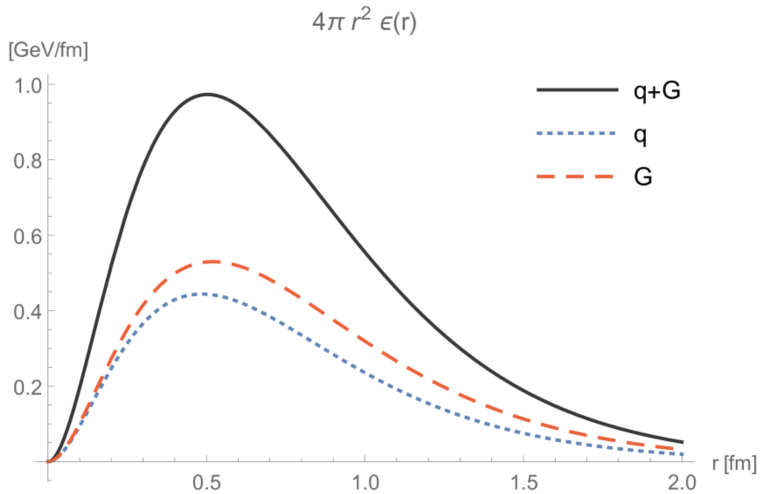
Energy distribution

[C.L., Moutarde, Trawinski, arXiv:1810.xxxxx]

Multipole model for the GFFs

$$F(t) = \frac{F(0)}{(1 + t/\Lambda^2)^n}$$

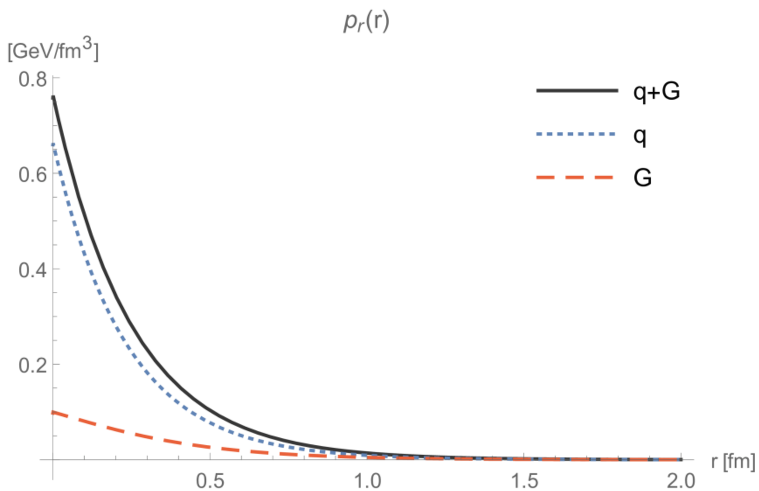


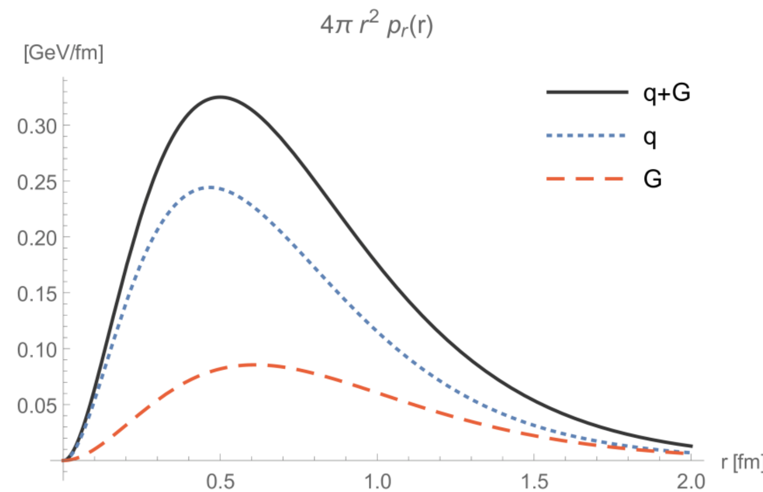


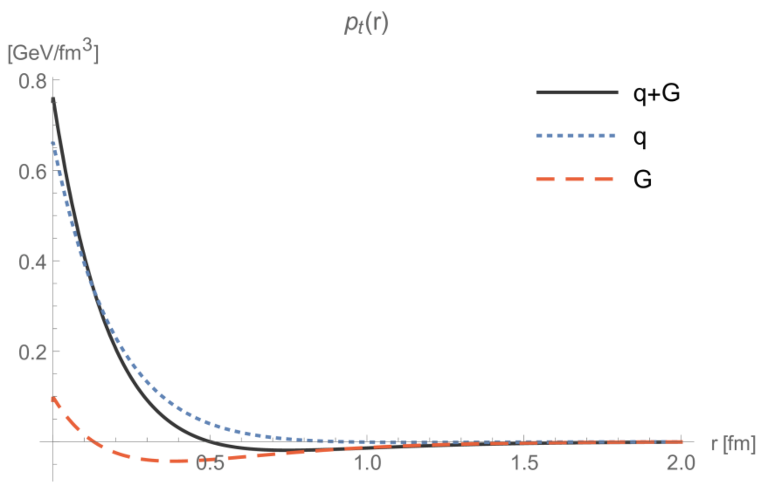
$$\sqrt{\langle r^2\rangle_M} = 0.91\,\mathrm{fm}$$
$$\sqrt{\langle r^2\rangle_Q} = 0.84 - 0.88\,\mathrm{fm}$$

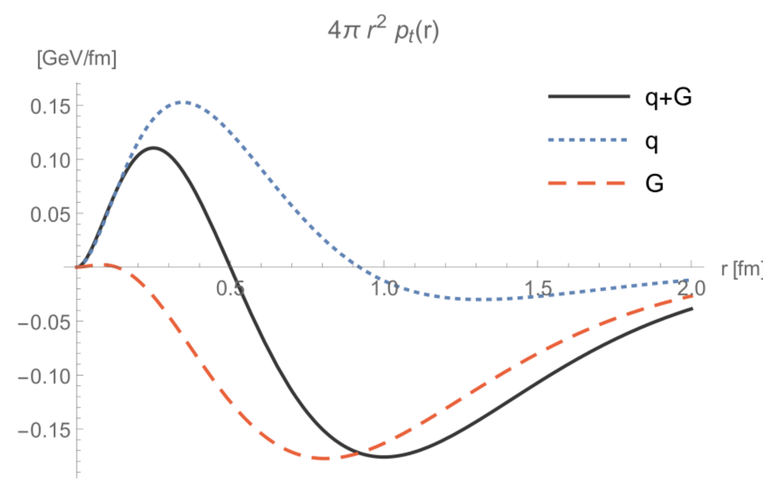
Pressure distribution

[C.L., Moutarde, Trawinski, arXiv:1810.xxxxx]



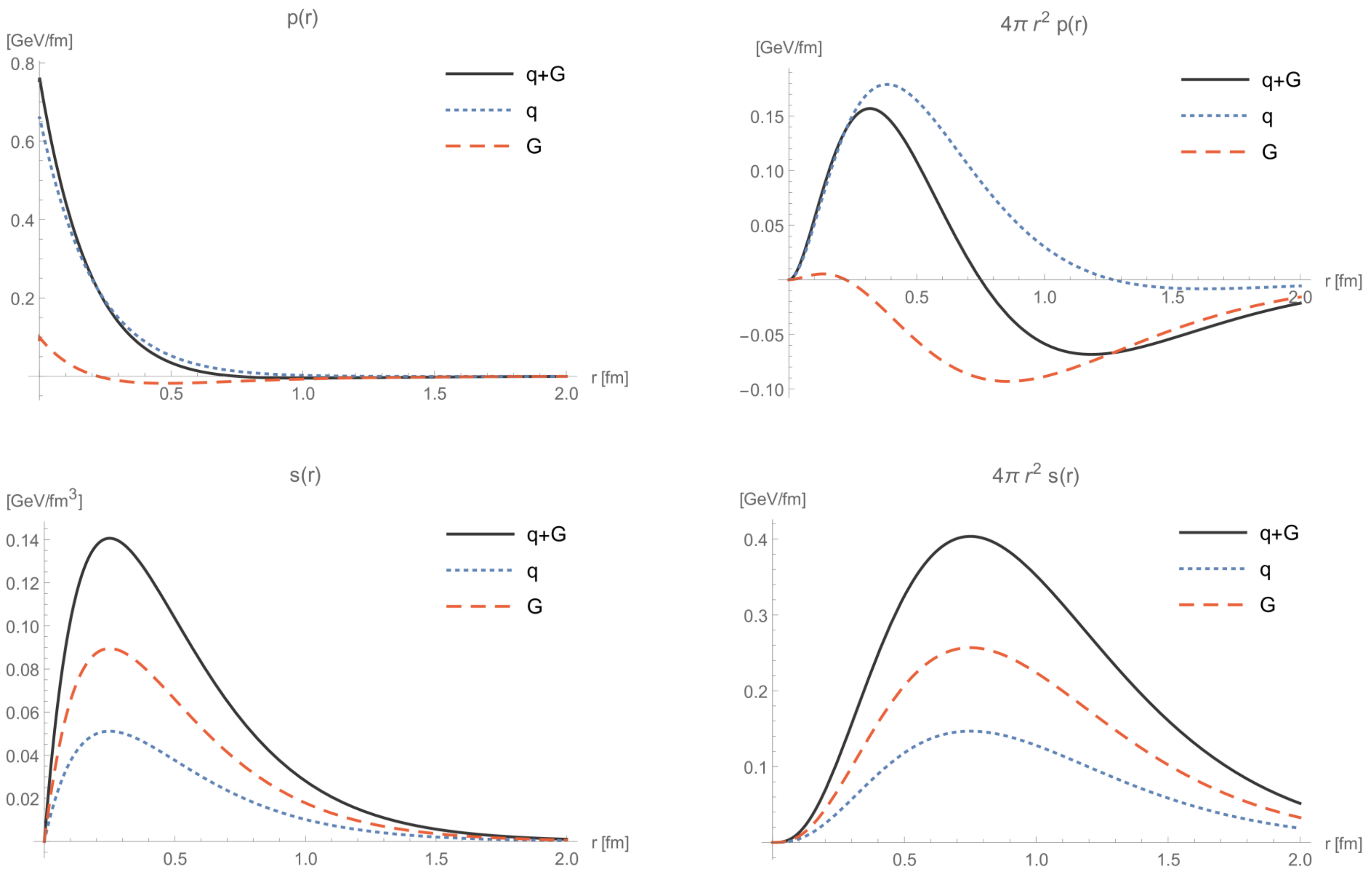






Pressure distribution

[C.L., Moutarde, Trawinski, arXiv:1810.xxxxx]



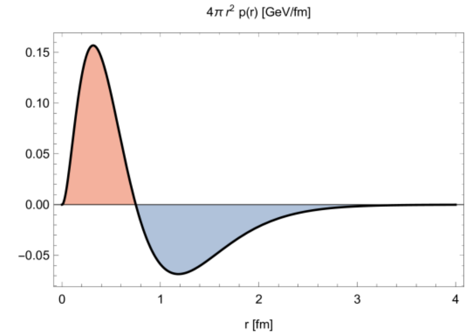
Hydrostatic equilibrium

[Polyakov (2003)]
[Goeke *et al.* (2007)]
[Polyakov, Schweitzer (2018)]
[C.L., Moutarde, Trawinski, arXiv:1810.xxxxx]

$$\nabla^i T^{ij}(\vec{r}) = 0 \quad \Rightarrow \quad \frac{\mathrm{d}p_r(r)}{\mathrm{d}r} = -\frac{2s(r)}{r}$$

von Laue relation [von Laue (1911)]

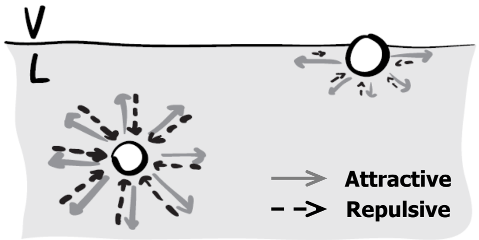
$$\int_0^\infty \mathrm{d}r\, r^2\, p(r) = 0$$



Surface tension

[Bakker (1928)]
[Kirkwood, Buff (1949)]
[Marchand *et al.* (2011)]

$$\gamma = \int \mathrm{d}r\, s(r)$$



Generalized Young-Laplace relation

[Thomson (1858)]

$$p(0) = 2\int_0^\infty \mathrm{d}r\, \frac{s(r)}{r}$$

$$s(r) \approx \gamma\, \delta(r-R) \quad \Rightarrow \quad p(0) = \frac{2\gamma}{R}$$

Compact stars

White dwarfs, neutron stars, quark stars, strange stars, gravastars, black holes, …

Highest energy densities and strongest gravitational fields!

Tests under extreme conditions

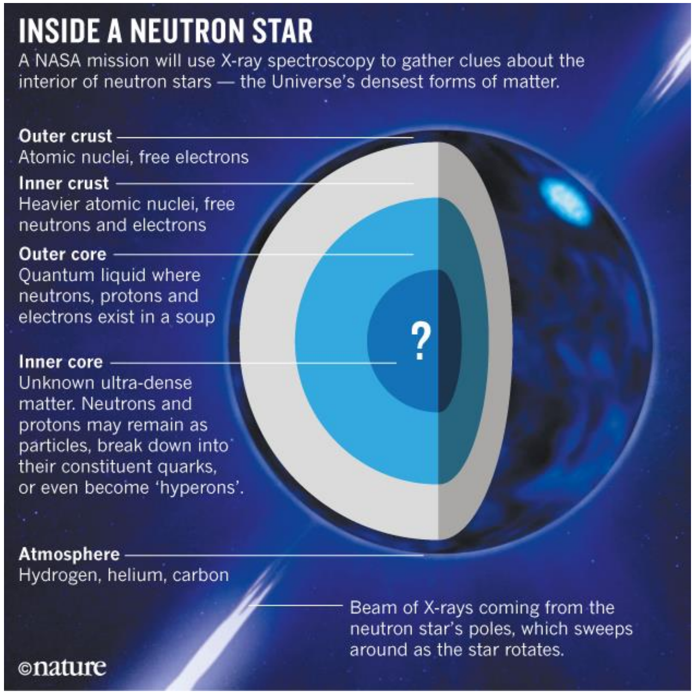
- Nuclear matter
- General relativity & alternatives

[Berti *et al.* (2015)]
[Lattimer, Prakash (2016)]

EMT is likely anisotropic

- Relativistic nuclear interactions
- Mixture of fluids of different types
- Presence of superfluid
- Existence of solid core
- Phase transitions
- Presence of magnetic field
- Viscosity
- ...

[Ruderman (1972)]
[Canuto (1974)]
[Bowers, Liang (1974)]
[Herrera, Santos (1997)]



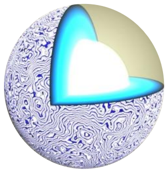
$$M \sim 1.4\, M_\odot \qquad M_\odot = 2\times 10^{33}\,\mathrm{g}$$
$$R \sim 10\,\mathrm{km}$$
$$\rho \sim 3\,\rho_0 \qquad \rho_0 = 2.8\times 10^{14}\,\mathrm{g/cm^3}$$
$$g \sim 2.4\times 10^{12}\,\mathrm{m/s^2}$$

[Potekhin (2010)]

$$M_p \sim 1.67\times 10^{-24}\,\mathrm{g}$$
$$R_p \sim 0.84\,\mathrm{fm}$$
$$\rho_p \sim 2.4\,\rho_0$$

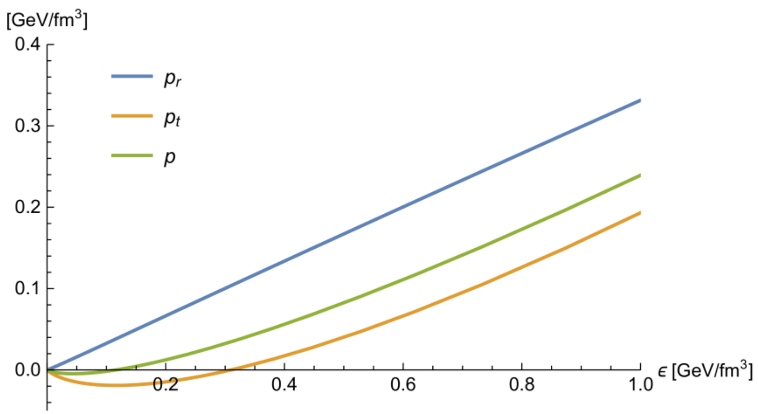
What can we learn?

Nucleon

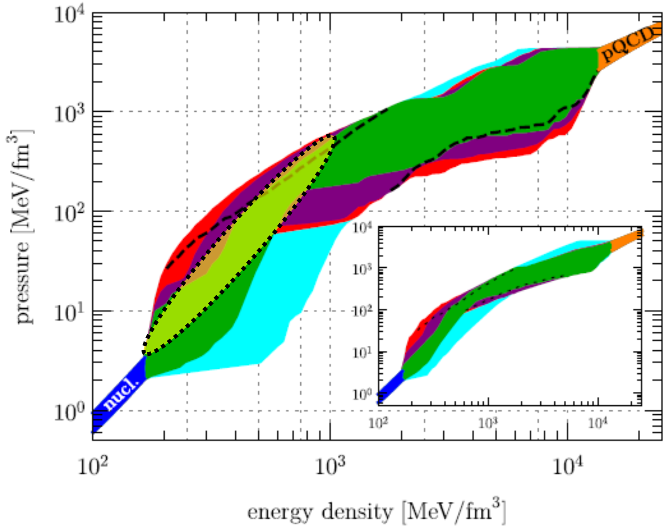


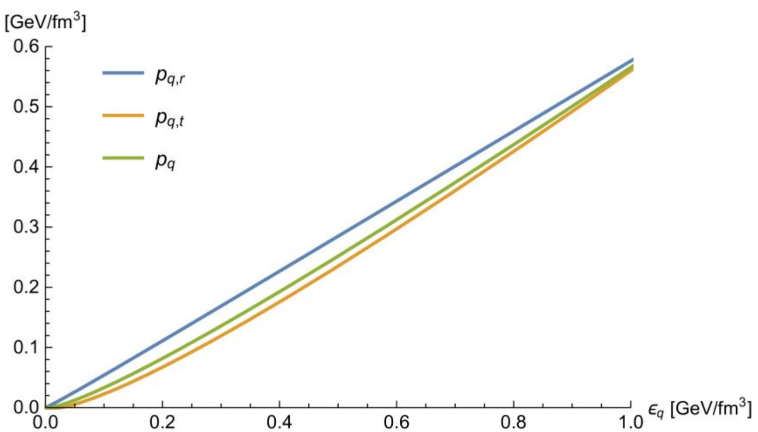
Neutron star

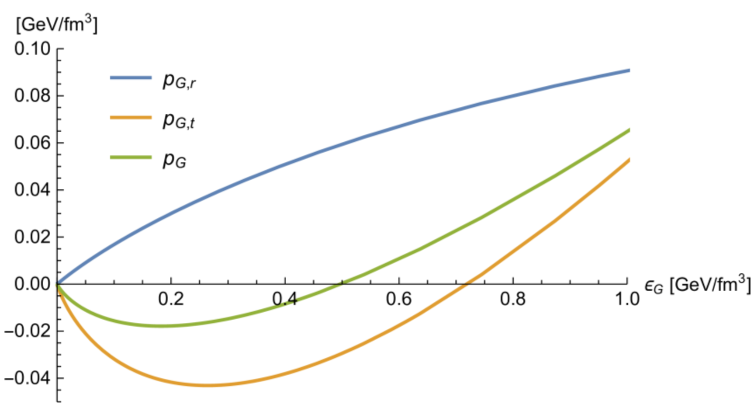
Equation of state



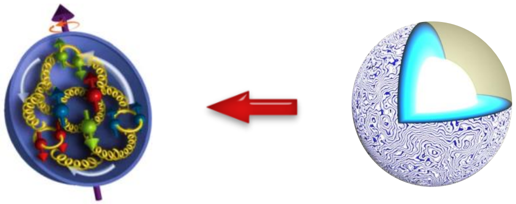
[Annala *et al.* (2018)]







What can we learn?

Nucleon  Neutron star

Stability constraints

[Wald (1984)]
[Herrera, Santos (1997)]
[Poisson (2004)]
[Abreu, Hernandez, Nunez (2007)]
[Hawking, Ellis (2011)]

Mechanical regularity

(i) $\varepsilon(0) < \infty$, $p(0) < \infty$ and $s(0) = 0$;

(ii) $\varepsilon(r) > 0$ and $p_r(r) > 0$;

(iii) $\frac{\mathrm{d}\varepsilon(r)}{\mathrm{d}r} < 0$ and $\frac{\mathrm{d}p_r(r)}{\mathrm{d}r} < 0$.

Speed of sound

(iv) $0 \leq v_{sr}^2(r) \leq 1$ and $0 \leq v_{st}^2(r) \leq 1$;

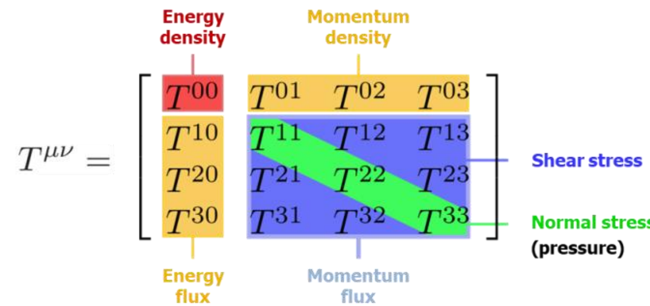
(v) $|v_{st}^2(r) - v_{sr}^2(r)| \leq 1$;

(vi) $\Gamma(r) = \frac{\varepsilon(r)+p_r(r)}{p_r(r)}\, v_{sr}^2 > \frac{4}{3}$.

Energy conditions

$$
\begin{aligned}
&\varepsilon(r) + p_i(r) \geq 0, \\
&\varepsilon(r) + p_i(r) \geq 0 \quad \text{and} \quad \varepsilon(r) \geq 0, \\
&\varepsilon(r) + p_i(r) \geq 0 \quad \text{and} \quad \varepsilon(r) + 3\,p(r) \geq 0, \\
&\qquad \varepsilon(r) \geq |p_i(r)|,
\end{aligned}
$$

Summary



- **Mass, spin and pressure all encoded in EMT**

- **Nucleon energy density and pressure are extremely high**

  → **Large anisotropy**

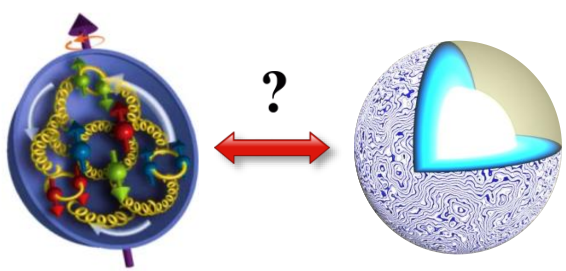
- **Same ballpark as interior of compact stars**

- **Exciting cross-talk between hadronic physics and neutron star physics!**

  → **Equation of state, stability constraints, ...**

- **EIC will play a key role**

  → **Gluon contributions, imaging, target versatility, ...**



Save the date!

Backup slides

Gravitational form factors

Mellin moment of twist-2 vector GPDs $\langle p', s'|T^{++}(0)|p, s\rangle$

$$\int \mathrm{d}x\, x\, H(x,\xi,t) = A(t) + 4\xi^2 C(t)$$

$$\int \mathrm{d}x\, x\, E(x,\xi,t) = B(t) - 4\xi^2 C(t)$$

[Ji (1996)]

Poincaré covariance

$$\langle p', s'|T_q^{[\alpha\beta]}(0)|p, s\rangle = -i\Delta_\mu \langle p', s'|S_q^{\mu\alpha\beta}(0)|p, s\rangle$$

$$D_q(t) = -G_A^q(t)$$

[C.L., Mantovani, Pasquini (2018)]

EMT trace

[Ji (1995)]
[C.L. (2018)]

$$\langle p', s'|G^2(0)|p, s\rangle, \quad \langle p', s'|\overline{\psi}(0)\psi(0)|p, s\rangle \quad \longrightarrow \quad \bar{C}(t)$$

Recent Lattice QCD results

Mass

Spin

Momentum

[Yang *et al.*, arXiv:1808.08677]

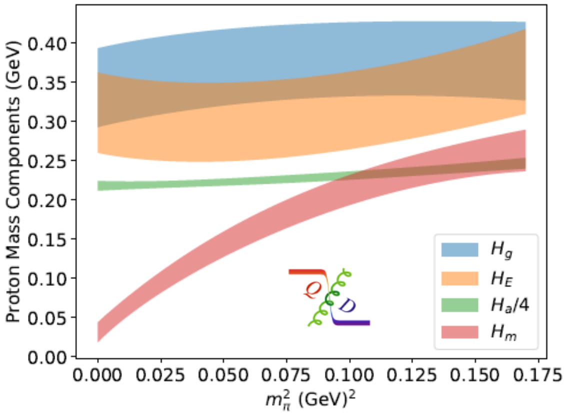


FIG. 3. The valence pion mass dependence of the proton mass decomposition in terms of the quark condensate ($\langle H_m \rangle$), quark energy $\langle H_E \rangle$, glue field energy $\langle H_g \rangle$ and trace anomaly $\langle H_a \rangle/4$.

[Alexandrou *et al.*, arXiv:1807.11214]

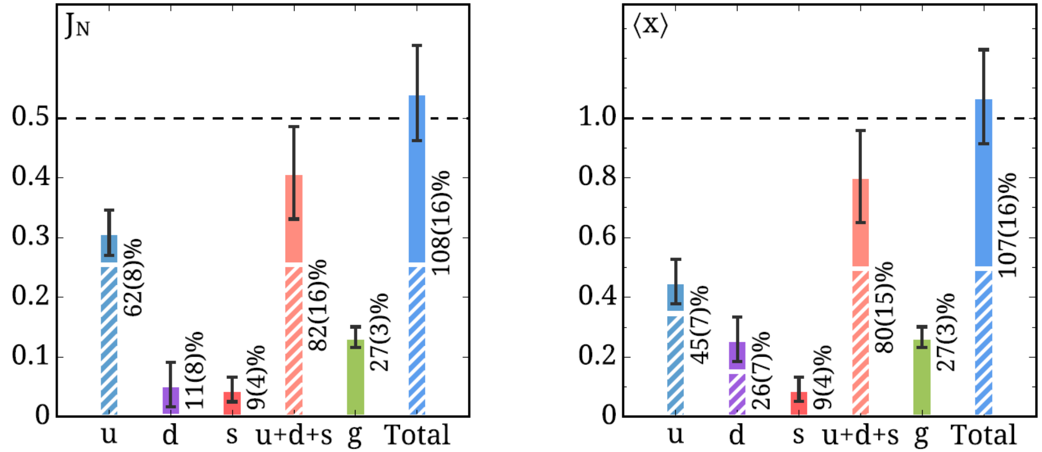


Figure 2: Left: Nucleon spin decomposition. Right: Nucleon momentum decomposition. The striped segments show valence quark contributions (connected) and the solid segments the sea quark and gluon contributions (disconnected). Results are given in $\overline{\text{MS}}$-scheme at 2 GeV.