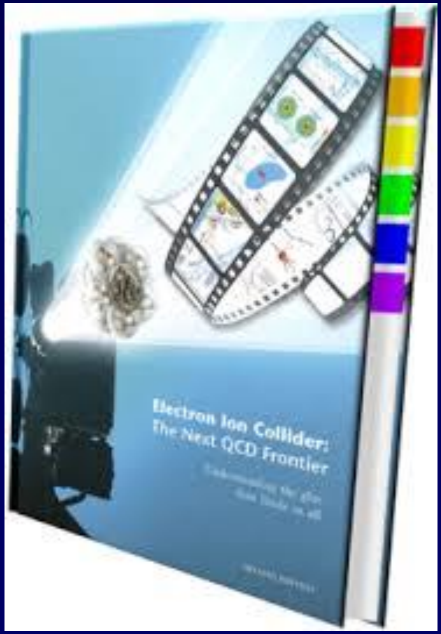
Proton Mass and Spin Decomposition

- Quark spin – proton spin crisis
- Anomalous Ward Identity, Goldberger-Treiman relation and renormalization
- Cluster decomposition error reduction
- Energy-momentum tensor -- proton spin and mass decomposition
- Renormalization of glue operators

χ QCD Collaboration



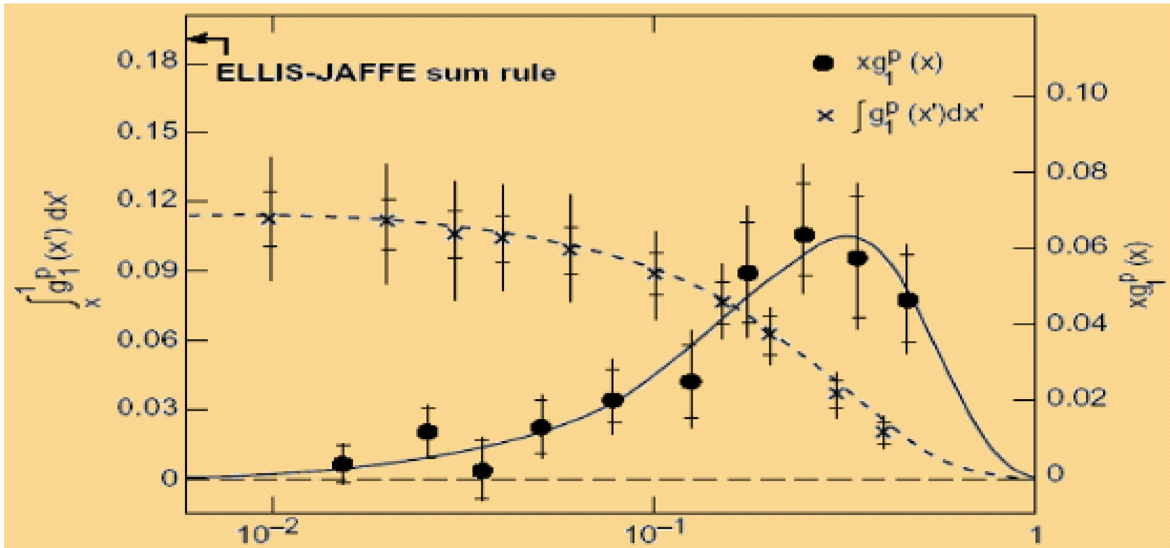
INT, Oct. 16, 2018

Where does the spin of the proton come from?

Quark spins according to the quark model

Twenty 9 years since the “spin crisis”

- EMC experiment in 1988/1989 – “the plot”:



$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

$$\Delta q = \int_0^1 dx\,\Delta q(x) = \langle P, s_{||}|\overline{\psi}_q(0)\gamma^+\gamma_5\psi_q(0)|P, s_{||}\rangle$$

- “Spin crisis” or puzzle: $\mathrm{DS} = \sum_q \mathrm{D}q + \mathrm{D}\overline{q} \sim 0.3$

Proton Spin Crisis

- What's wrong with the quark model?
- Mixture from the glue spin?

Anomalous Ward Identity

$$\partial_\mu A_\mu^0 = i2 \sum_{i=u.d.s} m_i P_i - \frac{iN_f}{8\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu}$$

Take

$$q(x) = \frac{1}{16\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu} = \partial_\mu K_\mu, \quad K_\mu = \frac{1}{8\pi^2}\varepsilon_{\mu\nu\rho\sigma} tr[A_\nu(\partial_\rho A_\sigma + \frac{2}{3}A_\rho A_\sigma)]$$

$$\partial_\mu (A_\mu^0 + 2iN_f K_\mu) = i2 \sum_{i=u.d.s} m_i P_i$$

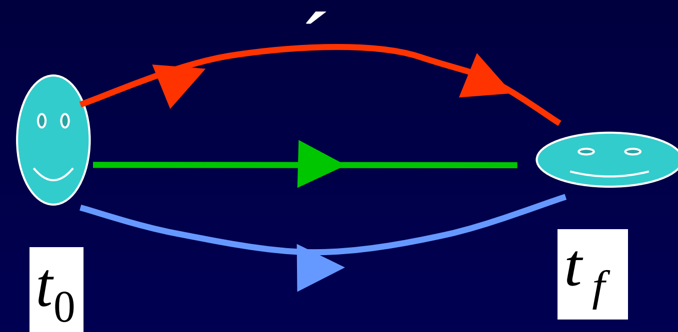
However, the Chern-Simons current is not gauge invariant.

- ` Proton spin crisis is the graveyard of all hadronic models'

Lattice Calculations of Quark and Glue Spins

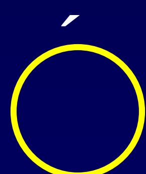
- Quark and Glue Momentum and Angular Momentum in the Nucleon

$(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d)(t)$



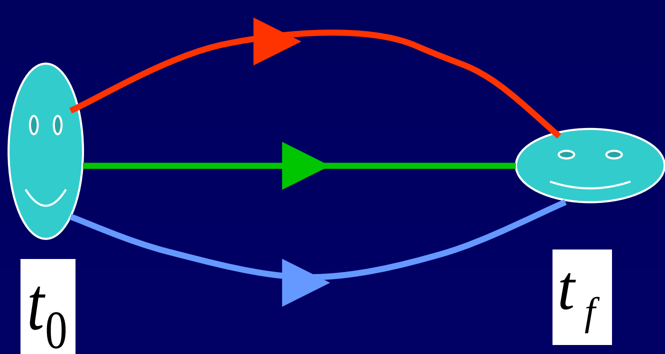
Connected insertion (CI)

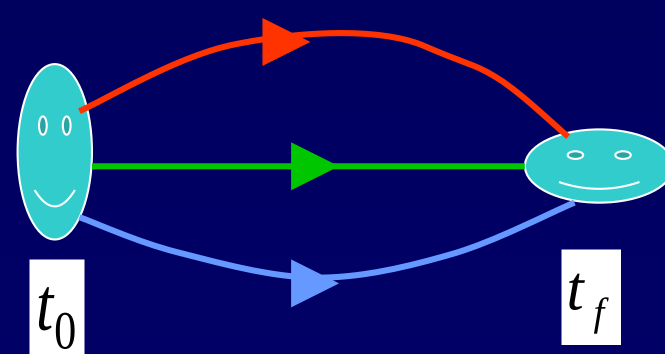
Disconnected insertion (DI)



$\bar{\Psi}\gamma_\mu D_\nu \Psi(t)(u,d,s)$

$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$





Quark Spin and Anomalous Ward Identify

- Calculation with a local singlet axial-vector current needs a normalization.
- AWI needs to be satisfied. $\partial_\mu A_\mu^0 = i2mP - \frac{iN_f}{8\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu}$
- Normalized AWI for overlap fermion for local current

$$\kappa_A \partial_\mu A_\mu^0 = i2mP - iN_f 2q(x)$$

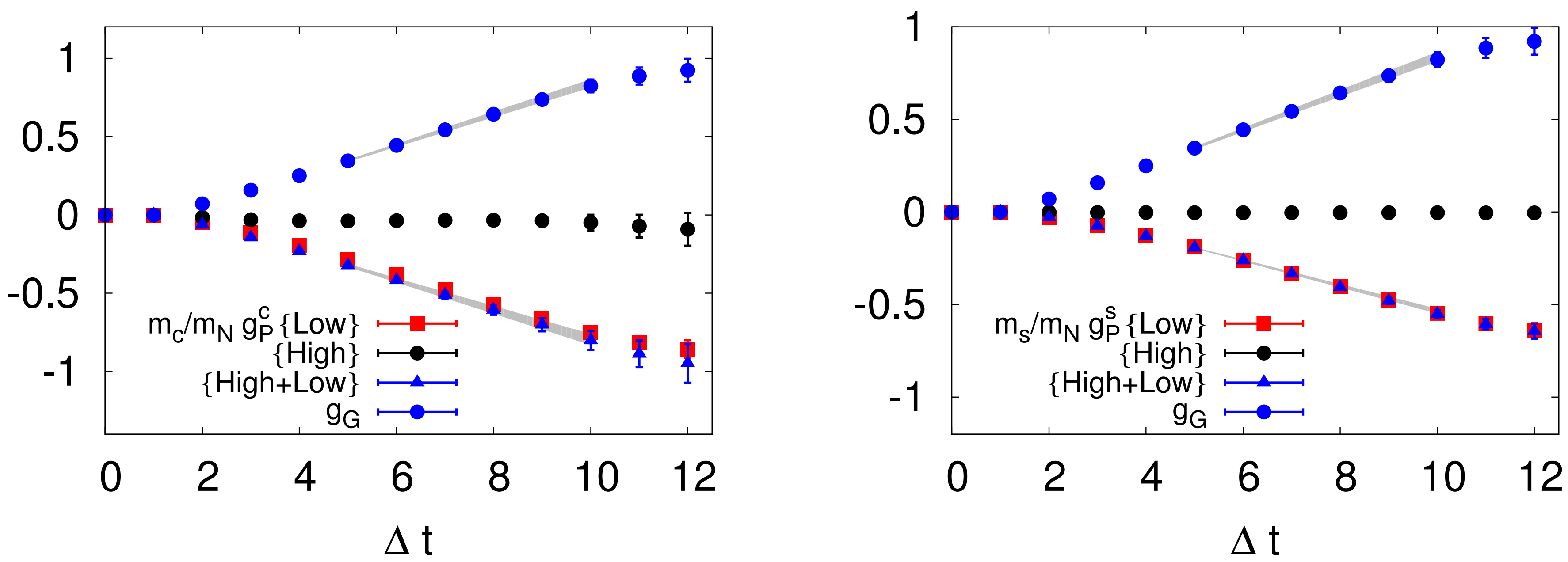
Renormaliztion and mixing:

$$Z_A^0 \kappa_A \partial_\mu A_\mu^0 = i2Z_m m Z_P P - iN_f 2(Z_q q(x) + \lambda \partial_\mu A_\mu^0)$$

  - Overlap fermion --> mP is RGI ($Z_m Z_P$=1)
  - Overlap operator for $q(x) = -1/2\, \mathrm{Tr}\gamma_5 D_{ov}(x,x)$ has no multiplicative renormalization.
  - Espriu and Tarrach (1982) $Z_A^0(2-\text{loop}) = 1 - \left(\frac{\alpha_s}{\pi}\right)^2 \frac{3}{8} C_2(R) N_f \frac{1}{\varepsilon},$

$$\lambda = -\left(\frac{\alpha_s}{\pi}\right)^2 \frac{3}{16} C_2(R) \frac{1}{\varepsilon}$$

Low-mode average for quark loop

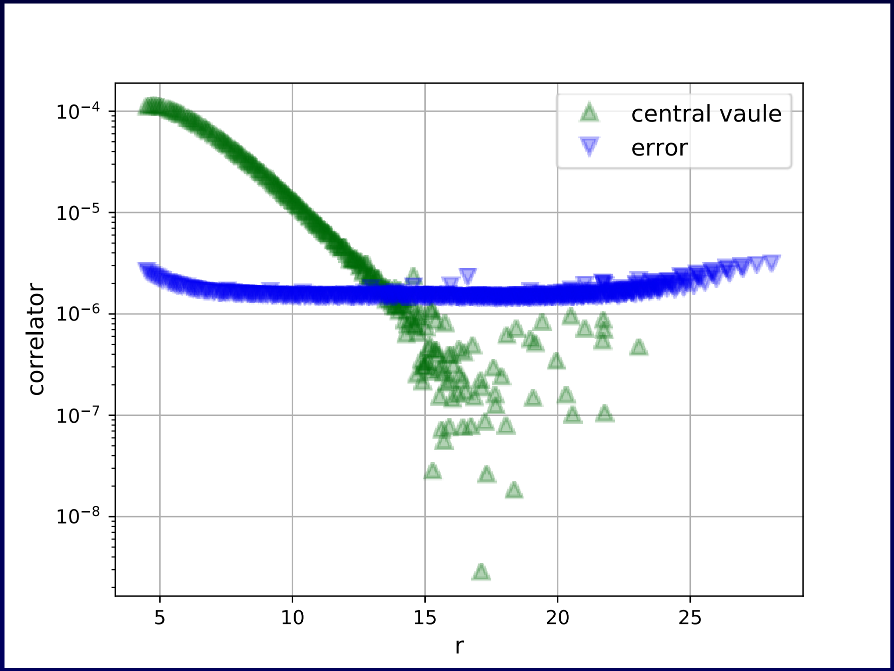


$$R(\Delta t, q^2) = \frac{\sum_{t_i+1}^{t_f-1} G_{NON}(t, \Delta t, \vec{p}, \vec{q})}{G_{NN}(\Delta t, \vec{p})} = \text{const.} + \Delta\text{t} \langle \vec{p}' s \,|\, O \,|\, \vec{p} s \rangle \frac{i\,|\vec{s}\,|}{\vec{q}\cdot\vec{s}}$$

$$\text{Tr loop} = \text{Tr} \sum_n \frac{f_n^\dagger O f_n}{m + i l_n} + \text{Tr}\; h_H^\dagger O D^{-1} h_H$$

Cluster Decomposition Error Reduction

Noise Problem with Large Volume



- Single falls off exponentially, but noise remains constant
  → sign problem, such as in glueball mass.
- nEDM with the θ term is noisy for large volume, because the topological charge fluctuation as $(V)^{1/2}$.

Cluster Decomposition Principle

H. Araki, K. Hepp, and D. Ruelle, Helv. Phys. Acta 35, 164 (1962);
S. Weinberg, Quantum Theory of Fields,Vol 1, pp. 169

$\left|\langle 0 | O_1(x) O_2(y) | 0 \rangle\right|_s$ £ $Ar^{-3/2} e^{-Mr}$, r = x-y (space like),

O_1 and O_2 are color-singlet operators,

Asympotic behavior of a boson propagator $K_1(r)/r$.

This point-to-point relation should hold for color-singlet operators separated by large Euclidean distance.

Variance Reduction via Cluster Decomposition -- Disconnected Insertions

- Variance of disconnected insertion
- Vacuum insertion

KFL, J. Liang, Y.B. Yang, arXiv:1706.06358

$$\mathrm{Var}(R,t) = \frac{1}{V^2} \sum_{\vec{x},\vec{y}} \left(\left\langle \sum_{r_1<R} O_1(\vec{x}+\vec{r}_1{}',t) \sum_{r_2<R} O_1^\dagger(\vec{y}+\vec{r}_2{}',t) \right\rangle \times \left\langle O_2(\vec{x},0) O_2^\dagger(\vec{y},0) \right\rangle \right) + \ldots$$

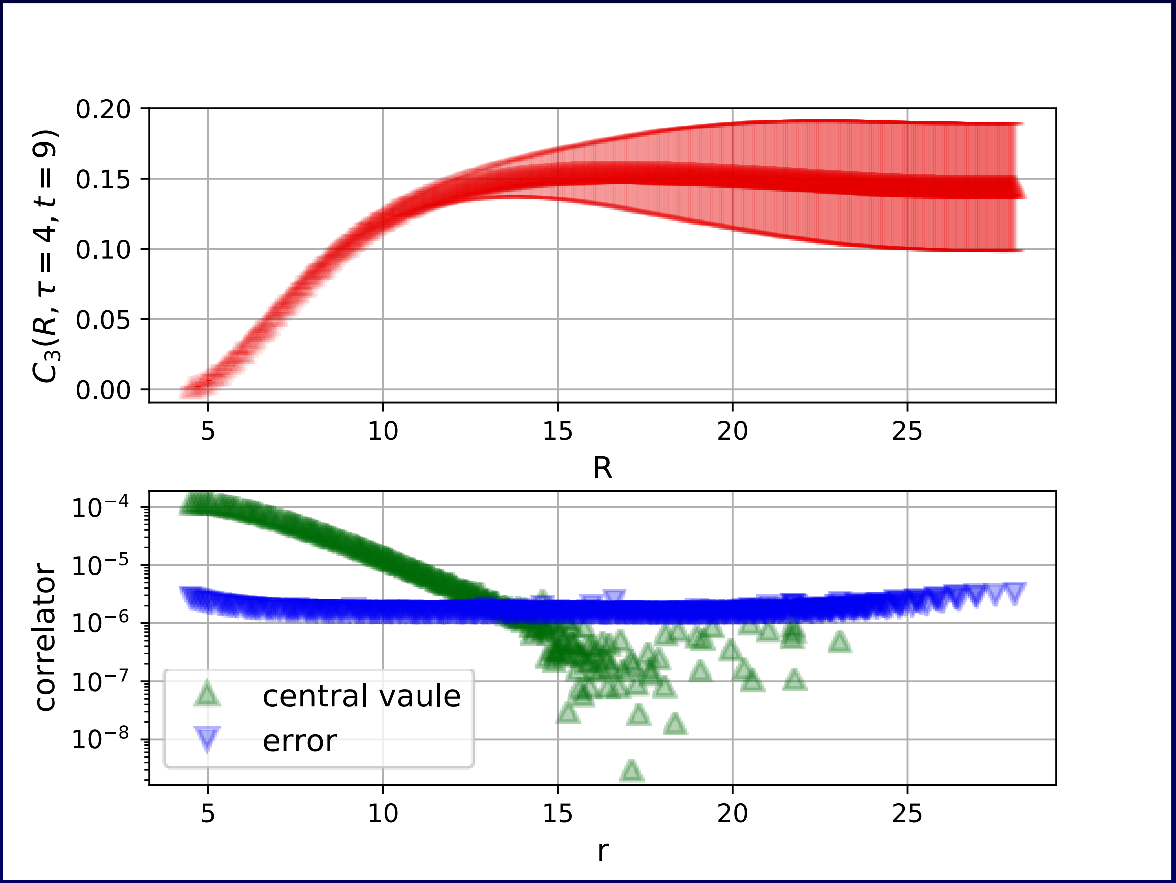
- $\mathrm{Var}(R_{max},t)$ =1 (indep of t), but $\mathrm{Var}(R_s,t) = V_s/V$.
- Gains singal to noise ratio:

$$\frac{\mathrm{S/N}(R_S,t)}{\mathrm{S/N}(L,t)} = \sqrt{\frac{V}{V_S}}$$

- Fast Fourier transform to calculate the truncated sum in relative coordinates ~ V log V operations.

Strangeness in the Nucleon

$$C_3(R,\tau,t) = \left\langle \sum_{\vec{x}} \sum_{r<R} O_N(\vec{x},t) S(\vec{x}+\vec{r}\,',\tau) \bar{O}_N(\text{grid},0) \right\rangle, \quad \text{r} = \sqrt{(\vec{r}_x - \vec{x})^2 + (\tau - t)^2}$$



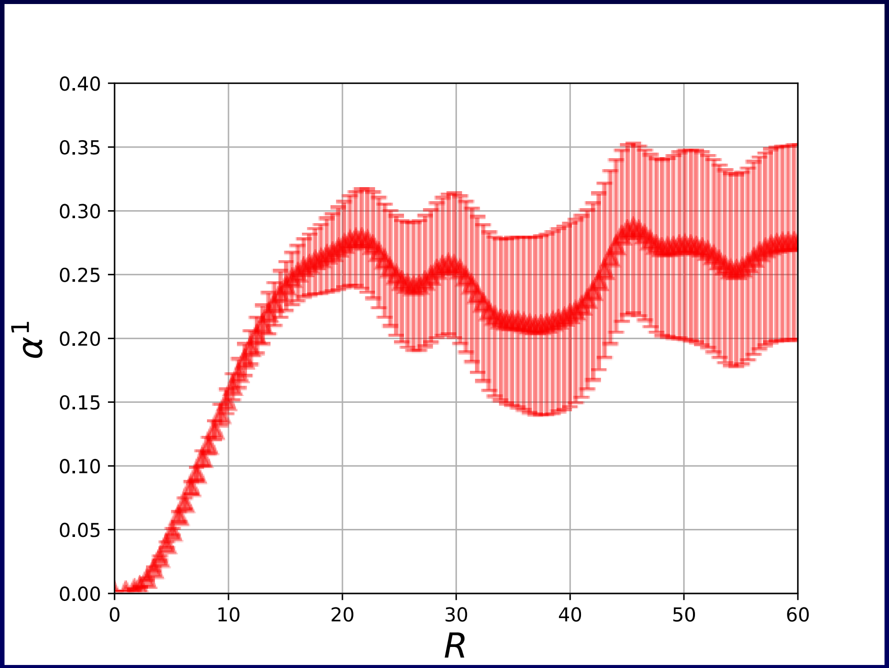
32^3 x 64 (32ID) RBC (4.6 fm, m_π=170 MeV)

CP violation angle in the nucleon

$$C_{3Q}(R,t) = \left\langle \sum_{\vec{x}} O_N(\vec{x},t)\bar{O}_N(\text{grid},0)Q \right\rangle$$

$$= \left\langle \sum_{\vec{x}} O_N(\vec{x},t)\bar{O}_N(\text{grid},0) \sum_{y(r\le R)} q(y) \right\rangle, \quad \text{r} = \sqrt{(\vec{y}-\vec{x})^2 + (t_y - t)^2}$$

$$a^1 = \frac{Tr(C_{3Q}(t)g_5)}{Tr(C_2(t)\Gamma_e)}$$



48^3 x 96 RBC lattice (a = 0.114 fm, m_π = 139 MeV),
m_π(valence) = 280 MeV, r (cut off) = 16, S/N increases by ~ 3.6

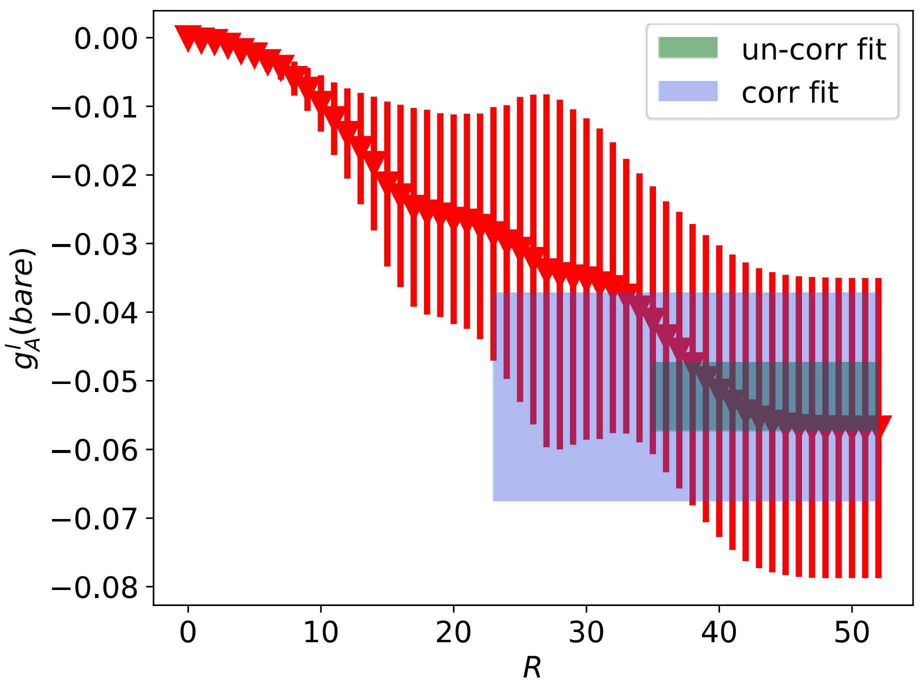
Quark spin with overlap fermion on domain-wall fermion configurations

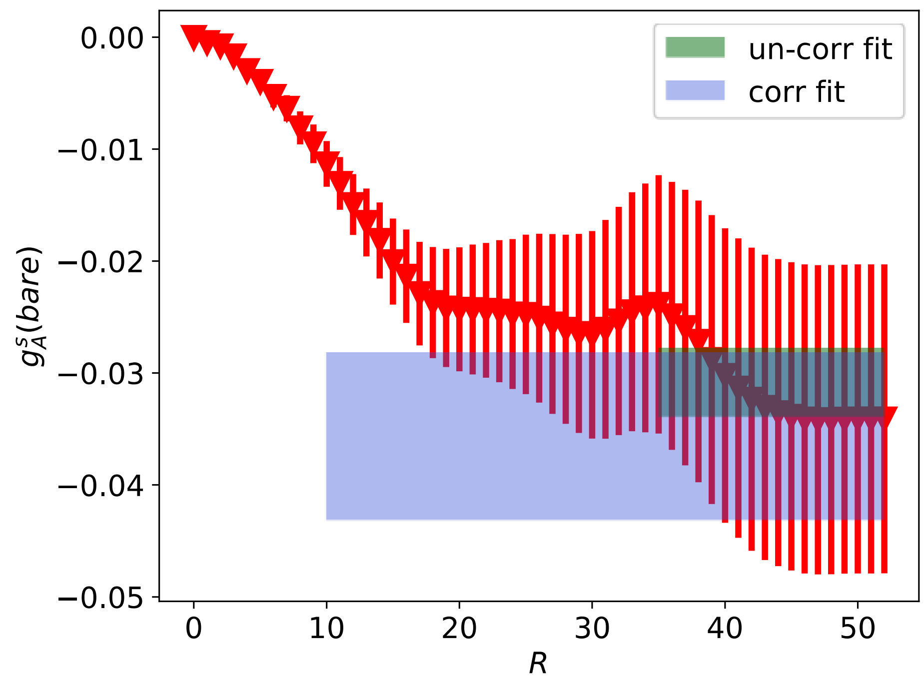
RBC/UK CD 2+1 flavor DWF Configurations

label	$L^3 \times T$	a^{-1} (GeV)	$m_l^{(s)}a$	$m_s^{(s)}a$	m_π (MeV)	$N_{\rm cfg}$
32I	$32^3 \times 64$	2.3833(86)	0.004	0.03	302	309
24I	$24^3 \times 64$	1.7848(50)	0.005	0.04	337	203
32ID	$32^3 \times 64$	1.3784(68)	0.001	0.045	171	200

- Overlap fermion is many times more expensive than Wilson fermion to invert
- It has chiral symmetry at finite a (Ginsparg-Wilson relation)
- Renormalization is easier with Ward identities
- Multi-mass inversion with deflation with the same eigenvectors
- Typically 5-6 valence quarks for each lattice ensemble

g_A in DI for the 32ID lattice with cluster decomposition error reduction (CDER)





light quark

strange quark

$$C_3(R) = C_3(\infty) + k\sqrt{R}\,\frac{e^{-MR}}{M}$$

Non-perturbative Renormalization

$$\begin{pmatrix} \Delta u^N(\mathrm{CI}) \\ \Delta d^N(\mathrm{CI}) \\ \Delta u^{\overline{\mathrm{MS}}}(\mathrm{DI})(\mu) \\ \Delta d^{\overline{\mathrm{MS}}}(\mathrm{DI})(\mu) \\ \Delta s^{\overline{\mathrm{MS}}}(\mathrm{DI})(\mu) \end{pmatrix} = \begin{pmatrix} Z_A & 0 & 0 & 0 & 0 \\ 0 & Z_A & 0 & 0 & 0 \\ Z_A^{\mathrm{D},\overline{\mathrm{MS}}} & Z_A^{\mathrm{D},\overline{\mathrm{MS}}} & Z_A + Z_A^{\mathrm{D},\overline{\mathrm{MS}}} & Z_A^{\mathrm{D},\overline{\mathrm{MS}}} & Z_A^{\mathrm{D},\overline{\mathrm{MS}}} \\ Z_A^{\mathrm{D},\overline{\mathrm{MS}}} & Z_A^{\mathrm{D},\overline{\mathrm{MS}}} & Z_A^{\mathrm{D},\overline{\mathrm{MS}}} & Z_A + Z_A^{\mathrm{D},\overline{\mathrm{MS}}} & Z_A^{\mathrm{D},\overline{\mathrm{MS}}} \\ Z_A^{\mathrm{D},\overline{\mathrm{MS}}} & Z_A^{\mathrm{D},\overline{\mathrm{MS}}} & Z_A^{\mathrm{D},\overline{\mathrm{MS}}} & Z_A^{\mathrm{D},\overline{\mathrm{MSI}}} & Z_A + Z_A^{\mathrm{D},\overline{\mathrm{MS}}} \end{pmatrix} \begin{pmatrix} \Delta u(\mathrm{CI}) \\ \Delta d(\mathrm{CI}) \\ \Delta u(\mathrm{DI}) \\ \Delta d(\mathrm{DI}) \\ \Delta s(\mathrm{DI}) \end{pmatrix}$$

$$Z_A^{\mathrm{D},\overline{\mathrm{MS}}} = Z_A^{\mathrm{D,RI}} + f_m + N_f f_m Z_A^{\mathrm{D,RI}}.$$

$$Z_A^{\mathrm{D,RI}} \equiv -Z_A \left(\frac{\Sigma_D}{\Sigma_C + N_f \Sigma_D}\right), Z_A = \frac{1}{\Sigma_C}$$

RI/MOM vertex

$$f_m = \left(\frac{\alpha_s}{4\pi}\right)^2 4C_F \left(-\frac{3}{2}\log\left(\frac{\mu^2}{p^2}\right) + \frac{7}{2}\right) \text{ with } C_F = 4/3.$$

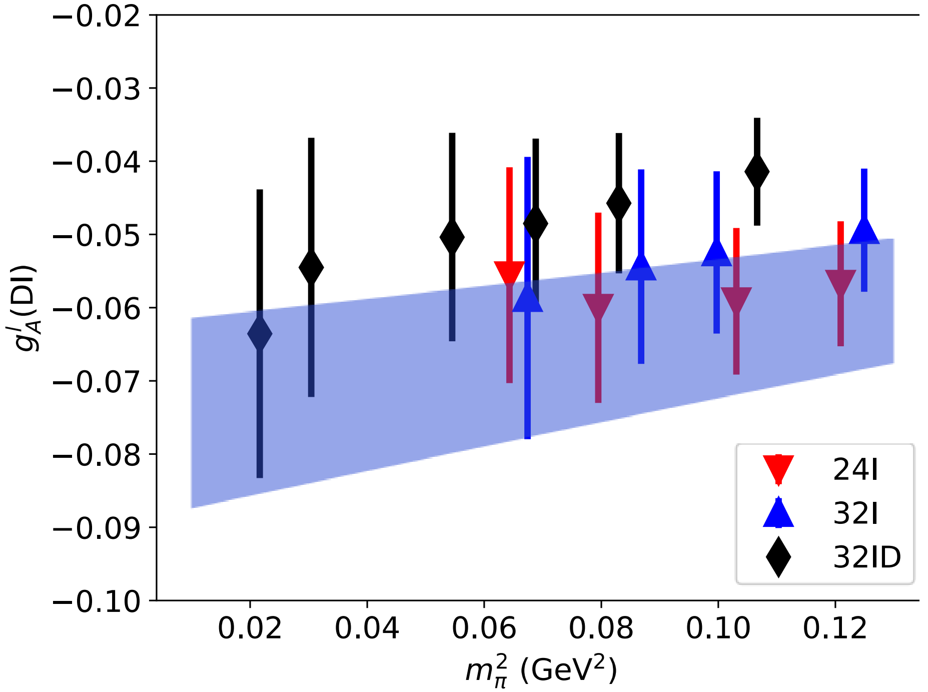
RI to MS-bar matching

The same as the conventional flavor irreducible representation due to linearity of the equations, but lattice classification is richer in structure and each components is discernible experimentally (KFL, 1703.04690).

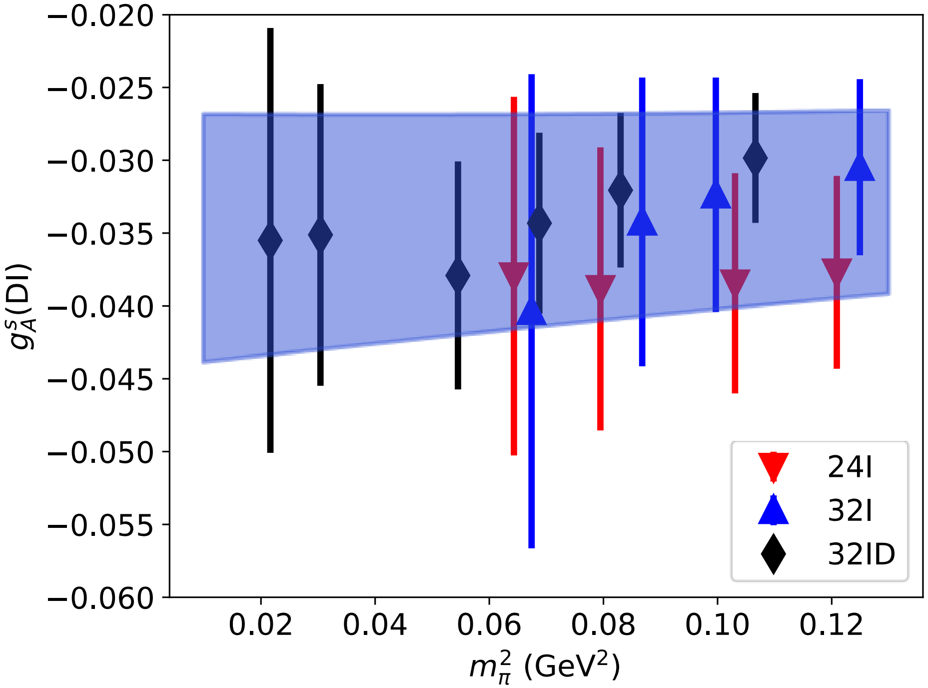
$$\begin{pmatrix} g_A^3 \\ g_A^8 \\ g_A^{0,\overline{\mathrm{MS}}}(\mu) \end{pmatrix} = \begin{pmatrix} Z_A & 0 & 0 \\ 0 & Z_A & 0 \\ 0 & 0 & Z_A + N_f Z_A^{\mathrm{D},\overline{\mathrm{MS}}}(\mu) \end{pmatrix} \begin{pmatrix} \Delta u - \Delta d \\ \Delta u + \Delta d - 2\Delta s \\ \Delta u + \Delta d + \Delta s \end{pmatrix}.$$

DI results

$$g_A = c_0 + c_1 a^2 + c_2(m_{p,v}^2 - m_{p,p}^2) + c_3(m_{p,s}^2 - m_{p,p}^2) + c_4 e^{-m_{p,v}L}$$



light quark

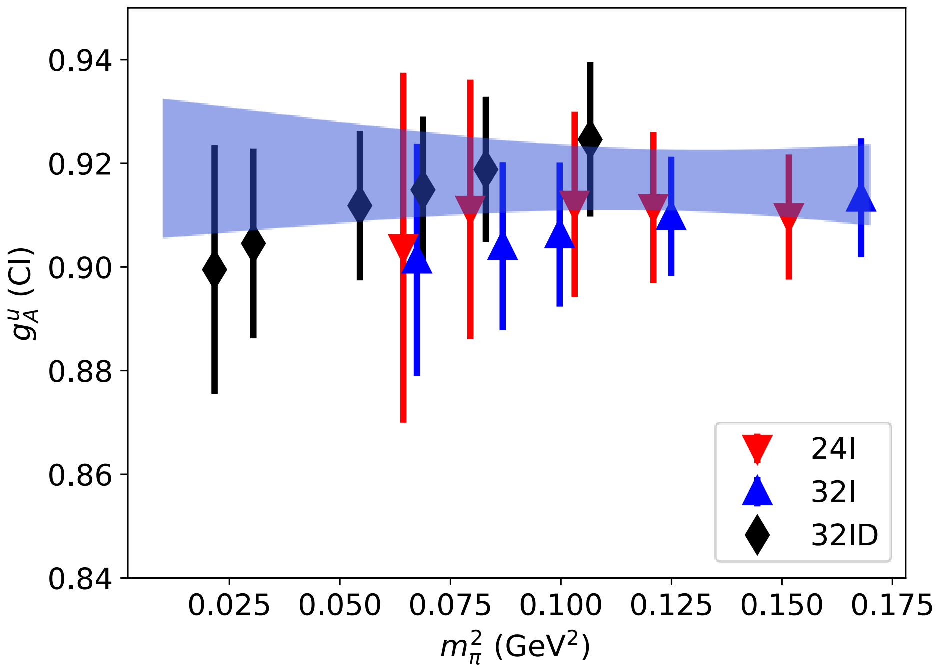


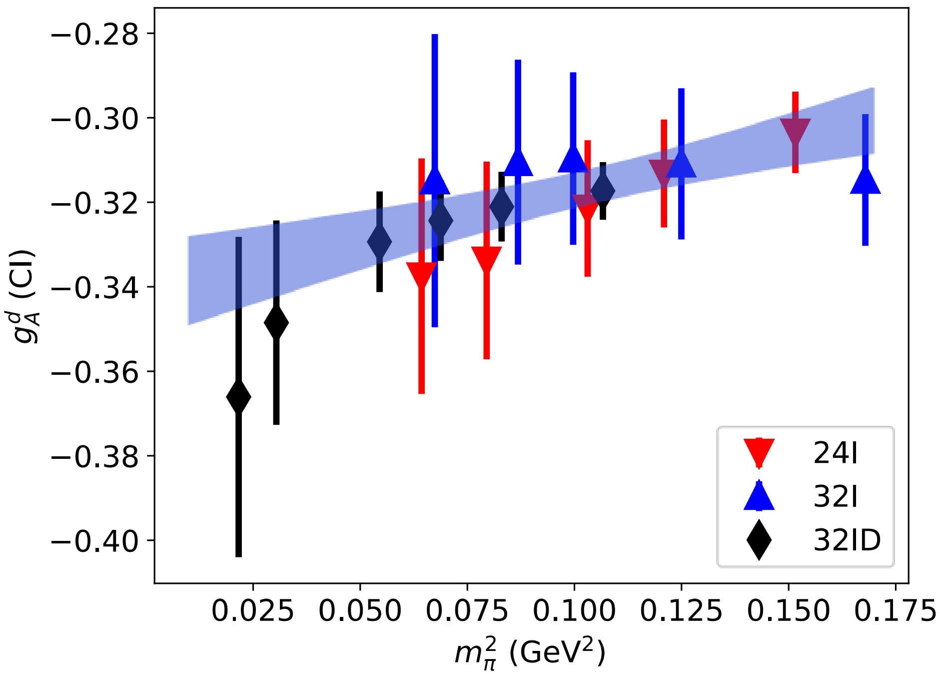
strange quark

No statistically significant O(a^2) dependence

CI results

$$g_A = c_0 + c_1 a^2 + c_2 (m^2_{p,v} - m^2_{p,p}) + c_3 (m^2_{p,s} - m^2_{p,p}) + c_4 e^{-m_{p,v} L}$$





u quark

d quark

No discernable O(a^2) dependence

Quark Spin Compoments $\overline{MS}$ (2 GeV)

g_A	$\Delta(u+d)$ CI	$\Delta(u/d)$ DI	Δs	Δu	Δd	$\Delta u-\Delta d$ (g_A^3)	DS
J. Green			-0.0240 (45)	0.863 (7)(14)	-0.345 (6)(9)	1.206 (20)	0.494 (11)(15)
C. Alexandrou	0.598 (24)(6)	-0.077 (15)(5)	-0.042 (10)(2)	0.830 (26)(4)	-0.386 (16)(6)	1.216 (31)(7)	0.402 (34)(10)
c QCD	0.580 (16)(30)	-0.070 (12)(15)	-0.035 (6)(7)	0.847 (18)(32)	-0.407 (16)(18)	1.254 (16)(30)	0.405 (25)(37)
NPPDFpol1.1 (Q^2=10 GeV^2)			-0.10 (8)	0.76 (4)	-0.41 (4)	1.17 (6)	0.25 (10)
DSSV (Q^2=10 GeV^2)			-0.012 +(56)-(62)	0.793 +(28)-(34)	-0.416 +(35)-(25)	1.209 +(45)-(42)	0.366 +(62)-(42)

J. Green et al., N_F=2+1, Clover fermion, m_π = 317 MeV, one lattice

C. Alexandrou et al., N_F=2, twisted mass fermion, , m_π = 131 MeV, one lattice

c QCD, N_F=2+1, Overlap fermion, , m_π = 170, 290, 330 MeV, 5 - 6 valence quarks for each of the three lattices

Expt. g_A^3 = 1.2723(23); CalLat: g_A^3 = 1.271(13)

Quark Spin

- Lattice calculation with chiral fermion is getting close in revealing the origin of the smallness of the quark spin – the disconnected insertion is large and negative.
- The interplay between the pseudoscalar and topological charge couplings in the anomalous Ward identity is the origin for the negative DI contribution – another example of U(1) anomaly at work.
- In the future it would be desirable to use the exponentially local chiral axial-vector current (P. Hazenfratz) to check the calculation where $Z_A{}^0{}_{,norm} = 1$.

Where does the rest of the spin of the proton come from?

Glue spin
Quark ortibal angular momentum
Glue orbital angular momentum

Momenta and Angular Momenta of Quarks and Glue

- Energy momentum tensor operators decomposed in quark and glue parts gauge invariantly --- Xiangdong Ji (1997)

$$T^q_{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi + (\mu \leftrightarrow \nu)\right] \rightarrow \vec{J}_q = \int d^3x \left[\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \vec{x}\times\bar{\psi}\gamma_4(-i\vec{D})\psi\right]$$

$$T^g_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} - \frac{1}{4}\delta_{\mu\nu}F^2 \qquad \rightarrow \vec{J}_g = \int d^3x\left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

- Nucleon form factors

$$\langle p,s \,|\, T_{\mu\nu} \,|\, p's'\rangle = \bar{u}(p,s)[T_1(q^2)\gamma_\mu\bar{p}_\nu - T_2(q^2)\bar{p}_\mu\sigma_{\nu\alpha}q_\alpha/2m$$

$$-\mathrm{i}T_3(q^2)(q_\mu q_\nu - \delta_{\mu\nu}q^2)/m + T_4(q^2)\delta_{\mu\nu}m/2]u(p's')$$

- Momentum and Angular Momentum

$$Z_{q,g}T_1(0)_{q,g}\ [\mathrm{OPE}] \rightarrow \langle x\rangle_{q/g}(\mu,\overline{\mathrm{MS}}),\quad Z_{q,g}\left[\frac{T_1(0)+T_2(0)}{2}\right]_{q,g} \rightarrow J_{q/g}(\mu,\overline{\mathrm{MS}})$$

Normalization, Renormalization and Quark-Glue Mixing

Momentum and Angular Momentum Sum Rules

$$\langle x\rangle_q^R = Z_q \langle x\rangle_q^L, \quad \langle x\rangle_g^R = Z_g \langle x\rangle_g^L,$$

$$J_q^R = Z_q J_q^L, \quad J_g^R = Z_g J_g^L,$$

$$Z_q \langle x\rangle_q^L + Z_g \langle x\rangle_g^L = 1,$$

$$Z_q J_q^L + Z_g J_g^L = \frac{1}{2} \quad \Rightarrow \begin{cases} Z_q T_1^q(0) + Z_g T_1^g(0) = 1, \\ Z_q (T_1^q + T_2^q)(0) + Z_g (T_1^g + T_2^g)(0) = 1, \\ Z_q T_2^q(0) + Z_g T_2^g(0) = 0 \end{cases}$$

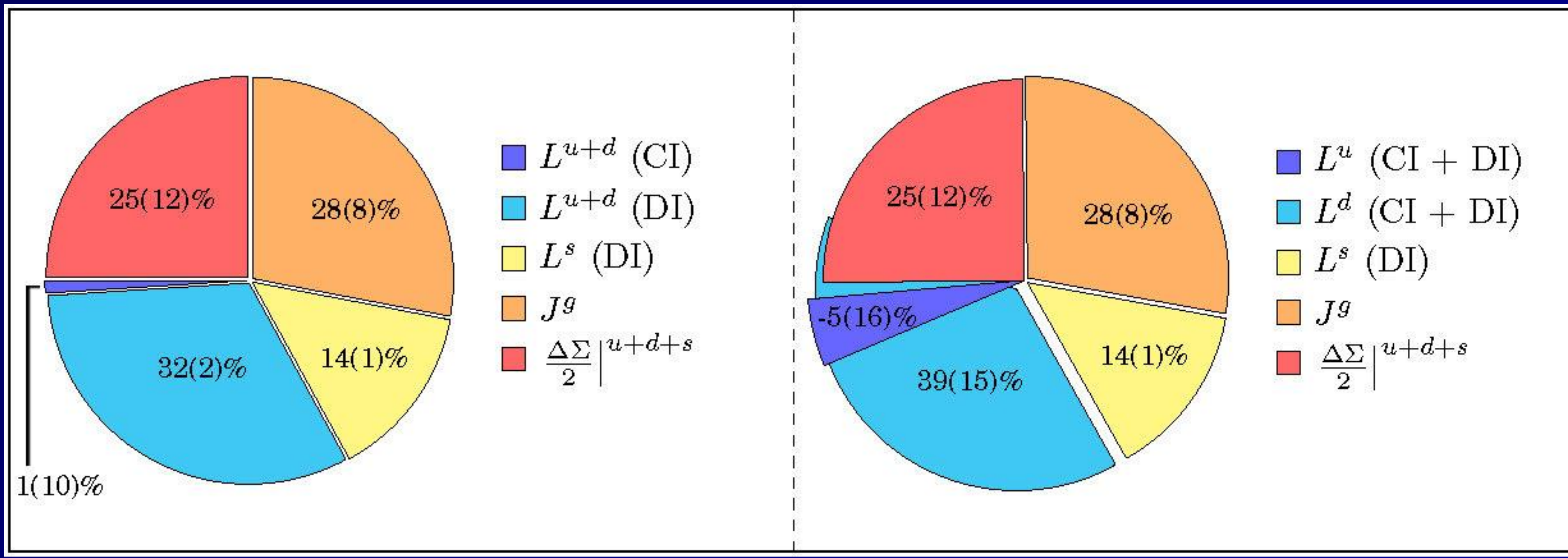
Mixing

$$\begin{bmatrix} \langle x\rangle_q^{\overline{MS}}(\mu) \\ \langle x\rangle_g^{\overline{MS}}(\mu) \end{bmatrix} = \begin{bmatrix} C_{qq}(\mu) & C_{qg}(\mu) \\ C_{gq}(\mu) & C_{gg}(\mu) \end{bmatrix} \begin{bmatrix} \langle x\rangle_q^R \\ \langle x\rangle_g^R \end{bmatrix}$$

M. Glatzmaier, KFL
arXiv:1403.7211

Quark Spin, Orbital Angular Momentum, and Gule Angular Momentum (M. Deka *et al*, 1312.4816, PRD)

pizza cinque stagioni



$$\mathrm{D}q \approx 0.25;$$

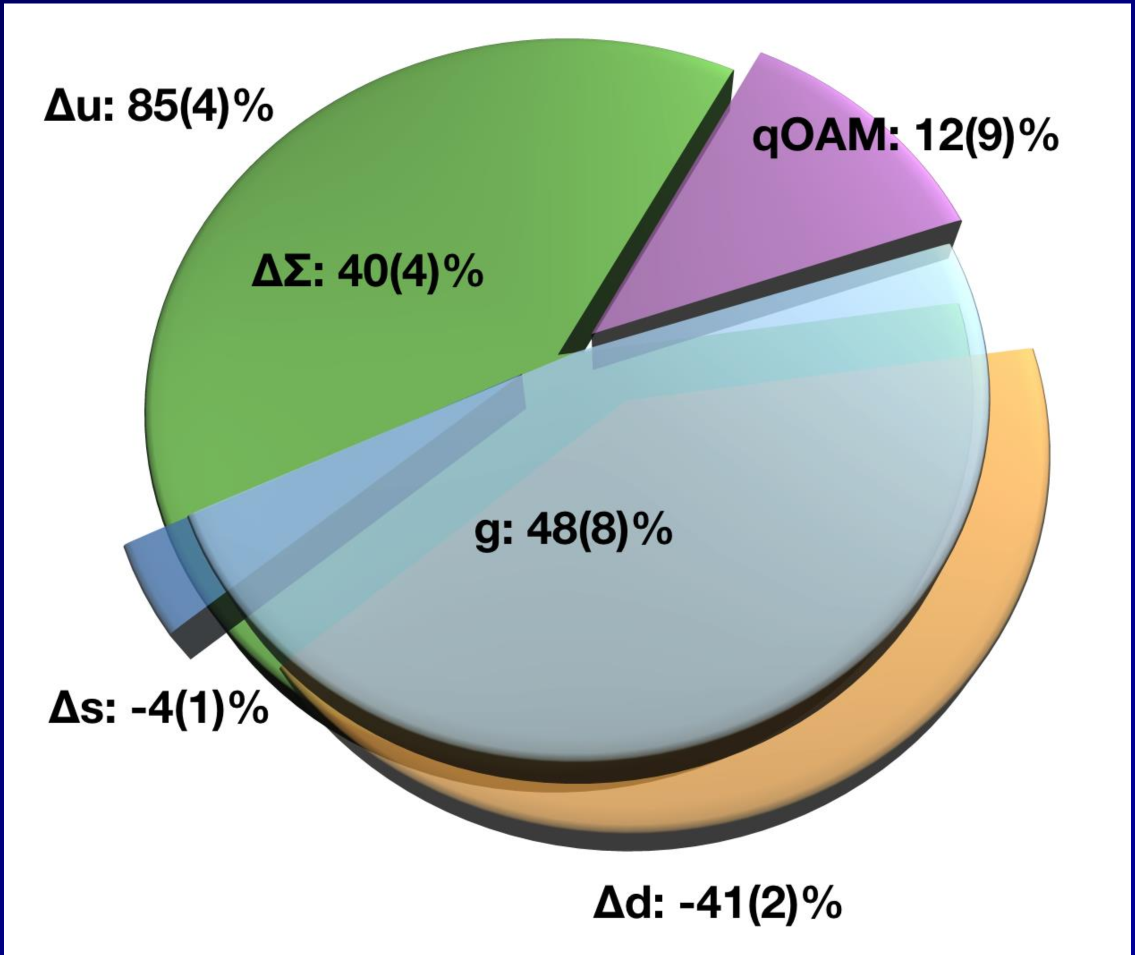
$$2\,L_q \approx 0.47\ (0.01(\mathrm{CI})+0.46(\mathrm{DI}));$$

$$2\,\mathrm{J}_g \approx 0.28$$

These are quenched results so far.

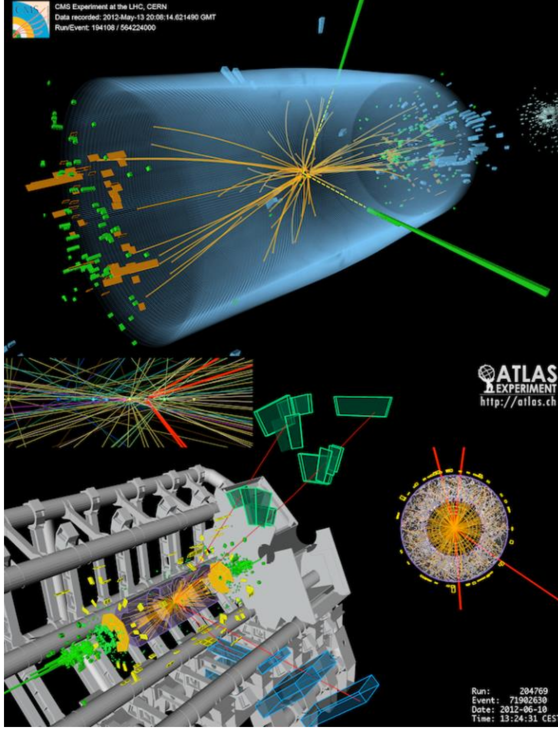
Proton Spin Decomposition (2+1 Flavor)

Preliminary

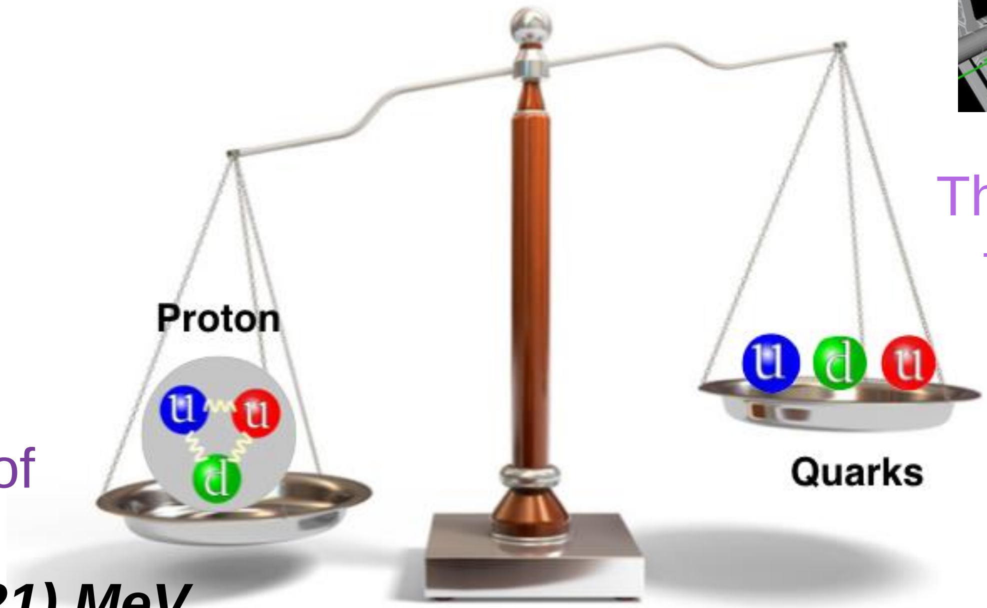


Approximate by setting $T_2 = 0$

Motivation



Where does the proton mass come from, and how ?



But the mass of the proton is

938.272046(21) MeV.

~100 times of the sum of the quark masses!

The Higgs boson make the u/d quark having masses (2GeV MS-bar):

$m_u = 2.08(9)$ *MeV*

$m_d = 4.73(12)$ *MeV*

Laiho, Lunghi, & Van de Water, Phys.Rev.D81:034503,2010

Quark and Glue Components of Hadron Mass

- Energy momentum tensor

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2 \qquad \langle P|T_{\mu\nu}|P\rangle = P_\mu P_\nu / M$$

- Trace anomaly

$$T_{mm} = - m(1+g_m)\bar{y}y + \frac{b(g)}{2g}G^2$$

- Separate into traceless part $\bar{T}_{\mu\nu}$ and trace part $\hat{T}_{\mu\nu}$

$$\langle P|\bar{T}^{q,g}_{mm}|P\rangle = \langle x\rangle_{q,g}(m^2)(P_m P_n - \frac{1}{4}d_{mm}P^2)/M, \quad \langle x\rangle_q(m^2) + \langle x\rangle_g(m^2) = 1$$

$$\langle \bar{T}_{44}\rangle = -3/4M; \qquad \langle \hat{T}_{mm}\rangle = -M$$

Decomposition of hadron mass

Xiangdong Ji, PRL 74, 1071 (1995);
PRD 52, 271 (1995)

$$M = -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_a\rangle/4 = \langle H_E\rangle(\mu) + \langle H_m\rangle + \langle H_g\rangle(\mu) + \langle H_a\rangle/4;$$

$$M = -\langle \hat{T}_{\mu\mu}\rangle = \langle H_m\rangle + \langle H_a\rangle;$$

where

$$H_q = \sum_{u,d,s\ldots}\int d^3x - \bar{\psi}(\gamma_4 D_4)\psi;\;\; H_E = \sum_{u,d,s\ldots}\int d^3x\; \bar{\psi}(\vec{\gamma}\cdot\vec{D})\psi;\;\; H_m = \sum_{u,d,s\ldots} m_f \int d^3x\; \bar{\psi}\psi;$$

$$H_g = \int d^3x\,(B^2 - E^2);\;\; H_a = \int d^3x\; \frac{-\beta(g)}{2g}(B^2 + E^2) + \gamma_m H_m$$

– Equation of motion

$$\sum_z (D_c + m)(x,z)\frac{1}{D_c + m}(z,y) = d_{x,y} \Rightarrow \begin{cases} 0 \text{ for CI} \\ \text{constant for DI} \end{cases} \qquad D_c = \frac{r\,D_{ov}}{1 - D_{ov}/2}$$

Therefore,

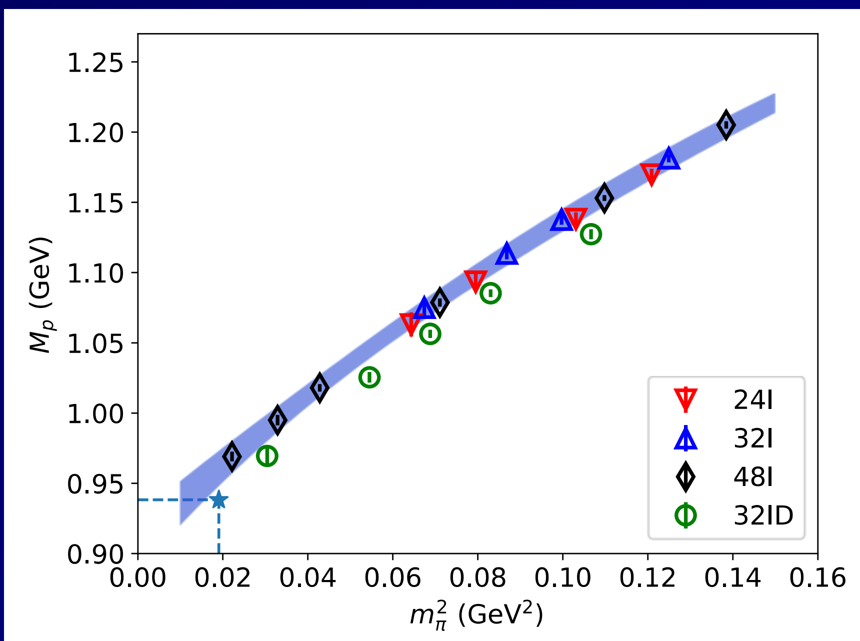
$$\langle H_q\rangle - \langle H_E\rangle = \langle H_m\rangle + O(a^2)$$

Overlap fermion on DWF configurations

RBC/UK CD 2+1 flavor DWF Configurations

Y.B. Yang et al., 1808.08677

Symbol	$L^3 \times T$	a (fm)	$m_s^{(s)}$(MeV)	m_π(MeV)	N_{cfg}
32ID	$32^3 \times 64$	0.1431(7)	89.4	171	200
24I	$24^3 \times 64$	0.1105(3)	120	330	203
48I	$48^3 \times 96$	0.1141(2)	94.9	139	81
32I	$32^3 \times 64$	0.0828(3)	110	300	309



SU(4|2) mixed action HBCPT

$$
\begin{aligned}
M(m_\pi^v, m_\pi^{sea}, a, L) = {} & M_0 + C_1 (m_\pi^v)^2 + C_2 (m_\pi^{sea})^2 \\
& - \frac{(g_A^2 - 4 g_A g_1 - 5 g_1^2)\pi}{3(4\pi f_\pi)^2} (m_\pi^v)^3 \\
& - \frac{(8 g_A^2 + 4 g_A g_1 + 5 g_1^2)\pi}{3(4\pi f_\pi)^2} (m_\pi^{pq})^3 \\
& + C_3^{I/ID} a^2 + C_4 \frac{(m_\pi^v)^2}{L} e^{-m_\pi^v L},
\end{aligned}
$$

$M_N = 960(13)$ MeV, c^2 / dof = 0.52

Fix $g_A = 1.2723$, $M_N = 931(8)$ MeV, c^2 / dof = 1.5

Non-perturbative Renormalization

- Renormalized $\langle x\rangle_q$ and $\langle x\rangle_g$ in MS-bar at μ

$$\langle x\rangle^R_{u,d,s} = Z^{\overline{\rm MS}}_{QQ}(\mu)\langle x\rangle_{u,d,s} + \delta Z^{\overline{\rm MS}}_{QQ}(\mu)\sum_{q=u,d,s}\langle x\rangle_q + Z^{\overline{\rm MS}}_{QG}(\mu)\langle x\rangle_g,\ \ \langle x\rangle^R_g = Z^{\overline{\rm MS}}_{GQ}(\mu)\sum_{q=u,d,s}\langle x\rangle_q + Z^{\overline{\rm MS}}_{GG}\langle x\rangle_g,$$

$$\begin{pmatrix} Z^{\overline{\rm MS}}_{QQ}(\mu)+N_f\delta Z^{\overline{\rm MS}}_{QQ}(\mu) & N_f Z^{\overline{\rm MS}}_{QG}(\mu) \\ Z^{\overline{\rm MS}}_{GQ}(\mu) & Z^{\overline{\rm MS}}_{GG}(\mu)\end{pmatrix} = \left\{\left[\begin{pmatrix} Z_{QQ}(\mu_R)+N_f\delta Z_{QQ} & N_f Z_{QG}(\mu_R) \\ Z_{GQ}(\mu_R) & Z_{GG}(\mu_R)\end{pmatrix}\begin{pmatrix} R_{QQ}(\frac{\mu}{\mu_R})+\mathcal{O}(N_f\alpha_s^2) & N_f R_{QG}(\frac{\mu}{\mu_R}) \\ R_{GQ}(\frac{\mu}{\mu_R}) & R_{GG}(\frac{\mu}{\mu_R})\end{pmatrix}\right]\Big|_{a^2\mu_R^2\to 0}\right\}^{-1}$$

- Renormalization of glue operator in gluon propagator is very noisy.

Glue Renormalization

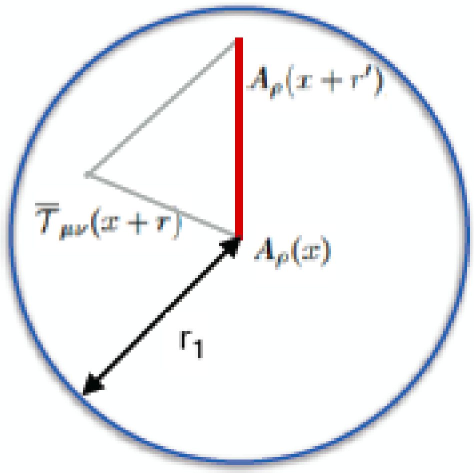
- Off-diagonal and traceless renormalization in RI/MOM

$$Z^{-1}(\mu_R^2) = \left(\frac{N_c^2-1}{2} Z_g^{\rm RI}(\mu_R^2)\right)^{-1} \times \frac{V\langle \overline{\mathcal{T}}_{g,\mu\nu}\,{\rm Tr}[A_\rho(p)A_\rho(-p)]\rangle}{2p_\mu p_\nu \langle {\rm Tr}[A_\rho(p)A_\rho(-p)]\rangle^2}\Bigg|_{\substack{p^2=\mu_R^2,\\ \rho\neq\mu\neq\nu,\\ p_\rho=0}} = \frac{p^2\langle \overline{\mathcal{T}}_{g,\mu\nu}\,{\rm Tr}[A_\rho(p)A_\rho(-p)]\rangle}{2p_\mu p_\nu \langle {\rm Tr}[A_\rho(p)A_\rho(-p)]\rangle}\Bigg|_{\substack{p^2=\mu_R^2,\\ \rho\neq\mu\neq\nu,\\ p_\rho=0}},$$

$$Z_T^{-1}(\mu_R^2) = \frac{p^2\langle(\overline{\mathcal{T}}_{\mu\mu}-\overline{\mathcal{T}}_{\nu\nu})\,{\rm Tr}[A_\rho(p)A_\rho(-p)]\rangle}{2p_\mu^2\langle {\rm Tr}[A_\rho(p)A_\rho(-p)]\rangle}\Bigg|_{\substack{p^2=\mu_R^2,\\ \rho\neq\mu\neq\nu,\\ p_\rho=0,\\ p_\nu=0}}.$$

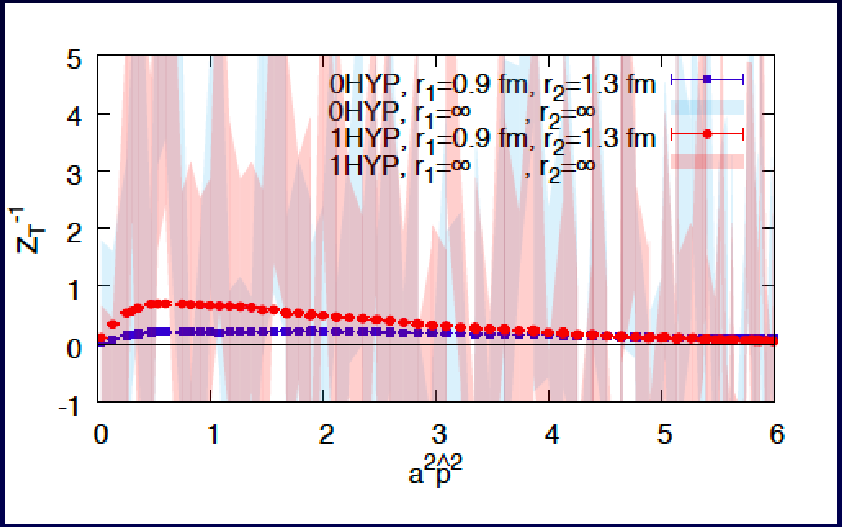
- CDER

$$C_3^{\rm CDER}(p) \equiv \left\langle \int_{|r|<r_1} d^4r \int_{|r'|<r_2} d^4r' \int d^4x \; e^{ip\cdot r'}\,\overline{\mathcal{T}}_{\mu\nu}(x+r)\,{\rm Tr}[A_\rho(x)A_\rho(x+r')]\right\rangle.$$



r_1: Cut on the $T_{\mu\nu}$ - A_ρ correlation

Cluster Decomposition Error Reduction (CDER)



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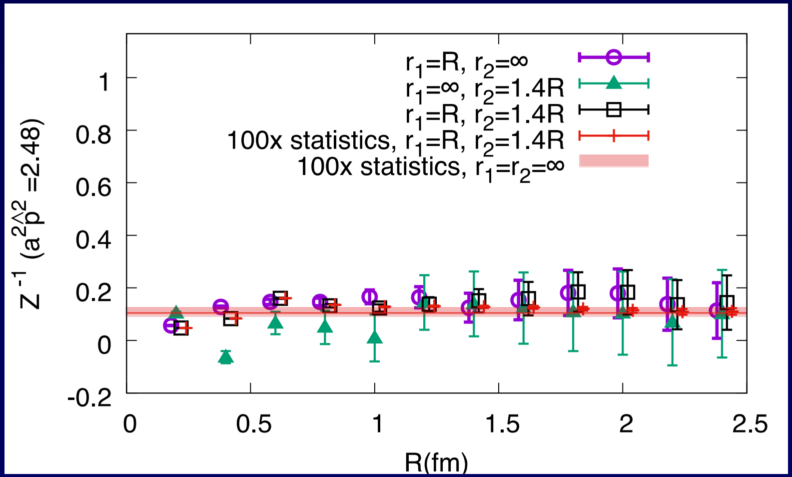
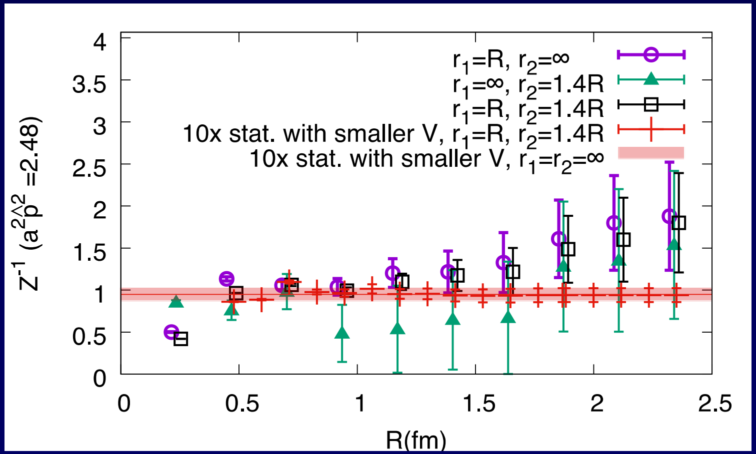
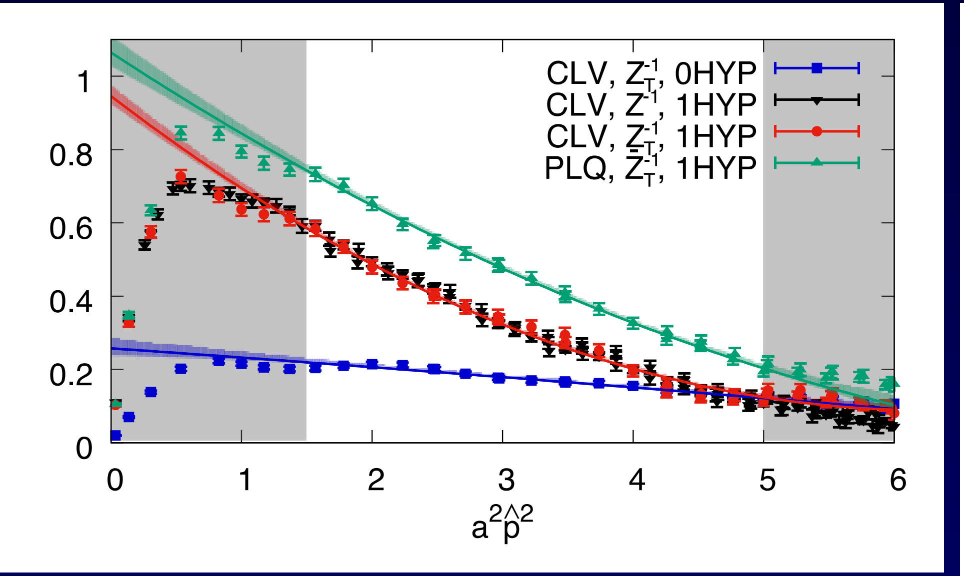


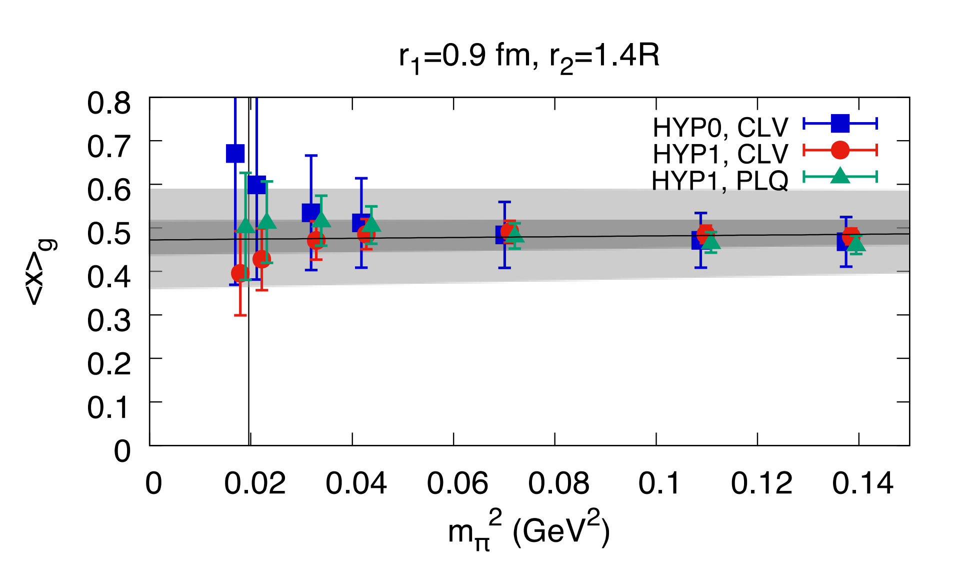


70,834 $24^3 \times 64$ quenched configs, a=0.098 fm, use 1% for CDER

21,166 $24^3 \times 64$ N_f =2 Clover configs, m_π = 450 MeV, a =0.117 fm, use 10% for CDER

Z_{GG}^{-1} and <x>$_g$ with CDER





R_1= 0.9 fm, R_2=1.4 R_1, the error is reduced by ~ 300 times on the 48I lattice with L = 5.5 fm.

$(V/V_{R1})^{½}$ $(V/V_{R2})^{½}$ ~ 300

Perturbative mixing with quark

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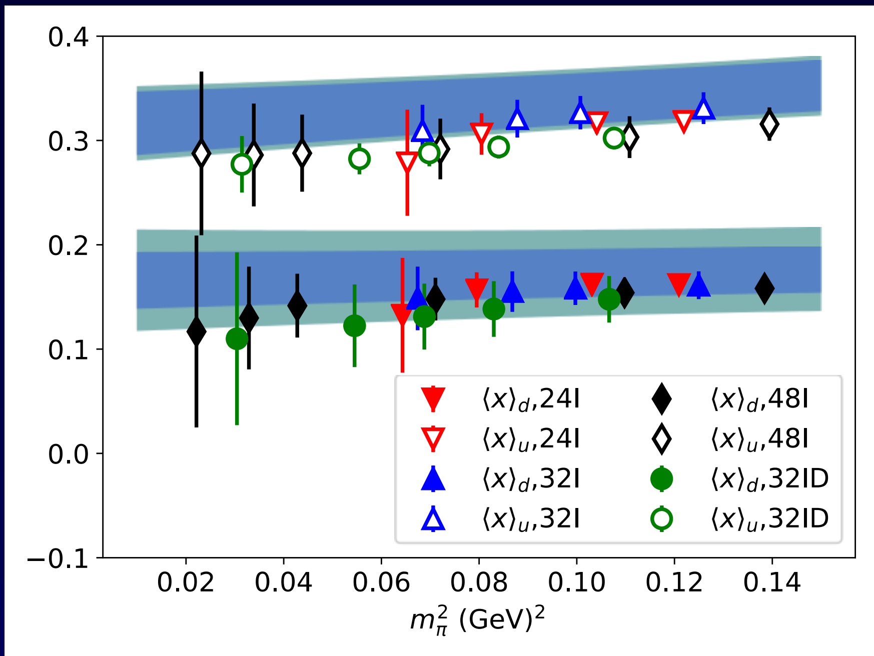
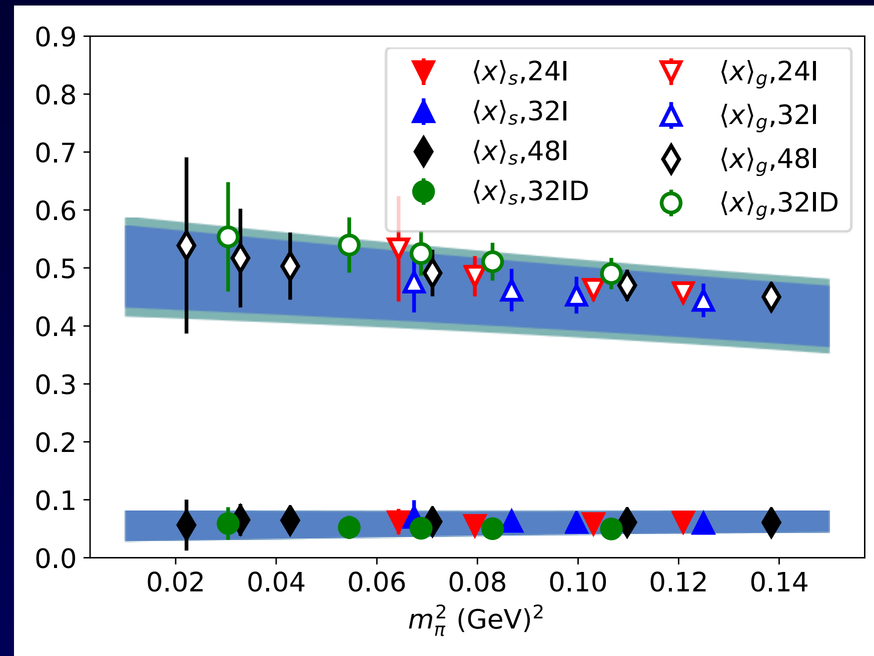
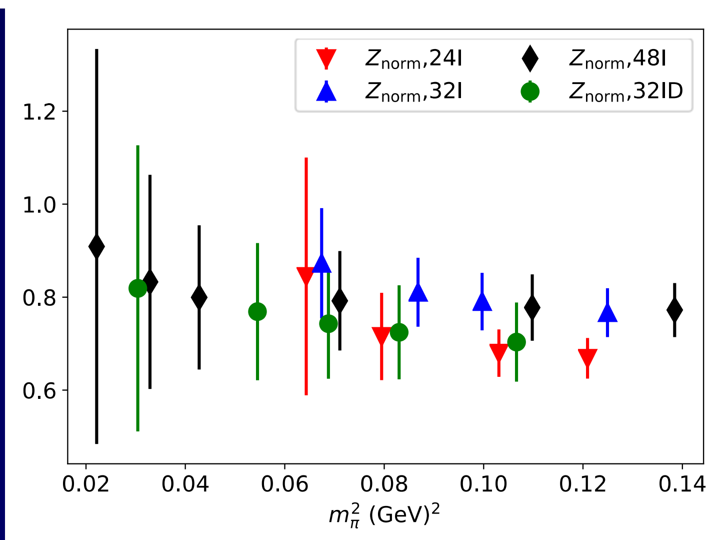
$<x>_q$, $<x>_g$ and Normalization







$$Z_{\text{norm}}\left(\sum_{f=u,d,s} <x>_f + <x>_g\right) = 1$$

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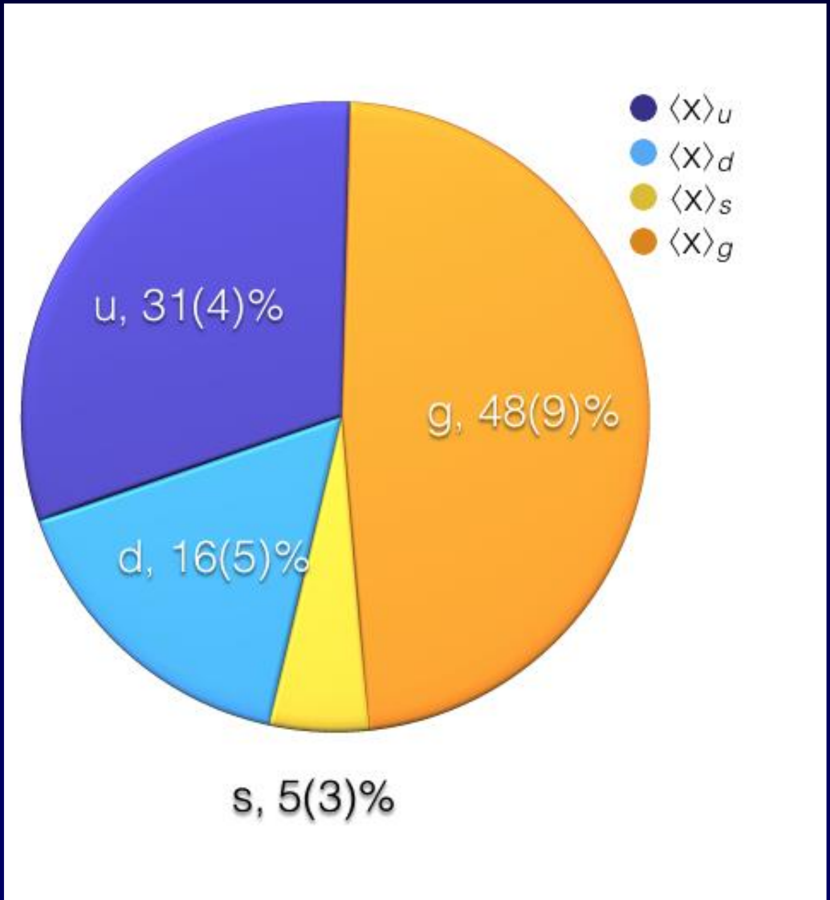
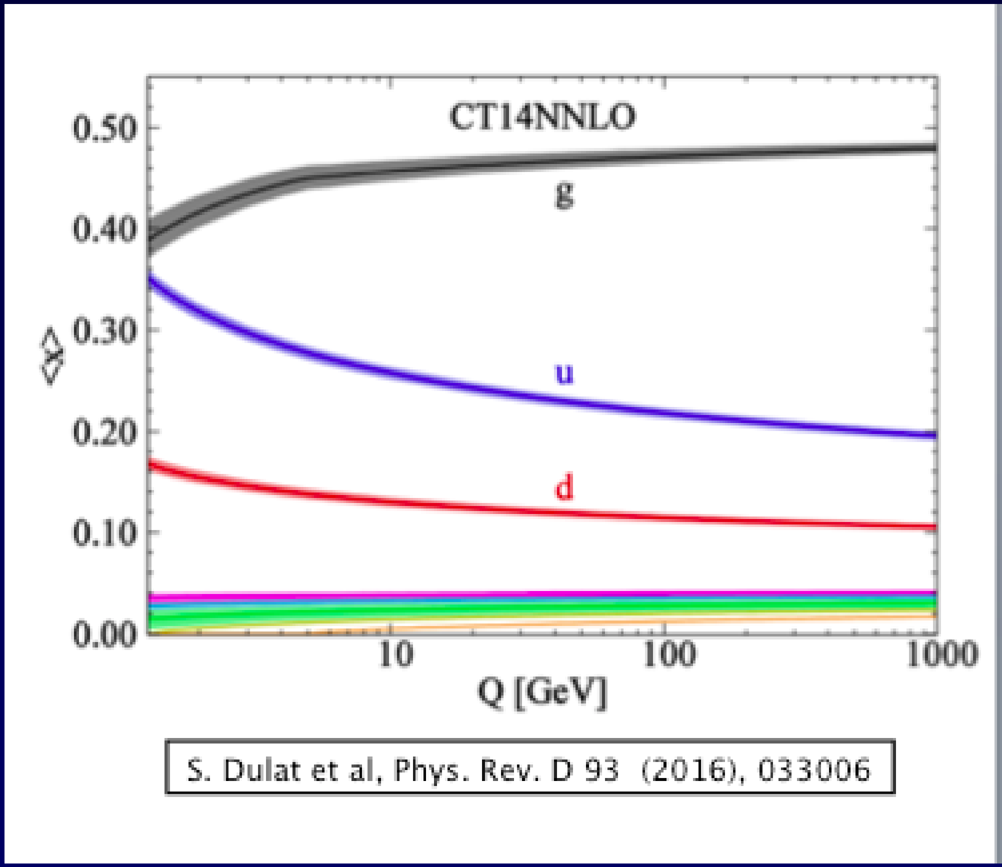
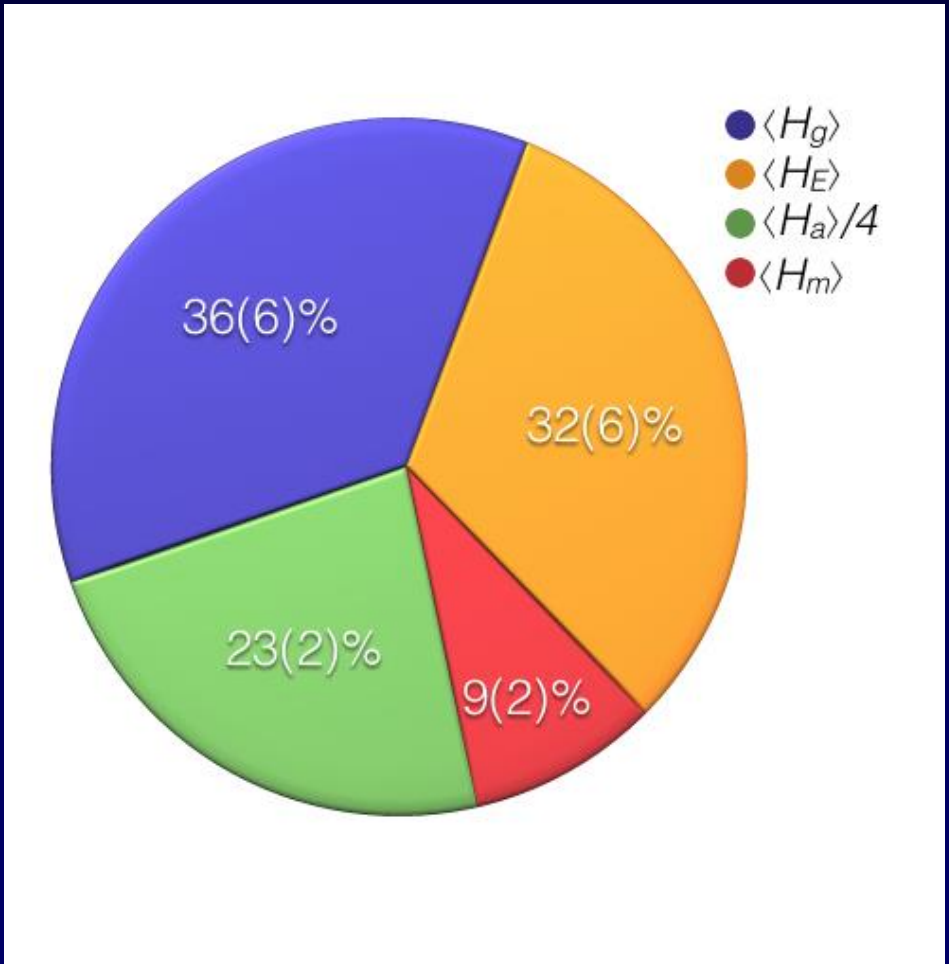
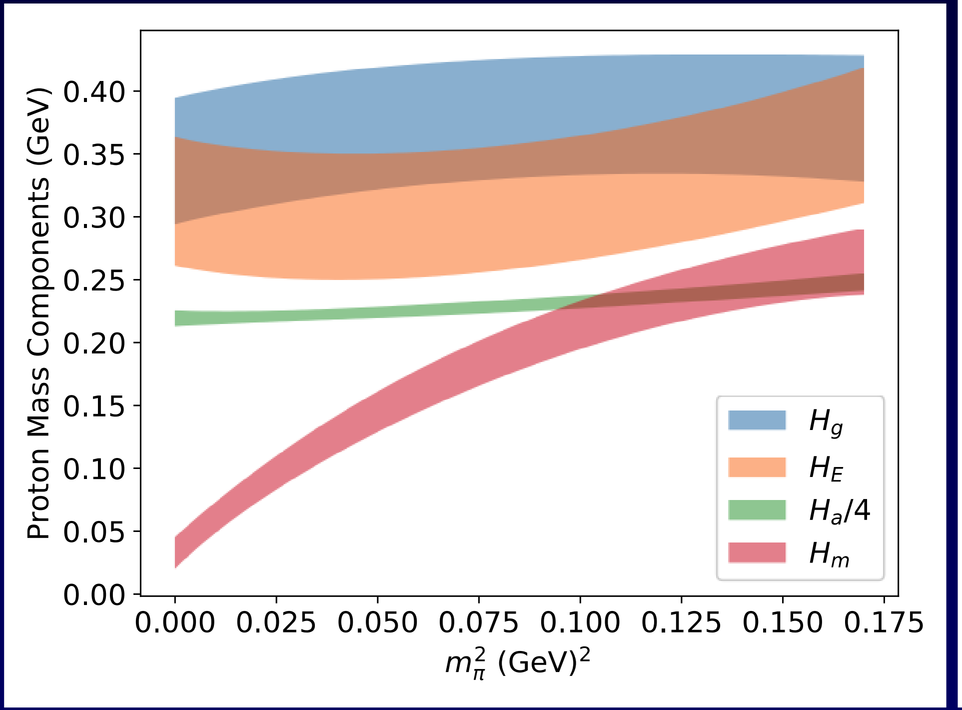
Comparison with Global Fitting of <x> MS-bar at 2 GeV

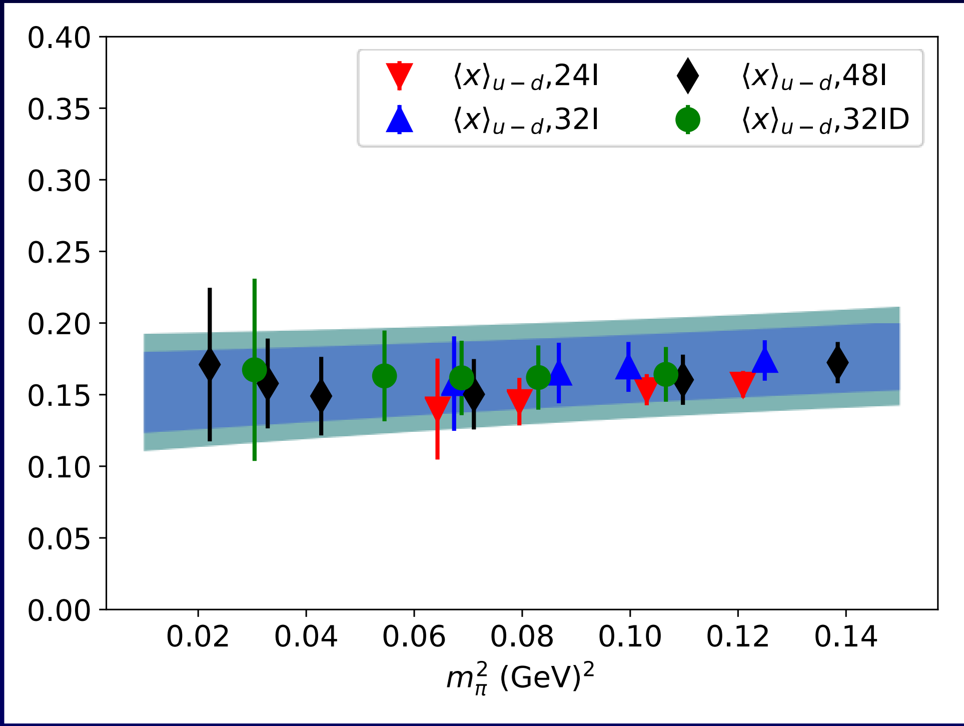




Proton Mass Decomposition







$$<x>_{u-d} = 0.151(28)(29)$$

CT14 --- 0.158(6)(23)

Summary and Challenges

- Lattice calculations of the physical 2+1 flavor dynamical fermions at the physical pion point and with extrapolations to continuum and infinite volume limits are becoming available even with chiral fermions.

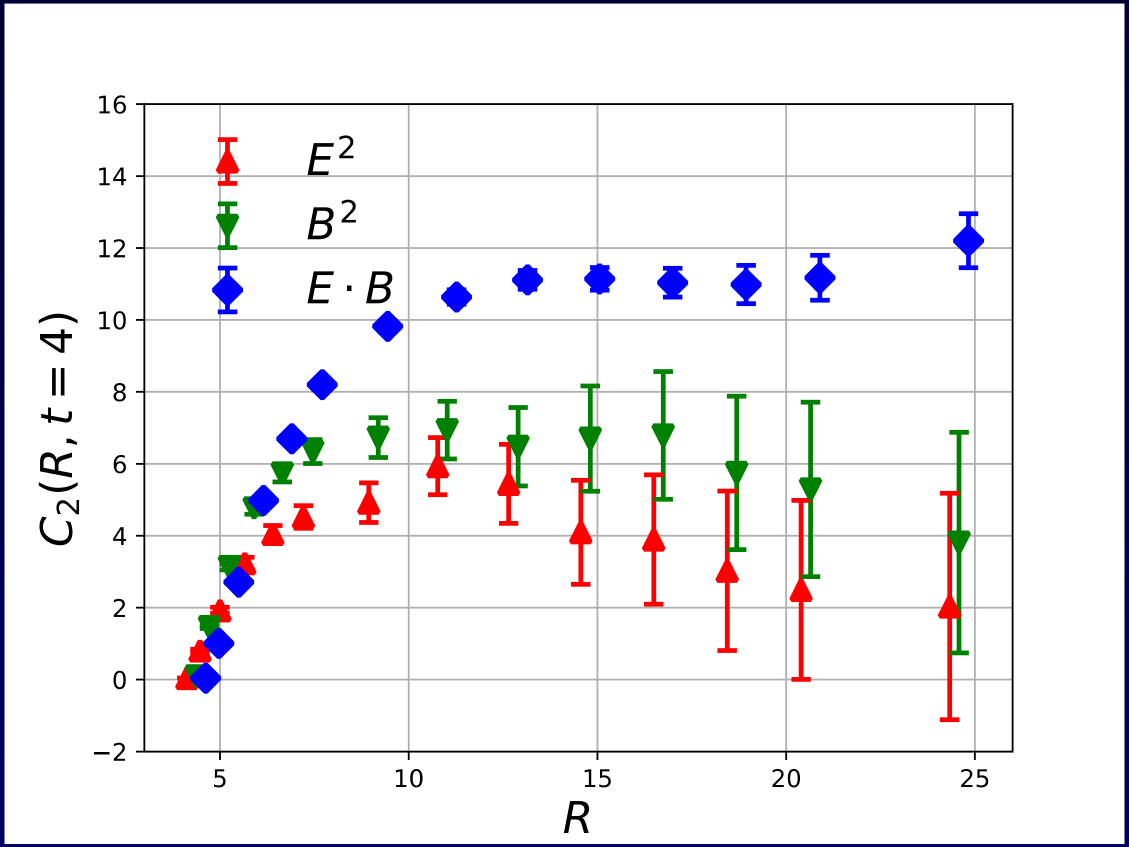
- Decomposition of proton spin and hadron masses into quark and glue components on the lattice is feasible, pending reasonalbe statistics of non-perturbative renormalization. Large momentum frame for the proton to calculate glue helicity remains a challenge.

- Together with evolution, factorization, perturbative QCD, lattice QCD results with small enough statistical and systematic errors can compare directly with experiments and have an impact in advancing our understanding of the underline physics of the hadron structure (form factors, PDF, neutron electric dipole moment, muon g-2, etc).

Status of Proton Spin Problem

- The crisis is over and most of the puzzles are solved. However, challenges still remain in experiments and lattice calculations to totally understand the proton spin decomposition.

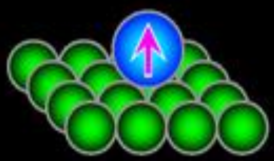
Glueball Masses on 48^3 x 96 lattice (L= 5.5 fm)



Scalar: cutoff at R=9 reduces error by ~ 4 which is ≈ $(25/9)^{3/2}$

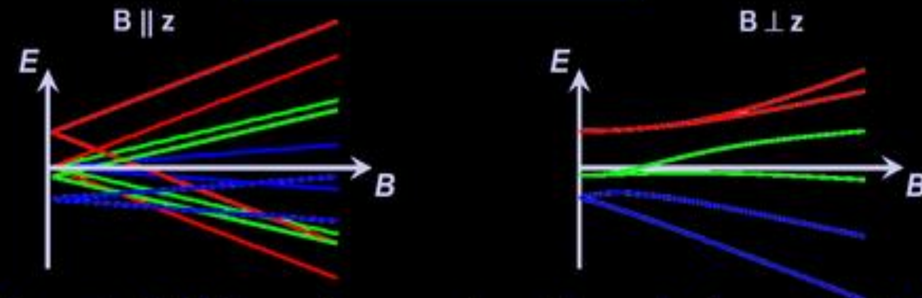
Pseudoscalar: cutoff at R=11 reduces error by ~ 3 which is ≈ $(25/11)^{3/2}$

Anisotropy at a surface



- Free atomic spin is rotationally invariant: all spin orientations are degenerate.
- Loss of rotational symmetry breaks degeneracy of spin orientations.

$$H = -g\mu_B \vec{B} \cdot \vec{S} + DS_z^2$$



Magnetic field dependence varies with angle of magnetic field.

AWI and Goldberger-Treiman Relation

- Generalized Goldberger-Treiman relation is violated badly at small momentum transfer

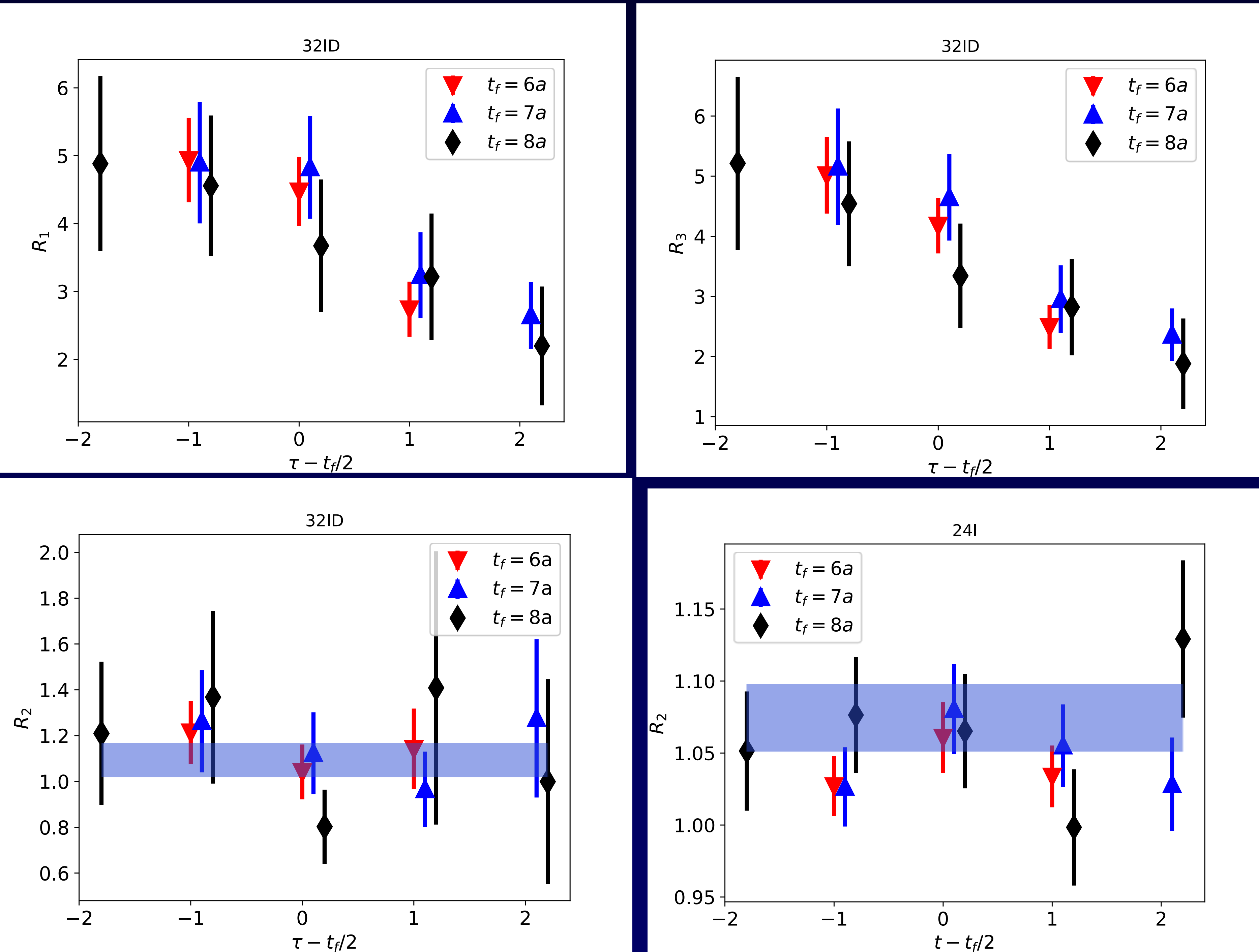
$$g_A(q^2)+q^2/2m_N h_A(q^2)^1 \; m_q/m_N g_P(q^2)+2g_Q(q^2)$$

- Consider $|\vec{q}|=2\pi/L$

$$R_1=\frac{m_q/m_N g_P(q^2)+2g_Q(q^2)}{g_A(q^2)+q^2/2m_N h_A(q^2)}$$ --> GTR

$$\mathrm{R}_2=\frac{m_q/m_N\langle P(\tau,\vec{q})\rangle-2i\langle q(\tau,\vec{q})\rangle}{iq_i\langle A_i(\tau,\vec{q})\rangle+\langle A_4(\tau,\vec{q})-A_4(\tau-1,\vec{q})\rangle}$$ --> AWI

$$\mathrm{R}_3=\frac{m_q/m_N\langle P(\tau,\vec{q})\rangle-2i\langle q(\tau,\vec{q})\rangle}{iq_i\langle A_i(\tau,\vec{q})\rangle+(E'-E)\langle A_4(\tau,\vec{q})\rangle}$$ --> GTR


32ID
R_1
$\tau - t_f/2$
32ID
R_3
$\tau - t_f/2$
32ID
R_2
$\tau - t_f/2$
24I
R_2
$t - t_f/2$
$t_f = 6a$
$t_f = 7a$
$t_f = 8a$

Orbital Angular Momentum

skyrmion

Trinacria, Erice

AWI vs Goldberger-Treiman Relation

- AWI is an operator relation which should be satisfied in any state.
- GTR is based on applying the derivative on the nucleon state which is susceptible to excited state contamination.
- $Z_A{}^0{}_{,norm}$ the same as $Z_A{}^3{}_{,norm}$ in CI.