NLO calculations for dilute-dense processes in the CGC picture

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Outline

Outline of this talk

- Dilute-dense processes, power counting
- NLO DIS: with massless quarks
  Balitsky & Chirilli 2010, Beuf 2017, Hänninen, T.L. , Paatelainen 2017
  + 1st numerical implementation Ducloué, Hänninen, T.L., Zhu 2017
- Loops in LCPT with massive quarks Beuf, T.L. Paatelainen, in progress

Trinity of dilute-dense CGC calculations

- Evolution equation (BK)
- Total DIS cross section
- Single inclusive hadron production in pA-collisions

+ essential question: doing the three consistently.

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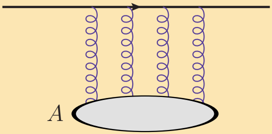
Trinity of dilute-dense CGC calculations

- Evolution equation (BK)
- **Total DIS cross section $\Longleftarrow$ this talk**
- Single inclusive hadron production in pA-collisions

+ essential question: doing the three consistently.

Dilute-dense processes

Eikonal scattering off target of glue



How to measure small-x glue?

- Dilute probe through target color field
- At high energy interaction is eikonal

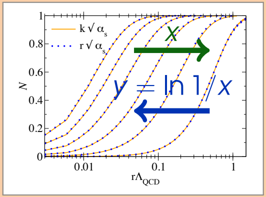
Eikonal scattering amplitude: Wilson line V

$$V = \mathbb{P}\exp\left\{-ig\int^{x^+} \mathrm{d}y^+ A^-(y^+, x^-, \mathbf{x})\right\} \underset{x^+\to\infty}{\approx} V(\mathbf{x}) \in \mathrm{SU}(N_\mathrm{c})$$

- Amplitude for color dipole

$$\mathcal{N}(r = |\mathbf{x}-\mathbf{y}|) = 1 - \left\langle \frac{1}{N_\mathrm{c}} \operatorname{Tr} V^\dagger(\mathbf{x}) V(\mathbf{y}) \right\rangle$$

from color transparency to saturation

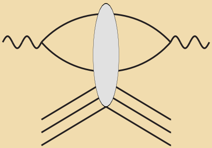


- $1/Q_\mathrm{s}$ is Wilson line **correlation length**

Dilute-dense process at LO

Physical picture at small x

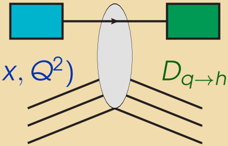
DIS



- $\gamma^* \to q\bar{q}$ dipole interacts with target color field
- Total cross section is $2\times$Im-part of amplitude

"Dipole model": Nikolaev, Zakharov 1991
Fits to HERA data:
e.g. Golec-Biernat, Wüsthoff 1998

Forward hadrons



- q/g from probe: collinear pdf
- $|\text{amplitude}|^2 \sim$ dipole
- Indep. fragmentation

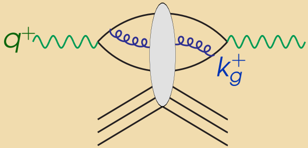
"Hybrid formalism";
Dumitru, Jalilian-Marian 2002

Both involve same dipole amplitude $\mathcal{N} = 1 - S$

Dilute-dense process at LL

Add one **soft** gluon: large logarithm of energy/x

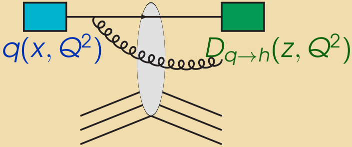
DIS



- Soft gluon: large logarithm

$$\int_{x_{Bj}} \frac{dk_g^+}{k_g^+} \sim \ln \frac{1}{x_{Bj}}$$

Forward hadrons



- Soft gluon $k^+ \to 0$:
  same large log
- Collinear gluon $k_T \to 0$:
  DGLAP evolution of pdf, FF

Dumitru et al 2005

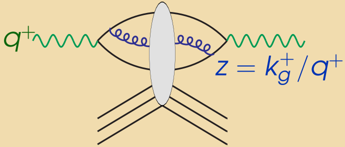
Absorb large log into renormalization of target:

BK equation Balitsky 1995, Kovchegov 1999

Dilute-dense process at NLO

Add one gluon, but **not** necessarily soft

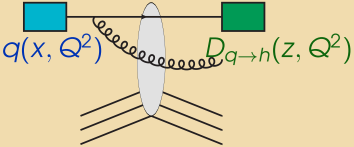
DIS



- DIS impact factor

  Balitsky & Chirilli 2010, Beuf 2017

Forward hadrons



- NLO single inclusive

  Chirilli et al 2011

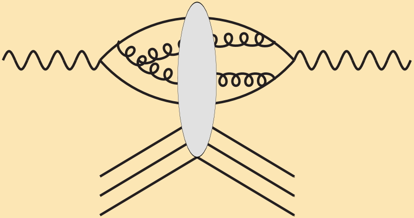
- Leading small-k^+ gluon already in BK-evolved target
- Need to **subtract** leading log from cross section:

$$\sigma_{NLO} = \int \mathrm{dz}\left[\overbrace{\sigma(z) - \sigma(z=0)}^{\sigma_{\text{sub}}} + \overbrace{\sigma(z=0)}^{\text{absorb in BK}}\right] \qquad z = \frac{k_g^+}{P_{\text{tot}}^+}$$

NLO to NLL

NLO evolution equation:

- Consider NNLO DIS
- Extract leading soft logarithm
- Lengthy calculation:
  Balitsky & Chirilli 2007
- But additional resummations needed for practical phenomenology



(+ many diagrams at same order)

- $\alpha_s^2 \ln^2(1/x)$: two iterations of LO BK
- $\alpha_s^2 \ln 1/x$: NLO BK
- α_s^2: part of NNLO impact factor (not calculated)

Summary: power counting

$$\sigma \sim \overbrace{\mathcal{O}(1)}^{\text{LO}} + \underbrace{\mathcal{O}(\alpha_s \ln 1/x)}_{\text{LL}} + \overbrace{\mathcal{O}(\alpha_s)}^{\text{NLO}} + \underbrace{\mathcal{O}(\alpha_s^2 \ln 1/x)}_{\text{NLL}}$$

- Current phenomenology LL
- Theory recently becoming understood at NLO & NLL
- Moving to phenomenology, numerical implementations:
  - Fit to DIS data with (approx) NLL evolution (but not NLO) : Albacete 2015, Iancu et al 2015
  - Single inclusive hadrons at NLO (but not NLL) : Stasto et al 2013, Ducloué et al 2015
  - Full NLL evolution (Not yet NLO) Mäntysaari 2015
  - NLO DIS cross section (Not yet NLL) Ducloué et al 2017

DIS at NLO, massless quarks

DIS at NLO: Fock state expansion

Balitsky & Chirilli 2010, Beuf 2016, 2017, H. Hänninen, T.L., Paatelainen 2017

To be specific: want total γ^*-target cross section

$$\sigma_\lambda^{\gamma^*} = 2\text{Re}\left[(-i)\mathcal{M}^{\text{fwd}}_{\gamma^*_\lambda \to \gamma^*_\lambda}\right],$$

using the optical theorem, with the total elastic amplitude

$$_i\langle \gamma_\lambda(\vec{q}', Q^2)|(\hat{\mathcal{S}}_E - \mathbf{1})|\gamma_\lambda(\vec{q}, Q^2)\rangle_i = 2q^+(2\pi)\delta(q'^+ - q^+) i\mathcal{M}^{\text{fwd}}_{\gamma^*_\lambda \to \gamma^*_\lambda}.$$

$\hat{\mathcal{S}}_E$: eikonal scattering $\Longrightarrow$ Wilson line in coordinate space.
At NLO need Fock state decomposition of $|\gamma_\lambda(\vec{q}, Q^2)\rangle_i$ up to g^2:

$$|\gamma_\lambda(\vec{q}, Q^2)\rangle_i = \sqrt{Z_{\gamma^*}}\Bigg[|\gamma_\lambda(\vec{q}, Q^2)\rangle + \sum_{q\bar{q}} \Psi^{\gamma^*\to q\bar{q}}|q(\vec{k}_0, h_0)\bar{q}(\vec{k}_1, h_1)\rangle$$
$$+ \sum_{q\bar{q}g} \Psi^{\gamma^*\to q\bar{q}g}|q(\vec{k}_0, h_0)\bar{q}(\vec{k}_1, h_1)g(\vec{k}_2, \sigma)\rangle + \cdots\Bigg]$$

with **Light Cone Wave Functions** $\Psi^{\gamma^*\to q\bar{q}}$ **and** $\Psi^{\gamma^*\to q\bar{q}g}$

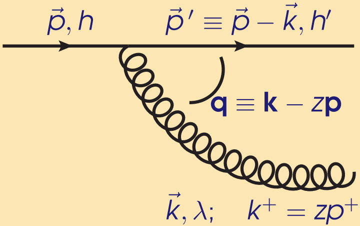
DIS at NLO: procedure

Beuf 2016, 2017, H. Hänninen, T.L., Paatelainen 2017

1. Evaluate LCPT diagrams
   - $\psi^{\gamma^* \to q\bar{q}}$ to 1 loop
   - $\psi^{\gamma^* \to q\bar{q}g}$ at tree level
2. Fourier-transform to transverse coordinate space
3. Square to get ${}_i\langle \gamma_\lambda(\vec{q}\,', Q^2)|(\hat{\mathcal{S}}_E - \mathbf{1})|\gamma_\lambda(\vec{q}, Q^2)\rangle_i$

LCPT rules:

- Intermediate (∋ "final") state k^- denominators
- On-shell vertices, most importantly $q\bar{q}g$

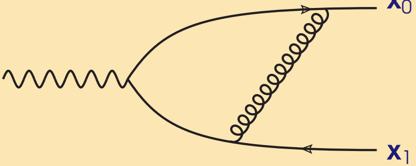


$$\left[\bar{u}_{h'}(p')\not{\varepsilon}^*_\lambda(k)u_h(p)\right] = \frac{-2}{z\sqrt{1-z}}\left[\left(1 - \frac{z}{2}\right)\delta_{h',h}\delta^{ij} + \frac{z}{2}ih\delta_{h',h}\varepsilon^{ij}\right]\mathbf{q}^i\varepsilon^{*j}_\lambda,$$

(This is in $d = 4$, generalize for $d < 4$)

Note 2 index structures for massless quarks.

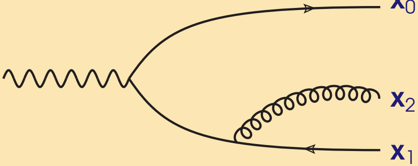
DIS at NLO: real and virtual corrections



Virtual corrections,
interaction with target

$$\mathcal{N}(\mathbf{x}_0, \mathbf{x}_1)$$

+ UV divergence in loop



Real corrections,
interaction with target

$$\mathcal{N}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)$$

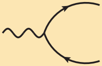
UV divergence in $\mathbf{x}_2$-integral

These UV-divergences cancel because

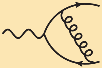
$$\mathcal{N}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2 \to \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2 \to \mathbf{x}_1) = \mathcal{N}(\mathbf{x}_0, \mathbf{x}_1)$$

DIS at NLO: subtraction of BK

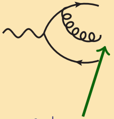
Evaluate cross section as $\sigma^{\text{NLO}}_{L,T} = \sigma^{\text{LO}}_{L,T} + \sigma^{\text{dip}}_{L,T} + \sigma^{qg}_{L,T,\text{sub.}}$



$$\Rightarrow \quad \sigma^{\text{LO}} \sim \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} |\psi^{\text{LO}}_{\gamma^*\to q\bar{q}}(z_1,\mathbf{x}_0,\mathbf{x}_1)|^2 \mathcal{N}_{01}(x_{Bj})$$



$$-\,* \Rightarrow \quad \sigma^{\text{dip}} \sim \alpha_s C_F \int_{\mathbf{x}_0,\mathbf{x}_1,z_1} \left|\psi^{\text{LO}}_{\gamma^*\to q\bar{q}}\right|^2 \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right] \mathcal{N}_{01}(x_{Bj})$$



$$+\,* \Rightarrow \quad \sigma^{qg}_{\text{sub.}} \sim \alpha_s C_F \int_{z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2))$$

- LL

$k_g^+ \sim z_2$

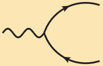
$$- \left|\psi_{\gamma^*\to q\bar{q}g}(z_1,0,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2)) \Big].$$

* UV-divergence

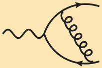
- LL: subtract leading log, already in BK-evolved $\mathcal{N}$

DIS at NLO: subtraction of BK

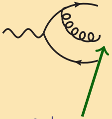
Evaluate cross section as $\sigma^{\mathrm{NLO}}_{L,T} = \sigma^{\mathrm{LO}}_{L,T} + \sigma^{\mathrm{dip}}_{L,T} + \sigma^{qg}_{L,T,\mathrm{sub.}}$



$$\Rightarrow \quad \sigma^{\mathrm{LO}} \sim \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} |\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}(z_1,\mathbf{x}_0,\mathbf{x}_1)|^2 \mathcal{N}_{01}(x_{Bj})$$



$$- * \Rightarrow \quad \sigma^{\mathrm{dip}} \sim \alpha_s C_\mathrm{F} \int_{\mathbf{x}_0,\mathbf{x}_1,z_1} \left|\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}\right|^2 \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right] \mathcal{N}_{01}(x_{Bj})$$



- LL

$k_g^+ \sim z_2$

$$+ * \Rightarrow \quad \sigma^{qg}_{\mathrm{sub.}} \sim \alpha_s C_\mathrm{F} \int_{z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2)) - \left|\psi_{\gamma^*\to q\bar{q}g}(z_1,0,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2)) \Big].$$

* UV-divergence

- LL: subtract leading log, already in BK-evolved $\mathcal{N}$
- Parametrically $X(z_2) \sim x_{Bj}$, but $X(z_2) \sim 1/z_2$ essential!

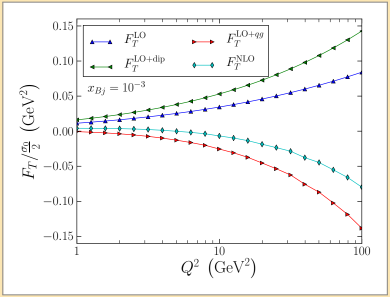
  ($X(z_2)$ =momentum fraction to which the target is evolved)

Numerical implementation

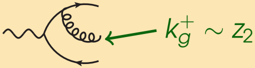
Ducloué, Hänninen, T.L., Zhu 2017

$$\sigma_{L,T}^{qg,\text{sub.}} \sim \alpha_s C_F \int_{z_1,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \int_{x_{Bj}/x_0}^{1} \frac{dz_2}{z_2}\left[\mathcal{K}_{L,T}^{\text{NLO}}(z_2, X(z_2)) - \mathcal{K}_{L,T}^{\text{NLO}}(0, X(z_2))\right].$$

- Target fields at scale $X(z_2)$:
  - $X(z_2) = x_{Bj}$: unstable
    (like single inclusive)



$X(z_2) = x_{Bj}$



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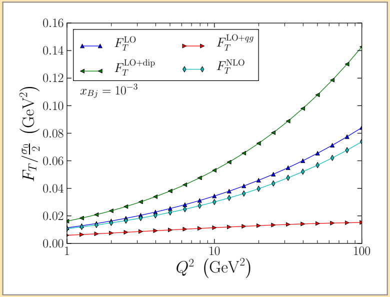
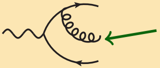
- Target fields at scale $X(z_2)$:
  - $X(z_2) = x_{Bj}$: unstable (like single inclusive)
  - $X(z_2) = x_{Bj}/z_2$ OK



$X(z_2) = x_{Bj}/z_2$



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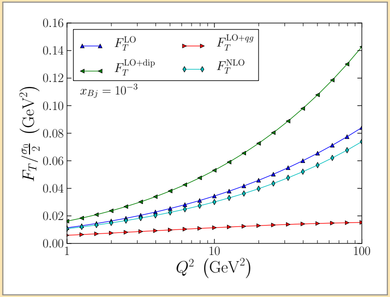
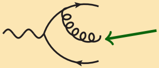
- Target fields at scale $X(z_2)$:
  - $X(z_2) = x_{Bj}$: unstable (like single inclusive)
  - $X(z_2) = x_{Bj}/z_2$ OK
- Lower limit of z_2
  - $z_2 > \frac{x_{Bj}}{x_0}$ from target k^- (assuming ${k_T}^2 \sim Q^2$)
  - Strict k^+ factorization: $z_2 > \frac{x_{Bj}}{x_0} \frac{M_p^2}{Q^2}$ $\Longrightarrow$ would require kinematical constraint
  - For "dipole" term integrate to $z_2 = 0$



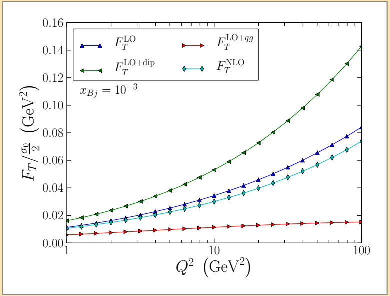
$X(z_2) = x_{Bj}/z_2$



1st numerical implementation: general features

Ducloué, Hänninen, T.L., Zhu 2017

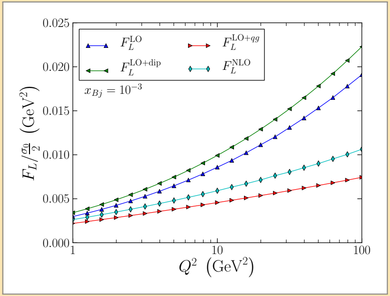
- Major cancellation between different NLO terms



1st numerical implementation: general features

Ducloué, Hänninen, T.L., Zhu 2017

- Major cancellation between different NLO terms (similar for F_L)



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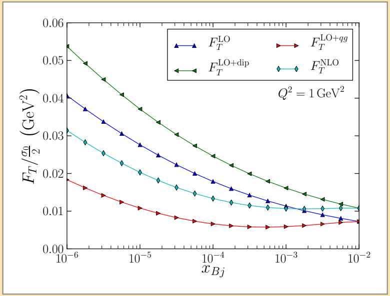
- Major cancellation between different NLO terms (similar for F_L)
- qg-term explicitly zero at $x_{Bj} = x_0 \implies$ transient effect



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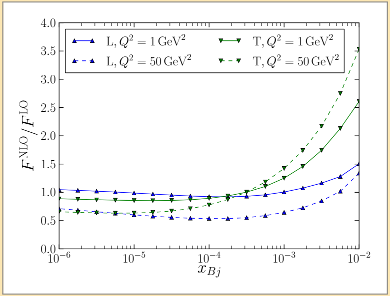
- Major cancellation between different NLO terms (similar for F_L)
- qg-term explicitly zero at $x_{Bj} = x_0 \implies$ transient effect
- Running coupling (parent dipole)
  - Transient effect larger

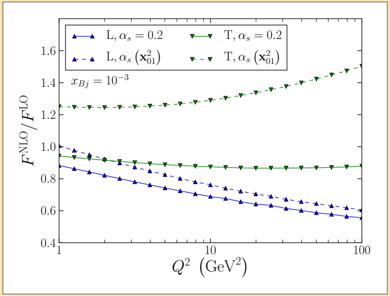


NLO/LO ratio

1st numerical implementation: general features

Ducloué, Hänninen, T.L., Zhu 2017

- Major cancellation between different NLO terms (similar for F_L)
- qg-term explicitly zero at $x_{Bj} = x_0 \implies$ transient effect
- Running coupling (parent dipole)
  - Transient effect larger
  - But Q^2-dependence stable



NLO/LO ratio

Overall conclusions

- NLO corrections of expected relative magnitude
- Need to think about $X(z_2)$, z_2 limits for actually fitting data

DIS at NLO, massive quarks

Motivation, issues

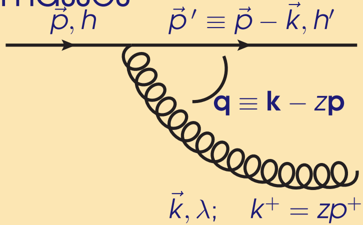
- There is data! F_2^c from HERA, charm will be measured at EIC, both inclusive and exclusive
- LO F_2^c problematic. Dirty little secret: heavy quarks in rcBK fits do not actually work!
  - AAMQS fit has separate proton area σ_0 for q and Q: good χ^2 but clearly unphysical
  - Fit by T.L., Mäntysaari 2013 : only light quarks: straightforward generalization does not work
  - Collinear resummed fit by Iancu et al 2015 better, but only uses old HERA data with large errors
  - b-dependent JIMWLK Mäntysaari, Schenke 2018 : F_2 and F_{2c} not described simultaneously

LCPT loops with massive quarks are also fun!

- New Lorentz structures: rotational invariance constraints
- Approach in this talk: start with same regularization (cutoff in k^+ + $\perp$ dim. reg.) that was used for massless case

Elementary vertex with masses

$$\left[\bar{u}_{h'}(p')\not{\varepsilon}^*_\lambda(k)u_h(p)\right] \sim$$



$$\overbrace{\bar{u}_{h'}\gamma^+ u_h}^{\sim\delta_{h,h'}} \delta^{ij} q^i \varepsilon^{*j}_\lambda + \overbrace{\bar{u}_{h'}\gamma^+[\gamma^i,\gamma^j]u_h}^{\sim\delta_{h,h'}} q^i \varepsilon^{*j}_\lambda + \overbrace{\bar{u}_{h'}\gamma^+\gamma^j u_h}^{\sim\delta_{h,-h'}} m_q \varepsilon^{*j}_\lambda$$

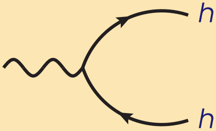
- New 3rd spin-flip structure (light cone helicity flip if you wish)
- Note: no $\perp$ momentum in spin-flip vertex

What are new *UV*-divergent and finite contributions?

1. "Vertex correction" diagrams: calculation more complicated, but conceptually simpler
2. "Propagator correction" diagrams: calculation simple, interpretation not!

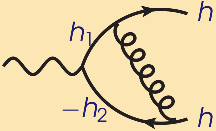
Vertex corrections to spin flip vertex

Look at spin flip part



$\sim m_q$

Corrections from

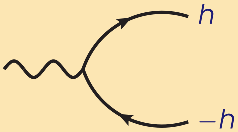


, 2 options

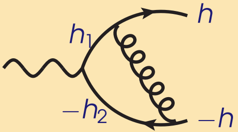
- 1 spin-flip vertex: $h_1 \neq h$, $h_2 \neq h_1$ or $h_2 \neq h$
  $\Longrightarrow$ log-divergent $\sim m_q \frac{1}{\varepsilon}$ (2 ED's $\sim \mathbf{k}^2$ each, 2 vertices $\mathbf{k}$ each)
  $\Longrightarrow$ absorb into **vertex mass** counterterm δm_v,
  same as δm_q in conventional perturbation theory
- 3 spin-flip vertices: $h_1 \neq h$, $h_2 \neq h_1$ and $h_2 \neq h$
  $\Longrightarrow$ finite NLO contribution

Vertex corrections to non-spin flip vertex

Non-spin flip part



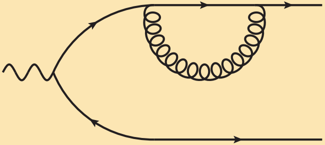
Corrections from

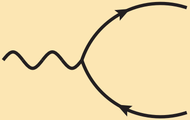


2 options

- no spin-flip vertex: $h_1 = h$, $h_2 = h_1$ and $h_2 \neq -h$
  mass only modifies ED's $\Longrightarrow$ not new contribution
- 2 spin-flip + 1 non-flip $h_1 = h$ or $h_2 = h_1$ or $h_2 = -h$
  $\Longrightarrow$ again finite NLO contribution
  (2 ED's $\sim \mathbf{k}^2$ each, 1 vertex $\sim \mathbf{k}$, finite integral
  $\sim \int d^2\mathbf{k} \frac{\mathbf{k}}{((\mathbf{k}-\dots)^2+\dots)((\mathbf{k}-\dots)^2+\dots)}$)

Quark propagator corrections

In  can have 0 or 2 spin-flip vertices

- m_q-dependent divergence $\sim$  $\times \frac{m_q^2}{\Delta k_{\text{LO}}^-} \frac{1}{\varepsilon}$
- Can absorb into a renormalization of m_q in ED of LO LCWF
- But problem: this **kinetic mass** counterterm δm_k is **not** same as the previous δm_v
- In fact δm_v is same as in covariant theory, δm_k different
- This has been known for a long time e.g. Haridranath, Zhang, also Burkardt in Yukawa th.

Mass renormalization

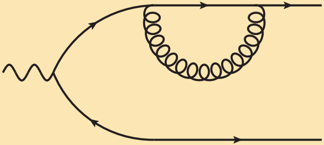
- 2 conceptually different masses:
  - Kinetic mass: relates energy and momentum
  - Vertex mass: amplitude of spin-flip in gauge boson vertex
- 1 parameter in Lagrangian, but 2 parameters in LCPT Hamiltonian — and thus in quantization
- Lorentz-invariance requires they stay the same
- In practical LCPT calculations so far used k^+-cutoff and $\perp$ dim. reg. violates rot. inv. $\implies m_v \neq m_k$ at loop level.

There are 3 options to deal with this

1. Regularize as before, but use additional renormalization condition to set separately m_v and m_k $\implies$ discuss next
2. Use some other regularization $\implies$ finite parts hard!
3. Smartly combine with instantaneous "normal ordering" diagrams before integrating $\implies$ can explicitly keep $m_k = m_v$; also doable. For details see Beuf @ Hard Probes 2018

Two mass renormalization conditions

- One condition: pole mass scheme, require mass term in



$\to 0$ when $ED_{\mathrm{LO}} \to 0$

$\Longrightarrow$ determines δm_k

  - For γ_L this is the only mass renormalization needed

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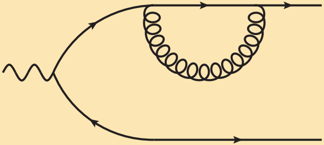

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- Other condition? Rotational invariance!
  Calculate one loop $\mathcal{M}(\gamma^* \to q\bar{q})$ for
  - Timelike virtual γ^*, $q^2 = M^2$
  - On shell final state $M^2 = (\mathbf{k}_q^2 + m^2)/(z(1-z))$

  Same diagrams as for spacelike γ^*. Then require

$$\mathcal{M}(\gamma_L^* \to q\bar{q}) = \mathcal{M}(\gamma_T^* \to q\bar{q})$$

$\Longrightarrow$ fixes δm_v to conventional value.

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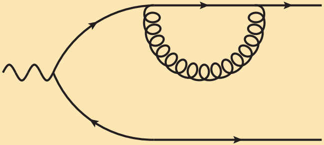

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Not happy?

Fiddling around with normal ordering diagrams gives same result + gauge invariance, $m_g = 0 \ldots$, but that's another talk

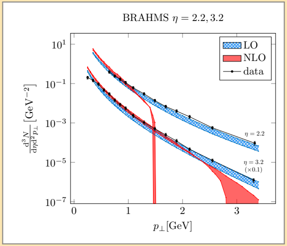
In stead of conclusions: to do for NLO DIS

$$\sigma \sim \underbrace{\mathcal{O}(1)}_{\text{LO}} + \underbrace{\mathcal{O}(\alpha_s \ln 1/x)}_{\text{LL}} + \underbrace{\mathcal{O}(\alpha_s)}_{\text{NLO}} + \underbrace{\mathcal{O}(\alpha_s^2 \ln 1/x)}_{\text{NLL}}$$

- Next: fit to HERA data with NLO impact factor
  (with LL or NLL evolution)
- Needs implementation (both DIS and single inclusive) :
  match NLL evolution with NLO cross section:
  - Evolution variable k^+ vs k^-
  - Kinematical constraint vs
    rapidity local resummation of double logs
  - Corresponding different subtractions from cross sections
- Loop calculation ongoing: quark masses
- Other:
  - Exclusive processes
  - Dihadron correlations

Backups: single inclusive

Negative cross sections

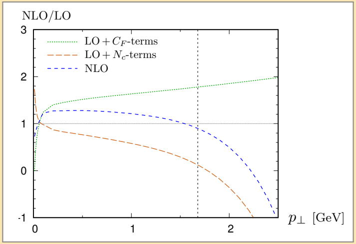


- Analytical calculation
  Chirilli, Xiao, Yuan 2012
- Numerics: Stasto, Xiao, Zaslavsky 2013
  $\Longrightarrow$ cross section negative
  (large N_c; mix C_F and N_c terms)
- Kinematics? Large k_T logs?? Beuf et al 2014, Watanabe, Xiao & Zaslavsky 2015

Ducloué, T.L., Zhu 2016: q channel at finite N_c

also Kang et al 2014

- Problem is in the rapidity divergence
- Most easily identified by color factor



Unsubtracted cross section, N_c-term

Discussion here following Iancu et al 2016 leave out C_F/DGLAP-terms

$$\frac{\mathrm{d}N^{\mathrm{LO}+N_c}}{\mathrm{d}^2\mathbf{k}\,\mathrm{d}y} \sim \mathcal{S}_0(k_T) + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi}\mathcal{K}(k_T, \xi, X(\xi))$$

- Dipole operator $\mathcal{S}_0$ is "bare"
- Rapidity at which dipoles are evaluated $X(\xi)$
- x_g: the target momentum fraction for LO kinematics
- Multi-Regge-kinematics: $X(\xi) = x_g/(1-\xi)$
- Only target $X(\xi) < x_0 \implies$ phase sp. limit $\xi < 1 - x_g/x_0$:

$$\text{BK:} \quad \mathcal{S}(k_T, x_g) = \mathcal{S}(k_T, x_0) + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi}\mathcal{K}(k_T, 1, X(\xi))$$

Combine these, taking $\mathcal{S}(k_T, x_0) \equiv \mathcal{S}_0(k_T) \ldots$

Subtracted form for cross section

Unsubtracted form

$$\mathcal{S}_0(k_T) + \alpha_s \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi} \mathcal{K}(k_T, \xi, X(\xi))$$

$$= \mathcal{S}(k_T, x_g) + \alpha_s \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi} \left[\mathcal{K}(k_T, \xi, X(\xi)) - \mathcal{K}(k_T, 1, X(\xi))\right]$$

subtracted form

(Recall: dipoles evaluated at rapidity $X(\xi)$)

- These are strictly equivalent, perfectly positive at all k_T
- Subtracted form is a true perturbative series
  unsubtracted has $\alpha_s \ln 1/x$ and α_s together

Origin of negativity in CXY

$$\frac{\mathrm{d}N^{\mathrm{LO}+N_c}}{\mathrm{d}^2\mathbf{k}\,\mathrm{d}y} \sim \mathcal{S}(k_T, x_g) + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi} \left[\mathcal{K}(k_T, \xi, X(\xi)) - \mathcal{K}(k_T, 1, X(\xi))\right]$$

How do CXY get a negative cross section?

- $\mathcal{K}(k_T, \xi, X(\xi)) - \mathcal{K}(k_T, 1, X(\xi))$ dominated by $\xi \ll 1$
- Replace $X(\xi) \to X(\xi = 0) = x_g$
- Change ξ integration limit to 1 (+ distribution!)

This gives CXY subtraction scheme

$$\frac{\mathrm{d}N^{\mathrm{LO}+N_c}}{\mathrm{d}^2\mathbf{k}\,\mathrm{d}y} \sim \mathcal{S}(k_T, x_g) + \alpha_s \int_0^1 \frac{\mathrm{d}\xi}{1-\xi} \Big[\overbrace{\mathcal{K}(k_T, \xi, x_g)}^{\sim \xi/k_T{}^4 \text{ for } k_T \gg Q_s} - \mathcal{K}(k_T, 1, x_g) \Big]$$

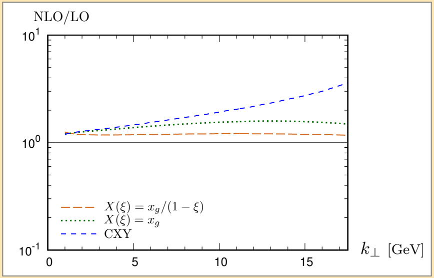
- Formally ok in α_s expansion
- Nice factorized form: only dipoles at x_g, like LO
- But subtraction no longer integral form of BK

Comparing subtraction procedures

First:
must also make choice for $X(\xi)$ in the C_F-term:
scheme dependence

▼

Take same $X(\xi)$ & limits as N_c-term



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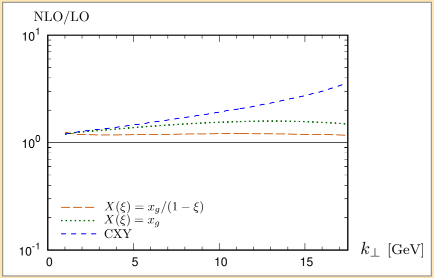

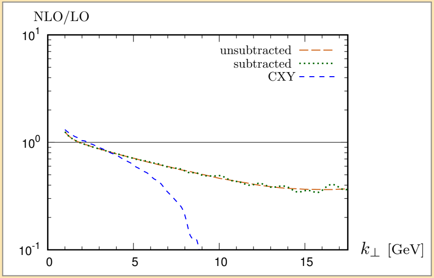

Two forms for NLO cross section

- Explicitly equivalent
- Positive, although $\ll$ LO