Status and prospects of GPD extraction from DVCS

Krešimir Kumerički

University of Zagreb, Croatia

Probing Nucleons and Nuclei in High Energy Collisions

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Outline

1. **Introduction and status of fits to DVCS data**

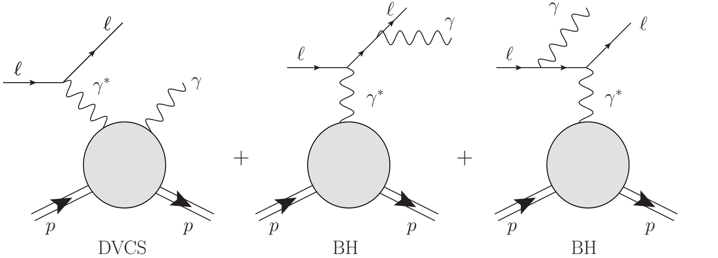
2. **Uncertainties?**

3. **Neural net fits**

4. **Summary**

Access to GPDs via DVCS

- Deeply virtual Compton scattering (DVCS) — "gold plated" process of exclusive physics
- DVCS is measured via leptoproduction of a photon



- Interference with Bethe-Heitler process gives unique access to both real and imaginary part of DVCS amplitude.

DVCS cross section

$$d\sigma \propto |\mathcal{T}|^2 = |\mathcal{T}_{\rm BH}|^2 + |\mathcal{T}_{\rm DVCS}|^2 + \mathcal{I} \, .$$

- where *e. g.* interference term is

$$\mathcal{I} \quad \propto \quad \frac{-e_\ell}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\},$$

- where *e. g.* $c_1^{\mathcal{I}}$ harmonic for unpolarized target is

$$c_{1,\rm unpol.}^{\mathcal{I}} \propto \left[F_1 \,\mathfrak{Re}\, \mathcal{H} - \frac{t}{4M_p^2} F_2 \,\mathfrak{Re}\, \mathcal{E} + \frac{x_{\rm B}}{2 - x_{\rm B}} (F_1 + F_2) \,\mathfrak{Re}\, \widetilde{\mathcal{H}} \right]$$

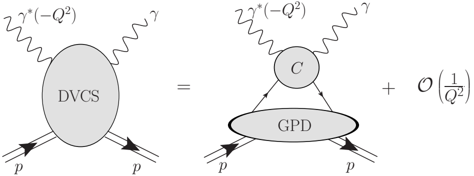
- and at leading order everything depends on four complex

Compton form factors (CFFs)

$$\mathcal{H}(\xi, t, Q^2), \quad \mathcal{E}(\xi, t, Q^2), \quad \widetilde{\mathcal{H}}(\xi, t, Q^2), \quad \widetilde{\mathcal{E}}(\xi, t, Q^2)$$

Factorization of DVCS $\longrightarrow$ GPDs

- [Collins et al. '98]



- CFFs are convolution:

$$ {}^{a}\mathcal{H}(\xi, t, Q^2) = \int \mathrm{d}x \; C^a(x, \xi, \frac{Q^2}{Q_0^2}) \; H^a(x, \xi, t, Q_0^2) $$

$a{=}q, G$

- $H^a(x, \eta, t, Q_0^2)$ — Generalized parton distribution (GPD)

Hybrid GPD models for global fits

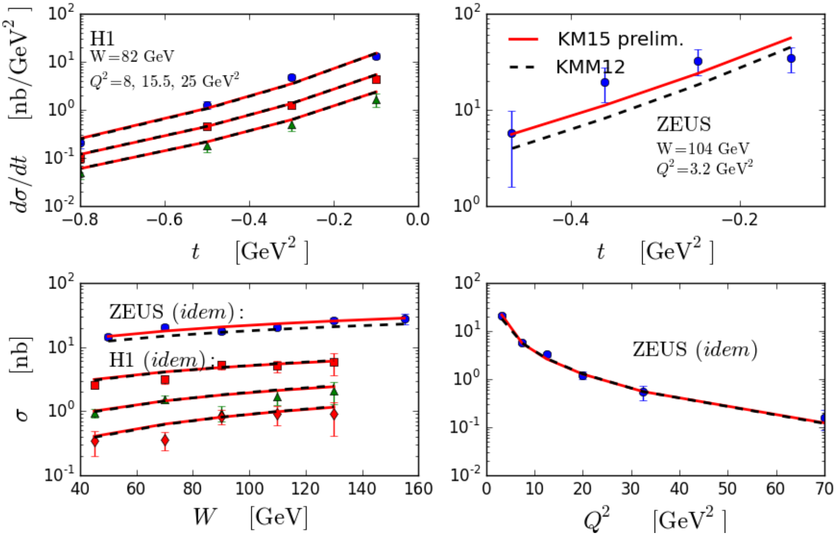
- [K.K., Müller '09-'15]
- Sea quarks and gluons modelled using SO(3) partial wave expansion in conformal GPD moment space + $\mathcal{Q}^2$ evolution.
- Valence quarks — model CFFs directly (ignoring $\mathcal{Q}^2$ evolution):

$$\mathfrak{Im}\,\mathcal{H}(\xi,t) = \pi\left[\frac{4}{9}H^{u_{\rm val}}(\xi,\xi,t) + \frac{1}{9}H^{d_{\rm val}}(\xi,\xi,t) + \frac{2}{9}H^{\rm sea}(\xi,\xi,t)\right]$$

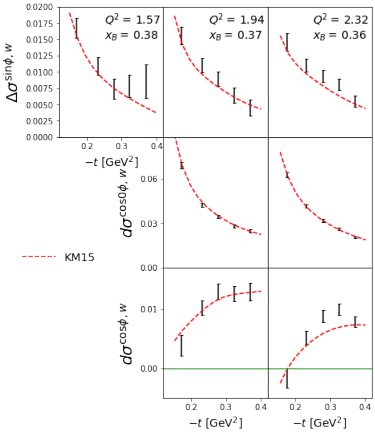
$$H(x,x,t) = n\, r\, 2^{\alpha}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}\,.$$

- $\mathfrak{Re}\,\mathcal{H}$ determined by dispersion relations
- 15 free parameters in total for H, $\tilde{H}$, E, $\tilde{E}$.

Fit examples (1/2): H1/ZEUS



Fit examples (2/2): JLab's Hall A (2015)



- KM15 global fit is fine. $\chi^2/n_{\rm d.o.f.} = 240./275$ ✓

Global fit χ^2 values: KM and PARTONS

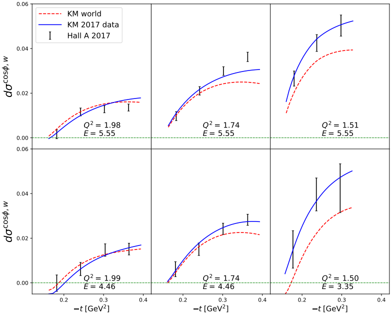
[K.K., Müller '09-'15]

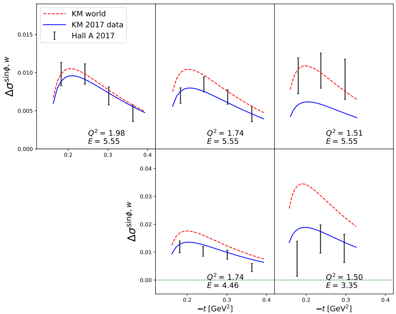
Collaboration	Observable	Ref.	$n_{\rm pts}$	KMM12		KM15	
				$\chi^2/n_{\rm pts}$	pull	$\chi^2/n_{\rm pts}$	pull
ZEUS	$\sigma_{\rm DVCS}$	[19, 20]	11	0.49	-1.76	0.51	-1.74
ZEUS,H1	$d\sigma_{\rm DVCS}/dt$	[19, 21, 22]	24	0.97	0.85	1.04	1.37
HERMES	$A_{\rm C}^{\cos 0\phi}$	[23]	6	1.31	0.49	1.24	0.29
HERMES	$A_{\rm C}^{\cos\phi}$	[23]	6	0.24	-0.56	0.07	-0.20
HERMES	$A_{\rm LU,I}^{\sin\phi}$	[23]	6	2.08	-2.52	1.34	-1.28
CLAS	$A_{\rm LU}^{\sin\phi}$	[24]	4	1.28	2.09		
CLAS	$A_{\rm LU}^{\sin\phi}$	[4, 25]	13			1.24	0.63
CLAS	$\Delta\sigma^{\sin\phi,w}$	[7]	48			0.41	-1.66
CLAS	$d\sigma^{\cos 0\phi,w}$	[7]	48			0.16	-0.21
CLAS	$d\sigma^{\cos\phi,w}$	[7]	48			1.16	6.36
Hall A	$\Delta\sigma^{\sin\phi,w}$	[5]	12	1.06	-2.55		
Hall A	$d\sigma^{\cos 0\phi,w}$	[5]	4	1.21	2.14		
Hall A	$d\sigma^{\cos\phi,w}$	[5]	4	3.49	-0.26		
Hall A	$\Delta\sigma^{\sin\phi,w}$	[6]	15			0.81	-2.84
Hall A	$d\sigma^{\cos 0\phi,w}$	[6]	10			0.40	0.92
Hall A	$d\sigma^{\cos\phi,w}$	[6]	10			2.52	-2.42
HERMES,CLAS	$A_{\rm UL}^{\sin\phi}$	[18, 26]	10	1.90	-1.89	1.10	-1.94
HERMES	$A_{\rm LL}^{\cos 0\phi}$	[26]	4	3.44	2.17	3.19	1.99
HERMES	$A_{\rm UT,I}^{\sin(\phi-\phi_S)\cos\phi}$	[27]	4	0.90	0.61	0.90	0.71
CLAS	$A_{\rm UL}^{\sin\phi}$	[4]	10			0.76	0.38
CLAS	$A_{\rm LL}^{\cos 0\phi}$	[4]	10			0.50	-0.22
CLAS	$A_{\rm LL}^{\cos\phi}$	[4]	10			1.54	2.40

[Moutarde, Sznajder, Wagner, '18]

No.	Collab.	Year	Ref.	χ^2	n	χ^2/n
1	HERMES	2001	13	9.8	10	0.98
2		2006	114	2.9	4	0.72
3		2008	115	24.2	18	1.35
4		2009	116	40.1	35	1.15
5		2010	117	40.3	18	2.24
6		2011	118	14.5	24	0.60
7		2012	119	25.4	35	0.73
8	CLAS	2001	14	—	0	—
9		2006	120	0.9	2	0.47
10		2008	121	371.1	283	1.31
11		2009	122	36.4	22	1.66
12		2015	123	351.4	311	1.13
13		2015	124	937.9	1333	0.70
14	Hall A	2015	112	220.2	228	0.97
15		2017	113	258.8	276	0.94
16	COMPASS	2018	55	10.7	1	10.67

Including Hall A 2017 data in global world fit: fail ✗





Global world DVCS data fit

before 2017	including 2017 Hall A
$\chi^2/n_{\rm d.o.f} = 240./275$ ✓	$\chi^2/n_{\rm d.o.f} = 545./337$ ✗

Uncertainties?

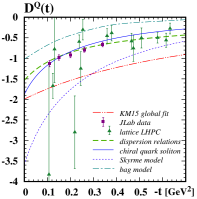
Example: Uncertainty of D-term

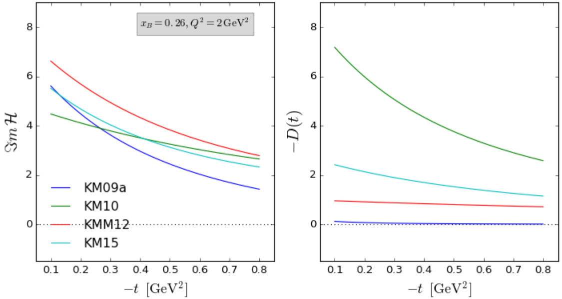
- D-term is a part of GPD that has nice physical interpretation (related to pressure distribution inside nucleon [M. Polyakov '03]), see talk by [P. Schweitzer] tomorrow
- within some approximations and up to a charge-related prefactor of order one it is equal to subtraction constant in dispersion relation for CFFs $\mathcal{H}$ and $\mathcal{E}$ [O. Teryaev '05]:

$$D(t) \sim \Delta(t) = \mathfrak{Re}\,\mathcal{H}(\xi, t) - \frac{1}{\pi}\mathrm{P.V.}\int_0^1 dx \frac{2x}{\xi^2 - x^2}\,\mathfrak{Im}\,\mathcal{H}(x, t)$$

- In KM fits [K.K., D. Müller], $D(t) = D/(1 - t/M_D^2)^2$ and is extracted directly by fits (where $\mathfrak{Re}\,\mathcal{H}$ is then determined by dispersion relations)

Extractions of D-term





- Fit parameter uncertainties of $D(t)$ are $\sim$ 20%, but systematic uncertainty due to model selection is unknown and presumably much larger!
- [V. D. Burkert, F.-X. Girod and L. Elouadrhiri '18] use just CLAS $d\sigma$ and $\Delta\sigma$ DVCS data to extract very precise value of D-term

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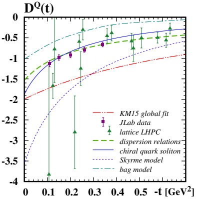

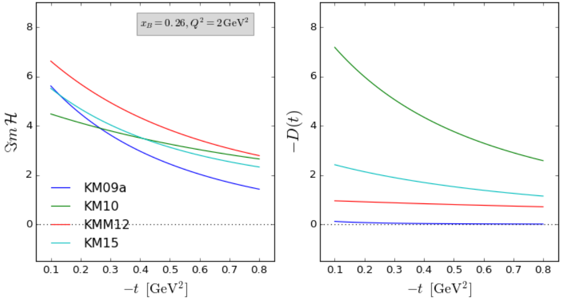

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- Data science: total error = bias + variance

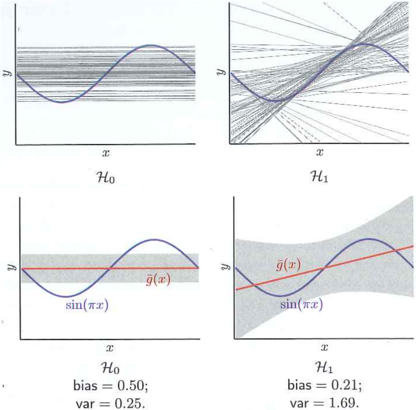
Bias-variance tradeoff: toy example

- "Unknown" $f(x) = sin(\pi x)$ "measured" at two points.

$\mathcal{H}_0$ - rigid (biased) $\mathcal{H}_1$ - flexible

$h(x) = a$ $h(x) = ax + b$

[Abu-Mostafa et al. '12]



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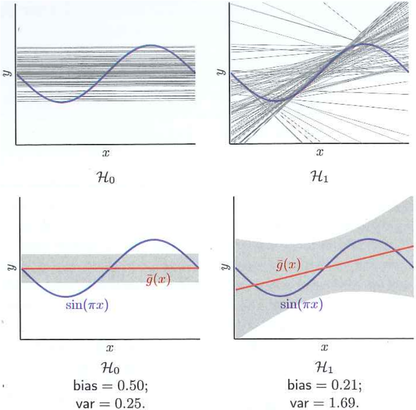

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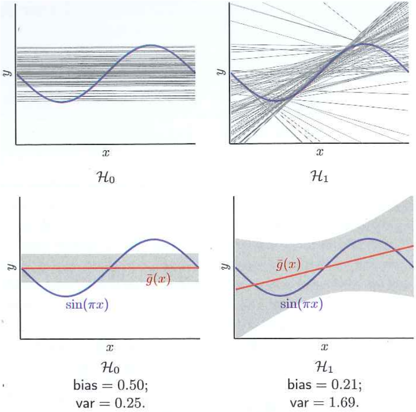
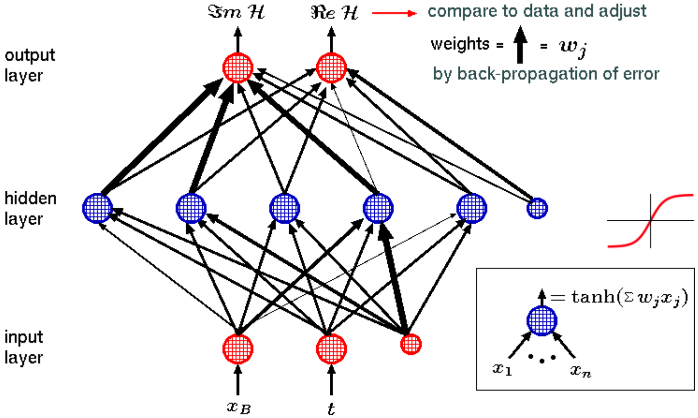


error = bias + variance

- In DVCS situation is the opposite! We need to decrease bias.
- Neural networks are proven to be unbiased

Neural net fits

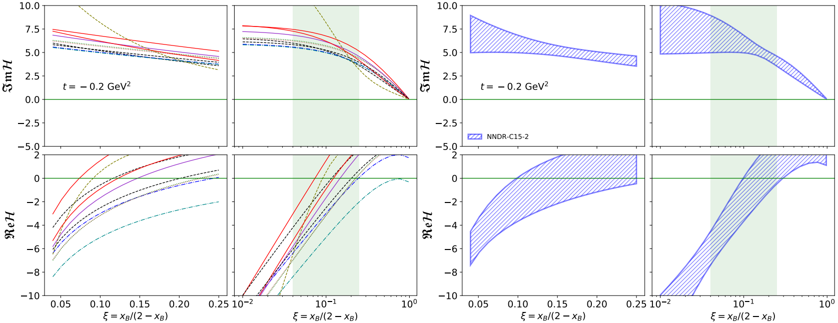
Fitting with neural networks



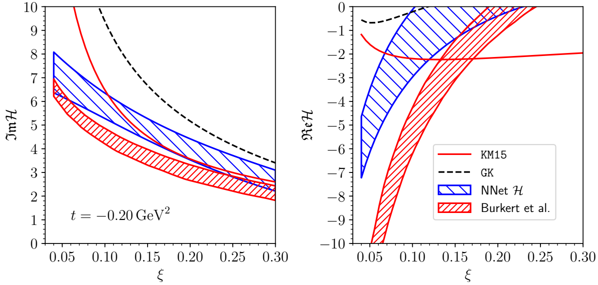
- Essentially a least-square fit of a complicated many-parameter function. $f(x) = \tanh(\sum w_i \tanh(\sum w_j \cdots))$ $\Rightarrow$ no theory bias

Study A: NN fit to CLAS 2015 data

- We start by fitting just to the CLAS 2015 $d\sigma$ and $\Delta\sigma$ measurements [Jo et al. '15], and just $\mathcal{H}$
- We utilize dispersion relations (one NNet represents $\mathfrak{Im}\,\mathcal{H}$, another represents $D(t)$)
- Uncertainty is estimated by averaging over ensemble of neural nets:



Comparison to [Burkert et al. '18]



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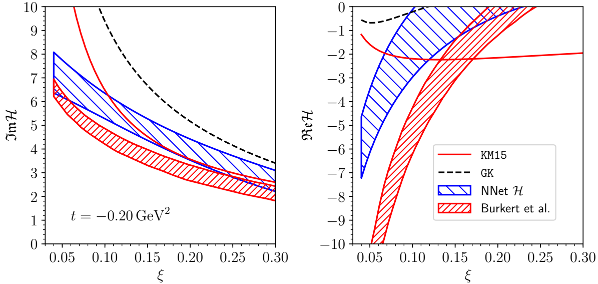
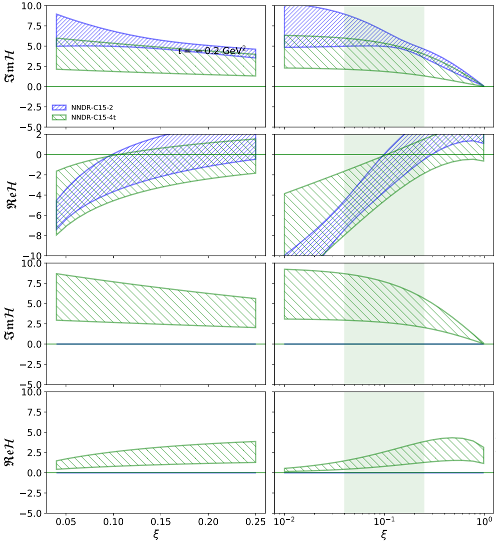
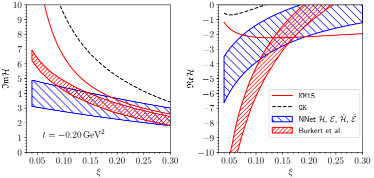


- But what is the effect of using the limited set of data, and assumption of $\mathcal{H}$ dominance?
- $\chi^2/n_{\rm pts} = 1725/2028$ (this NNet), $1912/2028$ (KM15), $2322/2028$ (GK)

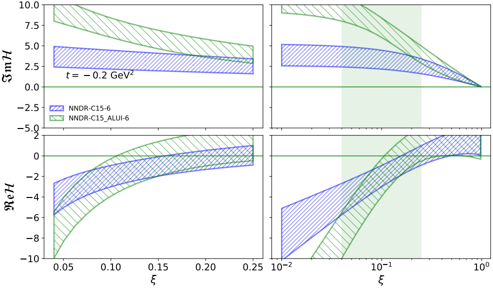
More flexible model: $\mathcal{H} + \widetilde{\mathcal{H}}$





Adding more data points

- Adding HERMES $A_{LU,I}$ data. (Model now includes $\mathcal{H}$ and $\mathcal{E}$)



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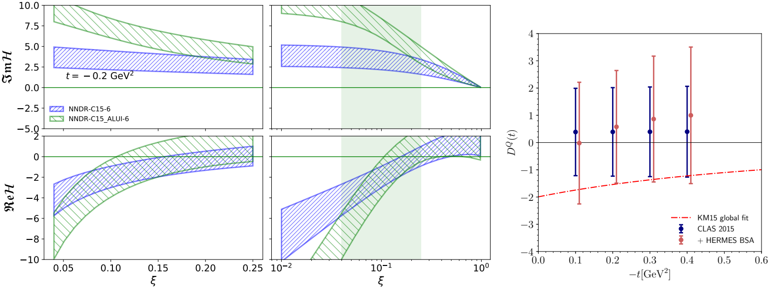
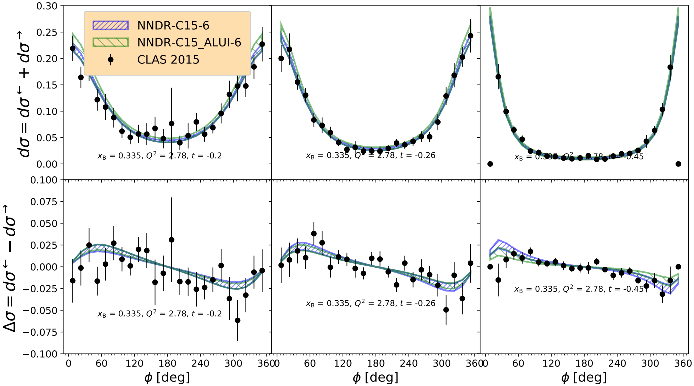


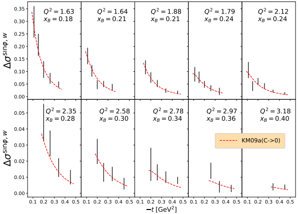
- CLAS15 data alone is still consistent with zero D-term.

Propagating uncertainties back to $d\sigma$ and $\Delta\sigma$

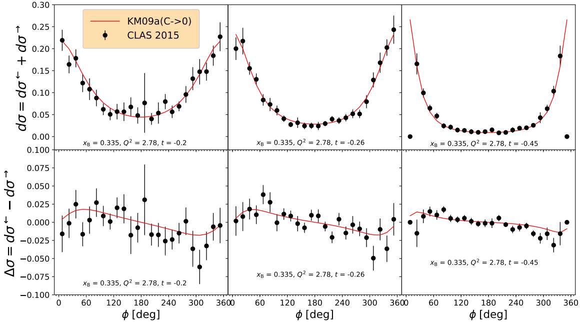


- Small propagated error is due to small sensitivity of these observables to CFFs (and D-term).

Independent simple way to the same conclusion



- `KM09a` model with subtraction constant $C \to 0$ describes CLAS 2015 $d\sigma$ and $\Delta\sigma$ data correctly
- $\chi^2/n_{\rm pts} = 1746/2023$ ✓



Study B: NN fit to world fixed target data

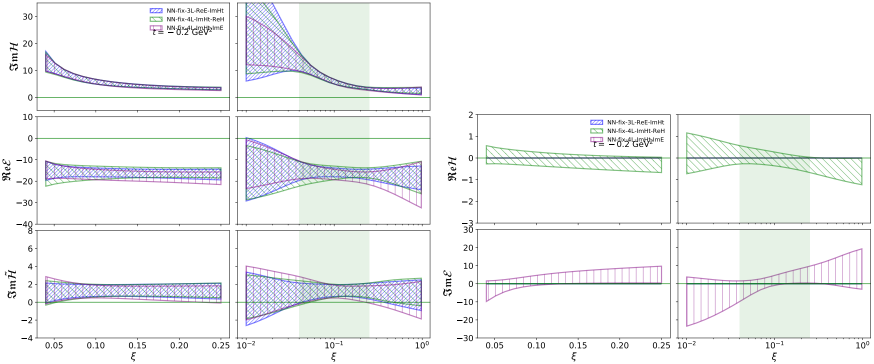
- Representative subset of world DVCS fixed target data:

```
npt x obs    collab  harm.  ref.
------------------------------------------
 6 x ALUI   HERMES  -1.0    arXiv:1203.6287
12 x AUTDVCS  HERMES  0     arXiv:0802.2499
12 x AUTI   HERMES  1.0     arXiv:0802.2499
 6 x BCA    HERMES  0.0     arXiv:1203.6287
 6 x BCA    HERMES  1.0     arXiv:1203.6287
12 x BSDw   CLAS    -1      arXiv:1504.02009
15 x BSDw   HALLA   -1      arXiv:1504.05453
12 x BSSw   CLAS    0.0     arXiv:1504.02009
12 x BSSw   CLAS    1.0     arXiv:1504.02009
10 x BSSw   HALLA   0.0     arXiv:1504.05453
10 x BSSw   HALLA   1.0     arXiv:1504.05453
 6 x BTSA   HERMES  0.0     arXiv:1004.0177v1
 3 x TSA    CLAS    -1      arXiv:hep-ex/0605012
 6 x TSA    HERMES  -1.0    arXiv:1004.0177v1
------------------------------------------
TOTAL = 128
```

- We now use completely unconstrained neural nets representing $\mathfrak{Im}\,\mathcal{H}$, $\mathfrak{Re}\,\mathcal{H}$, $\mathfrak{Im}\,\mathcal{E}$, $\mathfrak{Re}\,\mathcal{E}$, ... (do not assume dispersion relations)

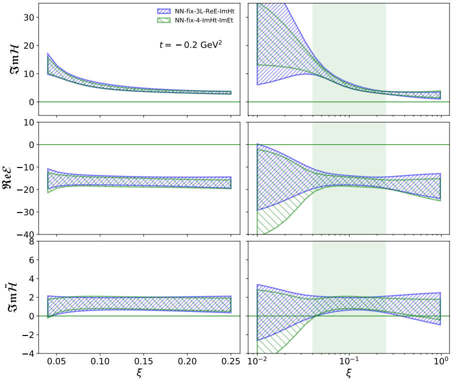
Results (1/2)

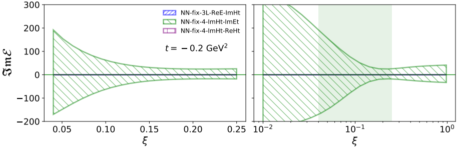
- Only $\mathfrak{Im}\,\mathcal{H}$, $\mathfrak{Im}\,\widetilde{\mathcal{H}}$ and $\mathfrak{Re}\,\mathcal{E}$ consistently extracted as different from zero, and, with somewhat smaller significance, $\mathfrak{Re}\,\mathcal{H}$ and $\mathfrak{Im}\,\mathcal{E}$:



Results (2/2)

- Other CFFs come out consistent with zero. Only bounds on their size are obtained. *E. g.* for $\mathfrak{Im}\,\widetilde{\mathcal{E}}$:

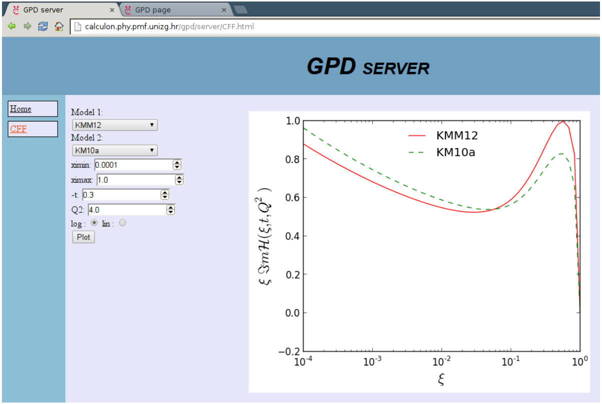




Summary

- Neural network method has a unique capability of extraction of Compton form factors (and, later, GPDs) with reliable uncertainties

GPD/CFF server



- Plots of all CFFs available; numerical values soon to come . . .