

GPDs from meson electroproduction and applications

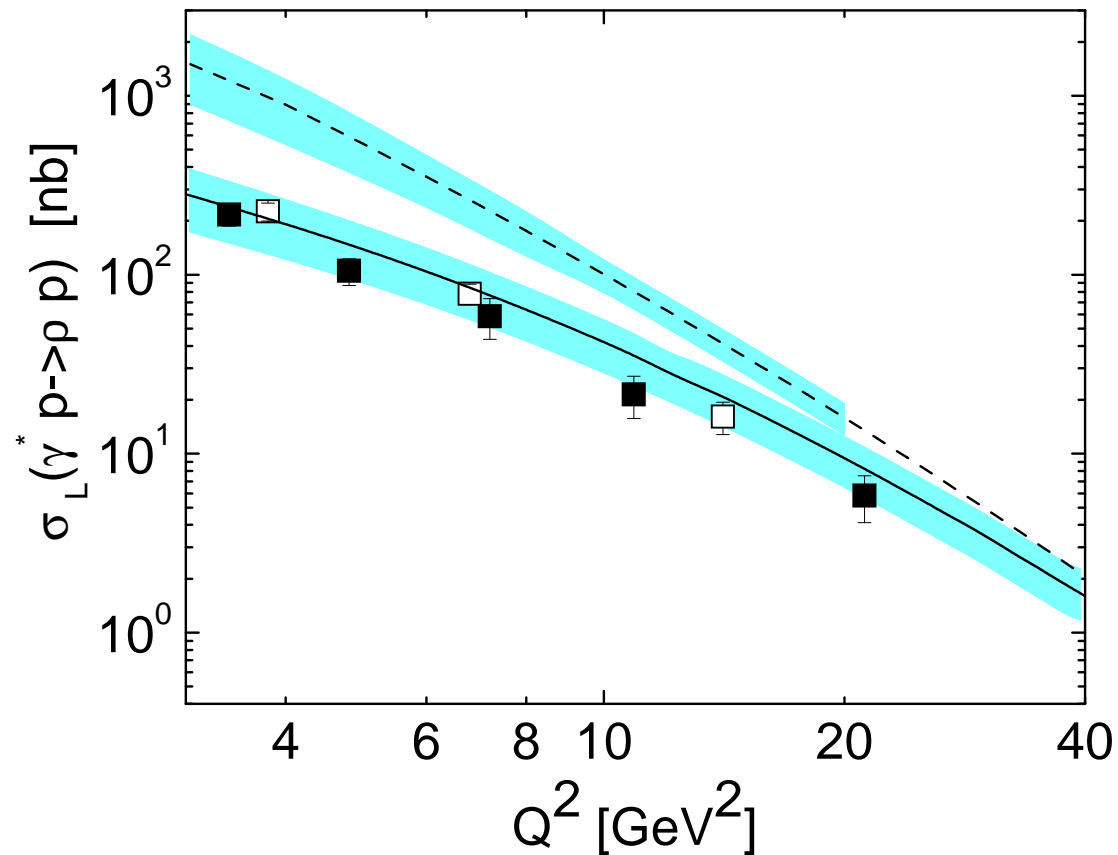
P. Kroll

Univiversitaet Wuppertal and Universitaet Regensburg
INT Seattle, October 2018

Outline:

- Introduction: handbag approach, subprocess amplitudes
Extraction of GPDs from meson electroproduction
- GPDs in **transverse position space**
- Predictions for DVCS, E and **parton angular momentum**
- **Universality:** Lepton pair production in exclusive processes
Predictions for other mesons (ω, K, η)
Meson neutrino production
- Summary

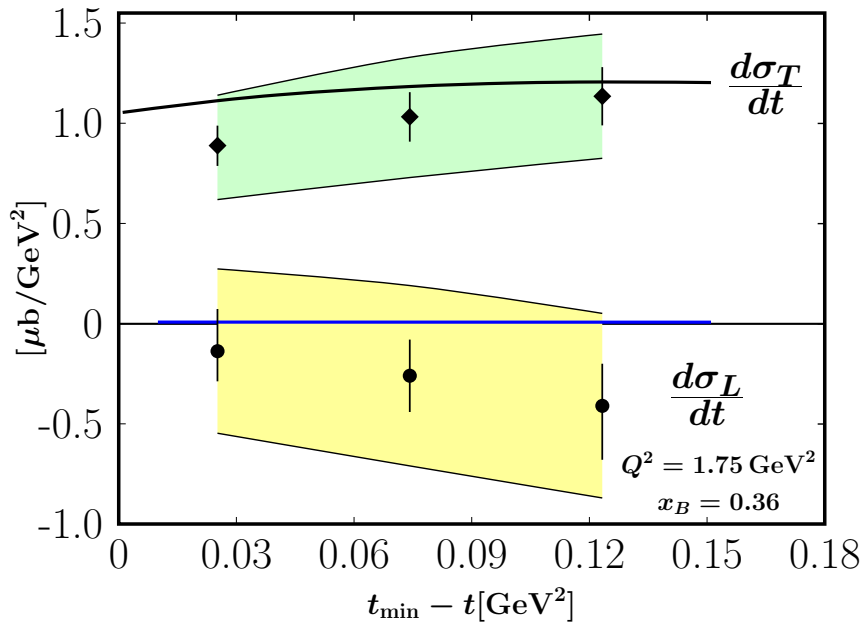
Leading-twist calculations of meson electroproduction fail



$W = 75 \text{ GeV}$

HERA, ZEUS

Pion production: contributions from γ_T^* are large



Hall A collaboration π^0 production
 Defurne et al (1608.01003)

(predictions from
 Goloskokov-K. (1106.4897))

$d\sigma_T \gg d\sigma_L$ ($d\sigma \simeq d\sigma_T$)
 like expectation for $Q^2 \rightarrow 0$

to be contrasted with

QCD expectation for $Q^2 \rightarrow \infty$: $d\sigma_T \ll d\sigma_L$ ($d\sigma \simeq d\sigma_L$)

leading twist does not dominate (much larger Q^2 required for it)

Further evidence for contribution from transverse photons:

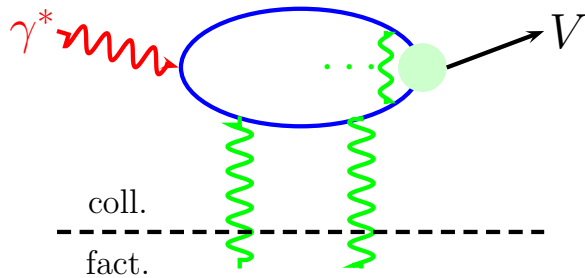
$A_{UT}^{\sin \phi_S}(\pi^+)$ HERMES(09); $d\sigma_{TT}/dt(\pi^0)$ CLAS(12)

The subprocess amplitude for DVMP

mod. pert. approach - quark trans. momenta in subprocess

(emission and absorption of partons from proton collinear to proton momenta)

transverse separation of color sources \implies gluon radiation



Sudakov factor [Sterman et al\(93\)](#)

$$S(\tau, \mathbf{b}, Q^2) \propto \ln \frac{\ln(\tau Q / \sqrt{2} \Lambda_{\text{QCD}})}{-\ln(b \Lambda_{\text{QCD}})} + \text{NLL}$$

resummed gluon radiation to NLL $\implies \exp[-S]$

provides sharp cut-off at $b = 1/\Lambda_{\text{QCD}}$

LO pQCD

+ quark trans. mom.

+ Sudakov supp.

\implies asymp. fact. formula

(lead. twist) for $Q^2 \rightarrow \infty$

Sudakov factor generates series of power corr. $\sim (\Lambda_{\text{QCD}}^2/Q^2)^n$

(from soft regions $\tau, \bar{\tau} \rightarrow 0$) and suppresses higher order Gegenbauer terms strongly

for HERA kinematics: similar to leading-log appr., color dipole model

[Frankfurt et al \(96\)](#), [Nemcik et al \(97\)](#),...

unintegrated gluon GPD [Martin et al \(99\)](#)

$$\mathcal{H}_{0\lambda,0\lambda}^M = \int d\tau d^2b \hat{\Psi}_M(\tau, -\mathbf{b}) e^{-S} \hat{\mathcal{F}}_{0\lambda,0\lambda}(\bar{x}, \xi, \tau, Q^2, \mathbf{b})$$

$\hat{\Psi}_M \sim \exp[\tau \bar{\tau} b^2 / 4a_M^2]$ LC wave fct of meson

$\hat{\mathcal{F}}$ FT of hard scattering kernel

e.g. $\propto 1/[k_\perp^2 + \tau(\bar{x} + \xi)Q^2/(2\xi)] \implies$ Bessel fct

Transverse photons in the handbag approach

need subprocess amplitude for $\gamma_T^* \rightarrow \pi$, non-vanishing for $t \rightarrow 0$

there is only one $\mathcal{H}_{0-,++}$ (angular momentum conservation:

$$\mathcal{H}_{\nu'\mu'\nu\mu} \sim \sqrt{-t}^{|\nu-\mu-\nu'+\mu'|} \text{ for } t \rightarrow 0)$$

\implies parton helicity flip transv. GPDs $H_T, E_T, \tilde{H}_T, \tilde{E}_T$ are required

go along with twist-3 pion wf. (q and \bar{q} forming the pion, have same helicity)

twist-3 DAs $\Phi_P \equiv 1, \Phi_\sigma = 6\tau(1-\tau)$ in WW approx.

$\mathcal{H}_{0-++} \neq 0$ for $t \rightarrow 0$ (from Φ_P , contr. from $\Phi_\sigma \propto t/Q^2$ neglected)

$\mathcal{H}_{0-++} \propto \mu_\pi/Q$ $\mu_\pi = m_\pi^2/(m_u + m_d) \simeq 2 \text{ GeV}$ at scale 2 GeV

$$\mathcal{M}_{0-++} = e_0 \sqrt{1-\xi^2} \int dx \mathcal{H}_{0-++}^{\text{twist}-3} H_T \quad \mathcal{M}_{0+\pm+} = -e_0 \frac{\sqrt{-t'}}{4m} \int dx \mathcal{H}_{0-++}^{\text{twist}-3} \bar{E}_T$$

(suppr. by μ_π/Q as compared to $L \rightarrow L$) $\mathcal{M}_{0--+} = 0$

prominent role of transversity GPDs also claimed by Ahmad et al (09)

analysis and results different

Parametrizing the GPDs

double distribution representation

Mueller *et al* (94), Radyushkin (99)

$$K^i(x, \xi, t) = \int_{-1}^1 d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta \delta(\rho + \xi\eta - x) K^i(\rho, \xi = 0, t) w_i(\rho, \eta) + D_i \Theta(\xi^2 - \bar{x}^2)$$

weight fct $w_i(\rho, \eta) \sim [(1 - |\rho|)^2 - \eta^2]^{n_i}$ ($n_g = n_{sea} = 2, n_{val} = 1$, generates ξ dep.)

zero-skewness GPD $K^i(\rho, \xi = 0, t) = k^i(\rho) \exp[(B_{ki} - \alpha'_{ki} \ln(\rho))t]$

$$k = q, \Delta q, \delta^q \text{ for } H, \tilde{H}, H_T \text{ or } N_{ki} \rho^{-\alpha_{ki}(0)} (1 - \rho)^{\beta_{ki}} \text{ for } E, \tilde{E}, \bar{E}_T$$

Regge-like t dep. (for small $-t$ reasonable appr.), D -term neglected

advantage: polynomiality and reduction formulas automatically satisfied
positivity bounds respected (checked numerically)

What has been done?

- analysis of FF with help of sum rules (DFJK(04), update: Diehl-K 1302.4604) using CTEQ6 (ABM11, DSSV11) PDFs, fixes H, E, \tilde{H} for valence quarks
- analysis of $d\sigma_L/dt$ for ρ^0 and ϕ production Goloskokov-K, hep-ph/0611290 data from H1, ZEUS, E665, HERMES for $Q^2 \gtrsim 3 \text{ GeV}^2$ and $W \gtrsim 4 \text{ GeV}$ ($\xi \lesssim 0.1$, $-t \lesssim 0.5 \text{ GeV}^2$) fixes H for sea quarks and gluons for given H^{val} (E negligible, others don't contr.) (only free parameters a_V)
- analysis of π^+ production, Goloskokov-K, 0906.0460 $d\sigma/dt$ and A_{UT} data from HERMES ($W \simeq 4 \text{ GeV}$, $Q^2 \simeq 2 - 5 \text{ GeV}^2$) evidence for strong contr. from γ_T^* (H_T) fixes pion pole and $H_T^{(3)}$ (no clear signal for \tilde{E})
- π^0 cross section and η/π^0 cross section ratio from CLAS (large skewness!), SDME and A_{UT} for ρ^0 prod. HERMES, Goloskokov-K, 1106.4897, 1310.1472 fixes H_T and $\bar{E}_T = 2\tilde{H}_T + E_T$ for valence quarks
- $\tilde{H}, \tilde{E}, H_T, \bar{E}_T$ for gluons and sea quarks unknown as yet, E_{sea} see below E_T, \tilde{E}_T unknown

FT to transverse position space

$$k^a(x, \mathbf{b}) = \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} e^{-i\mathbf{b}\Delta_{\perp}} K^a(x, \xi = 0, t = -\Delta_{\perp}^2)$$

$$K^a = k_a(x) \exp[-t f_a(x)]$$

$$k^a(x, \mathbf{b}) = \frac{1}{4\pi} \frac{k_a(x)}{f_a(x)} \exp[-b^2/(4f_a(x))]$$

more general profile fct. [DFJK\(04\)](#), [Diehl-K 1302.4604](#), [\(de Teramond et al 1801.09154\)](#),

$$f_a = (B_a + \alpha'_a \ln 1/x)(1-x)^3 + A_a x(1-x)^2 \quad \text{Moutarde et al 1807.07620}$$

density interpretation of FT: [Burkhardt\(00\)](#), [Diehl-Hägler\(05\)](#)

$q(x, \mathbf{b})$ density of unpolarized quark in an unpolarized proton

$q^{\pm} = \frac{1}{2}[q(x, \mathbf{b}) \pm \Delta q(x, \mathbf{b})]$ quarks with helicity (anti)parallel to proton helicity

$q_X(x, \mathbf{b}) = q(x, \mathbf{b}) - \frac{b_y}{m} \frac{\partial}{\partial b^2} e(x, \mathbf{b})$ unpolarized quark in proton
polarized along X direction

$q_T^x(x, \mathbf{b}) = \frac{1}{2}[q(x, \mathbf{b}) - \frac{b_y}{m} \frac{\partial}{\partial b^2} \bar{e}_T(x, \mathbf{b})]$ transversely pol. quark (x direction)
in unpolarized proton

...

Estimate of proton radius

consider Fourier transform of H

work in hadron's center of momentum frame

$$\sum x_i \mathbf{b}_i = 0$$

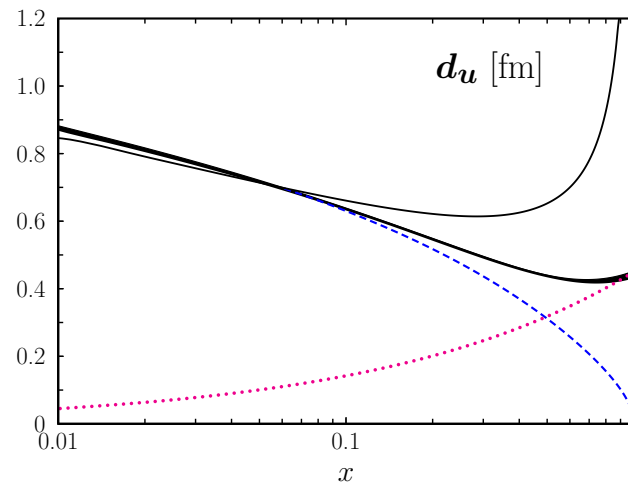
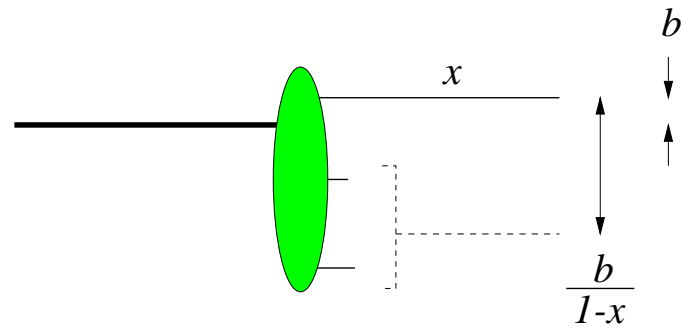
distance between active parton and cluster of spectators:

$$d_q(x) = \frac{\sqrt{\langle b^2 \rangle_x^q}}{1-x} = \frac{2\sqrt{f_q(x)}}{1-x} \rightarrow 2\sqrt{A_q}$$

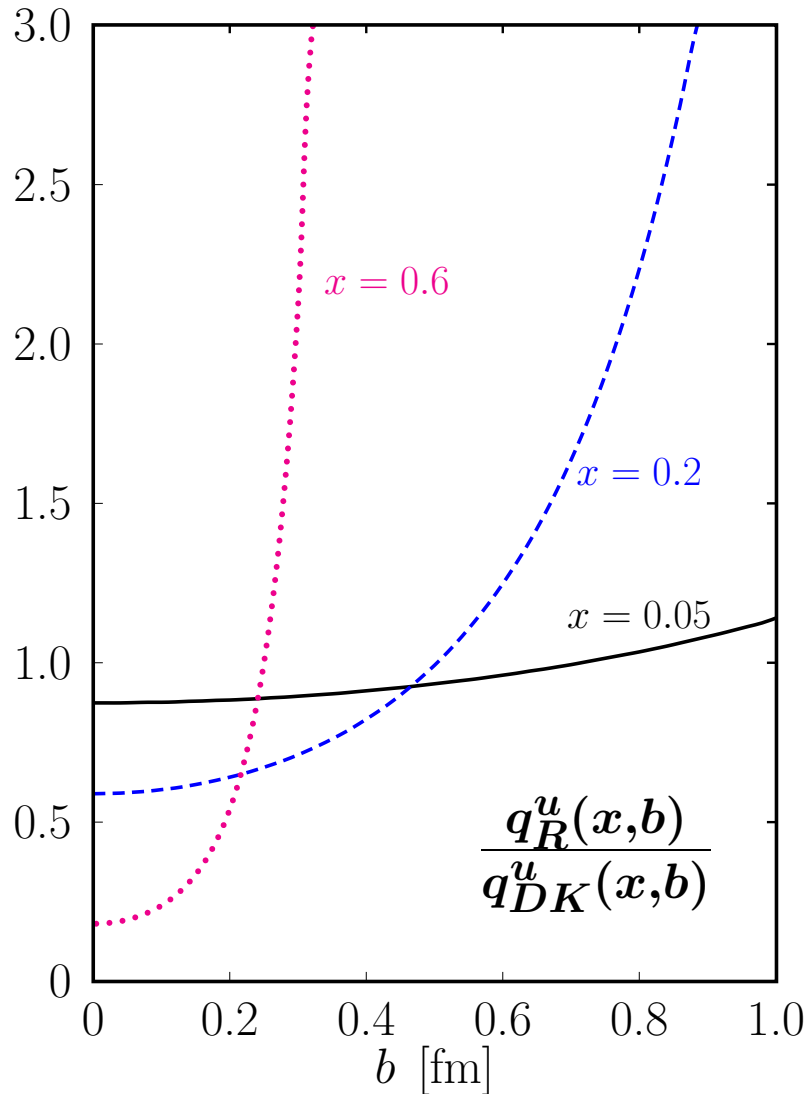
for $x \rightarrow 1$

Regge-type term, **A term**, full profile fct
 Regge-like profile fct can (only) be used
 at small x (small $-t$)

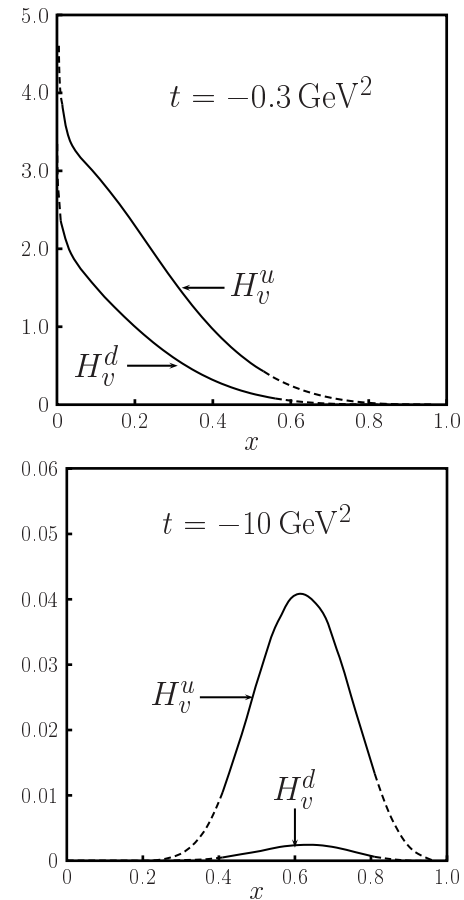
(Regge-like: $A = 0$ and $(1-x)^3 \rightarrow 1$)



FT with Regge-like profile function



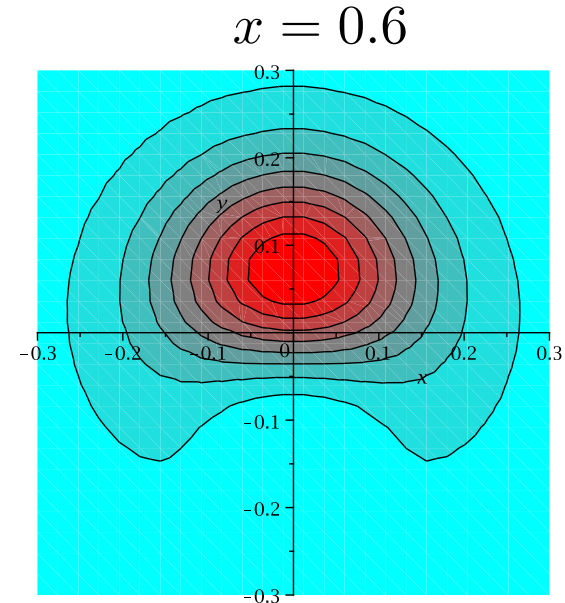
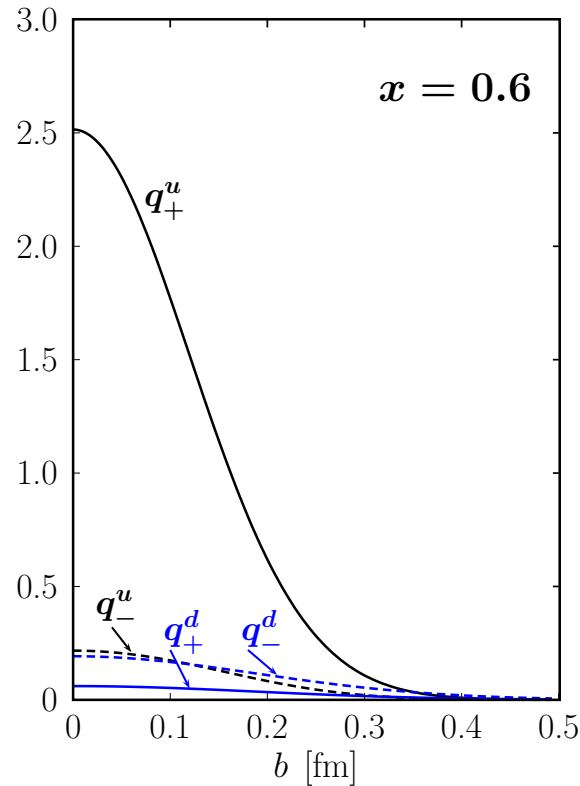
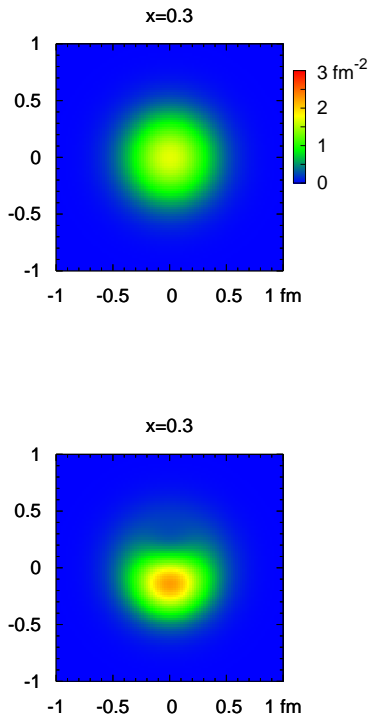
only reasonable for
 $x \lesssim 0.1$



large x -region not explored by
electroproduction

Densities in transverse position space at large x

only for valence quarks as yet; for gluons and strange quarks?



d quark density in unpolarized and polarized proton (in X direction)

proton f.f.

u quarks with same helicity as the proton dominates in agreement with pQCD

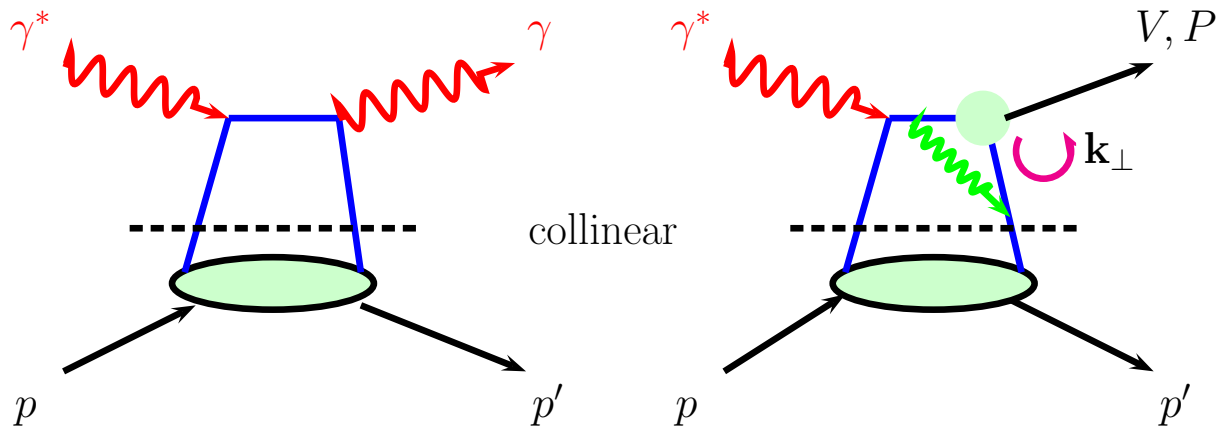
Brodsky et al(95)

F_A and WACS K_{LL}

transversely pol. u quark (in x direction) in unpolarized proton

wide-angle photopro. of π^0

DVCS



leading-twist, LO accuracy, collinear for consistency

with H most of the DVCS observables can be computed

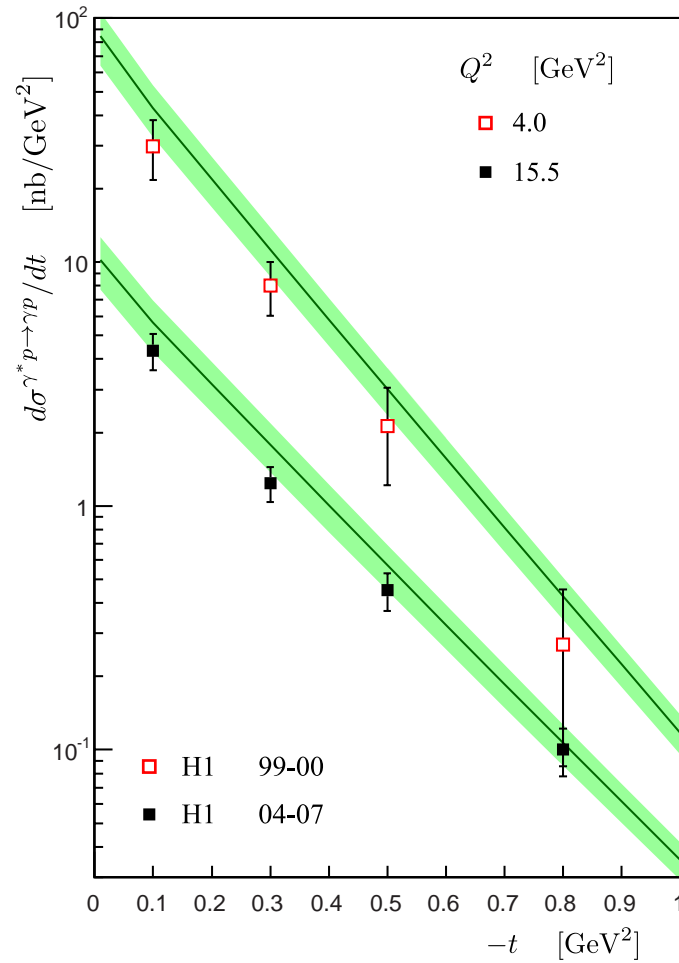
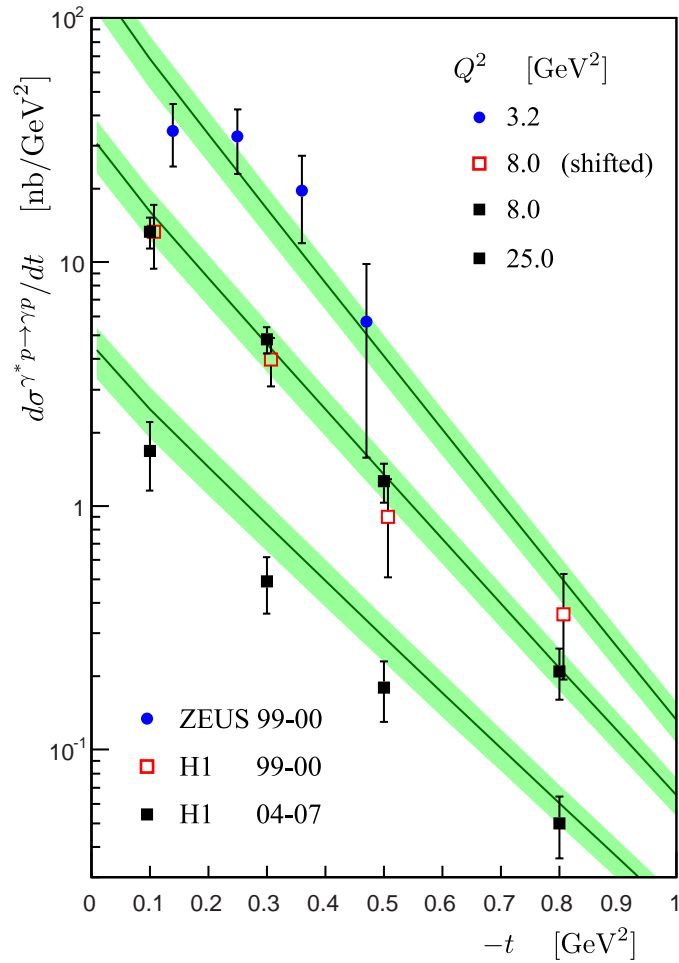
good agreement with all data from HERMES, HERA

except for Jlab6 kinematics (large skewness)

power corrections needed

Braun et al 1401.7621

DVCS at HERA



$W \simeq 90$ GeV data from ZEUS, H1

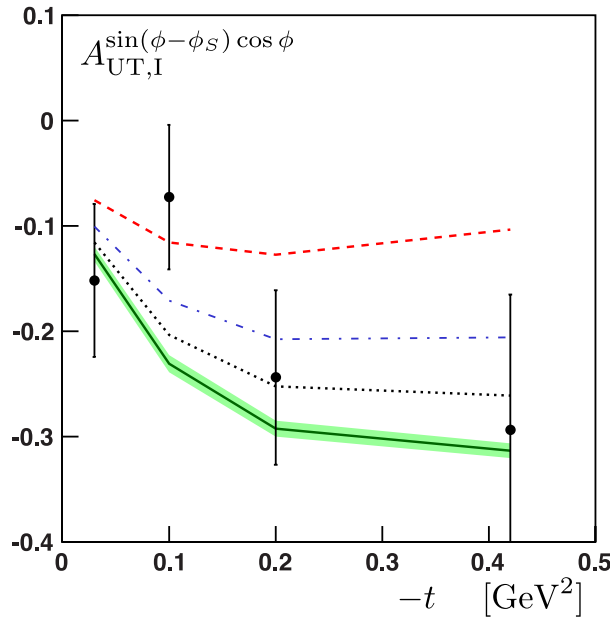
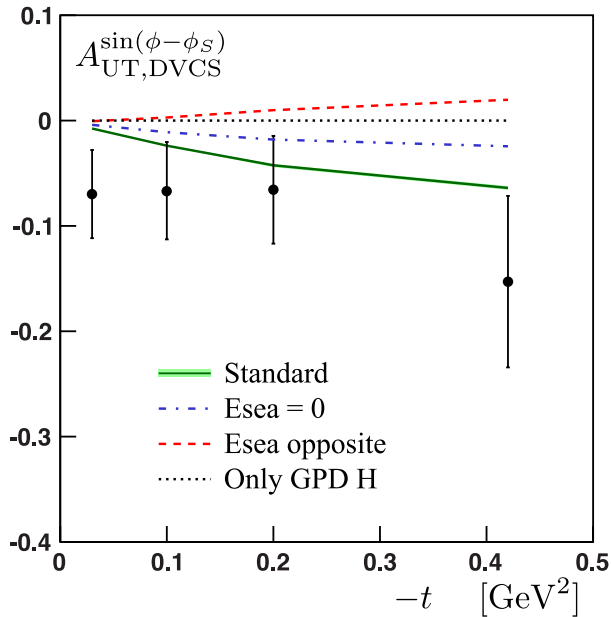
K-Moutarde-Sabatie (1210.6975)

leading-twist accuracy

parameter-free computation

UNIVERSALITY

Target asymmetry in DVCS



data: HERMES(08)

$$\langle Q^2 \rangle \simeq 2.5 \text{ GeV}^2$$

$$\langle x_B \rangle \simeq 0.09$$

theory:

K-Moutarde-

Sabatie(12)

$$A_{UT,DVCS}^{\sin(\phi-\phi_S)} \sim \text{Im}[\mathcal{E}^* \mathcal{H}]$$

$$\Rightarrow \mathcal{E}^{\text{sea}} \text{ seen}$$

from BH-DVCS interference

separate contr. from

$\text{Im } \mathcal{H}$ and $\text{Im } \mathcal{E}$

positivity bound for FTs forbids large sea \Rightarrow gluon small too (Diehl-Kugler(07))

$$\frac{b^2}{m^2} \left(\frac{\partial e_s(x,b)}{\partial b^2} \right)^2 \leq s^2(x,b) - \Delta s^2(x,b)$$

negative \mathcal{E}^{sea} favored in both cases

Application: Angular momenta of partons

$$J^a = \frac{1}{2} \left[q_{20}^a + e_{20}^a \right] \quad J^g = \frac{1}{2} \left[g_{20} + e_{20}^g \right] \quad (\xi = t = 0)$$

q_{20}^a, g_{20} from ABM11 (NLO) PDFs ($a = u, d, s, \bar{u}, \bar{d}, \bar{s}$)

$e_{20}^{a_v}$ (=0.163, -0.122) from form factor analysis Diehl-K. (13):

$e_{20}^s \simeq 0 \dots -0.024$ from analysis of A_{UT} in DVCS and positivity. bound

e_{20}^g (= $-\sum e_{20}^{a_v} - 6e_{20}^s$) (Goloskokov-K (09), K. 1410.4450)

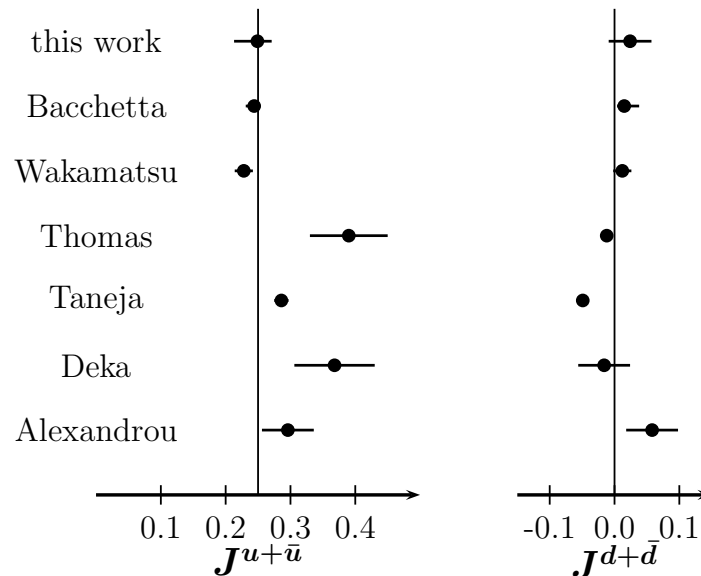
at scale 2 GeV:

$$J^{u+\bar{u}} = 0.249_{-0.036}^{+0.022};$$

$$J^{d+\bar{d}} = 0.024_{-0.033}^{+0.033};$$

$$J^{s+\bar{s}} = 0.005_{-0.014}^{+0.014};$$

$$J^g = 0.221_{+0.084}^{-0.067}.$$

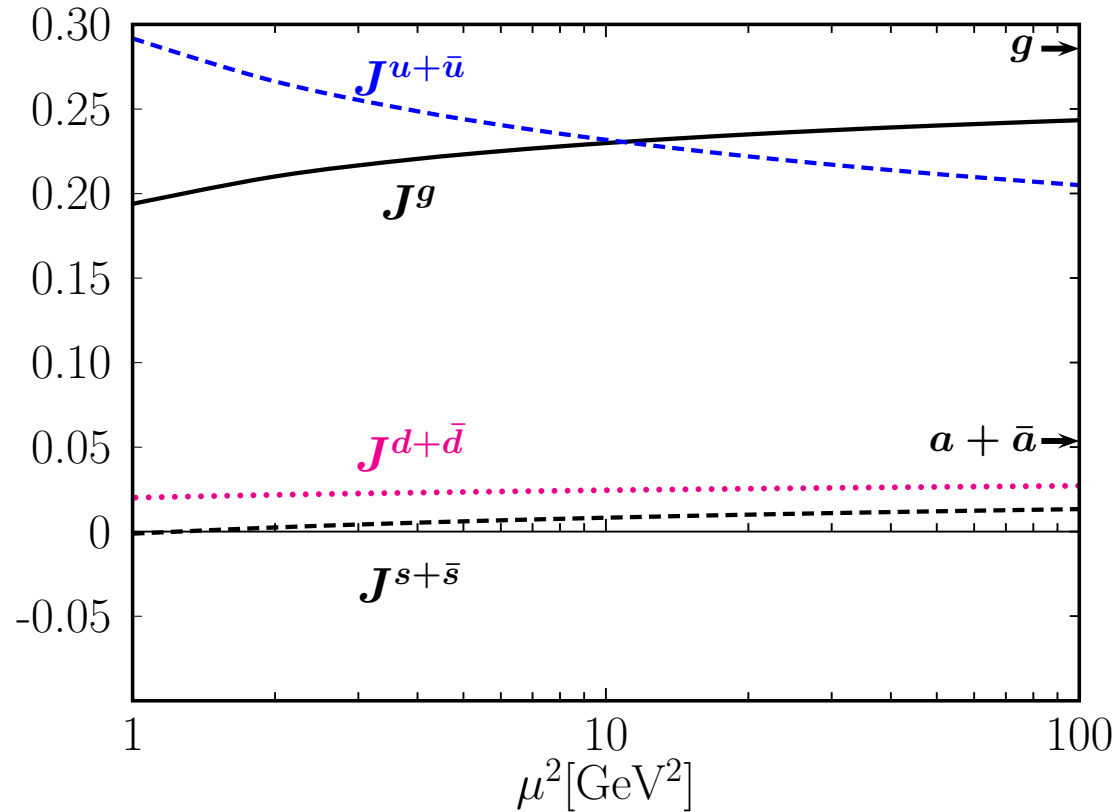


$\sum J^i = 1/2$ (spin of the proton)

need better determination of E^s and/or E^g

e.g. smaller errors of A_{UT} in DVCS or in J/Ψ production Koempel et al(11) PK 15

Evolution of the angular momenta



$$e_{20}^{a_v}(Q^2) = e_{20}^{a_v}(Q_0^2) e^{-d_{qq}s} \quad d_{qq} = \frac{32}{75} \quad d_+ = \frac{56}{75}$$

$$\Sigma_e(Q^2) = \sum_a (e_{20}^a + e_{20}^{\bar{a}}) = -e_{20}^g(Q^2) = \Sigma_e(Q_0^2) e^{-d_+s}$$

$$n_f = 4 \quad s = \ln \frac{\ln(Q^2/\Lambda_{QCD}^2)}{\ln(Q_0^2/\Lambda_{QCD}^2)}$$

small charm contribution not shown

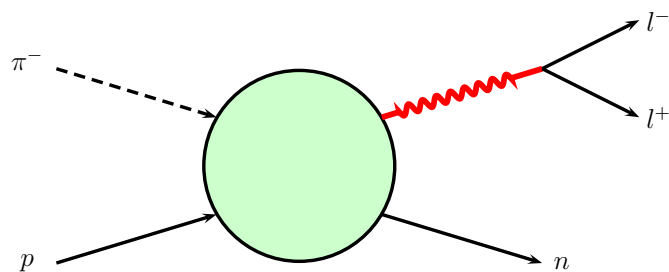
Lepton-pair production in exclusive processes

related to electroproduction

- same GPDs
- $\hat{s} - \hat{u} (l - P)$ crossed subprocess ($P = \gamma, \pi, K$)
- $\mathcal{H}^{P \rightarrow \gamma^*}(\hat{u}, \hat{s}) = -\mathcal{H}^{\gamma^* \rightarrow P}(\hat{s}, \hat{u})$
- equivalent to $Q^2 \rightarrow -Q'^2$

- timelike DVCS Pire et al, 1203.4392, 1407.0413, 1407.1990
- $\pi^- p \rightarrow l^+ l^- n$ Goloskokov-K, 1506.04619
- $p(\pi) p \rightarrow l^+ l^- p(\pi) p$ double handbag Pivovarov-Teryaev(14)

The exclusive Drell-Yan process



[Berger-Diehl-Pire \(01\)](#): leading-twist, LO analysis of long. cross section

(i.e. exploiting asymp. factor. formula)

(detailed reanalysis [Sawada et al, 1605.00364](#))

we know that leading-twist analysis of π^+ production fails with [JLAB](#), [HERMES](#) data by order of magnitude

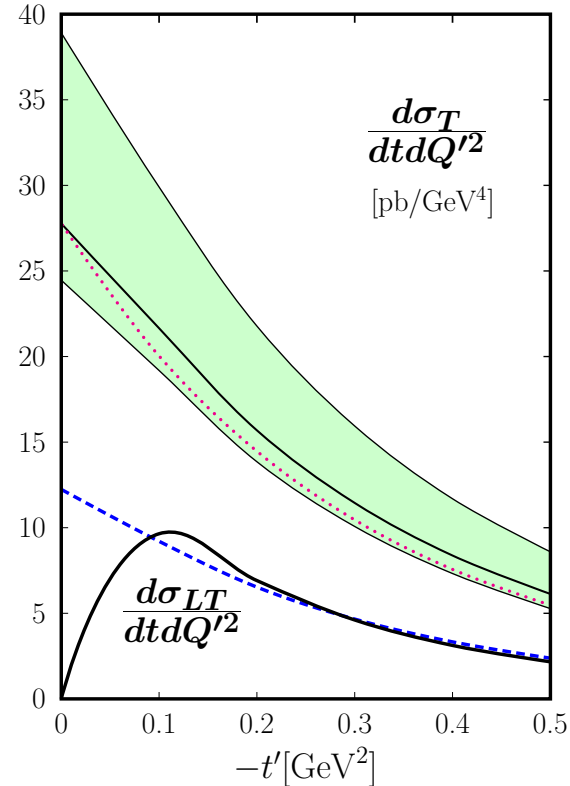
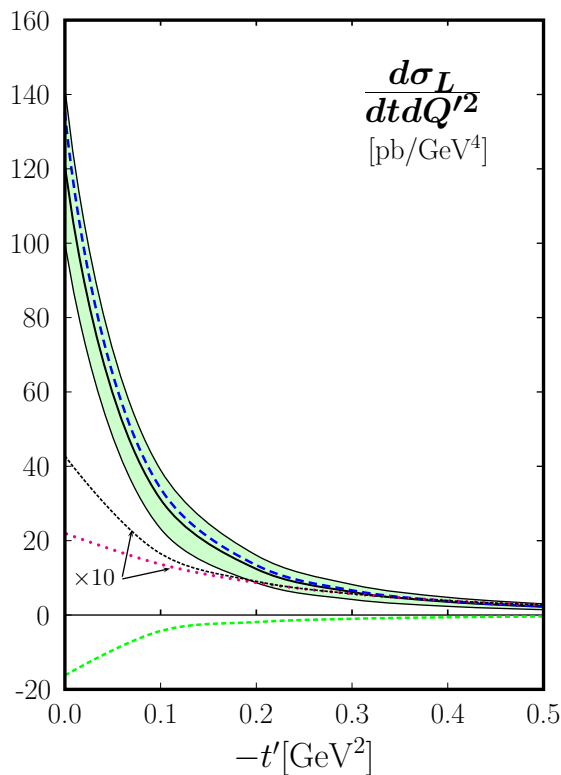
Therefore ...

([Goloskokov-K. 1506.04619](#))

a reanalysis of the exclusive Drell-Yan process seems appropriate making use of what we have learned from analysis of pion production

- take into account transverse photons and transversity GPDs
- retaining quark transverse momenta in the subprocess (the MPA)

Results on the Drell-Yan cross sections



$Q'^2 = 4 \text{ GeV}^2$ and $s = 20 \text{ GeV}^2$

solid lines with error bands: full result

pion pole, $|\langle \tilde{H}^{(3)} \rangle|^2$, interference, short dashed: leading-twist contribution

time-like pion FF: $Q'^2 |F_\pi(Q'^2)| = 0.88 \pm 0.04 \text{ GeV}^2$ (CLEO, BaBar, $J/\Psi \rightarrow \pi^+ \pi^-$)

phase ($\exp[i\delta(Q'^2)]$) from disp. rel. Belicka et al(11) for $Q'^2 < 8.9 \text{ GeV}^2$

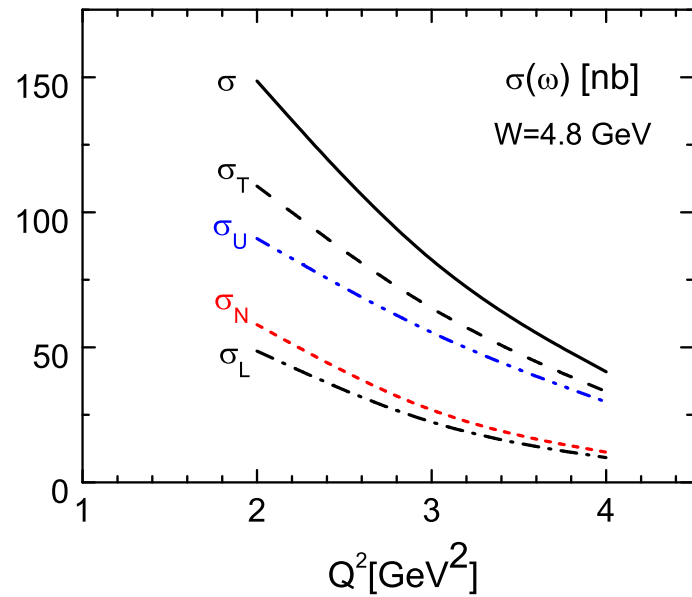
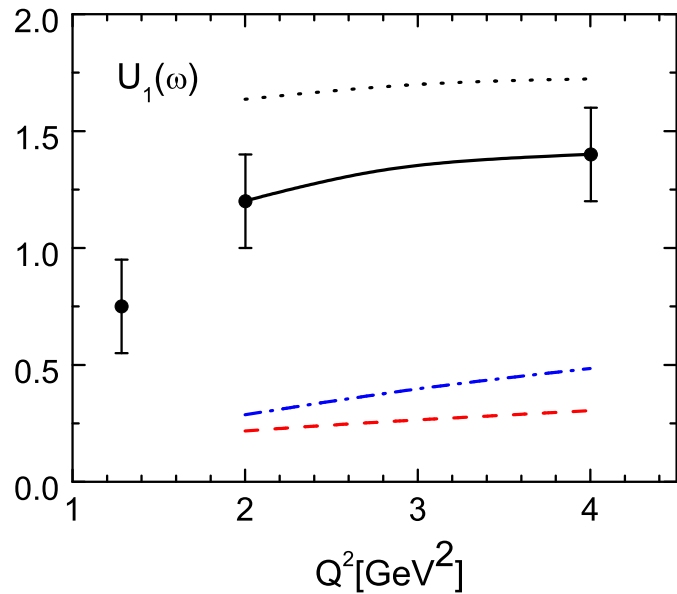
$$\delta = 1.014\pi + 0.195(Q'^2/\text{GeV}^2 - 2) - 0.029(Q'^2/\text{GeV}^2 - 2)^2$$

for $Q'^2 \geq 8.9 \text{ GeV}^2$: $\delta = \pi$, the LO pQCD result

Remarks on processes with time-like virtual photons

- time-like excl. processes difficult to understand theoretically
e.g. no satisfactory explanation of time-like elm form factors within pert. QCD
- Drell-Yan process $\pi^- p \rightarrow l^+ l^- X$
large K -factor needed (larger than NLO corr. [Sutton et al \(92\)](#))
now understood as 'threshold logs' $(Q'^2/(x_1 x_2 s) \rightarrow 1)$
(gluon radiation resummed to NLL [Sterman\(87\)](#), [Catani-Trentadue\(89\)](#))
leading finally to reasonable fits of data and extraction of PDFs for the pion with plausible behavior for $x \rightarrow 1$ [Aicher-Schäfer-Vogelsang \(11\)](#)
- hard exclusive scattering processes with time-like virtual photons
no data as yet but predictions
[experimental verification of predictions important](#)

ω SDMEs



unnatural parity contribution

$$U_1 = 1 - r_{00}^{04} + 2r_{1-1}^{04} - 2r_{11}^1 - 2r_{1-1}^1 = 2 \frac{d\sigma_U}{d\sigma}$$

$W = 4.8(8) \text{ GeV}$, without pion pole,

dotted 3.5 GeV , $t' = -0.08 \text{ GeV}^2$

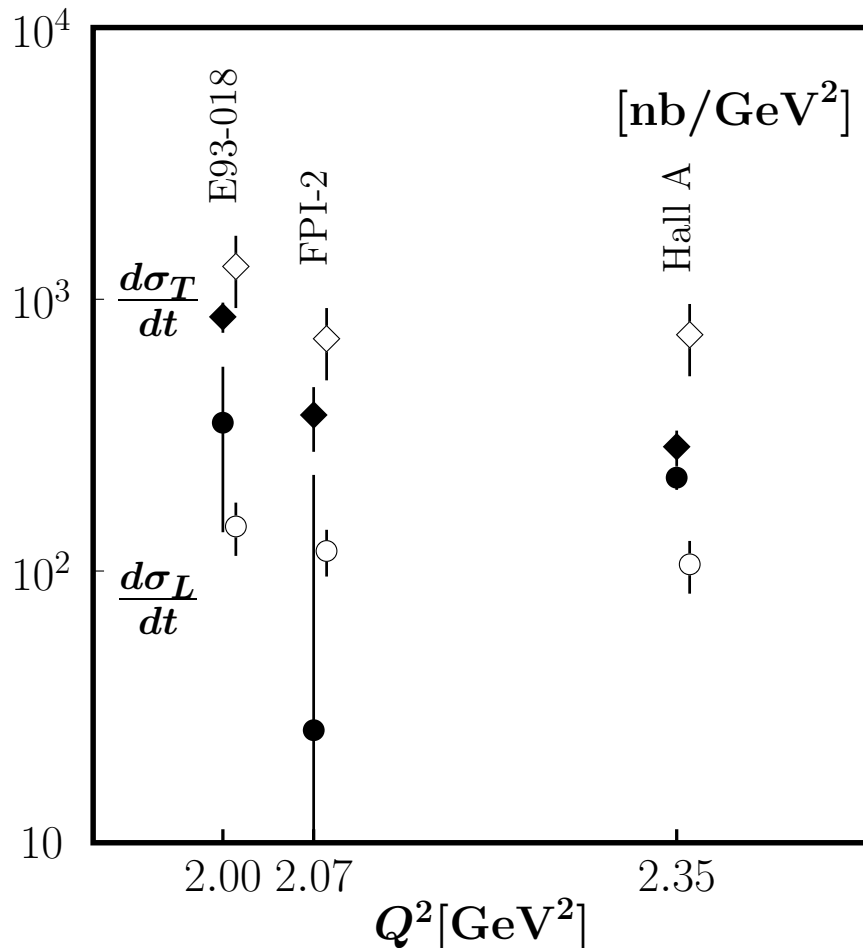
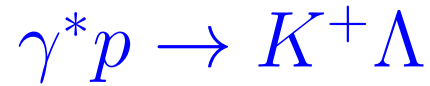
strong unnat. parity contr. - pion pole

allows for extraction of $|g_{\pi\omega}|$

various cross sections

different from ρ^0 and

from $Q^2 \rightarrow \infty$ expectation



$$K_{p \rightarrow \Lambda} = \frac{1}{\sqrt{6}} [2K^u - K^d - K^s]$$

E93-018 $W = 1.85$ GeV

$$t = t_0 = -0.74 \text{ GeV}^2$$

FPI-2 $W = 2.39$ GeV, $t = -0.4$ GeV²

Hall A $W = 2.08$ GeV

$$t = t_0 = -0.57 \text{ GeV}^2$$

same GPDs as for pions, no fits

flavor symmetric sea assumed

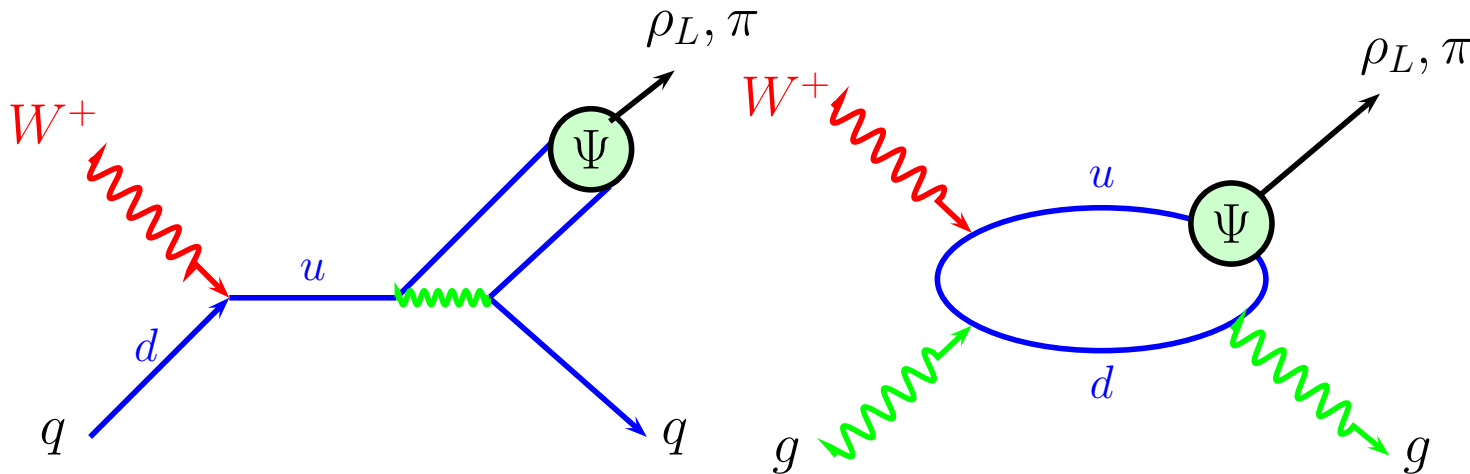
$$\text{kaon/pion pole} \sim [(t - m_\pi^2)/(t - m_k^2)]^2$$

also prediction for π^0 production off neutrons, η production, ρ^0 prod. with transversity GPDs (SDME)

Meson neutrino production

V-A theory

all 4 helicity non-flip GPDs contribute



H_g contribution dominant [Pire et al\(17\)](#)

π and ρ_L amplitudes are proportional
twist-3 contribution (H_T, \bar{E}_T) small

[Kopeliovich et al\(14\)](#)

[MINERVA 1409.3835](#)

$\nu A \rightarrow \mu \pi A$

$E_\nu = 1.5 - 20 \text{ GeV}, Q^2?$

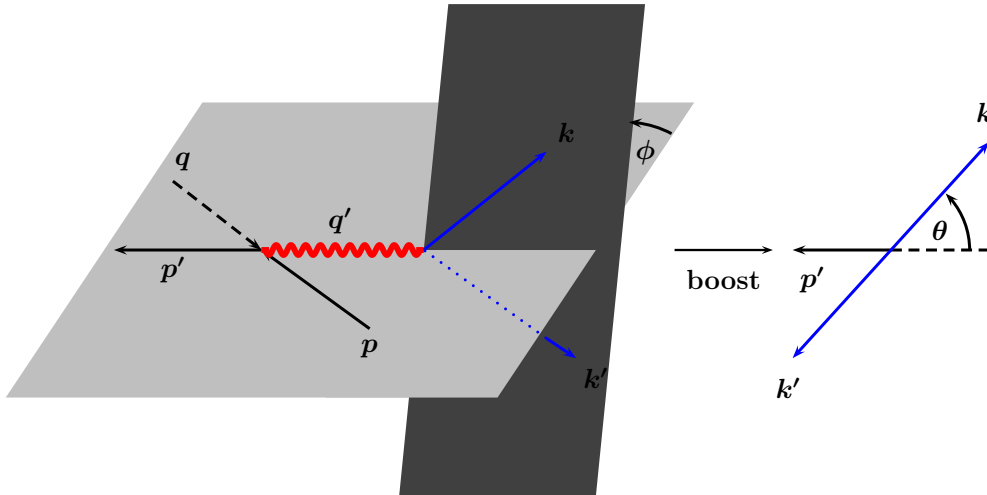
NLO corrections for π and K production [Siddikov-Schmidt\(16\)](#)

nuclear effects only substantial for $x_B \lesssim 0.1$ [Schmidt-Siddikov\(15\)](#)

Summary

- The handbag approach, generalized to transverse photons and with meson size corrections, describes all DVMP data for $Q^2 \gtrsim 2 \text{ GeV}^2$ and $W \gtrsim 4 \text{ GeV}$ for ρ^0 ($\gtrsim 2 \text{ GeV}$ for ϕ, π)
- From the combined analysis of nucleon form factors, DVMP (and DVCS for E^{sea}) a set of GPDs has been extracted ($H, E, \tilde{H}, H_T, \bar{E}_T$ for valence quarks, gluon and sea quarks only for H)
- This set of GPDs allows for calculations of other hard exclusive processes (DVCS, ω , *Kaon* and η lepton-pair production ...) **test of universality**
- and to obtain first results on **parton angular momenta**
- Evaluation of **transverse localization of partons** in the proton only possible for valence quarks as yet. For others large $-t$ behaviour unknown
- The GPDs need **improvements**: (of course) possible (and necessary) with new data from COMPASS, JLAB12 and EIC framework **PARTONS** [Berthou et al\(1512.06174\)](#)

Cross section



k momentum of l^-
 $\tau = Q'^2 / (s - m^2)$
 the time-like analogue
 of x_B

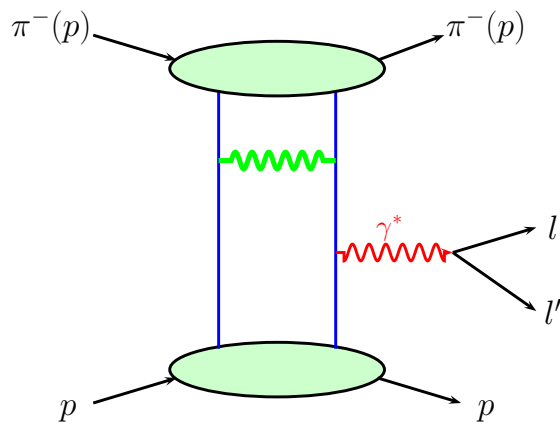
$$\frac{d\sigma}{dt dQ'^2 d\cos\theta d\phi} = \frac{3}{8\pi} \left\{ \sin^2\theta \frac{d\sigma_L}{dt dQ'^2} + \frac{1 + \cos^2\theta}{2} \frac{d\sigma_T}{dt dQ'^2} \right. \\ \left. + \frac{1}{\sqrt{2}} \sin(2\theta) \cos\phi \frac{d\sigma_{LT}}{dt dQ'^2} + \sin^2\theta \cos(2\phi) \frac{d\sigma_{TT}}{dt dQ'^2} \right\}$$

$$\frac{d\sigma_L}{dt dQ'^2} = \frac{\alpha_{\text{elm}}}{48\pi^2} \frac{\tau^2}{Q'^6} \sum_{\nu'} |\mathcal{M}_{0\nu',0+}|^2 \quad \frac{d\sigma_T}{dt dQ'^2} = \frac{\alpha_{\text{elm}}}{48\pi^2} \frac{\tau^2}{Q'^6} \sum_{\mu=\pm 1, \nu'} |\mathcal{M}_{\mu\nu',0+}|^2$$

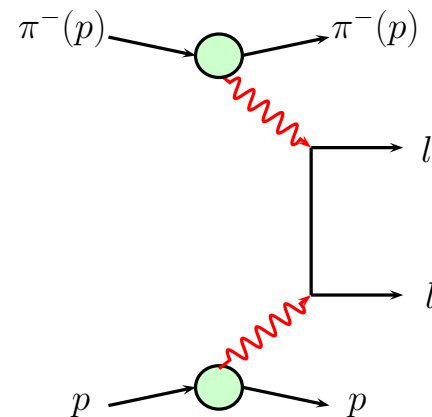
partial cross sections analogous to pion production

Lepton-pair production in exclusive hadron-hadron collisions

Pivovarov-Teryaev (14): double handbag



access to pion GPD



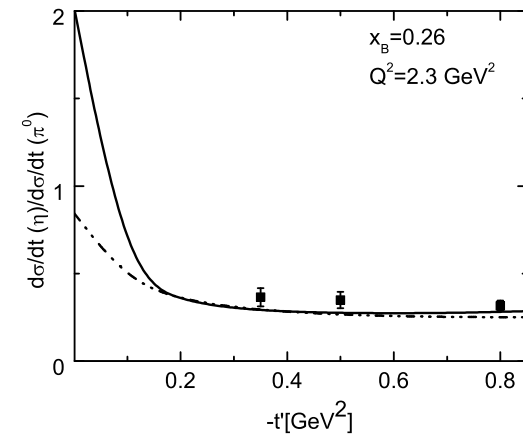
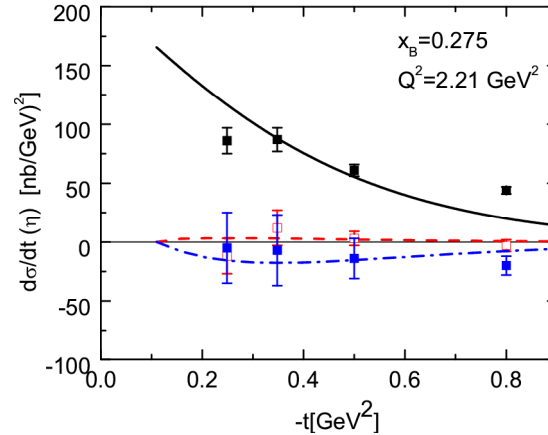
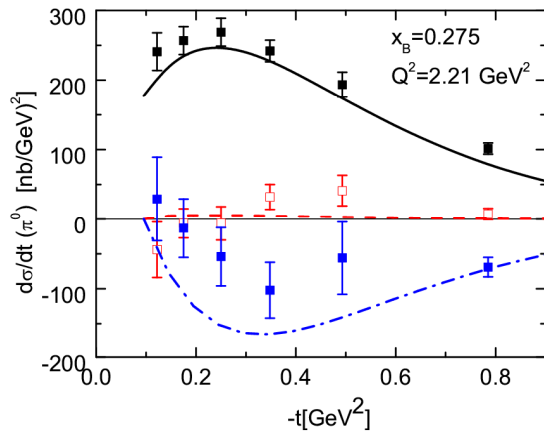
elm. contribution

$$\sim F_{\text{elm}}^{\pi(p)} F_{\text{elm}}^p$$

η production

Bedlinsky et al (1405.0988)

theory: Goloskokov-K (1106.4897)



unseparated (longitinal, transverse) cross sections

$$\frac{d\sigma(\eta)}{d\sigma(\pi^0)} \simeq \left(\frac{f_\eta}{f_\pi}\right)^2 \frac{1}{3} \left| \frac{e_u \langle K^u \rangle + e_d \langle K^d \rangle}{e_u \langle K^u \rangle - e_d \langle K^d \rangle} \right|^2 \quad (f_\eta = 1.26 f_\pi)$$

if K^u and K^d have opposite sign: $\eta/\pi^0 \simeq 1$ ($\eta = (\cos \theta_8 - \sqrt{2} \sin \theta_1) \eta_q$)

if K^u and K^d have same sign: $\eta/\pi^0 < 1$ (FKS scheme)

$t' \simeq 0$ \tilde{H}, H_T dominant (see also Eides et al(98) assuming dominance of \tilde{H} for all t')

$t' \neq 0$ \bar{E}_T dominant