Twist-3 Observables for Deeply Virtual Exclusive Reactions

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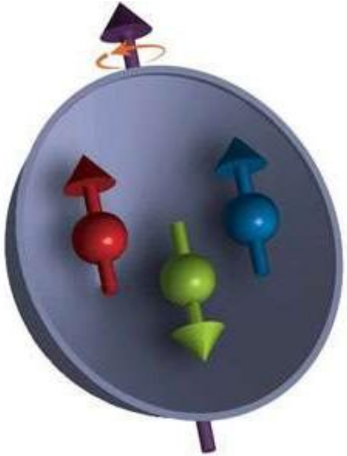
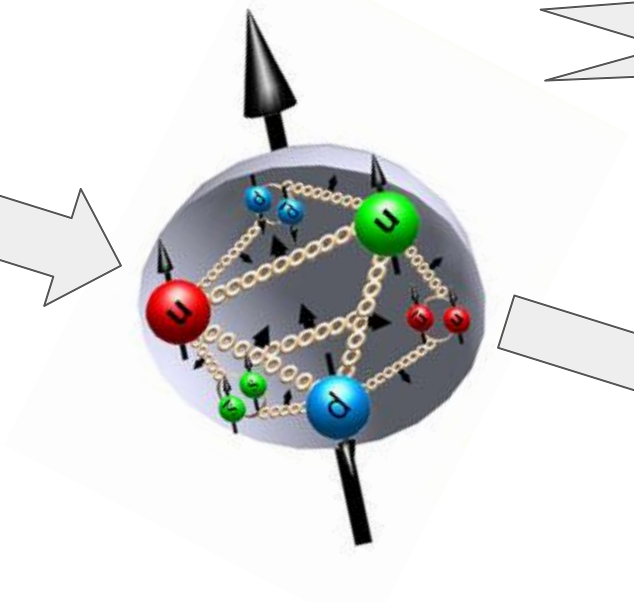




Partonic Orbital Momentum

The naive parton model cannot explain the spin of the proton.

Gluon contribution is negligible

Proton Spin Crisis!

$$\frac{\Delta\Sigma}{2} = \frac{1}{2}\sum_{q} \Delta q + \Delta\bar{q} \neq \frac{1}{2}$$

$$\frac{\Delta\Sigma}{2} + \Delta G \neq \frac{1}{2}$$

$$\frac{\Delta\Sigma}{2} + \Delta G + L_{q,g} \overset{???}{=} \frac{1}{2}$$

How do we describe the orbital angular momentum of the partons?

$$\vec{L} = \vec{r} \times \vec{p}$$

classically

$$L_z^q = -\left(k_T \times b_T\right)_z^q$$

Partonic

b_T Relative average transverse position from the center of momentum of the system

k_T Relative average transverse momentum

$$l_z^q = \int dx\, d^2k_T\, d^2b_T \left(b_T \times k_T\right)_z^q \rho^{[\gamma^+]}(b_T, k_T, x)$$

$$l_z^q = -\int dx\, d^2k_T \frac{k_T^2}{M^2} F_{1,4}^q$$

Observable?

Sum Rules for OAM

No framework yet for GTMD observables

Rajan, Englehardt, Liuti arXiv:1709.05770

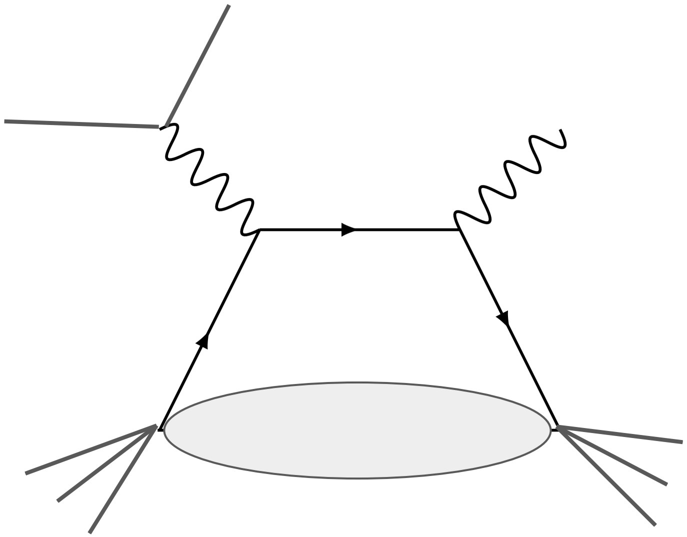
$$\frac{d}{dx}\int d^2k_T \frac{k_T^2}{M^2} F_{1,4} = H + E + \tilde{E}_{2T}$$

Twist-2 **Twist-3**

Abha’s talk on Friday

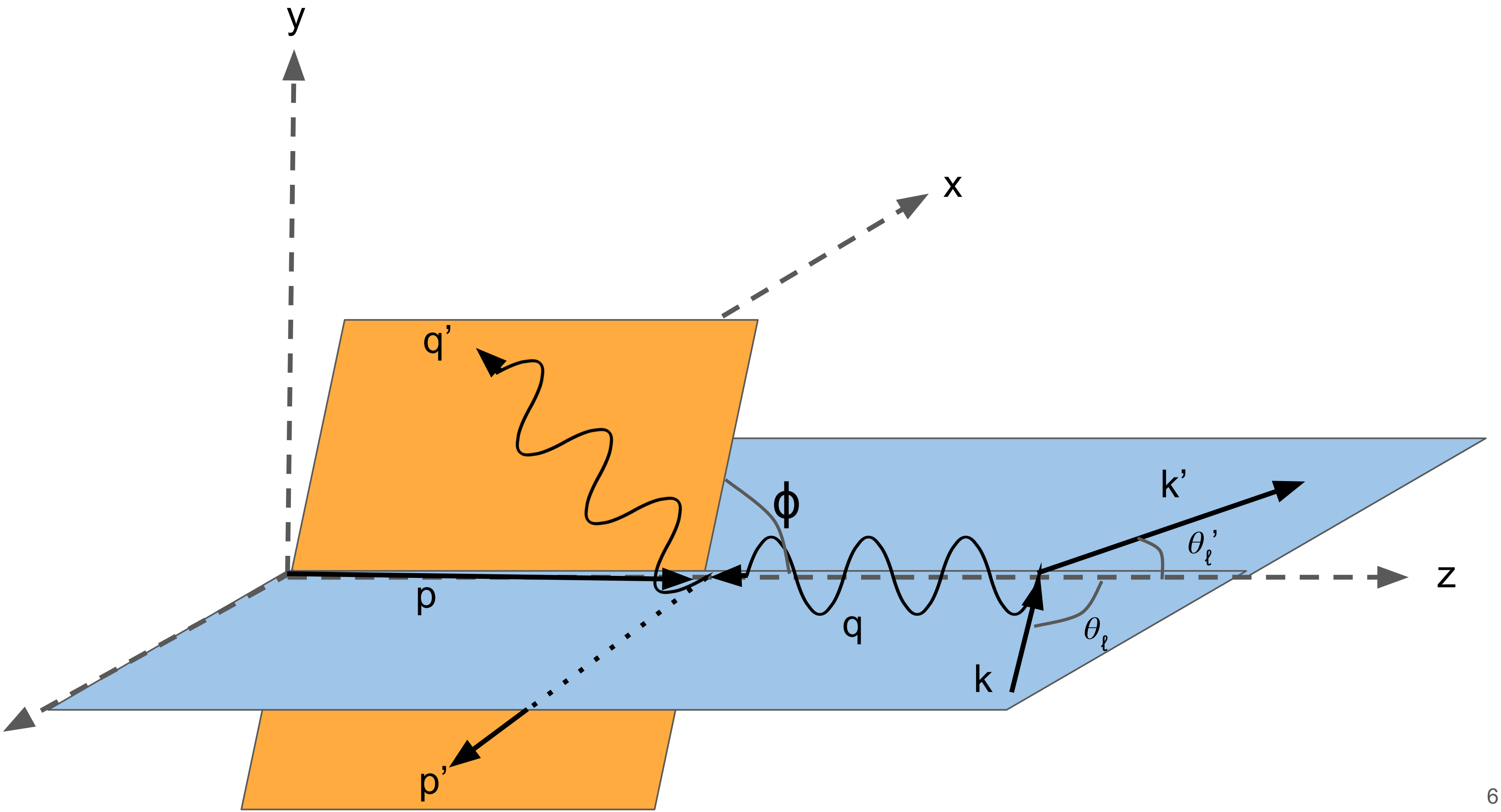
Can we disentangle the Twist-3 GPDs from data?

DVCS



Twist - 2 (blue)
Twist - 3 (green)

$$W^{\mu\nu} \propto \gamma^\mu \gamma^+ \gamma^\nu = \begin{bmatrix} \gamma^- & \gamma^1 - i\gamma^2\gamma_5 & \gamma^2 + i\gamma^1\gamma_5 & i\gamma^-\gamma_5 \\ \gamma^1 + i\gamma^2\gamma_5 & \gamma^+ & i\gamma^+\gamma_5 & -\gamma^1 - i\gamma^2\gamma_5 \\ \gamma^2 - i\gamma^1\gamma_5 & -i\gamma^+\gamma_5 & \gamma^+ & -\gamma^2 + i\gamma^1\gamma_5 \\ -i\gamma^-\gamma_5 & -\gamma^1 + i\gamma^2\gamma^5 & -\gamma^2 - i\gamma^1\gamma_5 & \gamma^- \end{bmatrix}$$



y
x
q'
ϕ
k'
θℓ'
z
p
q
θℓ
k
p'

$$\frac{d^5\sigma_{DVCS}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}|T_{DVCS}|^2$$

Unpolarized

$$= \frac{\Gamma}{Q^2(1-\epsilon)}\Big\{\Big[F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)}\left[\cos\phi F_{UU}^{\cos\phi} + \sin\phi F_{UU}^{\sin\phi}\right]$$

LU polarized

$$+ (2h)F_{LU} + (2h)\sqrt{2\epsilon(1-\epsilon)}\sin\phi F_{LU}^{\sin\phi} + (2h)\sqrt{2\epsilon(1-\epsilon)}\cos\phi F_{LU}^{\cos\phi}\Big]$$

UL polarized

$$+ (2\Lambda)\Big[F_{UL} + \sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi} + \sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UL}^{\cos\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi}$$

LL polarized

$$+ (2h)\sqrt{1-\epsilon^2}F_{LL} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\cos\phi F_{LL}^{\cos\phi} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\sin\phi F_{LL}^{\sin\phi}\Big]$$

UT polarized

$$+ |\vec{S}_\perp|\Big[\sin(\phi-\phi_S)\left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right)$$
$$\epsilon\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)} + \epsilon\sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)}$$
$$+\sqrt{2\epsilon(1+\epsilon)}\left(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\right)\Big]$$

LT polarized

$$+ (2h)|\vec{S}_\perp|\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}$$
$$+\sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big]\Big\}$$

$$
\begin{aligned}
\frac{d^5\sigma_{DVCS}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} &= \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}\left|T_{DVCS}\right|^2 \\
&= \frac{\Gamma}{Q^2(1-\epsilon)}\left\{\left[F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi}\right.\right. \\
&+ \sqrt{\epsilon(\epsilon+1)}\left[\cos\phi F_{UU}^{\cos\phi} + \sin\phi F_{UU}^{\sin\phi}\right] \\
&+ (2h)F_{LU} + (2h)\ \sqrt{2\epsilon(1-\epsilon)}\sin\phi F_{LU}^{\sin\phi} + (2h)\sqrt{2\epsilon(1-\epsilon)}\cos\phi F_{LU}^{\cos\phi}\Big] \\
&+ (2\Lambda)\left[F_{UL} + \sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi} + \sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UL}^{\cos\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi}\right. \\
&+ (2h)\ \sqrt{1-\epsilon^2}F_{LL} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\cos\phi F_{LL}^{\cos\phi} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\sin\phi F_{LL}^{\sin\phi}\Big] \\
&+ |\ \vec{S}_\perp\ |\left[\sin(\phi-\phi_S)\left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right)\right. \\
&\quad\ \epsilon\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)} + \epsilon\sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)} \\
&\quad +\sqrt{2\epsilon(1+\epsilon)}\left(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\right)\Big] \\
&+ (2h)\,|\ \vec{S}_\perp\ |\left[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}\right. \\
&\quad\ \left.\left. +\sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\right]\right\}
\end{aligned}
$$

Twist - 2

$$
\begin{aligned}
\frac{d^5\sigma_{DVCS}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} &= \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}|T_{DVCS}|^2 \\
&= \frac{\Gamma}{Q^2(1-\epsilon)}\Big\{F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi} \\
&+ \sqrt{\epsilon(\epsilon+1)}\left[\cos\phi F_{UU}^{\cos\phi} + \sin\phi F_{UU}^{\sin\phi}\right] \\
&+ (2h)F_{LU} + (2h)\sqrt{2\epsilon(1-\epsilon)}\sin\phi F_{LU}^{\sin\phi} + (2h)\sqrt{2\epsilon(1-\epsilon)}\cos\phi F_{LU}^{\cos\phi}] \\
&+ (2\Lambda)\Big[F_{UL} + \sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi} + \sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UL}^{\cos\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi} \\
&+ (2h)\sqrt{1-\epsilon^2}F_{LL} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\cos\phi F_{LL}^{\cos\phi} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\sin\phi F_{LL}^{\sin\phi}\Big] \\
&+ |\vec{S}_\perp|\Big[\sin(\phi-\phi_S)\left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right) \\
&\quad \epsilon\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)} + \epsilon\sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)} \\
&\quad + \sqrt{2\epsilon(1+\epsilon)}\left(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\right)\Big] \\
&+ (2h)|\vec{S}_\perp|\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LT}^{\cos\phi_S} \\
&\quad + \sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big]\Big\}
\end{aligned}
$$

Twist - 2

Twist - 3

$$
\begin{aligned}
\frac{d^5\sigma_{DVCS}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} &= \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}|T_{DVCS}|^2 \\
&= \frac{\Gamma}{Q^2(1-\epsilon)}\Big\{F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi} \\
&+ \sqrt{\epsilon(\epsilon+1)}\left[\cos\phi F_{UU}^{\cos\phi} + \sin\phi F_{UU}^{\sin\phi}\right] \\
&+ (2h) F_{LU} + (2h)\sqrt{2\epsilon(1-\epsilon)}\sin\phi F_{LU}^{\sin\phi} + (2h)\sqrt{2\epsilon(1-\epsilon)}\cos\phi F_{LU}^{\cos\phi}] \\
&+ (2\Lambda)\Big[F_{UL} + \sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi} + \sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UL}^{\cos\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi} \\
&+ (2h)\sqrt{1-\epsilon^2}F_{LL} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\cos\phi F_{LL}^{\cos\phi} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\sin\phi F_{LL}^{\sin\phi}\Big] \\
&+ |\vec{S}_\perp|\Big[\sin(\phi-\phi_S)\left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right) \\
&\quad \epsilon\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)} + \epsilon\sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)} \\
&\quad + \sqrt{2\epsilon(1+\epsilon)}\left(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\right)\Big] \\
&+ (2h)|\vec{S}_\perp|\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LT}^{\cos\phi_S} \\
&\quad + \sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big]\Big\}
\end{aligned}
$$

Twist - 2

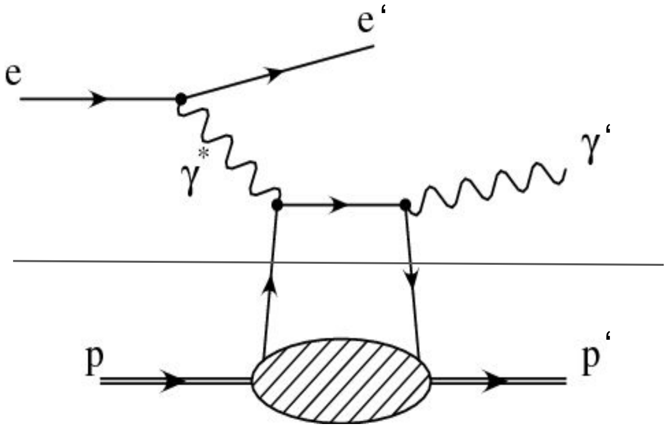
Twist - 3

Twist - 4

$$
\begin{aligned}
\frac{d^5\sigma_{DVCS}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} &= \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}|T_{DVCS}|^2 \\
&= \frac{\Gamma}{Q^2(1-\epsilon)}\Big\{F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi} \\
&+ \sqrt{\epsilon(\epsilon+1)}\left[\cos\phi F_{UU}^{\cos\phi} + \sin\phi F_{UU}^{\sin\phi}\right] \\
&+ (2h)\left[F_{LU} + (2h)\sqrt{2\epsilon(1-\epsilon)}\sin\phi F_{LU}^{\sin\phi} + (2h)\sqrt{2\epsilon(1-\epsilon)}\cos\phi F_{LU}^{\cos\phi}\right] \\
&+ (2\Lambda)\left[F_{UL} + \sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi} + \sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UL}^{\cos\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi}\right. \\
&+ (2h)\sqrt{1-\epsilon^2}F_{LL} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\cos\phi F_{LL}^{\cos\phi} + \left.2(2h)\sqrt{\epsilon(1-\epsilon)}\sin\phi F_{LL}^{\sin\phi}\right] \\
&+ |\vec{S}_\perp|\Big[\sin(\phi-\phi_S)\left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right) \\
&\quad \epsilon\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)} + \epsilon\sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)} \\
&\quad + \sqrt{2\epsilon(1+\epsilon)}\left(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\right)\Big] \\
&+ (2h)|\vec{S}_\perp|\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LT}^{\cos\phi_S} \\
&\quad + \sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big]\Big\}
\end{aligned}
$$

S. Ahmad, G. R. Goldstein, S. Liuti, Phys. Rev. D79, 054014 (2009).

Photon/Proton Helicity Amplitudes



$$f_{\Lambda,\Lambda'}^{\Lambda_\gamma^*,\Lambda_\gamma'} = \sum_{\lambda,\lambda'} g_{\lambda,\lambda'}^{\Lambda_\gamma^*,\Lambda_\gamma'}(x,\xi,t;Q^2) \otimes A_{\Lambda,\lambda,\Lambda',\lambda'}(x,\xi,t)$$

Tree Level

$$g_{\lambda,\lambda'}^{\Lambda_\gamma,\Lambda_\gamma'} = ie^2\overline{u}(k,\lambda')\gamma^\mu\gamma^+\gamma^\nu\left[\frac{\varepsilon_\mu^*(q,\Lambda_\gamma')\varepsilon_\nu(q,\Lambda_\gamma)}{(k+q)^2+i\epsilon}+\frac{\varepsilon_\nu^*(q,\Lambda_\gamma')\varepsilon_\mu(q,\Lambda_\gamma)}{(k-q)^2+i\epsilon}\right]u(k,\lambda)q^-$$

$\mu,\nu \in \{1,2\}$

$\mu \in \{1,2\},\ \nu \in \{0,3\}$

Jakob, Wick Annals Phys 7, 404 (1959)

Photon/Proton Structure Functions and Phase

Definition of a Helicity Amplitude →

$$f_{\Lambda\Lambda'}^{\Lambda_{\gamma*}\Lambda'_\gamma}(\theta,\phi) = e^{-i(\Lambda_{\gamma*}-\Lambda-\Lambda'_\gamma+\Lambda')\phi}\,\tilde{f}_{\Lambda\Lambda'}^{\Lambda_{\gamma*}\Lambda'_\gamma}(\theta),$$

$$e^{i(\Lambda_{\gamma*}^{(1)}-\Lambda_{\gamma*}^{(2)})\,\phi}\,F_{\Lambda\Lambda'}^{\Lambda_{\gamma*}^{(1)}\Lambda_{\gamma*}^{(2)}} = e^{i(\Lambda_{\gamma*}^{(1)}-\Lambda_{\gamma*}^{(2)})\,\phi}\sum_{\Lambda_{\gamma'}}\left(\tilde{f}_{\Lambda\Lambda'}^{\Lambda_{\gamma*}^{(1)}\Lambda'_\gamma}\right)^*\tilde{f}_{\Lambda\Lambda'}^{\Lambda_{\gamma*}^{(2)}\Lambda'_\gamma} = \sum_{\Lambda_{\gamma'}}\left(f_{\Lambda\Lambda'}^{\Lambda_{\gamma*}^{(1)}\Lambda'_\gamma}\right)^* f_{\Lambda\Lambda'}^{\Lambda_{\gamma*}^{(2)}\Lambda'_\gamma}$$

Enter the observables in your cross section.

$$e^{i(\Lambda_{\gamma*}^{(1)}-\Lambda_{\gamma*}^{(2)}-2\Lambda)\,\phi}\,F_{T,\Lambda\Lambda'}^{\Lambda_{\gamma*}^{(1)}\Lambda_{\gamma*}^{(2)}} =$$

$$\sum_{\Lambda_{\gamma'}}\left(f_{\Lambda\Lambda'}^{\Lambda_{\gamma*}^{(1)}\Lambda'_\gamma}\right)^* f_{-\Lambda\Lambda'}^{\Lambda_{\gamma*}^{(2)}\Lambda'_\gamma} = e^{i(\Lambda_{\gamma*}^{(1)}-\Lambda_{\gamma*}^{(2)}-2\Lambda)\,\phi}\sum_{\Lambda_{\gamma'}}\left(\tilde{f}_{\Lambda\Lambda'}^{\Lambda_{\gamma*}^{(1)}\Lambda'_\gamma}\right)^*\tilde{f}_{-\Lambda\Lambda'}^{\Lambda_{\gamma*}^{(2)}\Lambda'_\gamma}$$

Helicity Decomposition of DVCS Observables

$$\sqrt{2}\sqrt{\epsilon(1+\epsilon)}\sum_{\Lambda'_\gamma,\,\Lambda'}\left[-\left(f^{1,\Lambda'_\gamma}_{\Lambda,\Lambda'}\right)^* f^{0,\Lambda'_\gamma}_{\Lambda,\Lambda'} - \left(f^{0,\Lambda'_\gamma}_{\Lambda,\Lambda'}\right)^* f^{1,\Lambda'_\gamma}_{\Lambda,\Lambda'} + \left(f^{-1,\Lambda'_\gamma}_{\Lambda,\Lambda'}\right)^* f^{0,\Lambda'_\gamma}_{\Lambda,\Lambda'} + \left(f^{0,\Lambda'_\gamma}_{\Lambda,\Lambda'}\right)^* f^{-1,\Lambda'_\gamma}_{\Lambda,\Lambda'}\right]$$

$$= \left(e^{i\phi}+e^{-i\phi}\right)\Re e\left(-\widetilde{F}^{01}_{\Lambda+}-\widetilde{F}^{01}_{\Lambda-}+\widetilde{F}^{0-1}_{\Lambda+}+\widetilde{F}^{0-1}_{\Lambda-}\right)+i\left(e^{i\phi}-e^{-i\phi}\right)\Im m\left(\widetilde{F}^{01}_{\Lambda+}+\widetilde{F}^{01}_{\Lambda-}+\widetilde{F}^{0-1}_{\Lambda+}+\widetilde{F}^{0-1}_{\Lambda-}\right)$$

$$= 2\cos\phi\,\Re e\left(-\widetilde{F}^{01}_{\Lambda+}-\widetilde{F}^{01}_{\Lambda-}+\widetilde{F}^{0-1}_{\Lambda+}+\widetilde{F}^{0-1}_{\Lambda-}\right)+2\sin\phi\,\Im m\left(\widetilde{F}^{01}_{\Lambda+}+\widetilde{F}^{01}_{\Lambda-}+\widetilde{F}^{0-1}_{\Lambda+}+\widetilde{F}^{0-1}_{\Lambda-}\right)$$

$$\sigma_{UU} = \sigma_{++} + \sigma_{--}$$

$$F_{UU}^{\cos\phi} = -2\,\Re\mathrm{e}\left(F^{01}_{++}+F^{01}_{+-}+F^{01}_{-+}+F^{01}_{--}\right)$$

$$F_{UU}^{\sin\phi} = 0$$

In terms of GPDs

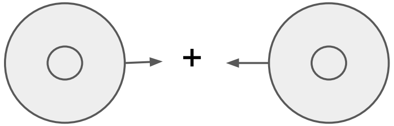
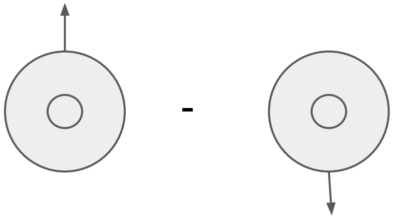
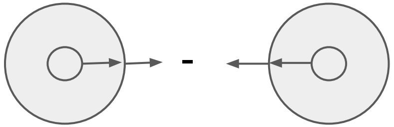
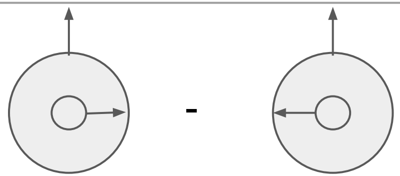
Twist-2 Observables

$$F_{UU,T} = 4\Big[(1-\xi^2)\left(|\,\mathcal{H}\,|^2 + |\,\tilde{\mathcal{H}}\,|^2\right) + \frac{t_o - t}{2M^2}\left(|\,\mathcal{E}\,|^2 + \xi^2\,|\,\tilde{\mathcal{E}}\,|^2\right) - \frac{2\xi^2}{1-\xi^2}\,\Re\mathrm{e}\left(\mathcal{H}\mathcal{E} + \widetilde{\mathcal{H}}\widetilde{\mathcal{E}}\right)\Big]$$

$$F_{LL} = 2\left[2(1-\xi^2)\ |\,\mathcal{H}\,\widetilde{\mathcal{H}}\,| + 4\xi\,\frac{t_o - t}{2M^2}\ |\,\mathcal{E}\widetilde{\mathcal{E}}\,| + \frac{2\xi^2}{1-\xi^2}\Re\mathrm{e}\left(\mathcal{H}\widetilde{\mathcal{E}} + \widetilde{\mathcal{H}}\mathcal{E}\right)\right]$$

$$F_{UT,T}^{\sin(\phi-\phi_S)} = -\frac{\sqrt{t_0 - t}}{2M}\Big[\Re e\left(\widetilde{\mathcal{H}} - \tfrac{\xi^2}{1-\xi^2}\widetilde{\mathcal{E}}\right)\Im m\mathcal{E} - \xi\,\Re e\left(\mathcal{H} - \tfrac{\xi^2}{1-\xi^2}\mathcal{E}\right)\Im m\widetilde{\mathcal{E}}$$

$$-\,\Im m\left(\widetilde{\mathcal{H}} - \tfrac{\xi^2}{1-\xi^2}\widetilde{\mathcal{E}}\right)\Re e\mathcal{E} + \xi\,\Im m\left(\mathcal{H} - \tfrac{\xi^2}{1-\xi^2}\mathcal{E}\right)\Re e\widetilde{\mathcal{E}}\Big]$$

GPD	Phase	Helicity Composition
H	1	
$\Delta_T E$	$e^{i\phi}$	
$\widetilde{H}$	1	
$\xi \Delta_T \widetilde{E}$	$e^{i\phi}$	

Twist-3 Observables

$$F_{UU}^{\cos\phi} = -2\,(1-\xi^2)\Re\mathrm{e}\Big[\left(2\widetilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T} + 2\widetilde{\mathcal{H}}'_{2T} + \mathcal{E}'_{2T}\right)\left(\mathcal{H} - \frac{\xi^2}{1-\xi^2}\mathcal{E}\right)$$
$$- 2\xi\left(\widetilde{\mathcal{E}}_{2T} + \widetilde{\mathcal{E}}'_{2T}\right)\left(\widetilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2}\widetilde{\mathcal{E}}\right) + \frac{t_0 - t}{16M^2}\left(\widetilde{\mathcal{H}}_{2T} + \widetilde{\mathcal{H}}'_{2T}\right)\left(\mathcal{E} + \xi\widetilde{\mathcal{E}}\right)$$
$$+ \left(\mathcal{H}_{2T} + \mathcal{H}'_{2T} + \frac{t_0 - t}{4M^2}\left(\widetilde{\mathcal{H}}_{2T} + \widetilde{\mathcal{H}}'_{2T}\right) + \frac{\xi}{1-\xi^2}\left(\widetilde{\mathcal{E}}_{2T} + \widetilde{\mathcal{E}}'_{2T}\right)\right.$$
$$\left. - \frac{\xi^2}{1-\xi^2}\left(\mathcal{E}_{2T} + \mathcal{E}'_{2T}\right)\right)\left(\mathcal{E} - \xi\widetilde{\mathcal{E}}\right)\Big]$$

What are these linear combinations of GPDs?

$$F_{LU}^{\sin\phi} = -2\,(1-\xi^2)\Im\mathrm{m}\Big[\left(2\widetilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T} + 2\widetilde{\mathcal{H}}'_{2T} + \mathcal{E}'_{2T}\right)\left(\mathcal{H} - \frac{\xi^2}{1-\xi^2}\mathcal{E}\right)$$
$$- 2\xi\left(\widetilde{\mathcal{E}}_{2T} + \widetilde{\mathcal{E}}'_{2T}\right)\left(\widetilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2}\widetilde{\mathcal{E}}\right) + \frac{t_0 - t}{16M^2}\left(\widetilde{\mathcal{H}}_{2T} + \widetilde{\mathcal{H}}'_{2T}\right)\left(\mathcal{E} + \xi\widetilde{\mathcal{E}}\right)$$
$$+ \Big[\left(\mathcal{H}_{2T} + \mathcal{H}'_{2T} + \frac{t_0 - t}{4M^2}\left(\widetilde{\mathcal{H}}_{2T} + \widetilde{\mathcal{H}}'_{2T}\right) + \frac{\xi}{1-\xi^2}\left(\widetilde{\mathcal{E}}_{2T} + \widetilde{\mathcal{E}}'_{2T}\right)\right.$$
$$\left. - \frac{\xi^2}{1-\xi^2}\left(\mathcal{E}_{2T} + \mathcal{E}'_{2T}\right)\right)\left(\mathcal{E} - \xi\widetilde{\mathcal{E}}\right)\Big]$$

R. L. Jaffe [hep-ph/9602236]

Good / Bad Components Definition

$$P^{\pm} = \frac{1}{2}\gamma^{\mp}\gamma^{\pm} \quad \psi_+ = \phi$$
$$\psi_{\pm} = P^{\pm}\psi \quad \psi_- = \chi$$
$$A^+ = 0$$

$$\left(i\not{D} - m\right)\psi = 0$$

$$D_{\pm} = \frac{\partial}{\partial \xi^{\pm}} - igA^{\mp}$$

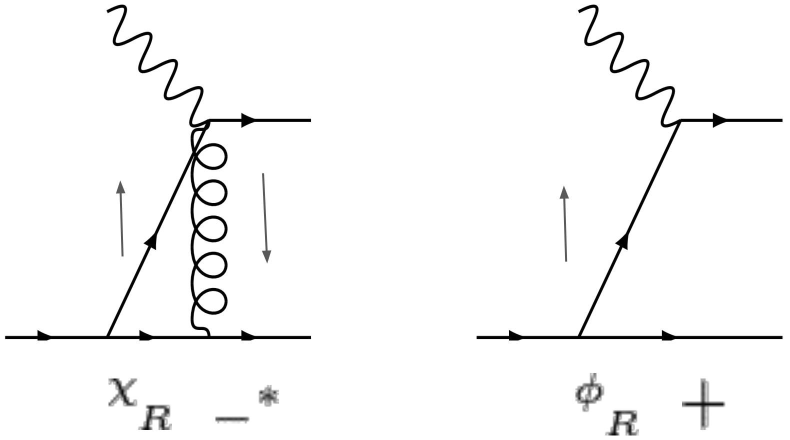
$$P^-\left(i\not{D} - m\right)\psi = 0$$
$$\frac{1}{2}\gamma^+\gamma^-\left(i\gamma^+ D_+ + i\gamma^- D_- + i\gamma^{\perp}\cdot D_{\perp} - m\right)\psi = 0$$
$$\left(i\gamma^+ P^+ D_+ + iP^-\gamma^{\perp}\cdot D_{\perp} - mP^-\right)\psi = 0$$
$$i\gamma^+ D_+\phi = \left(-i\gamma^{\perp}\cdot D_{\perp} + m\right)\chi$$

$$P^+\left(i\not{D} - m\right)\psi$$
$$\frac{1}{2}\gamma^-\gamma^+\left(i\gamma^+ D_+ + i\gamma^- D_- + i\gamma^{\perp}\cdot D_{\perp} - m\right)\psi = 0$$
$$\left(i\gamma^- P^- D_- + iP^+\gamma^{\perp}\cdot D_{\perp} - mP^+\right)\psi = 0$$
$$i\gamma^- D_-\chi = \left(-i\gamma^{\perp}\cdot D_{\perp} + m\right)\phi$$

Dynamically Independent Fields → Good Components

Dynamically Dependent Fields → Bad Components

$$\begin{pmatrix} \phi_R^\dagger \phi_R & \chi_R^\dagger \phi_R & \chi_L^\dagger \phi_R & \phi_L^\dagger \phi_R \\ \phi_R^\dagger \chi_R & \chi_R^\dagger \chi_R & \chi_L^\dagger \chi_R & \phi_L^\dagger \chi_R \\ \phi_R^\dagger \chi_L & \chi_R^\dagger \chi_L & \chi_L^\dagger \chi_L & \phi_L^\dagger \chi_L \\ \phi_R^\dagger \phi_L & \chi_R^\dagger \phi_L & \chi_L^\dagger \phi_L & \phi_L^\dagger \phi_L \end{pmatrix}$$



$$\gamma^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$= \chi_R^\dagger \phi_R + \phi_R^\dagger \chi_R - \phi_L^\dagger \chi_L - \chi_L^\dagger \phi_L$$

Tangerman (1996) *Higher-twist correlations in polarized hadrons*

OAM

$$\frac{1}{\sqrt{1-\xi^2}}\frac{\Delta_\perp}{P^+}\left(\widetilde{E}_{2T}-\xi E_{2T}\right)e^{i\phi}=W_{++}^{\gamma^1}+iW_{++}^{\gamma^2}-W_{--}^{\gamma^1}-iW_{--}^{\gamma^2}$$

$$\frac{1}{\sqrt{1-\xi^2}}\frac{\Delta_\perp}{P^+}\left(E_{2T}-\xi\widetilde{E}_{2T}+2\widetilde{H}_{2T}\right)e^{i\phi}=W_{++}^{\gamma^1}+iW_{++}^{\gamma^2}+W_{--}^{\gamma^1}+iW_{--}^{\gamma^2}$$

$$\frac{1}{\sqrt{1-\xi^2}}\frac{\Delta_\perp^2}{MP^+}2\widetilde{H}_{2T}=\left(W_{-+}^{\gamma^1}-iW_{-+}^{\gamma^2}\right)e^{2i\phi}-\left(W_{+-}^{\gamma^1}+iW_{+-}^{\gamma^2}\right)e^{-2i\phi}$$

$$\frac{1}{\sqrt{1-\xi^2}}\frac{4M}{P^+}\left(\widetilde{E}_{2T}-\xi E_{2T}-(1-\xi^2)H_{2T}-\frac{\Delta_\perp^2}{4M^2}\widetilde{H}_{2T}\right)=W_{+-}^{\gamma^1}-iW_{+-}^{\gamma^2}-W_{-+}^{\gamma^1}-iW_{-+}^{\gamma^2}$$

$$\frac{1}{\sqrt{1-\xi^2}}\frac{\Delta_\perp}{P^+}\left(\widetilde{E}'_{2T}-\xi E'_{2T}\right)e^{i\phi}=W^{\gamma^1\gamma_5}_{++}+iW^{\gamma^2\gamma_5}_{++}-W^{\gamma^1\gamma_5}_{--}-iW^{\gamma^2\gamma_5}_{--}$$

Spin-Orbit

$$\frac{1}{\sqrt{1-\xi^2}}\frac{\Delta_\perp}{P^+}\left(E'_{2T}-\xi\widetilde{E}'_{2T}+2\widetilde{H}'_{2T}\right)e^{i\phi}=W^{\gamma^1\gamma_5}_{++}+iW^{\gamma^2\gamma_5}_{++}+W^{\gamma^1\gamma_5}_{--}+iW^{\gamma^2\gamma_5}_{--}$$

$$-\frac{1}{\sqrt{1-\xi^2}}\frac{\Delta_\perp^2}{MP^+}2\widetilde{H}'_{2T}=\left(W^{\gamma^1\gamma_5}_{+-}+iW^{\gamma^2\gamma_5}_{+-}\right)e^{-2i\phi}+\left(W^{\gamma^1\gamma_5}_{-+}-iW^{\gamma^2\gamma^5}_{-+}\right)e^{2i\phi}$$

$$\frac{1}{\sqrt{1-\xi^2}}\frac{4M}{P^+}\left(\widetilde{E}'_{2T}-\xi E'_{2T}+(1-\xi^2)H'_{2T}+\frac{\Delta_T^2}{4M^2}\widetilde{H}'_{2T}\right)=W^{\gamma^1\gamma_5}_{+-}-iW^{\gamma^2\gamma_5}_{+-}+W^{\gamma^1\gamma_5}_{-+}+iW^{\gamma^2\gamma_5}_{-+}$$

Twist-4 Observables

$$F_{UU,L} = \frac{1}{(P^+)^2}\Bigg[2\left|\widetilde{\mathcal{H}}'_{2T} - \widetilde{\mathcal{H}}_{2T}\right|^2 + \frac{(1+\xi)^2}{4}\left|\mathcal{E}'_{2T} - \mathcal{E}_{2T} + \widetilde{\mathcal{E}}_{2T} - \widetilde{\mathcal{E}}'_{2T}\right|^2$$
$$+ \frac{(1-\xi)^2}{4}\left|\mathcal{E}'_{2T} - \mathcal{E}_{2T} - \widetilde{\mathcal{E}}_{2T} + \widetilde{\mathcal{E}}'_{2T}\right|^2 + (1+\xi)\Re e\left(\widetilde{\mathcal{H}}'_{2T} - \widetilde{\mathcal{H}}_{2T}\right)\left(\mathcal{E}'_{2T} - \mathcal{E}_{2T}\right)$$
$$+ (1+\xi)\Im m\left(\widetilde{\mathcal{H}}'_{2T} - \widetilde{\mathcal{H}}_{2T}\right)\left(\mathcal{E}'_{2T} - \mathcal{E}_{2T}\right)\Bigg]$$

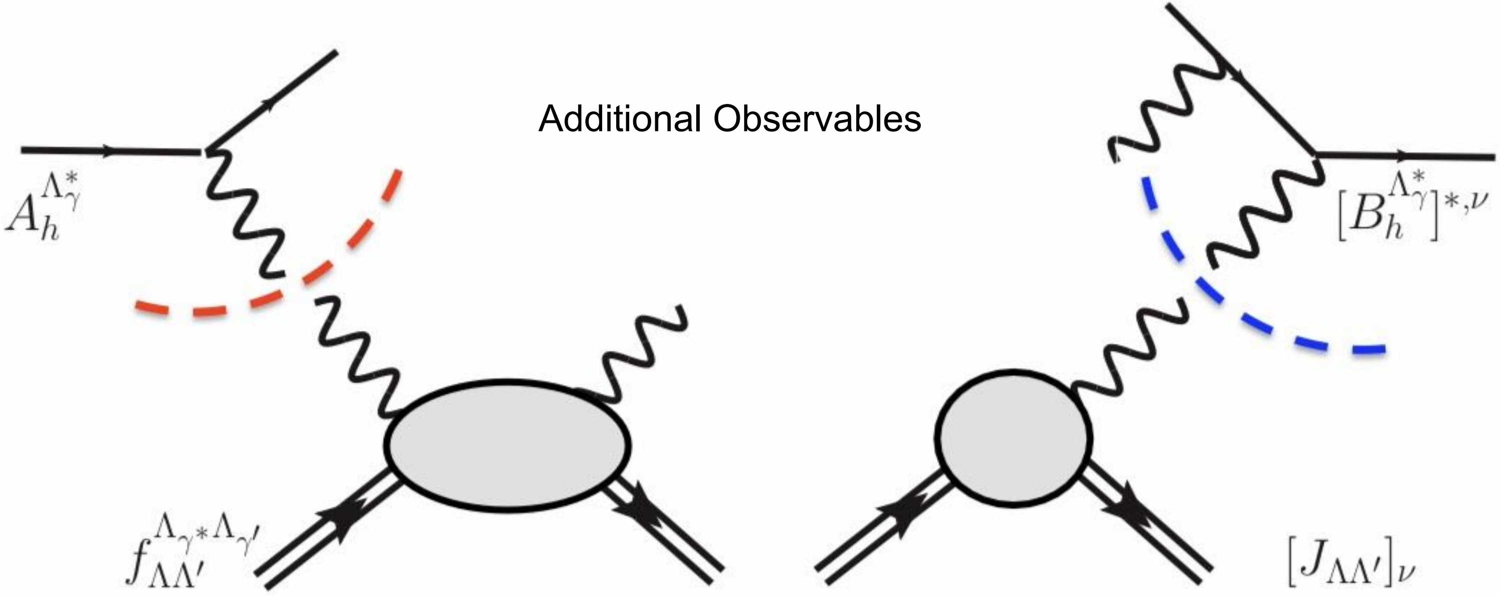
The Twist-4 observables are constructed from Twist-3 GPDs. This is because the DVCS process cannot access the Twist-4 sector of the hadronic tensor.

DVCS/BH Interference

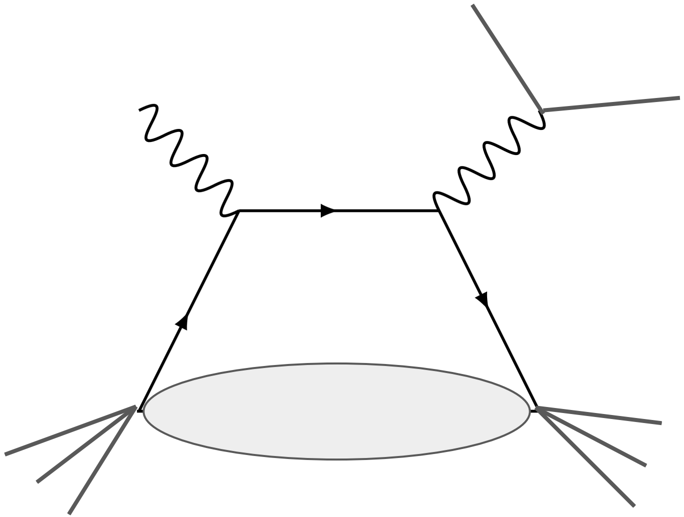
$$|T|^2 = |T_{\mathrm{BH}} + T_{\mathrm{DVCS}}|^2 = |T_{\mathrm{BH}}|^2 + |T_{\mathrm{DVCS}}|^2 + \mathcal{I}\ .$$

$$\mathcal{I} = T^*_{BH} T_{DVCS} + T^*_{DVCS} T_{BH}.$$

Additional Observables



TCS



Twist - 2
Twist - 3

$$e^{i\phi} \rightarrow e^{-i\phi}$$

$$W^{\mu\nu} \propto \gamma^{\mu}\gamma^{+}\gamma^{\nu} = \begin{bmatrix} \gamma^- & \gamma^1 - i\gamma^2\gamma_5 & \gamma^2 + i\gamma^1\gamma_5 & i\gamma^-\gamma_5 \\ \gamma^1 + i\gamma^2\gamma_5 & \gamma^+ & i\gamma^+\gamma_5 & -\gamma^1 - i\gamma^2\gamma_5 \\ \gamma^2 - i\gamma^1\gamma_5 & -i\gamma^+\gamma_5 & \gamma^+ & -\gamma^2 + i\gamma^1\gamma_5 \\ -i\gamma^-\gamma_5 & -\gamma^1 + i\gamma^2\gamma^5 & -\gamma^2 - i\gamma^1\gamma_5 & \gamma^- \end{bmatrix}$$

TCS

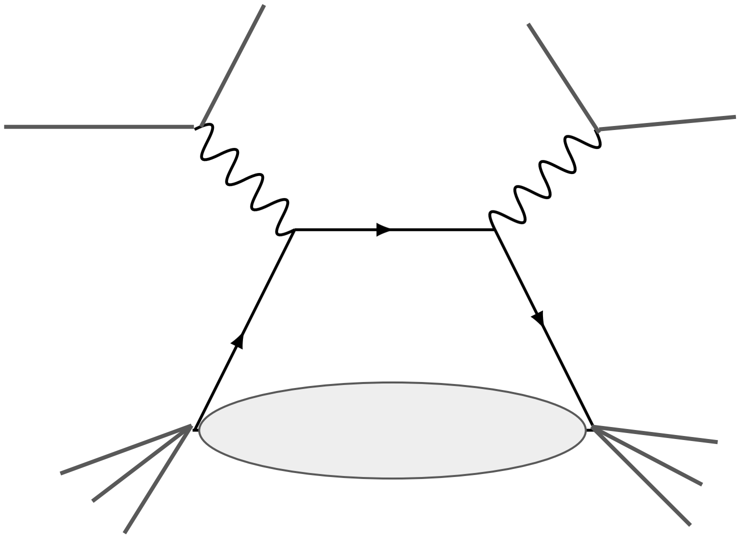
$$F_{UU}^{\cos\phi} = -2(1-\xi^2)\Re\mathrm{e}\Big[\Big(2\widetilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T} - 2\widetilde{\mathcal{H}}'_{2T} - \mathcal{E}'_{2T}\Big)\Big(\mathcal{H} - \frac{\xi^2}{1-\xi^2}\mathcal{E}\Big)$$
$$- 2\xi\Big(\widetilde{\mathcal{E}}_{2T} - \widetilde{\mathcal{E}}'_{2T}\Big)\Big(\widetilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2}\widetilde{\mathcal{E}}\Big) + \frac{t_0 - t}{16M^2}\Big(\widetilde{\mathcal{H}}_{2T} - \widetilde{\mathcal{H}}'_{2T}\Big)\Big(\mathcal{E} + \xi\widetilde{\mathcal{E}}\Big)$$
$$+ \Big(\mathcal{H}_{2T} - \mathcal{H}'_{2T} + \frac{t_0 - t}{4M^2}\Big(\widetilde{\mathcal{H}}_{2T} - \widetilde{\mathcal{H}}'_{2T}\Big) + \frac{\xi}{1-\xi^2}\Big(\widetilde{\mathcal{E}}_{2T} - \widetilde{\mathcal{E}}'_{2T}\Big)$$
$$- \frac{\xi^2}{1-\xi^2}\Big(\mathcal{E}_{2T} - \mathcal{E}'_{2T}\Big)\Big)\Big(\mathcal{E} - \xi\widetilde{\mathcal{E}}\Big)\Big]$$

$$-2\xi\widetilde{\mathcal{E}}_{2T}\widetilde{\mathcal{H}}$$

DVCS

$$F_{UU}^{\cos\phi} = -2(1-\xi^2)\Re\mathrm{e}\Big[\Big(2\widetilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T} + 2\widetilde{\mathcal{H}}'_{2T} + \mathcal{E}'_{2T}\Big)\Big(\mathcal{H} - \frac{\xi^2}{1-\xi^2}\mathcal{E}\Big)$$
$$- 2\xi\Big(\widetilde{\mathcal{E}}_{2T} + \widetilde{\mathcal{E}}'_{2T}\Big)\Big(\widetilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2}\widetilde{\mathcal{E}}\Big) + \frac{t_0 - t}{16M^2}\Big(\widetilde{\mathcal{H}}_{2T} + \widetilde{\mathcal{H}}'_{2T}\Big)\Big(\mathcal{E} + \xi\widetilde{\mathcal{E}}\Big)$$
$$+ \Big(\mathcal{H}_{2T} + \mathcal{H}'_{2T} + \frac{t_0 - t}{4M^2}\Big(\widetilde{\mathcal{H}}_{2T} + \widetilde{\mathcal{H}}'_{2T}\Big) + \frac{\xi}{1-\xi^2}\Big(\widetilde{\mathcal{E}}_{2T} + \widetilde{\mathcal{E}}'_{2T}\Big)$$
$$- \frac{\xi^2}{1-\xi^2}\Big(\mathcal{E}_{2T} + \mathcal{E}'_{2T}\Big)\Big)\Big(\mathcal{E} - \xi\widetilde{\mathcal{E}}\Big)\Big]$$

DDVCS



Twist - 2
Twist - 3
Twist - 4

Isolate OAM directly through a singular process?

Developed formalism through GPDs to isolate OAM.

$$W^{\mu\nu} \propto \gamma^\mu \gamma^+ \gamma^\nu = \begin{bmatrix} \gamma^- & \gamma^1 - i\gamma^2\gamma_5 & \gamma^2 + i\gamma^1\gamma_5 & i\gamma^-\gamma_5 \\ \gamma^1 + i\gamma^2\gamma_5 & \gamma^+ & i\gamma^+\gamma_5 & -\gamma^1 - i\gamma^2\gamma_5 \\ \gamma^2 - i\gamma^1\gamma_5 & -i\gamma^+\gamma_5 & \gamma^+ & -\gamma^2 + i\gamma^1\gamma_5 \\ -i\gamma^-\gamma_5 & -\gamma^1 + i\gamma^2\gamma^5 & -\gamma^2 - i\gamma^1\gamma_5 & \gamma^- \end{bmatrix}$$

Summary

- We see that DVCS *alone* cannot isolate the Twist-3 GPD connected to OAM.

- More observables through DVCS/BH Interference

- Can isolate Twist-3 GPDs of the vector and axial vector sector through observables of DVCS combined with TCS.

- DDVCS, processes to isolate GTMDs, so much more work to do ...

Backup Slides

GPCF / GTMD / GPDs

GPCFs

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{d^4 z}{(2\pi)^4} e^{ik\cdot z} \Big\langle P + \frac{\Delta}{2} \Big| \overline{\psi}(-\frac{z}{2}) \Gamma \mathcal{U}(-\frac{z}{2}, \frac{z}{2}) \psi(\frac{z}{2}) \Big| P - \frac{\Delta}{2} \Big\rangle$$

$$\int dk^-$$

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ik\cdot z} \Big\langle P + \frac{\Delta}{2} \Big| \overline{\psi}(-\frac{z}{2}) \Gamma \mathcal{U}(-\frac{z}{2}, \frac{z}{2}) \psi(\frac{z}{2}) \Big| P - \frac{\Delta}{2} \Big\rangle \Big|_{z^+=0}$$

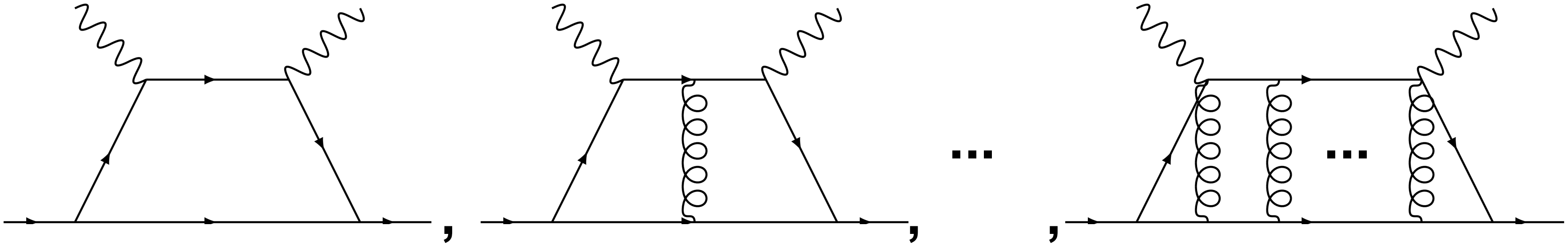
GTMDs

$$\int d^2 k_\perp$$

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-}{2\pi} e^{ixP^+ z^-} \Big\langle P + \frac{\Delta}{2} \Big| \overline{\psi}(-\frac{z}{2}) \Gamma \mathcal{U}(-\frac{z}{2}, \frac{z}{2}) \psi(\frac{z}{2}) \Big| P - \frac{\Delta}{2} \Big\rangle$$

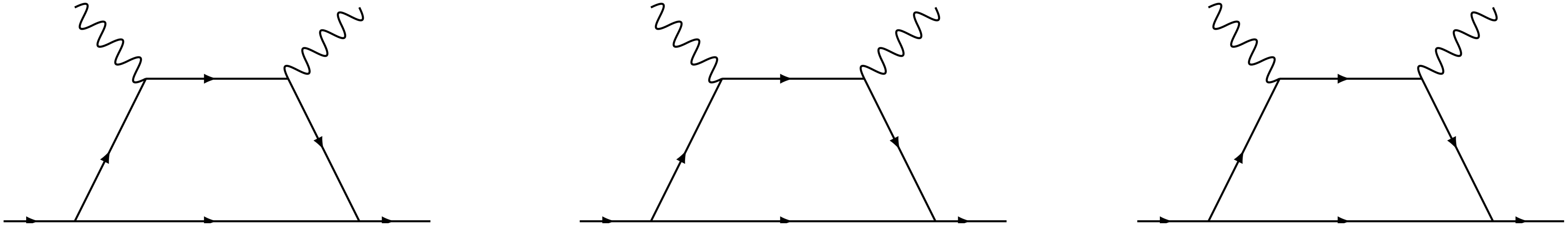
GPDs

Operator Twist vs. Kinematical Twist



Twist-2 | Twist-3 | Twist-(n+2)

$\gamma^{+}, \gamma^{+}\gamma_5, i\sigma^{i+}\gamma_5$ | $1, \gamma_5, \gamma^{i}, \gamma^{i}\gamma_5, i\sigma^{ij}\gamma_5, i\sigma^{+-}\gamma_5$ | $\gamma^{-}, \gamma^{-}\gamma_5, i\sigma^{i-}\gamma_5$

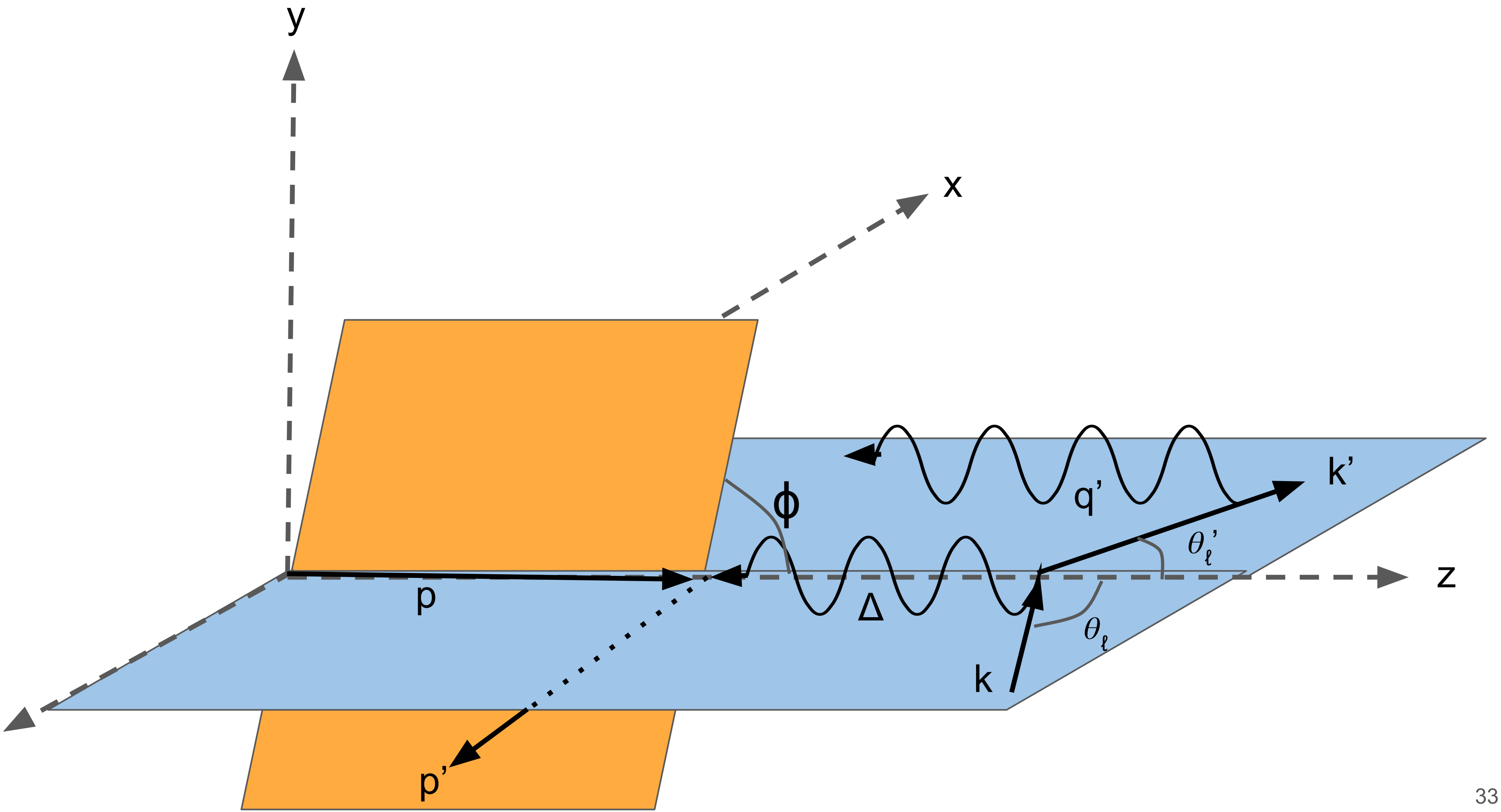


$O \sim 1$ | $O \sim \frac{1}{P^+}$ | $O \sim \frac{1}{(P^+)^2}$

$$
\begin{aligned}
W_{\Lambda'\Lambda}^{\gamma^+} &= \frac{1}{2P^+}\overline{U}(p',\Lambda')\left[\gamma^+ H + \frac{i\sigma^{+\Delta}}{2M}E\right]U(p,\Lambda) \\
&= \frac{1}{\sqrt{1-\xi^2}}\left[H(1-\xi^2) - \xi^2 E\right]\delta_{\Lambda,\Lambda'} + \frac{\Lambda}{\sqrt{1-\xi^2}}\frac{\Delta^1 + i\Lambda\Delta^2}{2M}E\,\delta_{\Lambda,-\Lambda'} \\
W_{\Lambda'\Lambda}^{\gamma^+\gamma^5} &= \frac{1}{2P^+}\overline{U}(p',\Lambda')\left[\gamma^+\gamma^5 \widetilde{H} + \frac{\Delta^+\gamma^5}{2M}\widetilde{E}\right]U(p,\Lambda) \\
&= \frac{1}{\sqrt{1-\xi^2}}\left[\Lambda\widetilde{H}(1-\xi^2) - \Lambda\xi^2\widetilde{E}\right]\delta_{\Lambda,\Lambda'} + \frac{1}{\sqrt{1-\xi^2}}\frac{\Delta^1 + i\Lambda\Delta^2}{2M}\xi\widetilde{E}\,\delta_{\Lambda,-\Lambda'}.
\end{aligned}
$$

$$
\begin{aligned}
W^{\gamma^i}_{\Lambda'\Lambda} &= \frac{M}{2(P^+)^2}\overline{U}(p',\Lambda')\left[i\sigma^{+i}H_{2T} + \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2M}E_{2T} + \frac{P^+\Delta^i}{M^2}\widetilde{H}_{2T} - \frac{P^+\gamma^i}{M}\widetilde{E}_{2T}\right]U(p,\Lambda) \\
&= \frac{1}{\sqrt{1-\xi^2}}\left[\frac{\Delta^i}{2P^+}\left(E_{2T} - \xi\widetilde{E}_{2T}\right) + \frac{i\Lambda\epsilon^{ij}\Delta^j}{2P^+}\left(\widetilde{E}_{2T} - \xi E_{2T}\right) + \frac{\Delta^i}{P^+}\widetilde{H}_{2T}\right]\delta_{\Lambda,\Lambda'} \\
&+ \frac{1}{\sqrt{1-\xi^2}}\left[\frac{-M(\Lambda\delta_{i1} + i\delta_{i2})}{P^+}\left((1-\xi^2)H_{2T} + \xi\widetilde{E}_{2T} - \xi^2 E_{2T}\right) - \Lambda\frac{\Delta^i(\Delta^1 + i\Lambda\Delta^2)}{2MP^+}\widetilde{H}_{2T}\right]\delta_{\Lambda,-\Lambda'}
\end{aligned} \tag{53}
$$

$$
\begin{aligned}
W^{\gamma^i\gamma^5}_{\Lambda'\Lambda} &= \frac{i\epsilon^{ij}M}{2(P^+)^2}\overline{U}(p',\Lambda')\left[i\sigma^{+j}H'_{2T} + \frac{\gamma^+\Delta^j - \Delta^+\gamma^j}{2M}E'_{2T} + \frac{P^+\Delta^j}{M^2}\widetilde{H}'_{2T} - \frac{P^+\gamma^j}{M}\widetilde{E}'_{2T}\right]U(p,\Lambda) \\
&= \frac{1}{\sqrt{1-\xi^2}}\left[\frac{i\epsilon^{ij}\Delta^j}{2P^+}\left(E'_{2T} - \xi\widetilde{E}'_{2T}\right) + \frac{i\epsilon^{ij}\Delta^j}{P^+}\widetilde{H}'_{2T} - \frac{\Lambda\Delta^i}{2P^+}\left(\widetilde{E}'_{2T} - \xi E'_{2T}\right)\right]\delta_{\Lambda,\Lambda'} \\
&+ \frac{1}{\sqrt{1-\xi^2}}\left[\frac{M(\delta_{i1} + i\Lambda\delta_{i2})}{P^+}\left((1-\xi^2)H'_{2T} + \xi\widetilde{E}'_{2T} - \xi^2 E'_{2T}\right) - \Lambda\frac{i\epsilon^{ij}\Delta^j(\Delta^1 + i\Lambda\Delta^2)}{2MP^+}\widetilde{H}'_{2T}\right]\delta_{\Lambda,-\Lambda'}
\end{aligned}
$$


y
x
ϕ
k’
q’
θ_ℓ’
z
p
Δ
θ_ℓ
k
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