Quark and Gluon Helicity at Small-x

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work with Dan Pitonyak and Matt Sievert,

arXiv:1706.04236 [nucl-th] and 7 other papers

Outline

- Goal: understanding the small-x asymptotics of TMDS
  - Helicity: helps evaluate the amount of proton spin coming from small x partons
  - Transversity: small-x contribution to the proton tensor charge
- Quark Helicity:
  - Quark helicity distribution at small x
  - Small-x evolution equations for quark helicity
  - Small-x asymptotics of quark helicity
- Gluon Helicity:
  - Gluon helicity distribution at small x
  - Small-x evolution equations for gluon helicity
  - Small-x asymptotics of quark helicity TMDs
- Valence Quark Transversity:
  - Quark transversity TMD at small x
  - Small-x evolution equation for quark transversity
  - Small-x asymptotics of quark transversity TMDs

Main Physical Results

- At large N_c we get for helicity

$$\Delta q(x,Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s\, N_c}{2\pi}} \approx 2.31 \sqrt{\frac{\alpha_s\, N_c}{2\pi}}$$

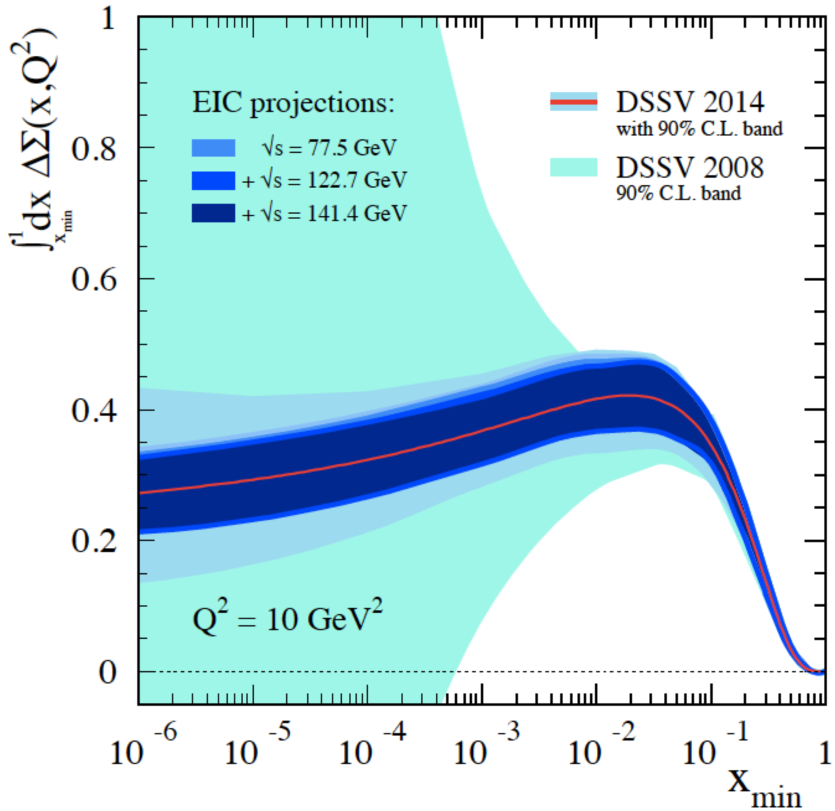
$$\Delta G(x,Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^G} \quad \text{with} \quad \alpha_h^G = \tfrac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s\, N_c}{2\pi}} \approx 1.88 \sqrt{\frac{\alpha_s\, N_c}{2\pi}}$$

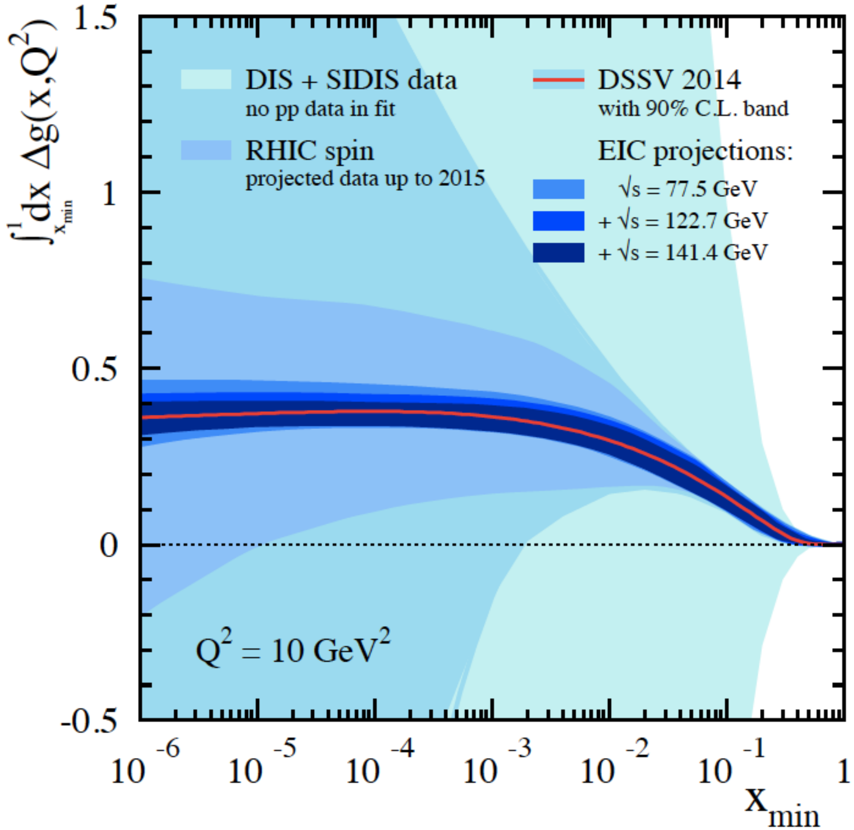
- For valence quark transversity TMDs we have (also at large N_c)

$$h_{1T}^{NS}(x,k_T^2) \sim h_{1T}^{\perp\, NS}(x,k_T^2) \sim \left(\frac{1}{x}\right)^{\alpha_t^q} \quad \text{with} \quad \alpha_t^q = -1 + 2\sqrt{\frac{\alpha_s\, C_F}{\pi}}$$

Our Goal: calculating all PDFs and TMDs at Small x

How much spin is at small x?





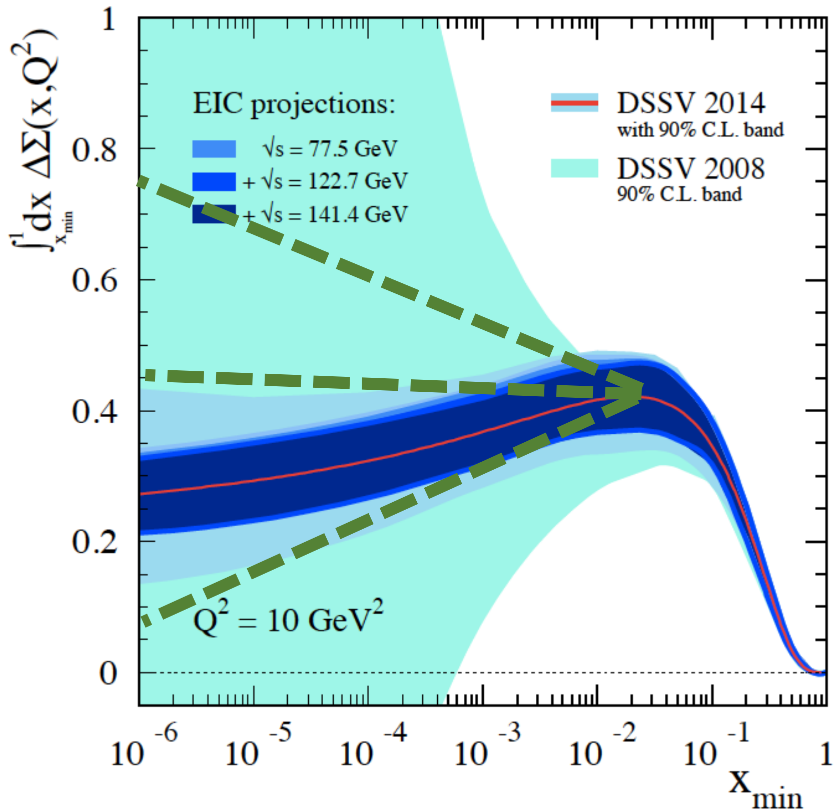
- E. Aschenaur et al, arXiv:1509.06489 **[hep-ph]**
- Uncertainties are very large at small x!

Spin at small x

- The goal of this project is to provide theoretical understanding of helicity PDF's at very small x.

- Our work would provide guidance for future hPDF's parametrizations of the existing and new data (e.g., the data to be collected at EIC).

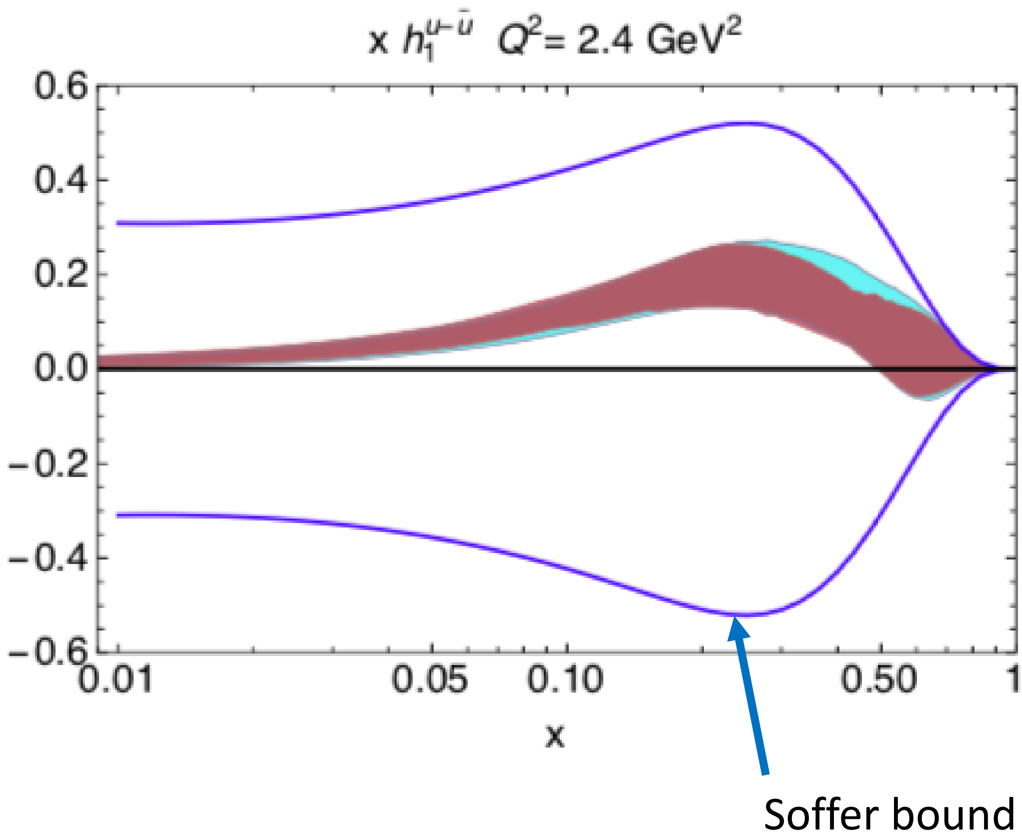
- Alternatively the data can be analyzed using our small-x evolution formalism.



Transversity and tensor charge

Transversity at small x is also poorly known, resulting in poor knowledge of the tensor charge:

$$\delta q(Q^2) = \int_0^1 dx\, h_1(x, Q^2)$$



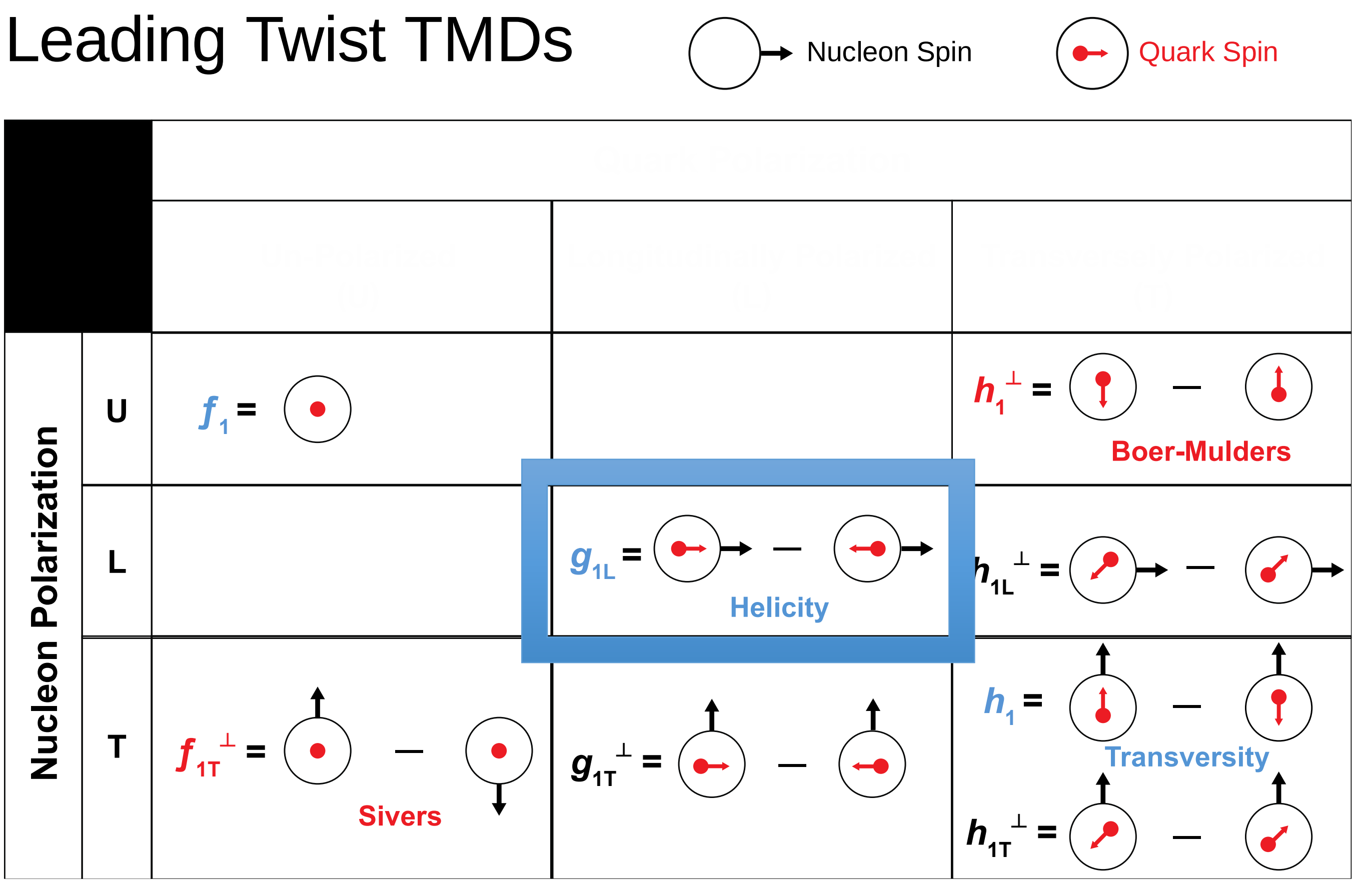
From the talk by M. Radici at DIS 2018 + see his talk at this meeting

Quark Helicity at Small x
(flavor-singlet case)

Yu.K., M. Sievert, arXiv:1505.01176 [hep-ph]
Yu.K., D. Pitonyak, M. Sievert, arXiv:1511.06737 [hep-ph], arXiv:1610.06197 [hep-ph], arXiv:1610.06188 [hep-ph], arXiv:1703.05809 [hep-ph], arXiv:1808.09010 [hep-ph]

Observables

- We want to calculate quark helicity PDF and TMD at small x.



Quark Helicity TMD

- We start with the definition of the quark helicity TMD with a future-pointing Wilson line staple.

$$g_{1L}^{q}(x,k_T^2) = \frac{1}{(2\pi)^3}\,\frac{1}{2}\sum_{S_L} S_L \int d^2 r\, dr^-\, e^{ik\cdot r} \langle p, S_L | \bar{\psi}(0)\,\mathcal{U}[0,r]\,\frac{\gamma^+\gamma^5}{2}\,\psi(r)|p,S_L\rangle_{r^+=0}$$

- At small-x, in anticipation of the shock-wave formalism, we rewrite the quark helicity TMD as (in A^-=0 gauge for the + moving proton)

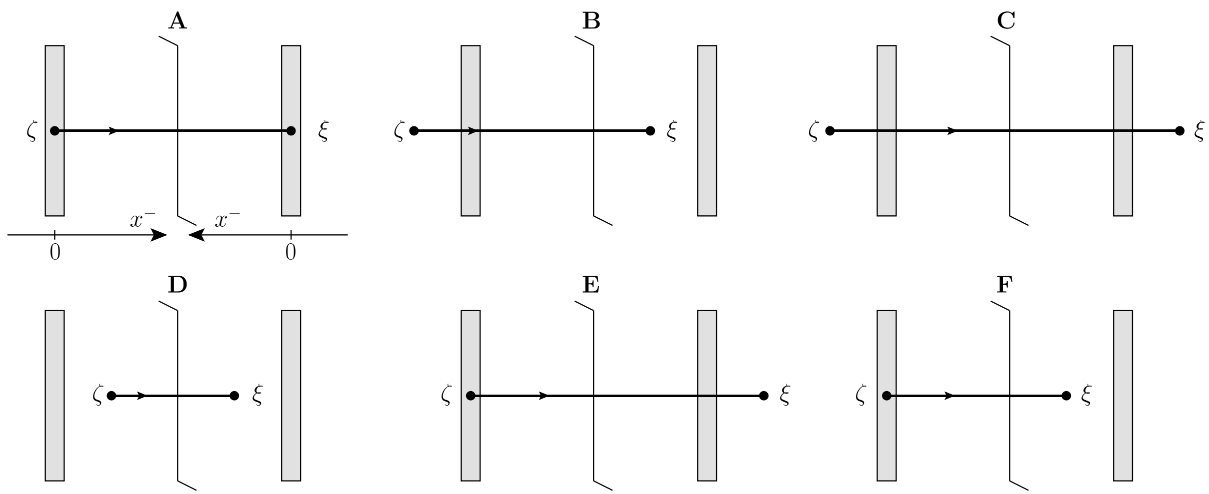
$$g_{1L}^{q}(x,k_T^2) = \frac{2p^+}{(2\pi)^3}\int d^2\zeta\, d\zeta^-\, d^2\xi\, d\xi^-\, e^{ik\cdot(\zeta-\xi)} \left(\frac{1}{2}\gamma^+\gamma^5\right)_{\alpha\beta} \left\langle \bar{\psi}_\alpha(\xi)\, V_{\underline{\xi}}[\xi^-,\infty]\;\; V_{\underline{\zeta}}[\infty,\zeta^-]\,\psi_\beta(\zeta)\right\rangle$$

where the fundamental light-cone Wilson line is

$$V_{\underline{x}}[b^-,a^-] = \mathrm{P}\exp\left\{ ig\int\limits_{a^-}^{b^-} dx^-\, A^+(x^-,\underline{x})\right\}$$

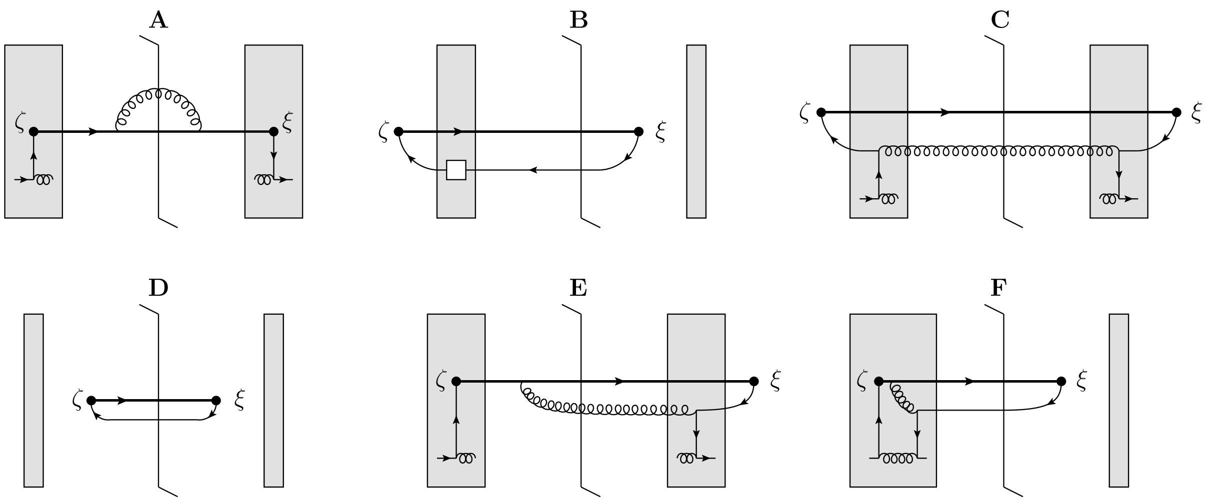
Quark Helicity TMD at Small x

- At high energy/small-x the proton is a shock wave, and we have the following contributions to the SIDIS quark helicity TMD:



$$g_{1L}^q(x, k_T^2) = \frac{2p^+}{(2\pi)^3} \sum_X \int d^2\zeta \, d\zeta^- \, d^2\xi \, d\xi^- \, e^{ik\cdot(\zeta-\xi)} \left(\tfrac{1}{2}\gamma^+\gamma^5\right)_{\alpha\beta} \left\langle \bar{\psi}_\alpha(\xi) \, V_{\underline{\xi}}[\xi^-, \infty] \, |X\rangle \; \langle X| \, V_{\underline{\zeta}}[\infty, \zeta^-] \, \psi_\beta(\zeta) \right\rangle$$

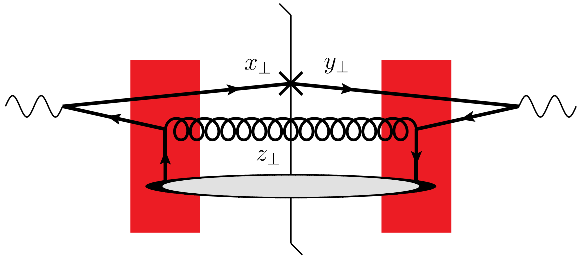
Quark Helicity TMD at Small x



- Diagram D does not transfer spin information from the target. Diagram C is canceled as we move t-channel quarks across the cut.
- Diagram F is energy-suppressed, since the gluon should have no time to be emitted and absorbed inside the shock wave.
- Diagrams of the types A and E++ can be shown to cancel each other at the leading (DLA) order (Ward identity).
- We are left with the diagram B.

Quark Helicity TMD at Small x

- Dominance of diagram B can also be obtained by applying crossing symmetry to the SIDIS process (KS ‘15):



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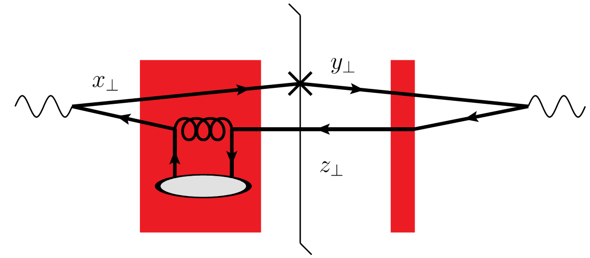
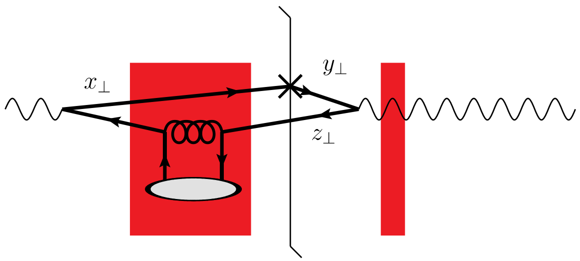


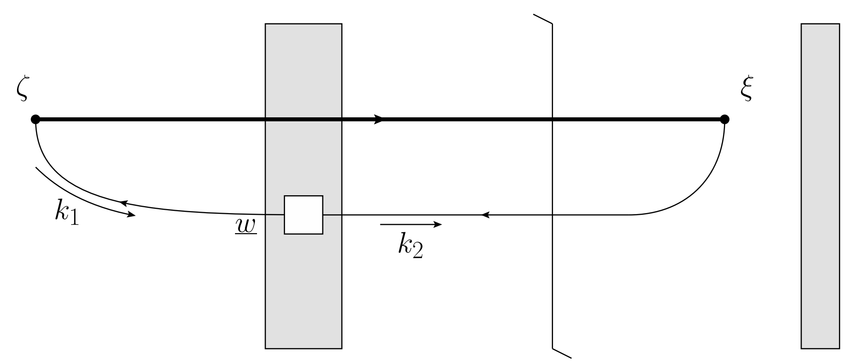
$+$ c.c. $=0$

$+$



$+$ c.c. = the answer

- Compare the last line to the diagram B: reflecting the cc amplitude into the amplitude reduces the above diagram to the one on the right.



Quark Helicity TMD at Small x

- Evaluating diagram B we arrive at

$$g_{1L}^{q}(x, k_T^2) = \frac{4N_c}{(2\pi)^6} \int d^2\zeta \, d^2w \, d^2y \, e^{-i\underline{k}\cdot(\underline{\zeta}-\underline{y})} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \, \frac{\underline{\zeta}-\underline{w}}{|\underline{\zeta}-\underline{w}|^2} \cdot \frac{\underline{y}-\underline{w}}{|\underline{y}-\underline{w}|^2} \, G_{\underline{w},\underline{\zeta}}(zs)$$

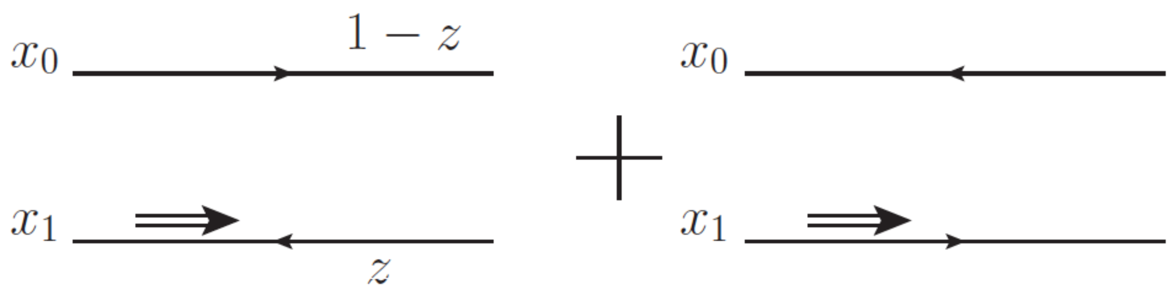
  where $G_{w\zeta}$ is the polarized dipole amplitude (defined on the next slide).

- Here s is the cms energy squared, Λ is some IR cutoff, underlining denotes transverse vectors, z = smallest longitudinal momentum fraction of the dipole momentum out of those carried by the quark and the antiquark

- The same result was previously obtained starting with the SIDIS process (KPS '15) instead of the operator definition of quark helicity TMD: we have thus shown that the two approaches are consistent at small x.

Polarized Dipole

- All flavor-singlet small-x helicity observables depend on one object, "polarized dipole amplitude":



$$G_{10}(z) \equiv \frac{1}{2N_c} \, \text{Re} \left\langle\!\!\left\langle \text{T} \, \text{tr} \left[V_{\underline{0}} V_{\underline{1}}^{pol \, \dagger} \right] + \text{T} \, \text{tr} \left[V_{\underline{1}}^{pol} V_{\underline{0}}^{\dagger} \right] \right\rangle\!\!\right\rangle (z)$$

unpolarized quark

polarized quark: eikonal propagation, non-eikonal spin-dependent interaction

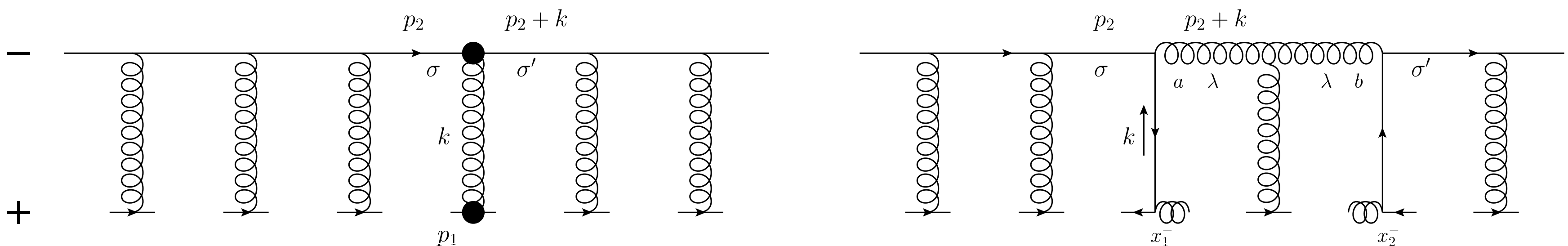
$$V_{\underline{x}} \equiv \mathcal{P} \exp \left[ig \int\limits_{-\infty}^{\infty} dx^+ \, A^-(x^+, 0^-, \underline{x}) \right]$$

- Double brackets denote an object with energy suppression scaled out:

$$\left\langle\!\!\left\langle \mathcal{O} \right\rangle\!\!\right\rangle (z) \equiv zs \left\langle \mathcal{O} \right\rangle (z)$$

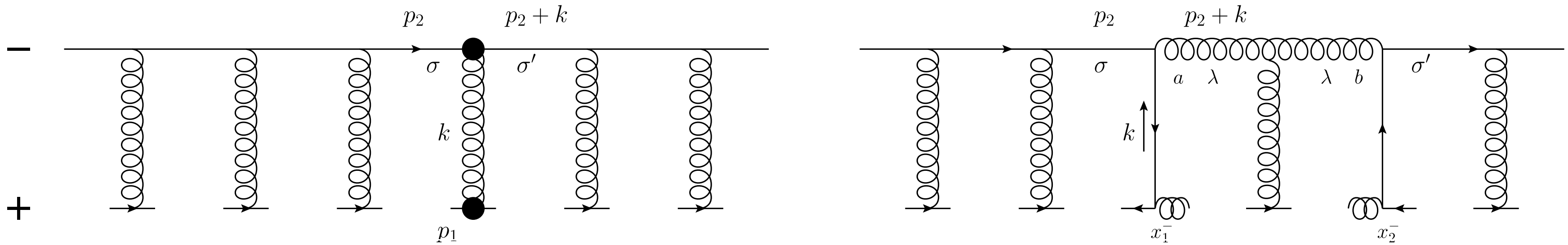
Polarized fundamental "Wilson line"

- To complete the definition of the polarized dipole amplitude, we need to construct the definition of the polarized "Wilson line" V^{pol}, which is the leading helicity-dependent contribution for the quark scattering amplitude on a longitudinally-polarized target proton.



- At the leading order we can either exchange one non-eikonal t-channel gluon (with quark-gluon vertices denoted by blobs above) to transfer polarization between the projectile and the target, or two t-channel quarks, as shown above.

Polarized fundamental "Wilson line"



- In the end one arrives at (cf. Chirilli '18)

$$V_{\underline{x}}^{pol} = \frac{igp_1^+}{s} \int\limits_{-\infty}^{\infty} dx^- \, V_{\underline{x}}[+\infty, x^-] \, F^{12}(x^-, \underline{x}) \, V_{\underline{x}}[x^-, -\infty]$$

$$- \frac{g^2 \, p_1^+}{s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \, V_{\underline{x}}[+\infty, x_2^-] \, t^b \, \psi_\beta(x_2^-, \underline{x}) \, U_{\underline{x}}^{ba}[x_2^-, x_1^-] \left[\frac{1}{2} \gamma^+ \gamma^5 \right]_{\alpha\beta} \bar{\psi}_\alpha(x_1^-, \underline{x}) \, t^a \, V_{\underline{x}}[x_1^-, -\infty].$$

- The first term on the right (the gluon exchange contribution) was known before (KPS '17), the second term (quark exchange) is new.
- We have employed an adjoint light-cone Wilson line

$$U_{\underline{x}}[b^-, a^-] = \mathcal{P} \exp \left[ig \int\limits_{a^-}^{b^-} dx^- \, \mathcal{A}^+(x^+ = 0, x^-, \underline{x}) \right]$$

Polarized adjoint “Wilson line”

- Quarks mix with gluons. Therefore, we need to construct the adjoint polarized Wilson line --- the leading helicity-dependent part of the gluon scattering amplitude on the longitudinally polarized target.

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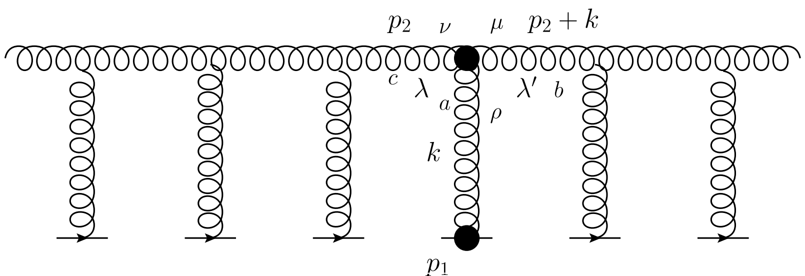
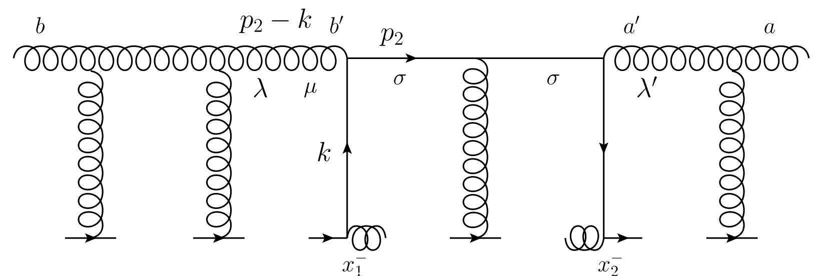




- The calculation is similar to the quark scattering case. It yields (cf. Chirilli ‘18)

$$(U_{\underline{x}}^{pol})^{ab} = \frac{2i\, g\, p_1^+}{s} \int\limits_{-\infty}^{+\infty} dx^- \ \left(U_{\underline{x}}[+\infty, x^-]\, \mathcal{F}^{12}(x^+ = 0, x^-, \underline{x})\, U_{\underline{x}}[x^-, -\infty] \right)^{ab}$$

$$- \frac{g^2\, p_1^+}{s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \ U_{\underline{x}}^{aa'}[+\infty, x_2^-]\, \bar{\psi}(x_2^-, \underline{x})\, t^{a'}\, V_{\underline{x}}[x_2^-, x_1^-]\, \frac{1}{2}\, \gamma^+ \gamma_5\, t^{b'}\, \psi(x_1^-, \underline{x})\, U_{\underline{x}}^{b'b}[x_1^-, -\infty] - \text{c.c.}$$

Small-x Evolution at large N_c

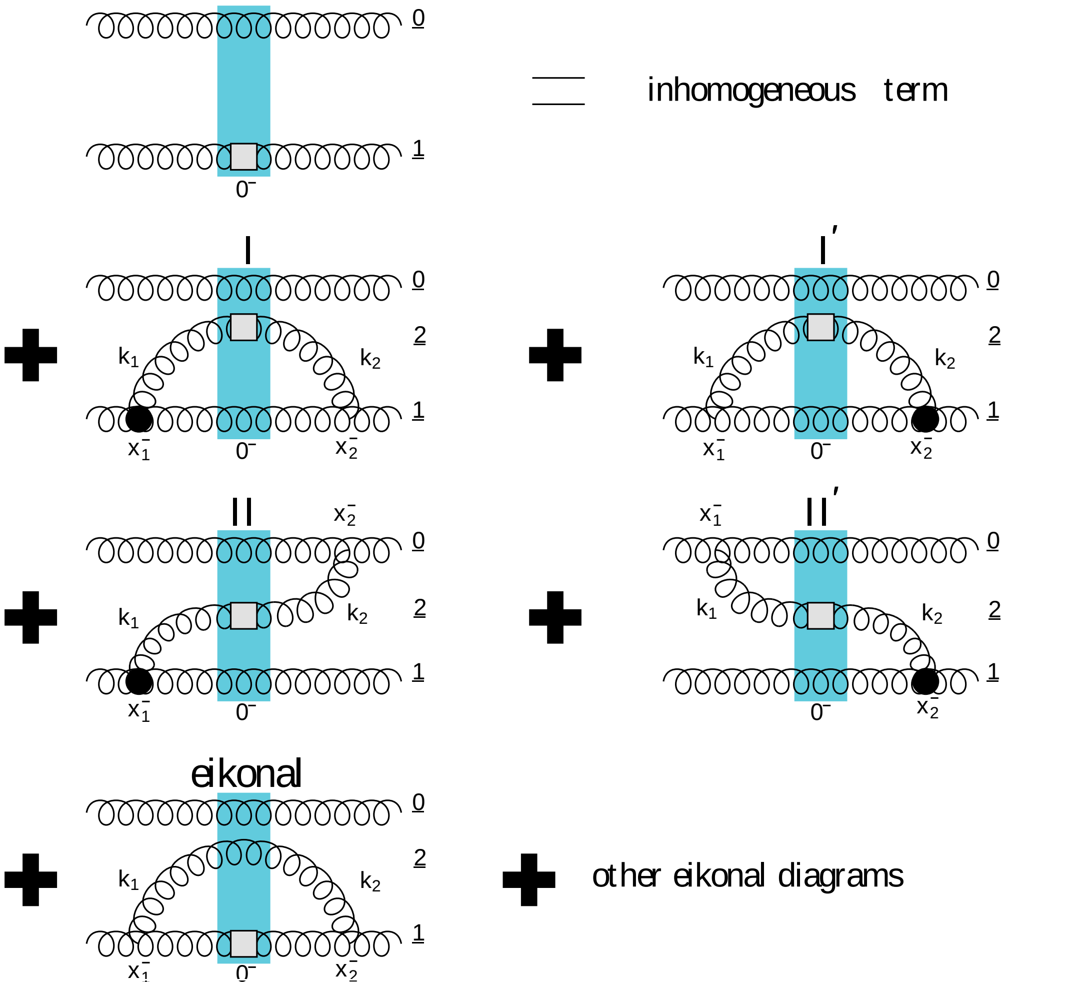
- At large N_c the evolution is gluon-driven. We will evolve a gluon dipole, remembering that at large N_c the relation between the adjoint and fundamental longitudinally-polarized gluon dipoles is

$$G_{10}^{adj}(z) = 4\, G_{10}(z)$$

(Note that the factor is 4, not 2 like in the unpolarized dipole case.)

Small-x Evolution at large N_c

- We need to sum the following diagrams (box denotes the polarized "Wilson lines"):



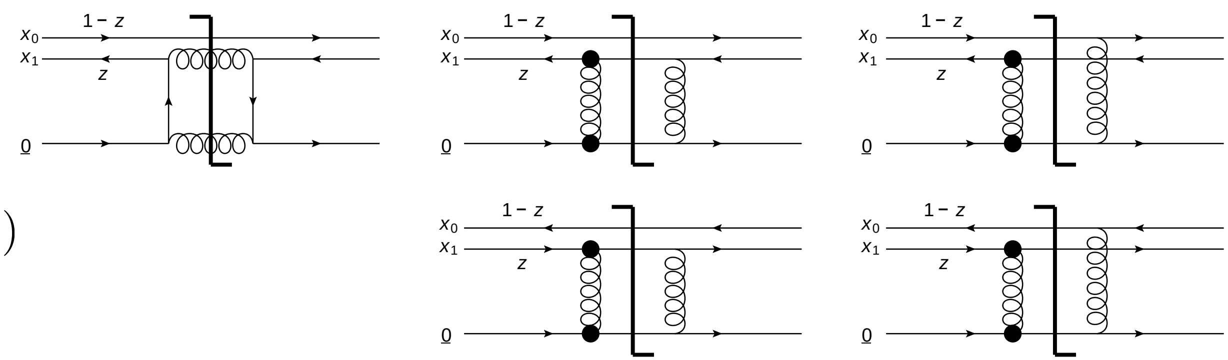
Large-N_c Evolution

- In the strict DLA limit (S=1) and at large N_c we get (here Γ is an auxiliary function we call the 'neighbour dipole amplitude') (KPS '15)

$$G(x_{10}^2, z) = G^{(0)}(x_{10}^2, z) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z') + 3\, G(x_{21}^2, z') \right]$$

$$\Gamma(x_{10}^2, x_{21}^2, z') = \Gamma^{(0)}(x_{10}^2, x_{21}^2, z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min\left\{ x_{10}^2,\, x_{21}^2 \frac{z'}{z''} \right\}} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma(x_{10}^2, x_{32}^2, z'') + 3\, G(x_{32}^2, z'') \right]$$

- The initial conditions are given by the Born-level graphs



$$\Gamma^{(0)}(x_{10}^2, x_{21}^2, z) = G^{(0)}(x_{10}^2, z)$$

$$G^{(0)}(x_{10}^2, z) = \frac{\alpha_s^2 C_F}{N_c} \pi \left[C_F \ln \frac{zs}{\Lambda^2} - 2\, \ln(zs\, x_{10}^2) \right]$$

Resummation Parameter

- For helicity evolution the resummation parameter is different from BFKL, BK or JIMWLK, which resum powers of leading logarithms (LLA)

$$\alpha_s \ \ln(1/x)$$

- Helicity evolution resummation parameter is double-logarithmic (DLA):

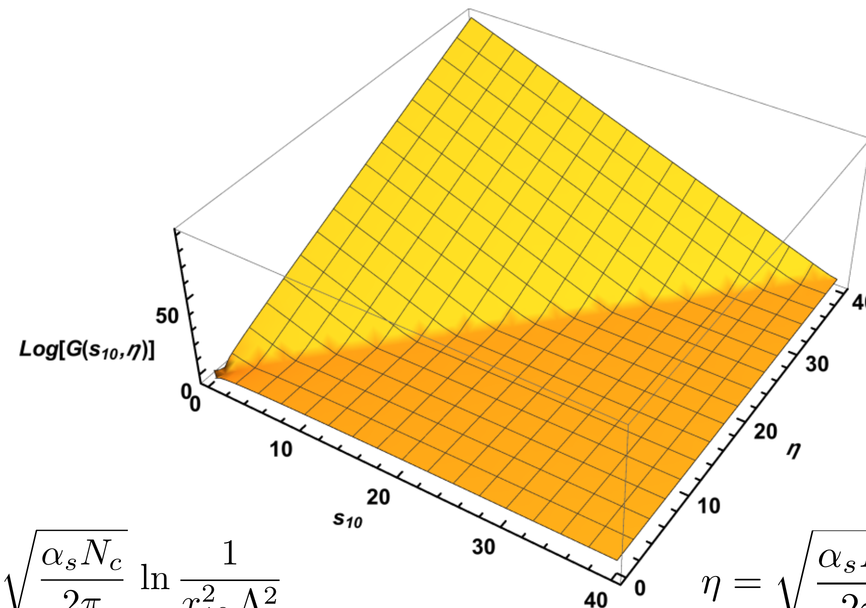
$$\alpha_s \ \ln^2 \frac{1}{x}$$

- The second logarithm of x arises due to transverse momentum (or transverse coordinate) integration being logarithmic both in the UV and IR.

- This was known before: Kirschner and Lipatov ’83; Kirschner ’84; Bartels, Ermolaev, Ryskin ‘95, ‘96; Griffiths and Ross ’99; Itakura et al ’03; Bartels and Lublinsky ‘03.

Quark Helicity at Small x

- These equations can be solved both numerically and analytically. (KPS ‘16-’17)



$$s_{10} = \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{1}{x_{10}^2 \Lambda^2} \qquad \eta = \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{zs}{\Lambda^2}$$

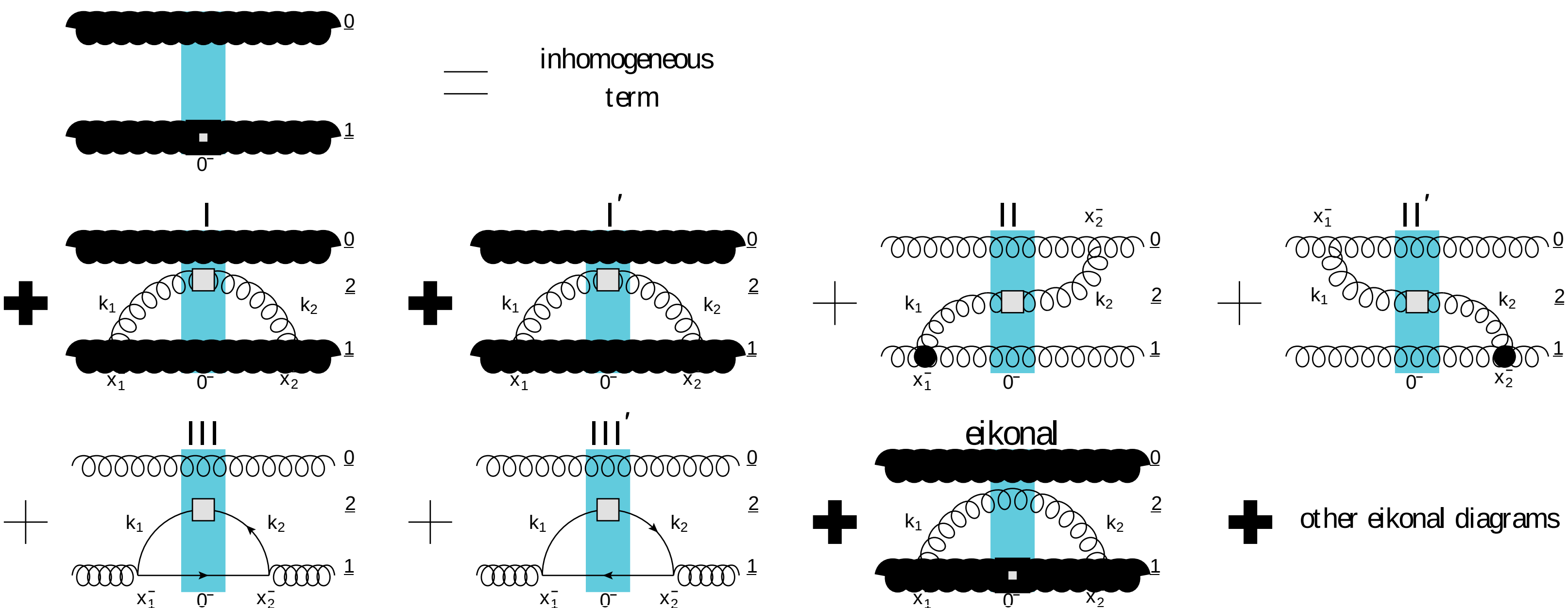
- The small-x asymptotics of quark helicity is (at large N_c)

$$\Delta q(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 2.31 \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

Small-x Evolution at large N_c&N_f

- At large N_c&N_f there is no simple relation between the adjoint and fundamental polarized dipole amplitudes. We need to construct coupled evolution equations mixing them with each other.

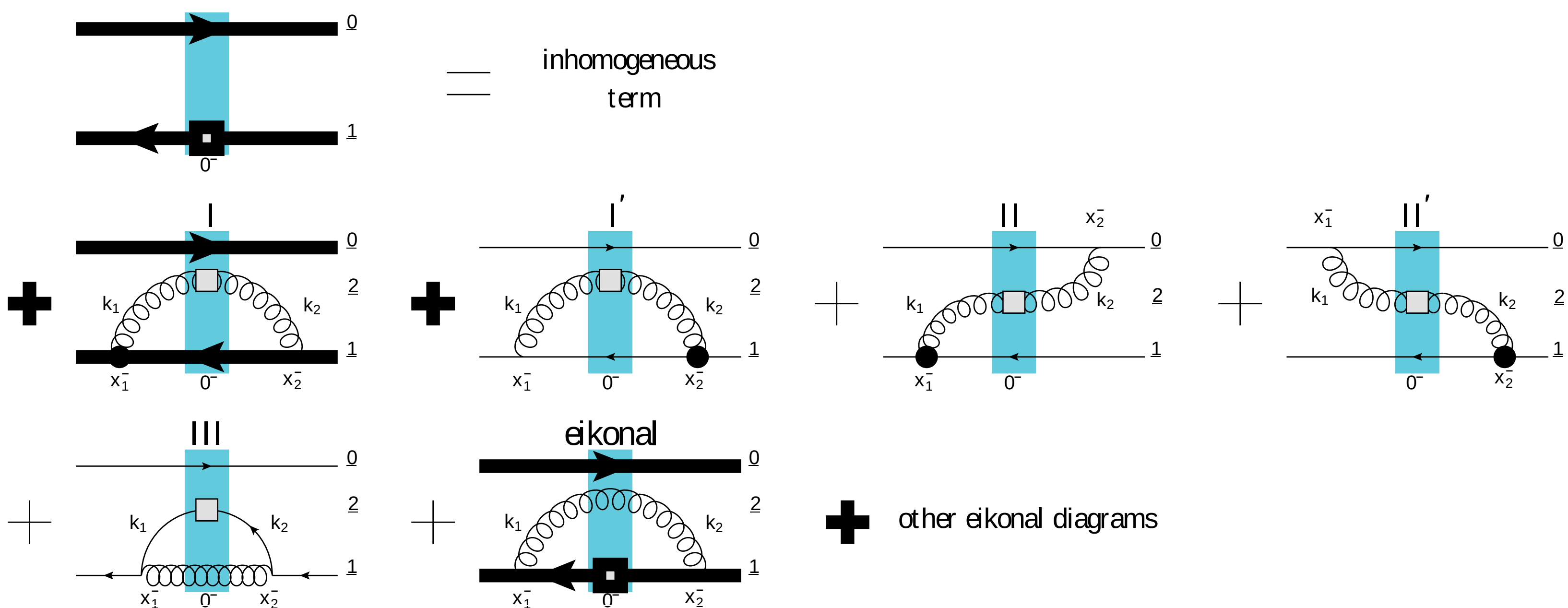
- Here's the adjoint dipole evolution:



Small-x Evolution at large N_c&N_f

- At large N_c&N_f there is no simple relation between the adjoint and fundamental polarized dipole amplitudes. We need to construct coupled evolution equations mixing them with each other.

- Here's the fundamental dipole evolution:



Small-x Evolution at large N_c&N_f

- The resulting equations are

$$Q_{10}(zs) = Q_{10}^{(0)}(zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{1/(z's)}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left\{ \frac{1}{2}\, \Gamma_{02,21}^{adj}(z') + \frac{1}{2}\, G_{21}^{adj}(z') + Q_{12}(z') - \bar{\Gamma}_{01,21}(z') \right\}$$

$$+ \frac{\alpha_s\, N_c}{4\,\pi} \int\limits_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int\limits_{1/(z's)}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2}\, Q_{21}(z'),$$

$$G_{10}^{adj}(z) = G_{10}^{adj\,(0)}(z) + \frac{\alpha_s\, N_c}{2\pi} \int\limits_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z} \frac{dz'}{z'} \int\limits_{1/(z'\,s)}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma_{10,21}^{adj}(z') + 3\, G_{21}^{adj}(z') \right]$$

$$- \frac{\alpha_s\, N_f}{2\,\pi} \int\limits_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int\limits_{1/(z'\,s)}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2}\, \bar{\Gamma}_{02;21}(z'),$$

These are yet to be solved.

$$\Gamma_{10,21}^{adj}(z') = \Gamma_{10,21}^{adj\,(0)}(z') + \frac{\alpha_s\, N_c}{2\pi} \int\limits_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z'} \frac{dz''}{z''} \int\limits_{1/(z''\,s)}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma_{10,32}^{adj}(z'') + 3\, G_{32}^{adj}(z'') \right]$$

$$- \frac{\alpha_s\, N_f}{2\,\pi} \int\limits_{\Lambda^2/s}^{z'} \frac{dz''}{z''} \int\limits_{1/(z''\,s)}^{x_{21}^2 z'/z''} \frac{dx_{32}^2}{x_{32}^2}\, \bar{\Gamma}_{03;32}(z''),$$

$$\bar{\Gamma}_{10,21}(z') = \bar{\Gamma}_{10,21}^{(0)}(z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z'} \frac{dz''}{z''} \int\limits_{1/(z''s)}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \left\{ \frac{1}{2}\, \Gamma_{03,32}^{adj}(z'') + \frac{1}{2}\, G_{32}^{adj}(z'') + Q_{32}(z'') - \bar{\Gamma}_{01,32}(z \right.$$

$$+ \frac{\alpha_s\, N_c}{4\,\pi} \int\limits_{\Lambda^2/s}^{z'} \frac{dz''}{z''} \int\limits_{1/(z''s)}^{x_{21}^2 z/z'} \frac{dx_{32}^2}{x_{32}^2}\, Q_{32}(z'').$$

Gluon Helicity at Small x

Yu.K., D. Pitonyak, M. Sievert, arXiv:1706.04236 [nucl-th]

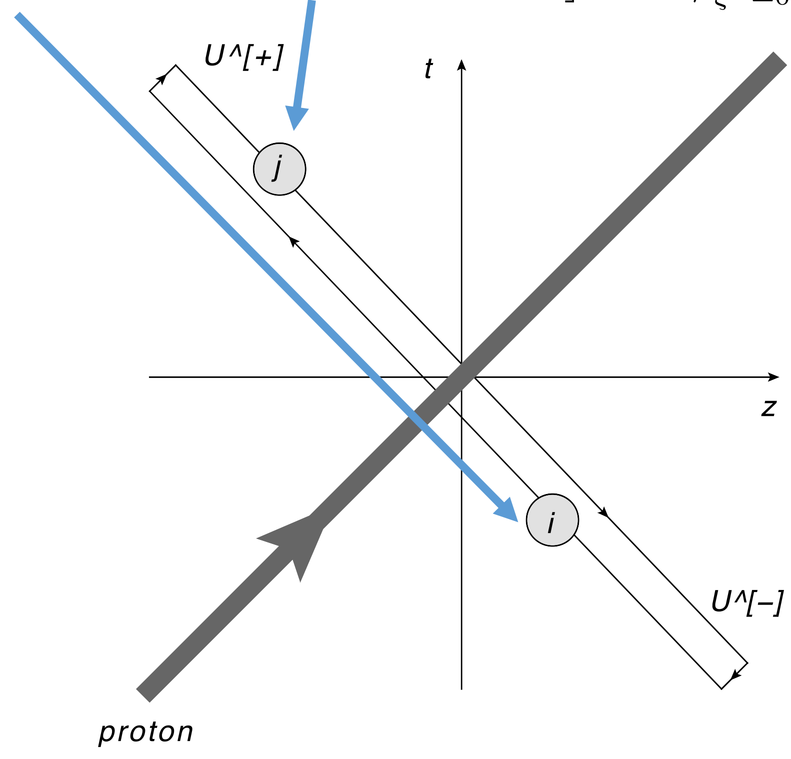
Dipole Gluon Helicity TMD

- Now let us repeat the calculation for gluon helicity TMDs.

- We start with the definition of the gluon dipole helicity TMD:

$$g_1^G(x, k_T^2) = \frac{-2i\, S_L}{x\, P^+} \int \frac{d\xi^- \, d^2\xi}{(2\pi)^3} \, e^{ixP^+\,\xi^- - i\underline{k}\cdot\underline{\xi}} \, \left\langle P, S_L | \epsilon_T^{ij} \, \mathrm{tr} \left[F^{+i}(0) \, \mathcal{U}^{[+]\dagger}[0, \xi] \, F^{+j}(\xi) \, \mathcal{U}^{[-]}[\xi, 0] \right] |P, S_L \right\rangle_{\xi^+=0}$$

- Here $U^{[+]}$ and $U^{[-]}$ are future and past Wilson line staples (hence the name `dipole' TMD, F. Dominguez et al '11 – looks like a dipole scattering on a proton):



Dipole Gluon Helicity TMD

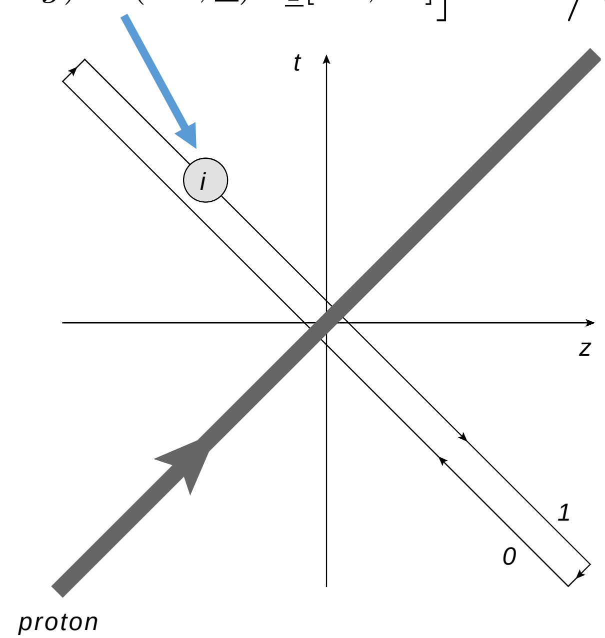
- At small x, the definition of dipole gluon helicity TMD can be massaged into

$$g_1^{G\,dip}(x, k_T^2) = \frac{8i\,N_c\,S_L}{g^2(2\pi)^3} \int d^2x_{10}\, e^{i\underline{k}\cdot\underline{x}_{10}}\; k_\perp^i \epsilon_T^{ij} \left[\int d^2b_{10}\, G_{10}^j(zs = \frac{Q^2}{x})\right]$$

- Here we obtain a new operator, which is a transverse vector (written here in A$^-$=0 gauge):

$$G_{10}^i(z) \equiv \frac{1}{4N_c} \int\limits_{-\infty}^{\infty} dx^- \left\langle \mathrm{tr}\left[V_{\underline{0}}[\infty, -\infty] V_{\underline{1}}[-\infty, x^-]\,(-ig)\,\tilde{A}^i(x^-, \underline{x})\,V_{\underline{1}}[x^-, \infty]\right] + \mathrm{c.c.}\right\rangle(z)$$

- Note that $k_\perp^i\, \epsilon_T^{ij}$ can be thought of as a transverse curl acting on $G_{10}^i(z)$ and not just on $\tilde{A}^i(x^-, \underline{x})$ -- different from the polarized dipole amplitude!



Dipole TMD vs dipole amplitude

- Note that the operator for the dipole gluon helicity TMD

$$G^i_{10}(z) \equiv \frac{1}{4N_c} \int\limits_{-\infty}^{\infty} dx^- \ \left\langle \mathrm{tr} \left[V_{\underline{0}}[\infty, -\infty] V_{\underline{1}}[-\infty, x^-] \, (-ig) \, \tilde{A}^i(x^-, \underline{x}) \, V_{\underline{1}}[x^-, \infty] \right] + \mathrm{c.c.} \right\rangle (z)$$

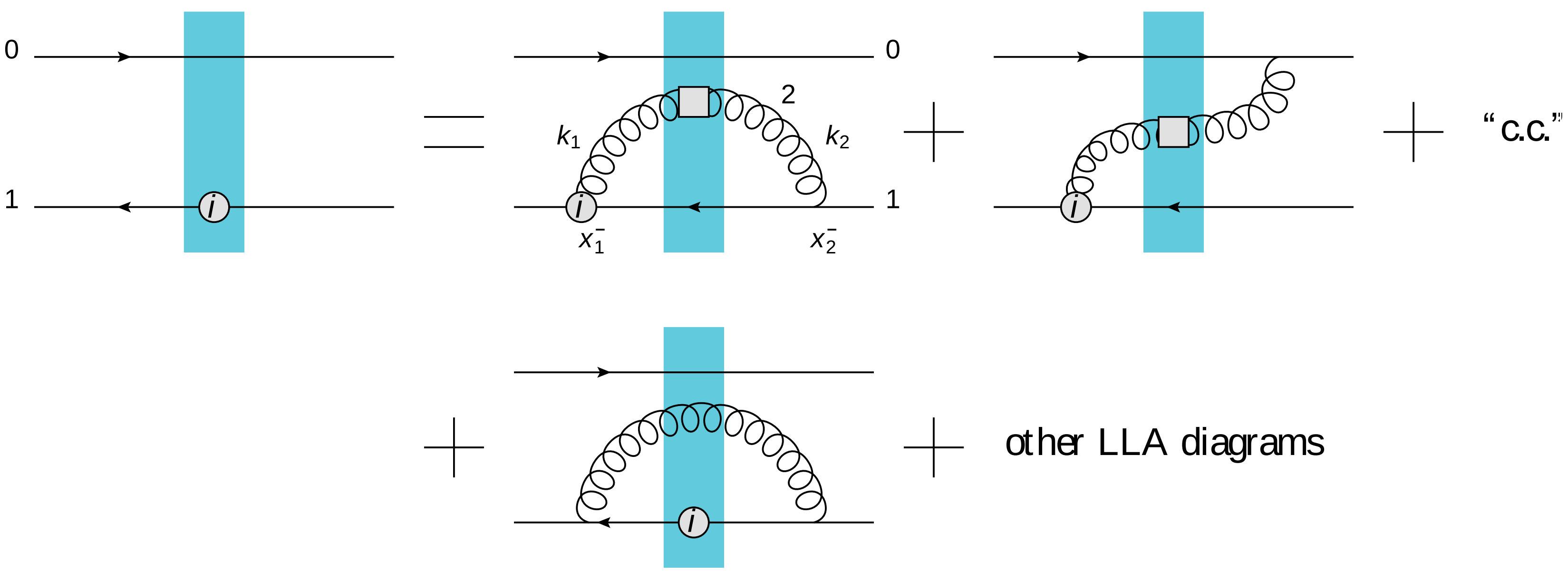
is different from the polarized dipole amplitude

$$G_{10}(z) \equiv \frac{1}{4N_c} \int\limits_{-\infty}^{\infty} dx^- \ \left\langle \mathrm{tr} \left[V_{\underline{0}}[\infty, -\infty] V_{\underline{1}}[-\infty, x^-] \, (-ig) \, \underline{\nabla} \times \underline{\tilde{A}}(x^-, \underline{x}) \, V_{\underline{1}}[x^-, \infty] \right] + \mathrm{c.c.} \right\rangle (z)$$

- We conclude that the dipole gluon helicity TMD does not depend on the polarized dipole amplitude! (Hence the ‘dipole’ name may not even be valid for such TMDs.)
- This is different from the unpolarized gluon TMD case.

Evolution Equation

- To construct evolution equation for the operator G^i governing the gluon helicity TMD we resum similar (to the quark case) diagrams:



Large-N_c Evolution: Equations

- This results in the following evolution equations:

$$G^i_{10}(zs) = G^{i\,(0)}_{10}(zs) + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \ln \frac{1}{x_{21}\Lambda} \, \frac{\epsilon_T^{ij} \, (x_{21})^j_\perp}{x_{21}^2} \, \Big[\Gamma^{gen}_{20\,,\,21}(z's) + G_{21}(z's) \Big]$$
$$- \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \ln \frac{1}{x_{21}\Lambda} \, \frac{\epsilon_T^{ij} \, (x_{20})^j_\perp}{x_{20}^2} \, \Big[\Gamma^{gen}_{20\,,\,21}(z's) + \Gamma^{gen}_{21\,,\,20}(z's) \Big]$$
$$+ \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Big[G^i_{12}(z's) - \Gamma^i_{10\,,\,21}(z's) \Big]$$

$$\Gamma^i_{10\,21}(z's) = G^{i\,(0)}_{10}(z's) + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z'} \frac{dz''}{z''} \int d^2 x_3 \, \ln \frac{1}{x_{31}\Lambda} \, \frac{\epsilon_T^{ij} \, (x_{31})^j_\perp}{x_{31}^2} \, \Big[\Gamma^{gen}_{30\,,\,31}(z''s) + G_{31}(z''s) \Big]$$
$$- \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z'} \frac{dz''}{z''} \int d^2 x_3 \, \ln \frac{1}{x_{31}\Lambda} \, \frac{\epsilon_T^{ij} \, (x_{30})^j_\perp}{x_{30}^2} \, \Big[\Gamma^{gen}_{30\,,\,31}(z''s) + \Gamma^{gen}_{31\,,\,30}(z''s) \Big]$$
$$+ \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z''s}}^{\min\left[x_{10}^2\,,\,x_{21}^2\frac{z'}{z''}\right]} \frac{dx_{31}^2}{x_{31}^2} \Big[G^i_{13}(z''s) - \Gamma^i_{10\,,\,31}(z''s) \Big].$$

Large-N_c Evolution: Equations

- Here

$$\Gamma^{gen}_{20,21}(z's) = \theta(x_{20} - x_{21})\,\Gamma_{20,21}(z's) + \theta(x_{21} - x_{20})\,G_{20}(z's)$$

is an object which we know from the quark helicity evolution, as the latter gives us G and Γ.

- Note that our evolution equations mix the gluon (G^i) and quark (G) small-x helicity evolution operators:

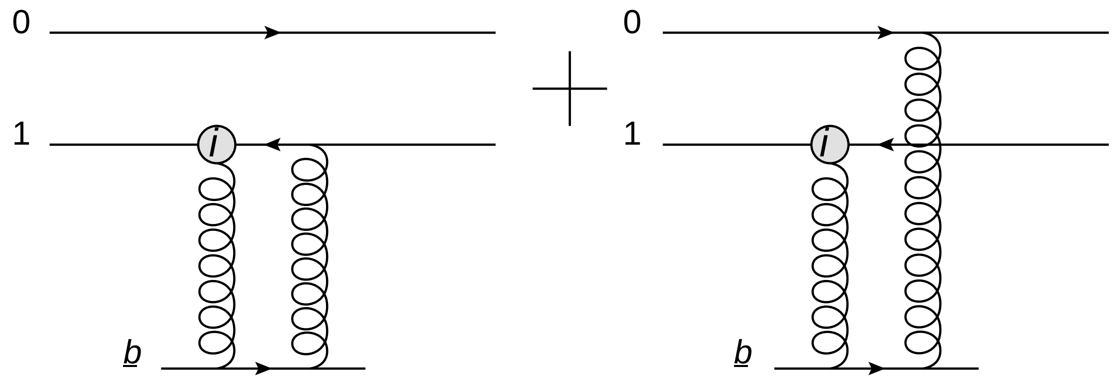
$$G^i_{10}(zs) = G^{i\,(0)}_{10}(zs) + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \ln \frac{1}{x_{21}\Lambda} \, \frac{\epsilon_T^{ij}\,(x_{21})^j_\perp}{x_{21}^2} \left[\Gamma^{gen}_{20\,,\,21}(z's) + G_{21}(z's)\right]$$

$$- \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \ln \frac{1}{x_{21}\Lambda} \, \frac{\epsilon_T^{ij}\,(x_{20})^j_\perp}{x_{20}^2} \left[\Gamma^{gen}_{20\,,\,21}(z's) + \Gamma^{gen}_{21\,,\,20}(z's)\right]$$

$$+ \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[G^i_{12}(z's) - \Gamma^i_{10\,,\,21}(z's)\right]$$

Initial Conditions

- Initial conditions for this evolution are given by the lowest order t-channel gluon exchanges:



$$\int d^2 b_{10}\, G^{i\,(0)}_{10}(zs) = \int d^2 b_{10}\, \Gamma^{i\,(0)}_{10,21}(zs) = -\frac{\alpha_s^2 C_F}{N_c}\,\pi\,\epsilon^{ij}\,x^j_{10}\,\ln\frac{1}{x_{10}\,\Lambda}$$

- Note that these initial conditions have no ln s, unlike the initial conditions for the quark evolution:

$$\int d^2 b_{10}\, G^{(0)}_{10}(zs) = \int d^2 b_{10}\, \Gamma^{(0)}_{10,21}(zs) = -\frac{\alpha_s^2 C_F}{N_c}\pi\,\ln(zs\,x^2_{10})$$

Large-N_c Evolution Equations: Solution

- These equations can be solved in the asymptotic high-energy region yielding the small-x gluon helicity intercept

$$\alpha_h^G = \frac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s\, N_c}{2\pi}} \approx 1.88 \sqrt{\frac{\alpha_s\, N_c}{2\pi}}$$

- We obtain the small-x asymptotics of the gluon helicity distributions:

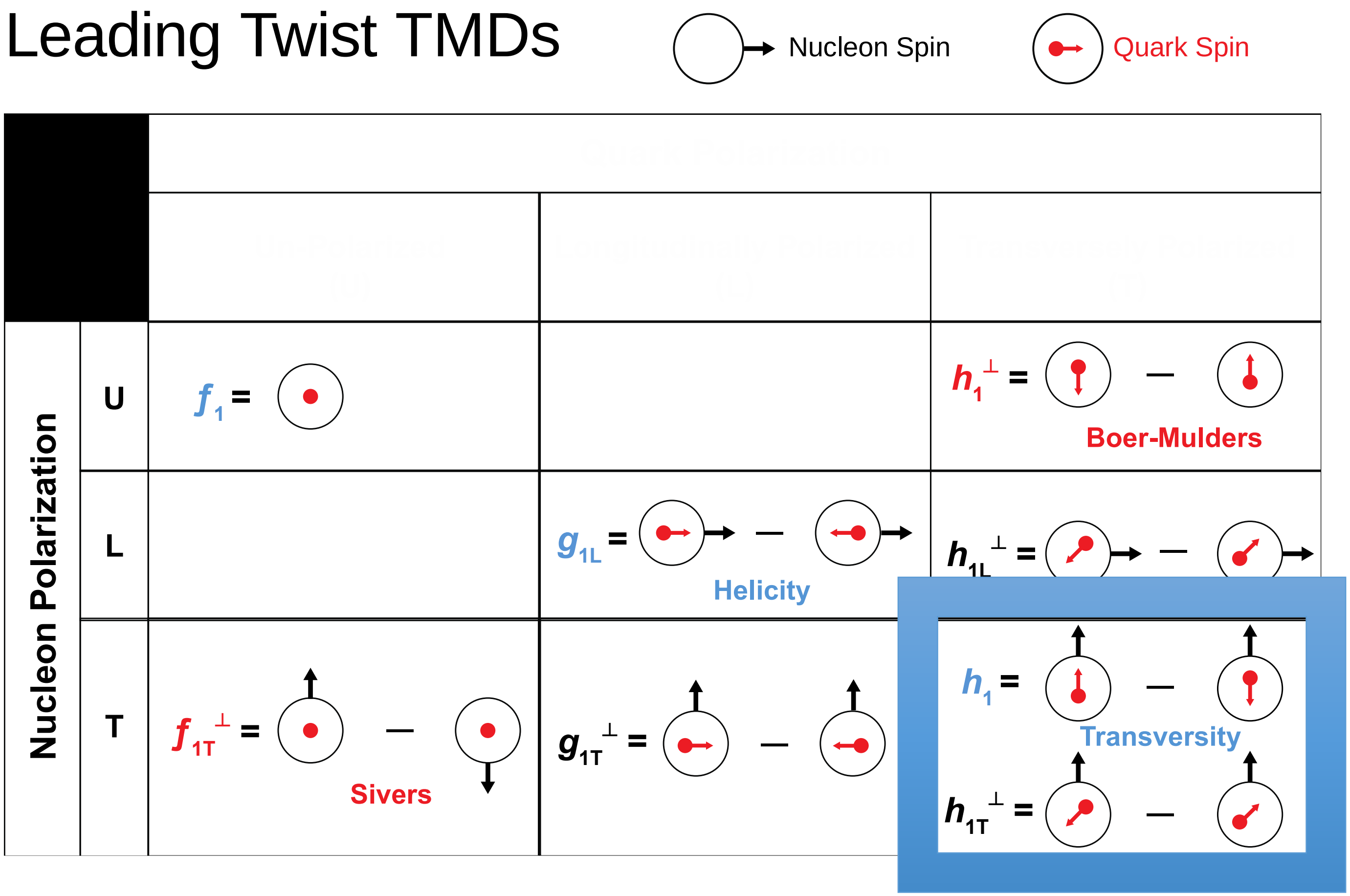
$$\Delta G(x, Q^2) \sim g_{1L}^{G\,dip}(x, k_T^2) \sim \left(\frac{1}{x}\right)^{\frac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s\, N_c}{2\pi}}}$$

Valence Quark Transversity at Small x

Yu.K., M. Sievert, arXiv:1808.10354 [hep-ph]

Observables

- We want to calculate quark transversity TMD at small x:

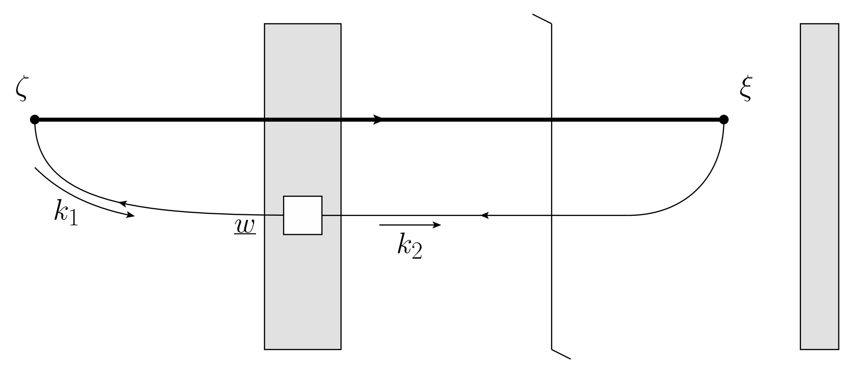


Quark Transversity Operator

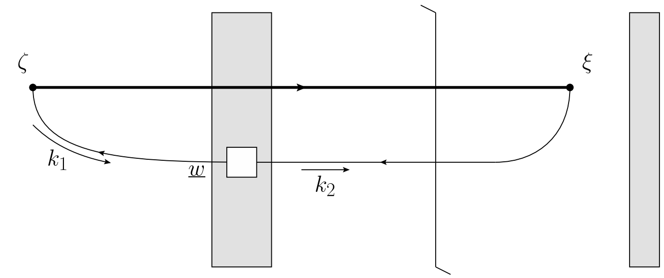
- Analysis of quark transversity is similar to quark helicity: we start with the operator definition (for proton spin in the x-direction)

$$h_{1T}^{q}(x,k_T^2) + \frac{k_x^2}{M^2}\, h_{1T}^{\perp\, q}(x,k_T^2) = \frac{1}{(2\pi)^3}\int d^2r\, dr^-\, e^{ik\cdot r}\langle p, S_x=+1|\bar{\psi}(0)\,\mathcal{U}[0,r]\,\frac{\gamma^5\,\gamma^+\,\gamma^1}{2}\,\psi(r)|p, S_x=+1\rangle_{r^+=0}$$

- Diagram analysis again shows that the B-type diagrams dominate:



Quark Transversity Operator



- Calculating the B-graph contribution we get (NS = flavor non-singlet)

$$h_{1T}^{NS}(x, k_T^2) + \frac{k_x^2}{M^2} h_{1T}^{\perp\, NS}(x, k_T^2) = - x \frac{8N_c}{(2\pi)^4} \int d^2 x_0 \, d^2 x_1 \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int \frac{d^2 k_1}{(2\pi)^2} e^{i(\underline{k}_1 + \underline{k})\cdot \underline{x}_{10}} \frac{1}{\underline{k}_1^2 \, \underline{k}^2} \left[\frac{1}{\underline{k}_1^2} + \frac{1}{\underline{k}^2} \right]$$
$$\times \left(-2 \, \underline{S} \cdot \underline{k}_1 \, \underline{S} \cdot \underline{k} + \underline{k}_1 \cdot \underline{k} - m^2 \right) T_{10}^{NS}(zs)$$

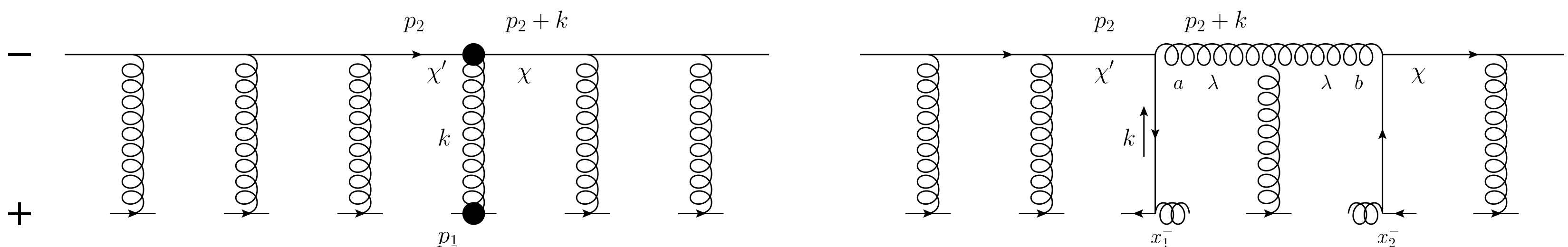
- We have defined a transversely polarized dipole operator

$$T_{10}^{NS}(zs) = \frac{(z\, s)^2}{2N_c} \, \mathrm{Re} \, \left\langle \mathrm{T\, tr} \left[V_{\underline{0}} \, V_{\underline{1}}^{pol,T\,\dagger} \right] - \mathrm{T\, tr} \left[V_{\underline{1}}^{pol,T} \, V_{\underline{0}}^{\dagger} \right] \right\rangle_{S_x = +1}$$

- Transverse-spin dependent interaction is suppressed by two (!) powers of energy (compared to unpolarized one), hence we rescaled T_{10} by s^2.

Transversely polarized "Wilson line"

- Next we need to calculate the fundamental transversely polarized "Wilson line" --- the leading transverse-spin dependent part of the quark scattering amplitude on a transversely polarized target.
- The diagrams are similar to the helicity case:



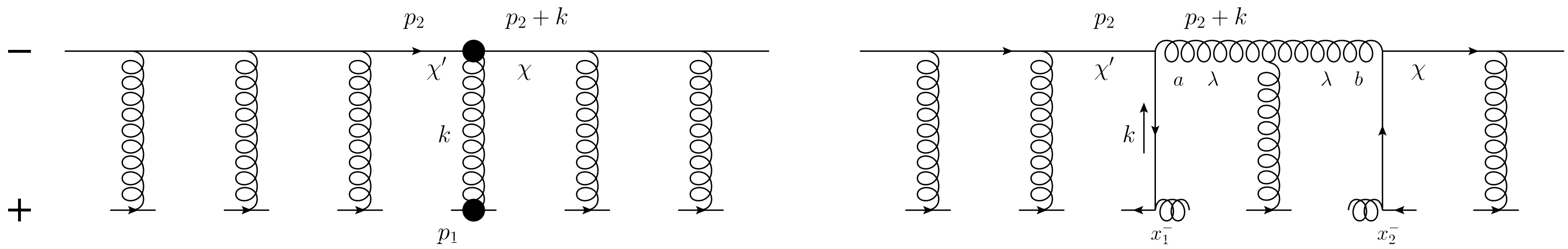
- The result is (S^i is a unit vector in the transverse spin direction)

$$V_{\underline{x}}^{pol,T} = \frac{2g\, m\, (p_1^+)^2}{s^2} \int\limits_{-\infty}^{+\infty} dx^- \, V_{\underline{x}}[+\infty, x^-] \, S^i \, \left[i\, \epsilon^{ij} \, F^{-j}(x^-, \underline{x}) \right] \, V_{\underline{x}}[x^-, -\infty]$$

$$- \frac{g^2\, (p_1^+)^2}{2\, s^2} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \, V_{\underline{x}}[+\infty, x_2^-] \, t^b \, \psi_\beta(x_2^-, \underline{x}) \, U_{\underline{x}}^{ba}[x_2^-, x_1^-] \Big[\left(i\, \gamma^5 \, \underline{S} \cdot \overleftarrow{\underline{D}} - \underline{S} \times \overleftarrow{\underline{D}} \right) \gamma^+ \gamma^-$$

$$+ \left(i\, \gamma^5 \, \underline{S} \cdot \underline{D} - \underline{S} \times \underline{D} \right) \gamma^- \gamma^+ \Big]_{\alpha\beta} \, \bar{\psi}_\alpha(x_1^-, \underline{x}) \, t^a \, V_{\underline{x}}[x_1^-, -\infty]$$

Transversely polarized “Wilson line”



- Note that the interaction with the transverse polarization carrying gluons enters with a factor of quark mass m, and hence does not give the double logarithmic contribution.

$$V_{\underline{x}}^{pol,T} = \frac{2g\, m\, (p_1^+)^2}{s^2} \int\limits_{-\infty}^{+\infty} dx^- \, V_{\underline{x}}[+\infty, x^-] \, S^i \, \left[i \, \epsilon^{ij} \, F^{-j}(x^-, \underline{x}) \right] \, V_{\underline{x}}[x^-, -\infty]$$

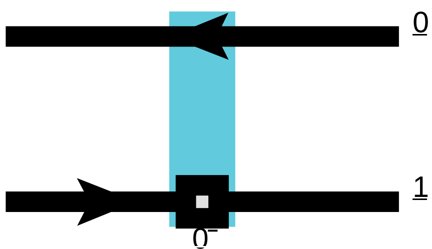
$$- \frac{g^2\,(p_1^+)^2}{2\,s^2} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \, V_{\underline{x}}[+\infty, x_2^-] \, t^b \, \psi_\beta(x_2^-, \underline{x}) \, U_{\underline{x}}^{ba}[x_2^-, x_1^-] \left[\left(i \, \gamma^5 \, \underline{S} \cdot \overleftarrow{\underline{D}} - \underline{S} \times \overleftarrow{\underline{D}} \right) \gamma^+ \, \gamma^- \right.$$

$$\left. + \left(i \, \gamma^5 \, \underline{S} \cdot \underline{D} - \underline{S} \times \underline{D} \right) \gamma^- \, \gamma^+ \right]_{\alpha\beta} \bar{\psi}_\alpha(x_1^-, \underline{x}) \, t^a \, V_{\underline{x}}[x_1^-, -\infty]$$

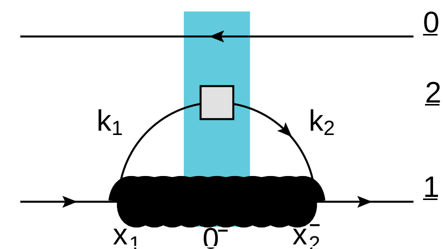
- Therefore, at DLA, quarks do not mix with gluons as they evolve to small x, and we do not need the gluon polarized “Wilson line”.

Evolution equation for quark transversity

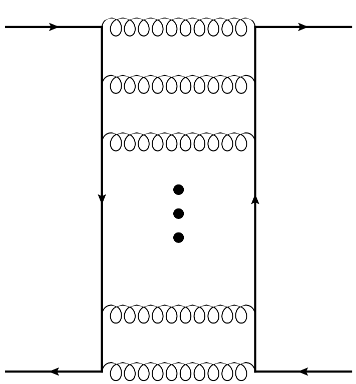
- Constructing the evolution equation for the transverse polarized fundamental dipole amplitude is straightforward (though a little cumbersome). Diagrammatically it is



= inhomogeneous term +



- We get



$$T_{10}^{NS}(zs) = T_{10}^{NS,\,(0)}(zs) + \frac{\alpha_s\, N_c}{2\,\pi} \int\limits_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int\limits_{1/z's}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2}\, T_{21}^{NS}(z's)$$

Small-x Asymptotics of Quark Transversity

- Solution of the transversity evolution equation is straightforward.
- The resulting small-x asymptotics is (cf. Kirschner et al, 1996)

$$h_{1T}^{NS}(x, k_T^2) \sim h_{1T}^{\perp\, NS}(x, k_T^2) \sim \left(\frac{1}{x}\right)^{\alpha_t^q} \quad \text{with} \quad \alpha_t^q = -1 + 2\sqrt{\frac{\alpha_s\, C_F}{\pi}}$$

- Note the suppression by x^2 compared to the unpolarized quark TMDs.
- For $\alpha_s = 0.3$ we get

$$h_{1T}^{NS}(x, k_T^2) \sim h_{1T}^{\perp\, NS}(x, k_T^2) \sim x^{0.243}$$

- This certainly satisfies the Soffer bound, but is not likely to produce much tensor charge from small x.

$$\delta q(Q^2) = \int\limits_0^1 dx\, h_1(x, Q^2)$$

Conclusions

- We now have a well-defined operator prescription for finding the small-x asymptotics of any TMD (either at large-N_c or at large N_c&N_f).
- We have

$$\Delta q(x,Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s\, N_c}{2\pi}} \approx 2.31 \sqrt{\frac{\alpha_s\, N_c}{2\pi}}$$

$$\Delta G(x,Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^G} \quad \text{with} \quad \alpha_h^G = \tfrac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s\, N_c}{2\pi}} \approx 1.88 \sqrt{\frac{\alpha_s\, N_c}{2\pi}}$$

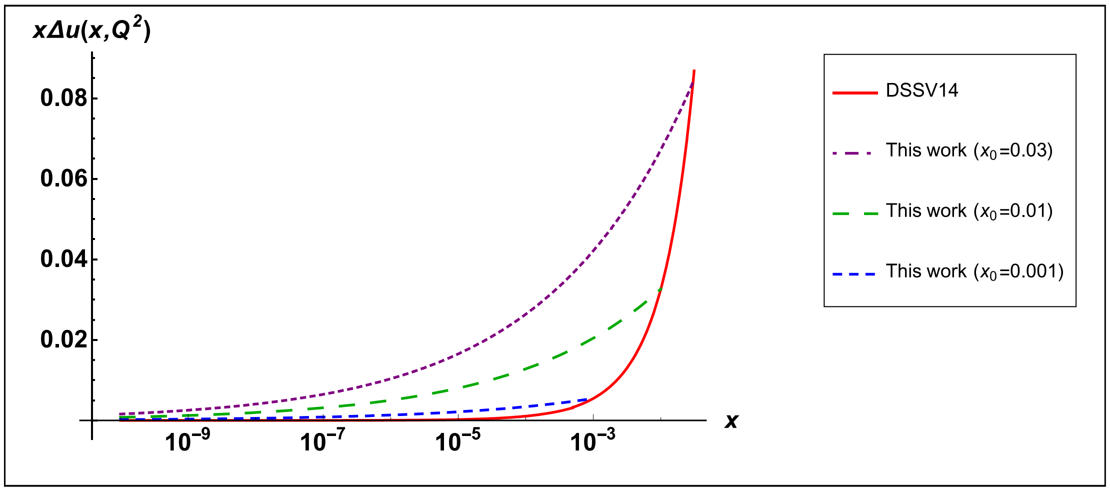
$$h_{1T}^{NS}(x,k_T^2) \sim h_{1T}^{\perp\, NS}(x,k_T^2) \sim \left(\frac{1}{x}\right)^{\alpha_t^q} \quad \text{with} \quad \alpha_t^q = -1 + 2\sqrt{\frac{\alpha_s\, C_F}{\pi}}$$

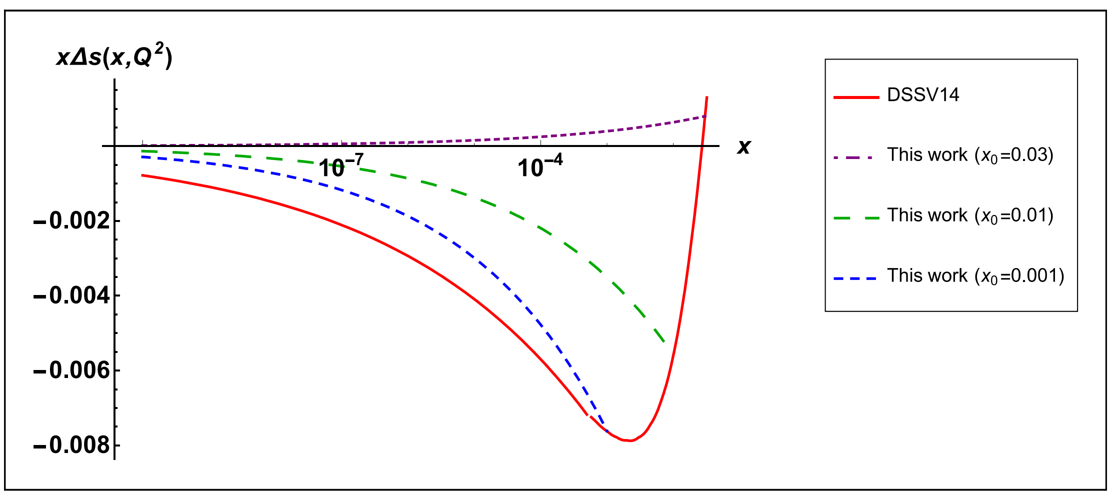
- Future helicity work will involve including running coupling corrections + solving the large-N_C&N_f equations + OAM at small x to constrain the spin+OAM coming from small-x quarks and gluons.
- EIC should be able to measure the above TMDs with high precision and down to fairly small x.

Backup Slides

Impact of our $\Delta\Sigma$ on the proton spin

- We have attached a $\Delta\tilde{\Sigma}(x,Q^2) = N\,x^{-\alpha_h}$ curve to the existing hPDF's fits at some ad hoc small value of x labeled x_0 :

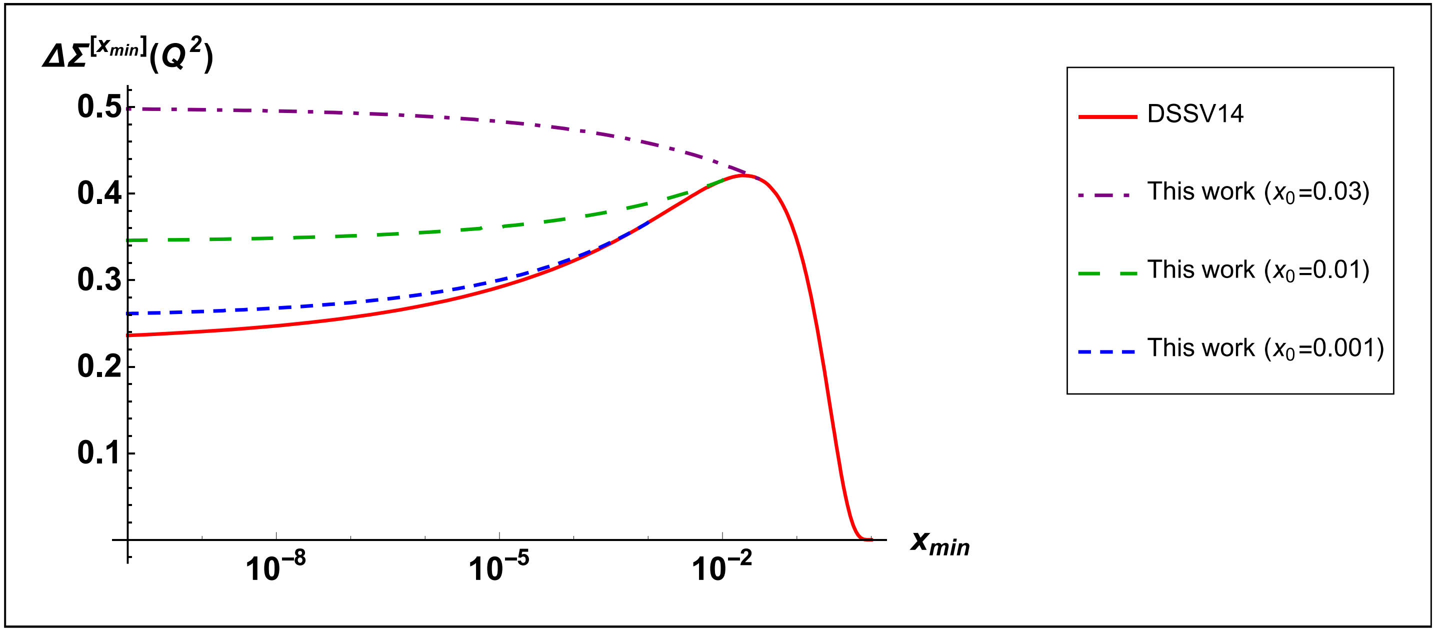




"ballpark" phenomenology

Impact of our ΔΣ on the proton spin

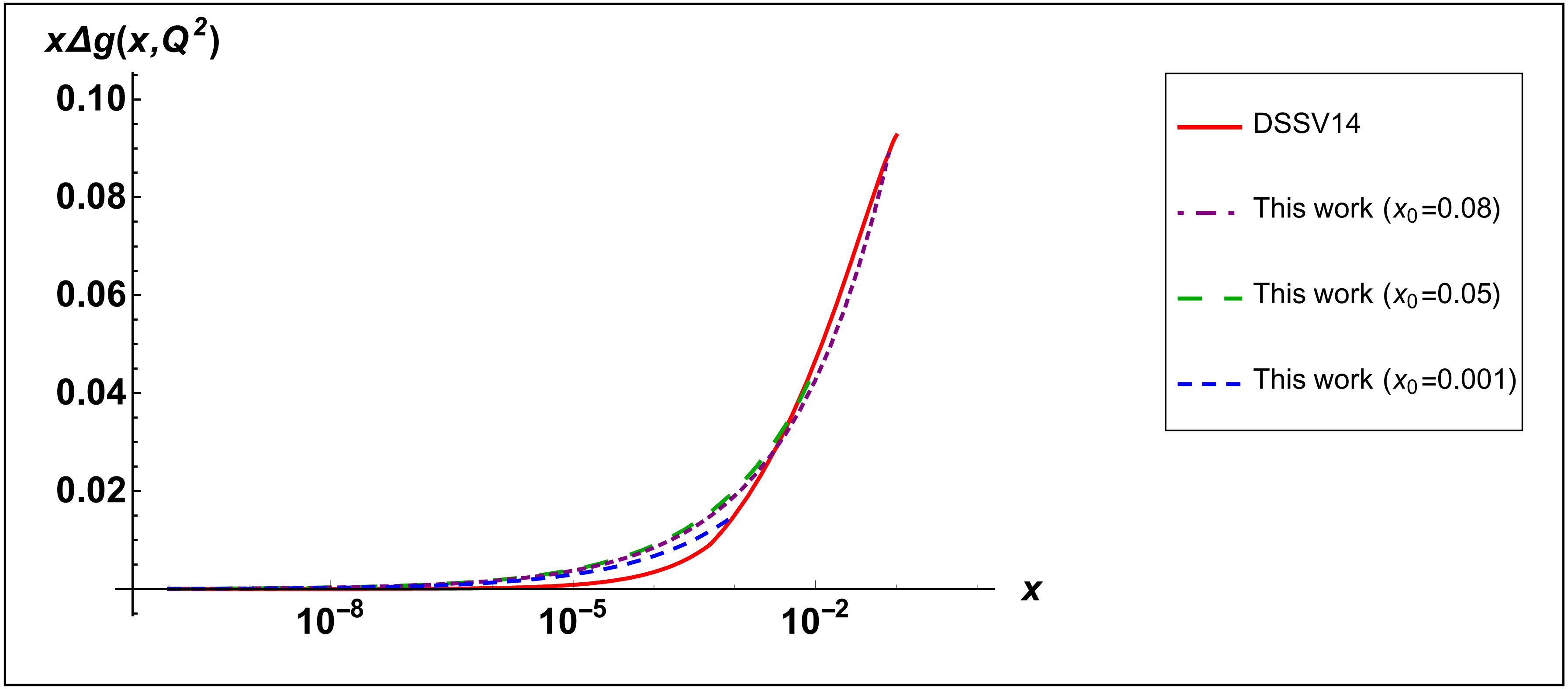
- Defining $$\Delta\Sigma^{[x_{min}]}(Q^2) \equiv \int_{x_{min}}^{1} dx\, \Delta\Sigma(x, Q^2)$$ we plot it for x_0=0.03, 0.01, 0.001:



- We observe a moderate to significant enhancement of quark spin.
- More detailed phenomenology is needed in the future.

Impact of our ΔG on the proton spin

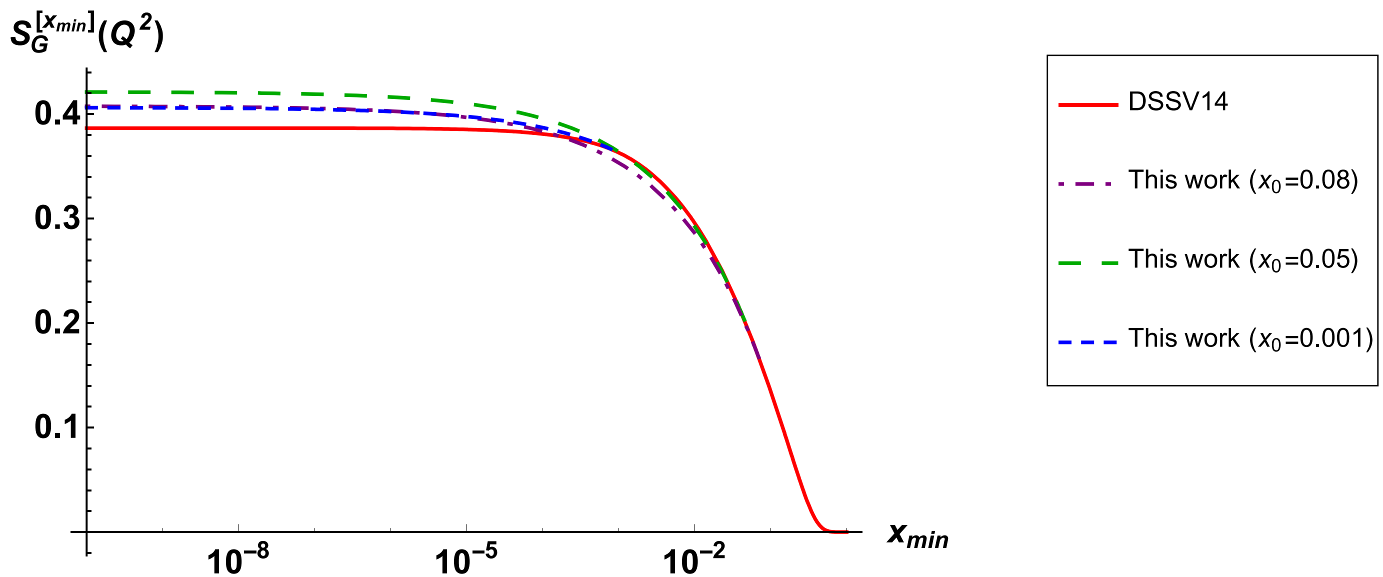
- We have attached a $\Delta\tilde{G}(x,Q^2) = N\,x^{-\alpha_h^G}$ curve to the existing hPDF's fits at some ad hoc small value of x labeled x_0 :



"ballpark" phenomenology

Impact of our ΔG on the proton spin

- Defining $S_G^{[x_{min}]}(Q^2) \equiv \int_{x_{min}}^{1} dx\, \Delta G(x, Q^2)$ we plot it for x_0=0.08, 0.05, 0.001:



- We observe a moderate enhancement of gluon spin.
- More detailed phenomenology is needed in the future.