



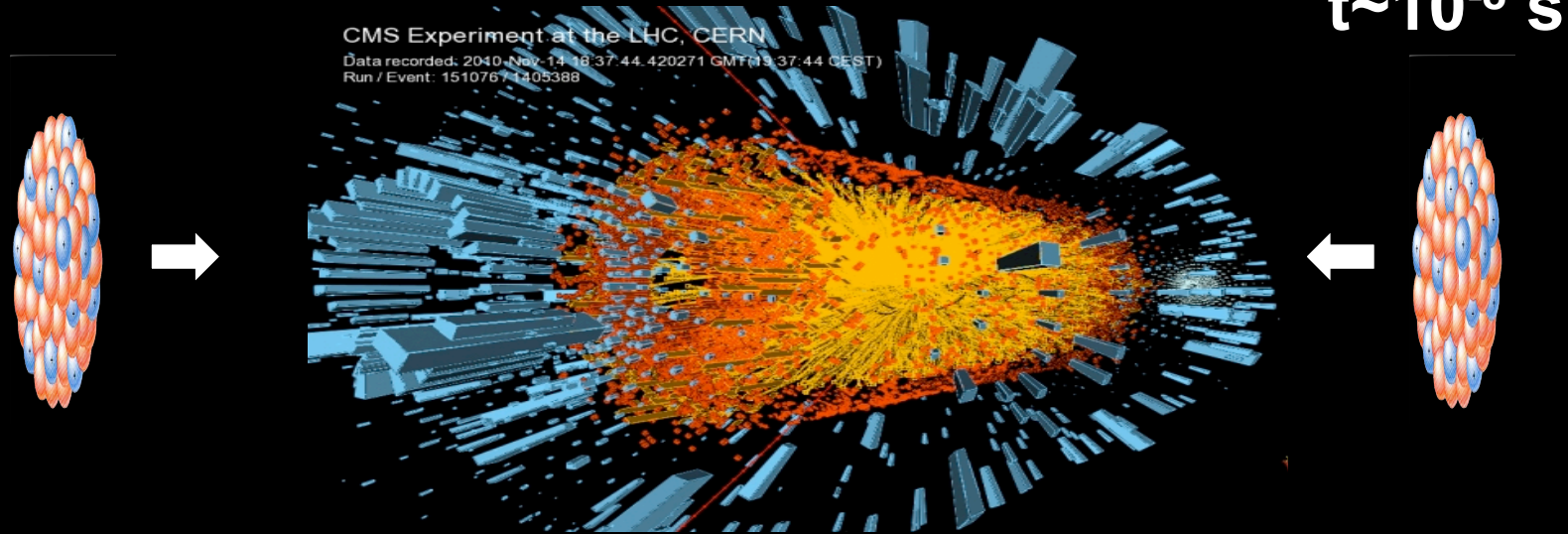
# Correlations and collectivity from large to small systems

Jiangyong Jia

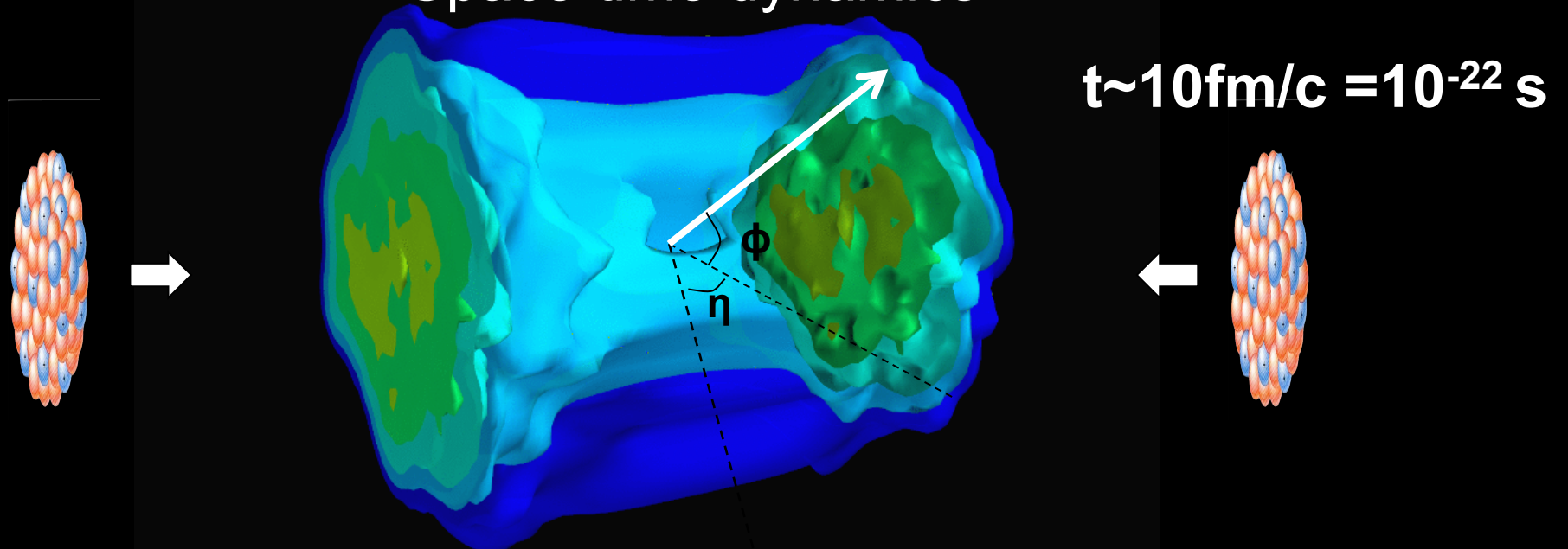
Stony Brook University and BNL

- Introduce collectivity with  $A+A$
- Collectivity in small systems
- Some future opportunities

# Seen by detector



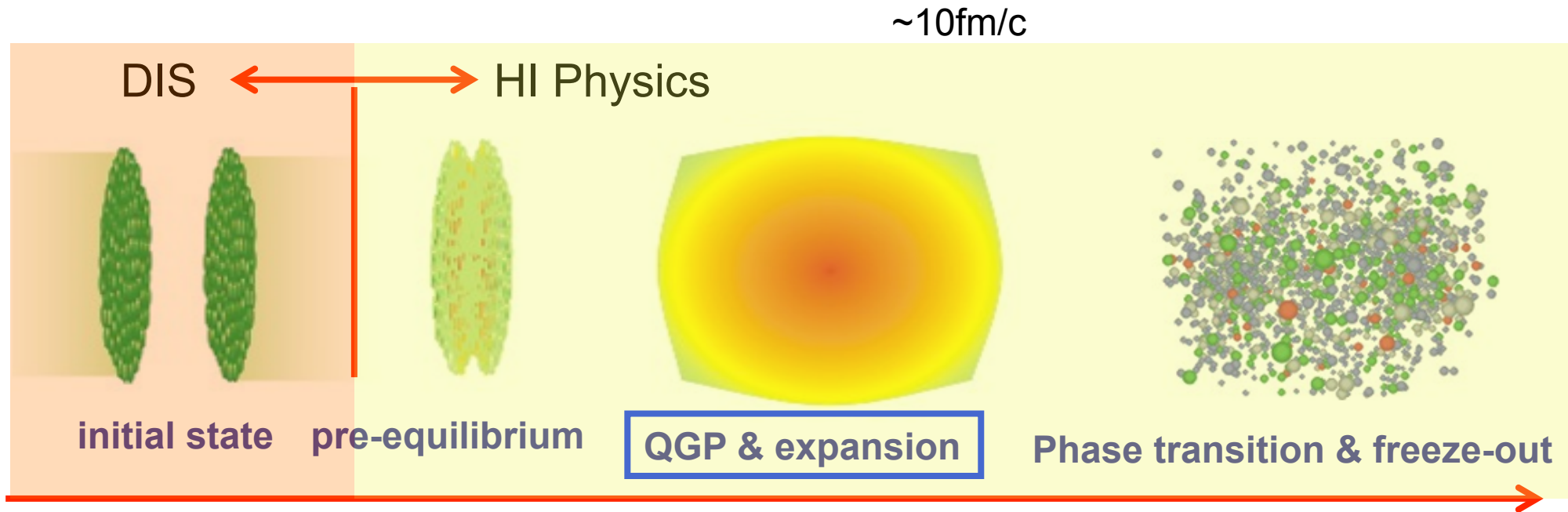
# Space-time dynamics



Model by 3D relativistic viscous hydrodynamics

Credit: Bjoern Schenke

# Space time history of heavy ion



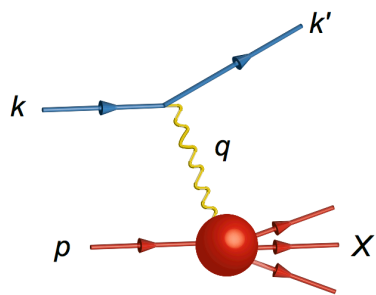
Large uncertainty in the **initial state**, DIS can help

**Space-time dynamics** successfully described by hydrodynamics

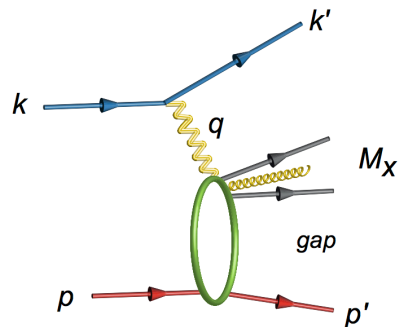
# DIS and Heavy Ion

- DIS: initial-state constituent distribution (one-body Wigner func.)
  - Precise control on kinematics

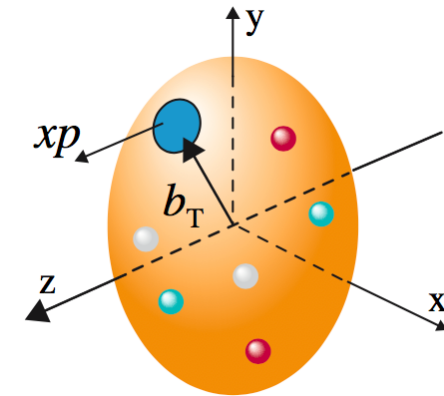
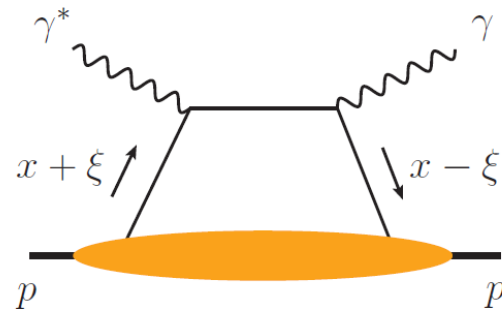
Inclusive



Semi-Inclusive

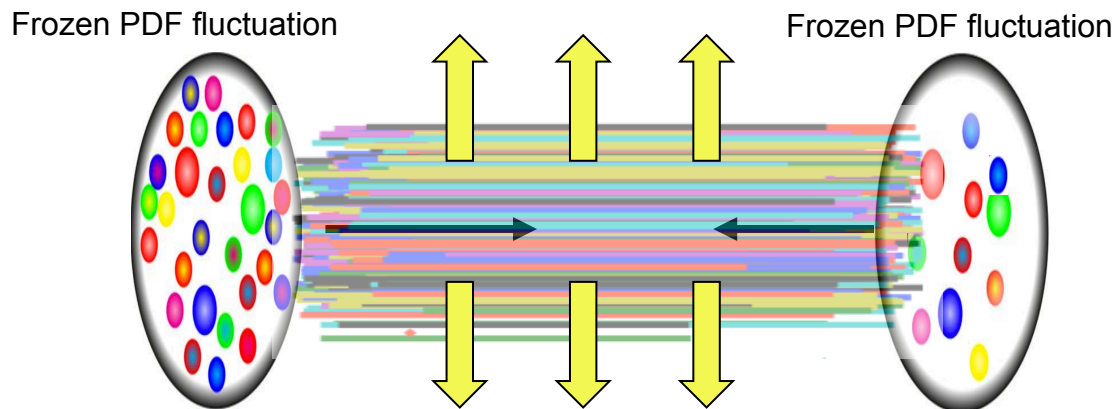


Exclusive

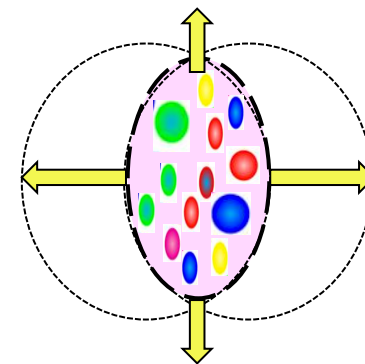


- Heavy-ion: Multi-Parton interactions (many-body Wigner function)

Longitudinal view



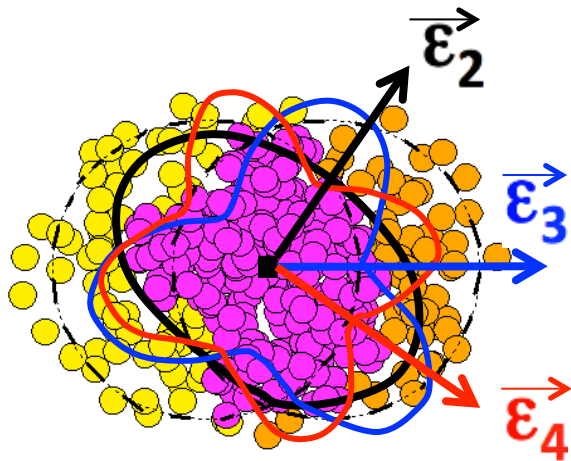
Transverse view



Require: 3D space-momentum distri. of initial-state partonic structure

# Collectivity in A+A collisions

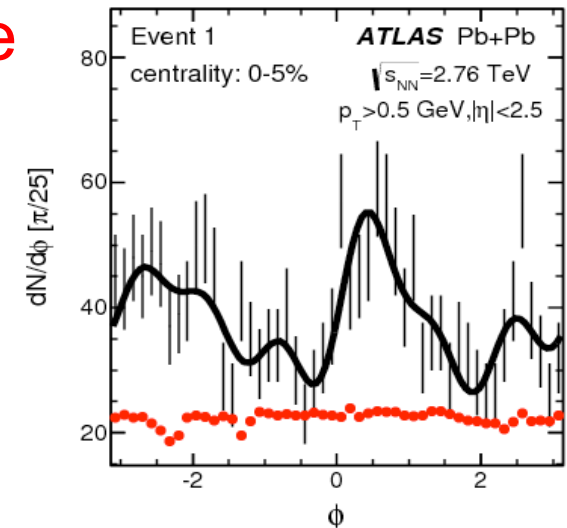
Initial state



Hydrodynamic response

Space-time dynamics

Final particle flow



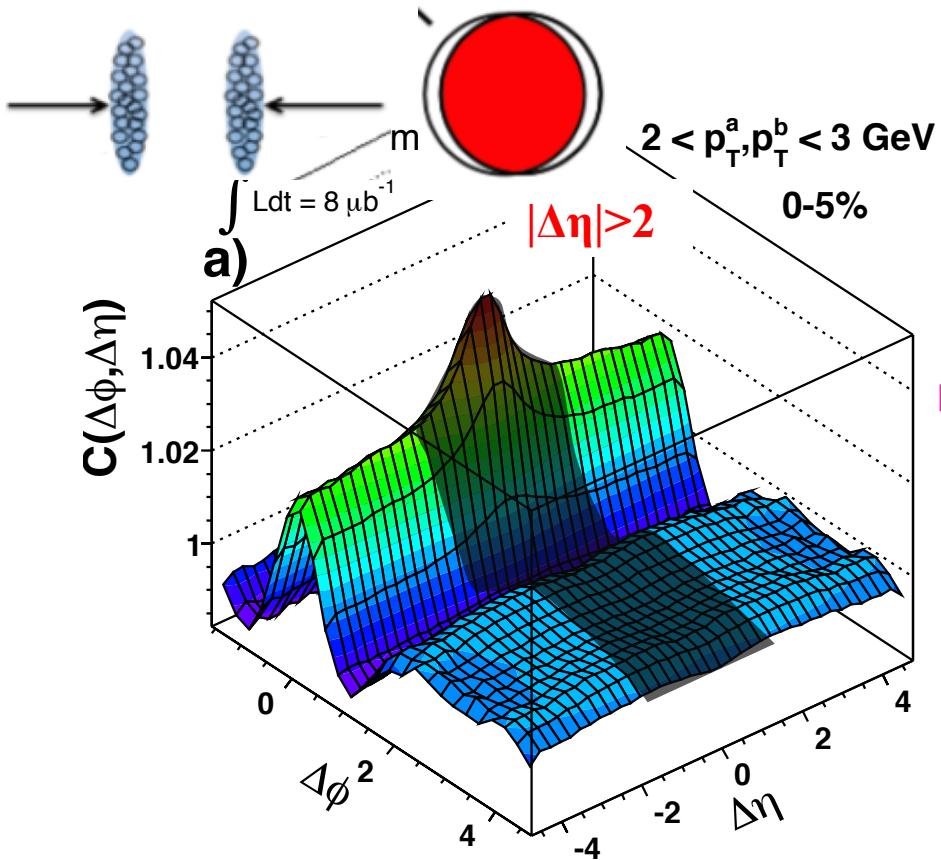
$$\vec{\epsilon}_n \equiv \epsilon_n e^{in\Phi_n^*} \equiv -\frac{\langle r^n e^{in\phi} \rangle}{\langle r^n \rangle}$$

$$\frac{dN}{d\phi} \propto 1 + 2 \sum_n v_n \cos n(\phi - \Phi_n)$$

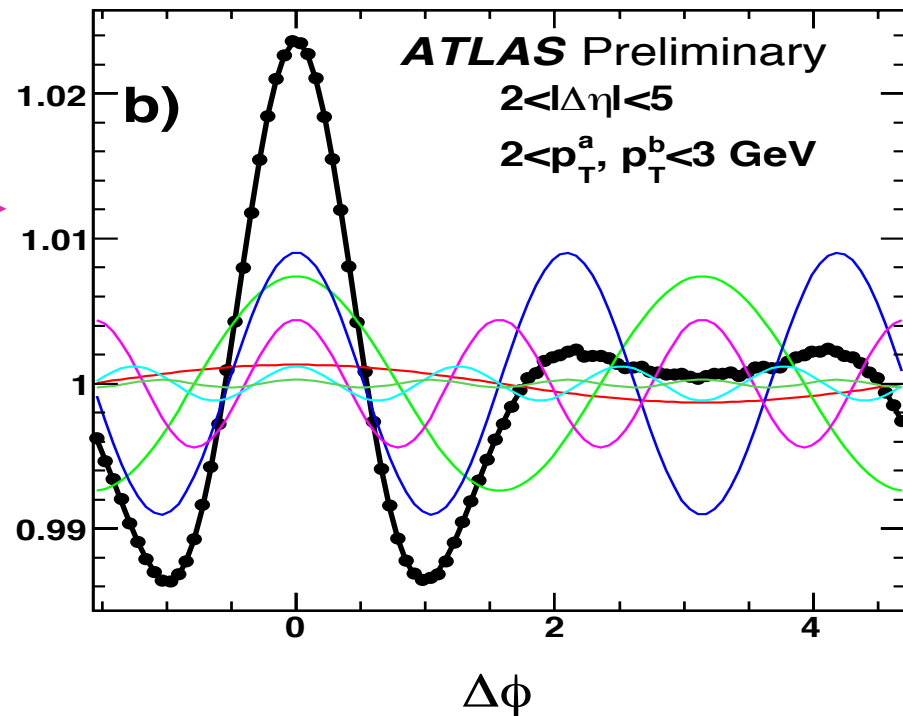
## ■ What we know:

- Large event-by-event **initial state fluctuation**
- Each event follows its own **hydrodynamic space-time evolution**
- **Small viscosity** ensure efficient transfer of  $(\epsilon_n, \Phi_n^*)$  to  $(v_n, \Phi_n)$

# Quantify long-range two-particle correlation

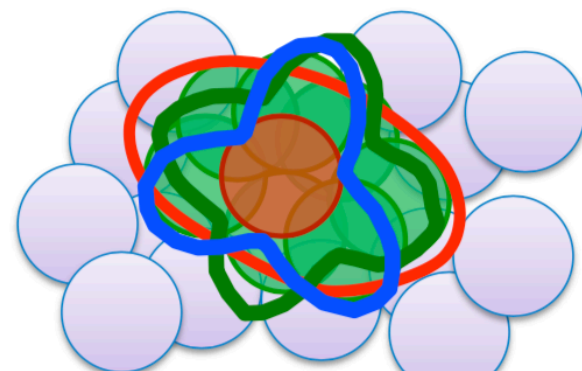


$$\frac{dN_{pairs}}{d\Delta\phi} \propto 1 + 2 \sum_n v_n^2 \cos(n\Delta\phi)$$



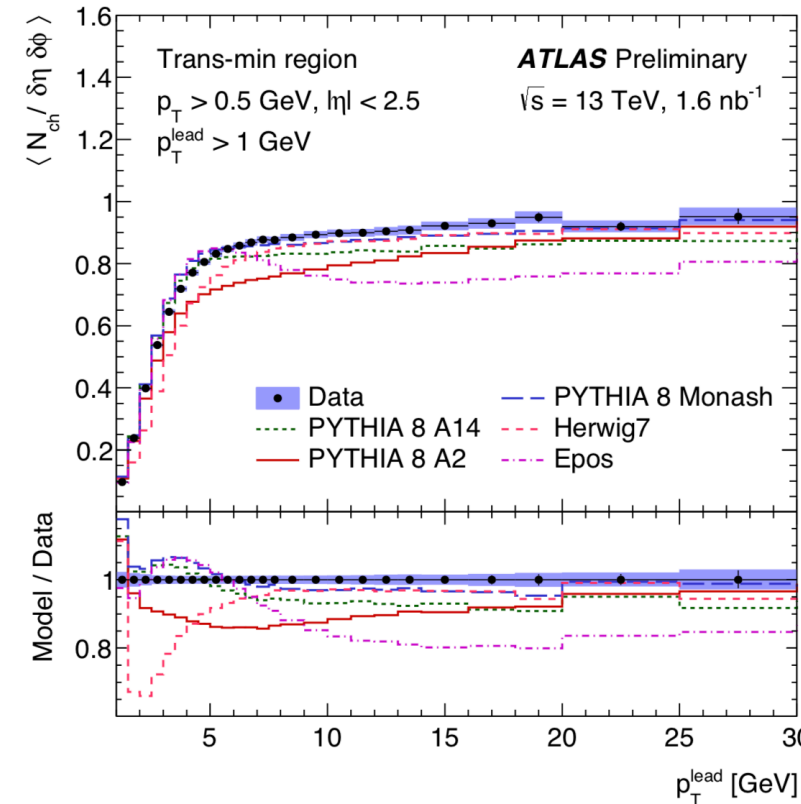
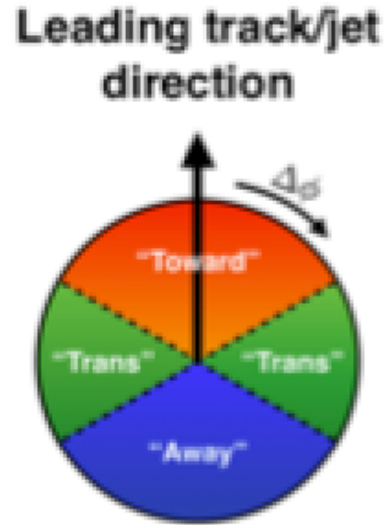
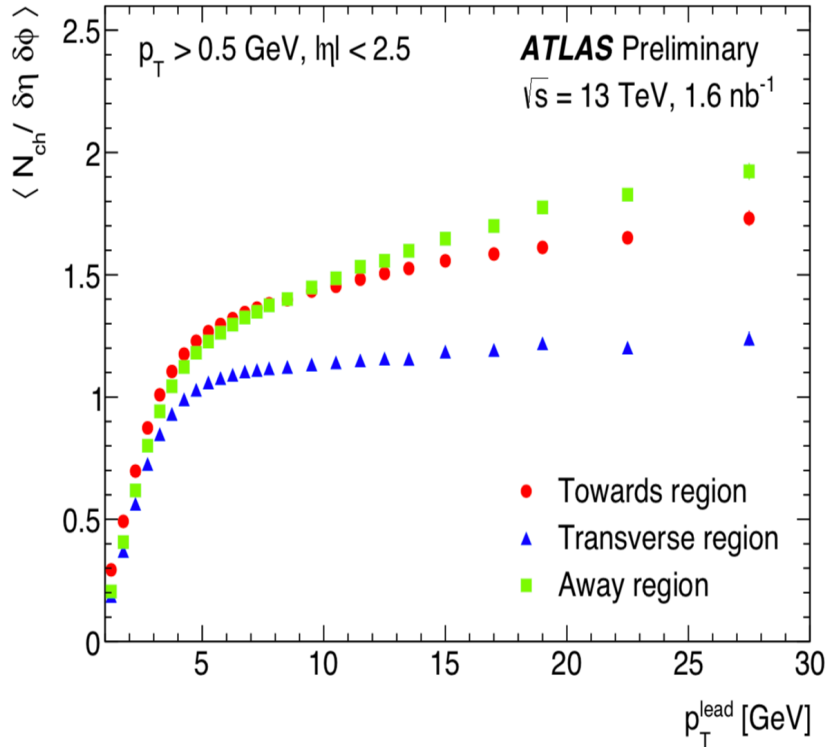
$$\varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \dots =$$

$\cos 2\Delta\phi$      $\cos 3\Delta\phi$      $\cos 4\Delta\phi$



# Connection to underlying event analysis

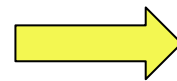
## Traditional UE studies



## Fourier decompose the “long-range” UE (ridge)

$$\frac{dN}{d\phi} \propto 1 + 2 \sum_n v_n \cos n(\phi - \Phi_n)$$

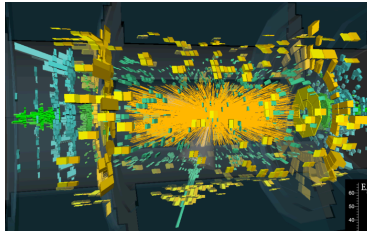
Single-particle distribution



$$\frac{dN_{\text{pairs}}}{d\Delta\phi} \propto 1 + 2 \sum_n v_n^2 \cos(n\Delta\phi)$$

Two-particle correlation with  $\Delta\eta$  gap

# Event-by-event fluctuations

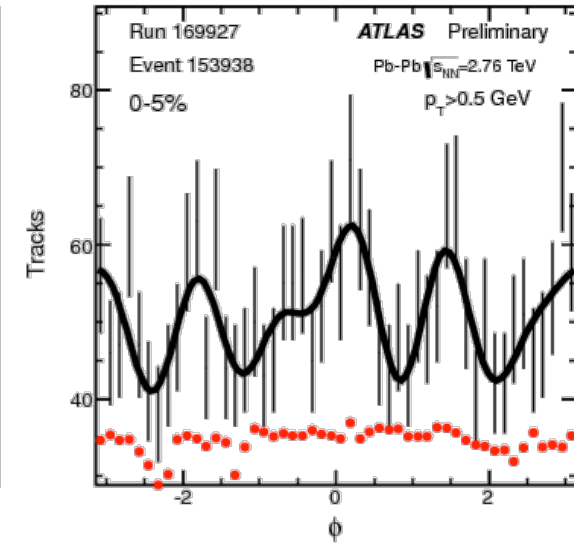
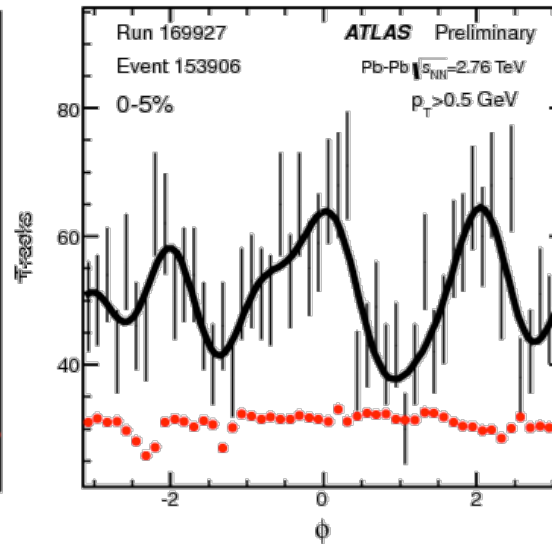
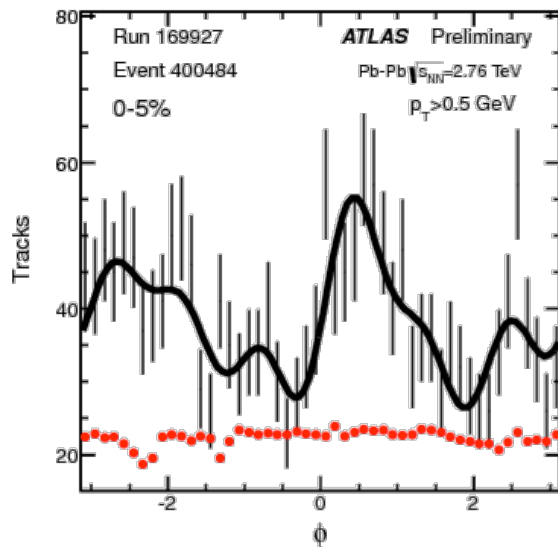


$$\frac{dN}{d\phi} \propto 1 + 2 \sum_n v_n \cos n(\phi - \Phi_n)$$

Event 1

Event 2

Event 3



Object of interest:

$$p(v_n, v_m, \dots, \Phi_n, \Phi_m, \dots) = \frac{1}{N_{\text{evts}}} \frac{dN_{\text{evts}}}{dv_n dv_m \dots d\Phi_n d\Phi_m \dots}$$



# Fluctuation observables

## Single particle distribution

$$\frac{dN}{d\phi} = N \left[ 1 + 2 \sum_n \mathbf{v}_n \cos n(\phi - \Phi_n) \right] = N \left[ \sum_{n=-\infty}^{\infty} \mathbf{V}_n e^{in\phi} \right]$$

Flow vector:  
 $\mathbf{V}_n = v_n e^{in\Phi_n}$

## Tools: Multi-particle correlations

$$\left\langle \frac{dN_1}{d\phi} \frac{dN_2}{d\phi} \dots \frac{dN_m}{d\phi} \right\rangle \Rightarrow \left\langle \left\langle e^{i(n_1\phi_1 + n_2\phi_2 + \dots + n_m\phi_m)} \right\rangle \right\rangle = \left\langle \mathbf{V}_{n_1} \mathbf{V}_{n_2} \dots \mathbf{V}_{n_m} \right\rangle$$

Average over events

$$n_1 + n_2 + \dots + n_m = 0$$

$$\left\langle v_{n_1} v_{n_2} \dots v_{n_m} \cos(n_1\Phi_{n_1} + n_2\Phi_{n_2} + \dots + n_m\Phi_{n_m}) \right\rangle$$

## Examples:

Moments of  $p(v_n, \Phi_n \dots)$

**2PC**  $\langle \langle \{2\}_n \rangle \rangle = \langle \langle e^{in(\phi_1 - \phi_2)} \rangle \rangle = \langle v_n^2 \rangle$

**4PC**  $\langle \langle \{4\}_n \rangle \rangle = \langle \langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle \rangle = \langle v_n^4 \rangle$

**4PC**  $\langle \langle \{4\}_{n,m} \rangle \rangle = \langle \langle e^{in(\phi_1 - \phi_2) + im(\phi_3 - \phi_4)} \rangle \rangle = \langle v_n^2 v_m^2 \rangle$

**3PC**  $\langle \langle \{3\}_n \rangle \rangle = \langle \langle e^{in(\phi_1 + \phi_2 - 2\phi_3)} \rangle \rangle = \langle v_n^2 v_{2n} \cos 2n(\Phi_n - \Phi_{2n}) \rangle$

# How to quantify nature of fluctuations?

- The shape of  $p(X)$  often quantified by cumulants

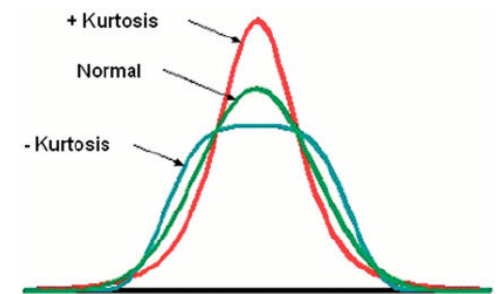
$$C_2 = \langle \delta X^2 \rangle \quad \delta X = X - \langle X \rangle$$

$$C_3 = \langle \delta X^3 \rangle$$

$$C_4 = \langle \delta X^4 \rangle - 3 \langle \delta X^2 \rangle^2$$

$$C_5 = \langle \delta X^5 \rangle - 10 \langle \delta X^3 \rangle \langle \delta X^2 \rangle$$

$C_2$  variance,  $C_3$  Skewness,  $C_4$  Kurtosis



- Substitute  $X=v_n e^{in\Phi_n}$ , one derive cumulants for flow, such as:

$$c_n\{4\} = \langle v_n^4 \rangle - 2 \langle v_n^2 \rangle^2 \quad \text{Probe } p(v_n)$$

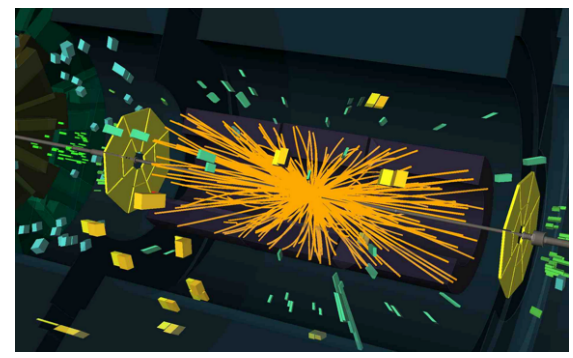
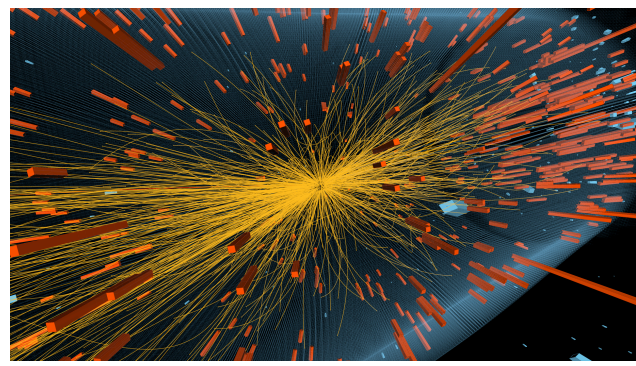
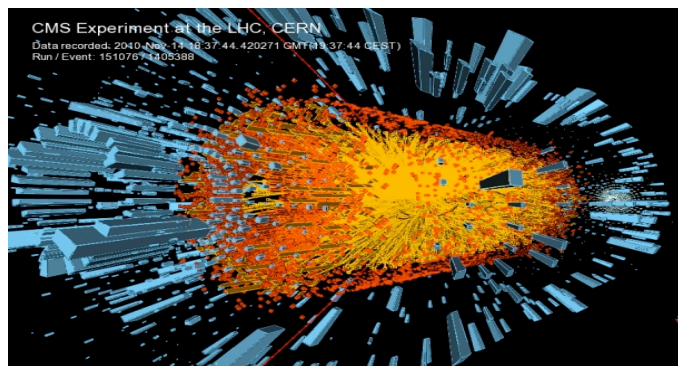
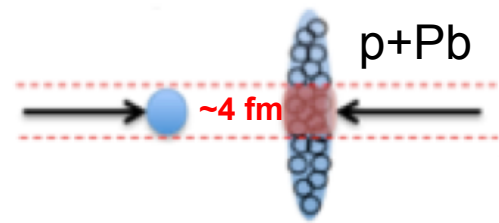
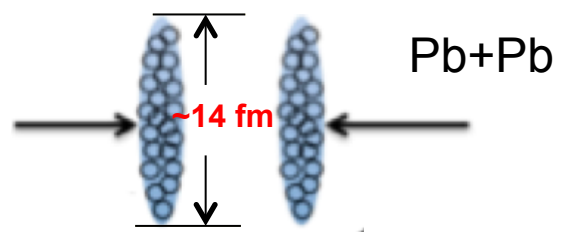
**Four-particle cumulants**

$$\text{if flow is constant, } c_n\{4\} = -v_n^4 < 0$$

$$sC_{n,m}\{4\} = \langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle \quad \text{Probe } p(v_n, v_m)$$

# Collectivity in small systems

# Collectivity in different systems



~30000 particles\*

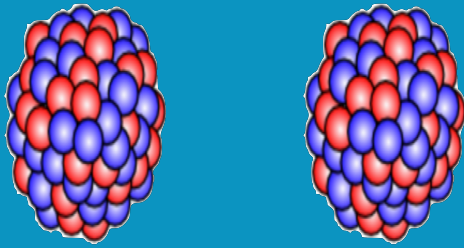
~2000 particles\*

~ 600 particles\*

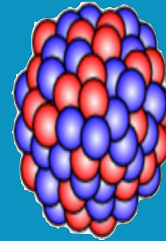
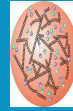
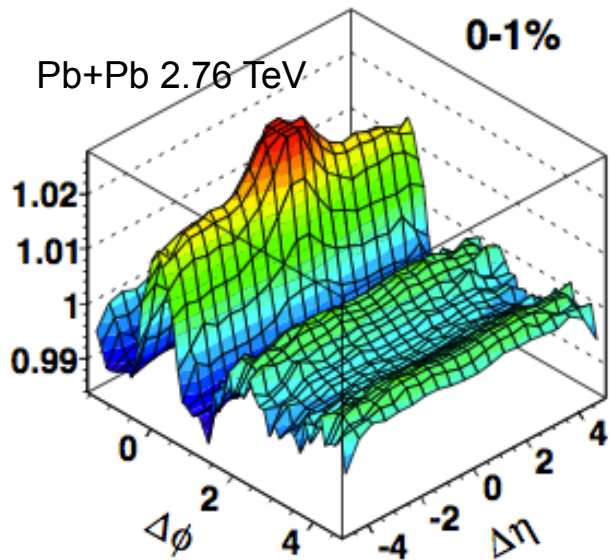
Change system size and shape at RHIC and LHC:  
 → Control space-time dynamics!

\* Rough number in very high-multiplicity events, integrated over full phase space at LHC

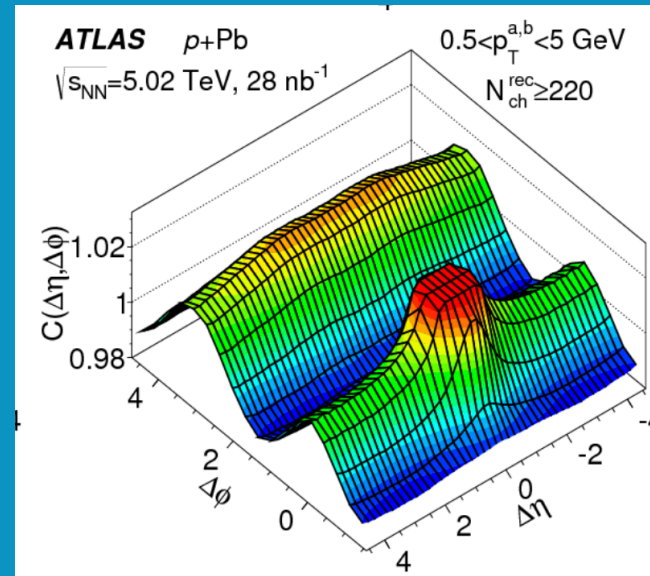
# Two-particle correlation in different systems



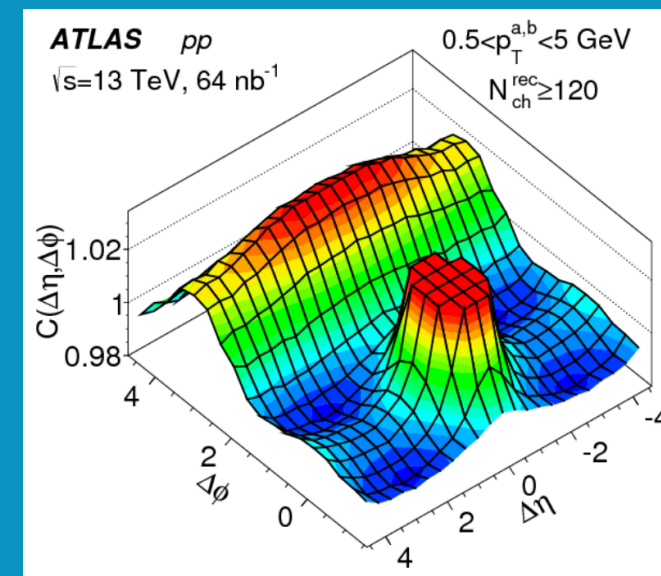
**Pb+Pb**



**p+Pb**



**p+p**

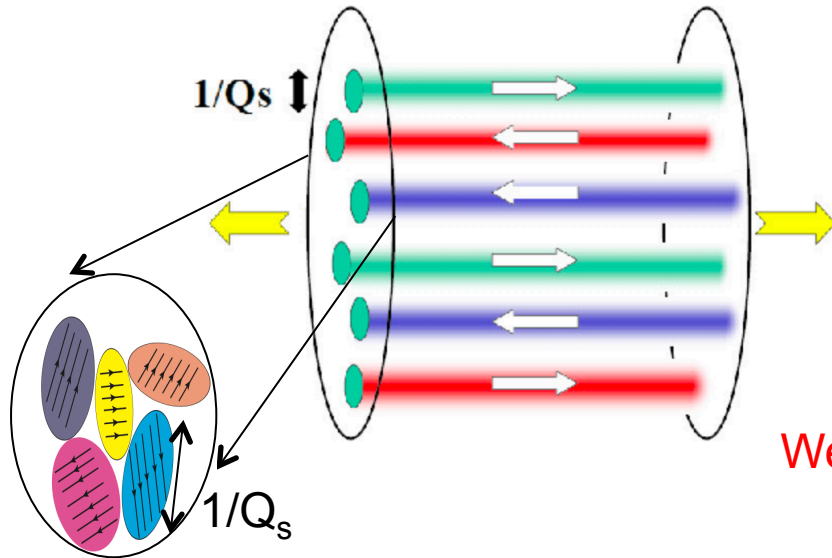


## ■ Long-range correlation comes

- directly from initial state momentum correlation (structure function, CGC)
- or it is a final state response to spatial fluctuation at  $t=0$  (hydro/transport).

**What is the timescale for emergence of collectivity?**

## Saturation/CGC



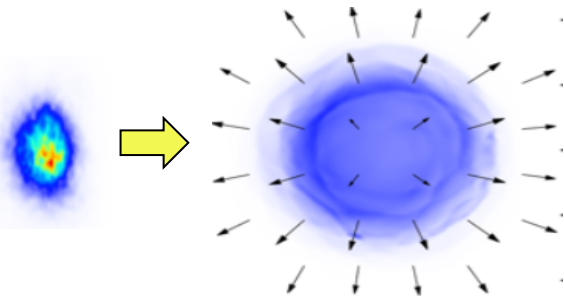
Domain of color fields of size  $1/Q_s$ , each produce multi-particles correlated across full  $\eta$ .

Uncorr. between domains, strong fluct. in  $Q_s$

More domains, smaller  $v_n$ , more  $Q_s$  fluct, stronger  $v_n$

Well motivated model framework, need systematic treatment

## Hydrodynamics



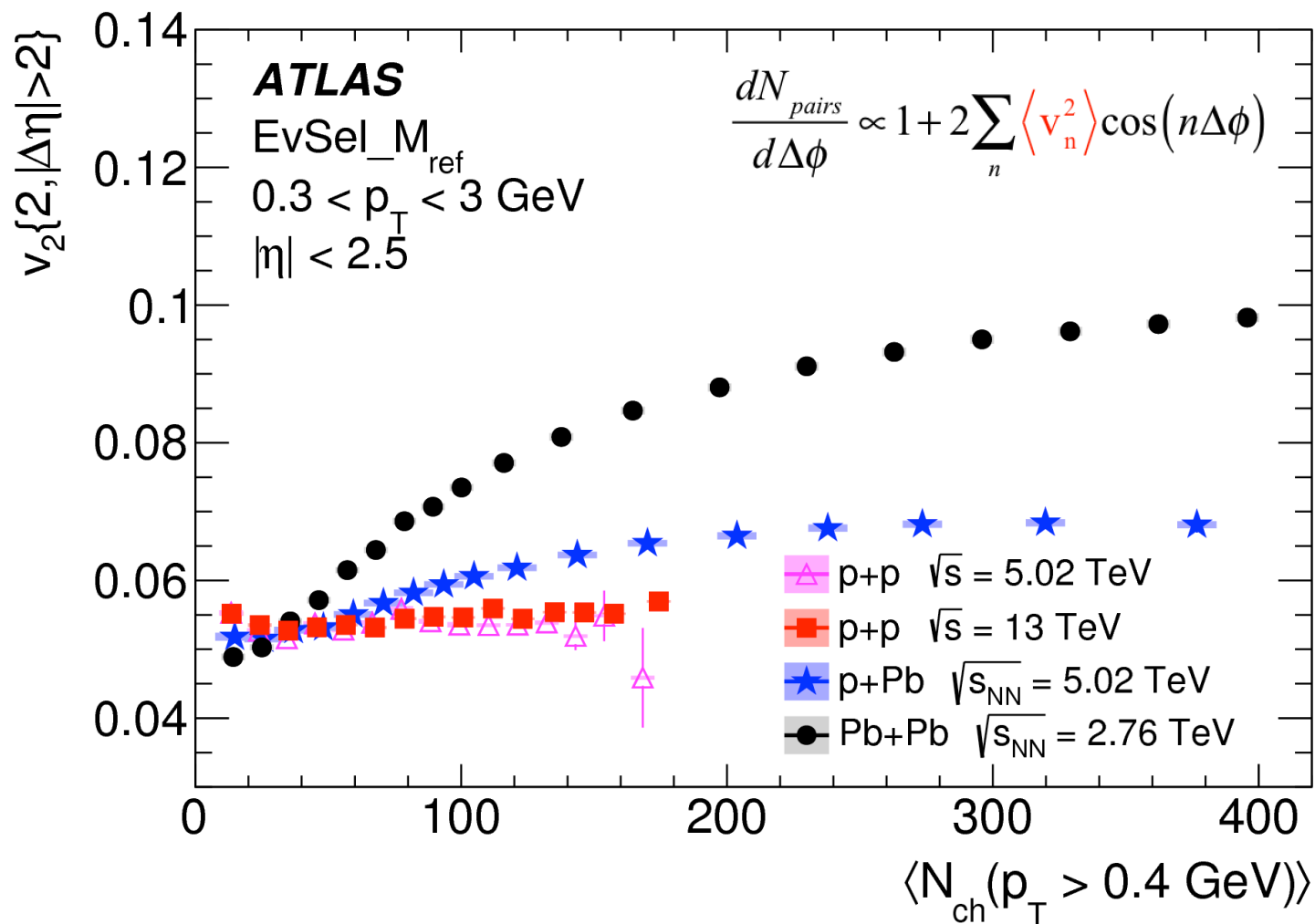
Hot spots in transverse plane e.g IP-plasma, boost-invariant geometry shape

Expansion and interaction of hot spots generate collectivity

$v_n$  depends on distribution of hot spots ( $\epsilon_n$ ) and transport properties.

Ongoing debate whether hydro is applicable in small systems

# $v_2$ in small systems



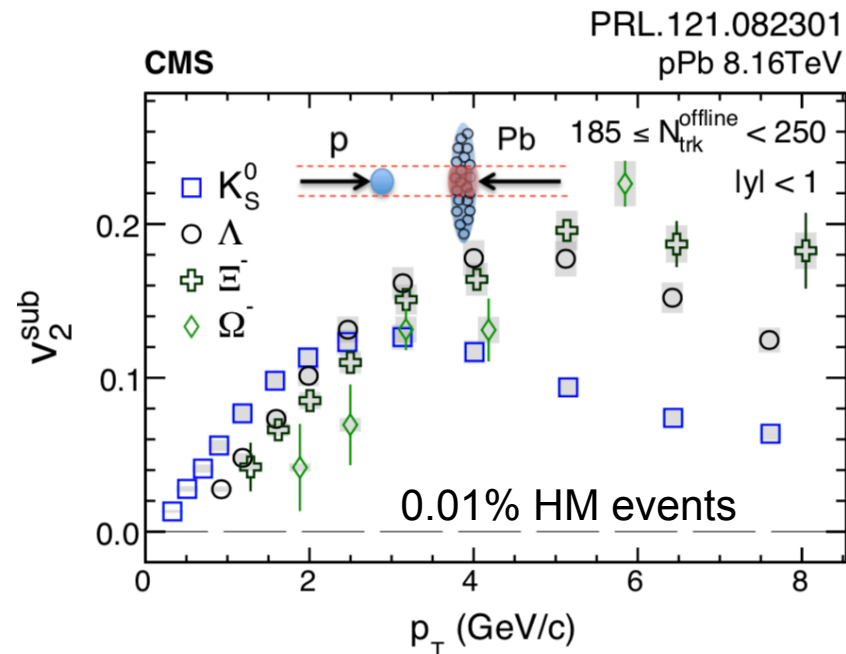
- pp  $v_2 \sim$  constant, pPb and PbPb  $v_2$  increase with  $N_{ch}$  (due to Geometry)
- $v_2$  persist to very low  $N_{ch}$  ( $\sim$ minbias  $N_{ch}$  value in pp and pPb)

Similar origin for the collectivity?

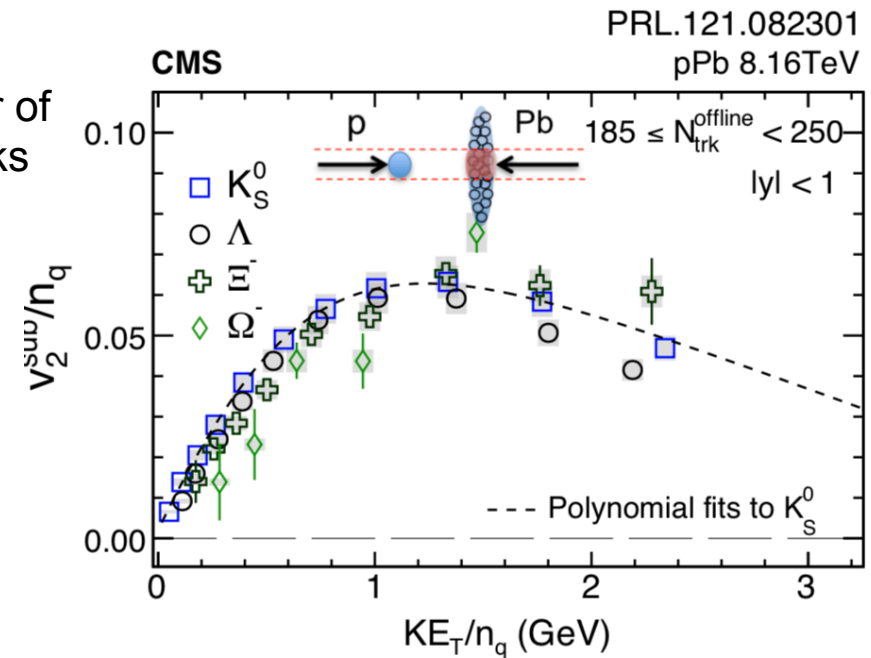
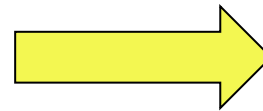
# PID $v_2$ in p+Pb

$v_2$  show mass ordering

$$\frac{v_2(\text{meson})}{2} \approx \frac{v_2(\text{baryon})}{3}$$



Scale by number of constituent quarks



Similar as observation in A+A

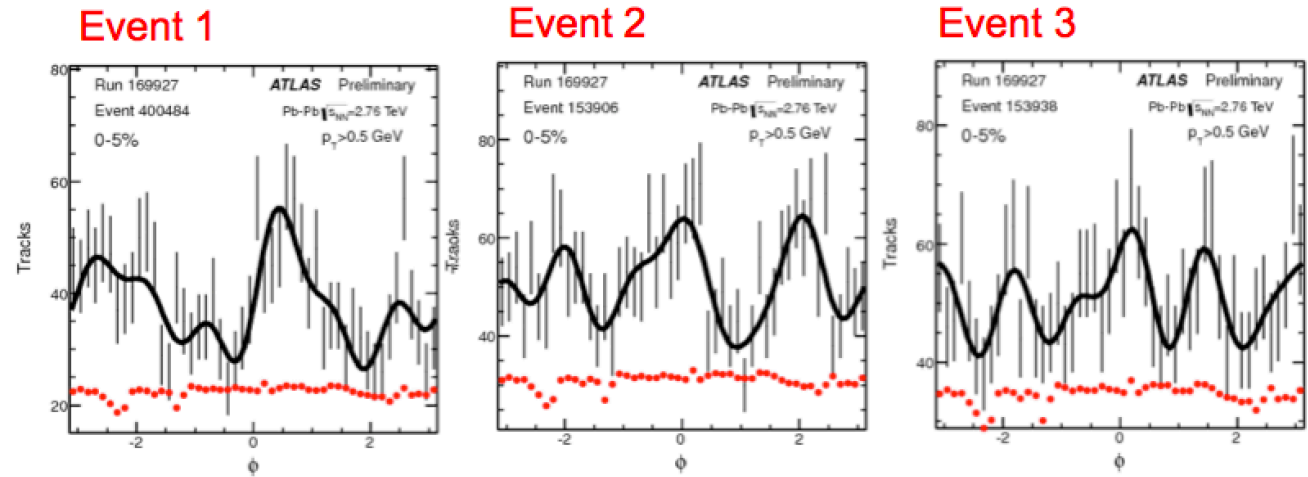
Collectivity occurs at partonic level?



# How about event-by-event fluctuations?

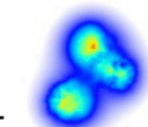
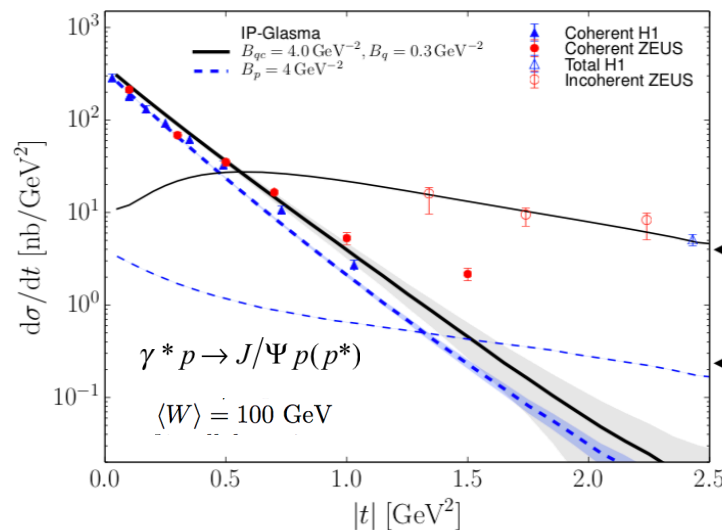
$$\frac{dN}{d\phi} = N \left[ 1 + 2 \sum_n v_n \cos n(\phi - \Phi_n) \right]$$

Observed directly  
in Pb+Pb collisions

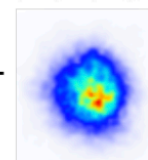


Must also has strong  
fluctuations in pp and pA

Indirect evidence from diffractive  
DIS ep  $J/\Psi$  cross-section data

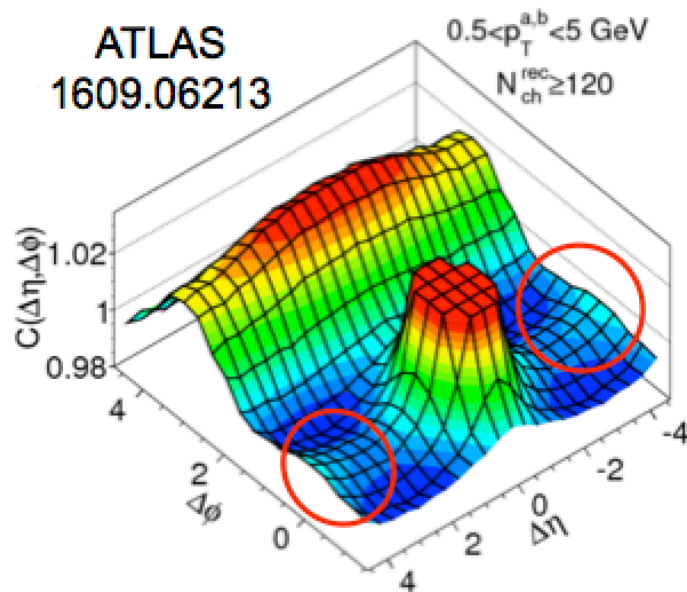


eccentric proton

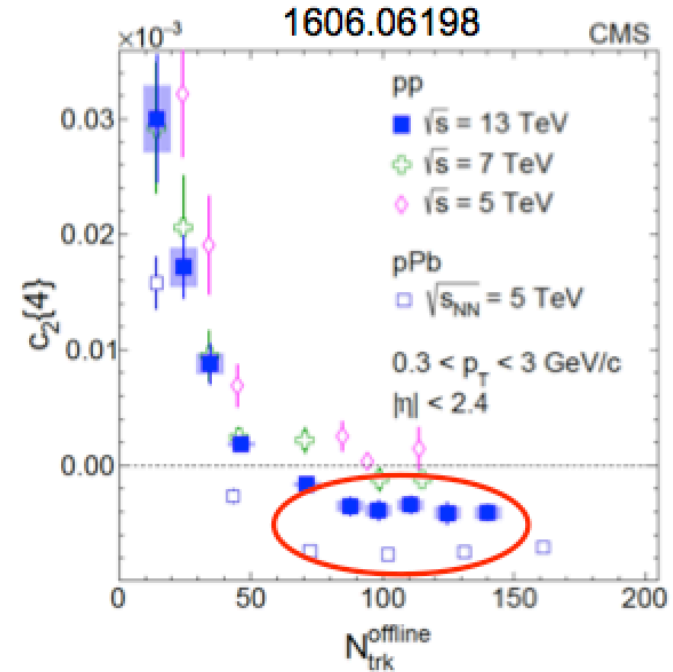


round proton

# Multi-particle nature of the ridge

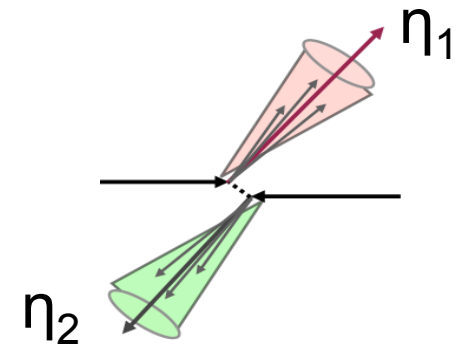


Long-range in  $\eta$

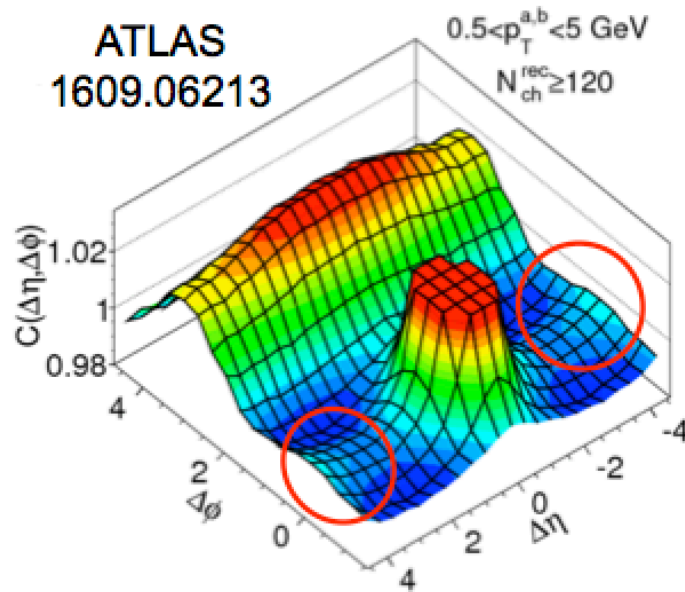


Multi-particle (3,4,5..) signal

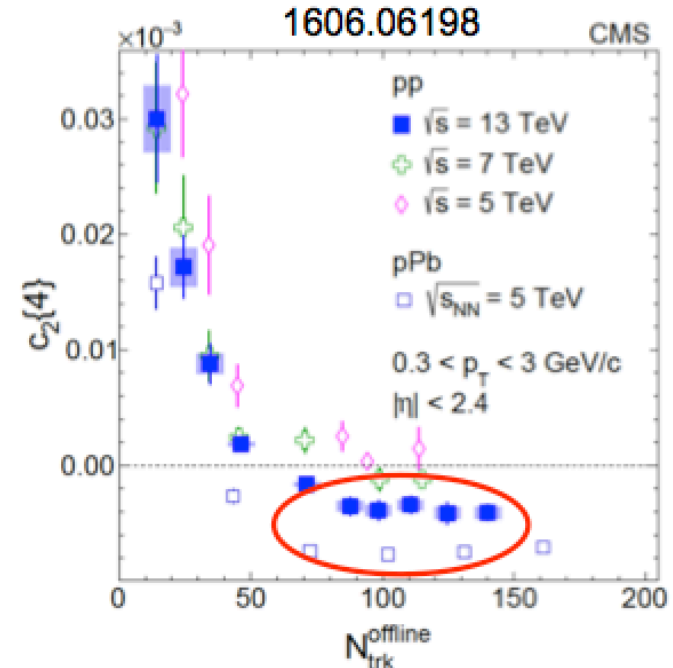
Very different from jets and dijet correlations,  
which are confined in **one or two  $\eta$  regions**



# Multi-particle nature of the ridge



Long-range in  $\eta$



Multi-particle (3,4,5..) signal

Suppress by requiring correlations between more than two  $\eta$  ranges

Example: WW TMD in  
DIS dijet production

$$E_1 E_2 \frac{d\sigma^{\gamma_L^* A \rightarrow q\bar{q}X}}{d^3k_1 d^3k_2 d^2b} = \alpha_{em} e_q^2 \alpha_s \delta(x_{\gamma^*} - 1) z^2 (1-z)^2 \frac{8\epsilon_f^2 P_\perp^2}{(P_\perp^2 + \epsilon_f^2)^4}$$

$$eA \rightarrow e' Q\bar{Q}X$$

$$\times \left[ xG^{(1)}(x, q_\perp) + \cos(2\phi) xh_\perp^{(1)}(x, q_\perp) \right]$$

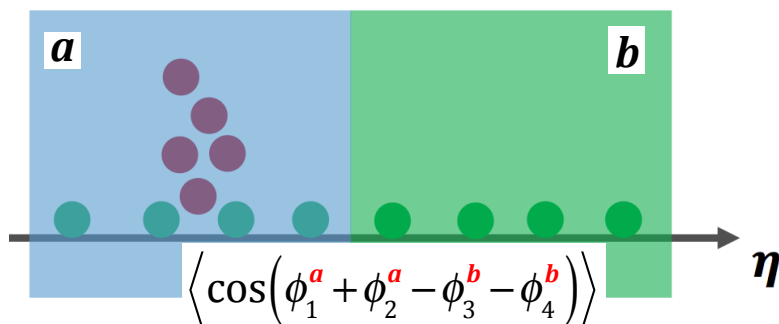
Dumitru et al Phys. Rev. D 94, 014030 (2016),  
Dumitru et al Phys. Rev. Lett. 115 (2015) 25, 252301

gluon distribution ( $G^{(1)}$ ) + linearly polarized  
partner ( $h^{(1)}$ )

# Long-range collectivity via subevent correlations<sup>20</sup>

arXiv:1701.03830

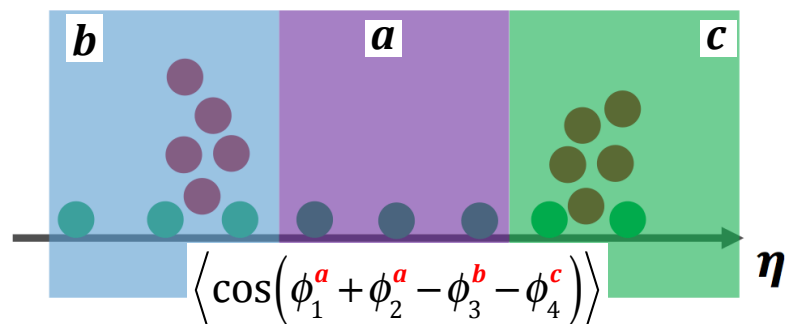
Event with jet



**2 sub-event**

suppress intra-jet correlations

Event with dijet



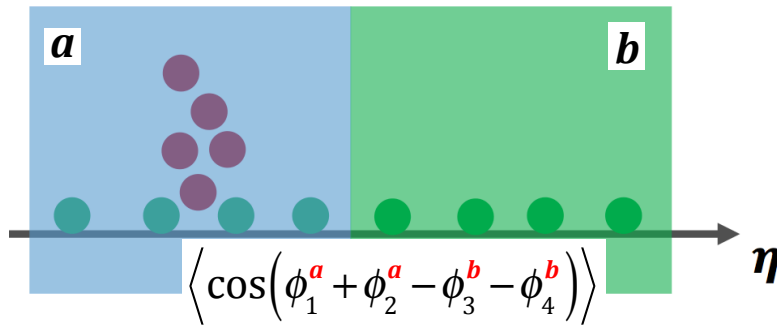
**3 sub-event**

suppress inter-jet correlations

# Long-range collectivity via subevent correlations <sup>21</sup>

arXiv:1701.03830

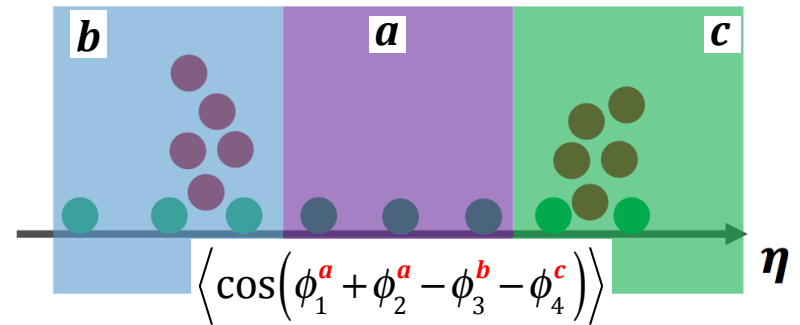
Event with jet



**2 sub-event**

suppress intra-jet correlations

Event with dijet

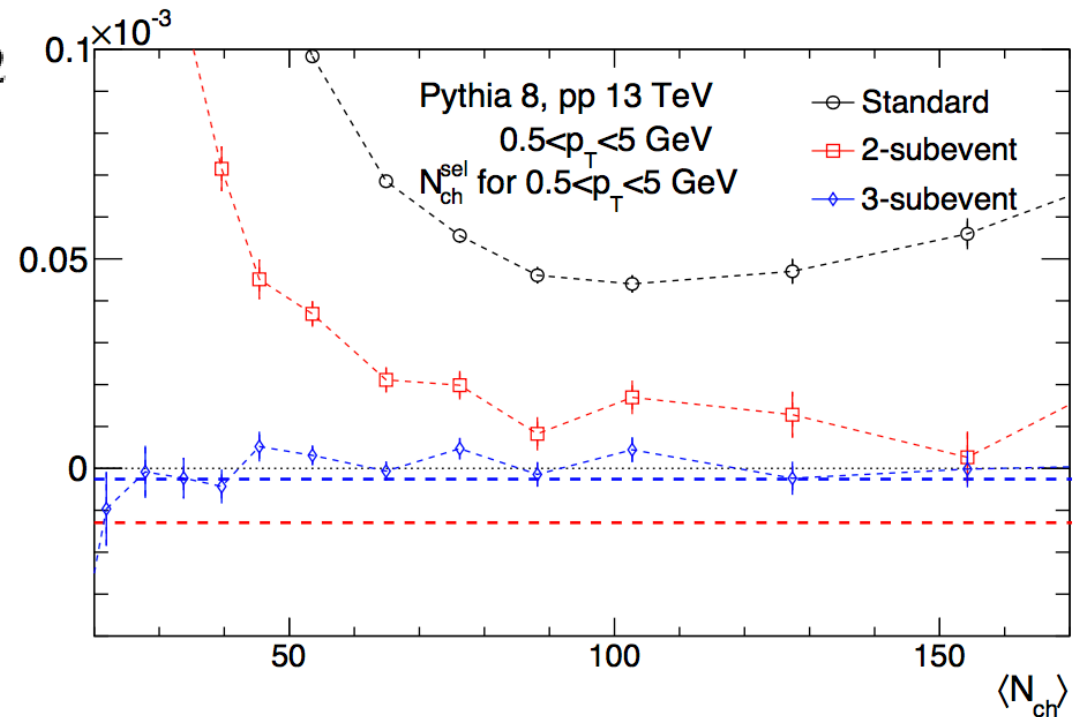


**3 sub-event**

suppress inter-jet correlations

$$c_2 \{4\} = \langle v_2^4 \rangle - 2 \langle v_2^2 \rangle^2$$

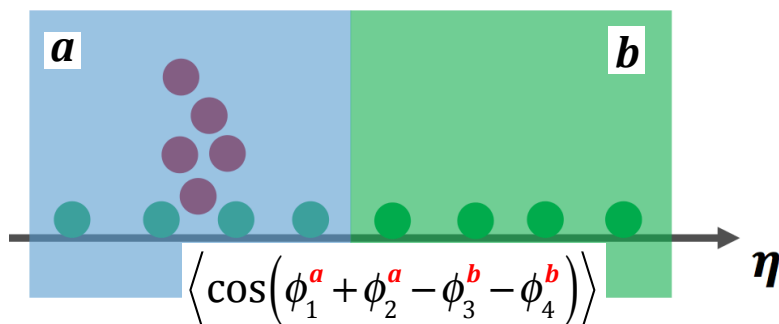
Mini-jet correlations  
suppressed by the method!



# Long-range collectivity via subevent correlations <sup>22</sup>

arXiv:1701.03830

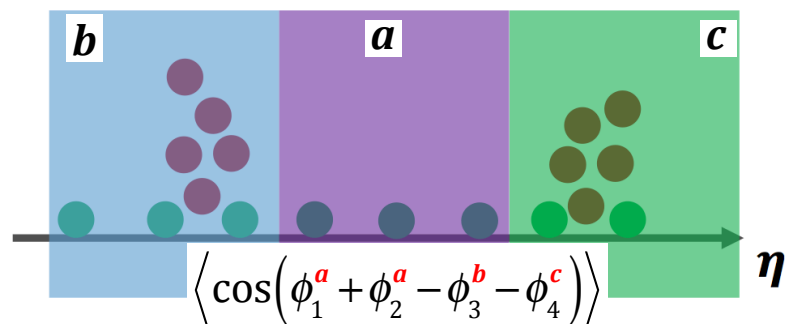
Event with jet



2 sub-event

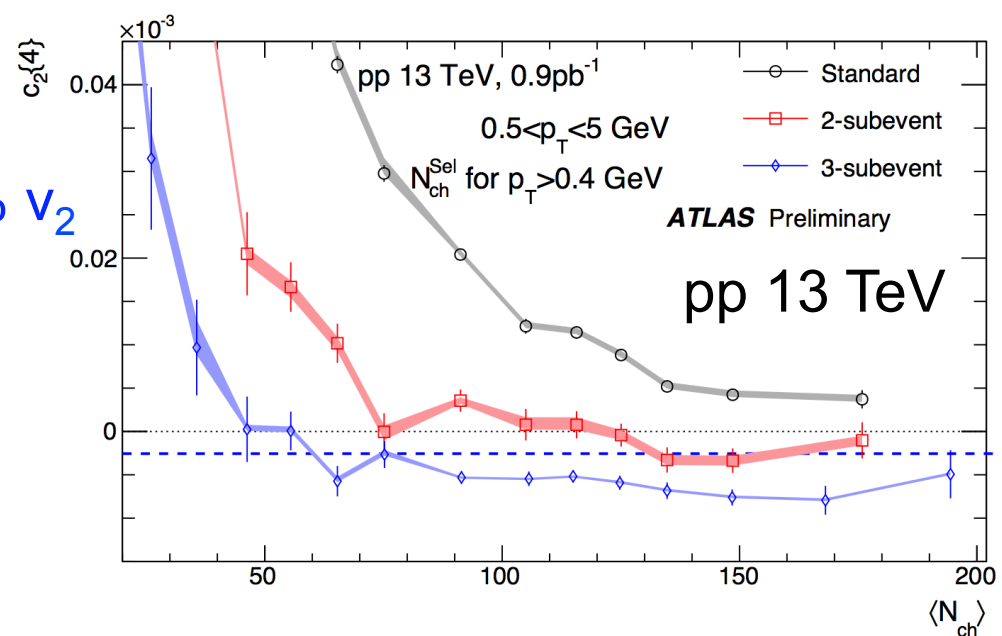
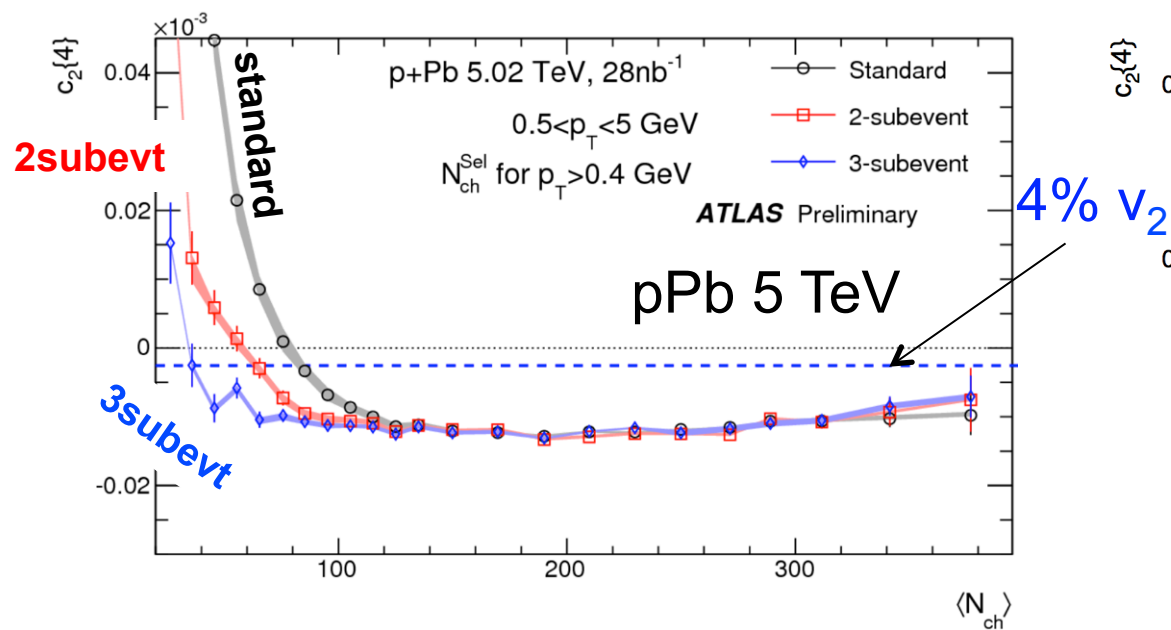
suppress intra-jet correlations

Event with dijet



3 sub-event

suppress inter-jet correlations



Jet correlation important at low  $N_{ch}$  for pPb, and over all  $N_{ch}$  in pp

Subevent method required to suppress the jet correlations in small system

# Sign-change of $c_2\{4\}$

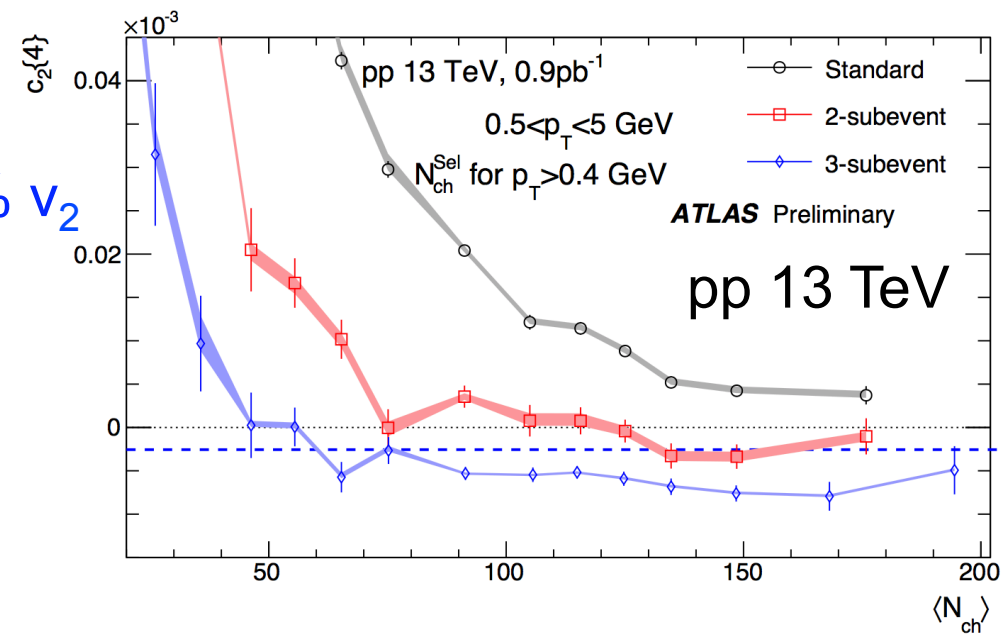
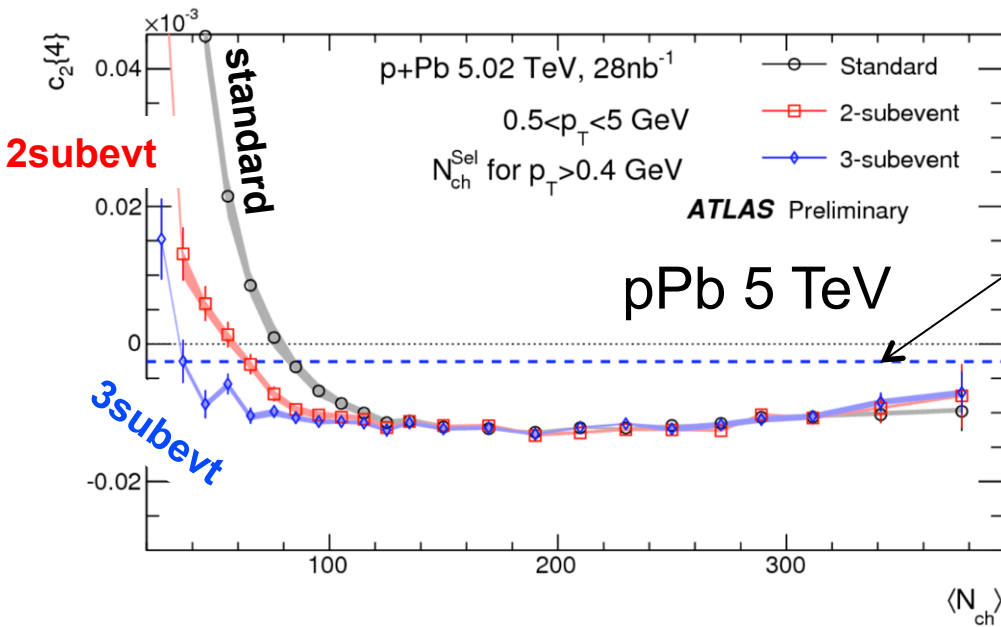
- Most positive  $c_2\{4\}$  in standard cumulants are jets and dijets.
  - Remaining positive  $c_2\{4\}$  in 3-subevent due to residual dijets.

- CGC expect sign change at low  $N_{ch}$ 

$$c_2\{4\} = \frac{1}{N_D^3} \left( \frac{1}{4(N_c^2 - 1)^3} - A^4 \right)$$

Dumitru, McLerran, Skokov
↑
↑

Glasma diagram
non-linear/non-Gaussian effects



Glasma diagram contribution is small?

# $p(v_2)$ in pp and pPb

- Constrain  $p(v_2)$  from two- and four-particle correlations

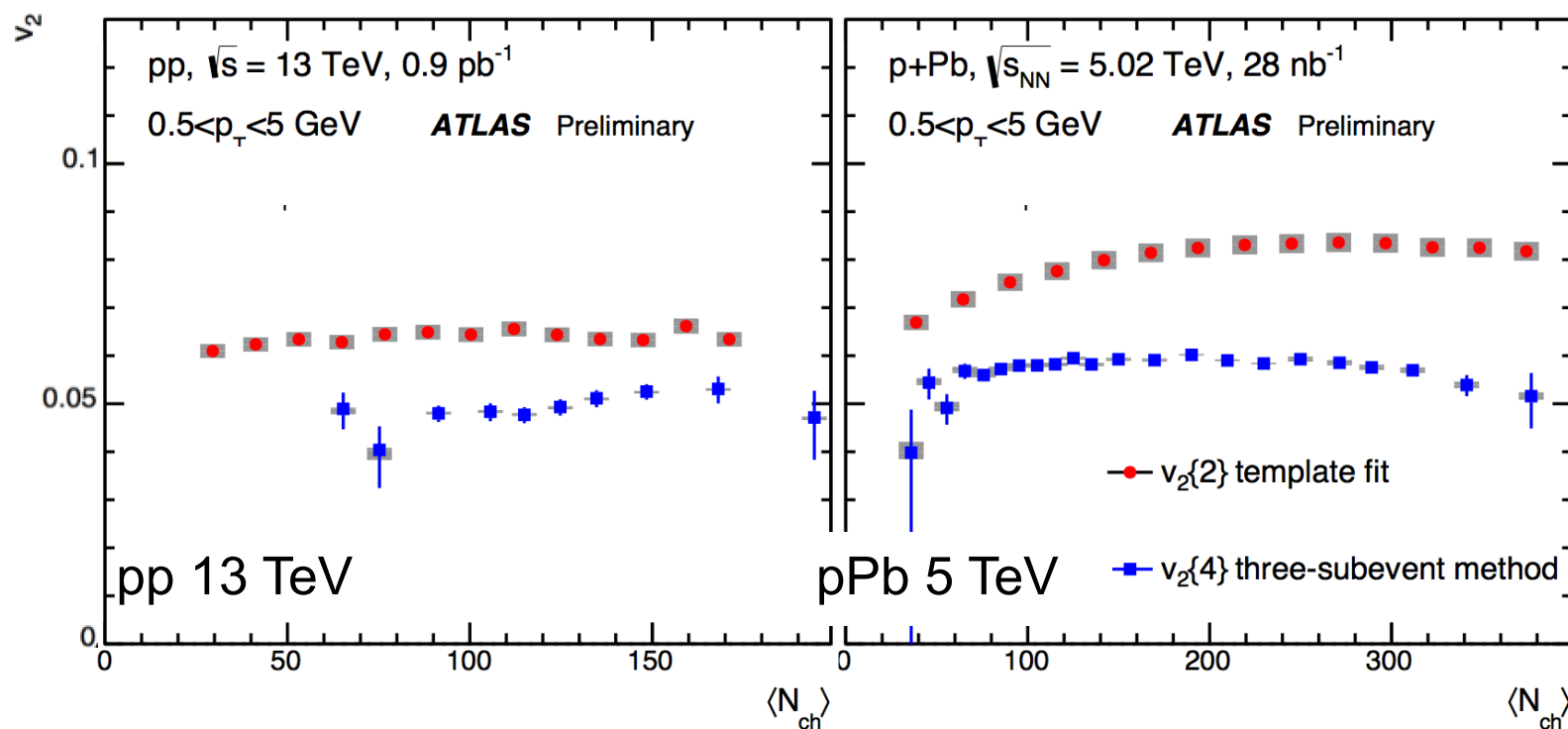
$$c_2\{2\} = \langle v_2^2 \rangle \equiv v_2\{2\}^2$$

Two-particle correlation

$$c_2\{4\} = \langle v_2^4 \rangle - 2 \langle v_2^2 \rangle^2 \equiv -v_2\{4\}^4$$

Four-particle correlation

- Results suggest significant non-Gaussian EbyE fluctuations



Both  $v_2\{2\}$  and  $v_2\{4\}$  show No hint of collectivity turning-off at low  $N_{ch}$ !

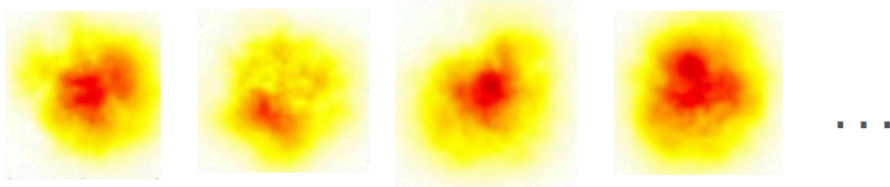
Challenge both initial and final state models?



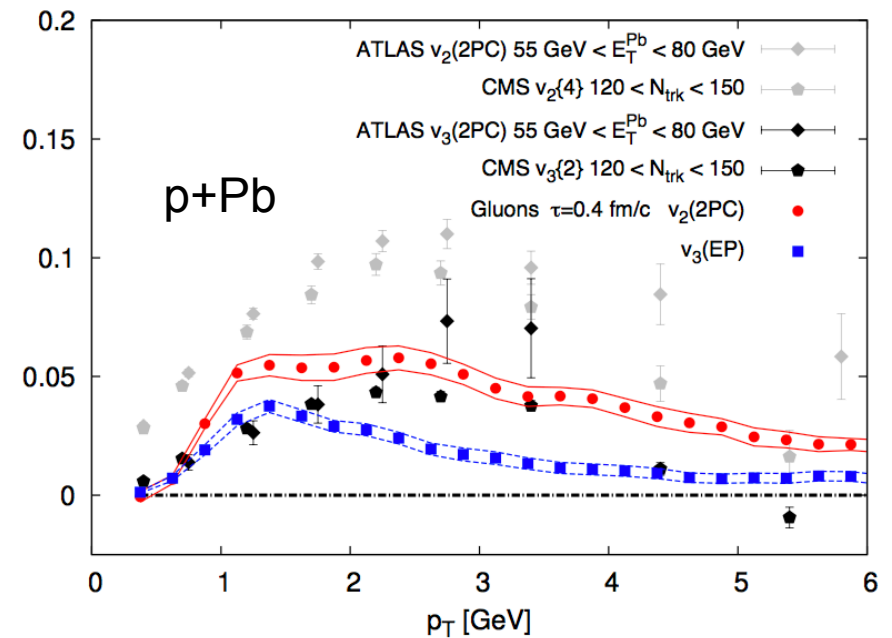
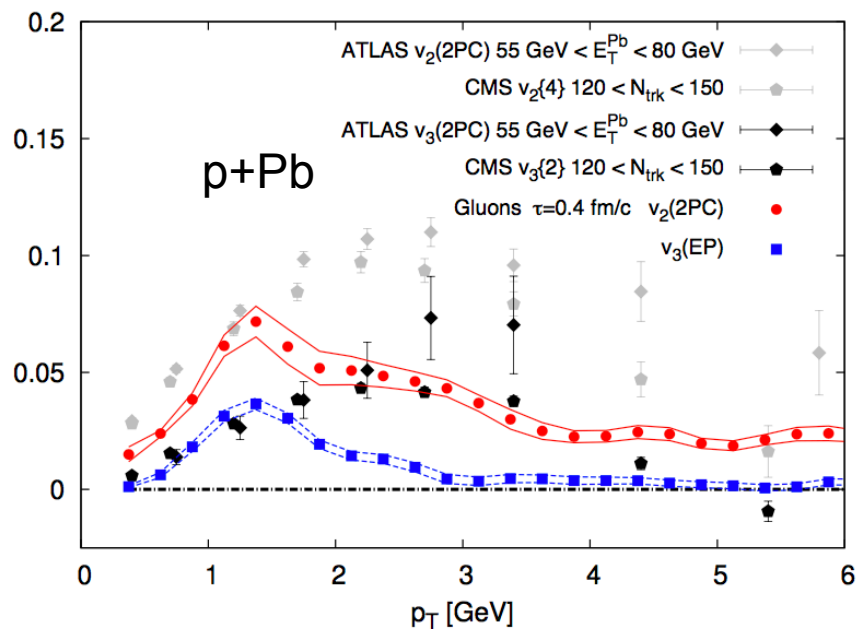
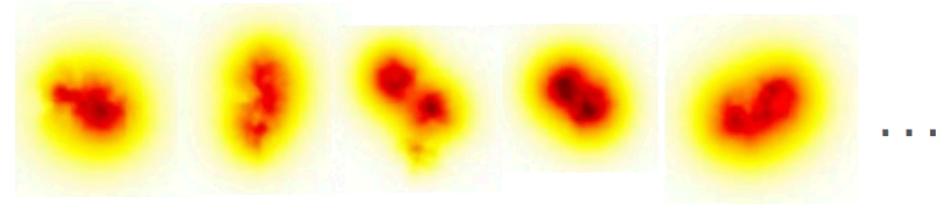
# Role of initial geometry in CGC

Schenke, Schlichting, Venugopalan

'Spherical' proton



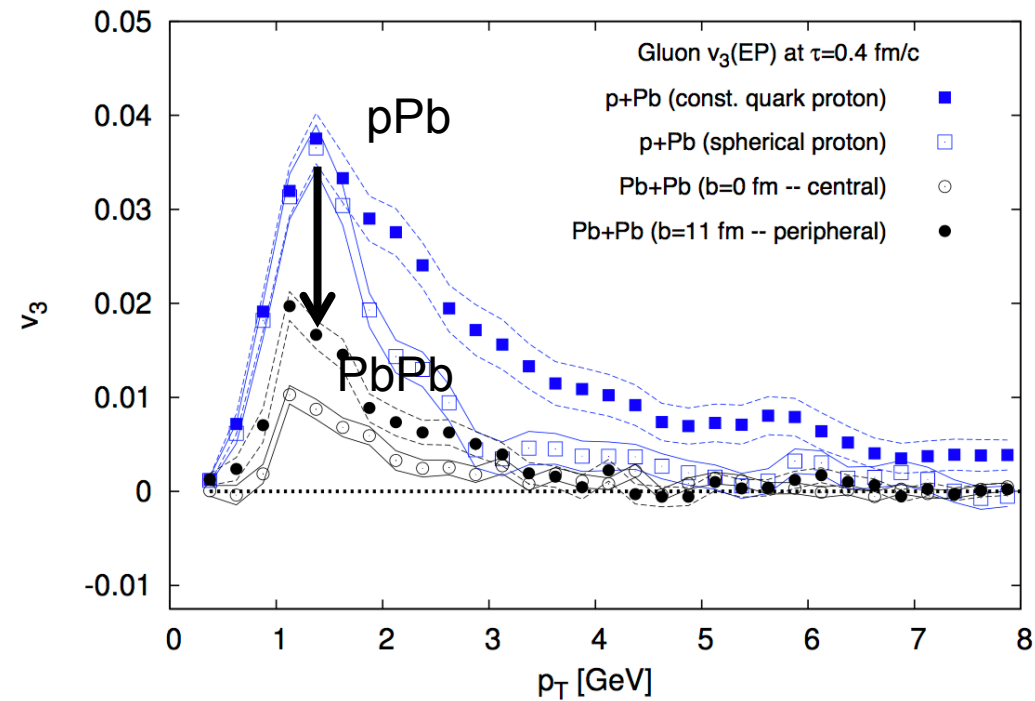
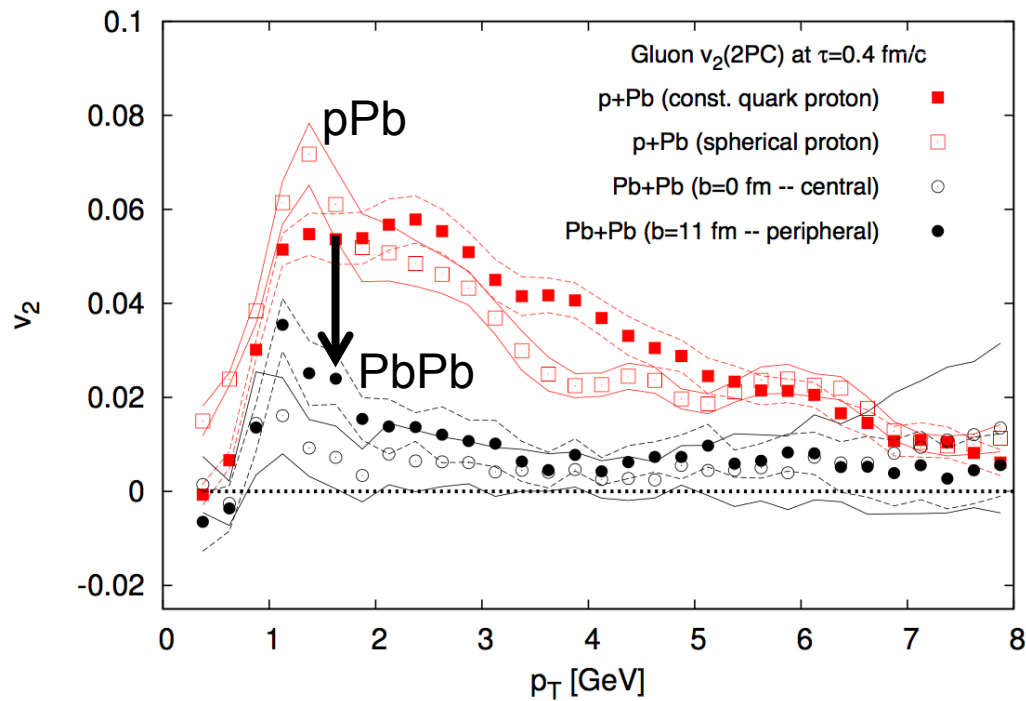
'Eccentric' proton



The orientation of collectivity is unrelated to initial eccentricity  
 $\rightarrow$  Very different from hydrodynamics

# Role of initial geometry in CGC

Schenke, Schlichting, Venugopalan

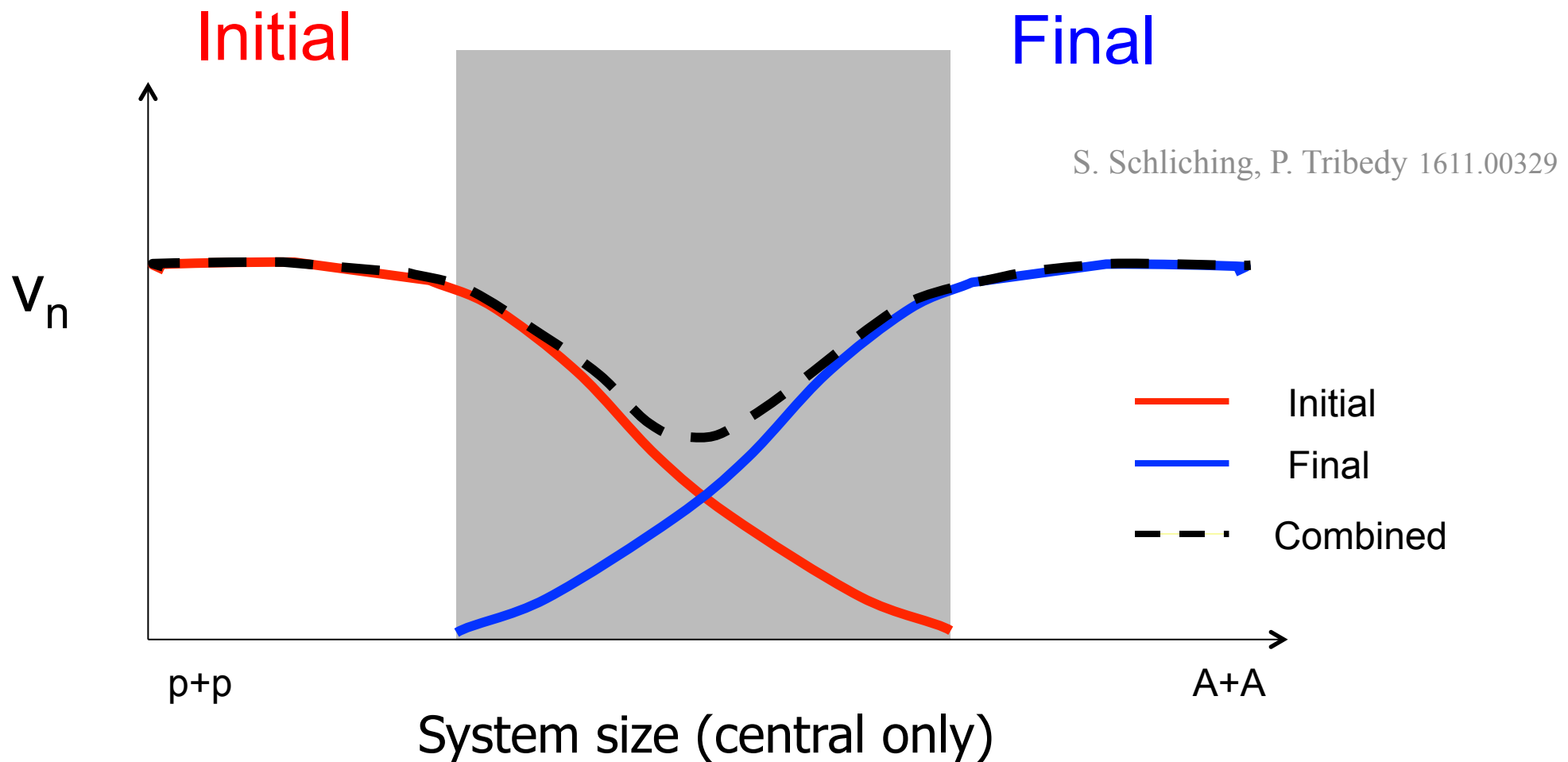


The orientation of collectivity is unrelated to initial eccentricity

→ Very different from hydrodynamics

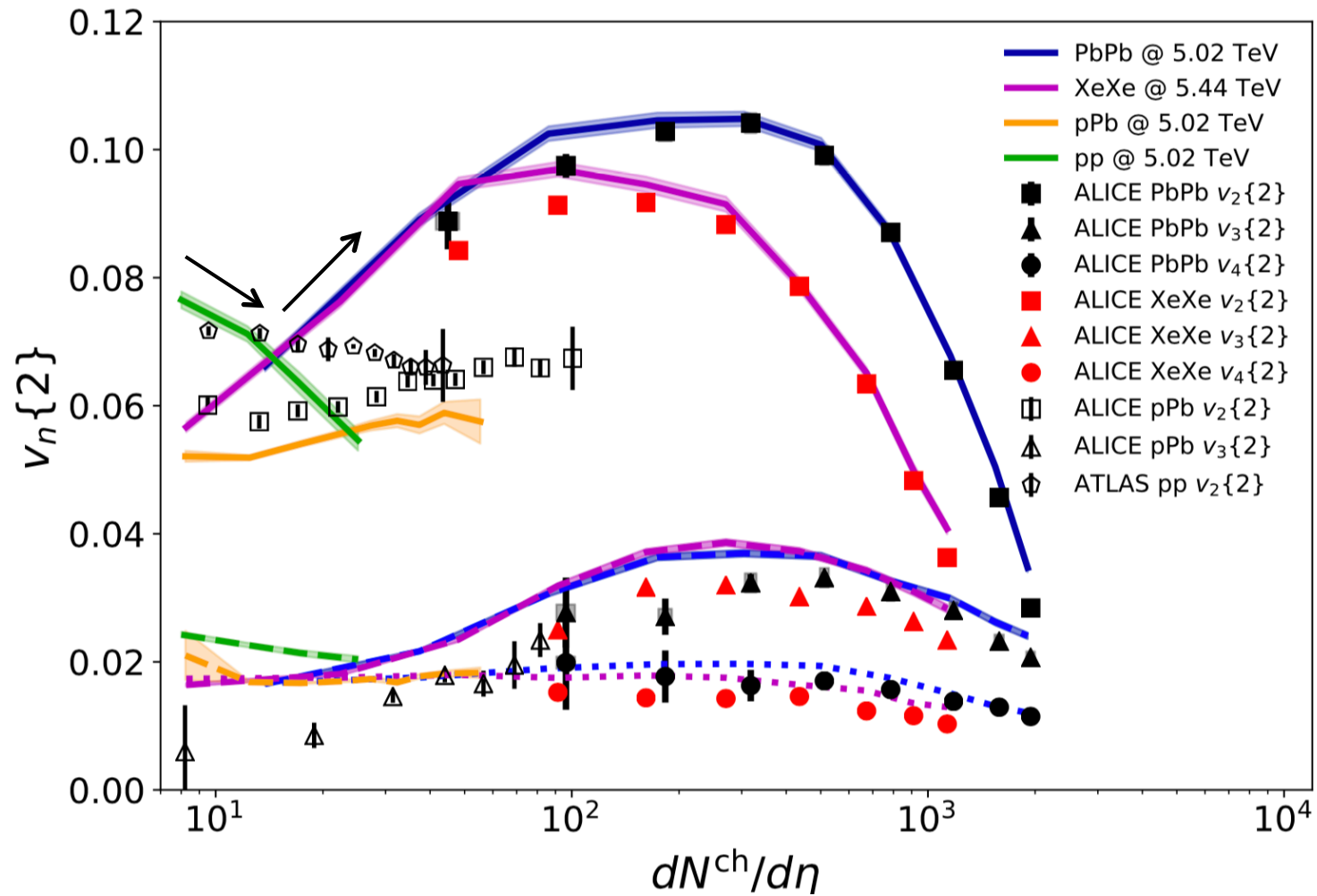
Expect contribution diminish as system size is increased

# Presence of both initial and final state scenarios?



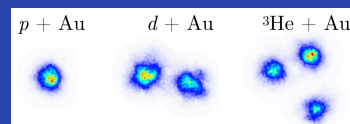
Phases of collectivity from CGC and hydro are unrelated  
→ a minimum of total  $v_n$  at certain system size?

B. Schenke, QM2018



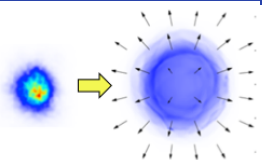
IP-Glasma + hydrodynamic calculation

# RHIC small system scan



S. Morrow, QM18

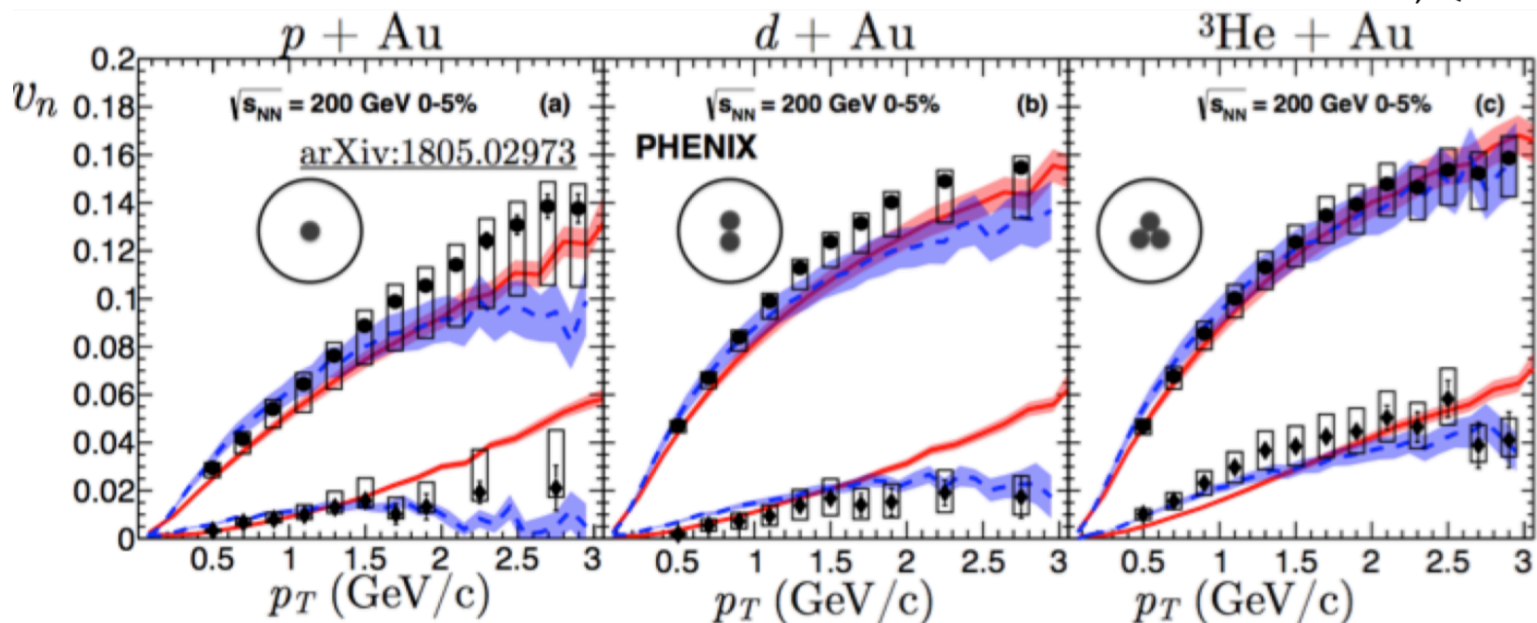
Hydro



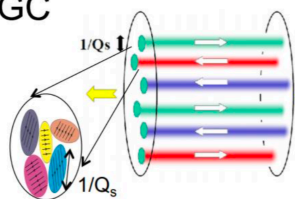
$$\varepsilon_2^{p+Au} < \varepsilon_2^{d+Au} \approx \varepsilon_2^{3He+Au}$$

$$\varepsilon_3^{p+Au} \approx \varepsilon_3^{d+Au} < \varepsilon_3^{3He+Au}$$

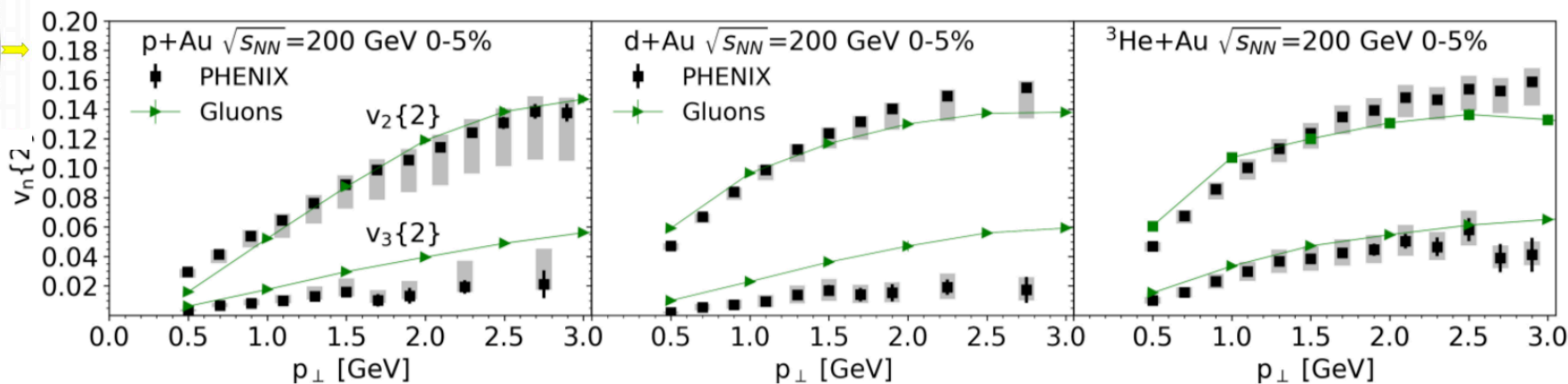
- $v_2$  Data
- ◆  $v_3$  Data
- $v_n$  SONIC
- -  $v_n$  iEBE-VISHNU



CGC

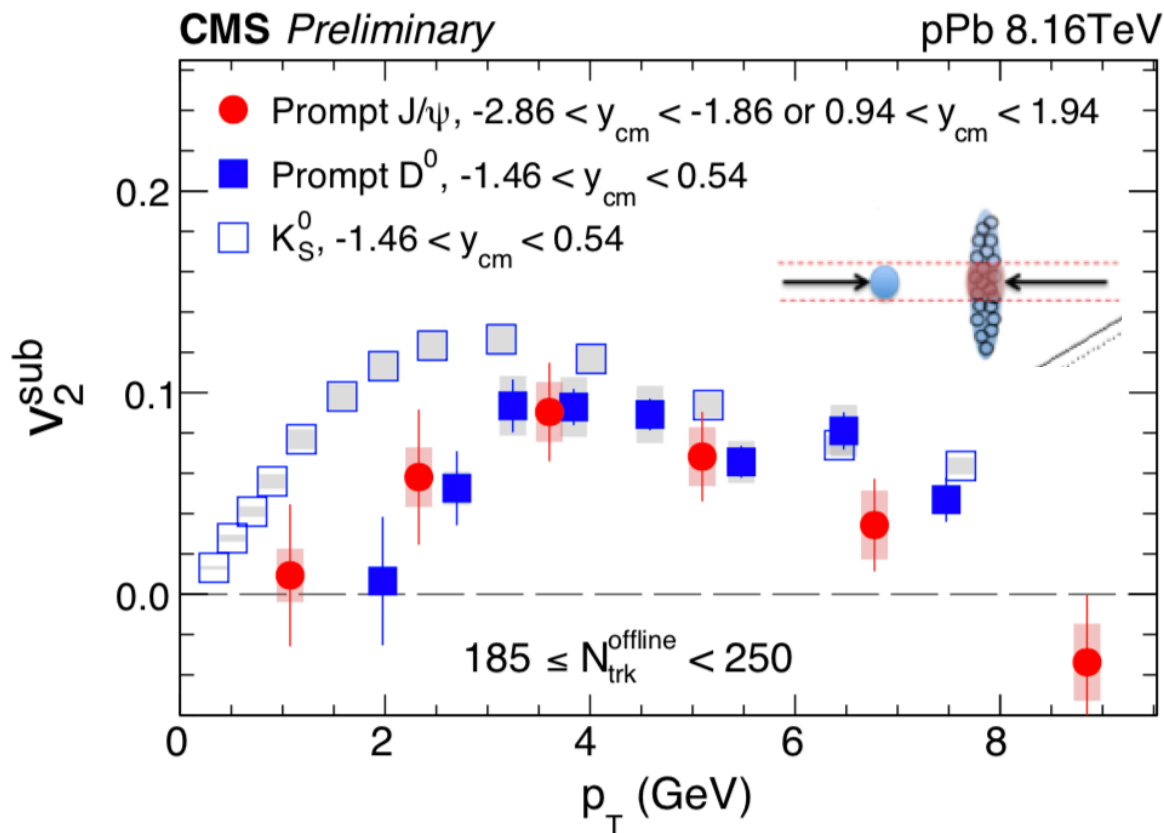


M.Mace, V.Skokov, P.Tribedy, R.Venugopalan 1805.09342



Can initial state correlation knows the shape of initial geometry?

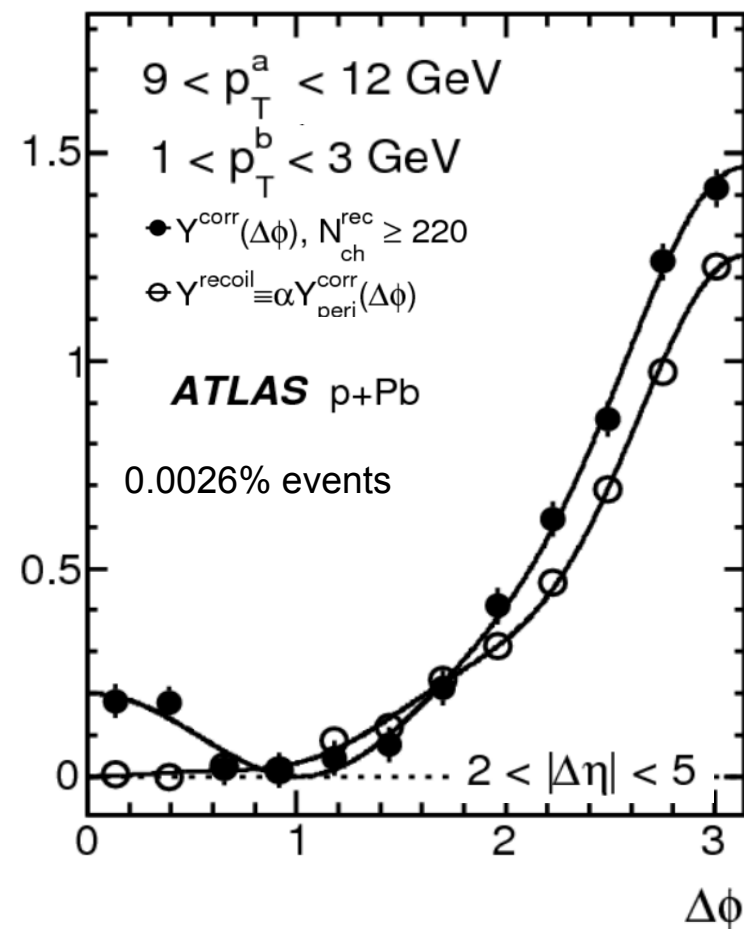
# Flow of heavy quark and high- $p_T$ hadron



Significant but smaller  $v_2$  for  $D^0$  and  $J/\Psi$ .



Charm **less** thermalized than light quarks?



Significant ridge for high  $p_T$  hadron



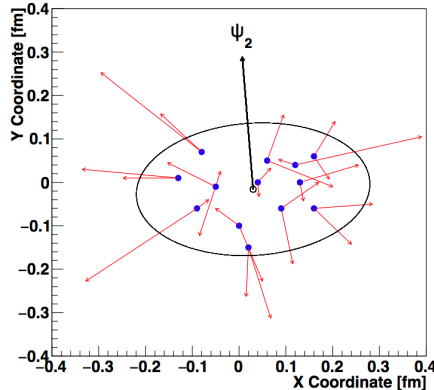
Hint of jet-medium interaction?

Implication for initial-state correlation models?

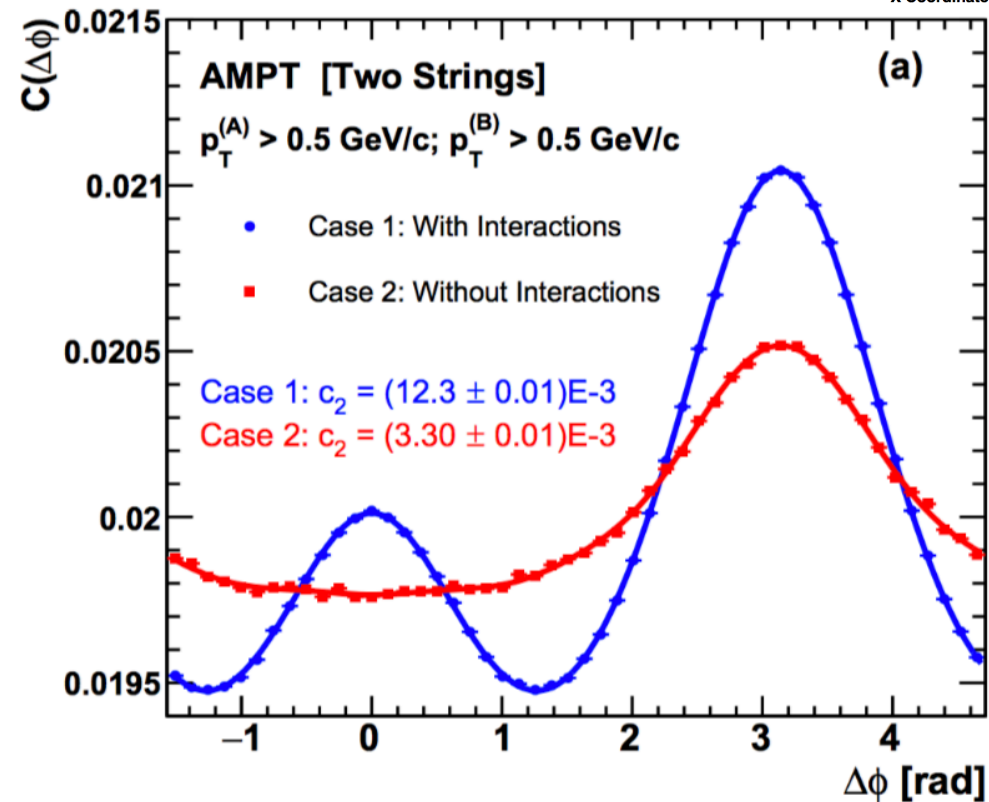
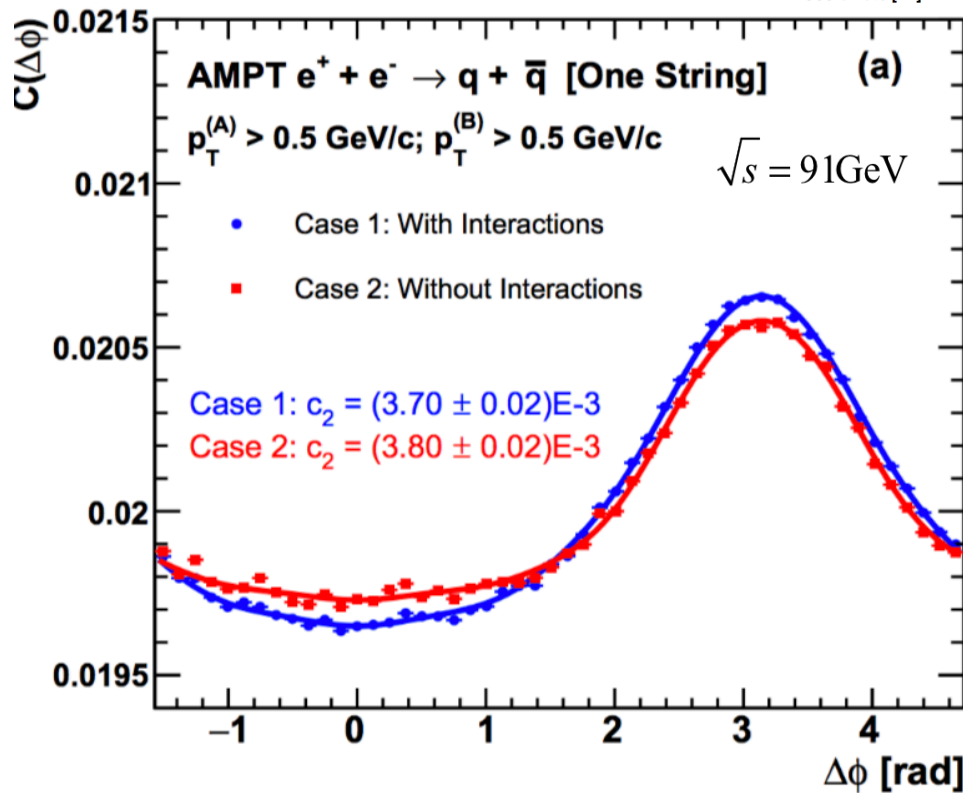
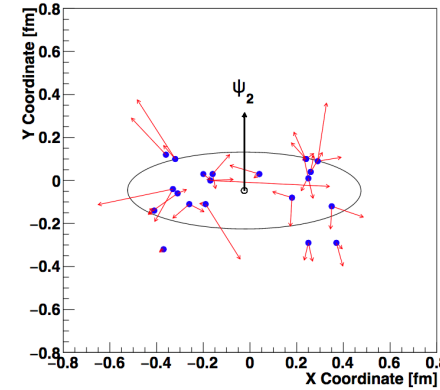
# Minimal requirement for long-range ridge

J. L Nagle et.al. 1707.02307

One String



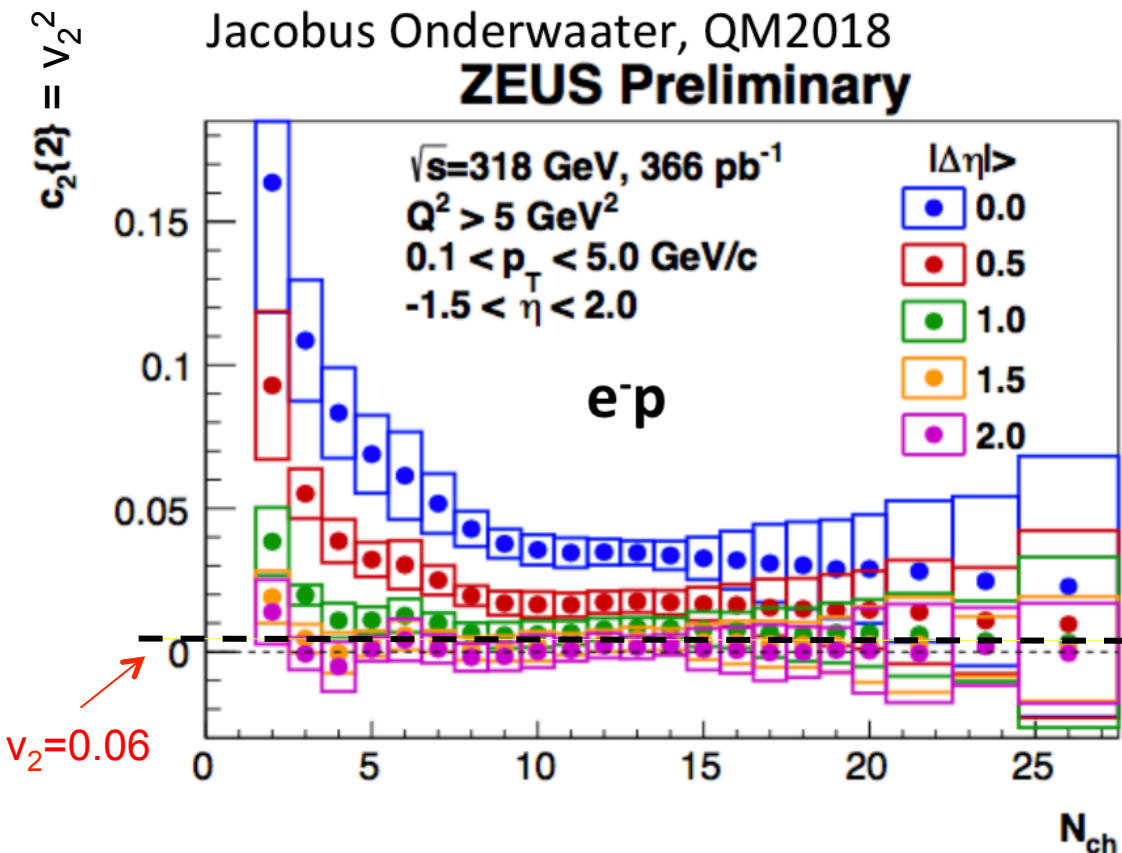
Two Strings



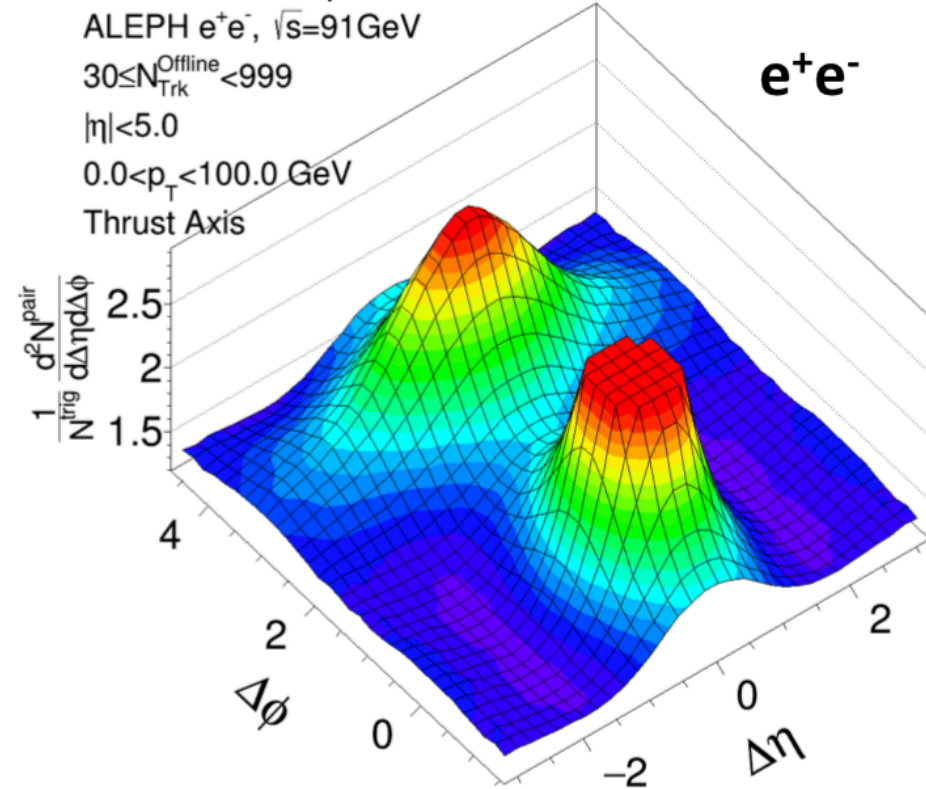
Two interacting strings are enough to create long-range correlation!

# Towards even smaller systems: ep, e+e-

Jacobus Onderwaater, QM2018  
**ZEUS Preliminary**



Yen-Jie Lee, QM2018

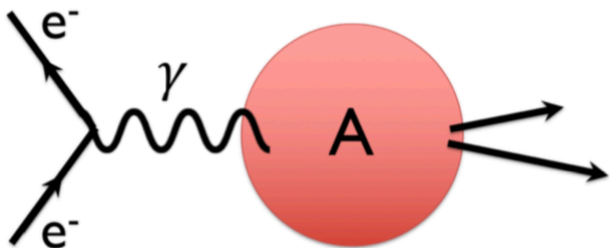


Ridge observation in ep or ee is currently limited by statistics

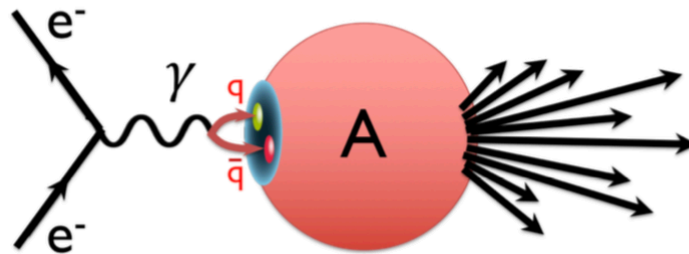


# High-multiplicity $e+A$ at EIC?

Typical  $e+A$

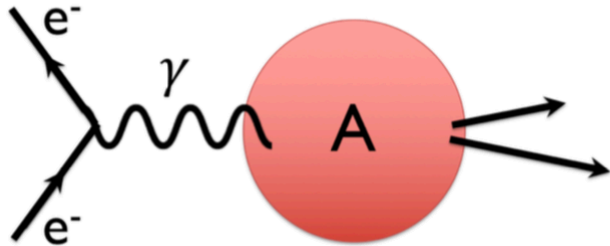


High-multiplicity  $e+A = (q\bar{q})+A$ ?

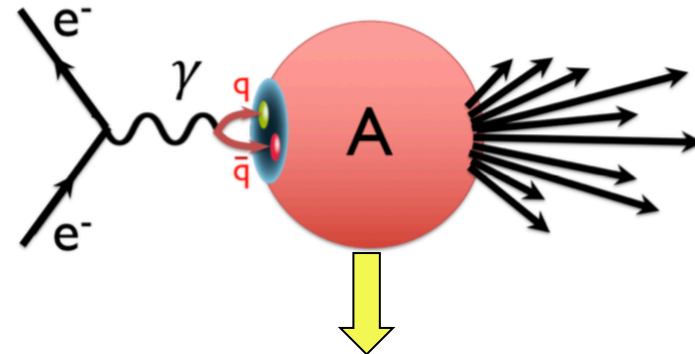


# High-multiplicity e+A at EIC?

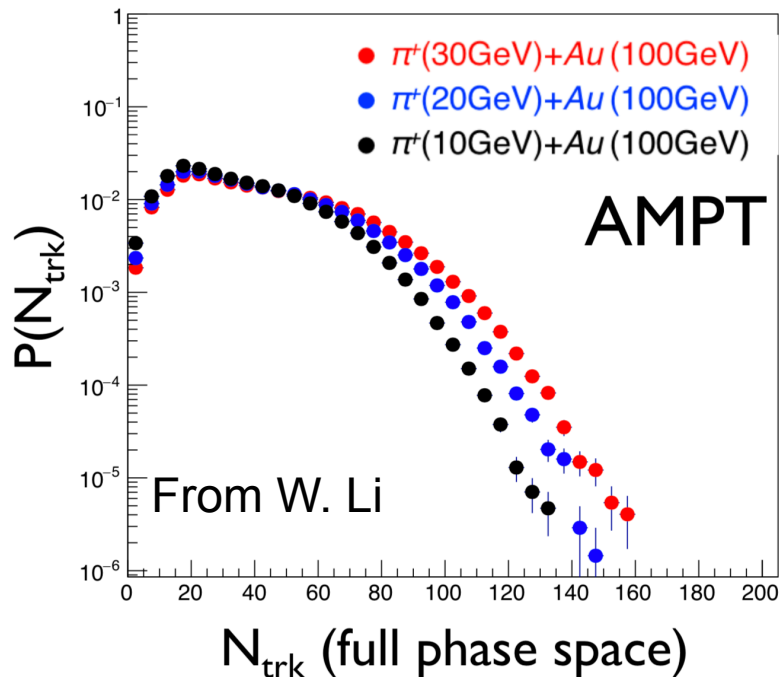
Typical e+A



High-multiplicity e+A = (q $\bar{q}$ )+A?

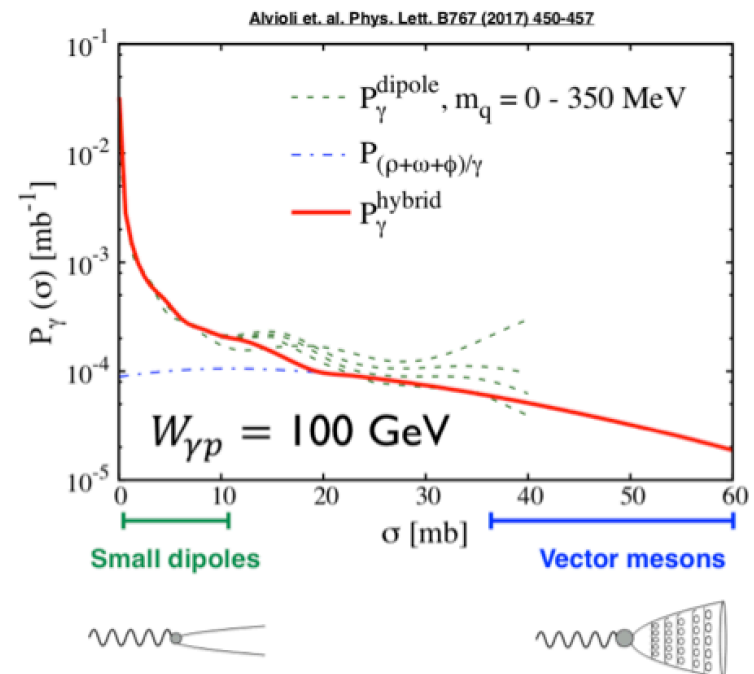


Multiplicity fluct.



Size/cross section fluct.

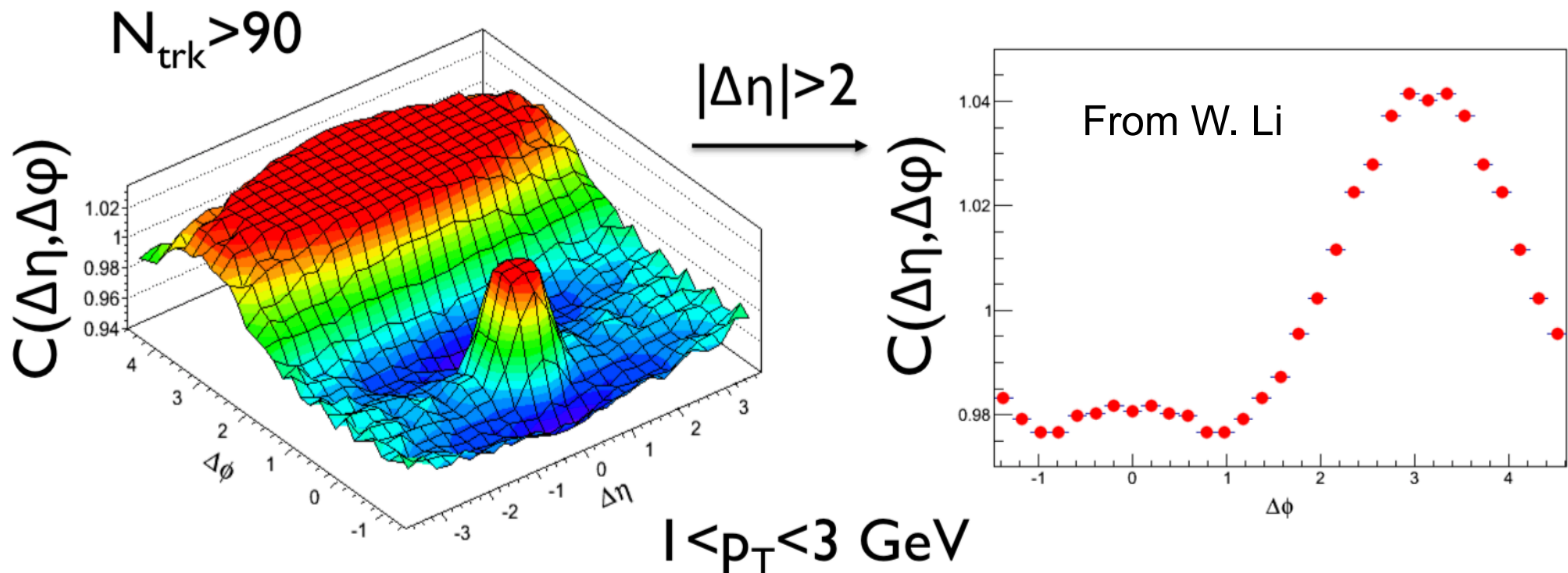
$$\sigma \sim N_{\text{trk}}$$



Control the size and energy of the probe via  $x$  and  $Q^2$

# High-multiplicity e+A at EIC?

$\pi^+$  (30GeV) + **Au** (100GeV) from AMPT

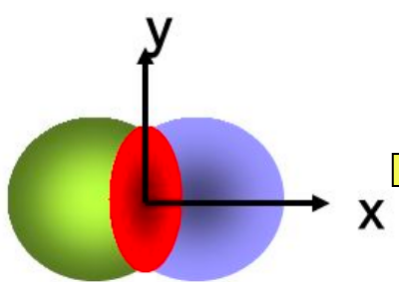


A long-range ridge can be observed at EIC  
in high-multiplicity e+Au events!

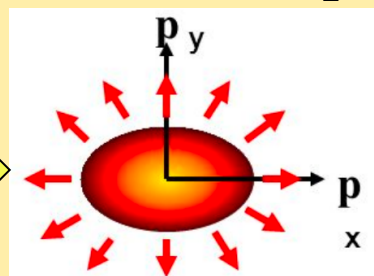
Can already check and confirm this idea in high-multiplicity UPC events

# Flow with polarized light-ion+A collisions

Initial shape ( $\epsilon_2$ )



Final flow ( $v_2$ )



Flow angle  $\Phi$  unknown.

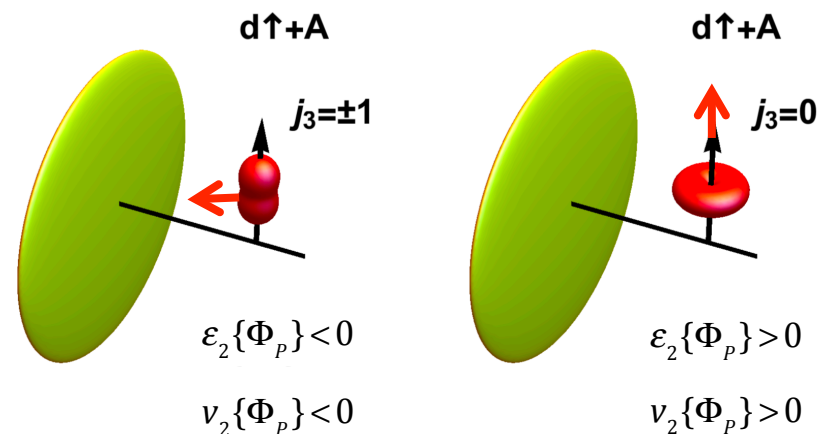
Has to be determined via momentum flow

Can't distinguish initial state flow or final state flow

Problem for small system

W. Broniowski and P. Bozek 1808.09840

Deuterium:  $J^P = 1^+$ , 5%,  $^3D_1$  wave, rest  $^3S_1$  wave



Polarization direction as absolute reference

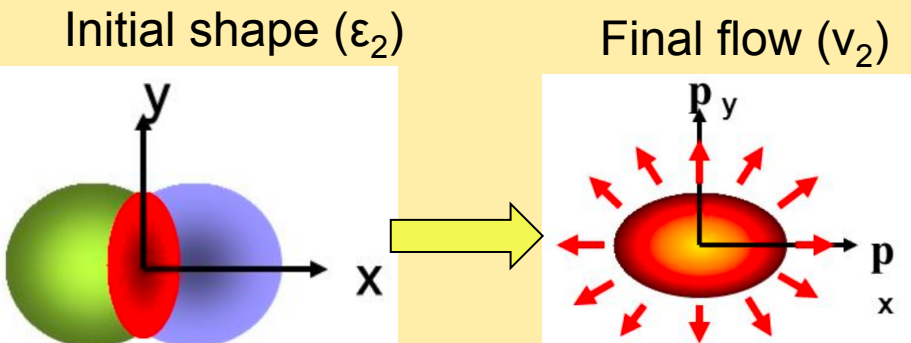
→ uncorrelated with jets

Eccentricity w.r.t.  $\Phi_p$  has opposite sign

# Flow with polarized light-ion+A collisions

W. Broniowski and P. Bozek 1808.09840

Deuterium:  $J^P = 1^+$ , 5%,  $^3D_1$  wave, rest  $^3S_1$  wave

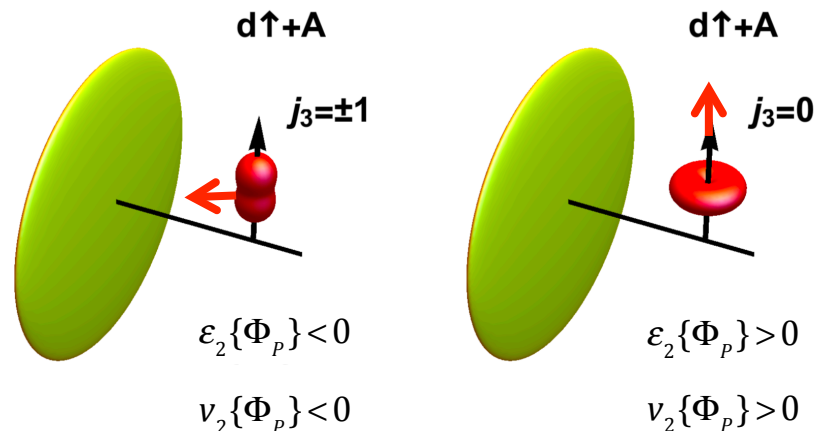


Flow angle  $\Phi$  unknown.

Has to be determined via momentum flow

Can't distinguish initial state flow or final state flow

Problem for small system



Polarization direction as absolute reference  
 $\rightarrow$  uncorrelated with jets  
 Eccentricity w.r.t.  $\Phi_P$  has opposite sign

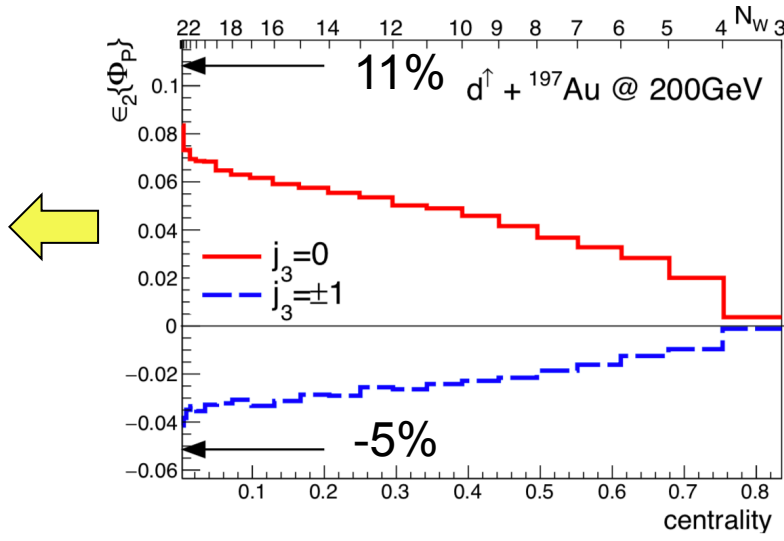
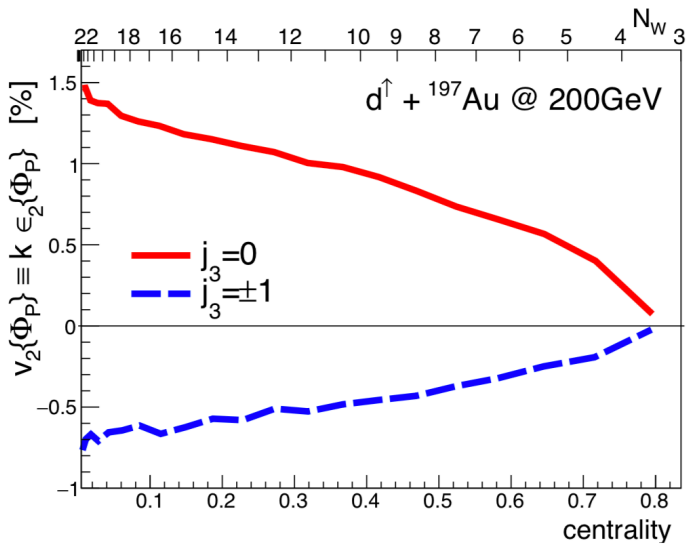


$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos [2(\phi - \Phi_P)]$$

$$v_2 \simeq k\epsilon_2, \quad k \sim 0.2$$

With 50% polarization:

$$-0.5\% \lesssim v_2\{\Phi_P\} \lesssim 1\%$$



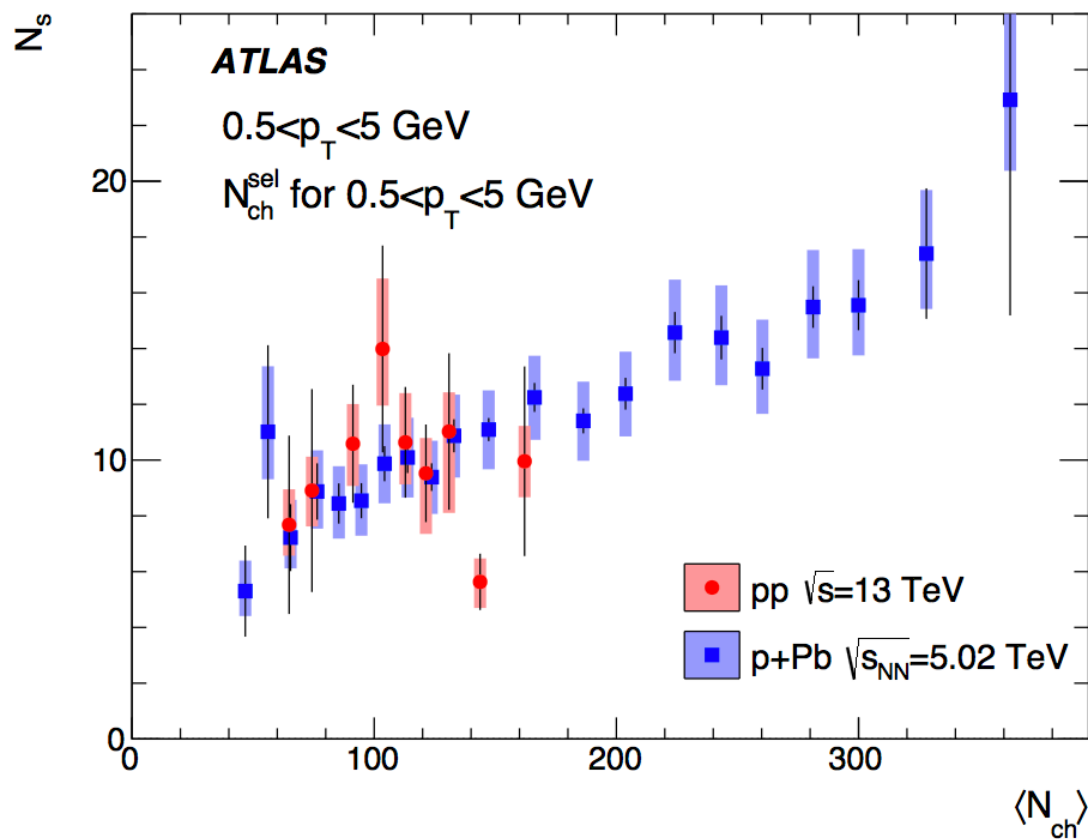
- Ridge observed in small collision systems at RHIC/LHC experiments
  - Implies a long-range and multi-particle collectivity qualitatively similar to those in AA collisions
  - Results of strong multi-parton dynamics in hadronic collisions
- From AA to pA to pp allow push the boundary between initial state correlations and final state interactions.
  - Both models are successful to some extents.
  - Would be useful to check the existing eP and e+e- data, but statistically limited
- A few unique opportunities to further test nature of collectivity.
  - Particle correlations in high-multiplicity e+P and e+A collisions.
  - Polarized light-ion + heavy-nucleus collisions offers a new way to disentangle initial vs final state effects.
- Can final-state physics shed some insight on initial-state physics i.e. EIC

# What do we learn from this?

PRL112,082301(2014)

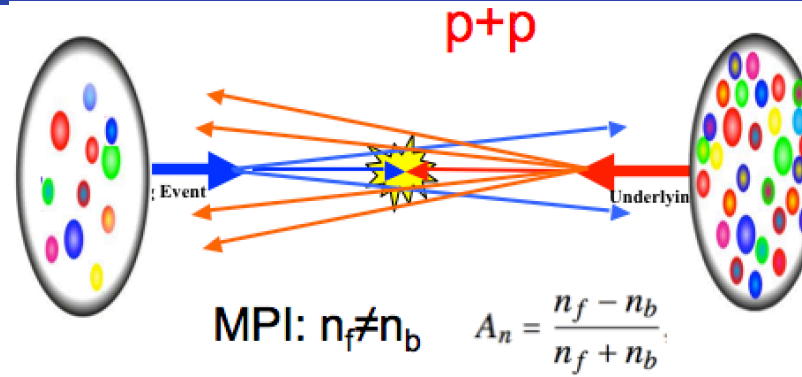
- $p(v_2)$  driven by fluc. of independent sources: multi-parton interactions (MPI)
- Number of sources  $N_s$  can be estimate from  $v_2\{4\}/v_2\{2\}$

$$\frac{v_2\{4\}}{v_2\{2\}} = \left[ \frac{4}{(3 + N_s)} \right]^{1/4}$$

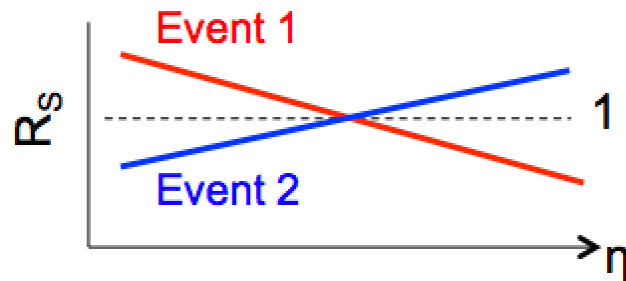


$N_s$  similar for pp and pPb at similar multiplicity.

# $N_s$ from forward-backward multiplicity fluc.



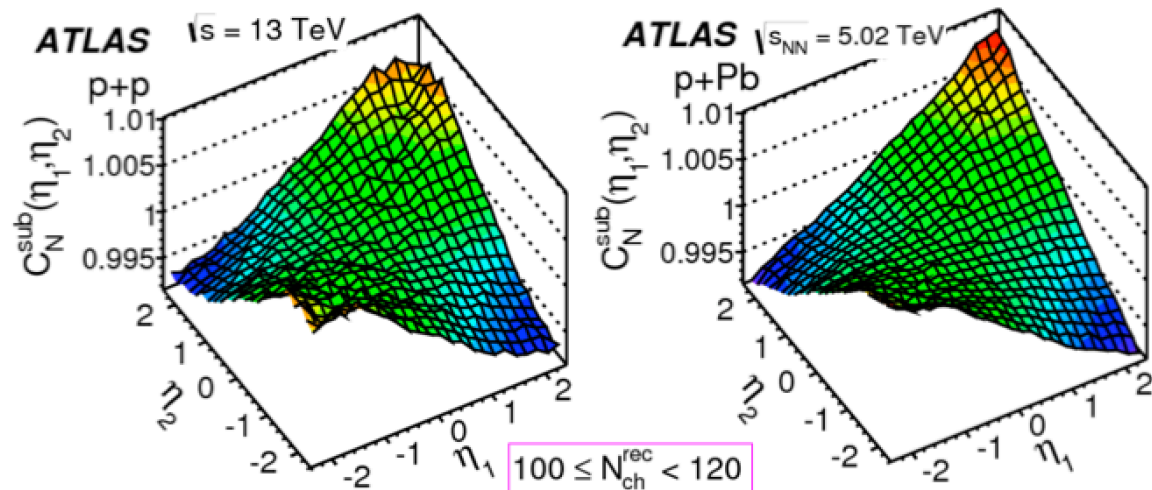
## Rapidity correlation:



$$R_s(\eta) \equiv \frac{N(\eta)}{\langle N(\eta) \rangle} \quad C = \frac{\langle N(\eta_1)N(\eta_2) \rangle}{\langle N(\eta_1) \rangle \langle N(\eta_2) \rangle} = \langle R_s(\eta_1)R_s(\eta_2) \rangle_{events}$$

$$R_s(\eta) \approx 1 + a_1 \eta \quad C = \langle R_s(\eta_1)R_s(\eta_2) \rangle \approx 1 + \langle a_1^2 \rangle \eta_1 \eta_2$$

## Confirmed by data:



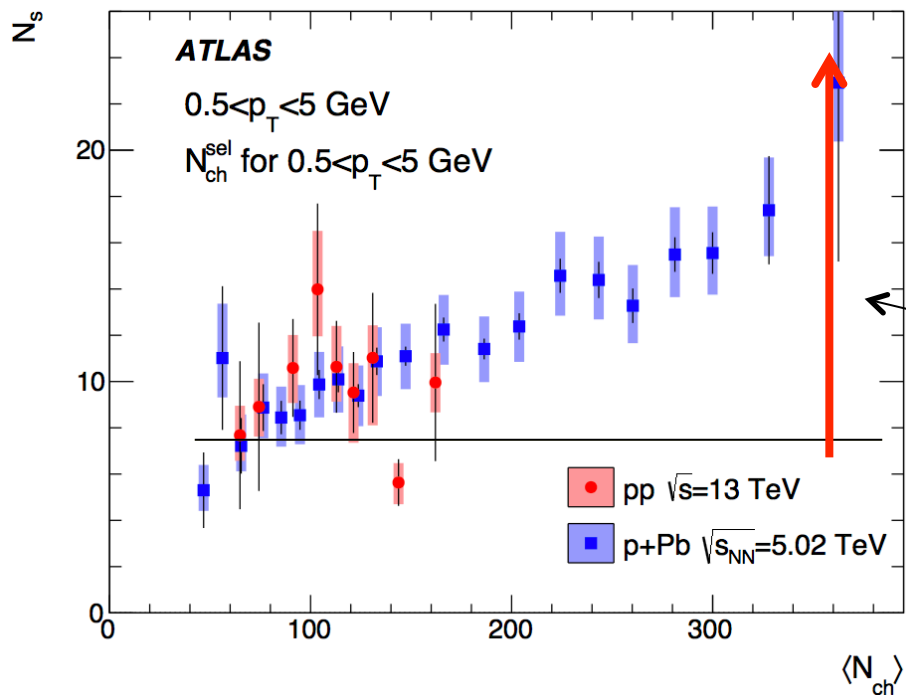


# Relate to the initial geometry

arXiv:1708.03559

Sources driving the transverse flow

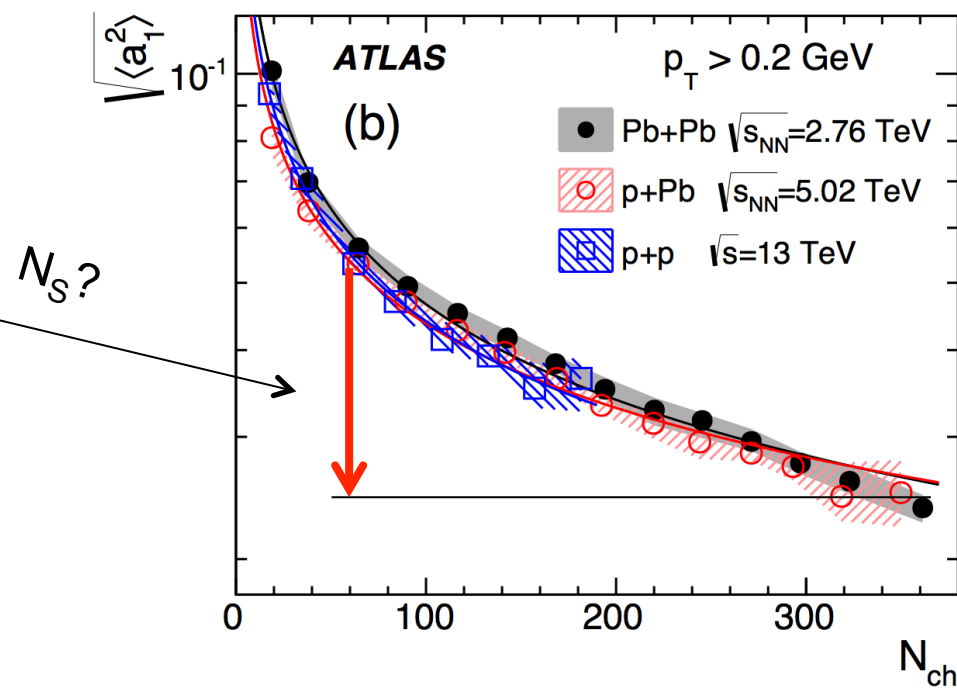
$$\frac{v_2\{4\}}{v_2\{2\}} = \left[ \frac{4}{(3 + N_s)} \right]^{1/4}$$



PRC 95, 064914 (2017)

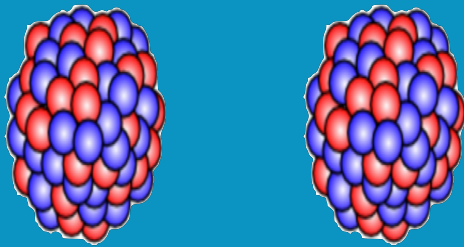
Source for particle production which drives FB multiplicity fluc.

$$\frac{N(\eta)}{\langle N(\eta) \rangle} \approx 1 + a_1 \eta \quad a_1 \propto \frac{1}{\sqrt{N_s}}$$

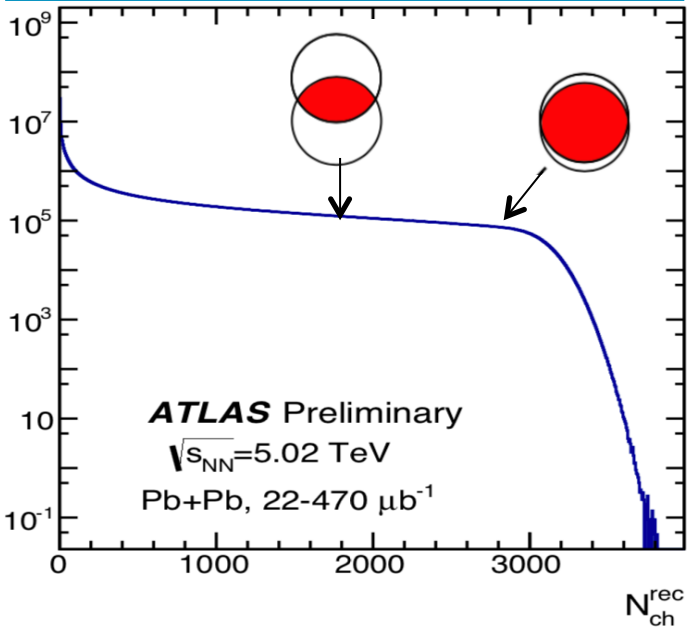


Same sources responsible for particle production and flow?

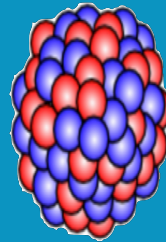
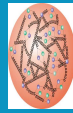
# Large event-by-event multiplicity fluctuations



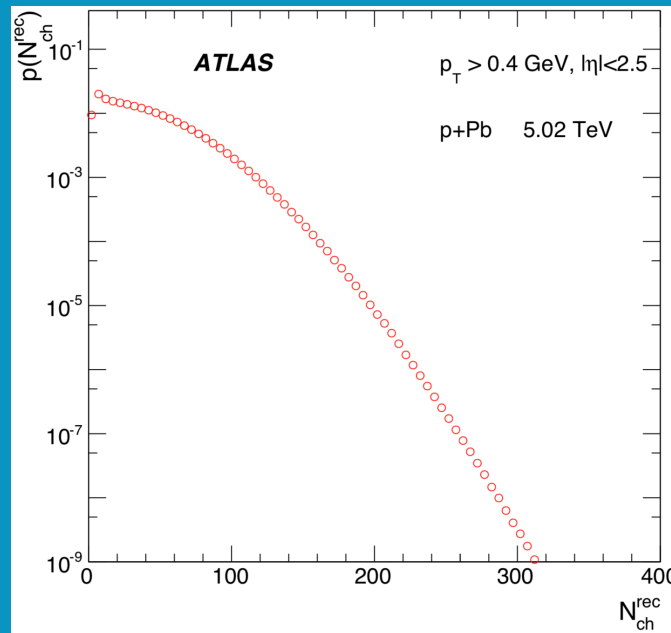
## Pb+Pb



shoulder & knee



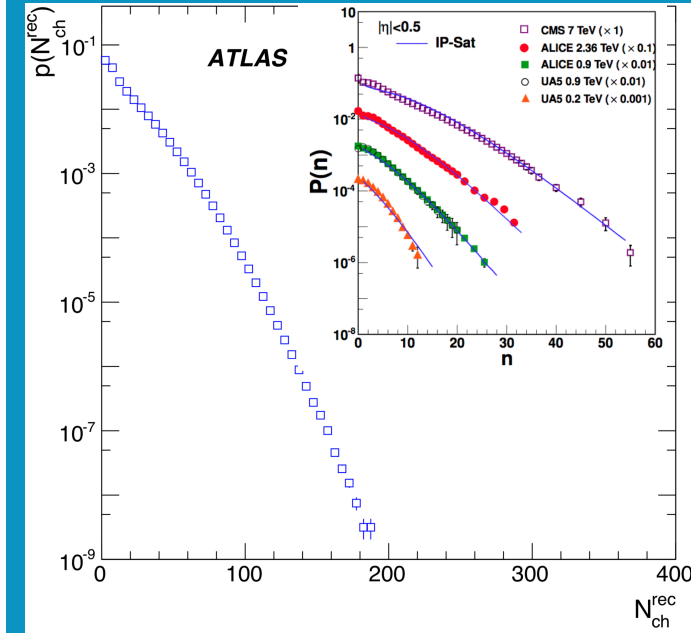
## p+Pb



~absent in pp, pPb



## p+p



signature of multiple sources for particle production