Particle production in dilute-dense collisions beyond leading order

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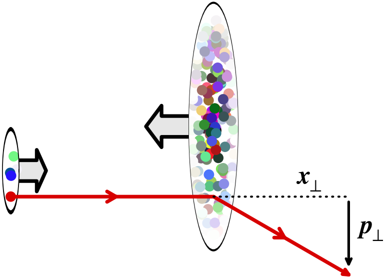
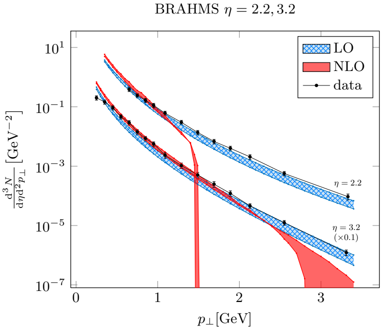
based on arXiv:1608.05293 (JHEP) & arXiv:1712.07480 (PRD)



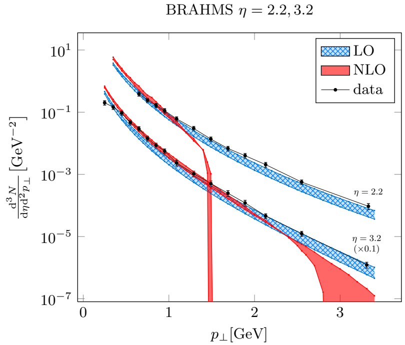


Outline

- Particle production at forward rapidities in pA collisions
  - dipole picture & hybrid factorization
- Tree-level (semi-classical) approximation
  - transverse momentum broadening in the MV model
- The leading-order (LO) approximation
  - BK evolution: dipole amplitude vs. particle production
- The next-to-leading (NLO) order approximation
  - NLO corrections to the impact factor *(Chirilli, Xiao, and Yuan, 2012)*
- A first puzzle (negative cross-section) and its solution *(E.I., A.H. Mueller and D.N. Triantafyllopoulos, arXiv:1608.05293)*
- A second puzzle (running coupling prescription) and its solution *(B. Ducloué, E.I., T. Lappi, A.H. Mueller, G. Soyez, D.N. Triantafyllopoulos and Y. Zhu, arXiv:1712.07480)*

The negative cross-section

- Single inclusive hadron production at forward rapidities
- Good agreement at low $p_\perp$ ☺ ... but negative cross-section at larger $p_\perp$ ☹



- NLO calculation by CXY, 2012
- Numerics by Stasto, Xiao, and Zaslavsky, 2013
- The problem occurs for semi-hard momenta $p_\perp \sim Q_s$
- CGC is expected to apply there

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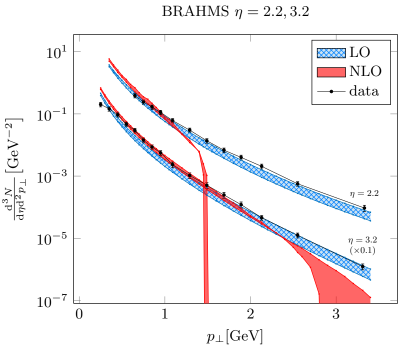
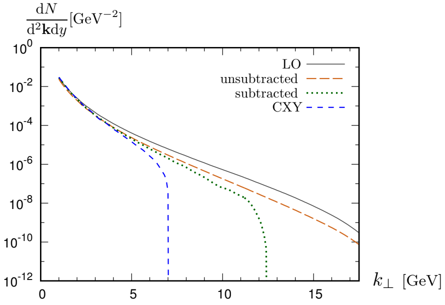




Ducloué, Lappi, and Zhu, arXiv:1703.04962

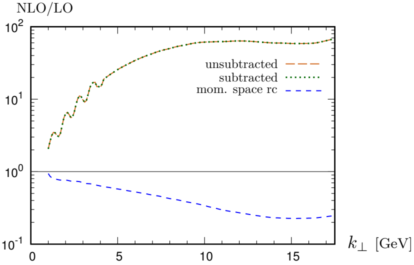
- An artefact of the 'k_T-factorization' commonly used at high energy
- New factorization scheme which avoids this problem

  (E.I., A. Mueller and D. Triantafyllopoulos, 2016)

The running-coupling puzzle

- Intermediate calculations naturally performed in transverse coordinate space (eikonal approximation)
- Commonly used prescriptions for RC in coordinate space lead to funny results

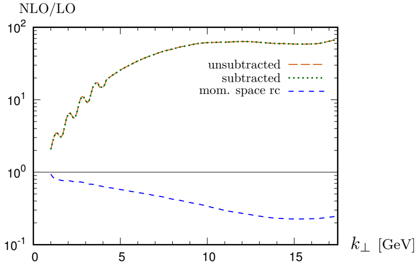
  (Ducloué, Lappi, and Zhu, arXiv:1703.04962)

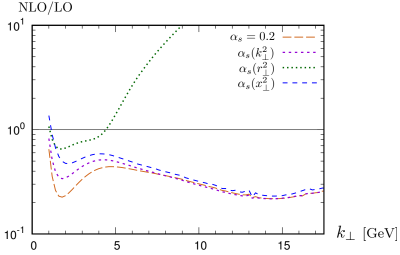


- NLO/LO ratio has the "wrong" sign (compared to the momentum-space prescriptions for RC)
- ... and is as large as ~ 100 !

The running-coupling puzzle

- Intermediate calculations naturally performed in transverse coordinate space (eikonal approximation)
- Commonly used prescriptions for RC in coordinate space lead to funny results *(Ducloué, Lappi, and Zhu, arXiv:1703.04962)*

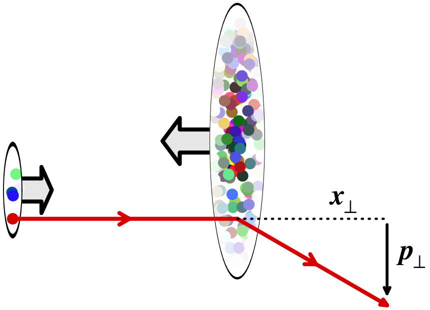


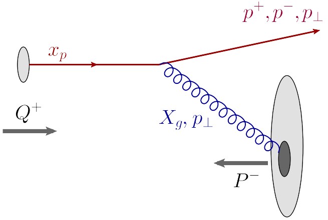


- Lack of commutativity between RC prescription and Fourier transform
- Alternative prescription in coordinate space which avoids this problem *(B. Ducloué et al, arXiv:1712.07480)*

Quark production in pA collisions

- The leading order picture: A quark initially collinear with the proton acquires a transverse momentum $p_\perp$ via scattering off the nuclear target
- Formally, a $2 \to 1$ process: $qg \to q$ (in general: multiple scattering)



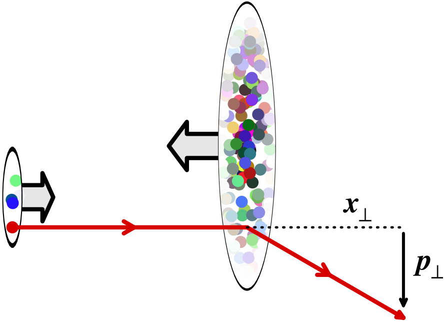


$$2p^-p^+ = p_\perp^2 \;\Rightarrow\; x_p X_g = \frac{p_\perp^2}{s}$$

- After the collision, the quark emerges with
  - a longitudinal momentum $p^+ = x_p Q^+$ from the parent proton
  - a light-cone energy $p^- = X_g P^-$ from a nucleon in the nucleus

Forward rapidities

- It is customary to use the proton-nucleon COM frame: $Q^+ = P^- = \sqrt{s/2}$
  - η : the (pseudo)-rapidity of the final quark in the COM frame



$$\eta = -\ln \tan \frac{\theta}{2}$$

$$x_p = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{\eta}$$

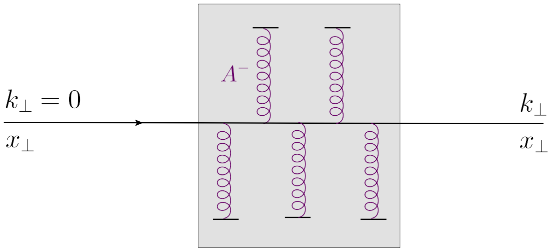
$$X_g = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{-\eta}$$

$X_g \ll x_p$ when $\eta > 0$

- Saturated gluons have large occupation numbers $\sim 1/\alpha_s$ and a typical transverse momentum $k_\perp \sim Q_s(X_g)$
- When $p_\perp \lesssim Q_s(X_g)$, multiple scattering becomes important

Eikonal approximation

- Multiple scattering can be resummed in the eikonal approximation
  - fixed transverse coordinate & color precession

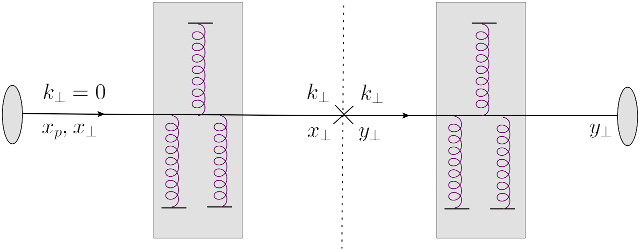


Amplitude: $$\mathcal{M}_{ij}(\boldsymbol{k}_\perp) \equiv \int \mathrm{d}^2\boldsymbol{x}_\perp \, \mathrm{e}^{-i\boldsymbol{x}_\perp\cdot\boldsymbol{k}_\perp} V_{ij}(\boldsymbol{x}_\perp)$$

Wilson line: $$V(\boldsymbol{x}_\perp) = \mathrm{P}\exp\left\{ig\int \mathrm{d}x^+ A_a^-(x^+,\boldsymbol{x}_\perp)t^a\right\}$$

- A_a^- : color field representing small-x gluons in the nucleus

Multiple scattering



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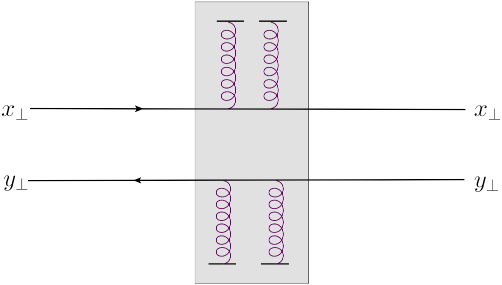
Cross-section: $$\frac{\mathrm{d}\sigma}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}_\perp} \simeq x_p q(x_p, Q^2) \, \frac{1}{N_c} \left\langle \sum_{ij} |\mathcal{M}_{ij}(\boldsymbol{k}_\perp)|^2 \right\rangle_{X_g}$$

- Average over the color fields A^- in the target (CGC)
- Two Wilson lines at different transverse coordinates, traced over color

Dipole picture

- Equivalently: the elastic S-matrix for a $q\bar{q}$ color dipole



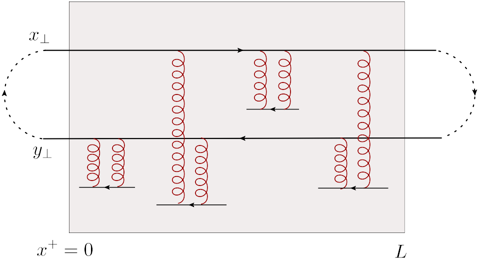
$$S(\boldsymbol{x},\boldsymbol{y};X_g) \equiv \frac{1}{N_c}\left\langle \mathrm{tr}\left[V(\boldsymbol{x})V^\dagger(\boldsymbol{y})\right]\right\rangle_{X_g}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} \simeq x_p q(x_p) \int_{\boldsymbol{x},\boldsymbol{y}} \mathrm{e}^{-\mathrm{i}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{k}}\, S(\boldsymbol{x},\boldsymbol{y};X_g)$$

- F. transform $\mathcal{S}(\boldsymbol{k},X_g)$: "unintegrated gluon distribution", or "dipole TMD"

Dipole scattering in the MV model

- A large nucleus $(A \gg 1)$ & not too small values of x $(\bar{\alpha}_s \ln 1/x \ll 1)$
- No quantum evolution: the only color sources are the $A \times N_c$ valence quarks
- Independent scatterings $\Longrightarrow$ the multiple scattering series exponentiates



$$S(r) = \mathrm{e}^{-T_0(r)}$$

$$S(r) = \exp\left\{-\frac{r^2 Q_{0A}^2}{4} \ln \frac{1}{r^2\Lambda^2}\right\}$$

$$Q_{0A}^2 \equiv \frac{2\alpha_s^2 C_F A^{1/3}}{R^2}$$

- Q_{0A}^2: color charge squared of the valence quarks per unit area
- $\ln(1/r^2\Lambda^2)$: gluon exchanges within the range $r < \Delta x_\perp < 1/\Lambda$

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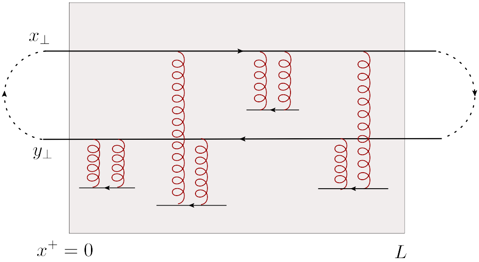

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- Saturation momentum Q_s: conventionally defined as $T_0(r) = 1$ for $\frac{2}{r} = Q_s$

$$Q_s^2(A) \,=\, Q_{0A}^2 \ln \frac{Q_s^2(A)}{4\Lambda^2} \,\propto\, A^{1/3} \ln A^{1/3}$$

Momentum broadening in the MV model

- At LO: forward quark production $\equiv$ transverse momentum broadening

$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{k}} = \mathcal{S}(\boldsymbol{k}) = \int \frac{\mathrm{d}^2\boldsymbol{r}}{(2\pi)^2}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\;\mathrm{e}^{-\frac{1}{4}r^2Q_0^2\ln\frac{1}{r^2\Lambda^2}}$$

- Would-be a Gaussian integration ... if there were not for the logarithm
- Two interesting situations which allow for simple results:
- Multiple scattering leading to a typical value $k_\perp \sim Q_s$
  - integral cut off at $r \sim 1/Q_s$ by the S-matrix $S(r)$
  - replace $1/r^2 \to Q_s^2$ within the argument of the log

$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{k}} \simeq \frac{1}{\pi Q_s^2}\,\mathrm{e}^{-k_\perp^2/Q_s^2}$$

  - a Gaussian distribution: random walk in $\boldsymbol{k}$

$$\langle k_\perp^2\rangle \equiv \int \mathrm{d}^2\boldsymbol{k}\; k_\perp^2\,\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{k}} = Q_s^2(A) \propto A^{1/3}$$

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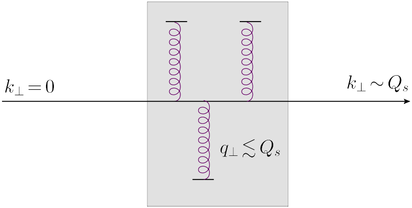
- Would-be a Gaussian integration ... if there were not for the logarithm
- Two interesting situations which allow for simple results:
- A single hard scattering yielding a much larger $k_\perp \gg Q_s$
  - integral cut off at $r \sim 1/k_\perp$ by the exponential
  - $rQ_s \ll 1 \implies$ one can expand $S \simeq 1 - T_0$ (one scattering)

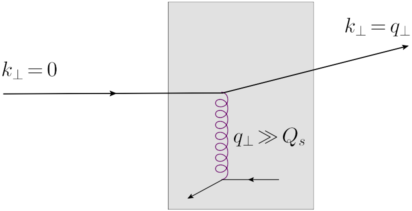
$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{k}} \simeq \frac{Q_{0A}^2}{\pi k_\perp^4}$$

  - approximate version of the collinear factorization: $qq \to qq$

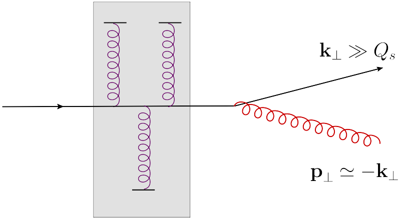
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}} \simeq x_p q(x_p)\,\frac{\alpha_s^2 C_F^2}{2k_\perp^4}\, x_A q_A(x_A)\,, \qquad x_A q_A(x_A) \equiv A N_c$$

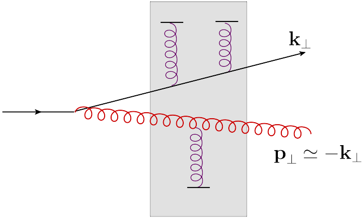
- Multiple soft scattering $\Longrightarrow$ $k_\perp \sim Q_s$





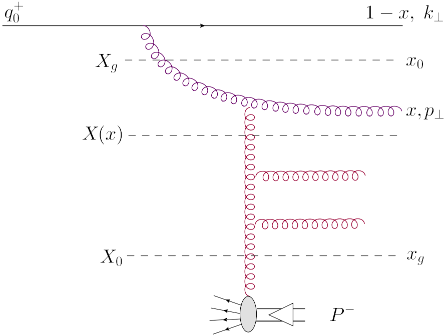
- Single hard scattering (like in the Rutherford experiment) $\Longrightarrow$ $k_\perp \gg Q_s$
- Emission of a hard recoil gluon: formally NLO but competitive at high $k_\perp$





Recoil gluon vs. evolution gluon

- The recoil gluon overlaps in phase-space with the first gluon in the high-energy evolution
  - the recoil gluon can take any longitudinal momentum fraction $x \leq 1$
  - the integration over $x \ll 1$ generates the rapidity log $\bar{\alpha}_s \ln(1/X_g)$



- LC energy conservation:

$$\frac{k_\perp^2}{2(1-x)q_0^+} + \frac{p_\perp^2}{2xq_0^+} = XP^-$$

- Simplifies when $x \ll 1$ & $k_\perp \sim p_\perp$

$$X(x) \simeq \frac{k_\perp^2}{xs} = \frac{X_g}{x}$$

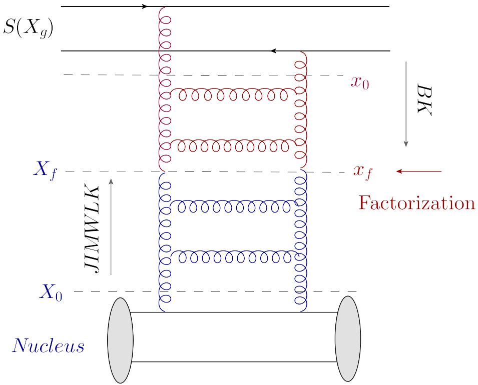
- $X \leq 1 \Longrightarrow x \geq X_g$

- The high-energy evolution must already be included in LO: $\bar{\alpha}_s \ln(1/X_g) \gtrsim 1$

LO Hybrid Factorization

(Dumitru, Hayashigaki, and Jalilian-Marian, arXiv:hep-ph/0506308).

$$\frac{\mathrm{d}\sigma_h}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{p}} = \int \frac{\mathrm{d}z}{z^2}\, x_p q(x_p, \mu^2) \left[\int_{\boldsymbol{x},\boldsymbol{y}} \mathrm{e}^{-\mathrm{i}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{k}}\, S(\boldsymbol{x},\boldsymbol{y};X_g)\right] D_{h/q}(z,\mu^2)$$



- LO evolution can be shared between dipole (BK) and nucleus (JIMWLK)

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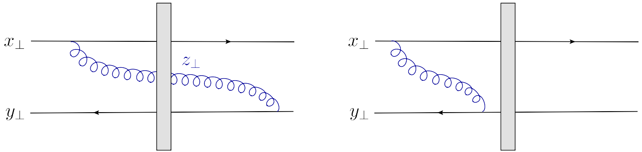
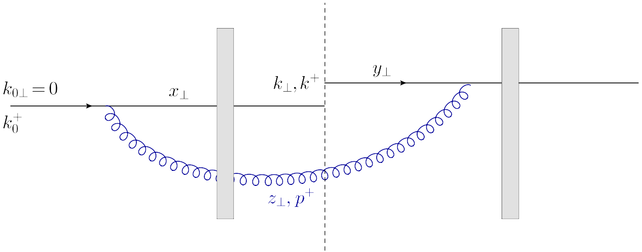
- Collinear factorization for the incoming proton/outgoing hadron
  - LO DGLAP evolution for quark distribution/ fragmentation
- High-energy (CGC) factorization for the quark-nucleus scattering
  - LO JIMWLK (BK) for target gluon distribution (dipole S-matrix)
- Natural, but non-trivial already at leading order
  - one needs to prove the factorization of the two types of evolution
- The dipole picture is preserved by the high-energy evolution up to NLO *(Kovchegov and Tuchin, 2002; Mueller and Munier, 2012)*

Dipole evolution (1): Real corrections

- One soft gluon ($x \ll 1$) & eikonal approx. for gluon scattering (fixed z)



- Exchange graphs for the dipole $\Longleftrightarrow$ "Real" corrections to particle production
  - dipole: gluon exchanged between the quark and the antiquark



  - production: gluon appears in the final state (it crosses the cut)

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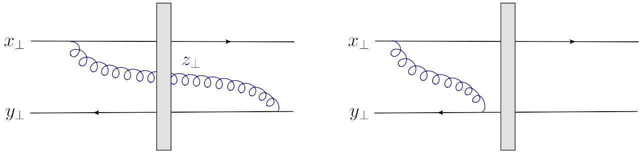
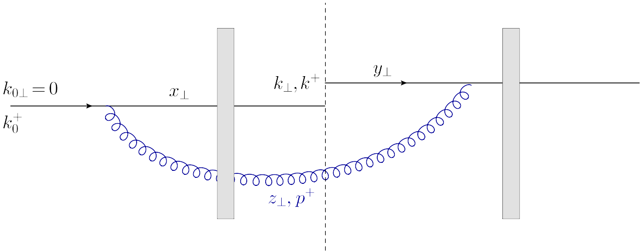


- Gluon scattering matters ("N_c-terms"), or not ("C_F-terms")
    - N_c-terms for dipole: gluon crosses the shockwave



    - N_c-terms for production: initial state times final state emissions

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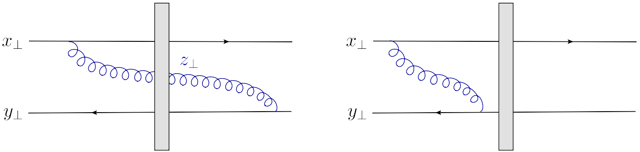

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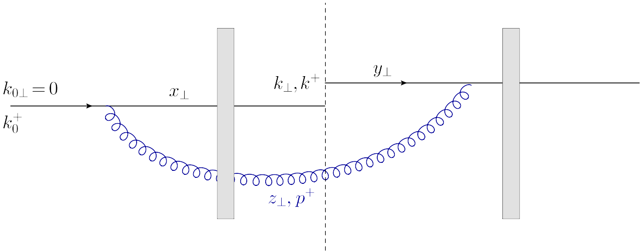

  - gluon Wilson line $\tilde{V}(\boldsymbol{z})$ in the DA ... but not also in the CCA

Dipole evolution (1): Real corrections

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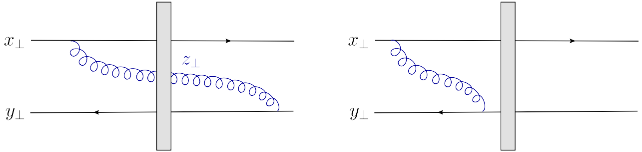
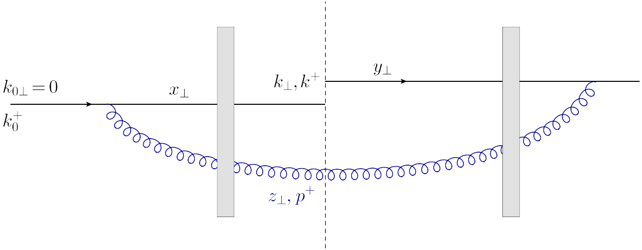


- Gluon scattering matters ("N_c-terms"), or not ("C_F-terms")
  - C_F-terms for dipole: gluon does not cross the shockwave



  - C_F-terms for production: only initial (or only final) state emissions

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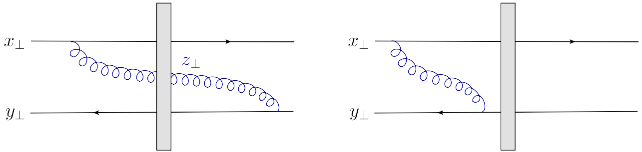

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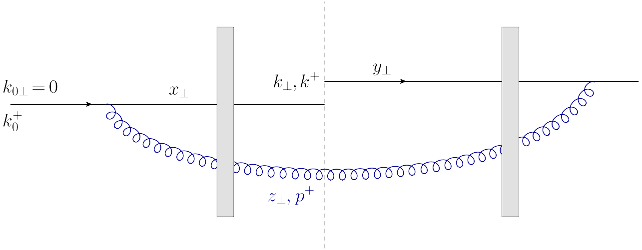
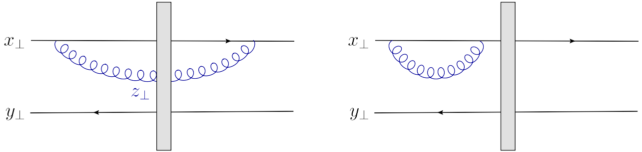


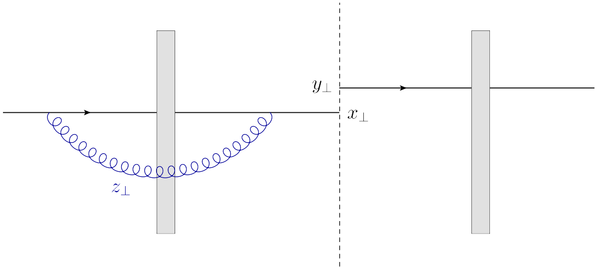
  - the gluon scattering is not measured: $\tilde{V}(\boldsymbol{z})\tilde{V}^\dagger(\boldsymbol{z}) = 1$

Dipole evolution (2): Virtual corrections

- Self-energy corrections to dipole evolution:
  - the gluon crosses the shockwave ("N_c-terms"), or not ("C_F-terms")



- "Virtual" corrections to quark production: the gluon does not cross the cut



  - N_c-terms for production: gluon Wilson line $\tilde{V}(\boldsymbol{z})$

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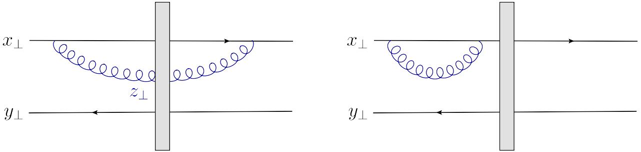

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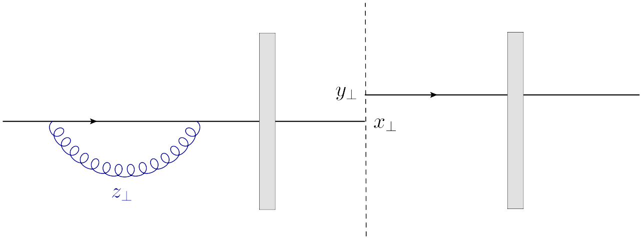
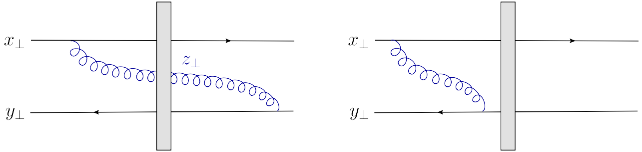


  - C_F-terms for production: no gluon Wilson line

- The exchange ("real") graphs in more detail: $Y = \ln(1/x)$ & $\alpha_s \mathrm{d}Y \ll 1$



$$\mathrm{d}_1 S_Y(\boldsymbol{x},\boldsymbol{y}) = -\frac{\alpha_s}{\pi^2}\,\mathrm{d}Y \int \mathrm{d}^2\boldsymbol{z}\, \frac{(\boldsymbol{x}-\boldsymbol{z})^i}{(\boldsymbol{x}-\boldsymbol{z})^2}\,\frac{(\boldsymbol{y}-\boldsymbol{z})^i}{(\boldsymbol{y}-\boldsymbol{z})^2}$$

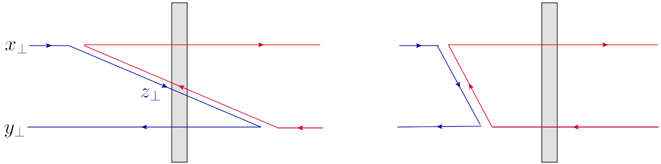
$$\left\langle \tilde{V}_{ab}(\boldsymbol{z})\,\frac{1}{N_c}\,\mathrm{tr}\Big(V(\boldsymbol{x})t^b\,V^\dagger(\boldsymbol{y})t^a\Big) - \frac{C_F}{N_c}\,\mathrm{tr}\big(V_{\boldsymbol{x}}V_{\boldsymbol{y}}^\dagger\big)\right\rangle_Y$$

- N_c-term: the gluon emitted at $\boldsymbol{x}$ hits the shockwave a $\boldsymbol{z}$

$$g\,\frac{(\boldsymbol{x}-\boldsymbol{z})^i}{(\boldsymbol{x}-\boldsymbol{z})^2}\ :\quad \text{amplitude for gluon emission and propagation from } \boldsymbol{x} \text{ to } \boldsymbol{z}$$

- Not a closed equation: evolution couples $S_{q\bar{q}}(\boldsymbol{x},\boldsymbol{y})$ to $S_{q\bar{q}g}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$

- A closed evolution equation can be obtained in the multi-color limit $N_c \to \infty$
  - at large N_c, a gluon can be replaced by a quark-antiquark pair
  - gluon emission by a dipole $\approx$ dipole splitting into 2 dipoles



$$\mathrm{d}_1 S_Y(\boldsymbol{x},\boldsymbol{y}) \simeq -\frac{\alpha_s N_c}{2\pi^2}\mathrm{d}Y \int_{\boldsymbol{z}} \frac{(\boldsymbol{x}-\boldsymbol{z})^i}{(\boldsymbol{x}-\boldsymbol{z})^2}\,\frac{(\boldsymbol{y}-\boldsymbol{z})^i}{(\boldsymbol{y}-\boldsymbol{z})^2}\Big\{S_Y(\boldsymbol{x},\boldsymbol{z})\,S_Y(\boldsymbol{z},\boldsymbol{y}) - S_Y(\boldsymbol{x},\boldsymbol{y})\Big\}$$

- At large N_c, expectation values of colorless operators factorize

$$\left\langle \frac{\mathrm{tr}\big(V_{\boldsymbol{x}}V_{\boldsymbol{z}}^\dagger\big)}{N_c}\,\frac{\mathrm{tr}\big(V_{\boldsymbol{z}}V_{\boldsymbol{y}}^\dagger\big)}{N_c}\right\rangle_Y \simeq S_Y(\boldsymbol{x},\boldsymbol{z})\,S_Y(\boldsymbol{z},\boldsymbol{y})$$

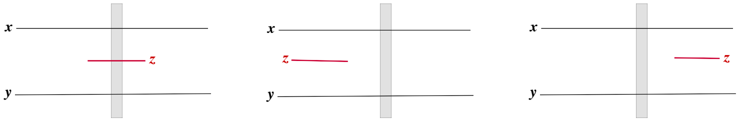
- Similar manipulations for the self-energy (“virtual”) graphs

The BK equation *(Balitsky, '96; Kovchegov, '99)*

- Evolution equation for the dipole S–matrix $S_{\boldsymbol{xy}}(Y)$ with $Y \equiv \ln(1/x)$

$$\frac{\partial S_{\boldsymbol{xy}}}{\partial Y} = \frac{\alpha_s N_c}{2\pi^2} \int \mathrm{d}^2\boldsymbol{z}\, \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \Big[\underbrace{S_{\boldsymbol{xz}}S_{\boldsymbol{zy}}}_{N_c\text{-term}} - \underbrace{S_{\boldsymbol{xy}}}_{C_F\text{-term}} \Big]$$

- Large N_c : the original dipole splits into two new dipoles



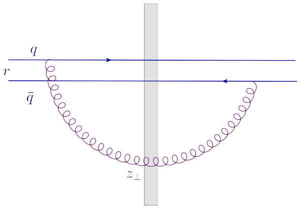
- Dipole kernel: BFKL kernel in the dipole picture *(Al Mueller, 1990)*

$$\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} = \Big[\underbrace{2\frac{(x^i-z^i)(y^i-z^i)}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{x}-\boldsymbol{z})^2}}_{\text{real}} \underbrace{-\frac{1}{(\boldsymbol{x}-\boldsymbol{z})^2} - \frac{1}{(\boldsymbol{z}-\boldsymbol{y})^2}}_{\text{virtual}} \Big]$$

- Finite N_c: Balitsky-JIMWLK hierarchy (n-point functions of Wilson lines)

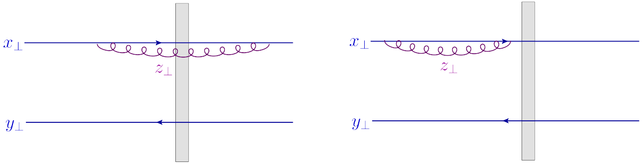
Good IR & UV behaviors

- Large-distance ($z_\perp \to \infty$) emissions cancel between "real" and "virtual":



$$\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \simeq \frac{r^2}{(\boldsymbol{z}-\boldsymbol{x})^4} \quad \text{when } |\boldsymbol{z}-\boldsymbol{x}| \simeq |\boldsymbol{z}-\boldsymbol{y}| \gg r$$

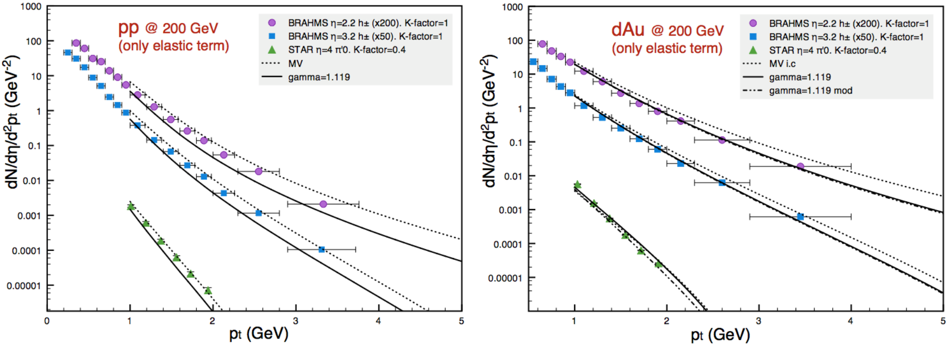
- Short-distance poles ($|\boldsymbol{z}-\boldsymbol{x}| \to 0$) cancel between N_c-terms and C_F-terms



LO phenomenology: rcBK

(Albacete, Dumitru, Fujii, Nara, arXiv:1209:2001)

- Fit parameters: initial condition for the rcBK equation + K-factors

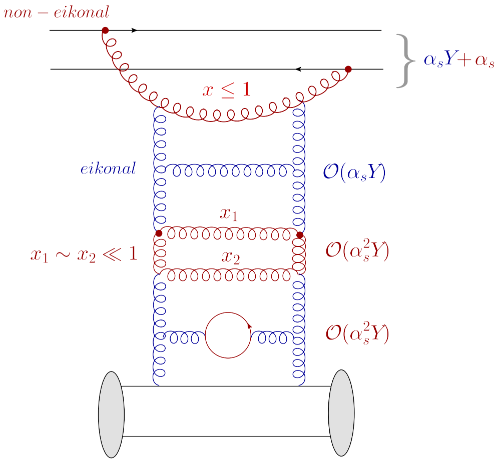


$$\left.\frac{\mathrm{d}N_h}{\mathrm{d}\eta\,\mathrm{d}^2\boldsymbol{k}}\right|_{\rm LO} = K_h \int_{x_p}^{1} \frac{\mathrm{d}z}{z^2}\, \frac{x_p}{z} q\left(\frac{x_p}{z}\right) \mathcal{S}\left(\frac{\boldsymbol{k}}{z}, X_g\right) D_{h/q}(z)$$

- What about the (other) NLO corrections ?

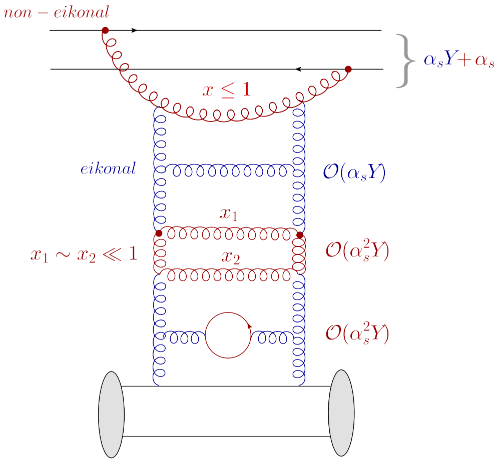
NLO corrections to particle production

- $\mathcal{O}(\alpha_s)$ corrections to the evolution *(cf. the talk by Dionysis T.)*
  - a pair of soft partons which are close in rapidity: $x_1 \sim x_2 \ll 1$
  - a contribution of $\mathcal{O}(\alpha_s^2 Y) \Rightarrow \mathcal{O}(\alpha_s)$ correction to the kernel



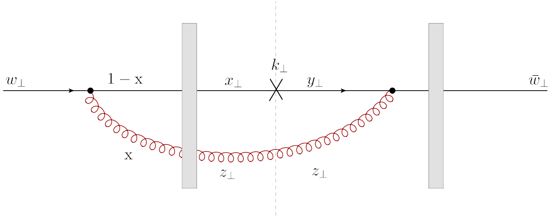
NLO corrections to particle production

- $\mathcal{O}(\alpha_s)$ corrections to the impact factor *(Chirilli, Xiao, and Yuan, 2012)*
  - the first emitted gluon is close in rapidity to the dipole: $x \sim \mathcal{O}(1)$
  - its emission must be computed with exact kinematics (beyond eikonal)



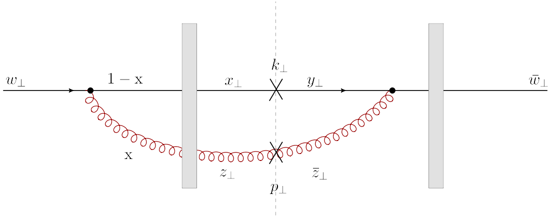
Some contributing graphs

- The same graphs as for one-step evolution, but with exact kinematics



  - $\boldsymbol{w} = x\boldsymbol{z} + (1-x)\boldsymbol{x}, \quad \bar{\boldsymbol{w}} = x\boldsymbol{z} + (1-x)\boldsymbol{y}$

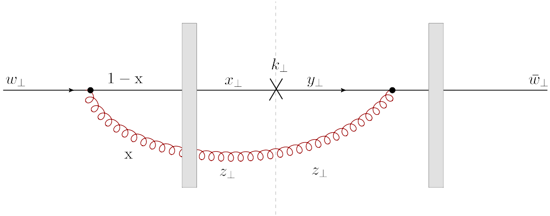
- Cyrille M. has computed quark-gluon production back in 2007



  - "integrate out the gluon" $\int \mathrm{d}^2\boldsymbol{p}_\perp \implies \boldsymbol{z} = \bar{\boldsymbol{z}}$

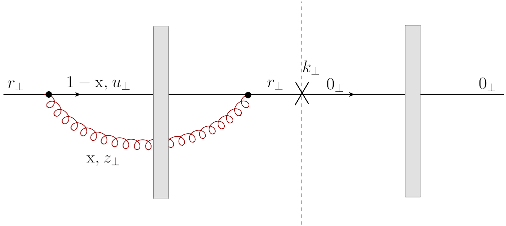
Some contributing graphs

- The same graphs as for one-step evolution, but with exact kinematics



  - $\boldsymbol{w} = x\boldsymbol{z} + (1-x)\boldsymbol{x}, \quad \bar{\boldsymbol{w}} = x\boldsymbol{z} + (1-x)\boldsymbol{y}$

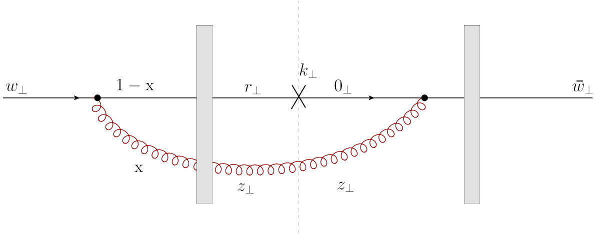
- The corresponding "virtual" graphs have been computed by CXY (2012)



- N.B. Scattering off the target is still eikonal $\Longrightarrow$ Wilson lines

A "real" N_c-term as an exemple

- Once again, expressions are most transparent in transverse coordinate space



$$\bar{\alpha}_s \int \mathrm{d}^2\boldsymbol{r}\, \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \int_{X_g}^1 \mathrm{d}x\, \frac{1+(1-x)^2}{2x} \int \mathrm{d}^2\boldsymbol{z}\, \frac{(\boldsymbol{z}-\boldsymbol{r})^i \boldsymbol{z}^i}{(\boldsymbol{z}-\boldsymbol{r})^2 \boldsymbol{z}^2}\; S(\boldsymbol{r}-\boldsymbol{z})S(\boldsymbol{z}-\bar{\boldsymbol{w}})$$

- Integral over $X_g < x < 1$ with exact DGLAP splitting function $P_{gq}(x)$
- Dipole S-matrices evaluated at target energy fraction $X(x) \simeq X_g/x$
- Same (Weiszäcker-Williams) emission kernel as in the eikonal approximation
- The gluon energy fraction x enters the coordinates of the parent quark: $\boldsymbol{w} = x\boldsymbol{z} + (1-x)\boldsymbol{r}$, $\bar{\boldsymbol{w}} = x\boldsymbol{z}$

N_c-terms: "real" + "virtual"

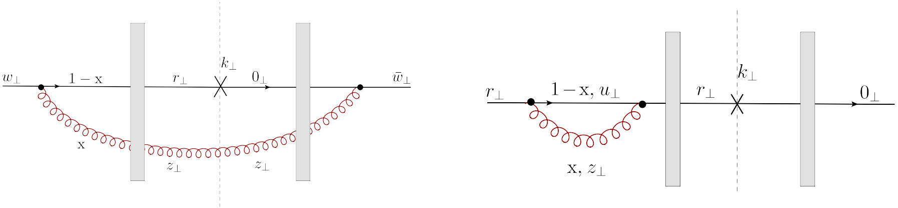
$$\bar{\alpha}_s \int \mathrm{d}^2\boldsymbol{r}\, \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \int_{X_g}^{1} \mathrm{d}x\, \frac{1+(1-x)^2}{2x} \int \mathrm{d}^2\boldsymbol{z}$$

$$\left\{ \frac{x_p}{1-x} q\left(\frac{x_p}{1-x}\right) \frac{(\boldsymbol{z}-\boldsymbol{r})\cdot\boldsymbol{z}}{(\boldsymbol{z}-\boldsymbol{r})^2\boldsymbol{z}^2}\, S(\boldsymbol{r}-\boldsymbol{z})S(\boldsymbol{z}-\bar{\boldsymbol{w}}) - \frac{x_p q(x_p)}{(\boldsymbol{z}-\boldsymbol{r})^2}\, S(\boldsymbol{u}-\boldsymbol{z})S(\boldsymbol{z}) \right\}$$

- For generic $x < 1$, the "real" and "virtual" terms ...
  - have different coordinate arguments for the S-matrices
  - and are differently weighted by the quark distribution
  - in the "real" terms, the produced quark takes an energy fraction $1 - x$
- For generic x, cancellations between "real" and "virtual" don't work anymore
- Complications when using a coordinate-space running coupling *(see below)*
- For small $x \ll 1$, one recovers one step in the BK evolution, as expected
  - how to avoid double counting of the small-x contribution ?

The C_F-terms: emergence of DGLAP

- The graphs in which the gluon is (effectively) non-interacting



$$-\frac{\alpha_s C_F}{2\pi}\int_{\boldsymbol{r}} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \int_{\boldsymbol{z}} \int_{X_g}^{1} \mathrm{d}x\, \frac{1+(1-x)^2}{2x}$$

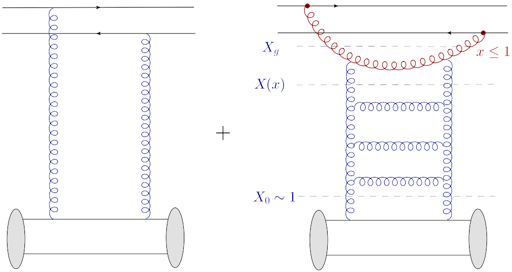
$$\left\{\frac{x_p}{1-x} q\left(\frac{x_p}{1-x}\right) \frac{(\boldsymbol{z}-\boldsymbol{r})\cdot\boldsymbol{z}}{(\boldsymbol{z}-\boldsymbol{r})^2\boldsymbol{z}^2}\, S(\boldsymbol{r}) - \frac{x_p q(x_p)}{(\boldsymbol{z}-\boldsymbol{r})^2}\, S(\boldsymbol{r})\right\}$$

- Large $z \gg r$ & generic $x < 1$: logarithmically IR-divergent integral over $\boldsymbol{z}$, recognized as one step in DGLAP (with $\xi \equiv 1 - x$)

$$-\frac{\alpha_s C_F}{2\pi}\mathcal{S}(\boldsymbol{k})\int_{x_p}^{1} \mathrm{d}\xi \left(\frac{1+\xi^2}{1-\xi}\right)_+ \frac{x_p}{\xi} q\left(\frac{x_p}{\xi}\right)\int_r \frac{\mathrm{d}\boldsymbol{z}^2}{\boldsymbol{z}^2} \equiv x_p \Delta q(x_p, 1/r^2)\mathcal{S}(\boldsymbol{k})$$

NLO cross-section w/o overcounting

(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)



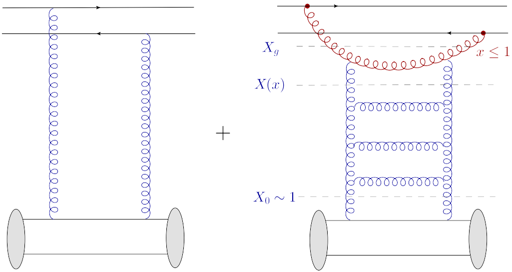
- Simply add the tree-level piece to the one-loop contribution with evolution

$$\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} = x_p q(x_p)\mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s \int_{\boldsymbol{r}} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \int_{X_g}^{1} \mathrm{d}x\, P_{gq}(x)$$

$$\times \int_{\boldsymbol{z}} \left\{ \frac{x_p}{1-x} q\Big(\frac{x_p}{1-x}\Big) \frac{(\boldsymbol{z}-\boldsymbol{r})\cdot\boldsymbol{z}}{(\boldsymbol{z}-\boldsymbol{r})^2\boldsymbol{z}^2}\, S(\boldsymbol{r}-\boldsymbol{z})S(\boldsymbol{z}-\bar{\boldsymbol{w}}) - \frac{x_p q(x_p)}{(\boldsymbol{z}-\boldsymbol{r})^2}\, S(\boldsymbol{u}-\boldsymbol{z})S(\boldsymbol{z}) \right\}$$

NLO cross-section w/o overcounting

(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)

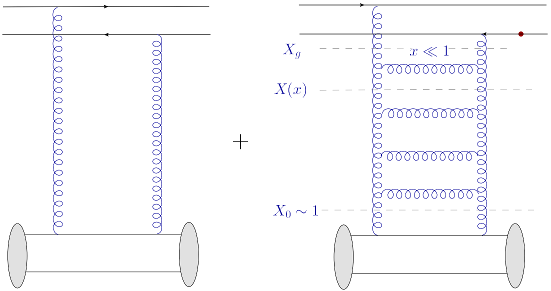


- In more compact but formal notations:

$$\frac{\mathrm{d}N}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}} = \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x}\, \mathcal{K}(x)\, \mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X(x)\big)\,; \quad X(x) \simeq \frac{X_g}{x}$$

- $\mathcal{K}(x)$: kernel for emitting a gluon with exact kinematics
- The quark distribution and the distinction "real" vs. "virtual" are implicit
- The evolution is evaluated at the target scale $X(x)$: non-local in rapidity

- The LO result is correctly included as the limit $x \ll 1 \Longrightarrow \mathcal{K}(x) \to \mathcal{K}(0)$



$$\left.\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}}\right|_{\mathrm{LO}} = \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x}\, \mathcal{K}(0)\, \mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X(x)\big) = \mathcal{S}(\boldsymbol{k}, X_g)$$

- This is just the (F.T. of the integral version of the) LO BK equation:

$$S_{\boldsymbol{xy}}(X_g) = S_{\boldsymbol{xy}}(X_0) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \int_{\boldsymbol{z}} \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \big[S_{\boldsymbol{xz}}S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}\big]\big(X(x)\big)$$

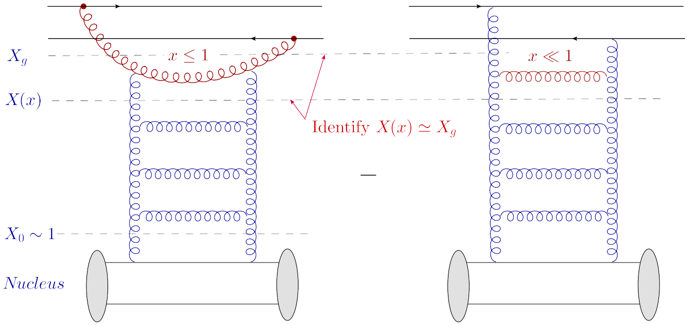
The negativity problem

- Return to our compact but formal expression for the cross-section at NLO:

$$\frac{\mathrm{d}N}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}} = \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x}\, \mathcal{K}(x)\, \mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X(x)\big)\,; \quad X(x) \simeq \frac{X_g}{x}$$

- This expression is positive semi-definite, by construction
- However, this is not the standard "k_T–factorization" at high energy
  - in k_T–factorization, the LO result is separated from NLO corrections
  - the NLO correction to the impact factor is local in rapidity
  - this requires additional approximations: "rapidity subtraction"
- One can subtract the LO piece from our NLO result ... but this is tricky
  - one adds and subtracts a large, LO, contribution
  - any error/additional approximation may lead to spurious results

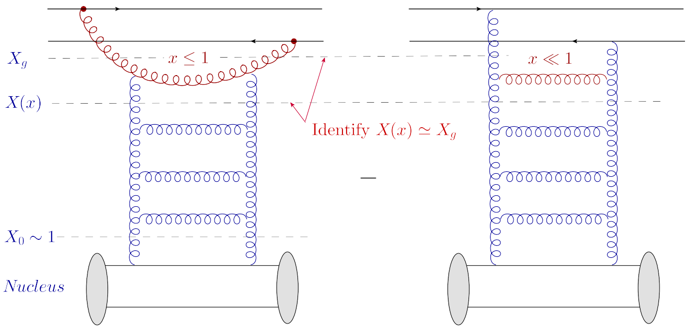
Recovering k_T–factorization



$$\frac{\mathrm{d}N}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}} = \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x}\left[\mathcal{K}(x) - \mathcal{K}(0)\right] \mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X(x)\big)$$

- The "subtracted" version of our NLO result: LO + NLO corrections
  - so far a mathematical identity (at least for fixed coupling)
- Not yet k_T–factorization: non-local in rapidity $(X(x))$

Recovering k_T–factorization

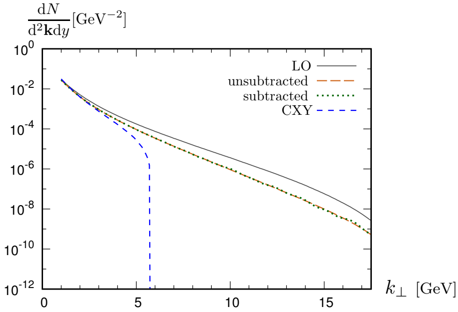


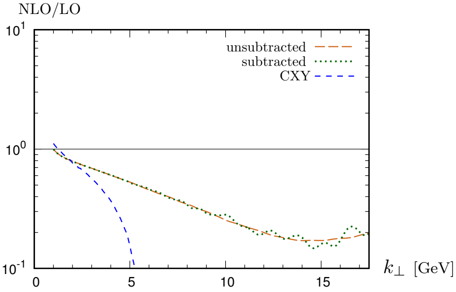
$$\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} = \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s \int_0^1 \frac{\mathrm{d}x}{x} \left[\mathcal{K}(x) - \mathcal{K}(0)\right] \mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X_g\big)$$

- To NLO accuracy, one can perform additional approximations:
  - replace $\mathcal{S}\big(X(x)\big) \simeq \mathcal{S}(X_g)$ (since integral dominated by $x \sim 1$)
  - set $X_g \to 0$ in the lower limit ('plus prescription')
- Local in rapidity : k_T–factorization as presented by CXY

Numerical results: Fixed coupling $\alpha_s = 0.2$

Ducloué, Lappi, Zhu, arXiv:1703.04962

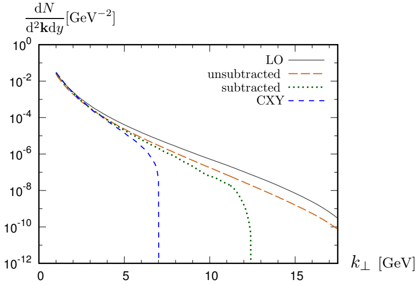


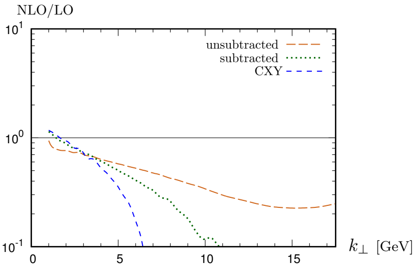


- Large NLO correction: $\gtrsim 50\,\%$ for $k_\perp \geq 5$ GeV
- The same results with and without subtracting the LO result
  - small oscillations in "subtracted" due to numerical errors
- $k_\perp$-factorization (CXY) rapidly becomes negative : over-subtraction

Numerical results: Running coupling

Ducloué, Lappi, Zhu, arXiv:1703.04962





- The running of the coupling renders the problem even more subtle:
  - already the "subtracted" result becomes negative
  - the "CXY" curve becomes negative even faster
- Mismatch between the running coupling prescriptions used ...
  - in coordinate space (for solving the BK equation)
  - ... and in momentum space (for computing the NLO impact factor)

Adding a running coupling

- The NLO impact factor is generally computed in momentum space
  - natural to use a running coupling $\bar{\alpha}_s(k_\perp^2)$ (at least for $k_\perp^2 \gtrsim Q_s^2$)

$$\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} = \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s(k_\perp^2) \int_{X_g}^1 \frac{\mathrm{d}x}{x}\, \mathcal{K}(x)\, \mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X(x)\big)$$

  - more generally: $\bar{\alpha}_s(k_{\max}^2)$

- Dipole S-matrix is computed by solving rcBK in coordinate space

$$S_{\boldsymbol{xy}}(X_g) = S_{\boldsymbol{xy}}(X_0) + \int_{X_g}^1 \frac{\mathrm{d}x}{x} \int_{\boldsymbol{z}} \bar{\alpha}_s(r_{\min}^2) \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \Big[S_{\boldsymbol{xz}} S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}\Big]$$

  - $r_{\min} \equiv \min\big\{|\boldsymbol{x}-\boldsymbol{y}|, |\boldsymbol{x}-\boldsymbol{z}|, |\boldsymbol{y}-\boldsymbol{z}|\big\}$

- Running coupling and Fourier transform do not "commute" with each other

The running coupling puzzle

- Why not work fully in coordinate space ? (BK equation and hard emission)
  (Ducloué, Lappi, Zhu, arXiv:1703.04962)

$$\int_{\boldsymbol{r}} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\ \bar{\alpha}_s(r_\perp)$$

$$\times \int_{\boldsymbol{z}} \left\{ \frac{x_p}{1-x} q\Big(\frac{x_p}{1-x}\Big) \frac{(\boldsymbol{z}-\boldsymbol{r})\cdot\boldsymbol{z}}{(\boldsymbol{z}-\boldsymbol{r})^2\boldsymbol{z}^2}\ S(\boldsymbol{r}-\boldsymbol{z})S(\boldsymbol{z}-\bar{\boldsymbol{w}}) - \frac{x_p q(x_p)}{(\boldsymbol{z}-\boldsymbol{r})^2} S(\boldsymbol{u}-\boldsymbol{z})S(\boldsymbol{z}) \right\}$$

- Remember: For generic x, there is no cancellation of the large daughter dipoles $(z_\perp \gg r_\perp)$ between "real" and "virtual" terms

- Not a problem of principle: the F.T. must select $z_\perp \sim r_\perp \sim 1/k_\perp$

  - the recoil gluon should be as hard as the produced quark: $p_\perp \simeq k_\perp$

$$\bar{\alpha}_s(k_\perp) \int_{\boldsymbol{r}} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \int_{\boldsymbol{z}\gg\boldsymbol{r}} \frac{1}{\boldsymbol{z}^2} S(-\boldsymbol{z})S((1-x)\boldsymbol{z}) = 0$$

The running coupling puzzle

- Why not work fully in coordinate space ? (BK equation and hard emission) *(Ducloué, Lappi, Zhu, arXiv:1703.04962)*

$$\int_{\boldsymbol{r}} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\ \bar{\alpha}_s(r_\perp)$$

$$\times \int_{\boldsymbol{z}} \left\{ \frac{x_p}{1-x} q\Big(\frac{x_p}{1-x}\Big) \frac{(\boldsymbol{z}-\boldsymbol{r})\cdot\boldsymbol{z}}{(\boldsymbol{z}-\boldsymbol{r})^2\boldsymbol{z}^2}\ S(\boldsymbol{r}-\boldsymbol{z})S(\boldsymbol{z}-\bar{\boldsymbol{w}}) - \frac{x_p q(x_p)}{(\boldsymbol{z}-\boldsymbol{r})^2} S(\boldsymbol{u}-\boldsymbol{z})S(\boldsymbol{z}) \right\}$$

- Remember: For generic x, there is no cancellation of the large daughter dipoles $(z_\perp \gg r_\perp)$ between "real" and "virtual" terms

- Not a problem of principle: the F.T. must select $z_\perp \sim r_\perp \sim 1/k_\perp$
  - the recoil gluon should be as hard as the produced quark: $p_\perp \simeq k_\perp$

$$\int_{\boldsymbol{r}} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\ \bar{\alpha}_s(r_\perp) \int_{\boldsymbol{z}\gg\boldsymbol{r}} \frac{1}{\boldsymbol{z}^2} S(-\boldsymbol{z})S((1-x)\boldsymbol{z}) \propto \int_{\boldsymbol{r}} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\ \bar{\alpha}_s(r_\perp) \sim -\frac{1}{k_\perp^2}$$

- Wrong sign & wrong power tail: $-1/k_\perp^2$ instead of $1/k_\perp^4$

Conclusions

- The hybrid factorization for pA collisions formally holds to NLO

  .... but this is rather subtle !

- On the nuclear side, one cannot 'automatically' apply k_T-factorization
  - one cannot enforce locality in rapidity in the NLO impact factor
  - with running coupling, the separation between LO and NLO contributions becomes dangerous

- But this is actually not needed: factorization is more general
  - no explicit separation between LO and NLO
  - non-local in rapidity

- Sensible physical results: positive cross-section, but smaller than at LO

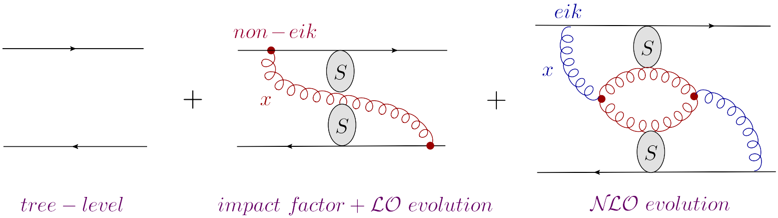
- A similar strategy is necessary when computing DIS at NLO (dipole picture)
  Ducloué, Hänninen, Lappi, and Zhu, arXiv:1708.07328

- See the subsequent talk by Tuomas Lappi !

Completing the NLO evolution

(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)

- Recall: the NLO BK evolution also involves 2-loop graphs



$$\frac{\mathrm{d}N}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}} = \mathcal{S}_0 + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x}\,\mathcal{K}(x)\,\mathcal{S}\big(X(x)\big) + \bar{\alpha}_s^2 \int_{X_g}^{1} \frac{\mathrm{d}x}{x}\,\mathcal{K}_2(0)\,\mathcal{S}\big(X(x)\big)$$

- $\mathcal{K}_2(0)$: NLO correction to the BK kernel with collinear improvement

  (Balitsky and Chirilli, 2008; Iancu et al, 2015)