#### F Hautmann

TMDs from low to high energies and the parton branching method

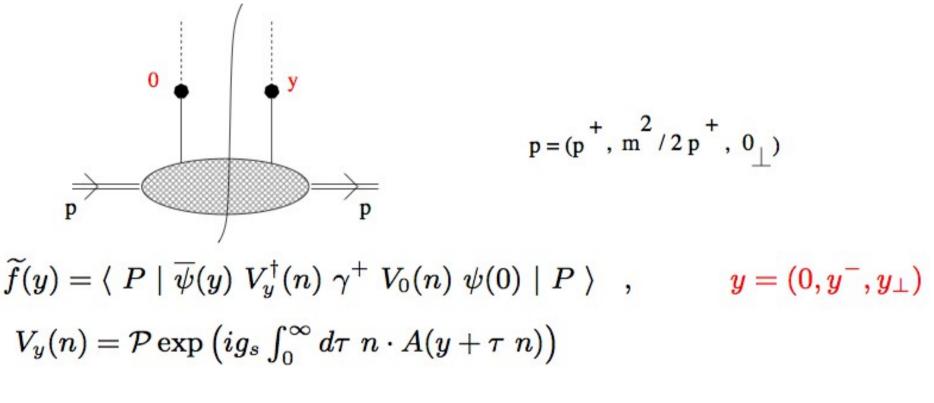
INT Program 18-3 "Probing Nucleons and Nuclei in High Energy Collisions" - EIC Symposium

Institute for Nuclear Theory, Seattle, October 2018

# Overview

#### TRANSVERSE MOMENTUM DEPENDENT (TMD) PARTON DISTRIBUTION FUNCTIONS

• Parton correlation functions at non-lightlike distances:

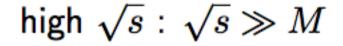


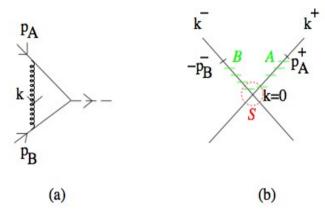
#### • TMD pdfs:

$$f(x,k_{\perp}) = \int \frac{dy^{-}}{2\pi} \frac{d^{d-2}y_{\perp}}{(2\pi)^{d-2}} e^{-ixp^{+}y^{-} + ik_{\perp} \cdot y_{\perp}} \tilde{f}(y)$$

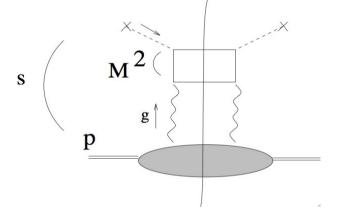
# Evolution equations for TMD parton distribution functions

low  $q_T$  :  $q_T \ll Q$ 









 $(lpha_s \ln \sqrt{s}/M)^n$ 

#### CSS evolution equation

#### CCFM evolution equation

R. Angeles-Martinez et al., "Transverse momentum dependent (TMD) parton distribution functions: status and prospects", Acta Phys. Polon. B46 (2015) 2501

## TMD distributions (unpolarized and polarized)

#### TABLE I

(Colour on-line) Quark TMD pdfs: columns represent quark polarization, rows represent hadron polarization. Distributions encircled by a dashed line are the ones which survive integration over transverse momentum. The shades of the boxes (light gray (blue) versus medium gray (pink)) indicate structures that are T-even or T-odd, respectively. T-even and T-odd structures involve, respectively, an even or odd number of spin-flips.

QUARKS	unpolarized	chiral	transverse
U	$(f_i)$		$h_1^{\perp}$
L		$(g_u)$	$h_{1L}^{\perp}$
т	$f_{ir}^{\perp}$	g <sub>17</sub>	$(h_{ir})h_{ir}^{\perp}$

#### TABLE II

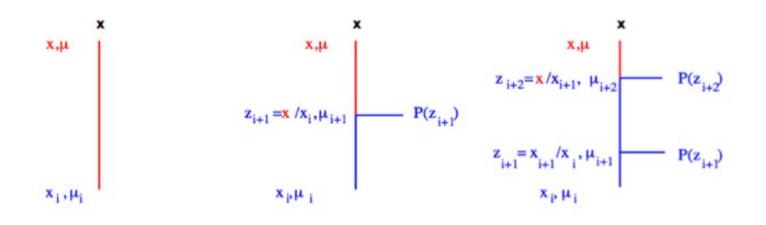
(Colour on-line) Gluon TMD pdfs: columns represent gluon polarization, rows represent hadron polarization. Distributions encircled by a dashed line are the ones which survive integration over transverse momentum. The shades of the boxes (light gray (blue) versus medium gray (pink)) indicate structures that are T-even or T-odd, respectively. T-even and T-odd structures involve, respectively, an even or odd number of spin-flips. Linearly polarized gluons represent a double spin-flip structure.

GLUONS	unpolarized	circular	linear
U	$(f_1^g)$		$h_1^{\perp g}$
L		$\left(g_{u}^{s}\right)$	$h_{1L}^{\perp g}$
т	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_{17}^g, h_{17}^{\perp g}$

R. Angeles-Martinez et al., "Transverse momentum dependent (TMD) parton distribution functions: status and prospects", Acta Phys. Polon. B46 (2015) 2501

## Parton Branching (PB) approach

Jung, Lelek, Radescu, Zlebcik & H, "Collinear and TMD quark and gluon densities from parton branching", JHEP 1801 (2018) 070



PB evolution equation motivated by

 applicability over large kinematic range from low to high transverse momenta

#### applicability to exclusive final states and Monte Carlo event generators

# Outline of this talk

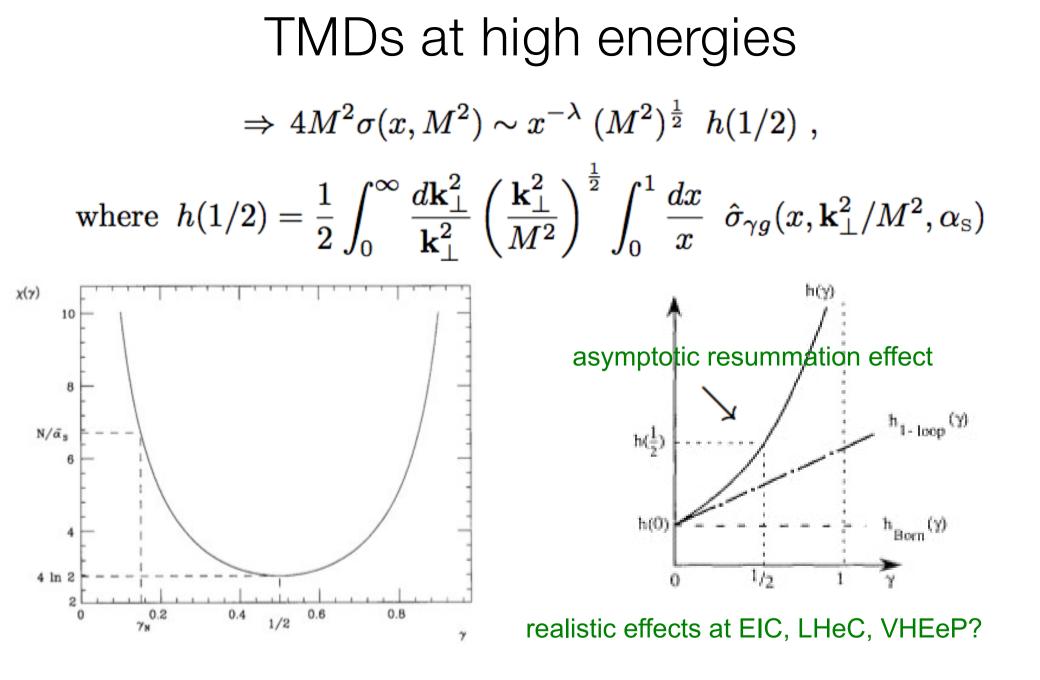
# • TMDs at high $\sqrt{s}$ and at low qT

# The parton branching (PB) method

## New results and applications

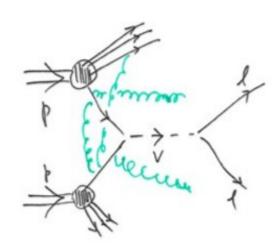
I. INTRODUCTION TMDs at high energies Ex.: heavy flavor electroproduction for  $s \gg M^2 \gg \Lambda_{\rm QCD}^2$  $\gamma + h \rightarrow Q + \bar{Q} + X$  $4M^2 \ \sigma(x,M^2) = \int d^2 \mathbf{k}_\perp \int_{-\infty}^1 rac{dz}{z} \ \hat{\sigma}_{\gamma g}(x/z,\mathbf{k}_\perp^2/M^2,lpha_{
m s}(M^2)) \ \mathcal{A}_{g/h}(z,\mathbf{k}_\perp)$ where TMD gluon distribution is given by Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution:

$$\mathcal{A}_{g/h}(x,\mathbf{k}_{\perp}) \sim \frac{1}{2\pi} e^{-\lambda \ln x} \, (\mathbf{k}_{\perp}^2)^{-\frac{1}{2}} \qquad \lambda = 4 \, C_A \, \frac{\alpha_{\rm s}}{\pi} \, \ln 2$$



- NB: incorporate sub-asymptotic, finite-x terms  $\rightarrow$  CCFM evolution
  - dense-medium modifications in nucleons and nuclei  $\rightarrow$  nonlinear evolution

## TMDs for low qT



Ex.: Drell-Yan production qT spectra for Q >> qT

$$\frac{d\sigma}{d^2\mathbf{q}_T dQ^2 dy} = \sum_{i,j} \frac{\sigma^{(0)}}{s} H(\alpha_{\rm S}) \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{q}_T \cdot \mathbf{b}} \mathcal{A}_i(x_1, \mathbf{b}, \mu, \zeta) \mathcal{A}_j(x_2, \mathbf{b}, \mu, \zeta) + \{\mathbf{q}_T - \text{finite}\} + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)$$

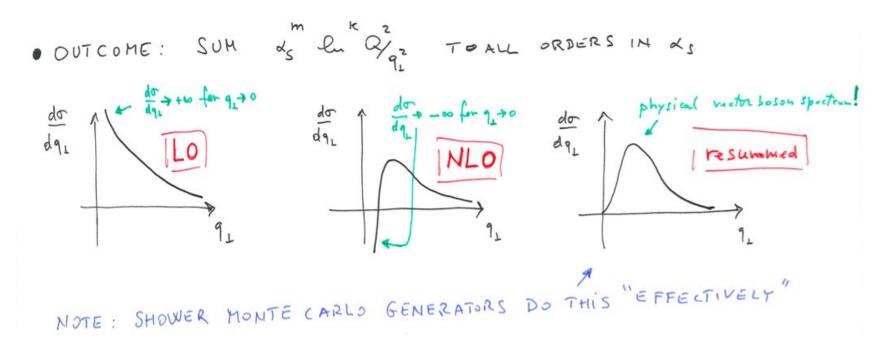
where 
$$\frac{\partial \ln \mathcal{A}}{\partial \ln \sqrt{\zeta}} = K(\mathbf{b}, \mu)$$

Collins-Soper-Sterman (CSS) evolution

and 
$$\frac{d\ln A}{d\ln \mu} = \gamma_f(\alpha_s(\mu), \zeta/\mu^2)$$
,  $\frac{dK}{d\ln \mu} = -\gamma_K(\alpha_s(\mu))$  RG evolution  
Cusp anomalous dimension

$$\Rightarrow -\aleph_{k} = \frac{2}{2 \exp 5} \&f \quad \text{i.e.} \quad \Im_{f}(\Im_{i}(\mu), \Im_{j+1}) = \Im_{f}(\Im_{i}(\mu), 1) - \frac{1}{2} \Im_{k} \&h \Im_{j}$$
• Soft Collinear Effective Theory (SCET) provides alternative approach leading to same results

## TMDs for low qT



• A MORE "PARTON-LIKE" FORMULATION is ACHIEVED BY ("OPE") DECOMPOSING THE THD PDF IN TERMS OF ORDINARY PDF'S ("OPE")

#### From color-neutral to color-charged final states

# KIKI KINI KINA HI

Color neutral:

Color charged:

• New long-time correlations in color-charged case:

$$\left(\frac{d\sigma}{d^4q}\right)_{t\bar{t}} = \sum_{ija_1a_2} \int d^2 \mathbf{b} \ e^{i\mathbf{q}_T \cdot \mathbf{b}} \ \int dz_1 \int dz_2 \ S(Q,\mathbf{b}) \ f_{a_1} \otimes [\operatorname{Tr}(H\Delta)C_1C_2]_{ija_1a_2} \otimes f_{a_2}$$

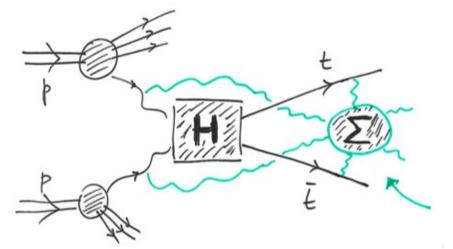
- Generate azimuthal asymmetries
- Observable for  $\Delta p_{\perp}$  high compared to  $\Lambda_{\rm QCD}$ ?

F Hautmann: Institute for Nuclear Theory, University of Washington, October 2018



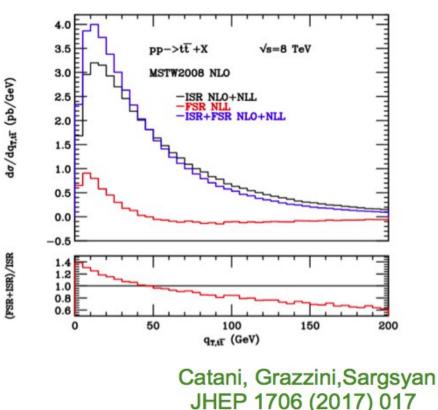
soft gluons coupling initial and final states

Color correlations in jet and heavy-flavor production



 Initial state / final state soft-gluon correlations
 → new "color entanglement" effects?

 A recent quantitative estimate of the size of color correlations for the top quark pair spectrum at the LHC:



# II. The Parton Branching (PB) method

#### MOTIVATION

- Provide evolution equation connected in a controllable way with DGLAP evolution of collinear parton distributions
- Applicable over broad kinematic range from low to high transverse momenta, for inclusive as well as non-inclusive observables

Implementable in Monte Carlo event generators

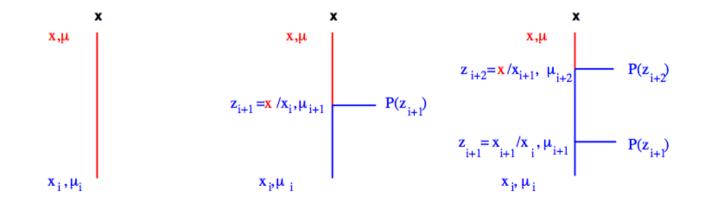
#### Parton Branching (PB) method: collinear PDFs

#### QCD evolution and soft-gluon resolution scale

[Jung, Lelek, Radescu, Zlebcik & H, PLB772 (2017) 446 + in progress]

$$\widetilde{f}_{a}(x,\mu^{2}) = \Delta_{a}(\mu^{2}) \ \widetilde{f}_{a}(x,\mu_{0}^{2}) + \sum_{b} \int_{\mu_{0}^{2}}^{\mu^{2}} \frac{d\mu'^{2}}{\mu'^{2}} \frac{\Delta_{a}(\mu^{2})}{\Delta_{a}(\mu'^{2})} \int_{x}^{z_{M}} dz \ P_{ab}^{(R)}(\alpha_{s}(\mu'^{2}),z) \ \widetilde{f}_{b}(x/z,\mu'^{2})$$

where 
$$\Delta_a(z_M,\mu^2,\mu_0^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz \ z \ P_{ba}^{(R)}(lpha_{
m S}(\mu'^2),z)
ight)$$



▷ soft-gluon resolution parameter  $z_M$  separates resolvable and nonresolvable branchings ▷ no-branching probability  $\Delta$ ; real-emission probability  $P^{(R)}$ 

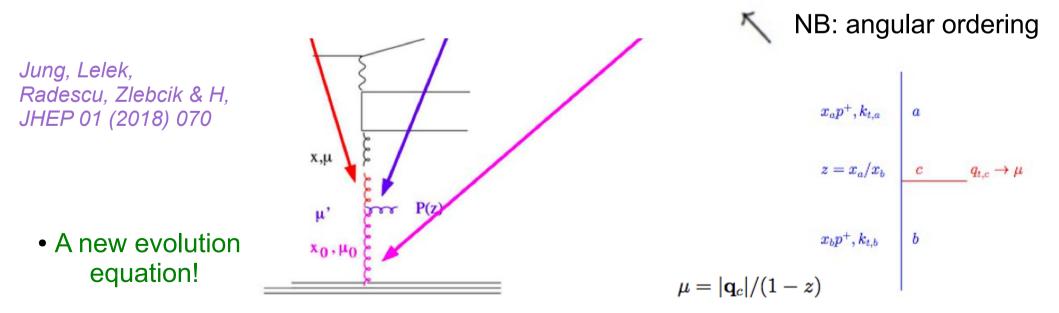
#### • Equivalent to DGLAP evolution equation for $zM \rightarrow 1$

#### Parton Branching (PB) method: TMD PDFs

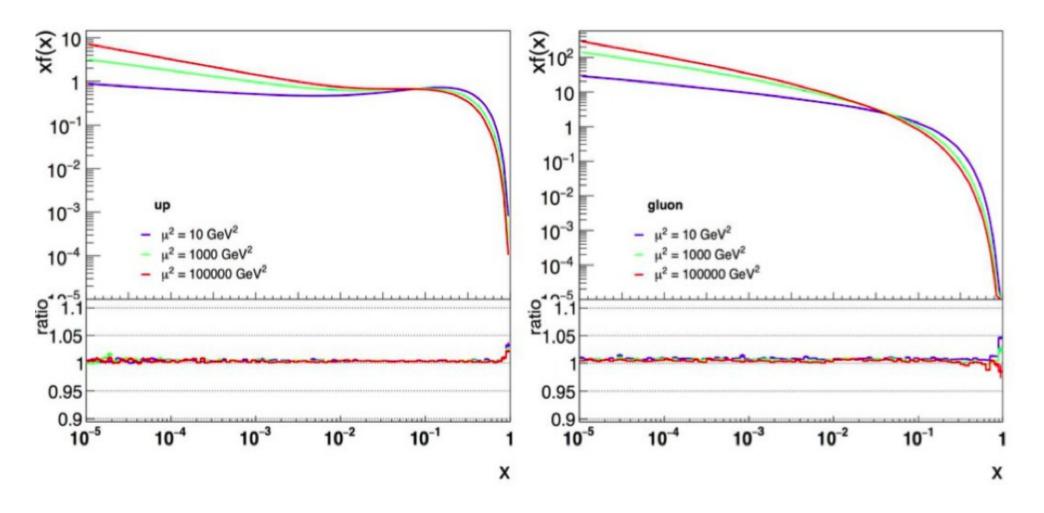
$$\begin{split} \widetilde{\mathcal{A}}_{a}(x,\mathbf{k},\mu^{2}) &= \Delta_{a}(\mu^{2}) \ \widetilde{\mathcal{A}}_{a}(x,\mathbf{k},\mu_{0}^{2}) + \sum_{b} \int \frac{d^{2}\mathbf{q}'}{\pi \mathbf{q}'^{2}} \ \frac{\Delta_{a}(\mu^{2})}{\Delta_{a}(\mathbf{q}'^{2})} \ \Theta(\mu^{2}-\mathbf{q}'^{2}) \ \Theta(\mathbf{q}'^{2}-\mu_{0}^{2}) \\ &\times \int_{x}^{z_{M}} dz \ P_{ab}^{(R)}(\alpha_{s}(\mathbf{q}'^{2}),z) \ \widetilde{\mathcal{A}}_{b}(x/z,\mathbf{k}+(1-z)\mathbf{q}',\mathbf{q}'^{2}) \end{split}$$

Solve iteratively :  $\widetilde{\mathcal{A}}_a^{(0)}(x,\mathbf{k},\mu^2) = \Delta_a(\mu^2) \ \widetilde{\mathcal{A}}_a(x,\mathbf{k},\mu_0^2) \ ,$ 

$$\begin{split} \widetilde{\mathcal{A}}_{a}^{(1)}(x,\mathbf{k},\mu^{2}) &= \sum_{b} \int \frac{d^{2}\mathbf{q}'}{\pi \mathbf{q}'^{2}} \; \Theta(\mu^{2}-\mathbf{q}'^{2}) \; \Theta(\mathbf{q}'^{2}-\mu_{0}^{2}) \\ \times \quad \frac{\Delta_{a}(\mu^{2})}{\Delta_{a}(\mathbf{q}'^{2})} \int_{x}^{z_{M}} dz \; P_{ab}^{(R)}(\alpha_{\mathrm{S}}(\mathbf{q}'^{2}),z) \; \widetilde{\mathcal{A}}_{b}(x/z,\mathbf{k}+(1-z)\mathbf{q}',\mu_{0}^{2}) \; \Delta_{b}(\mathbf{q}'^{2}) \end{split}$$

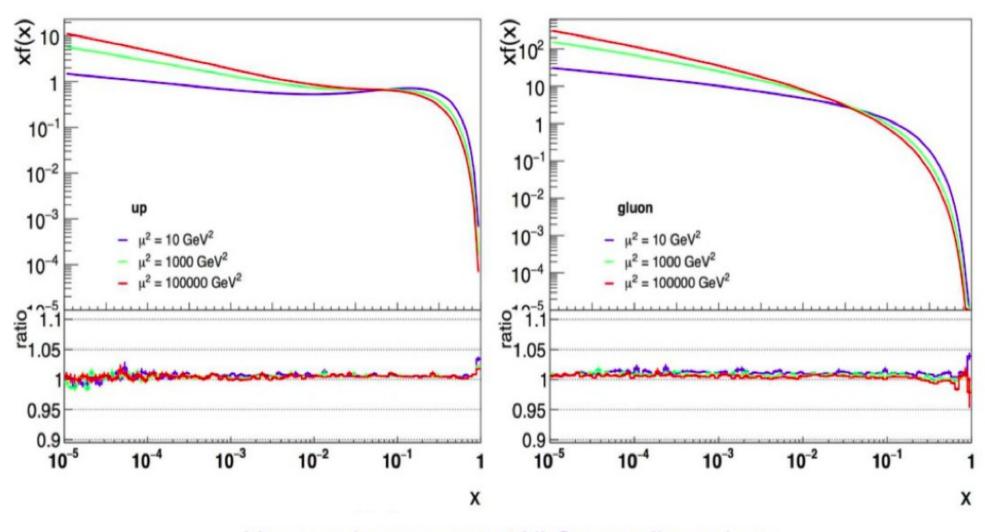


# Validation of the method with semi-analytic result from QCDNUM at LO



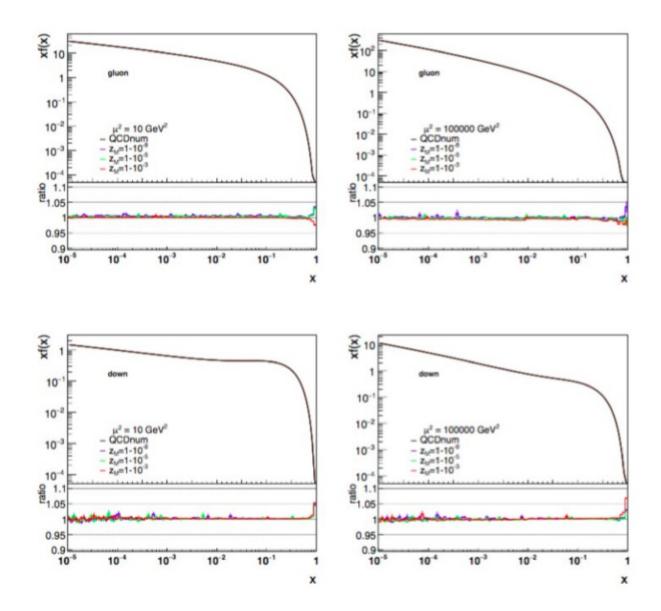
#### Agreement to better than 1 % over several orders of magnitude in x and mu

# Validation of the method with semi-analytic result from QCDNUM at NLO

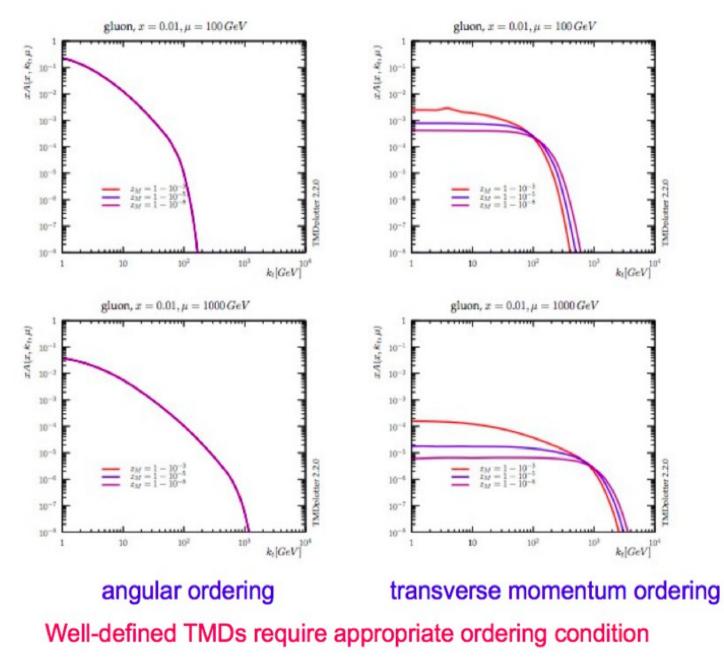


Very good agreement at NLO over all x and mu. NB: the same approach is designed to work at NNLO.

#### Stability with respect to resolution scale z\_M



#### TMDs and soft gluon resolution effects



#### PB method in xFitter

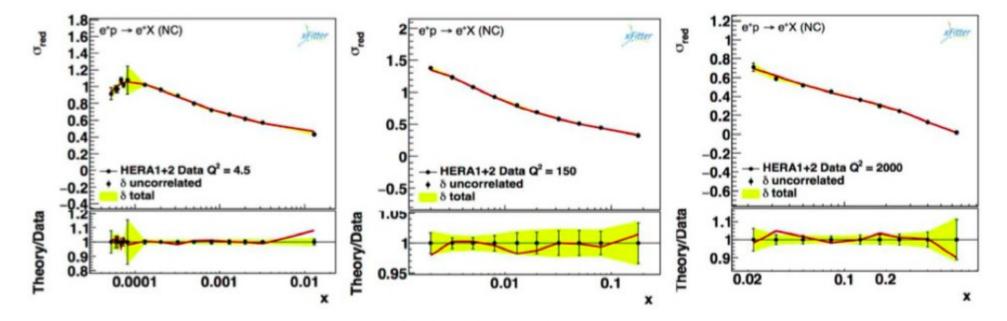
Determine starting distribution

A Bermudez et al, arXiv:1804.11152

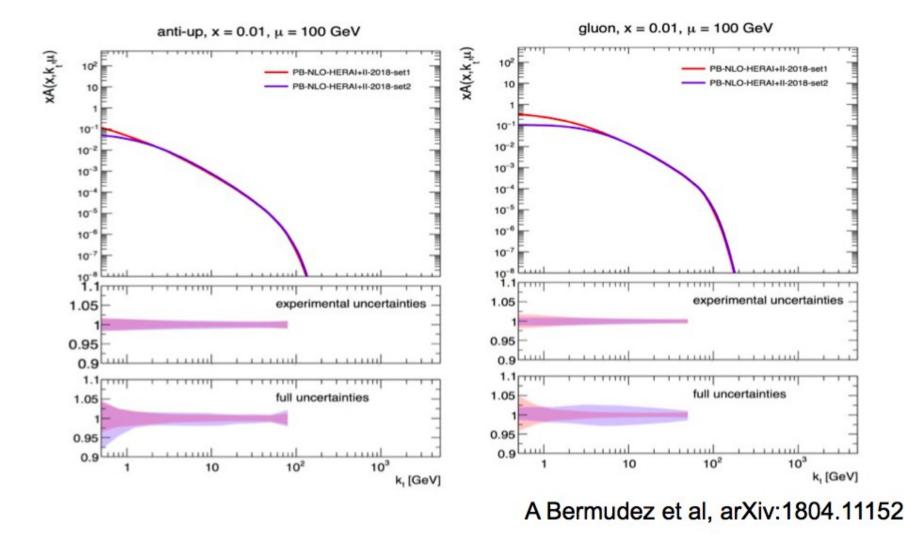
A. Lelek et al REF 2016

$$\begin{aligned} xf_a(x,\mu^2) &= x \int dx' \int dx'' \mathcal{A}_{0,b}(x') \tilde{\mathcal{A}}_a^b \left(x'',\mu^2\right) \delta(x'x''-x) \\ &= \int dx' \mathcal{A}_{0,b}(x') \cdot \frac{x}{x'} \; \tilde{\mathcal{A}}_a^b \left(\frac{x}{x'},\mu^2\right) \end{aligned}$$

• fit to HERA data (using xFitter) with  $Q^2 \ge 3.5~{\rm GeV^2}$  gives  $\chi^2/ndf \sim 1.2$ 



# TMD distributions from fits to precision HERA data



#### NLO determination of TMDs with uncertainties

### Where to find TMDs? TMDIib and TMDplotter

- TMDlib proposed in 2014 as part of the REF Workshop and developed since
- A library of parameterizations and fits of TMDs (LHAPDF-style)

http://tmdlib.hepforge.org http://tmdplotter.desy.de

 Also contains collinear (integrated) pdfs Eur. Phys. J. C (2014) 74:3220 DOI 10.1140/epjc/s10052-014-3220-9 THE EUROPEAN PHYSICAL JOURNAL C

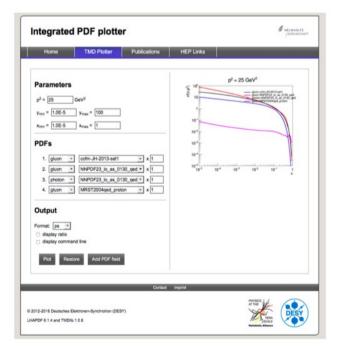
Special Article - Tools for Experiment and Theory

#### TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions

F. Hautmann<sup>1,2</sup>, H. Jung<sup>3,4</sup>, M. Krämer<sup>3</sup>, P. J. Mulders<sup>5,6</sup>, E. R. Nocera<sup>7</sup>, T. C. Rogers<sup>8,9</sup>, A. Signort<sup>5,6,a</sup>

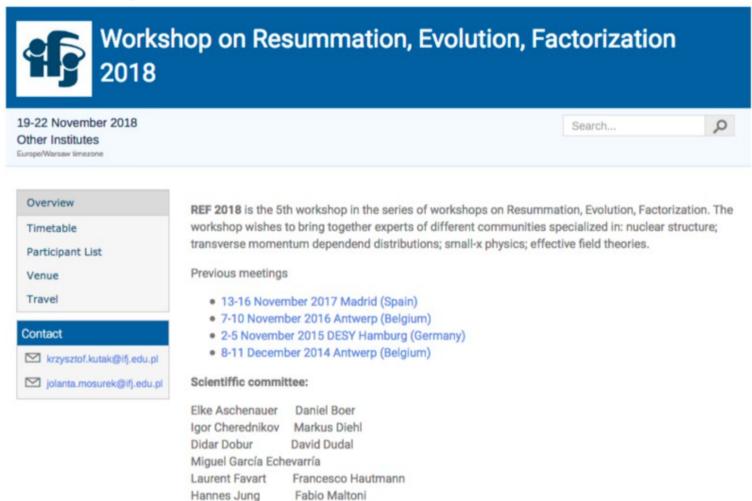
1 Rutherford Appleton Laboratory, Oxford, UK

- <sup>2</sup> Department of Theoretical Physics, University of Oxford, Oxford, UK
- 3 DESY, Hamburg, Germany
- <sup>4</sup> University of Antwerp, Antwerp, Belgium
- <sup>5</sup> Department of Physics and Astronomy, VU University Amsterdam, Amsterdam, The Netherlands
- <sup>6</sup> Nikhef, Amsterdam, The Netherlands
- <sup>7</sup> Università degli Studi di Genova, INFN, Genoa, Italy
- <sup>8</sup> C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, USA
- <sup>9</sup> Department of Physics, Southern Methodist University, Dallas, TX 75275, USA



#### Next REF Workshop: Cracow, 19-22 November 2018

#### https://indico.cern.ch/event/696311



**Gunar Schnell** 

Pierre Van Mechelen

Piet Mulders

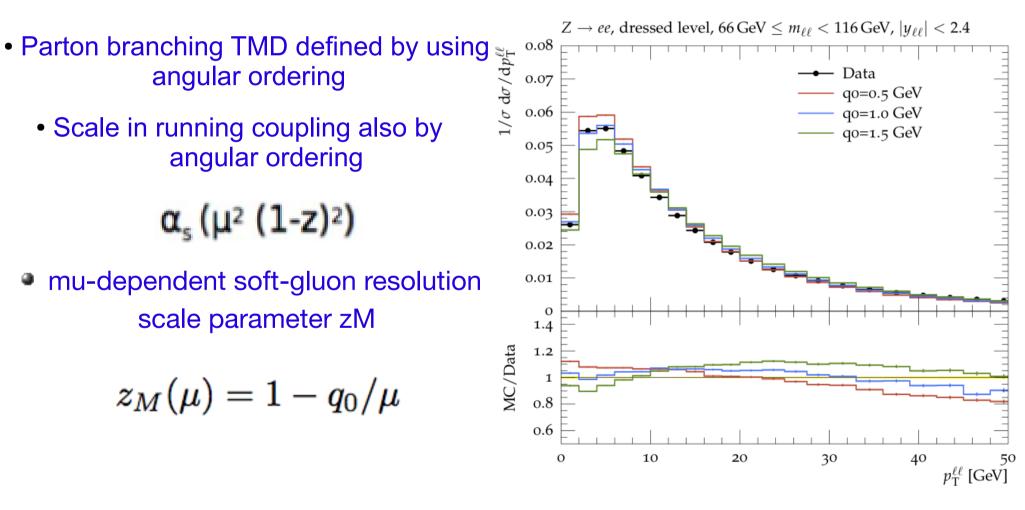
Andrea Signori

# III. New results and applications

## ONGOING WORK:

- Drell-Yan pT spectrum from convolution of two transverse momentum dependent distributions
- Comparison of parton branching results with analytic TMD resummation (Collins-Soper-Sterman method)
- First implementation for jets (using NLO matrix elements for color-charged final states)

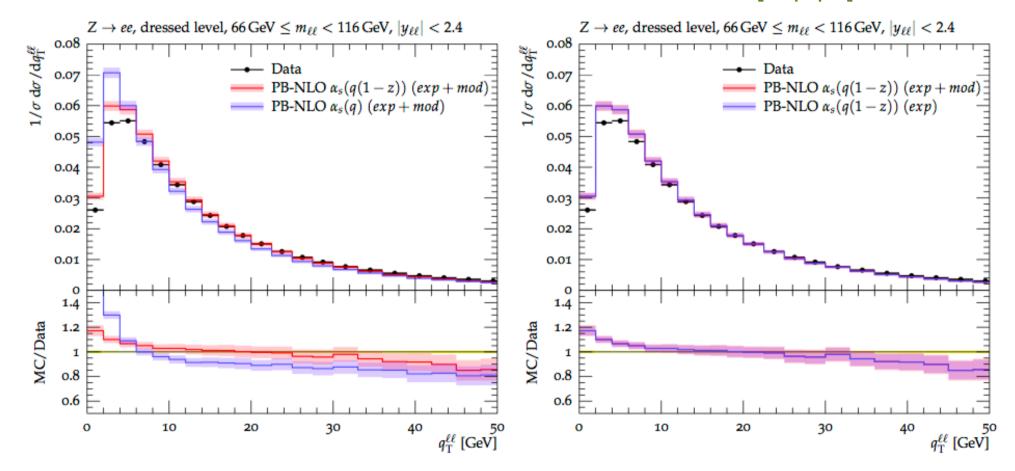
# Application of PB method to Z-boson transverse momentum spectrum in Drell-Yan production



#### LHC Electroweak WG Meeting, CERN, June 2018

# Z-boson transverse momentum spectrum: soft-gluon angular ordering effects

Zlebcik, Radescu, Lelek, Jung & H, JHEP 1801 (2018) 070; A Bermudez Martinez et al., arXiv:1804.11152 [hep-ph]



ATLAS data, EPJC 76 (2016) 291 F Hautmann: Institute for Nuclear Theory, University of Washington, October 2018

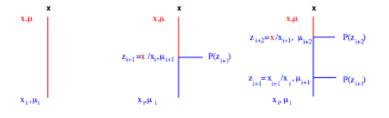
### Comparison with CSS (Collins-Soper-Sterman) resummation

 $\diamond$  The resummed DY differential cross section is given by

$$\frac{d\sigma}{d^2\mathbf{q}dQ^2dy} = \sum_{q,\bar{q}} \frac{\sigma^{(0)}}{s} H(\alpha_{\rm S}) \int \frac{d^2\mathbf{b}}{(2\pi)^2} \ e^{i\mathbf{q}\cdot\mathbf{b}} \mathcal{A}_q(x_1,\mathbf{b},Q) \mathcal{A}_{\bar{q}}(x_2,\mathbf{b},Q) + \mathcal{O}\left(\frac{|\mathbf{q}|}{Q}\right) \quad \text{where}$$

$$\begin{aligned} \mathcal{A}_{i}(x,\mathbf{b},Q) &= \exp\left\{\frac{1}{2}\int_{c_{0}/b^{2}}^{Q^{2}}\frac{d\mu'^{2}}{\mu'^{2}}\left[A_{i}(\alpha_{\mathrm{S}}(\mu'^{2}))\ln\left(\frac{Q^{2}}{\mu'^{2}}\right) + B_{i}(\alpha_{\mathrm{S}}(\mu'^{2}))\right]\right\}G_{i}^{(\mathrm{NP})}(x,\mathbf{b}) \\ &\times \sum_{j}\int_{x}^{1}\frac{dz}{z}C_{ij}\left(z,\alpha_{\mathrm{S}}\left(\frac{c_{0}}{\mathbf{b}^{2}}\right)\right)f_{j}\left(\frac{x}{z},\frac{c_{0}}{\mathbf{b}^{2}}\right)\end{aligned}$$

and the coefficients H, A, B, C have power series expansions in  $\alpha_S$ .  $\diamond$  The parton branching TMD is expressed in terms of real-emission  $P^{(R)}$ :



 $\triangleright$  via momentum sum rules, use unitarity to relate  $P^{(R)}$  to virtual emission  $\triangleright$  identify the coefficients in the two formulations, order by order in  $\alpha_S$ , at LL, NLL, ...

#### Comparison with CSS (Collins-Soper-Sterman) resummation

More precisely:

▷ The parton branching TMD contains Sudakov form factor in terms of

$$P^{(R)}_{ab}(lpha_{ ext{ iny S}},z) = K_{ab}(lpha_{ ext{ iny S}}) \; rac{1}{1-z} + R_{ab}(lpha_{ ext{ iny S}},z) \; \; ext{where}$$

$$K_{ab}(lpha_{
m S}) = \delta_{ab}k_{a}(lpha_{
m S}), \ \ k_{a}(lpha_{
m S}) = \sum_{n=1}^{\infty} \left(rac{lpha_{
m S}}{2\pi}
ight)^{n}k_{a}^{(n-1)}, \ \ R_{ab}(lpha_{
m S},z) = \sum_{n=1}^{\infty} \left(rac{lpha_{
m S}}{2\pi}
ight)^{n}R_{ab}^{(n-1)}(z)$$

Via momentum sum rules, use unitarity to re-express this in terms of

$$P^{(V)} = P - P^{(R)} , \quad \text{where} \quad$$

$$P_{ab}(lpha_{ ext{s}},z)=D_{ab}(lpha_{ ext{s}})\delta(1-z)+K_{ab}(lpha_{ ext{s}})\;rac{1}{(1-z)_+}+R_{ab}(lpha_{ ext{s}},z)$$

is full splitting function (at LO, NLO, etc.)

$$ext{with} \quad D_{ab}(lpha_{ ext{ iny S}}) = \delta_{ab} d_a(lpha_{ ext{ iny S}}) \;, \quad d_a(lpha_{ ext{ iny S}}) = \sum_{n=1}^\infty \left(rac{lpha_{ ext{ iny S}}}{2\pi}
ight)^n d_a^{(n-1)}$$

 $\triangleright$  Identify  $d_a(\alpha_s)$  and  $k_a(\alpha_s)$  with resummation formula coefficients (LL, NLL, . .)

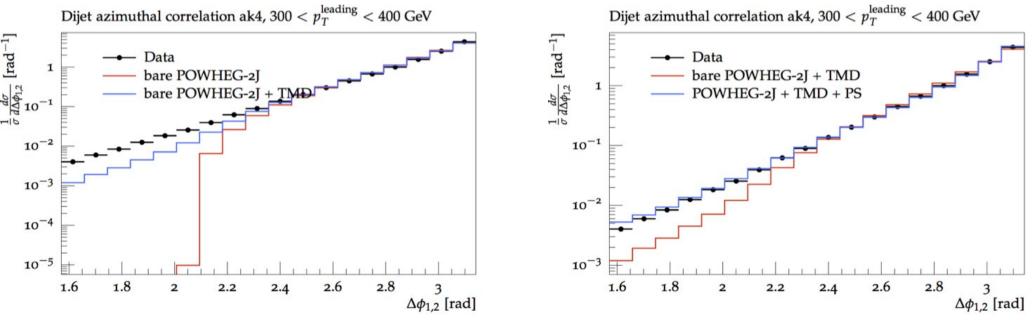
#### Comparison with CSS (Collins-Soper-Sterman) resummation

•  $d_a(lpha_{\scriptscriptstyle \mathrm{S}})$  and  $k_a(lpha_{\scriptscriptstyle \mathrm{S}})$  perturbative coefficients

$$\begin{aligned} & \text{one} - \text{loop} \ : \\ & d_q^{(0)} = \frac{3}{2} \, C_F \quad , \ k_q^{(0)} = 2 \, C_F \\ & \text{two} - \text{loop} \ : \\ & d_q^{(1)} = C_F^2 \left( \frac{3}{8} - \frac{\pi^2}{2} + 6 \, \zeta(3) \right) + C_F C_A \left( \frac{17}{24} + \frac{11\pi^2}{18} - 3 \, \zeta(3) \right) - C_F T_R N_f \left( \frac{1}{6} + \frac{2\pi^2}{9} \right) \ , \\ & k_q^{(1)} = 2 \, C_F \, \Gamma \ , \quad \text{where} \ \ \Gamma = C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - T_R N_f \frac{10}{9} \end{aligned}$$

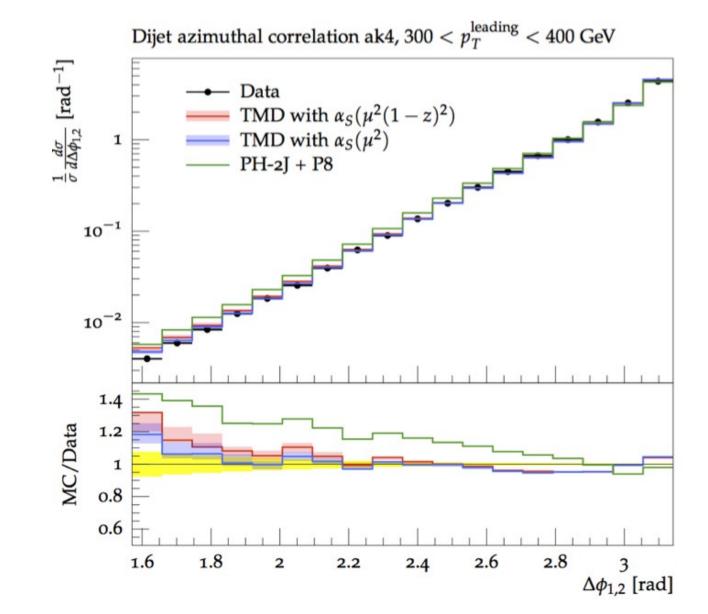
• The k and d coefficients of the PB formalism match, order by order, the A and B coefficients of the CSS formalism

#### Di-jets from PB method: towards NLO-matched parton-shower Monte Carlo generators with TMDs



- Events by NLO POWHEG 2 jets
- Parton branching TMD (with angular ordering)
- TMD parton shower

#### Di-jets from PB method: towards NLO-matched parton-shower Monte Carlo generators with TMDs



- Events by NLO
   POWHEG 2 jets
- Parton branching TMD (with angular ordering)
- TMD parton shower

## Conclusions

- PB method to take into account simultaneously soft-gluon emission at z → 1 and transverse momentum qT recoils in the parton branchings along the QCD cascade
- potentially relevant for calculations both in collinear factorization and in TMD factorization

 $\rightarrow$  cf. parton shower calculations and analytic resummation

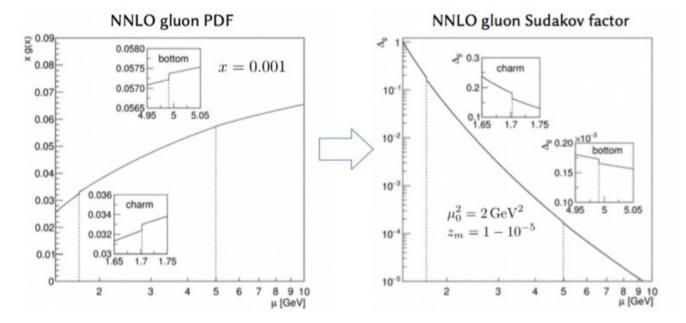
- terms in powers of In (1 zM) can be related to large-x resummation? → relevant to near-threshold, rare processes to be investigated at high luminosity
- systematic studies of ordering effects and color coherence

 $\rightarrow$  helpful to analyze long-time color correlations?

## **EXTRA SLIDES**

#### PB method at NNLO

- In NNLO VFNS discontinuities both in  $\alpha_S$  and PDFs
- These discontinuities ensure continuity of observables, e.g.  ${\it F}_2$



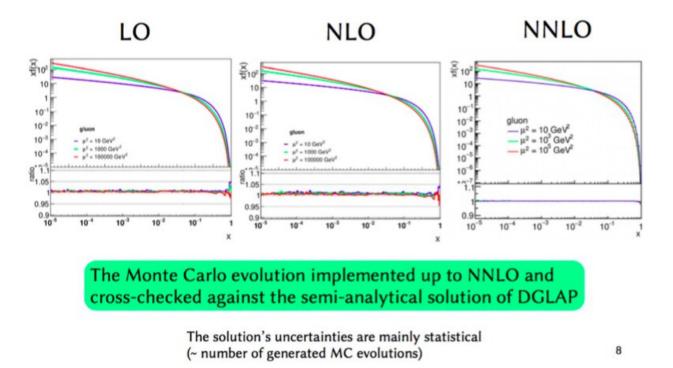
Discontinuities in the quark and gluon Sudakov factors

R. Zlebcik, talk at REF 2017, November 2017

Workshop REF2017, Universidad Complutense Madrid, 13-16 November 2017

#### PB method at NNLO

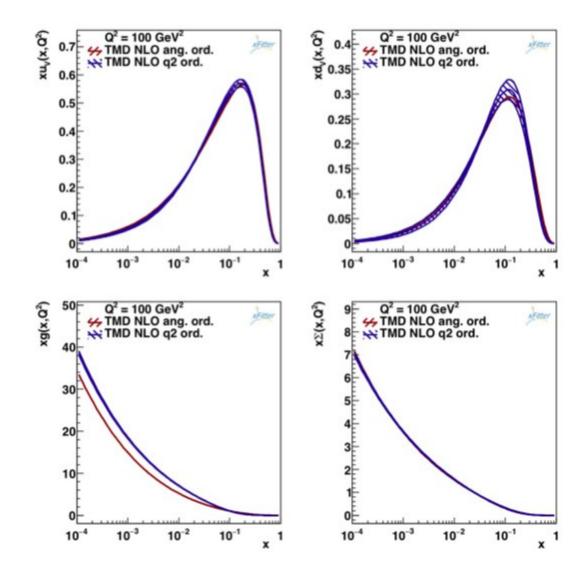
#### The Monte Carlo solution vs QCDNUM



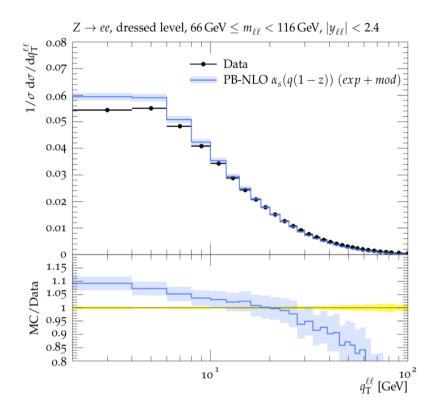
R. Zlebcik, talk at REF 2017, November 2017

Workshop REF2017, Universidad Complutense Madrid, 13-16 November 2017

# Effects of coupling's scale and angular ordering in integrated parton distributions



#### Z-boson pT spectrum including TMD uncertainties



 Cf. predictions from fixed-order + resummed calculations Bizon et al., arXiv:1805.05916

