Gauge invariant completion of Jaffe-Manohar

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

$$\langle PS|\epsilon^{ij}F^{i+}A^j_{phys}|PS\rangle = 2S^+\Delta G$$

$$\lim_{\Delta\to 0}\langle P'S|\bar{\psi}\gamma^+ i\overleftrightarrow{D}^i_{pure}\psi|PS\rangle = iS^+\epsilon^{ij}\Delta_{\perp j}L^q_{can}$$

$$\lim_{\Delta\to 0}\langle P'S|F^{+\alpha}\overleftrightarrow{D}^i_{pure}A^{phys}_\alpha|PS\rangle = -i\epsilon^{ij}\Delta_{\perp j}S^+L^g_{can}$$

where $A^\mu_{phys} = \frac{1}{D^+}F^{+\mu}$ $\quad D^\mu_{pure} = D^\mu - igA^\mu_{phys}$ YH (2011)

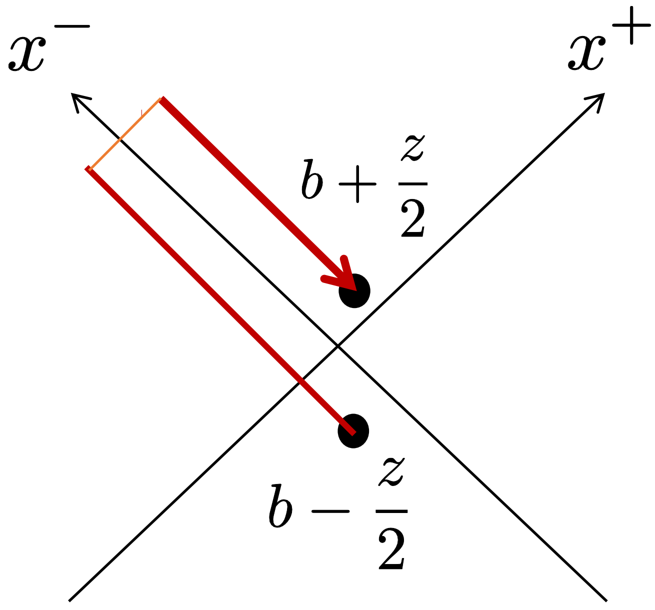
OAM from the Wigner distribution

Lorce, Pasquini (2011);
YH (2011);

Define

$$L^q = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^q(x, \vec{b}_\perp, \vec{k}_\perp)$$

Wigner distribution
with a staple Wilson line



`PDF' for OAM?

Take the staple-shaped Wilson line and define

$$L^{q,g} = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

$$L^{q,g}(x) = \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

Agrees with Harindranath, Kundu (1998); Hagler, Schafer (1998) in the light-cone gauge.

Warning: This is NOT the usual (twist-two) PDF.

OAM distribution from collinear operators

Ji's OAM canonical OAM `potential OAM'

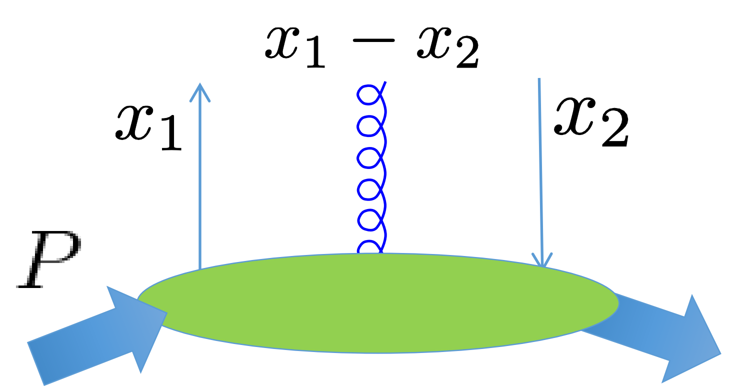
$$\langle \bar{\psi} \vec{b} \times \vec{D} \psi \rangle = \langle \bar{\psi} \vec{b} \times \vec{D}_{pure} \psi \rangle + \langle \bar{\psi} \vec{b} \times ig \vec{A}_{phys} \psi \rangle$$

$$A^{\mu}_{phys} = \frac{1}{D^+} F^{+\mu}$$

For a 3-body operator, it is natural to define the double density.

$$\int d\lambda d\mu e^{i\frac{\lambda}{2}(x_1+x_2)+i\mu(x_1-x_2)} \langle P'S'|\bar{\psi}(-\lambda/2) D^i(\mu) \psi(\lambda/2)|PS\rangle$$

$$= \epsilon^{ij} \Delta_j S^+ \Phi_D(x_1, x_2) + \cdots$$



Ji's OAM   canonical OAM   `potential OAM'

$$\langle \bar{\psi} \vec{b} \times \vec{D} \psi \rangle = \langle \bar{\psi} \vec{b} \times \vec{D}_{pure} \psi \rangle + \langle \bar{\psi} \vec{b} \times ig\vec{A}_{phys} \psi \rangle$$

doubly-unintegrate

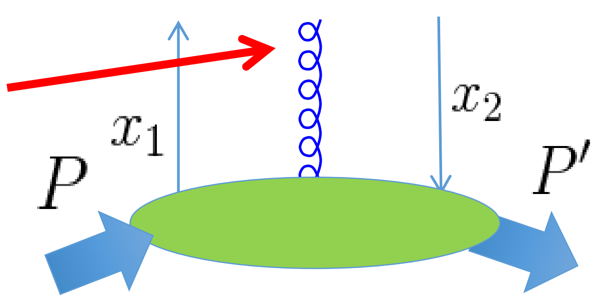
$$\Phi_D(x_1, x_2) = \delta(x_1 - x_2) L^q_{can}(x_1) + \mathcal{P}\frac{1}{x_1 - x_2}\Phi_F(x_1, x_2)$$

The gluon has zero energy,
partonic interpretation!

Canonical OAM density

YH, Yoshida (2012)

$$x_1 - x_2 = 0$$



It coincides with $L_{can}(x)$ defined via the Wigner distribution

Relation to twist-three GPD

YH. Yoshida (2012)

$$\int d\lambda e^{i\lambda x} \frac{d\lambda}{2\pi} \langle P'S' | \bar{\psi}(0) \gamma^\mu \psi(\lambda) | PS \rangle$$

$$= H_q(x) \bar{u}(P'S') \gamma^\mu u(PS) + E_q(x) \bar{u}(P'S') \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m} u(PS)$$ twist-2

$$+ G_3(x) \bar{u}(P'S') \gamma^\mu_\perp u(PS) + \cdots$$

twist-3

$$x(H_q(x) + E_q(x) + G_3(x))$$

$$= \Delta q(x) + L^q_{can}(x) + \int dx' \mathcal{P} \frac{1}{x - x'} \Big(\Phi_F(x, x') + \tilde{\Phi}_F(x, x') \Big)$$

integrate

$$\int dx\, x G_3(x) = -L^q_{\mathrm{Ji}}$$

Penttinen, Polyakov, Shuvaev, Strikman (2000)

Twist structure of OAM distributions

YH, Yoshida (2012)

Wandzura-Wilczek part

$$L^q_{can}(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'}(H_q(x')+E_q(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\tilde{H}_q(x')$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \Phi_F(x_1,x_2)\mathcal{P}\frac{3x_1-x_2}{x_1^2(x_1-x_2)^2}$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{\Phi}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^2(x_1-x_2)}\,.$$

Genuine twist-three part

$$L^g_{can}(x) = \frac{x}{2}\int_x^{\epsilon(x)} \frac{dx'}{x'^2}(H_g(x')+E_g(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\Delta G(x')$$
$$+2x\int_x^{\epsilon(x)} \frac{dx'}{x'^3}\int dX\,\Phi_F(X,x') + 2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{M}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^3(x_1-x_2)}$$
$$+2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 M_F(x_1,x_2)\mathcal{P}\frac{2x_1-x_2}{x_1^3(x_1-x_2)^2}$$

`DGLAP' equation for OAM PDF

Hagler, Schafer (1998)
YH, Nakagawa, Xiao, Yuan, Zhao (2016)

$$\frac{d}{d\ln Q^2}\begin{pmatrix} L_q(x) \\ L_g(x) \end{pmatrix} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\begin{pmatrix} \hat{P}_{qq}(z) & \hat{P}_{qg}(z) & \Delta\hat{P}_{qq}(z) & \Delta\hat{P}_{qg}(z) \\ \hat{P}_{gq}(z) & \hat{P}_{gg}(z) & \Delta\hat{P}_{gq}(z) & \Delta\hat{P}_{gg}(z) \end{pmatrix}\begin{pmatrix} L_q(x/z) \\ L_g(x/z) \\ \Delta q(x/z) \\ \Delta G(x/z) \end{pmatrix},$$

$$\begin{aligned}
\hat{P}_{qq}(z) &= C_F\left(\frac{z(1+z^2)}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right), \\
\hat{P}_{qg}(z) &= n_f z(z^2+(1-z)^2), \\
\hat{P}_{gq}(z) &= C_F(1+(1-z)^2), \\
\hat{P}_{gg}(z) &= 6\frac{(z^2-z+1)^2}{(1-z)_+} + \frac{\beta_0}{2}\delta(z-1), \\
\Delta\hat{P}_{qq}(z) &= C_F(z^2-1), \\
\Delta\hat{P}_{qg}(z) &= n_f(1-3z+4z^2-2z^3), \\
\Delta\hat{P}_{gq}(z) &= C_F(-z^2+3z-2), \\
\Delta\hat{P}_{gg}(z) &= 6(z-1)(z^2-z+2),
\end{aligned}$$

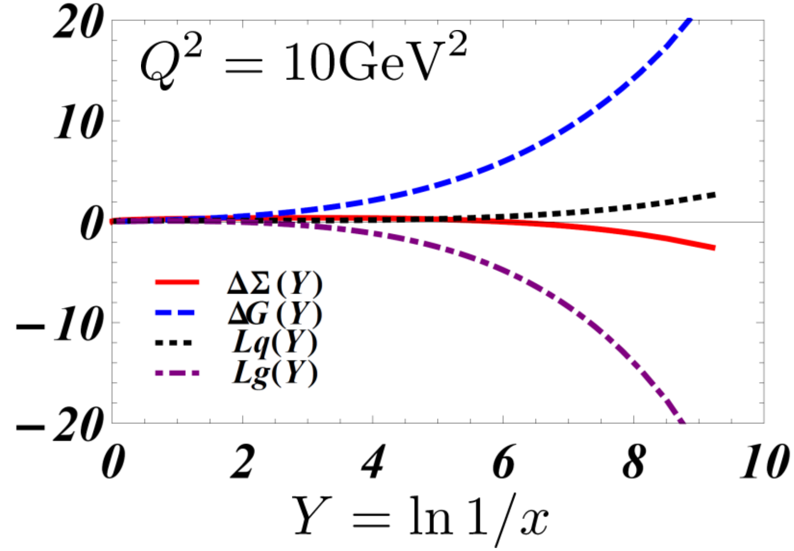
Numerical results

YH, Yang (2018)

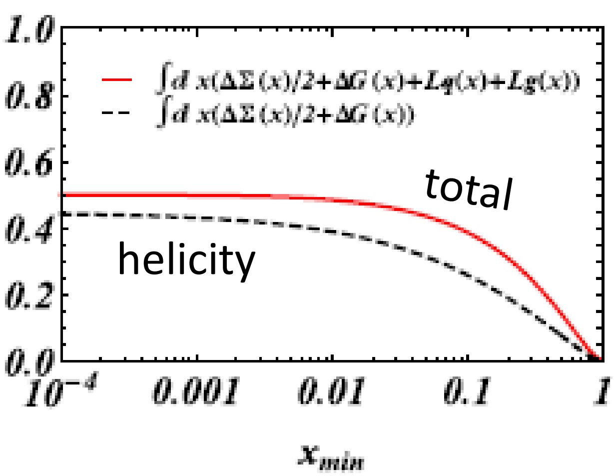
`Democratic model'

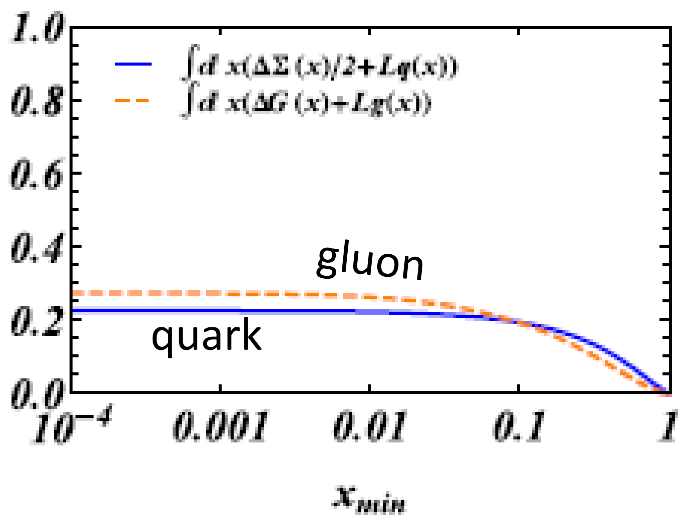
$$\Delta\Sigma(x,Q_0^2)=\frac{1}{4},\quad \Delta G(x,Q_0^2)=\frac{1}{8},$$

$$L_q(x,Q_0^2)=\frac{1}{8},\quad L_g(x,Q_0^2)=\frac{1}{8},$$



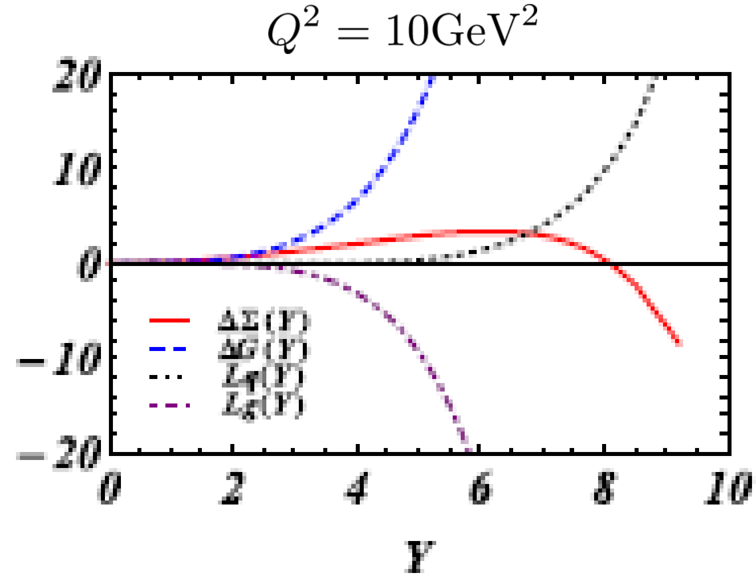
$$\int_{x_{min}}^{1} dx\left(\frac{1}{2}\Delta\Sigma(x)+\Delta G(x)+L_q(x)+L_g(x)\right)$$

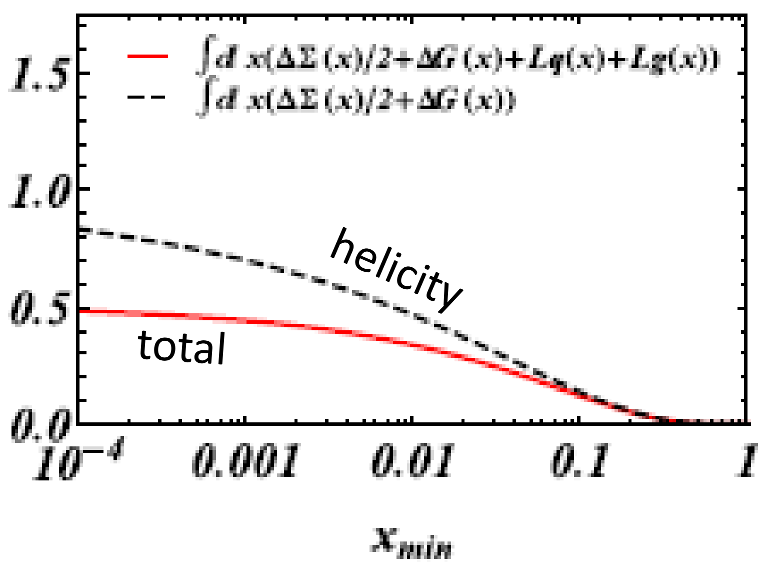


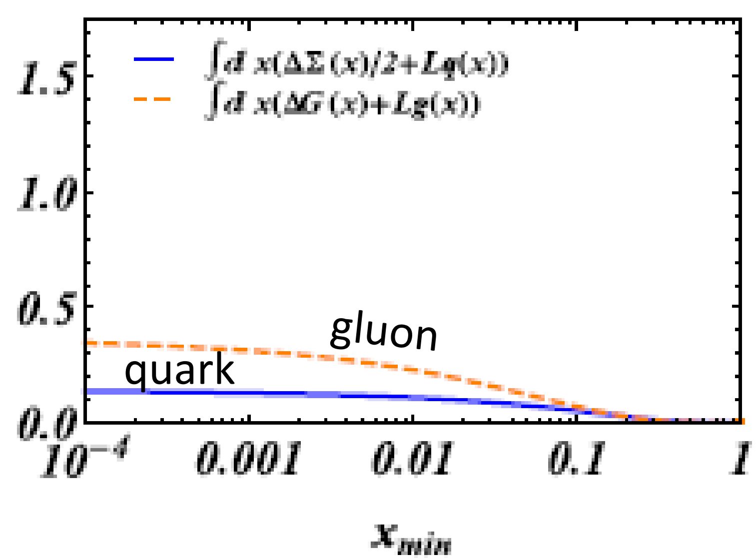


`Helicity dominance model’

$$\Delta\Sigma(x,Q_0^2) = A_q x^{-0.3}(1-x)^3$$
$$\Delta G(x,Q_0^2) = A_g x^{-0.6}(1-x)^3$$
$$L_q(x,Q_0^2) = L_g(x,Q_0^2) = 0$$







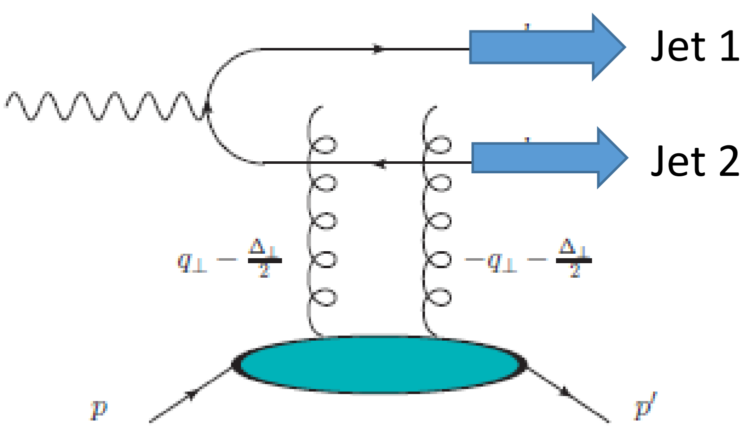
Observables for OAM

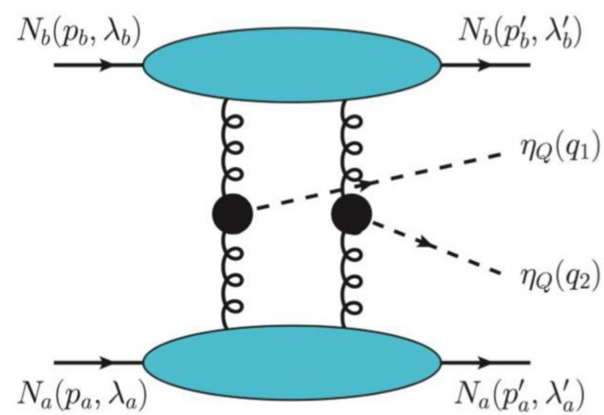
Essentially the measurement of Wigner/GTMD .
Tag two hadrons (jets) in the final state, together with the recoiling proton

Relative momentum between the two hadrons

$$W(x, k_\perp, \Delta_\perp)$$

Recoiling proton momentum

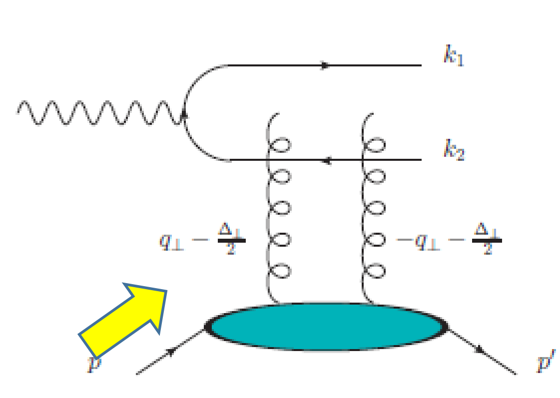




Ji, Yuan, Zhao (2017); YH, Nakagawa, Xiao, Yuan, Zhao (2017),
Bhattacharya, Metz, Zhou (2017); Bhattacharya, Metz, Ojha, Tsai, Zhou (2018)

Longitudinal single spin asymmetry in exclusive dijet production at EIC

$$d\Delta\sigma \sim$$



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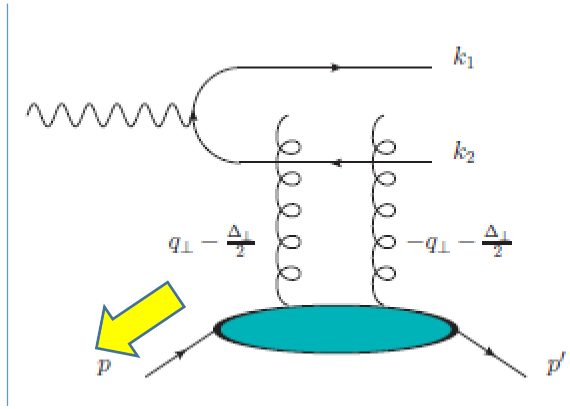

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medium-x gluon   Ji, Yuan, Zhao
small-x gluon   YH, Nakagawa, Xiao, Yuan, Zhao
quark   Bhattacharya, Metz, Ojha, Tsai, Zhou

Look for the asymmetry

$$d\Delta\sigma = \sin(\phi_P - \phi_\Delta) d\tilde{\sigma}$$

relative momentum of dijet   proton recoil momentum

From Ji, Yuan, Zhao, 1612.02438

$$\frac{d\Delta\sigma}{dydQ^2d\Omega} = \sigma_0\lambda_p\frac{2(\bar{z}-z)(\vec{q}_\perp\times\vec{\Delta}_\perp)}{\vec{q}_\perp^2+\mu^2}\left[(1-y)A_{fL}+\frac{1+(1-y)^2}{2}A_{fT}\right]$$

Compton form factor with gluon GPD

$$A_{fL} = 16\beta\ \mathrm{Im}\left(\left[\mathcal{F}_g^* + 4\xi^2\bar{\beta}\mathcal{F}_g'^*\right]\left[\mathcal{L}_g + 8\xi^2\bar{\beta}\mathcal{L}_g'\right]\right)\ ,$$

$$A_{fT} = 2\ \mathrm{Im}\left(\left[\mathcal{F}_g^* + 2\xi^2(1-2\beta)\mathcal{F}_g'^*\right]\left[\mathcal{L}_g + 2\bar{\beta}\left(\frac{1}{z\bar{z}}-2\right)\left(\mathcal{L}_g + 4\xi^2(1-2\beta)\mathcal{L}_g'\right)\right]\right)$$

$$\mathcal{L}_g(\xi,t) = \int dx\frac{x\xi}{(x+\xi-i\varepsilon)^2(x-\xi+i\varepsilon)^2}\ xL_g(x,\xi,t)$$

$$\mathcal{L}_g'(\xi,t) = \int dx\frac{x\xi}{(x+\xi-i\varepsilon)^3(x-\xi+i\varepsilon)^3}\ xL_g(x,\xi,t)$$

Extrapolation to $\xi\to 0$