QED corrections to TMD evolution

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MARIE CURIE ACTIONS

[Bacchetta, MGE arXiv:1810.02297]

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Week 2: Workshop on Transverse spin and TMDs
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1. ***Motivation***
2. ***TMDs in QCD***
3. ***Definition of TMDs in QCDxQED***
4. ***QED corrections to anomalous dimensions***
5. ***QED corrections to f_1^q at large p_T***
6. ***Conclusions & Outlook***

Motivation

Why care about QED corrections?

- Push theoretical precision to **better constrain nonperturbative Physics**
- Currently: **(N)NNLL TMD evolution** and **NNLO Wilson coefficients** for various TMDs
- Several recent theory/pheno improvements for **QED corrections to DGLAP evolution integrated PDFs and photon PDF**
- **Why not?** Formally it's in any case an interesting issue

- I will write the expansion of any quantity as:

$$A(\alpha_s, \alpha) = \sum_{n,m} A^{(n,m)} \left(\frac{\alpha_s}{4\pi}\right)^n \left(\frac{\alpha}{4\pi}\right)^m$$

QED corrections to DGLAP kernels (1/2)

Extending DGLAP equations

6 Introducing QED corrections

- *DGLAP equations dictate the evolution of PDFs*
- **EW** interactions connects **QCD** partons with **photons and leptons.**

New PDFs (leptons & photon) + *New splitting functions* → ***Modified DGLAP equations***

- **Extend original DGLAP equations to deal with new *objects*:**

$$\frac{dg}{dt} = \sum_f P_{gf} \otimes f + \sum_f P_{g\bar{f}} \otimes \bar{f} + P_{gg} \otimes g + P_{g\gamma} \otimes \gamma$$

$$\frac{d\gamma}{dt} = \sum_f P_{\gamma f} \otimes f + \sum_f P_{\gamma\bar{f}} \otimes \bar{f} + P_{\gamma g} \otimes g + P_{\gamma\gamma} \otimes \gamma$$

$$\frac{dq_i}{dt} = \sum_f P_{q_i f} \otimes f + \sum_f P_{q_i\bar{f}} \otimes \bar{f} + P_{q_i g} \otimes g + P_{q_i\gamma} \otimes \gamma$$

$$\frac{dl_i}{dt} = \sum_f P_{l_i f} \otimes f + \sum_f P_{l_i\bar{f}} \otimes \bar{f} + P_{l_i g} \otimes g + P_{l_i\gamma} \otimes \gamma$$

Kernels with fermions

Photon distributions

Lepton distributions

Kernels with photons

Kernels with leptons

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282 and arXiv:1606.02887 [hep-ph]

From a talk by GFR Sborlini

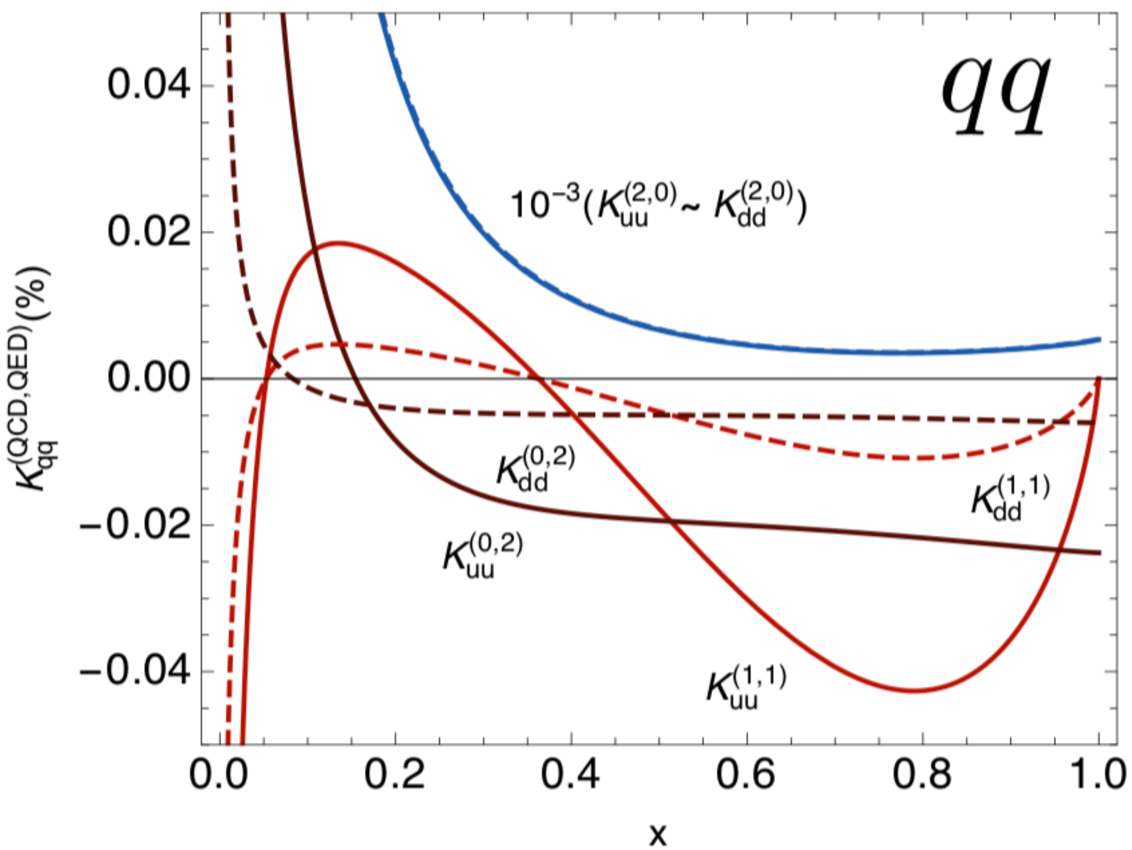
They have calculated QED corrections to DGLAP kernels at $\mathcal{O}(\alpha_s\alpha)$ and $\mathcal{O}(\alpha^2)$

QED corrections to DGLAP kernels (2/2)

- We define a ratio to quantify the effect of H.O. EW corrections

$$K_{ab}^{(i,j)} = a_S^i \, a^j \, \frac{P_{ab}^{(i,j)}(x)}{P_{ab}^{\mathrm{LO}}(x)} \quad \text{with} \quad P_{ab}^{\mathrm{LO}} = a_S \, P_{ab}^{(1,0)} + a \, P_{ab}^{(0,1)}$$

- Quark-quark splittings



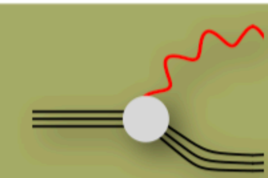
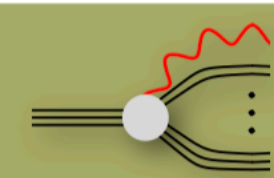
- Pure QCD contribution still dominant (x10^3)
- QED corrections introduce charge separation effects (specially at $\mathcal{O}(\alpha^2)$)
- Small corrections in intermediate x region

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282 and arXiv:1606.02887 [hep-ph]

From a talk by GFR Sborlini

Small corrections
Still, already useful at the LHC (and beyond)

Photon PDF from DIS structure functions (1/3)

	 elastic	 inelastic	LHAPDF public computer-readable form?
Gluck Pisano Reya 2002	dipole	model	✘
MRST2004qed	✘	model	✓
CT14qed_inc	dipole	model (data-constrained)	✓
Martin Ryskin 2014	dipole (only electric part)	model	✘
Harland-Lang, Khoze Ryskin 2016	dipole	model	✘
NNPDF23qed (& NNPDF30qed)	no separation; fit to data		

From a talk by G. Salam

Increasing interest on photon PDF
Several PDF groups already included it in their standard fits

Photon PDF from DIS structure functions (2/3)

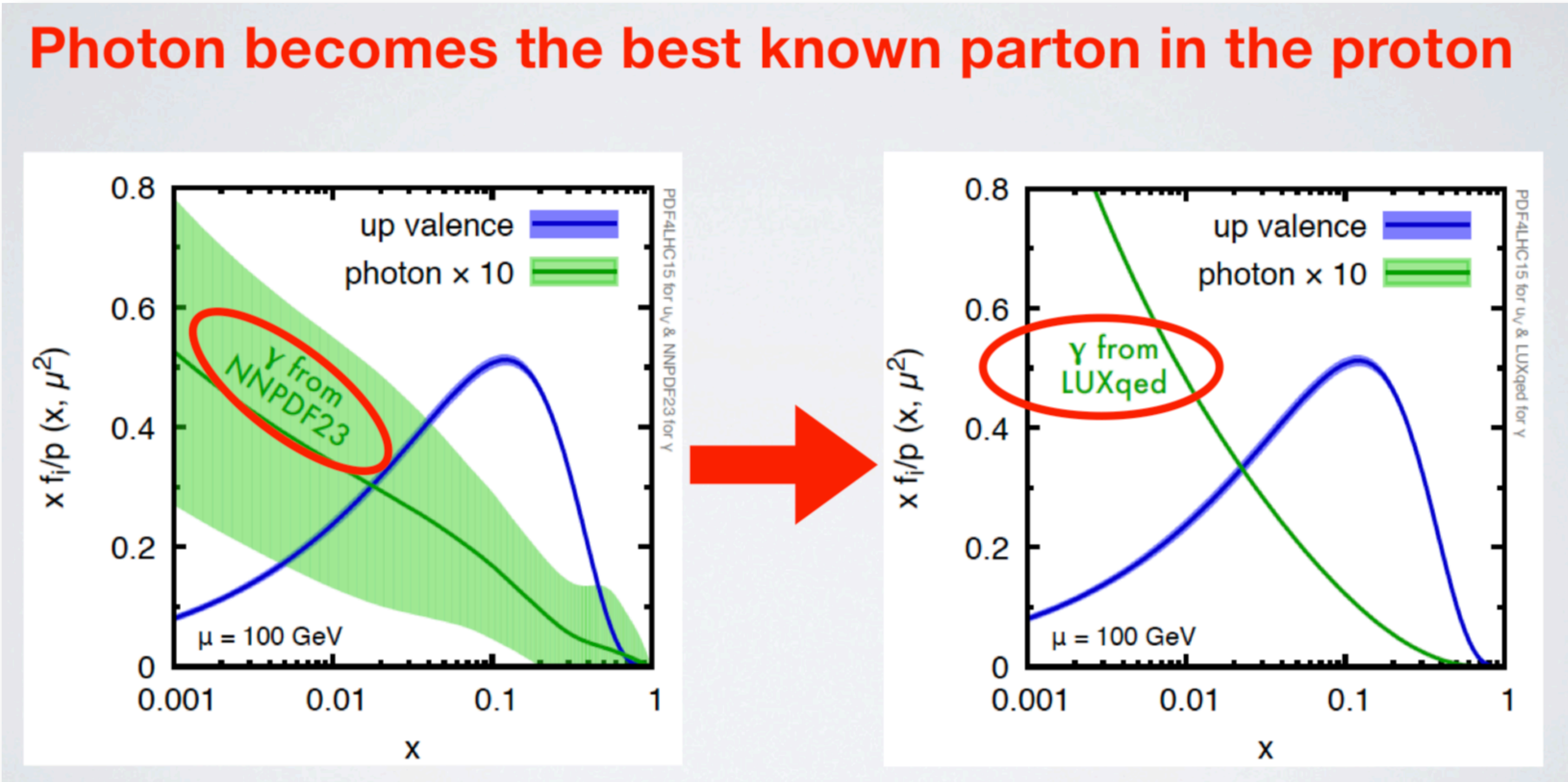
LUXqed approach

[Manohar, Nason, Salam, Zanderighi JHEP12(2017)046, PRL117(2016)]

- Main idea: write the cross section for an **imaginary BSM heavy-lepton production** process (which couples to SM electron and photon) **in two ways:**
  - in terms of **DIS structure functions F_2 and F_L**
  - in terms of a **photon PDF in collinear factorization**
- **Photon PDF can then be written in terms of F_2 and F_L** (model independent):

$$x f_{\gamma/p}(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \left\{ \int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \left[\left(z p_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, Q^2) - z^2 F_L\left(\frac{x}{z}, Q^2\right) \right] - \alpha^2(\mu^2) z^2 F_2\left(\frac{x}{z}, \mu^2\right) \right\}$$

Photon PDF from DIS structure functions (3/3)

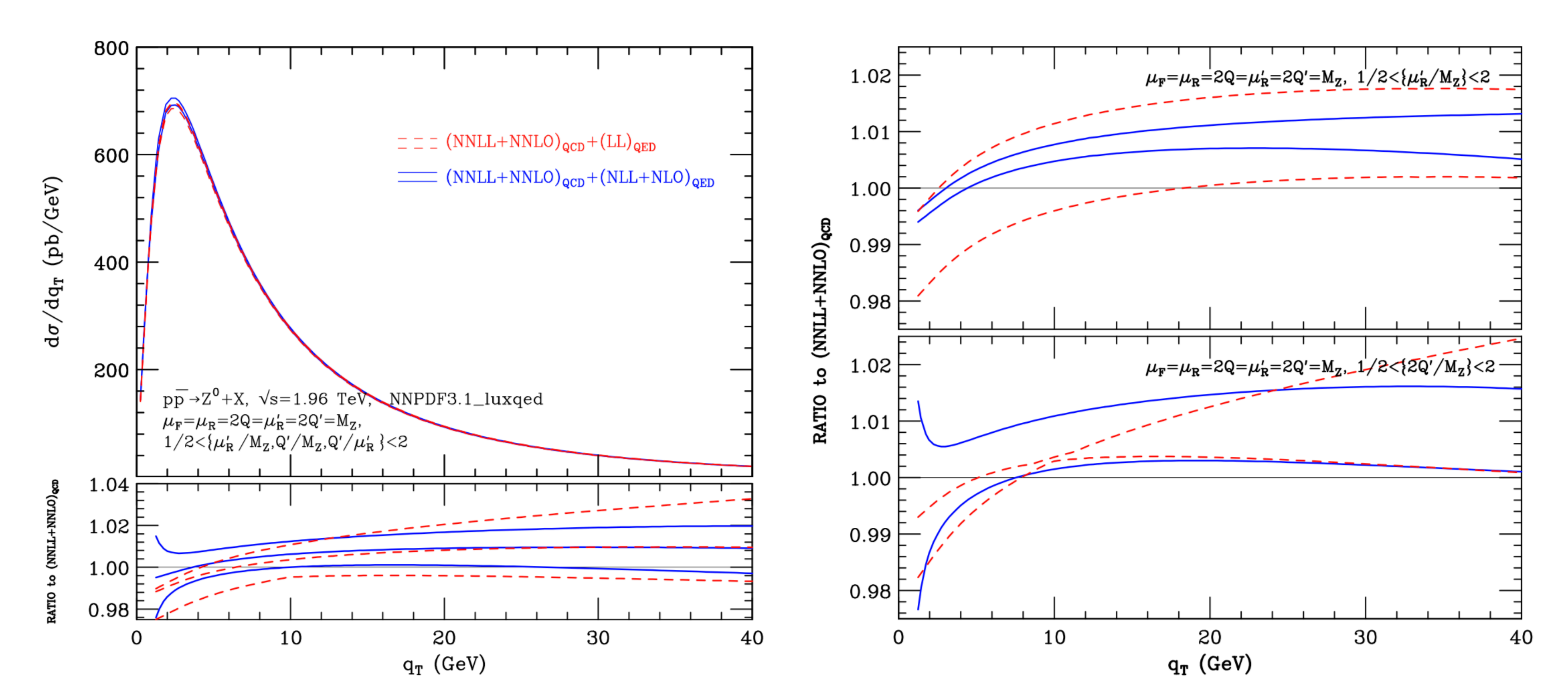


From a talk by G. Zanderighi

- Similar comparisons with other fits (e.g. CT14, MRST2004)

The **EIC** will potentially better measure **F_2 and F_L**,
and thus **better constrain the photon PDF**

Z boson p_T distribution



[Cieri, Ferrera, Sborlini JHEP08(2018)]

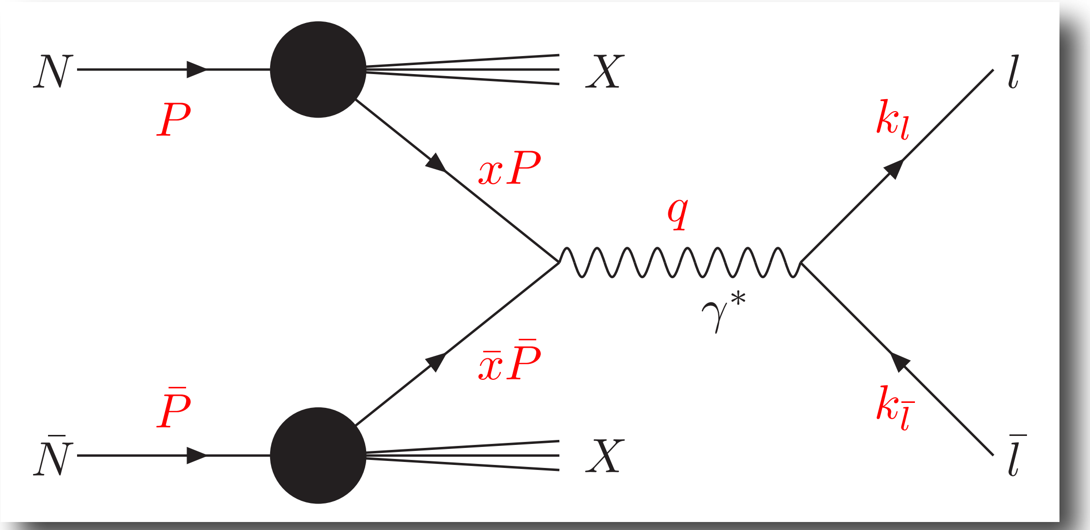
- They include QED corrections in their approach (corrections and photon PDF)
- They find a 1-4% impact on Z production at Tevatron/LHC

We do it systematically, by considering the operator definition of TMDs and obtaining the new pieces for all (un)polarized quark/gluon TMD PDFs and FFs

TMDs in QCD

Definition of TMDs in QCD (1/2)

I take Drell-Yan production as an example



$$d\sigma = \sigma_0(\mu)\, H(Q^2,\mu)\, dy\, \frac{d^2 q_\perp}{(2\pi)^2} \int d^2 y_\perp\, e^{-i q_\perp \cdot y_\perp}\, J_n(x_A, y_\perp, \mu)\, S(y_\perp,\mu)\, J_{\bar n}(x_B, y_\perp, \mu)$$

$$J_n(0^+, y^-, \vec{y}_\perp) = \frac{1}{2}\sum_{\sigma_1} \langle N_1(P,\sigma_1)|\, \bar\chi_n(0^+, y^-, \vec{y}_\perp)\, \frac{\bar{n}\!\!\!/}{2}\, \chi_n(0)\, |N_1(P,\sigma_1)\rangle|_{\text{zb subtracted}}$$

$$J_{\bar n}(y^+, 0^-, \vec{y}_\perp) = \frac{1}{2}\sum_{\sigma_2} \langle N_2(\bar P,\sigma_2)|\, \bar\chi_{\bar n}(0)\, \frac{n\!\!\!/}{2}\, \chi_{\bar n}(y^+, 0^-, \vec{y}_\perp)\, |N_2(\bar P,\sigma_2)\rangle|_{\text{zb subtracted}}$$

$$S(0^+, 0^-, \vec{y}_\perp) = \frac{Tr}{N_c} \langle 0|\, [S_n^{T\dagger} S_{\bar n}^T](0^+, 0^-, \vec{y}_\perp) [S_{\bar n}^{T\dagger} S_n^T](0)\, |0\rangle$$

$$\chi_n = W_n^\dagger \xi_n$$

$$W_n(x) = \bar P \exp\left[\int_{-\infty}^0 ds\, \bar n \cdot A_n^a(x + \bar n s) t^a\right]$$

$$S_n(x) = P \exp\left[\int_{-\infty}^0 ds\, n \cdot A_s^a(x + n s) t^a\right]$$

Definition of TMDs in QCD (2/2)

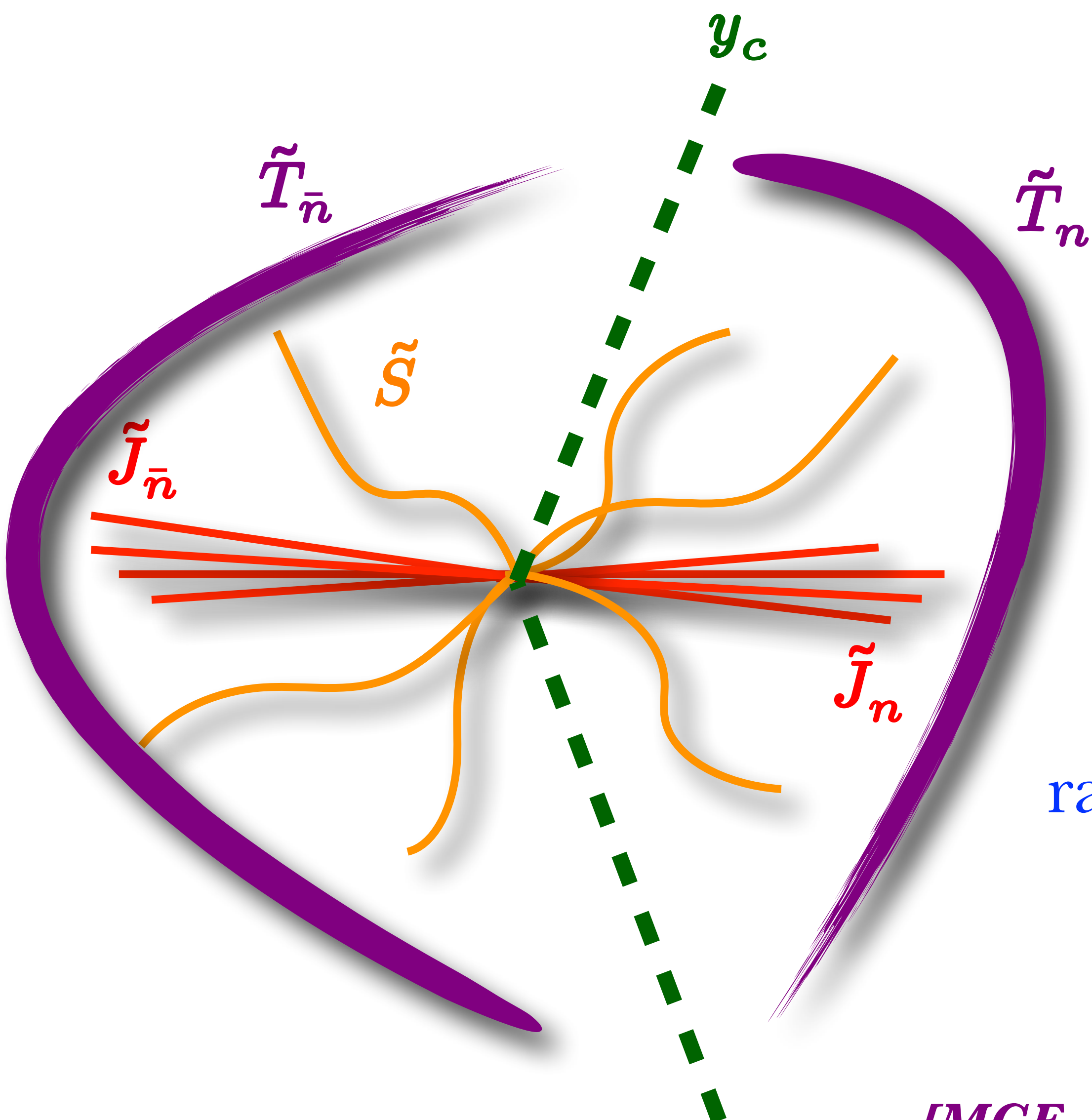
$$k_n \sim Q(1, \lambda^2, \lambda)$$
$$k_{\bar{n}} \sim Q(\lambda^2, 1, \lambda)$$
$$k_s \sim Q(\lambda, \lambda, \lambda)$$

$$y = \frac{1}{2} \ln \left| \frac{k^+}{k^-} \right|$$

Different rapidities
(mixed under boosts)

$$k_n^2 \sim k_{\bar{n}}^2 \sim k_s^2 \sim Q^2\lambda^2$$

Same invariant mass!



Cancel spurious rapidity divergences

$$\zeta_A = (p^+)^2 e^{-2y_c}$$

$$\tilde{T}_n(x_A, \vec{k}_{n\perp}, S_A; \zeta_A, \mu) = \tilde{J}_n \sqrt{\tilde{S}}$$

$$\tilde{T}_{\bar{n}}(x_B, \vec{k}_{\bar{n}\perp}, S_B; \zeta_B, \mu) = \tilde{J}_{\bar{n}} \sqrt{\tilde{S}}$$

$$\zeta_B = (\bar{p}^-)^2 e^{+2y_c}$$

[MGE, Idilbi, Scimemi JHEP1207(2012), PLB726(2013), PRD90(2014)]
[MGE, Kasemets, Mulders, Pisano JHEP1507(2015)]
[Collins' book '11]

Evolution of TMDs in QCD

TMDs depend on **two scales**: renormalization and rapidity scales

- The dependence on the **renormalization scale** is:

$$\frac{d}{d\ln\mu}\ln\tilde{T}^{[pol]}_{j\leftrightarrow A}(x,b_\perp,S_A;\zeta_A,\mu)=\gamma_j\left(\alpha_s(\mu),\ln\frac{\zeta_A}{\mu^2}\right)$$

Known at 3-loops. Numerical at 4-loops

$$\gamma_j\left(\alpha_s(\mu),\ln\frac{\zeta_A}{\mu^2}\right)=-\Gamma^j_{cusp}(\alpha_s(\mu))\ln\frac{\zeta_A}{\mu^2}-\gamma^j_{nc}(\alpha_s(\mu))$$

[Moch, Vermaseren, Vogt JHEP0508(2005), NPB688(2004)]
[Moch, Ruijl, Ueda, Vermaseren, Vogt JHEP10(2017)]

- The dependence on the **rapidity scale** is:

$$\frac{d}{d\ln\zeta_A}\ln\tilde{T}^{[pol]}_{j\leftrightarrow A}(x,b_\perp,S_A;\zeta_A,\mu)=-D_j(b_T;\mu)$$

Known at NNLO

$$\frac{dD_j}{d\ln\mu}=\Gamma^j_{cusp}(\alpha_s(\mu))$$

Cusp does **<u>not</u>** completely determine D_j

Indirect NLO: [Becher, Neubert EPJC71(2011)]
Direct NLO: [MGE, Scimemi, Vladimirov PRD93(2016)]

[Li, Zhu PRL118(2017)]
[Vladimirov PRL118(2017)]

TMDs in full glory

$$\tilde{T}_{i\leftrightarrow A}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \tilde{C}^T_{i\leftrightarrow j}(x,\hat{b}_T;\mu_b^2,\mu_b) \otimes t_{j\leftrightarrow A}(x;\mu_b)$$
$$\times \exp\left[\int_{\mu_b}^{\mu} \frac{d\hat{\mu}}{\hat{\mu}} \gamma_j\left(\alpha_s(\hat{\mu}), \ln\frac{\zeta}{\hat{\mu}^2}\right)\right] \left(\frac{\zeta}{\mu_b^2}\right)^{-D_j(\hat{b}_T;\mu_b)}$$
$$\times \tilde{T}^{NP}_{i\leftrightarrow A}(x,b_T;\zeta)$$

- General philosophy: **only parametrize what cannot be calculated**
- **Nonperturbative** part of **Dj is universal** (for all (un)polarized TMDs)
- Higher-order calculations allow better determination of nonperturbative ingredients
- At large and low bT we need cutoffs (qT<Lambda and qT>Q regions)
- There are subtleties with the evolution path: ***[Scimemi, Vladimirov JHEP08(2018)003]***
- The determination of nonperturbative pieces is not easy (Fourier transform mixes regions, overlap of perturbative and non-perturbative)

The higher the theoretical precision, the better!

TMDs in QCDxQED

Definition of quark TMDs in QCDxQED

- We can now consider Drell-Yan where colliding quarks exchange/emit also photons!
- Factorization follows the same steps as in pure QCD…

$$J_{i/P}(x,k_{nT}) = \frac{1}{2}\int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{-i(\frac{1}{2}y^- xP^+ - y_\perp\cdot k_{n\perp})}$$

$$\times \frac{1}{2}\sum_S \langle PS|\left[\bar\xi_n W_n^T \widehat{W}_{i,n}^T\right](0^+,y^-,y_\perp)\frac{\bar{n}\!\!\!/}{2}\left[\widehat{W}_{i,n}^{T\dagger} W_n^{T\dagger}\xi_n\right](0)\,|PS\rangle$$

new photon Wilson lines

$$S_i(k_{sT}) = \int\frac{d^2y_\perp}{(2\pi)^2}e^{iy_\perp\cdot k_{s\perp}}\frac{Tr_c}{N_c}\langle 0|\left[S_n^{T\dagger}S_{\bar n}^T \widehat{S}_{i,n}^{T\dagger}\widehat{S}_{i,\bar n}^T\right](0^+,0^-,y_\perp)\left[S_{\bar n}^{T\dagger}S_n^T\widehat{S}_{i,\bar n}^{T\dagger}\widehat{S}_{i,n}^T\right](0)\,|0\rangle$$

$$W_n(x) = \bar{P}\exp\left[ig_s\int_{-\infty}^{0} ds\,\bar{n}\cdot A_n(x+s\bar{n})\right]$$

photon field

$$\widehat{W}_{i,n}(x) = \exp\left[ieQ_i\int_{-\infty}^{0} ds\,\bar{n}\cdot B_n(x+s\bar{n})\right]$$

- New photon Wilson lines introduce rapidity divergences, which cancel in the TMDs as in QCD
- (Un)polarized TMDPDFs and TMDFFs defined similarly

QCDxQED evolution of quark TMDs

- Evolution equations are:

$$\frac{d}{d\ln\mu}\ln\tilde{F}_i(x,b_T;\zeta,\mu) \equiv \gamma_{F_i}\left(\alpha_s(\mu),\alpha(\mu),\ln\frac{\zeta}{\mu^2}\right) = -\gamma_i(\alpha_s(\mu),\alpha(\mu)) - \Gamma_i(\alpha_s(\mu),\alpha(\mu))\ln\frac{\zeta}{\mu^2}$$

$$\frac{d}{d\ln\zeta}\ln\tilde{F}_i(x,b_T;\zeta,\mu) = -D_i(L_\perp;\alpha_s(\mu),\alpha(\mu))$$

$$L_\perp = \ln(\mu^2 b_T^2 e^{2\gamma_E}/4)$$

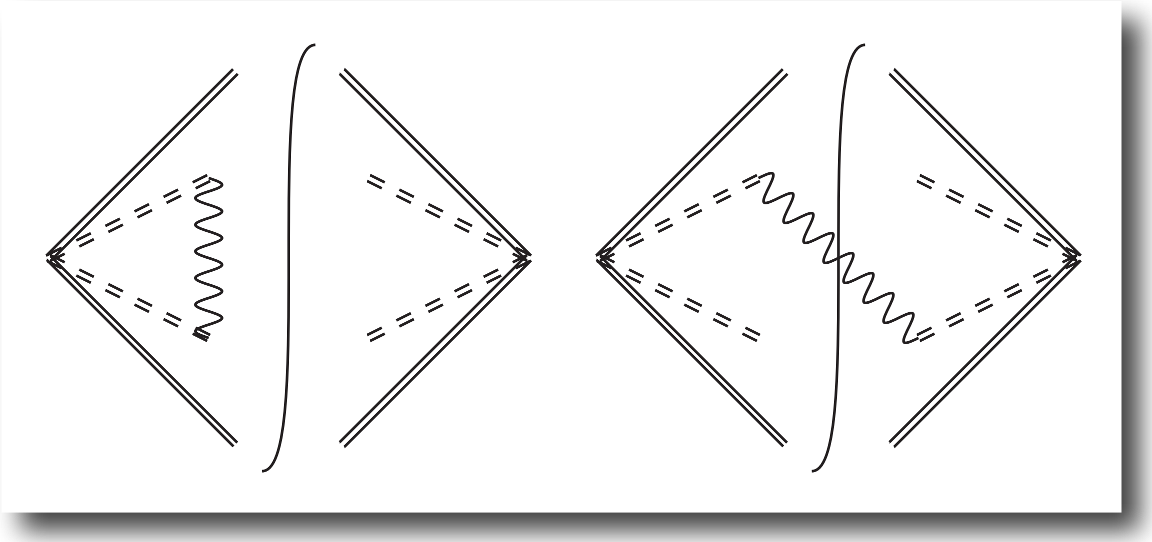
$$\frac{d}{d\ln\mu}D_i(L_\perp;\alpha_s(\mu),\alpha(\mu)) = \Gamma_i(\alpha_s(\mu),\alpha(\mu))$$

- QCD vs QED scales are not distinguished for simplicity
- QED corrections break flavor universality of pure QCD evolution
- There is no relation between alpha_s and alpha that holds for all scales (mub is integrated over)
- So either we fix a relation, either we consider each contribution independently

QCDxQED evolution of quark TMDs

- **Pure QED** corrections to the soft function at **leading order** give:

$$S_i(k_{sT}) = \int \frac{d^2 y_\perp}{(2\pi)^2} e^{i y_\perp \cdot k_{s\perp}} \frac{Tr_c}{N_c} \langle 0| \left[S_n^{T\dagger} S_{\bar{n}}^T \widehat{S}_{i,n}^{T\dagger} \widehat{S}_{i,\bar{n}}^T \right] (0^+, 0^-, y_\perp) \left[S_{\bar{n}}^{T\dagger} S_n^T \widehat{S}_{i,\bar{n}}^{T\dagger} \widehat{S}_{i,n}^T \right] (0) |0\rangle$$



Double lines: gluon Wilson lines
Double dashed: photon Wilson lines

$$\tilde{S}\left(b_T; \mu; \delta^+, \delta^-\right) = 1 + \frac{\alpha Q_i^2}{2\pi} \left[-\frac{2}{\varepsilon_{UV}^2} + \frac{2}{\varepsilon_{UV}} \ln\frac{\delta^+ \delta^-}{\mu^2} + L_T^2 + 2 L_T \ln\frac{\delta^+ \delta^-}{\mu^2} + \frac{\pi^2}{6} \right]$$

$$D_i^{(0,1)}(L_\perp) = \frac{\Gamma_i^{(0,1)}}{2} L_\perp \qquad \Gamma_i^{(0,1)} = 4 Q_i^2$$

QCDxQED evolution of quark TMDs

- **Pure QED** corrections to the soft function at **NLO** give:

$$D_i^{(0,2)}(L_\perp) = \frac{\Gamma_i^{(0,1)}}{4\hat{\beta}^{(0,1)}}\left(\hat{\beta}^{(0,1)}L_\perp\right)^2 + \left(\frac{\Gamma_i^{(0,2)}}{2\hat{\beta}^{(0,1)}}\right)\left(\hat{\beta}^{(0,1)}L_\perp\right) + D_i^{(0,2)}(0)$$

$$D^{(2,0)}(0) = C_F C_A\left(\frac{404}{27} - 14\zeta_3\right) - \left(\frac{112}{27}\right) C_F T_F n_f$$

$$D_i^{(0,2)}(0) = -\left(\frac{112}{27}\right) Q_i^2 [N_c \sum_j^{n_f} Q_j^2 + n_l Q_l^2],$$

$$\Gamma_i^{(0,2)}/\Gamma_i^{(0,1)} = -\frac{20}{9}[N_c \sum_j^{n_f} Q_j^2 + n_l Q_l^2]$$

$$\Gamma^{(2,0)}/\Gamma^{(1,0)} = \left(\frac{67}{9} - \frac{\pi^2}{3}\right) C_A - \frac{20}{9} T_F n_f$$

QCDxQED evolution of quark TMDs

- Beta functions:

$$\frac{d\ln\alpha_s}{d\ln\mu^2} \equiv \beta(\alpha_s(\mu),\alpha(\mu)) = -\sum_{n,m}\beta^{(n,m)}\left(\frac{\alpha_s}{4\pi}\right)^n\left(\frac{\alpha}{4\pi}\right)^m$$

$$\frac{d\ln\alpha}{d\ln\mu^2} \equiv \hat{\beta}(\alpha_s(\mu),\alpha(\mu)) = -\sum_{n,m}\hat{\beta}^{(n,m)}\left(\frac{\alpha_s}{4\pi}\right)^n\left(\frac{\alpha}{4\pi}\right)^m$$

Mixed QCDxQED and pure QED coefficients

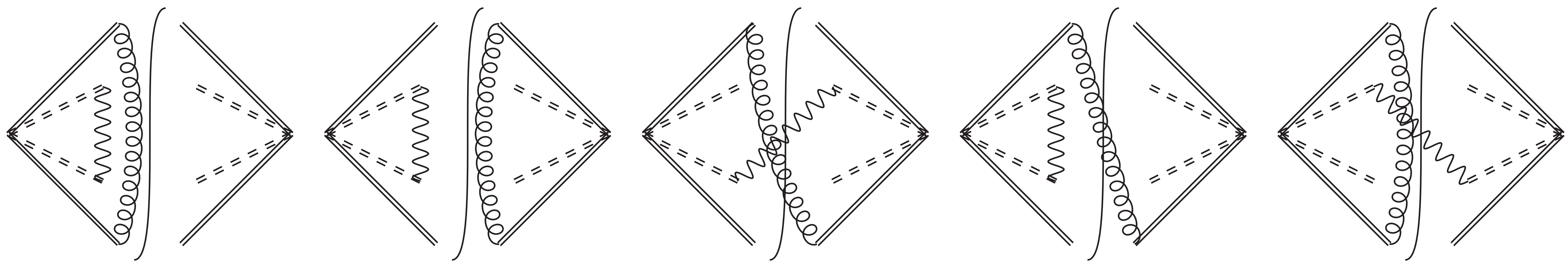
$$\beta^{(1,1)} = -2\sum_j^{n_f} Q_j^2$$

$$\hat{\beta}^{(0,1)} = -\frac{4}{3}[N_c\sum_j^{n_f} Q_j^2 + n_l Q_l^2]$$

$$\hat{\beta}^{(0,2)} = -4[N_c\sum_j^{n_f} Q_j^4 + n_l Q_l^4]$$

$$\hat{\beta}^{(1,1)} = -4C_F N_c\sum_j^{n_f} Q_j^2$$

QCDxQED evolution of quark TMDs



- **Soft function** can be partially ***factorized***:

$$S_i(k_{sT}) = \int \frac{d^2 y_\perp}{(2\pi)^2} e^{i y_\perp \cdot k_{s\perp}} \tilde{S}^{QCD}(y_T)\, \tilde{S}_i^{QED}(y_T) + O(\alpha_s^n \alpha^m)\Big|_{n\cdot m>1}$$

$$\tilde{S}^{QCD}(y_T) = \frac{Tr_c}{N_c} \langle 0| \left[S_n^{T\dagger} S_{\bar{n}}^T\right](0^+, 0^-, y_\perp) \left[S_{\bar{n}}^{T\dagger} S_n^T\right](0) |0\rangle$$

$$\tilde{S}_i^{QED}(y_T) = \langle 0| \left[\widehat{S}_{i,n}^{T\dagger} \widehat{S}_{i,\bar{n}}^T\right](0^+, 0^-, y_\perp) \left[\widehat{S}_{i,\bar{n}}^{T\dagger} \widehat{S}_{i,n}^T\right](0) |0\rangle$$

QCDxQED evolution of quark TMDs

- Cancellation of QCD & QED rapidity divergences only possible if:

$$\ln\tilde{S}_i(b_T) = A_i(L_\perp;\alpha_s,\alpha) + 2D_i(L_\perp;\alpha_s,\alpha)\ln\frac{\delta^+\delta^-}{\mu^2}$$

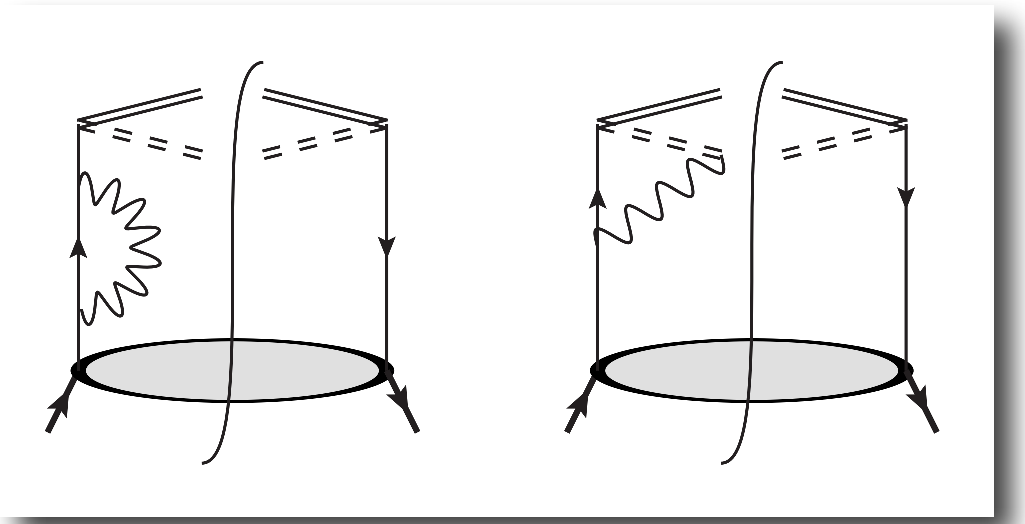
$$\ln\tilde{S}^{QCD}(b_T) = A^{QCD}(L_\perp;\alpha_s,\alpha) + 2D^{QCD}(L_\perp;\alpha_s,\alpha)\ln\frac{\delta^+\delta^-}{\mu^2}$$

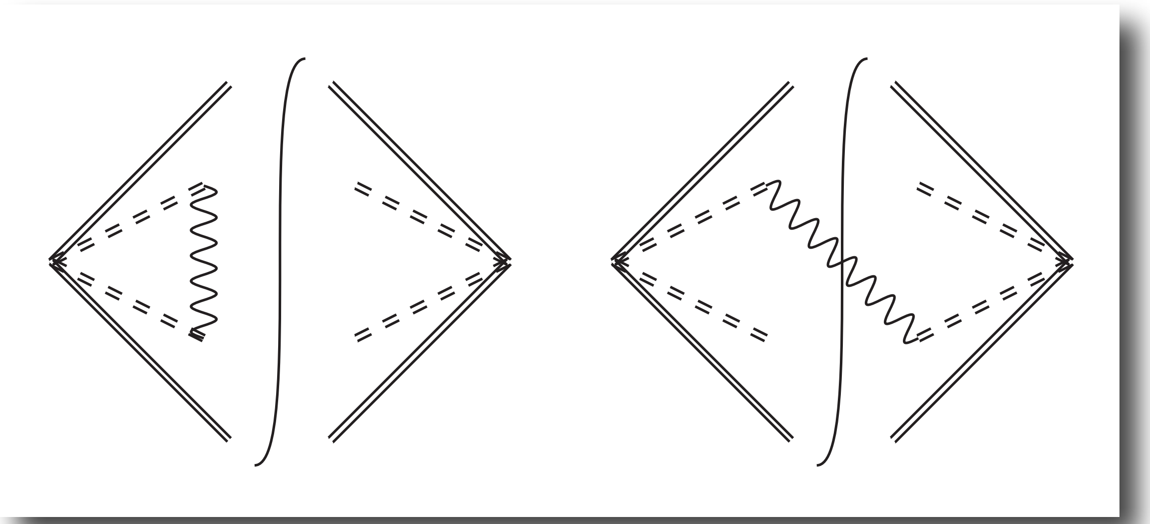
$$\ln\tilde{S}_i^{QED}(b_T) = A_i^{QED}(L_\perp;\alpha_s,\alpha) + 2D_i^{QED}(L_\perp;\alpha_s,\alpha)\ln\frac{\delta^+\delta^-}{\mu^2}$$

$$D_i^{(1,1)}(L_\perp) = 0$$

QCDxQED evolution of quark TMDs

- **Pure QED** virtual corrections at **1-loop**:





$$\gamma^{(1,0)} = -6C_F$$

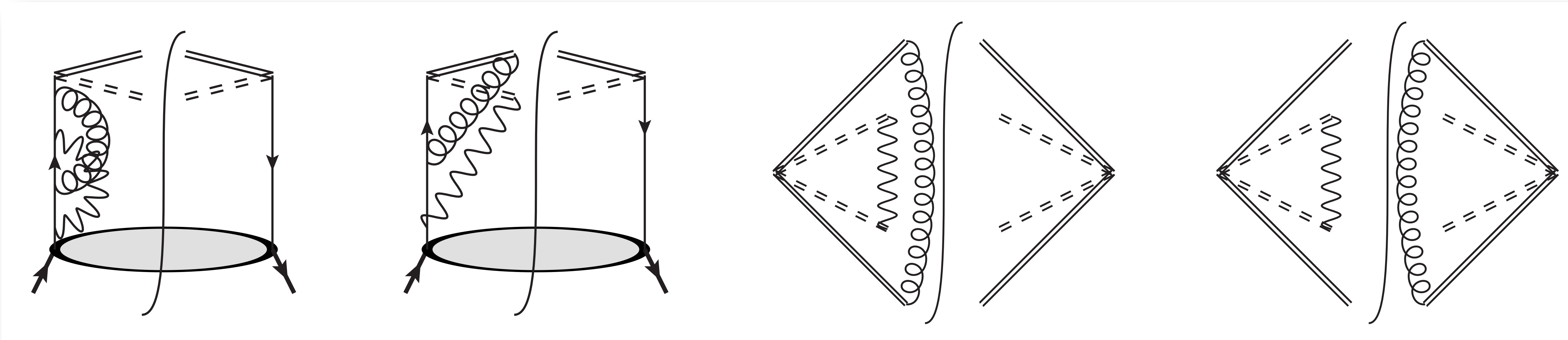
Calculation is analogous to the one in pure QCD

$$\gamma_i^{(0,1)} = -6Q_i^2$$
$$\Gamma_i^{(0,1)} = 4Q_i^2$$

$$\Gamma^{(1,0)} = 4C_F$$

QCDxQED evolution of quark TMDs

- **Mixed QCD-QED** virtual corrections for non-cusp:



$$\gamma^{(2,0)} = C_F^2\left(-3+4\pi^2-48\zeta_3\right) + C_F C_A\left(-\frac{961}{27}-\frac{11\pi^2}{3}+52\zeta_3\right)$$
$$+\,C_F T_F n_f\left(\frac{260}{27}+\frac{4\pi^2}{3}\right)$$

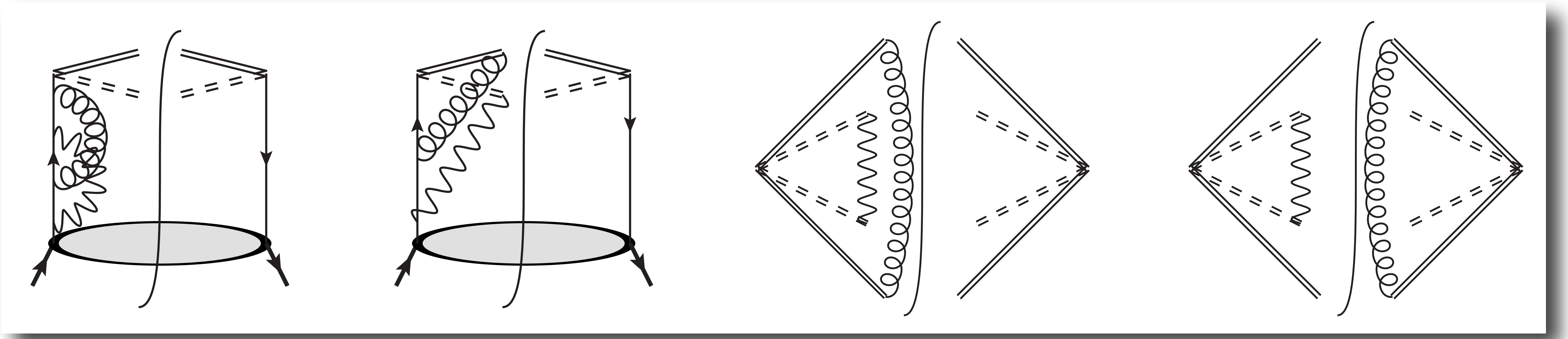
no fermion loops possible at this order

factor 2 needed because of 2 ways of replacing internal g-g with g-photon and photon-g

$$\gamma_i^{(1,1)} = 2C_F Q_i^2\left(-3+4\pi^2-48\zeta_3\right)$$

QCDxQED evolution of quark TMDs

- **Mixed QCD-QED** virtual corrections for cusp:



$$\Gamma^{(2,0)}/\Gamma^{(1,0)} = \left(\frac{67}{9} - \frac{\pi^2}{3}\right) C_A - \frac{20}{9} T_F n_f$$

$$\Gamma_i^{(1,1)} = 0$$

no fermion loops possible at this order

Consistent with

[Kilgore-Sturm PRD85(2012)]
[Kilgore EPJC73(2013)]

QCDxQED evolution of quark TMDs

- **Pure QED** virtual corrections at **2-loops**:

$$\gamma^{(2,0)} = C_F^2\left(-3+4\pi^2-48\zeta_3\right)+C_F C_A\left(-\frac{961}{27}-\frac{11\pi^2}{3}+52\zeta_3\right)$$
$$+C_F T_F n_f\left(\frac{260}{27}+\frac{4\pi^2}{3}\right)$$

$$\gamma_i^{(0,2)} = Q_i^4\left(-3+4\pi^2-48\zeta_3\right)+Q_i^2[N_c\sum_j^{n_f} Q_j^2+n_l Q_l^2]\left(\frac{260}{27}+\frac{4\pi^2}{3}\right)$$

$$\Gamma_i^{(0,2)}/\Gamma_i^{(0,1)} = -\frac{20}{9}[N_c\sum_j^{n_f} Q_j^2+n_l Q_l^2]$$

$$\Gamma^{(2,0)}/\Gamma^{(1,0)} = \left(\frac{67}{9}-\frac{\pi^2}{3}\right)C_A-\frac{20}{9}T_F n_f$$

QCDxQED evolution of quark TMDs: summary

$\mathcal{O}(\alpha)$

$$D_i^{(0,1)}(L_\perp) = \frac{\Gamma_i^{(0,1)}}{2} L_\perp$$

$$\gamma_i^{(0,1)} = -6 Q_i^2$$

$$\Gamma_i^{(0,1)} = 4 Q_i^2$$

$\mathcal{O}(\alpha_s \alpha)$

$$D_i^{(1,1)}(L_\perp) = 0$$

$$\gamma_i^{(1,1)} = 2 C_F Q_i^2 \left(-3 + 4\pi^2 - 48\zeta_3\right)$$

$$\Gamma_i^{(1,1)} = 0$$

$\mathcal{O}(\alpha^2)$

$$D_i^{(0,2)}(L_\perp) = \frac{\Gamma_i^{(0,1)}}{4\hat{\beta}^{(0,1)}} \left(\hat{\beta}^{(0,1)} L_\perp\right)^2 + \left(\frac{\Gamma_i^{(0,2)}}{2\hat{\beta}^{(0,1)}}\right) \left(\hat{\beta}^{(0,1)} L_\perp\right) + D_i^{(0,2)}(0)$$

$$D_i^{(0,2)}(0) = -\left(\frac{112}{27}\right) Q_i^2 [N_c \sum_j^{n_f} Q_j^2 + n_l Q_l^2],$$

$$\Gamma_i^{(0,2)}/\Gamma_i^{(0,1)} = -\frac{20}{9} [N_c \sum_j^{n_f} Q_j^2 + n_l Q_l^2]$$

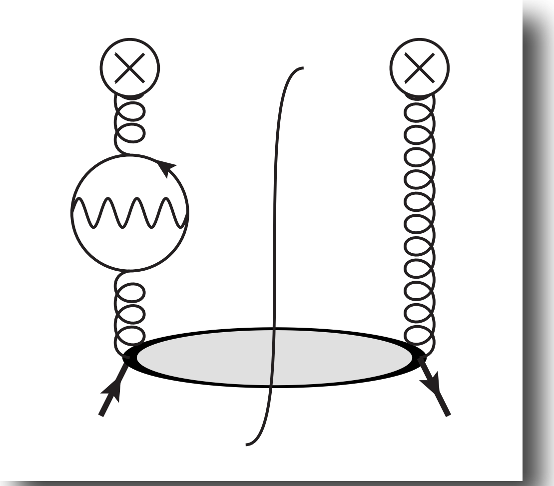
$$\gamma_i^{(0,2)} = Q_i^4 \left(-3 + 4\pi^2 - 48\zeta_3\right) + Q_i^2 [N_c \sum_j^{n_f} Q_j^2 + n_l Q_l^2] \left(\frac{260}{27} + \frac{4\pi^2}{3}\right)$$

$$\Gamma_i^{(0,2)}/\Gamma_i^{(0,1)} = -\frac{20}{9} [N_c \sum_j^{n_f} Q_j^2 + n_l Q_l^2]$$

Evolution of gluon TMDs in QCDxQED

- Gluon TMDs are **defined as in pure QCD**: **no photon Wilson lines**!
- Glue-Glue bilocal operator is already QED gauge invariant
- QED effects appear only as higher-order corrections

- **No pure QED** corrections for anomalous dimension nor D term
- **Mixed QCD-QED** virtual corrections for anomalous dimension:



No contribution of soft function at this order

$$\gamma_g^{(2,0)} = 2C_A^2\left(-\frac{692}{27}+\frac{11\pi^2}{18}+2\zeta_3\right)+2C_AT_Fn_f\left(\frac{256}{27}-\frac{2\pi^2}{9}\right)+8C_FT_Fn_f$$

$$\gamma_g^{(1,1)} = 8T_F\sum_j^{n_f}Q_j^2$$

(No factor Nc here)

- **No mixed QCD-QED** corrections for cusp nor D (soft function zero at this order)

QCDxQED evolution of gluon TMDs: summary

$\mathcal{O}(\alpha)$

$$D_g^{(0,1)}(L_\perp) = 0$$
$$\gamma_g^{(0,1)} = 0$$
$$\Gamma_g^{(0,1)} = 0$$

$\mathcal{O}(\alpha_s \alpha)$

$$D_g^{(1,1)} = 0$$
$$\gamma_g^{(1,1)} = 8T_F \sum_j^{n_f} Q_j^2$$
$$\Gamma_g^{(1,1)} = 0$$

$\mathcal{O}(\alpha^2)$

$$D_g^{(0,2)} = 0$$
$$\gamma_g^{(0,2)} = 0$$
$$\Gamma_g^{(0,2)} = 0$$

QED corrections to f_1^q at large p_T

Refactorization of TMDs in QCD

- TMDs contain perturbative information when transverse momentum is large:

$$\tilde{T}_{i\leftrightarrow A}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \tilde{C}^T_{i\leftrightarrow j}(x,b_T;\zeta,\mu) \otimes t_{j\leftrightarrow A}(x;\mu) + O(b_T\Lambda_{QCD})$$

- For each TMD we have a different OPE. For example:

$$\tilde{f}_1^{q/A}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^f_{q/j}(\bar{x},b_T;\zeta,\mu)\, f_{j/A}(x/\bar{x};\mu)$$

$$\tilde{h}_1^{\perp g/A\,(2)}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^h_{g/j}(\bar{x},b_T;\zeta,\mu)\, f_{j/A}(x/\bar{x};\mu)$$

$$\tilde{g}_{1L}^{g/A}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^g_{g/j}(\bar{x},b_T;\zeta,\mu)\, g_{j/A}(x/\bar{x};\mu)$$

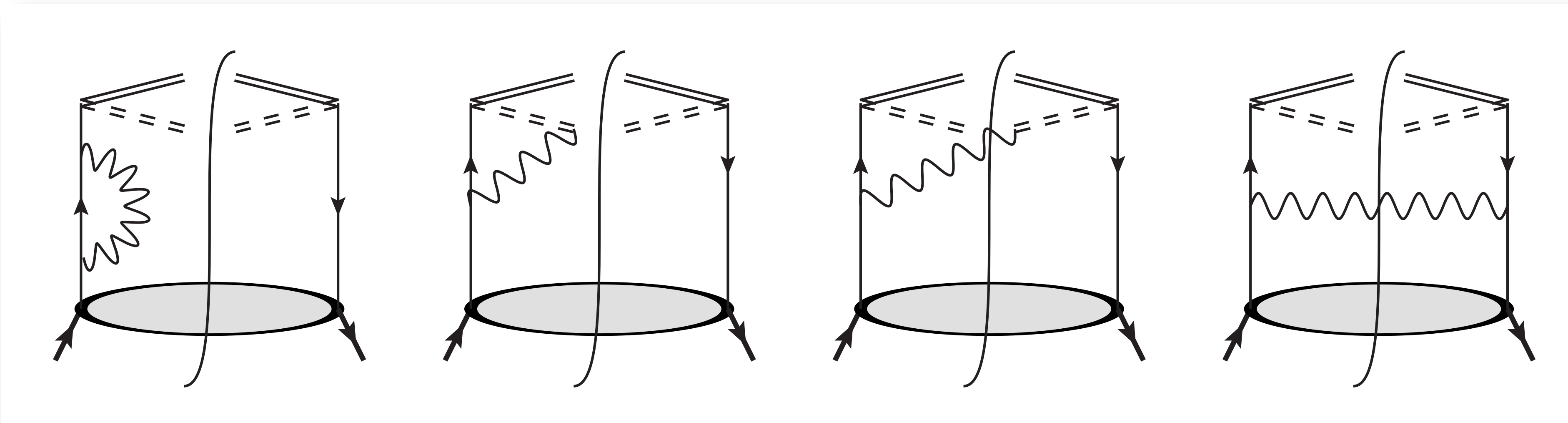
$$\tilde{f}_{1T}^{\perp g/A\,(1)}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}_1}{\bar{x}_1}\frac{d\bar{x}_2}{\bar{x}_2} \tilde{C}^{sivers}_{g/j}(\bar{x}_1,\bar{x}_2,b_T;\zeta,\mu)\, T_{F\,j/A}(x_1/\bar{x}_1,x_2/\bar{x}_2;\mu)$$

Unpolarized quark/gluon TMD distribution and fragmentation functions at NNLO in
[MGE, Scimemi, Vladimirov JHEP09(2016)004]

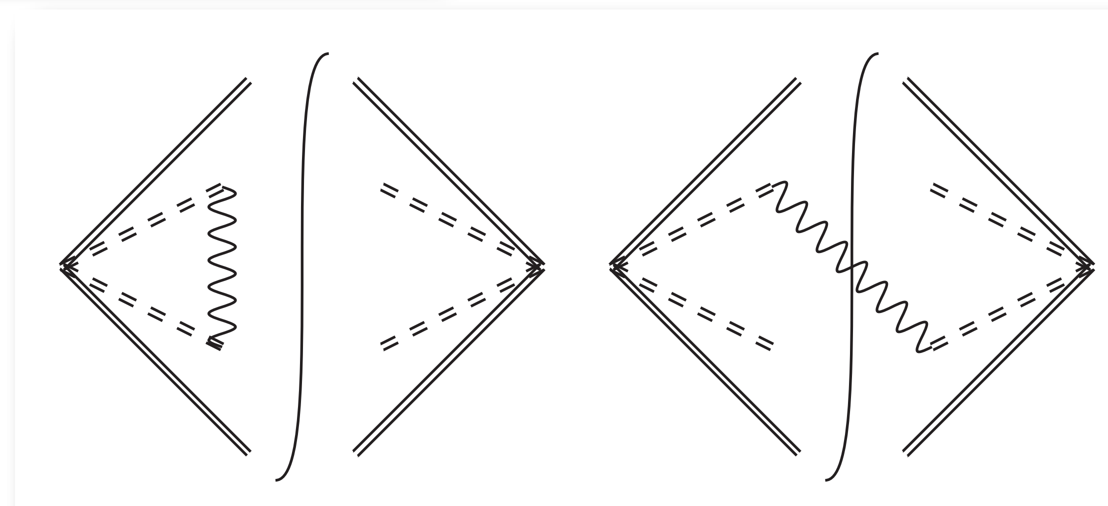
Transversely polarized TMDs at NNLO in
[Gutiérrez-Reyes, Scimemi, Vladimirov JHEP07(2018)172]
(more to come…)

OPE of unpol. q TMDPDF: QED contribution (1/2)

$$\tilde{f}_{i/A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g,\gamma} \tilde{C}_{i/j}(x, b_T; \zeta, \mu) \otimes f_{j/A}(x; \mu) + O\Big(b_T \Lambda_{QCD}\Big)$$



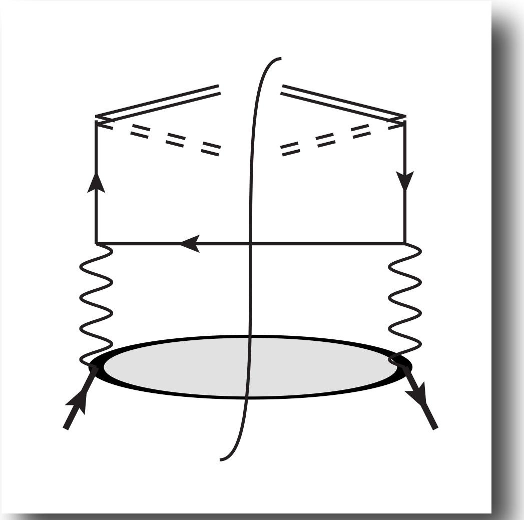
QED rapidity divergences cancel in the properly defined TMDs



$$\tilde{C}^{(0,1)}_{i/i}(x, b_T; \mu) = Q_i^2 \left[\delta(1-x)\left(-L_T^2 + 3L_T + 2L_T \ln\frac{\mu^2}{Q^2} - \frac{\pi^2}{6}\right) - 2L_T P_{i\leftarrow i} + 2(1-x)\right]$$

OPE of unpol. q TMDPDF: QED contribution (2/2)

$$\tilde{f}_{i/A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g,\gamma} \tilde{C}_{i/j}(x, b_T; \zeta, \mu) \otimes f_{j/A}(x; \mu) + O\Big(b_T \Lambda_{QCD}\Big)$$



$$\tilde{C}^{(0,1)}_{i/\gamma}(x, b_T; \mu) = N_c Q_i^2 \Big[-2L_T(x^2 + (1-x)^2) + 4x(1-x) \Big]$$

Factor Nc due to color multiplicity

Consistent with
[Cieri-Ferrera-Sborlini JHEP1808(2018)]

Conclusions & Outlook

- **Theoretical precision is needed at EIC (and beyond) to properly extract nonperturbative Physics**
- **EIC** will better constrain **photon PDF** through DIS structure functions (LUXqed)
- **QED corrections to TMD evolution** obtained at $\mathcal{O}(\alpha_s \alpha)$ and up to $\mathcal{O}(\alpha^2)$
- New results are **universal** for all (un)polarized quark/gluon TMDPDFs and TMDFFs **up to the flavor of the considered parton**
- **QED corrections** to f_1^q at large p_T obtained at $\mathcal{O}(\alpha)$
- Corrections to other TMDs can be obtained similarly
- "TMD community" is catching up with the "PDF community"!
- **ToDo: study numerical impact** on pheno (found to be around 1-4% for Z-boson p_T distribution at Tevatron/LHC ***[Cieri, Ferrera, Sborlini JHEP08(2018)]***)

Thank you!