PION NUCLEUS DRELL-YAN PROCESS

AND
PARTON TRANSVERSE MOMENTUM IN THE PION

INT Program INT-18-3

Probing Nucleons and Nuclei in High Energy Collisions

Week 2

A. Courtoy

Instituto de Física
Universidad Nacional Autónoma de México





OUTLINE

- Drell-Yan in πN scattering
- Focus on the W-term
 - Drell-Yan with transverse momentum
 - **→** Pion dynamics
 - **→** Effects on DY cross section

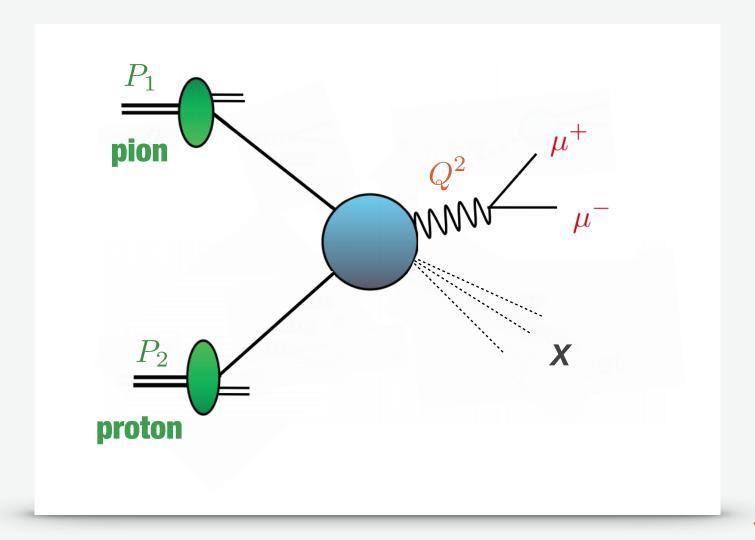
in collaboration with Federico Ceccopieri, Santiago Noguera & Sergio Scopetta

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Focus of the pion

in collaboration with Federico Ceccopieri, Santiago Noguera & Sergio Scopetta

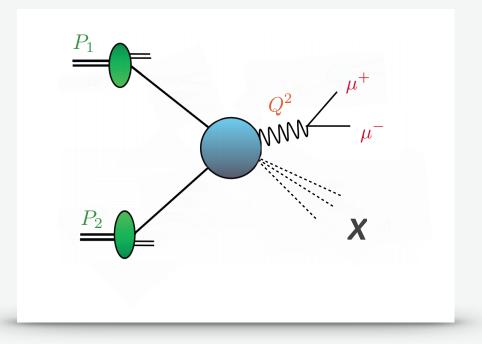


$$Q^2 = M^2$$
$$s = 2P_1 \cdot P_2$$

$$\tau = \frac{Q^2}{s} \equiv \text{finite as } Q^2, s \to \infty$$

Pion-proton Drell-Yan differential cross-section

$$d\sigma = \sum_{ab} \int dx_a \int dx_b \, f_a^{\pi}(x_a, \mu) f_b(x_b, \mu) \, d\hat{\sigma}_{ab}(x_a P_a, x_b P_b, Q, \alpha_s(\mu), \mu)$$



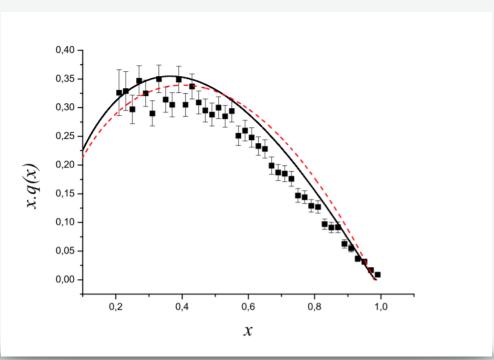
$$x_a x_b = \tau \qquad Q^2 = M^2$$
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Pion-proton Drell-Yan: main source of information on pion structure

E615 extraction (joint proton and pion PDF)

Momentum fraction carried by valence quarks \rightarrow allows Q_0 fixing



Nambu - Jona-Lasinio (NJL) with MSRS PDFs (1992)

$$Q_0 = 0.29 \text{GeV}$$
 , for the LO evolution;

$$Q_0 = 0.43 \text{GeV}$$
 , for the NLO evolution .

$$\Lambda_{\text{LO}} = 0.174 \, \text{GeV}$$

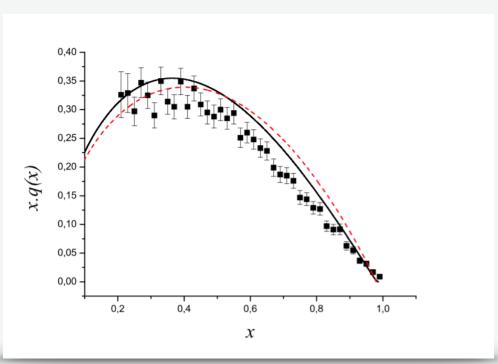
$$\Lambda_{\scriptscriptstyle \rm NLO} = 0.246\,GeV$$

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The exercice should be repeated for the new pion PDF of PRL121,152001.

THE PION IN NJL

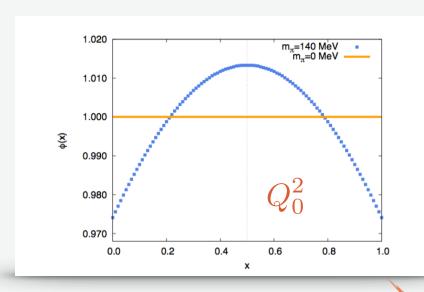
Successful results and predictions in the past
Pion means chiral low-energy model
Decent approach of QCD

Why NJL?

- Quarks dof
- Constituent quarks mass from gap equation
- Pion as a Goldstone mode
- Pion as a Bound-State in the sense of Bethe-Salpeter $\vec{\chi}_P(p) = -g_{\pi qq} \, i S(p) \, \gamma_5 \vec{\tau} \, i S(p-P)$
- Choice of a covariant regularization scheme (here we use Pauli-Villars)

Ruiz-Arriola, Broniowski, Gamberg, Noguera, Scopetta, Courtoy,...

THE PION IN NJL: DISTRIBUTION AMPLITUDE



Mind the scale of y-axis!

 $Q_0 = 0.29 \text{GeV}$,

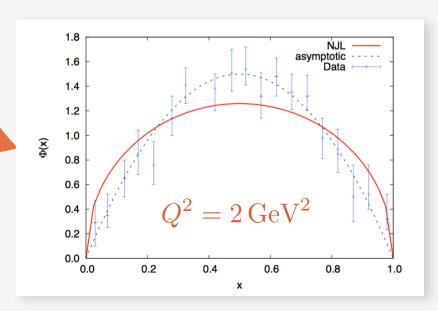
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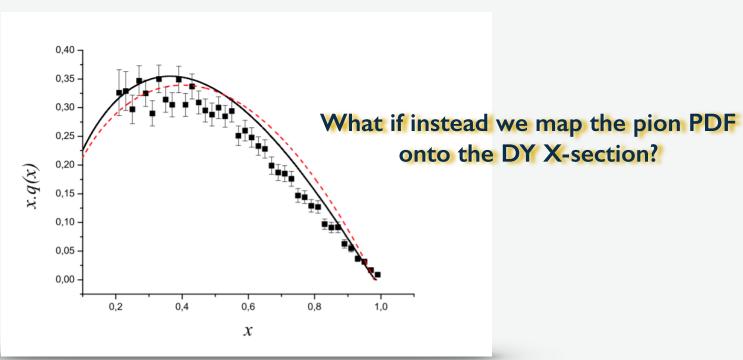
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Determination of NJL's scale

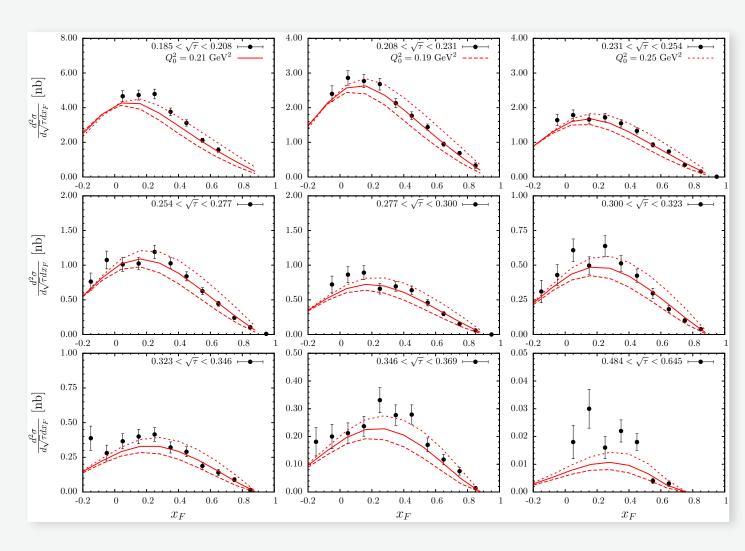
Comparison of integrated X-section with theory at NLO:

- pion from NJL
- proton from CTEQ06M

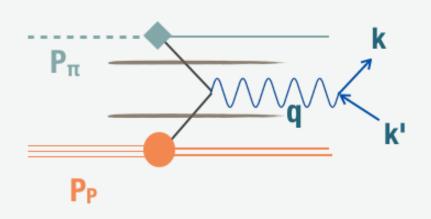
We find

 Q_0^2 =0.21 GeV²/ Q_0 =0.46 GeV

with $\chi^2/dof=2$



DRELL-YAN WITH TRANSVERSE MOMENTUM



See talks by Fulvio and Nobuo

With measured Q_T of order Q

$$\frac{d\sigma}{dQ^2 \, dy \, dQ_T^2} = \frac{4\pi^2 \alpha^2}{9Q^2 s} \sum_{a,b} \int_{x_{\pi}}^1 \frac{d\xi_{\pi}}{\xi_{\pi}} \int_{x_P}^1 \frac{d\xi_P}{\xi_P} \, T_{ab}(\cdots) f_{a/\pi}(\xi_{\pi},\mu) f_{b/P}(\xi_P,\mu)$$

$$q^{2} = (k + k')^{2}$$

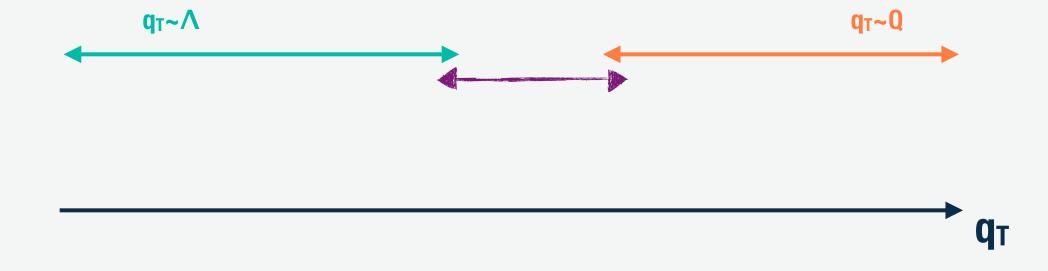
$$x_{\pi} = \frac{Q^{2}}{2P_{\pi} \cdot q}, \quad x_{P} = \frac{Q^{2}}{2P_{P} \cdot q}$$

$$\tau = \frac{Q^{2}}{s} \text{ fixed and finite as } Q^{2}, s \to \infty$$

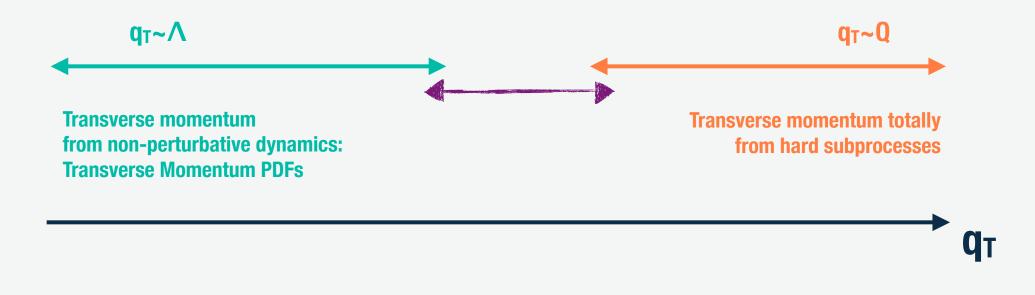
$$q^{\mu} = (x_{\pi}P_{\pi}^{+}, x_{P}P_{P}^{-}, \vec{q}_{T})$$

$$y = \frac{1}{2} \ln \frac{x_{\pi}}{x_{P}}$$

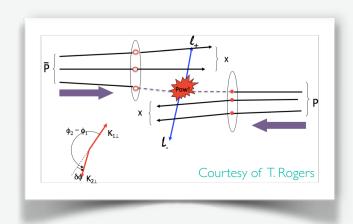


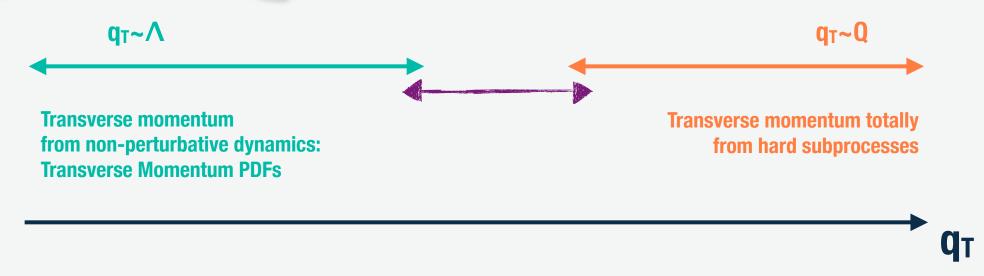


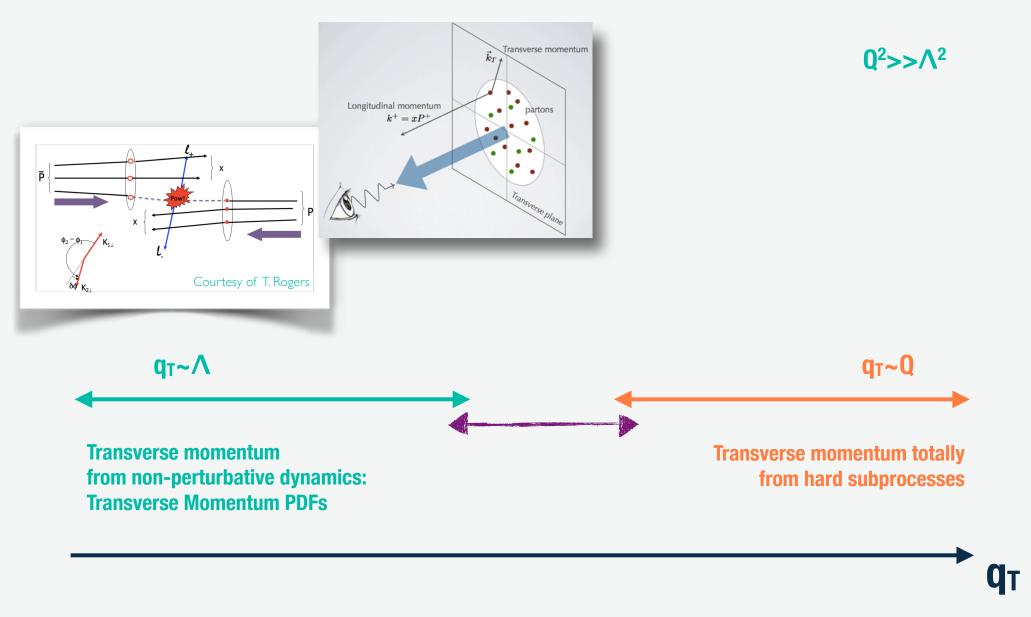
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$$\frac{d\sigma}{dQ^2 \, dy \, dQ_T^2} = \frac{4\pi^2 \alpha^2}{9Q^2 s} \int \frac{d^2 b}{2\pi^2} e^{i\vec{Q}_T \cdot \vec{b}} \sum_{a,b} \int_{x_{\pi}}^1 \frac{d\xi_{\pi}}{\xi_{\pi}} \int_{x_P}^1 \frac{d\xi_P}{\xi_P} f_{a/\pi}(\xi_{\pi}, \mu_b) f_{b/P}(\xi_P, \mu_b)$$

$$\times \exp\left(-C_F \frac{\alpha_s(q)}{2\pi} \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left[2\ln \frac{Q^2}{\mu^2} - 3\right] + \text{H.O.}\right)$$

$$\times \sum_{j} e_{j}^{2} C_{ja}(x_{\pi}/\xi_{\pi}, b; 2e^{-\gamma}; \mu_{b}) C_{jb}(x_{P}/\xi_{P}, b; 2e^{-\gamma}; \mu_{b})$$

Taming $b\sim 1/\Lambda$:

b-prescription

$$\mu_b = 2e^{-\gamma}/b^*$$
$$b^* = b/\sqrt{1 + b^2/b_{\text{max}}^2}$$

$$\times e^{S_{\mathrm{NP}}^{\pi}(b)} e^{S_{\mathrm{NP}}^{P}(b)}$$

NLL



$$\frac{d\sigma}{dQ^2 dy dQ_T^2} = \frac{4\pi^2 \alpha^2}{9Q^2 s} \int \frac{d^2 b}{2\pi^2} e^{i\vec{Q}_T \cdot \vec{b}} \sum_{a,b} \int_{x_\pi}^1 \frac{d\xi_\pi}{\xi_\pi} \int_{x_P}^1 \frac{d\xi_P}{\xi_P} f_{a/\pi}(\xi_\pi, \mu_b) f_{b/P}(\xi_P, \mu_b)$$

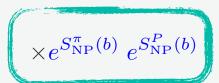
$$\times \exp\left(-C_F \frac{\alpha_s(q)}{2\pi} \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left[2\ln \frac{Q^2}{\mu^2} - 3\right] + \text{H.O.}\right)$$

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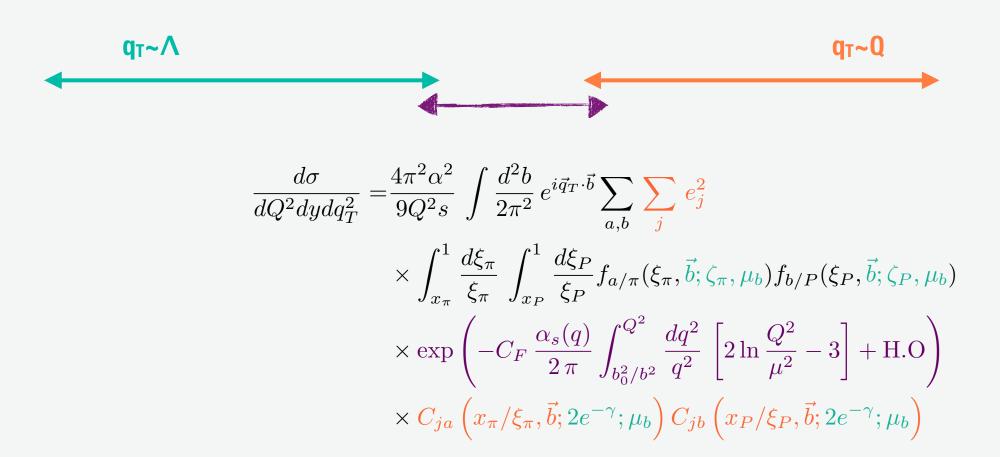
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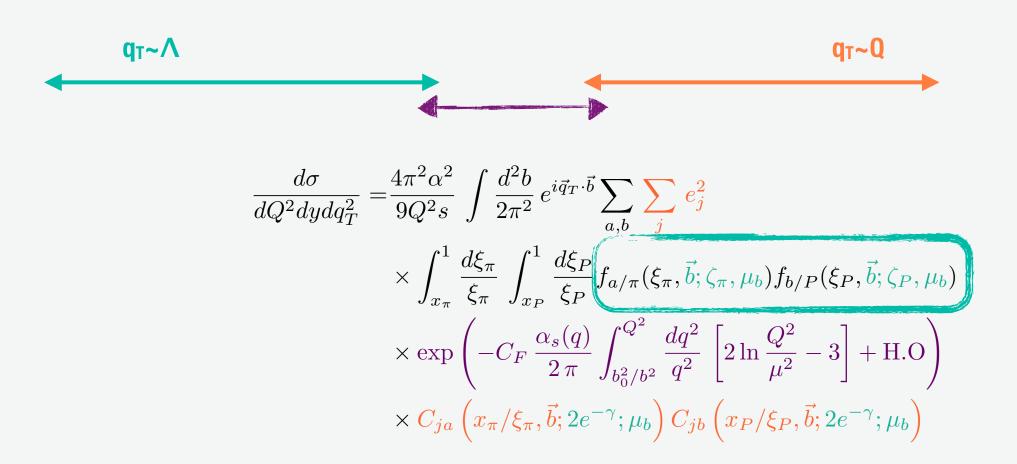
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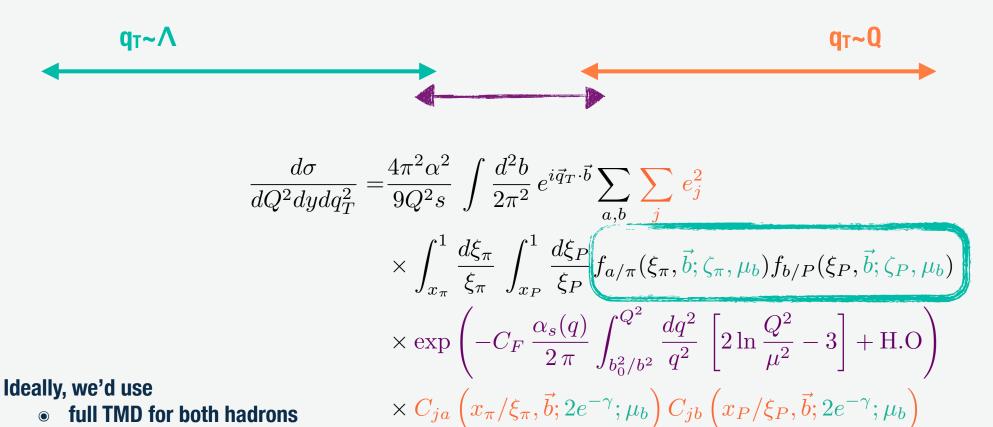












either from pheno. or similar models

But,

- no pheno proton TMD available (when we started this...)
- no model similar to NJL for the proton





$$\begin{split} \frac{d\sigma}{dQ^2 dy dq_T^2} &= \frac{4\pi^2 \alpha^2}{9Q^2 s} \, \int \frac{d^2 b}{2\pi^2} \, e^{i\vec{q}_T \cdot \vec{b}} \sum_{a,b} \sum_{j} e_j^2 \\ &\times \int_{x_\pi}^1 \frac{d\xi_\pi}{\xi_\pi} \, \int_{x_P}^1 \frac{d\xi_P}{\xi_P} \left[f_{a/\pi}(\xi_\pi,\vec{b};\zeta_\pi,\mu_b) f_{b/P}(\xi_P,\vec{b};\zeta_P,\mu_b) \right] \\ &\times \exp \left(-C_F \, \frac{\alpha_s(q)}{2\,\pi} \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \, \left[2 \ln \frac{Q^2}{\mu^2} - 3 \right] + \text{H.O} \right) \end{split}$$
 adrons
$$\times C_{ja} \left(x_\pi/\xi_\pi, \vec{b}; 2e^{-\gamma}; \mu_b \right) C_{jb} \left(x_P/\xi_P, \vec{b}; 2e^{-\gamma}; \mu_b \right) \end{split}$$

Ideally, we'd use

- full TMD for both hadrons
- either from pheno. or similar models

But,

- no pheno proton TMD available (when we started this...)
- → See talk by Andrea

no model similar to NJL for the proton



STRATEGY

- use a phenomenologically estimated $f_{b/P}(\xi_P;\mu_b) imes e^{S_{
 m NP}^P(b)}$
 - PDF from CTEQ6M
 - NP + b-prescription from [Konychev & Nadolsky, Phys. Lett. B 633, 710 (2006)]
- ullet use the pion TMD from the NJL model $f_{a/\pi}(\xi_\pi,ec b;\zeta_\pi,\mu_b)$
 - [Noguera, S. Scopetta, JHEP 1511, 102 (2015)]
 - redefine the hadronic scale of PDF from DY integrated data
 - interpret the k_T-dependence of the model onto the (unintegrated) DY data

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from now on: $e^{S_{NP}^P(b)} \to S_{NP}^P(b)$

THE NON-PERTURBATIVE PART

$$\frac{d\sigma}{dQ^2 dy dQ_T^2} - \frac{4\pi^2 \alpha^2}{9Q^2 s} \left\{ (2\pi)^{-2} \int d^2b \, e^{iQ_T \cdot b} \sum_j e_j^2 \tilde{W}_j(b_*; Q, x_A, x_B)_{pert} \right\}$$

$$\times \exp \left[-\ln \left(Q^2/Q_0^2 \right) g_1(b) - g_{j/A}(x_A, b) - g_{j/B}(x_B, b) \right]$$

One parameterization of the non-perturbative contribution

Here:
$$S_{NP}^{\pi W}(b) = S_{NP}^{\pi}(b) \sqrt{S_{NP}^{pp}(b)}$$



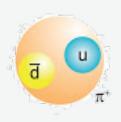
$$S_{NP}^{pp}(b)$$
= $\exp\{-[a_1 + a_2 \ln(M/(3.2 \text{ GeV})) + a_3 \ln(100x_1x_2)]b^2\}.$

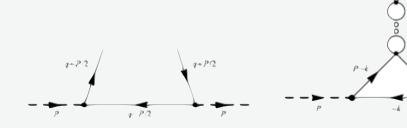
purely comes from the dynamics of the model

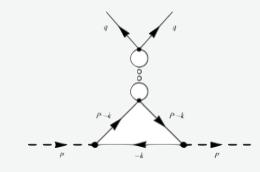
FULL TRANSVERSE MOMENTUM DEPENDENCE FOR THE PION

$$f(x; \mu) \times \exp(g_{j/P}(b)) = f(x, b; \mu)$$

TMD PDFs







$$\begin{split} f_{1,\pi}\left(x,k_{T}^{2}\right) = &\frac{3}{4\,\pi^{3}}\,g_{\pi qq}^{2}\,\theta\left(x\right)\,\theta\left(1-x\right)\,\sum_{i=0}^{2}c_{i} \\ &\times\left\{\frac{1}{k_{T}^{2}+M_{i}^{2}-m_{\pi}^{2}\,x\left(1-x\right)} + \frac{m_{\pi}^{2}\,x\left(1-x\right)}{\left[k_{T}^{2}+M_{i}^{2}-m_{\pi}^{2}\,x\left(1-x\right)\right]^{2}}\right\} \end{split}$$

$$f_{\pi}(x,b;\mu) \xrightarrow{chiral\ lim} f'_{\pi}(x;\mu)f''_{\pi}(b)$$

Our interpretation:
$$\exp(g_{j/\pi}(b)) = f_{\pi}''(b)$$

 \rightarrow no "g₁(b)" is this model picture

$$f_{\pi}''(b) = \frac{3}{2\pi^2} \left(\frac{m}{f_{\pi}}\right)^2 \sum_{i=0,2} \int dk_T \, k_T \, J_0(bk_T) \frac{a_i}{k_T^2 + m_i^2}$$
$$= \frac{3}{2\pi^2} \left(\frac{m}{f_{\pi}}\right)^2 \sum_{i=0,2} a_i K_0(m_i \, b)$$

$$f_{\pi}(x,b;\mu) \xrightarrow{chiral\ lim} f'_{\pi}(x;\mu)f''_{\pi}(b)$$

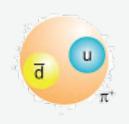
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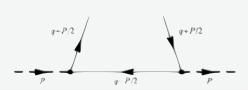
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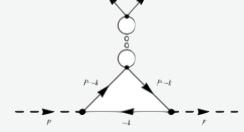
$$f_{\pi}^{"}(b) = \frac{3}{2\pi^{2}} \left(\frac{m}{f_{\pi}}\right)^{2} \sum_{i=0,2} \int dk_{T} k_{T} J_{0}(bk_{T}) \frac{a_{i}}{k_{T}^{2} + m_{i}^{2}}$$

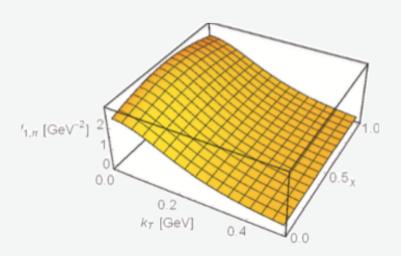
$$= \frac{3}{2\pi^{2}} \left(\frac{m}{f_{\pi}}\right)^{2} \sum_{i=0,2} a_{i} K_{0}(m_{i} b)$$

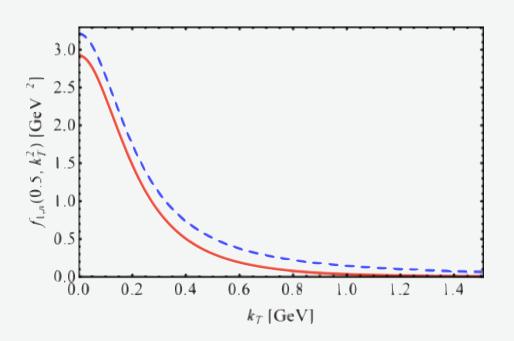
We assumed that factorization of the transverse momentum occurs at Q₀ only.

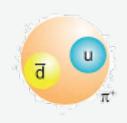




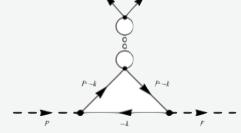


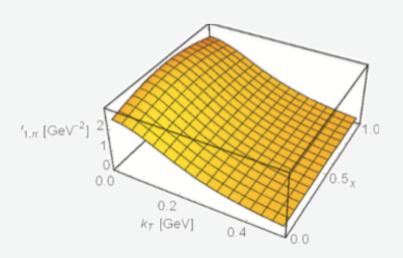


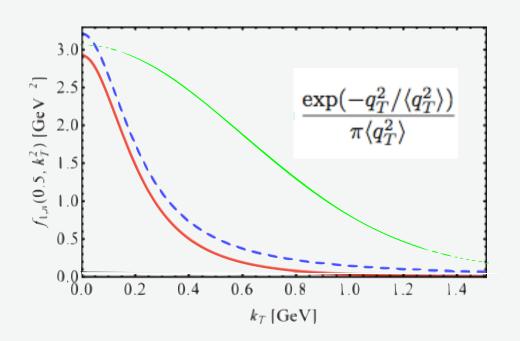


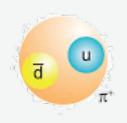


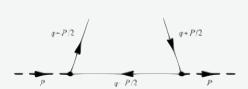


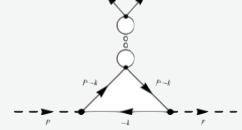


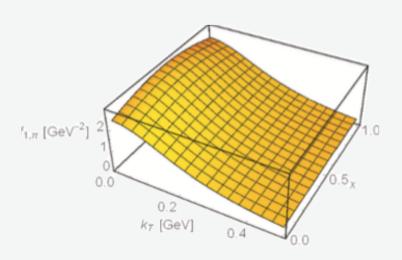


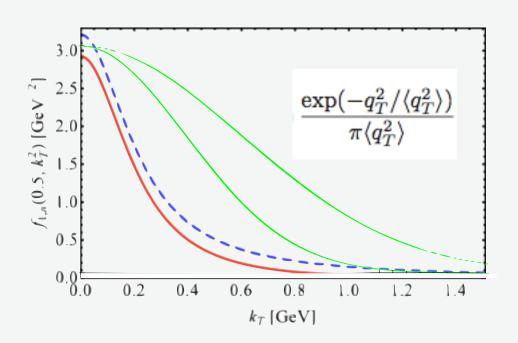


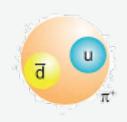


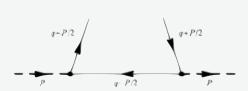


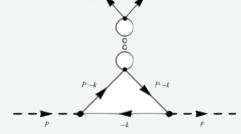


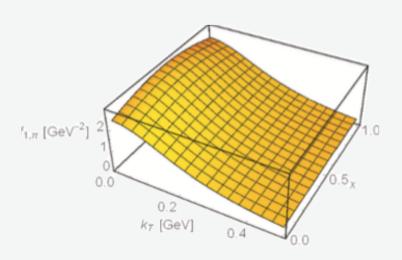


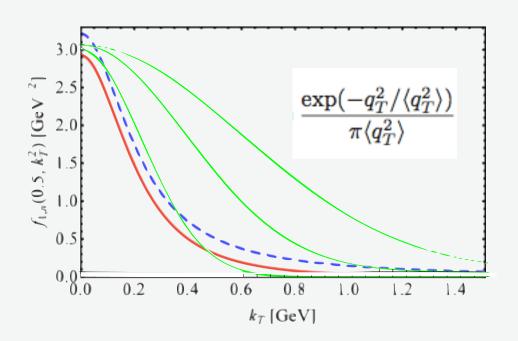




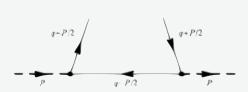


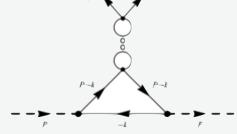


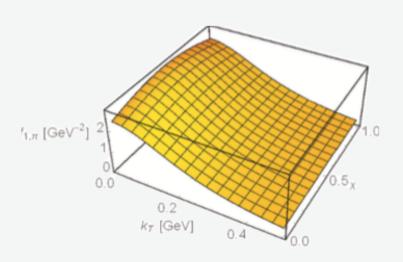




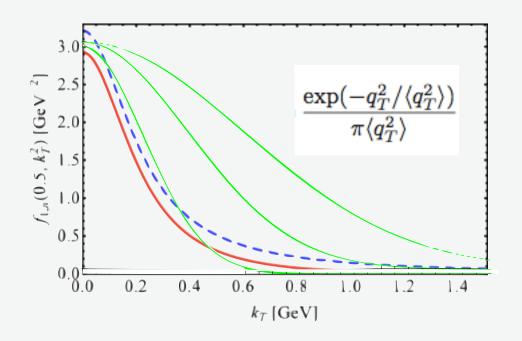


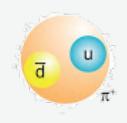


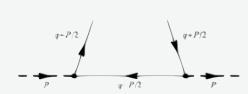


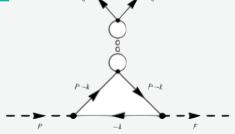


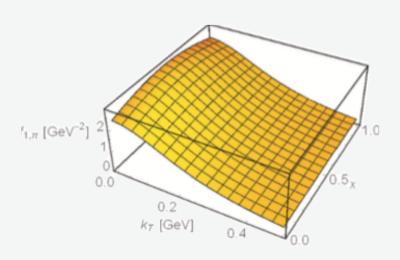
Pion dynamics → **differs from a gaussian**





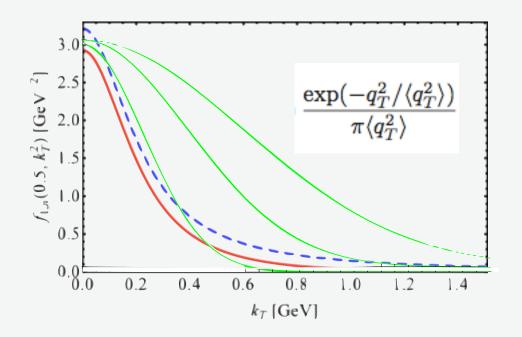




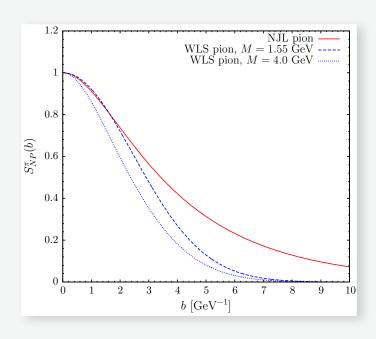


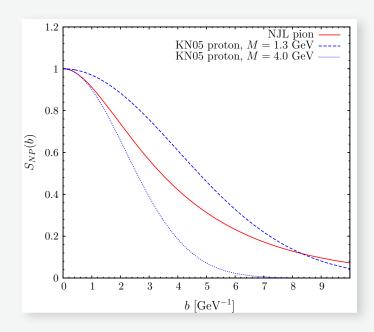
Pion dynamics → **differs from a gaussian**

Transverse profile → **no dpdce on x or M**



PION TRANSVERSE PROFILE





$$S_{\text{NP}}^{f_1^{q/\pi}} = g_1^{\pi} b^2 + g_2^{\pi} \ln \frac{b}{b_*} \ln \frac{Q}{Q_0}$$

Fit of pion Sudakovs' [Wang & et al, JHEP08-137]

$$S_{NP}^{pp}(b)$$
 (12)
= $\exp\{-[a_1 + a_2 \ln(M/(3.2 \,\text{GeV})) + a_3 \ln(100x_1x_2)]b^2\}.$

Fit of proton's [Konychev, P.M. Nadolsky, Phys. Lett. B 633, 710]

DRELL-YAN WITH PION DYNAMICS

Next-to-Leading Log

$$\sigma_{DY\pi N} \equiv \frac{d\sigma}{d\tau dy dp_T^2} = \sum_{q} \frac{\sigma_{q\bar{q}}^0}{2} \int_0^\infty db \, b \, J_0(bp_T) \, e^{S(b,b_{max},Q,C_1)} \, e^{S_{NP}^{\pi}(b)} \, e^{S_{NP}^{N}(b)} \cdot \left[\left(f_{q_a}^{\pi} \left(x_a, \mu_b \right) \otimes C_{aa'} \right) \left(F_{\bar{q}_b}^{N} \left(x_b, \mu_b \right) \otimes C_{bb'} \right) + q \leftrightarrow \bar{q} \right],$$

Wilson coeff. at order α s CTEQ6M PDFs evolved at NLO

Proton b_{max}=0.86 GeV⁻¹

Pion b_{max}=educated guess/adjusted to data
= b₀/Q₀=2.44 GeV⁻¹

stability upon variation of regulator

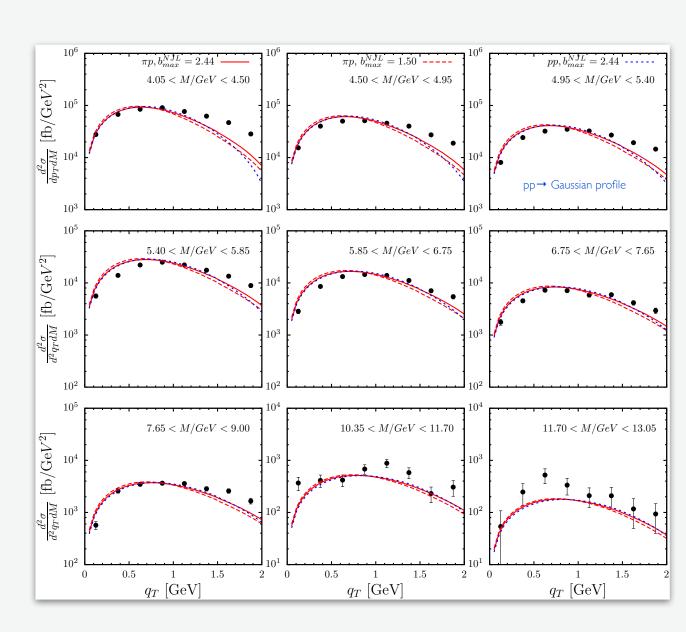
π^-W DRELL-YAN

Cross section in Q-bins

- **☑** overall magnitude
- **⊠** small qT
- ☐ higher qT
- ☐ Gaussian profile ~indistinguishable

No free parameters
Only Q₀ is fixed beforehand

with KN param.

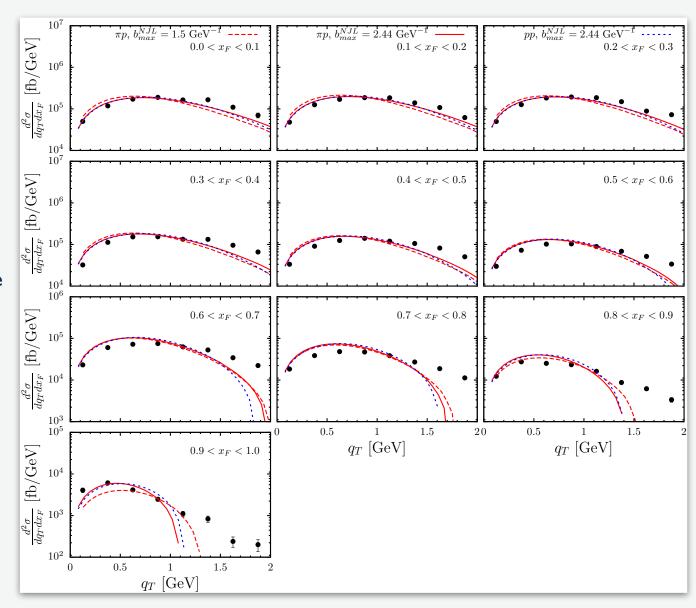


π^-W DRELL-YAN

Cross section in x-bins

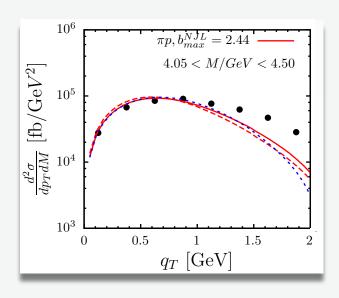
- **☑** overall magnitude
- **⊠** small qT
- ☐ higher qT
- ☐ Gaussian profile ~indistinguishable

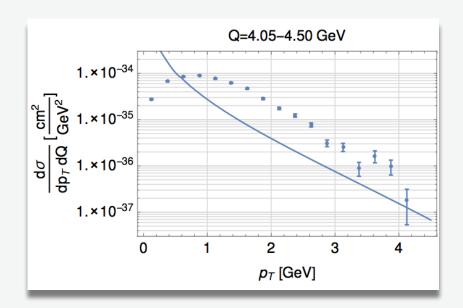
No free parameters
Only Q₀ is fixed beforehand



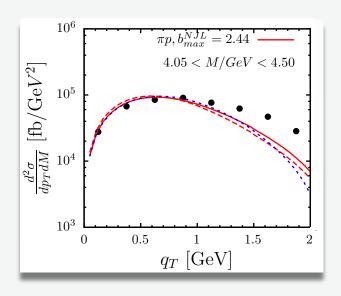
with KN param.

W WITHOUT 'FO' COMPLEMENTING FULVIO'S CONSIDERATIONS

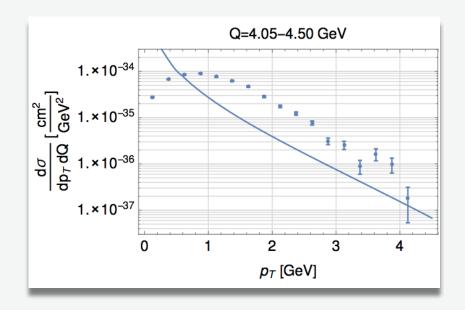




W WITHOUT 'FO' COMPLEMENTING FULVIO'S CONSIDERATIONS



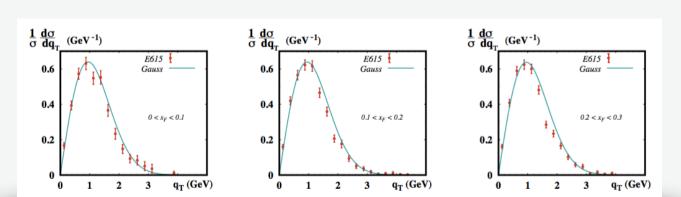
Pieces/hints/room to accomodate info in the low-qT regime...



...but we don't understand the pQCD regime properly.

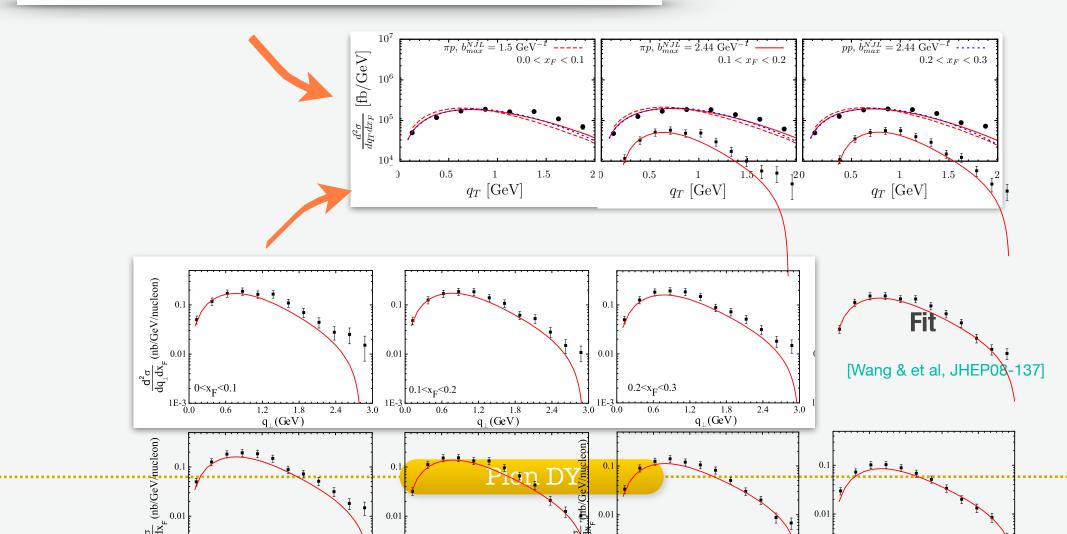
Possible piece of solution from Nobuo's talk.

π^-W DRELL-YAN

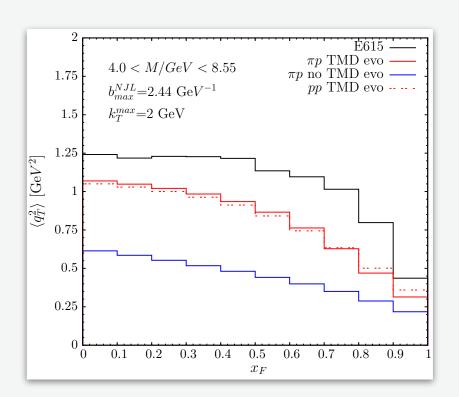


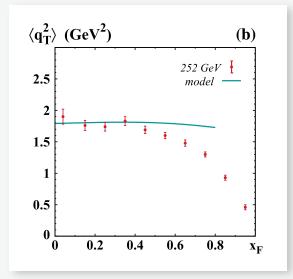
Only gaussian

[Pasquini & et al, Phys.Rev.D90]



$\pi^- W$ DRELL-YAN

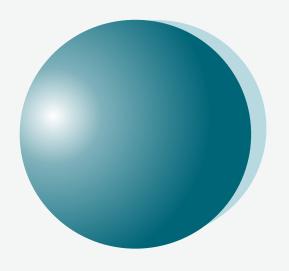


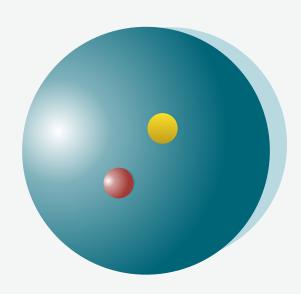


[Pasquini & et al, Phys.Rev.D90]

CSS evolution affects the transverse mmt distribution.

PION STRUCTURE FROM DY?



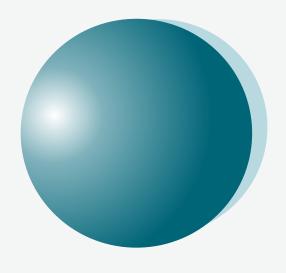


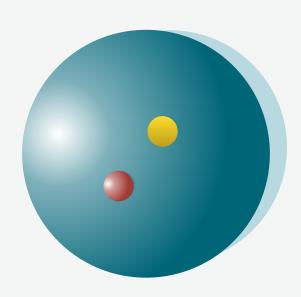
Degrees of freedom change governed by the chiral symmetry.



Resolution

CAN THE PION HELP THE PROTON?





- Fixing NP params. at low qT
- Less theoretical uncertainties from the pion
- Less "nice and symmetric" expression



Resolution

CONCLUSIONS

- Pion-proton collision to $\mu^+\mu^-$
- We have included pion nonperturbative dynamics in DY cross section
- Slight change in shape w.r.t. pure gaussians
- Need to understand another function: $g_K(b)$

Importance of nonperturbative inserts in perturbative evolution! Exciting physics ahead!

EIC PHYSICS

- Use knowledge on the pion to
 - lower proton uncertainties
 - disentangle possible symmetry effects
- Go to pion target SIDIS to test framework
 - relevant at JLab

- Relevant for COMPASS
- Use knowledge on pion to fix NP parameters
- Redefine/evaluate the hadronic scale from TMD pheno.