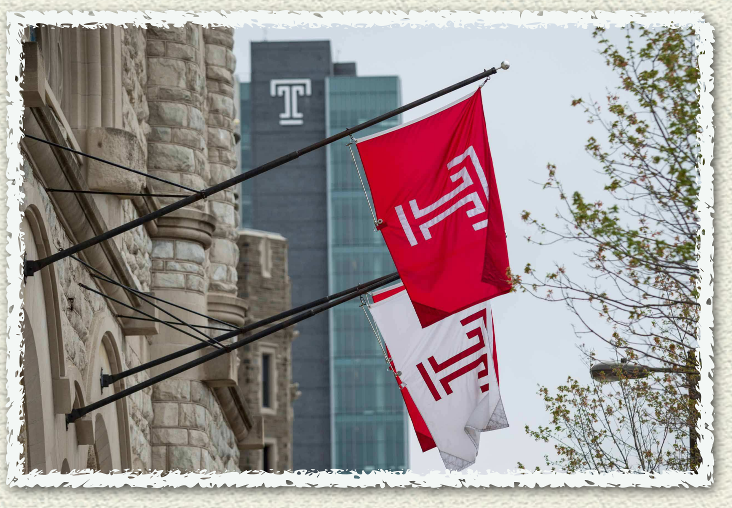
Proton Spin, Form Factors

from Lattice QCD

Martha Constantinou

Temple University

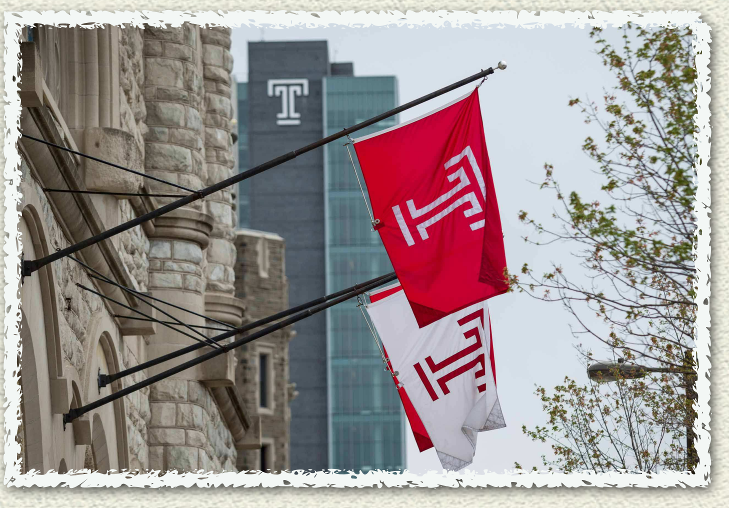


INT workshop on Probing
Nucleons and Nuclei in High Energy Collisions
October 15, 2018

Proton Spin, Form Factors
… and beyond
from Lattice QCD

Martha Constantinou

Temple University



INT workshop on Probing
Nucleons and Nuclei in High Energy Collisions
October 15, 2018

OUTLINE

A. Introduction

B. Nucleon on the Lattice

C. Nucleon Form Factors

   1. Electromagnetic
   2. Axial

D. Proton Spin

E. Access to x-dependence of PDFs

F. Discussion

In collaboration with

- C. Alexandrou [1,2]
- K. Cichy [3]
- K. Hadjiyiannakou [2]
- K. Jansen [4]
- C. Kallidonis [2]
- G. Koutsou [2]
- H. Panagopoulos [1]
- A. Scapellato [1]
- F. Steffens [5]
- A. Vaquero [6]

1. University of Cyprus
2. The Cyprus Institute
3. Adam Mickiewicz University
4. DESY, Zeuthen
5. Bonn University
6. University of Utah

OUTLINE

A. Introduction

B. Nucleon on the Lattice

C. Nucleon Form Factors

1. Electromagnetic
2. Axial

D. Proton Spin

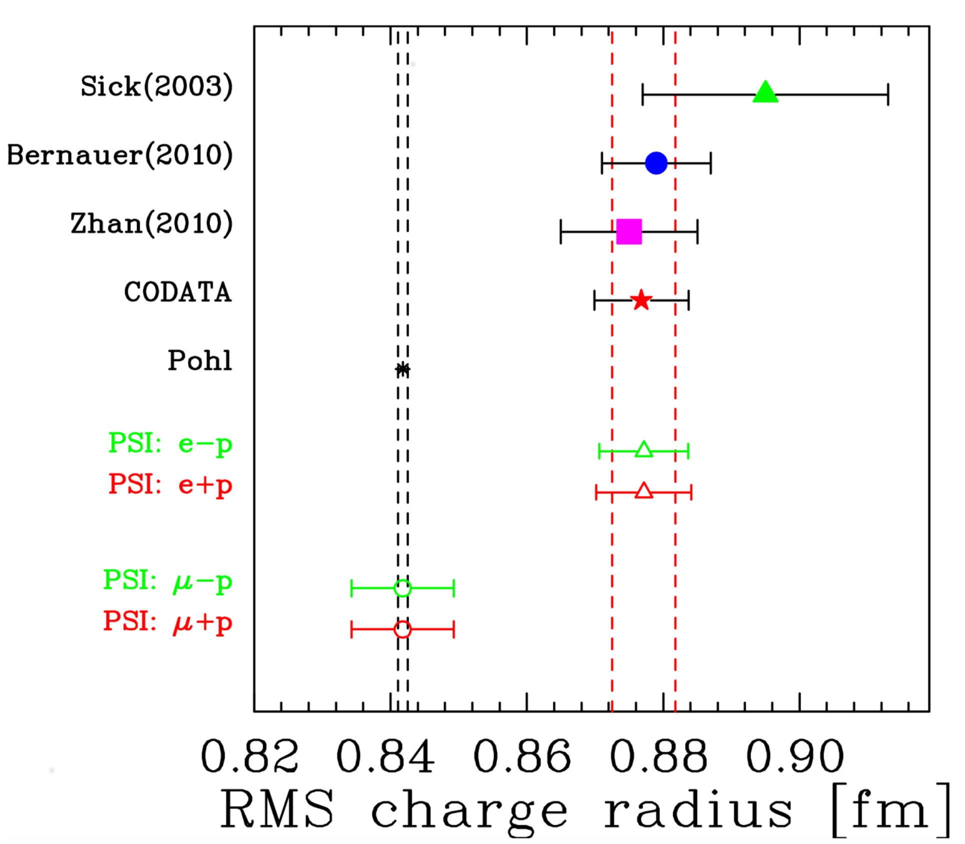
E. Access to x-dependence of PDFs

F. Discussion

Introduction - Motivation

FFs have been studied for decades as a tool to understand nucleon structure

- Nucleon electric & magnetic radii, magnetic moment extracted from Electromagnetic FFs
- Axial FFs relevant to experiments searching neutrino oscillations
- Intrinsic quark spin obtained from g_A ($G_A(Q^2{=}0)$)
- Total quark spin from <x>



[E. J. Downie, EPJ Conf. 113 (2016) 0502]

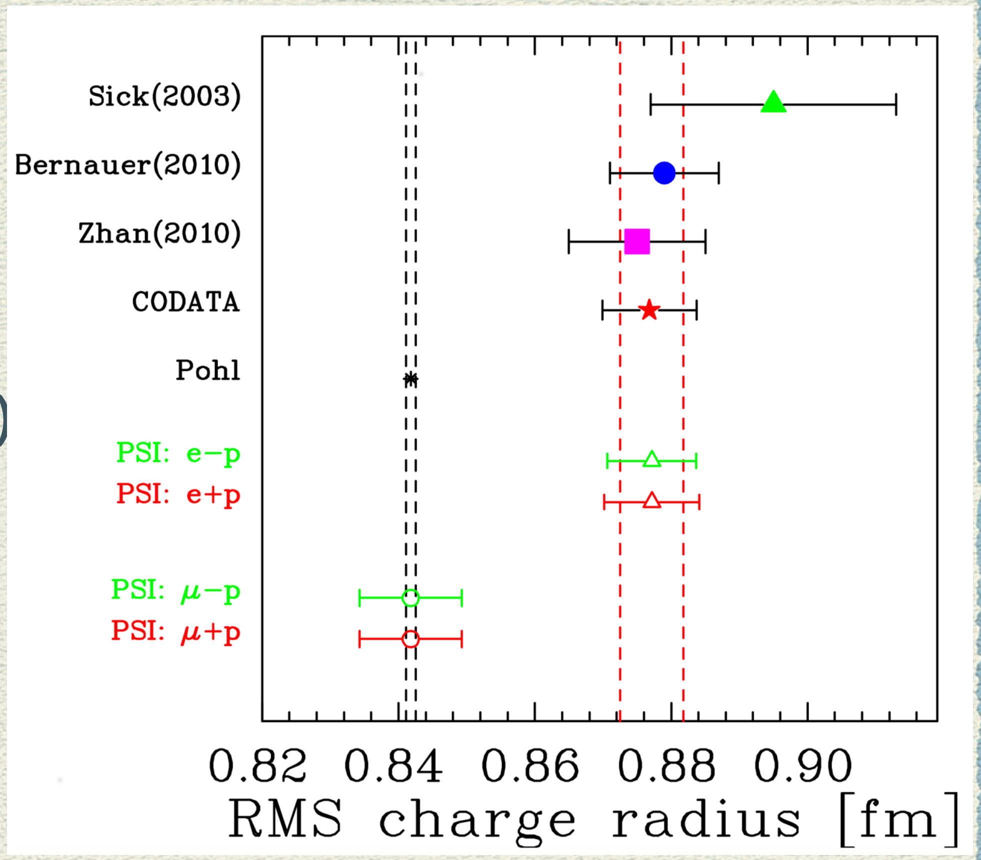
Information from experiments not without ambiguities

- Discrepancy of $\langle r_p^2 \rangle$ between electron scattering and muonic hydrogen Lamb shifts
- Large uncertainties in cross section of quasielastic neutrino-nucleon scattering: not well-constrained Axial FFs
- Strange E/M FFs are compatible with zero (HAPPEX collaboration, A4 exper., SAMPLE exper.)

Introduction - Motivation

FFs have been studied for decades as a tool to understand nucleon structure

- Nucleon electric & magnetic radii, magnetic moment extracted from Electromagnetic FFs
- Axial FFs relevant to experiments searching neutrino oscillations
- Intrinsic quark spin obtained from g_A ($G_A(Q^2=0)$)
- Total quark spin from <x>

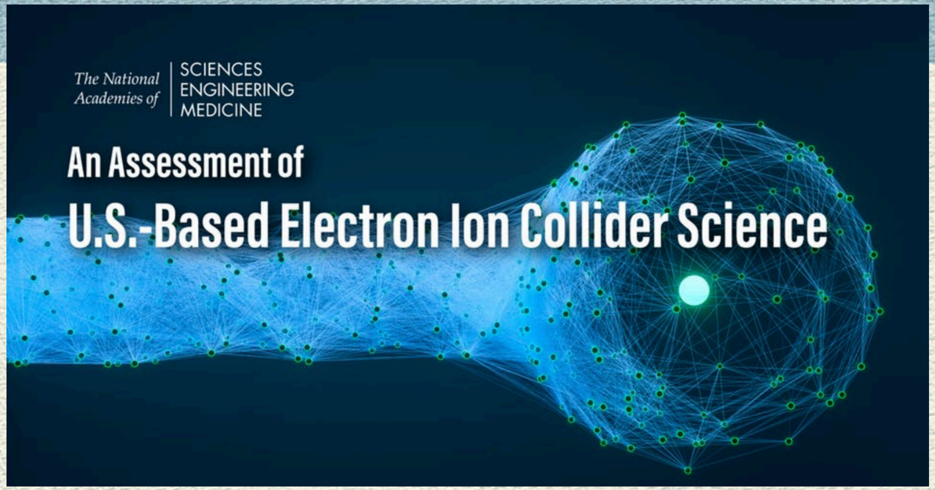


[E. J. Downie, EPJ Conf. 113 (2016) 0502]

Information from experiments not without ambiguities

- Discrepancy of $\langle r_p^2 \rangle$ between electron scattering and muonic hydrogen Lamb shifts
- Large uncertainties in cross section of quasielastic neutrino-nucleon scattering: not well-constrained Axial FFs
- Strange E/M FFs are compatible with zero (HAPPEX collaboration, A4 exper., SAMPLE exper.)

Lattice QCD ideal ab initio formulation to study nucleon form factors

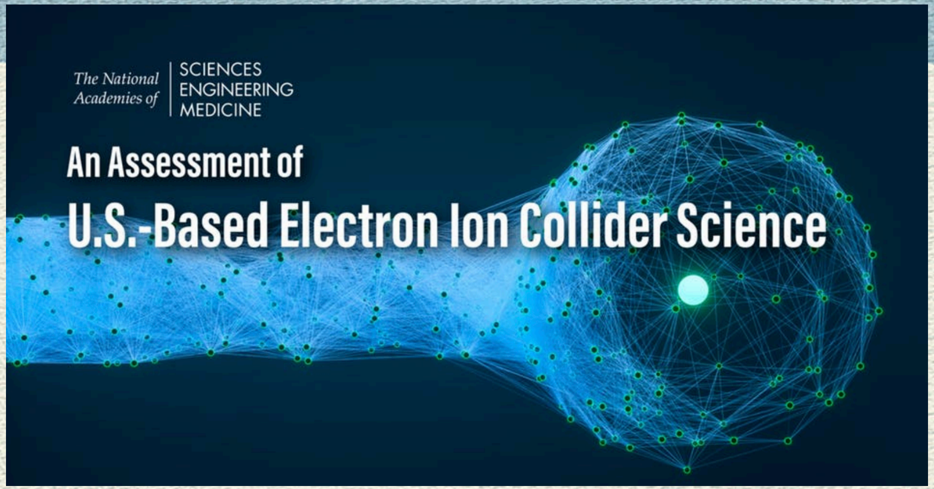


The EIC will address crucial questions in hadron structure:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?

... and measure PDFs in high accuracy

These questions can be addressed in Lattice QCD



The EIC will address crucial questions in hadron structure:

- ✱ How does the mass of the nucleon arise?
- ✱ How does the spin of the nucleon arise?

… and measure PDFs in high accuracy

These questions can be addressed in Lattice QCD

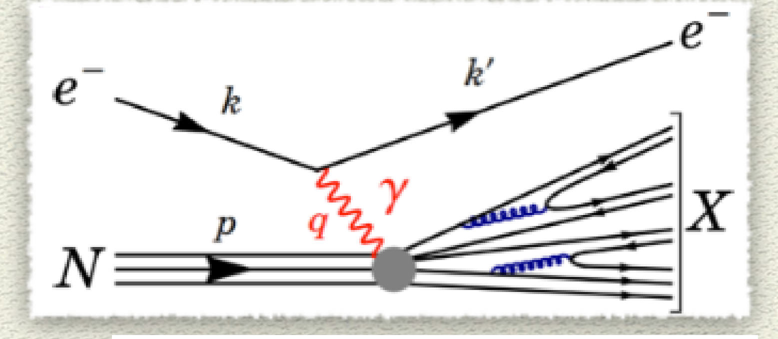
EIC will come at a time when lattice QCD:

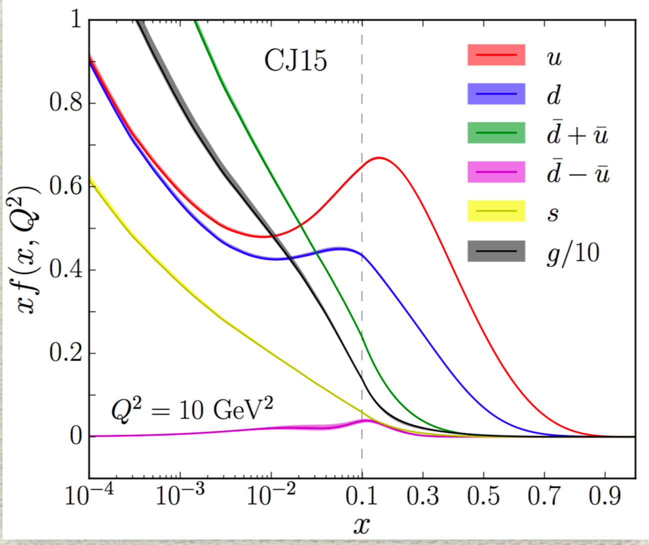
- ✱ will be well into the exa-scale computing era
- ✱ will reliably compute sea quark and gluon contributions
- ✱ is expected to reliably compute x-dependence quantities

Lattice QCD already showing promising results

Parton Distribution Functions

- Probe of hadron structure (1-D)
- Ideal for description of non-perturbative nature
- Probability densities for a given parton to carry a fraction-x of the hadron momentum
- Well-studied both experimentally and theoretically
- Necessary for analysis of DIS data
- Phenomenological input needed for data interpretation

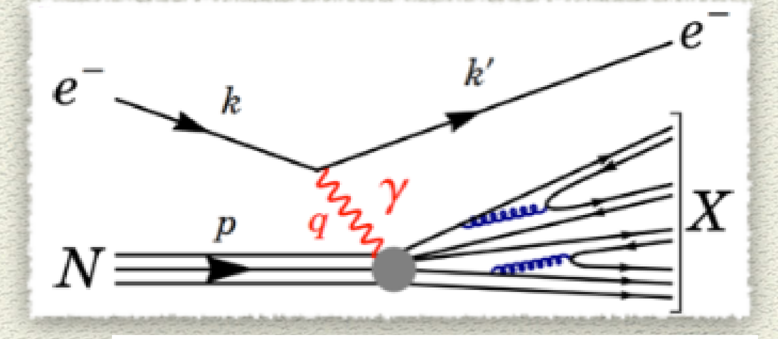


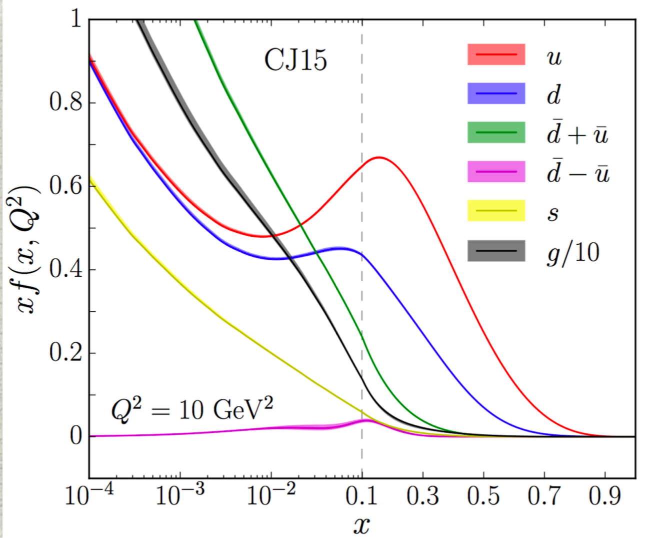


A. Accardi et al., arXiv:1602.03154]

Parton Distribution Functions

- Probe of hadron structure (1-D)
- Ideal for description of non-perturbative nature
- Probability densities for a given parton to carry a fraction-x of the hadron momentum
- Well-studied both experimentally and theoretically
- Necessary for analysis of DIS data
- Phenomenological input needed for data interpretation





Calculation from first principle imperative

A. Accardi et al., arXiv:1602.03154]

Parton Distribution Functions

- Lattice QCD is ideal ab initio formulation
- PDFs parameterized in terms of off-forward matrix elements of light-cone operators.

Parton Distribution Functions

- Lattice QCD is ideal ab initio formulation
- PDFs parameterized in terms of off-forward matrix elements of light-cone operators.

Not accessible in Euclidean lattice

Parton Distribution Functions

- Lattice QCD is ideal ab initio formulation
- PDFs parameterized in terms of off-forward matrix elements of light-cone operators.

Not accessible in Euclidean lattice

- On lattice: moments of PDFs (reconstructed via OPE)
- Reconstruction difficult task:
  - ★ n>3: operator mixing
  - ★ Statistical noise increases with high moments
- Moments of PDFs have physical interpretation and may serve as benchmark

OUTLINE

A. Introduction

B. Nucleon on the Lattice

C. Nucleon Form Factors

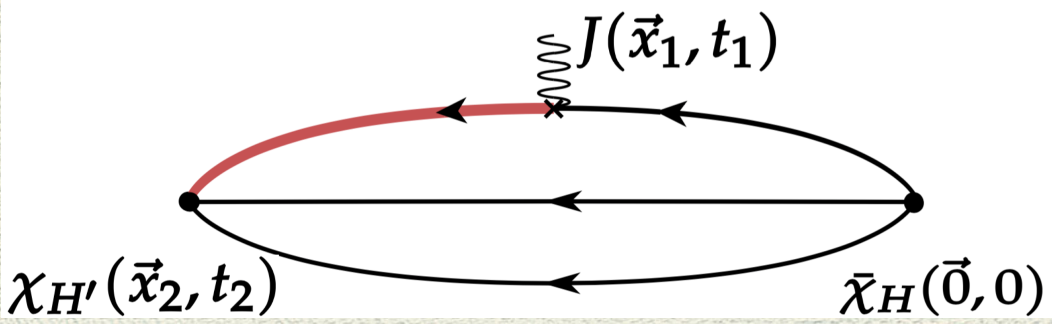
1. Electromagnetic
2. Axial

D. Proton Spin

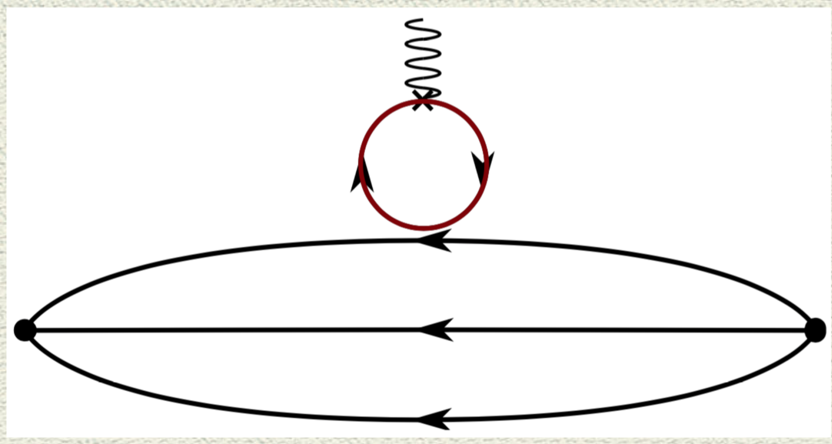
E. Access to x-dependence of PDFs

F. Discussion

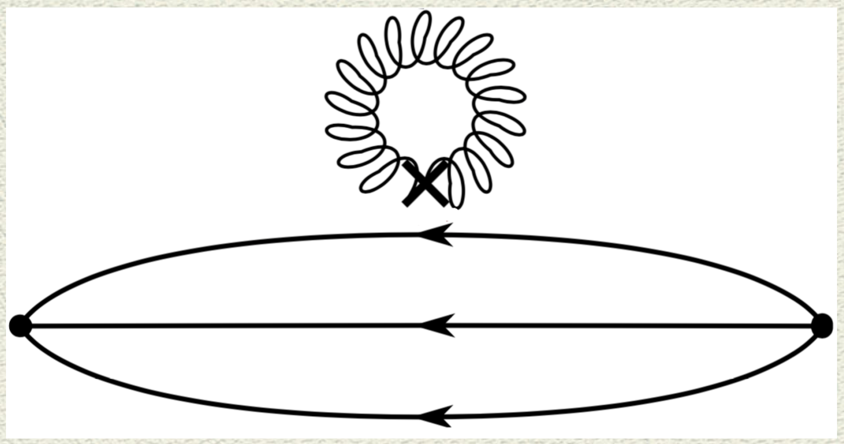
Nucleon on the Lattice



Connected

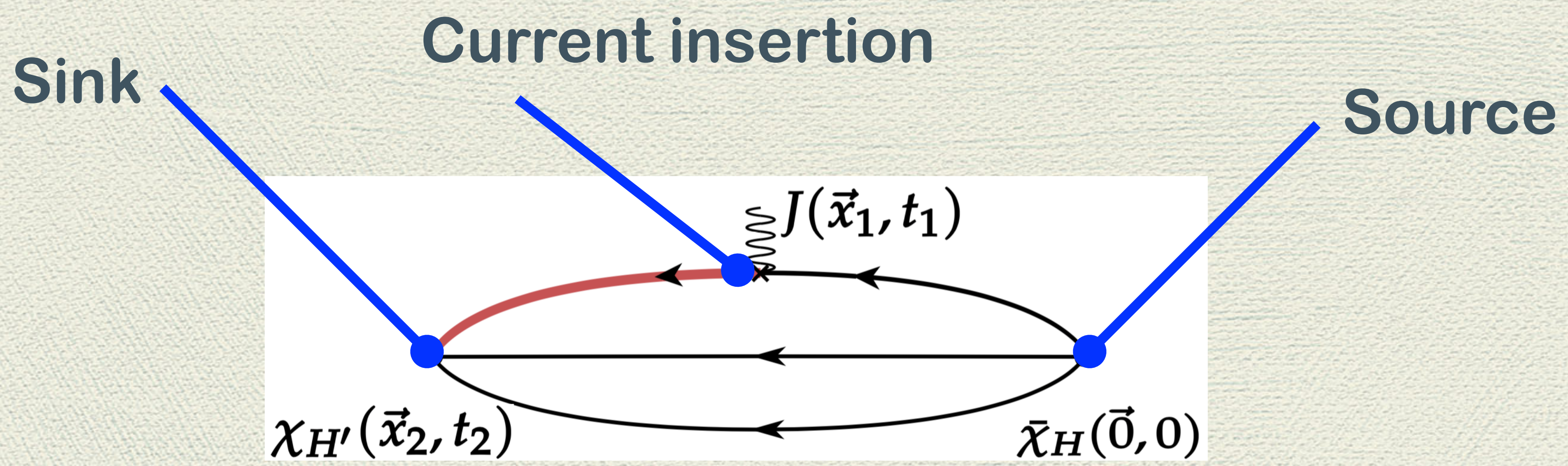


Disconnected
Quark loop

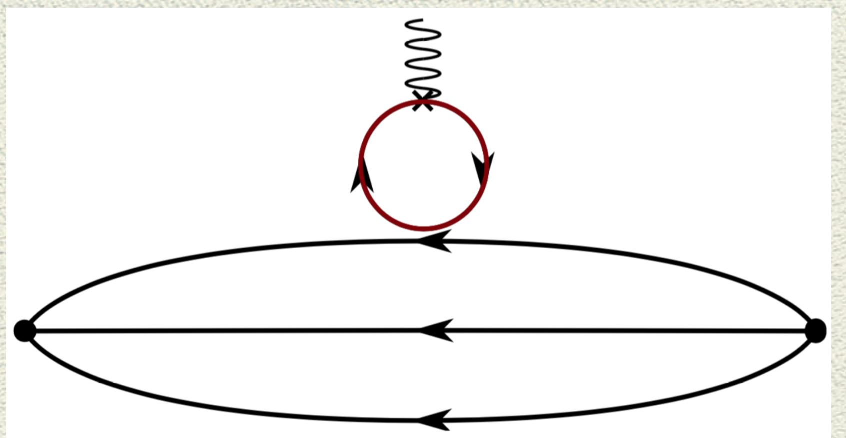


Disconnected
Gluon loop

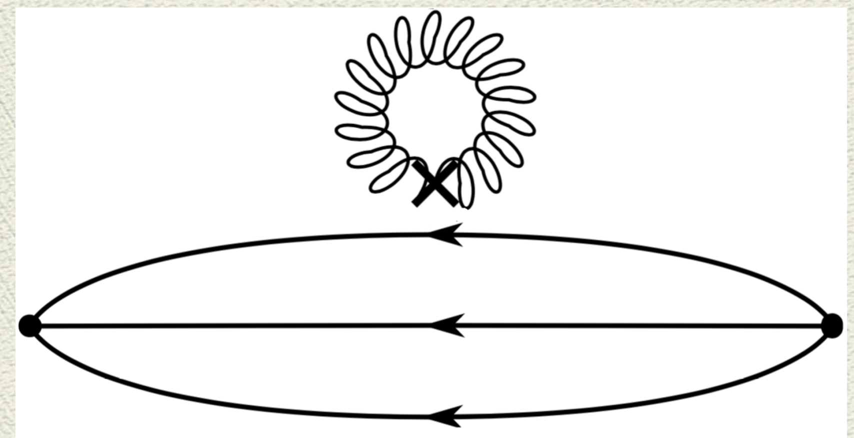
Nucleon on the Lattice



Connected



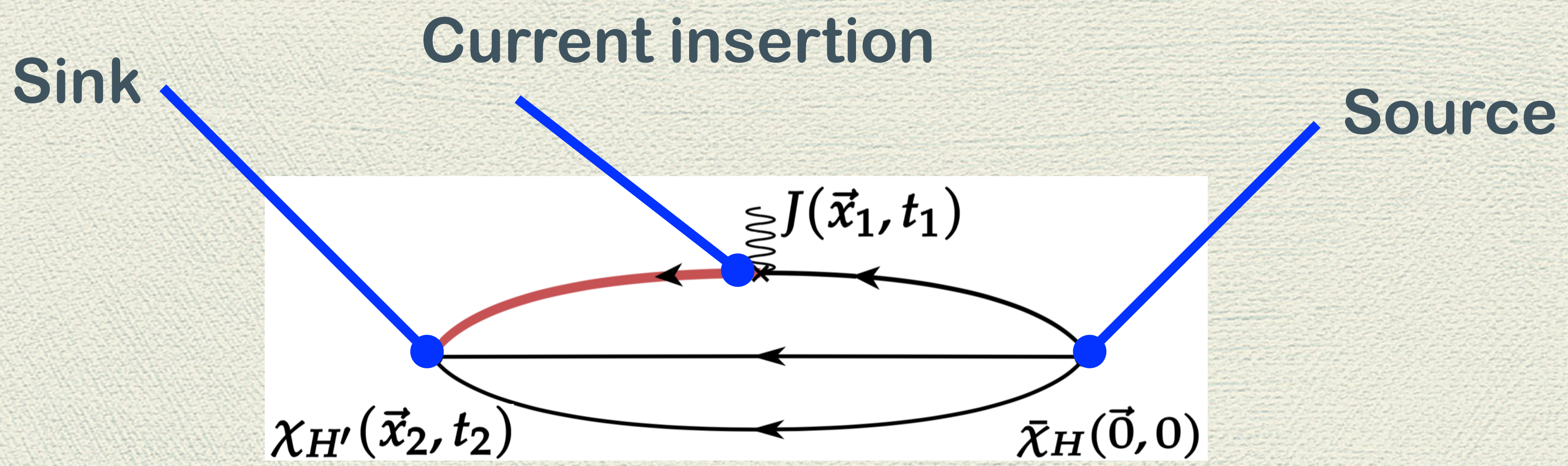
Disconnected
Quark loop



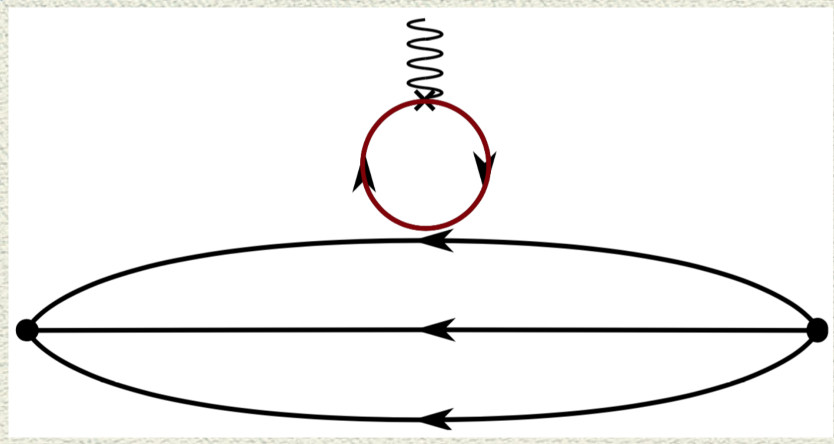
Disconnected
Gluon loop

- Separation between source and sink: excited states investigation
- Current insertion: ultra-local, covering derivative, non-local

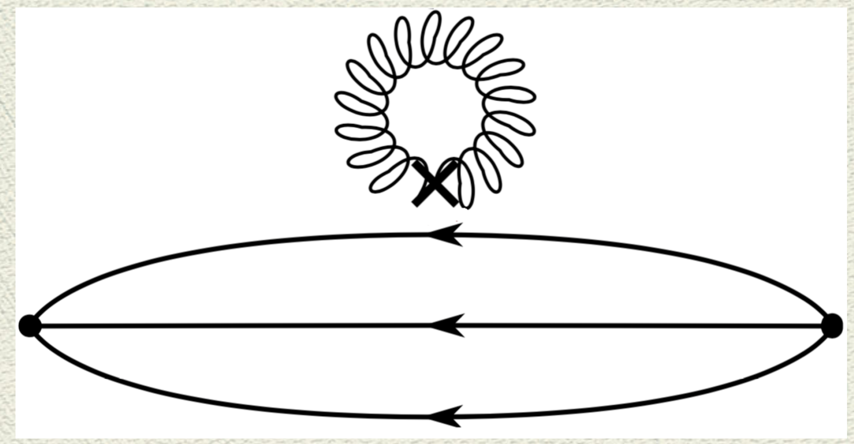
Nucleon on the Lattice



Connected



Disconnected
Quark loop



Disconnected
Gluon loop

Particularly interesting for EIC physics

- Separation between source and sink: excited states investigation
- Current insertion: ultra-local, covering derivative, non-local

Inherited Uncertainties

Laborious effort to eliminate uncertainties

Statistical errors significantly increase with:

* decrease of pion mass
* increase of momentum transfer between initial-final state
* increase of source-sink separation T_{sink}

Sources of systematic uncertainties:

* cut-off effects (finite lattice spacing)
* finite volume effects
* contamination from other hadron states
* chiral extrapolation for unphysical pion mass
* renormalization and mixing

Set up of Calculation

* Twisted Mass including a clover term (ETM Collaboration)

Ensemble	N_f	$L^3 \times T$	a(fm)	m_π (MeV)	Lm_π
Nf2.48c	2	$48^3 \times 96$	0.094	135	2.98
Nf2.64c	2	$64^3 \times 128$	0.094	132	3.97
Nf211.64c	2+1+1	$64^3 \times 128$	0.081	135	3.55

Nf211.64c (conn.)						Nf211.64c (disc.)		
$T_{\rm sink}$	$N_{\rm conf}$	$N_{\rm src}$	$T_{\rm sink}$	$N_{\rm conf}$	$N_{\rm src}$	Flavor	$N_{\rm conf}$	N_{src}
—	—	—	$16a$	625	16	u	750	200
$12a$	625	2	$18a$	625	32	d	750	200
$14a$	625	6	$20a$	625	32	s	750	200

Set up of Calculation

* Twisted Mass including a clover term (ETM Collaboration)

Ensemble	N_f	$L^3 \times T$	a(fm)	m_π (MeV)	Lm_π
Nf2.48c	2	$48^3 \times 96$	0.094	135	2.98
Nf2.64c	2	$64^3 \times 128$	0.094	132	3.97
Nf211.64c	2+1+1	$64^3 \times 128$	0.081	135	3.55

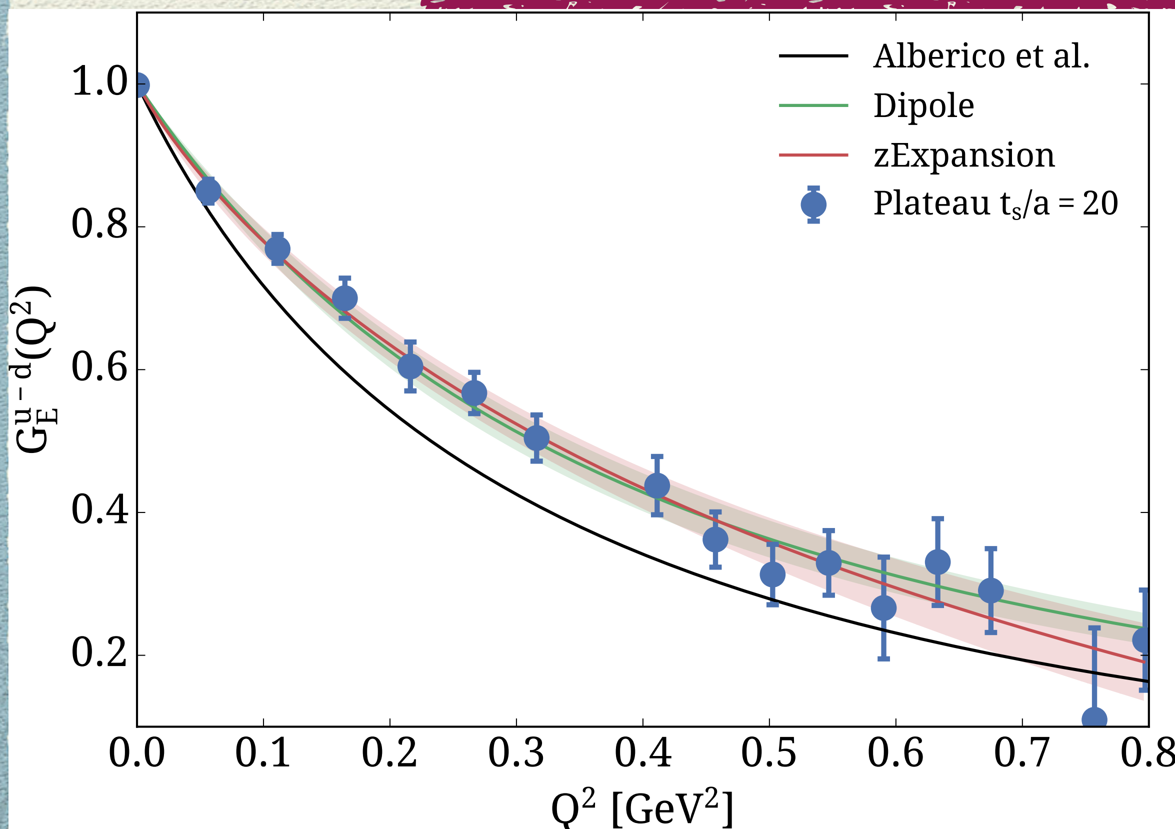
Nf211.64c (conn.)						Nf211.64c (disc.)		
$T_{\rm sink}$	$N_{\rm conf}$	$N_{\rm src}$	$T_{\rm sink}$	$N_{\rm conf}$	$N_{\rm src}$	Flavor	$N_{\rm conf}$	N_{src}
—	—	—	16a	625	16	u	750	200
12a	625	2	18a	625	32	d	750	200
14a	625	6	20a	625	32	s	750	200

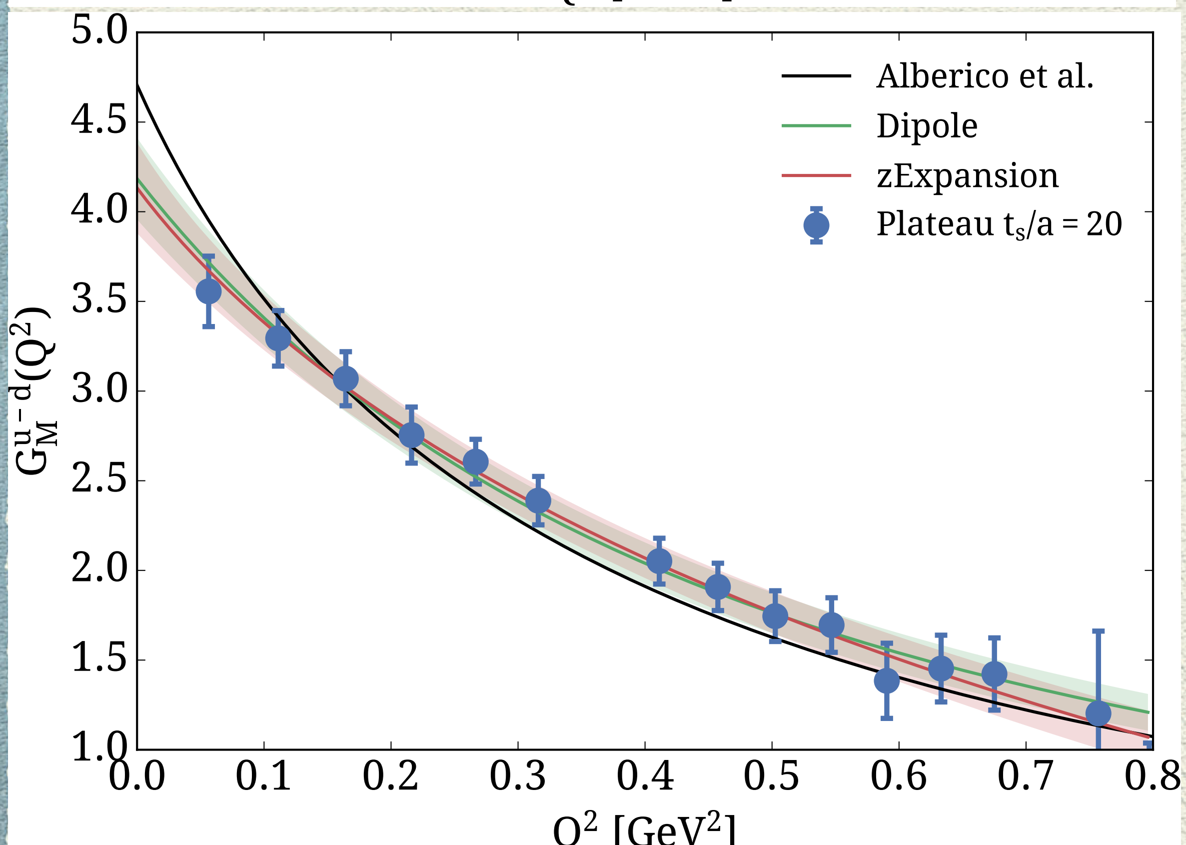
* Computer architecture (GPUs) and special techniques allow calculations at the physical point. In this work:
  - Hierarchical probing (HP)
  - One-end trick for Twisted Mass Fermions (OET)
  - Spin-color dilution (SCD)
  - Deflation (D)

* We study:
  - Isovector combination (u−d): only connected
  - Flavor decompositions: both connected and disconnected
  - Strange & charm contributions purely disconnected

OUTLINE

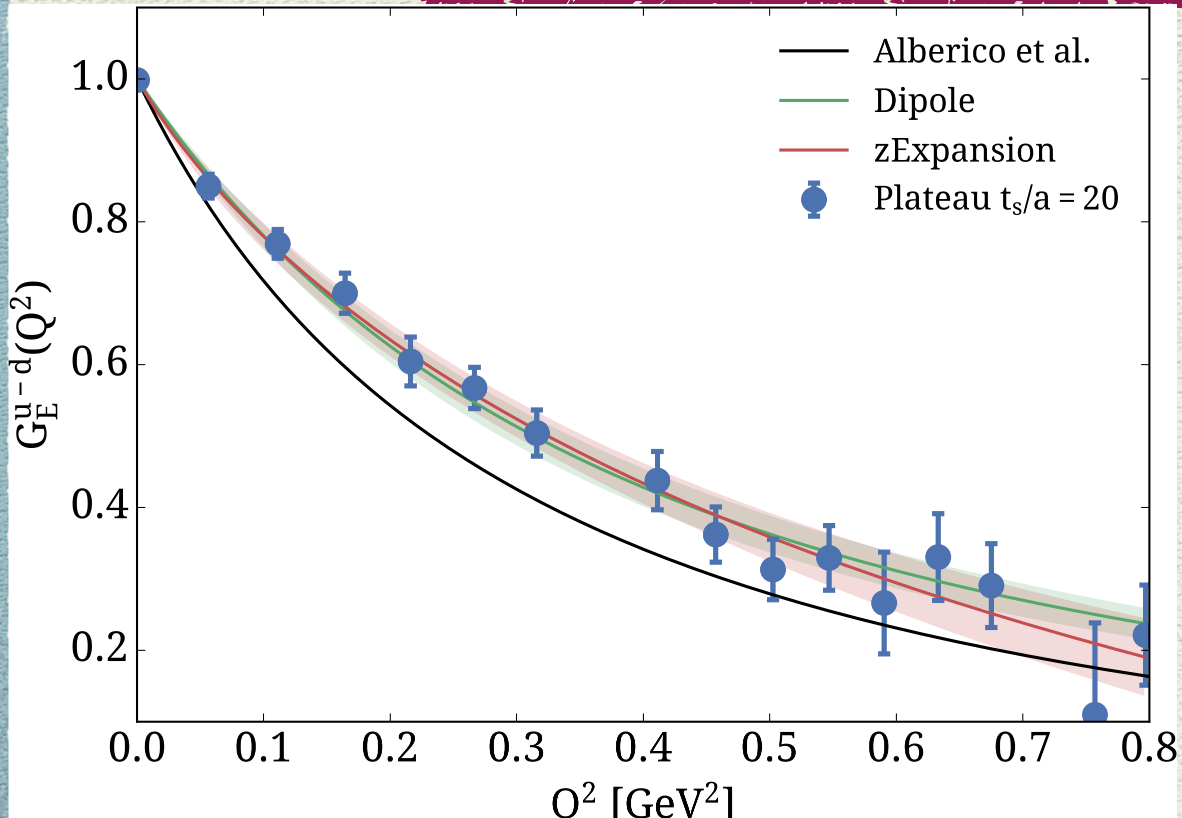
Nucleon E/M Form Factors

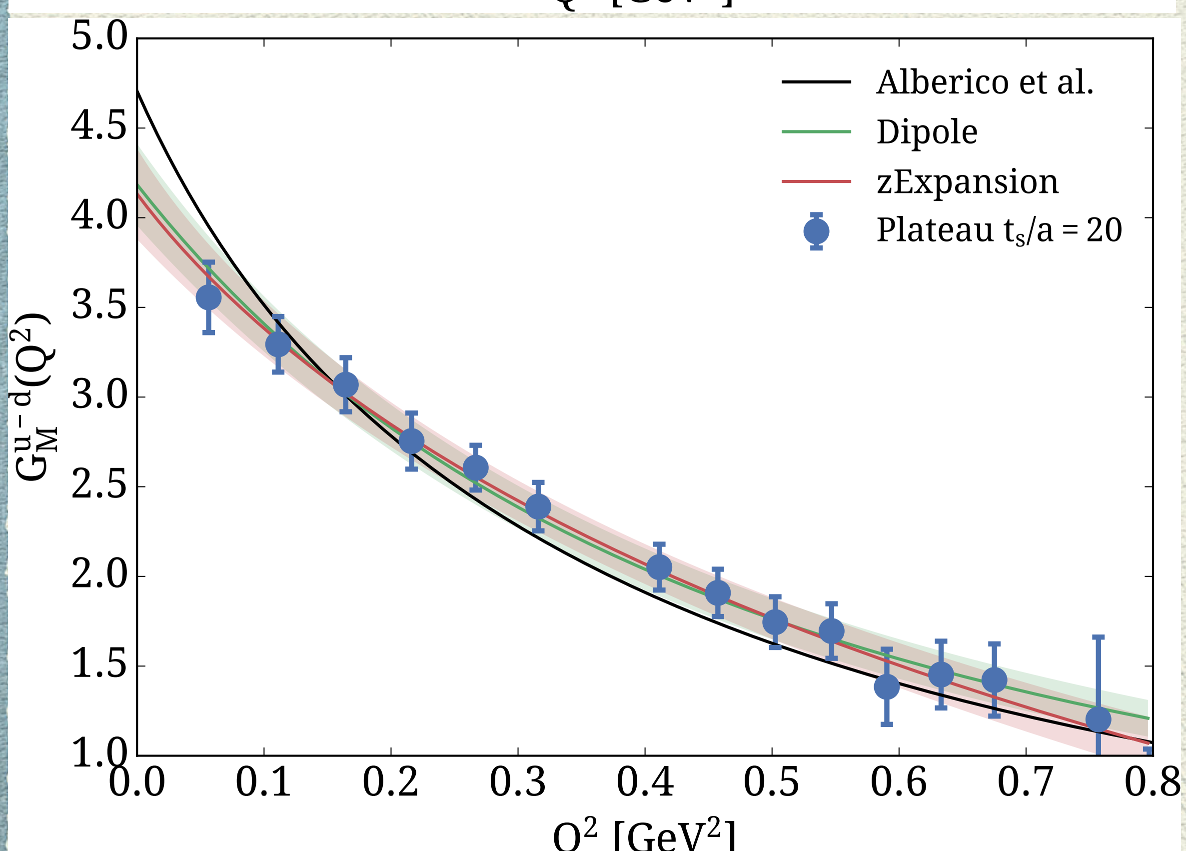




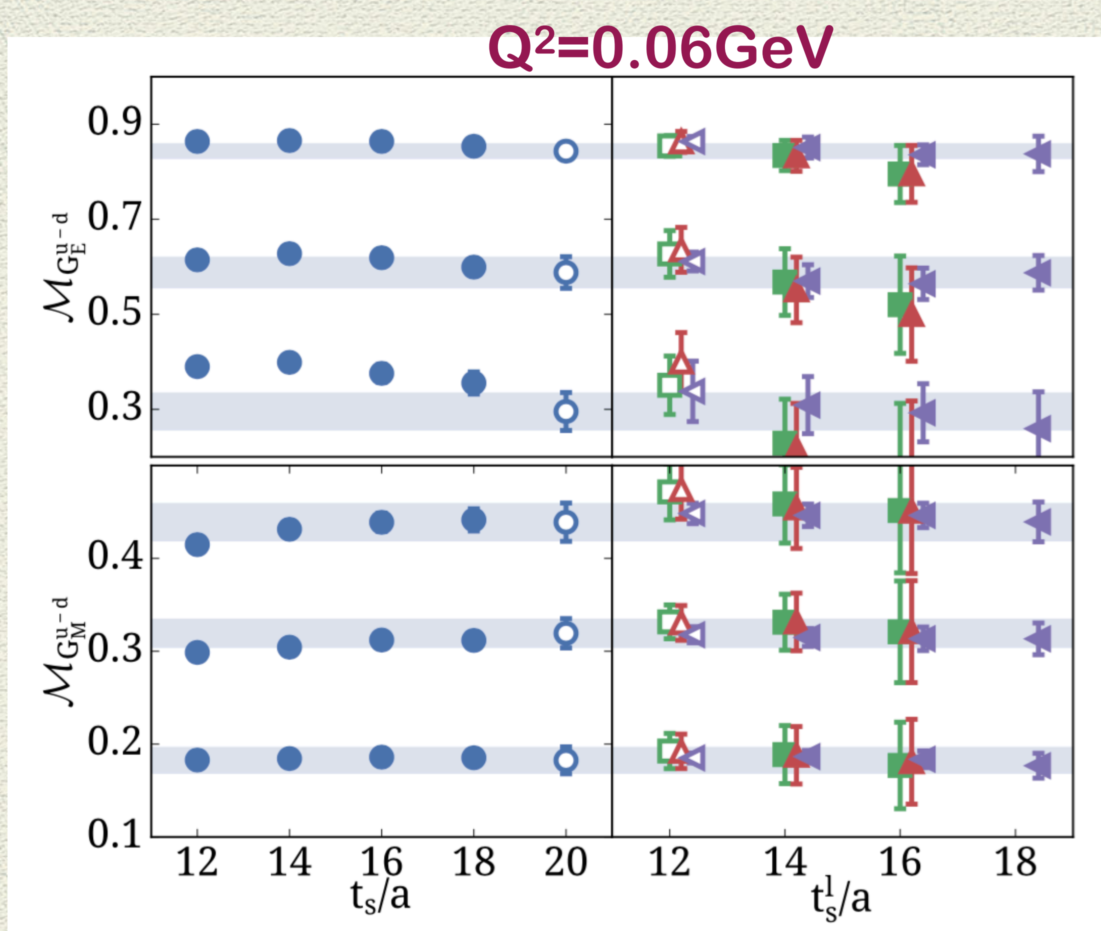
- G_E: slope of lattice data diff from experiments
- G_M: small-Q^2 has improved slope (Tsink =1.6fm)

Nucleon E/M Form Factors



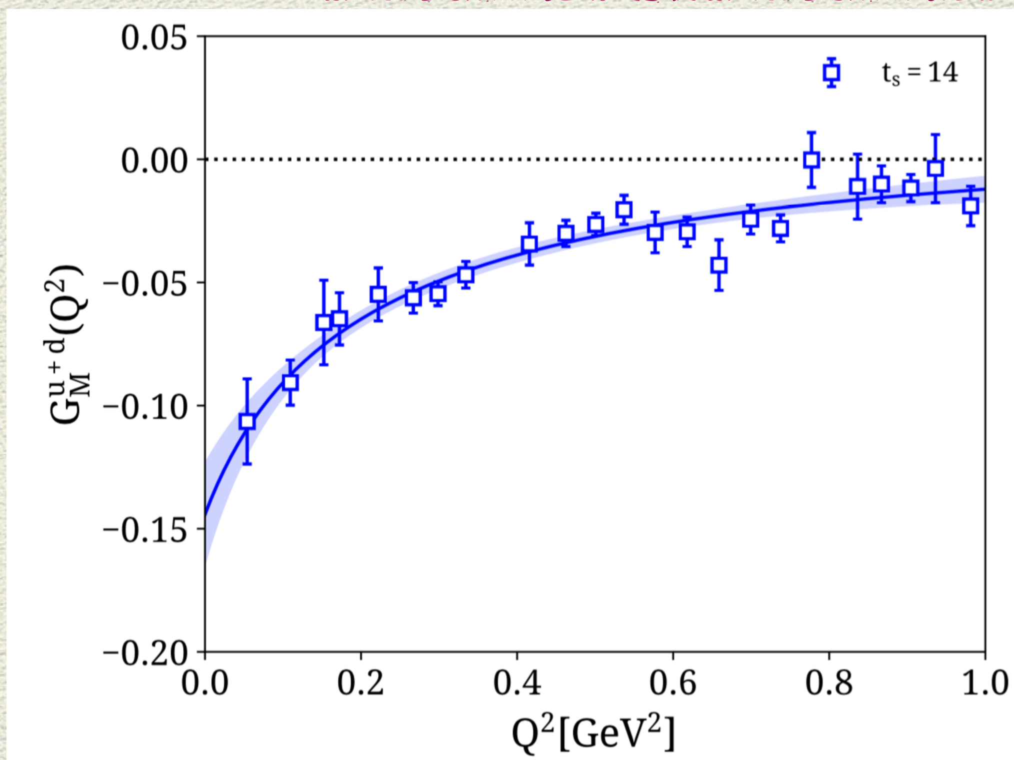


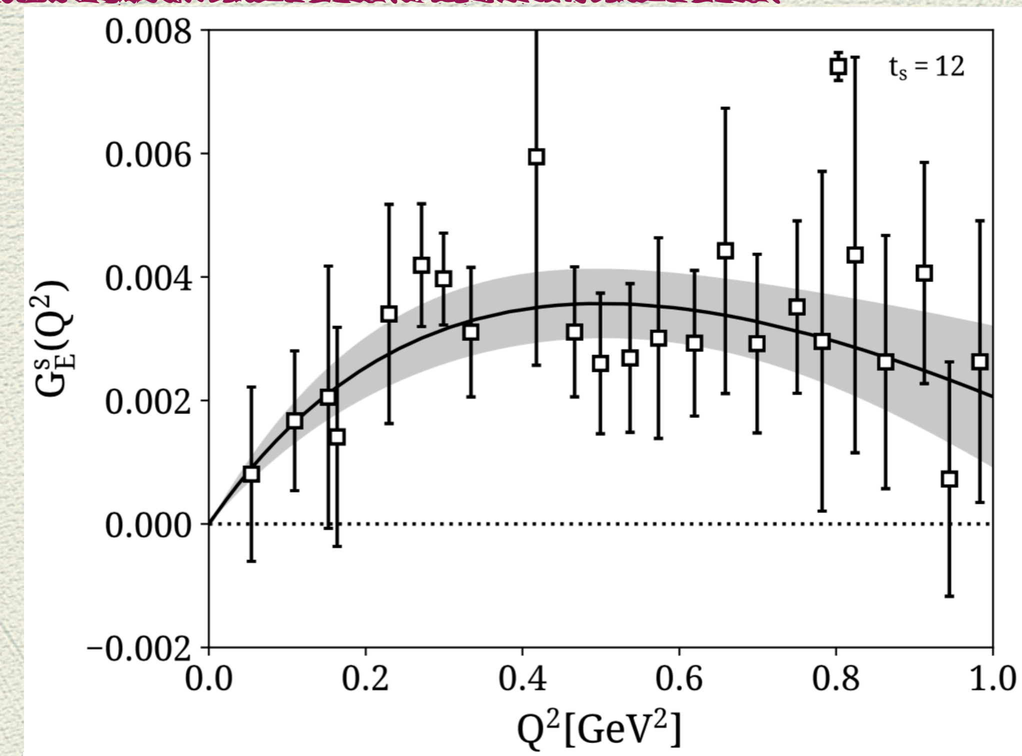
- G_E: slope of lattice data diff from experiments
- G_M: small-Q^2 has improved slope (Tsink =1.6fm)
- Excited states investigations: Tsink=1 – 1.6fm

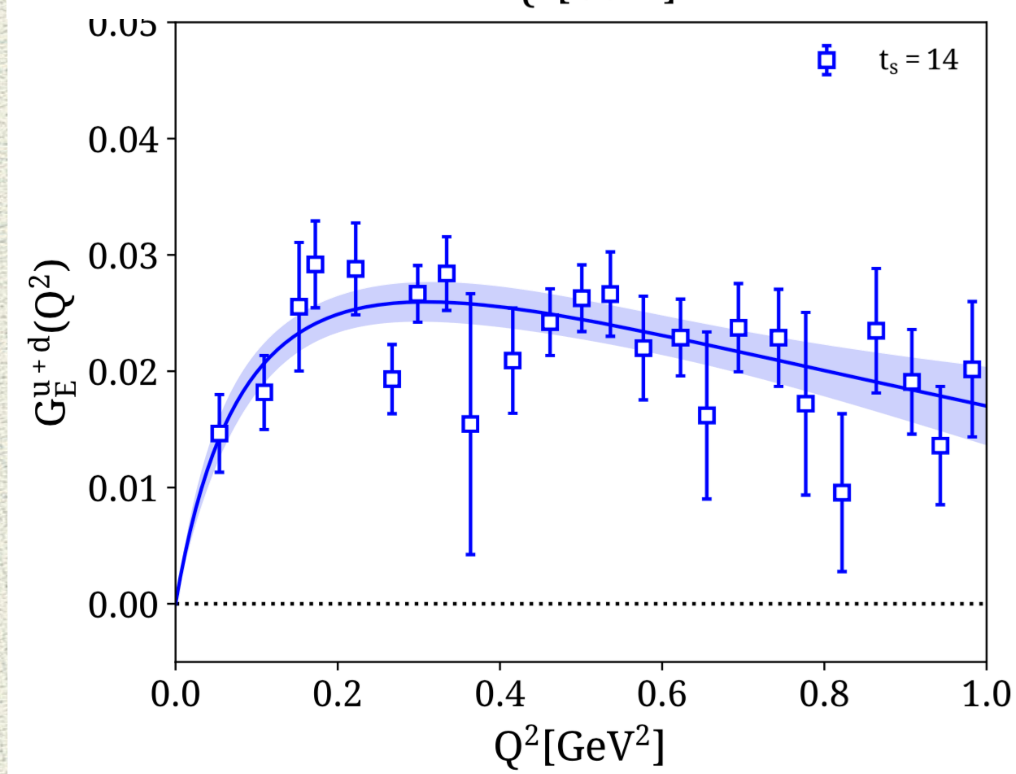


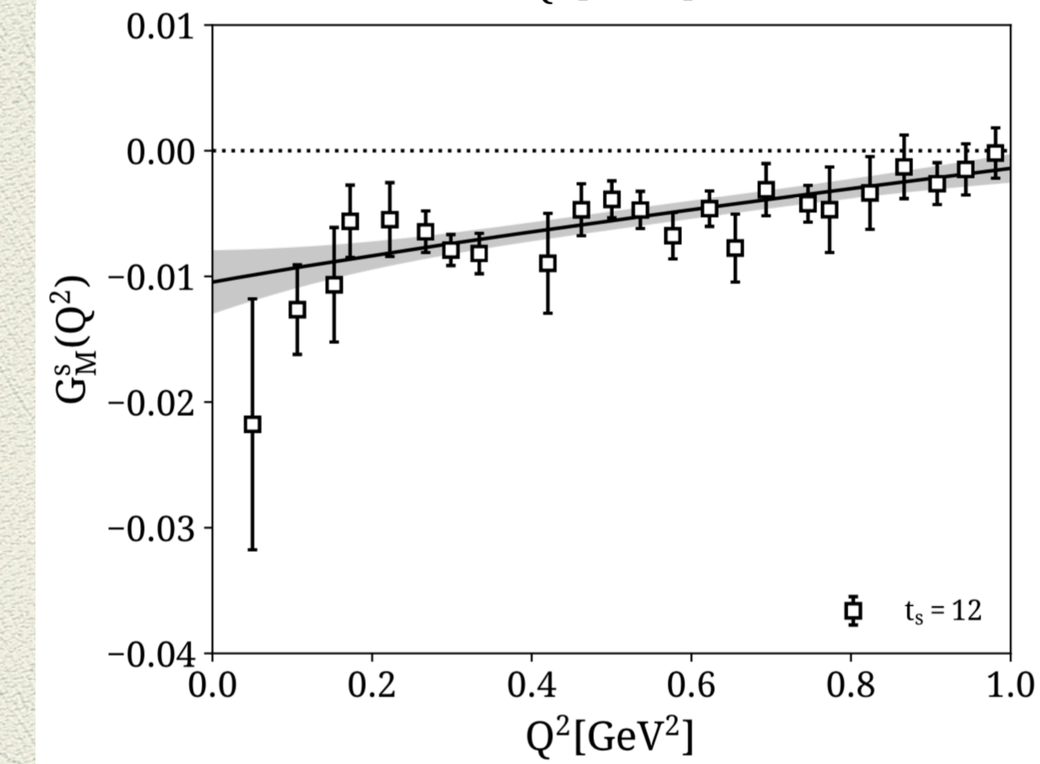
Plateau T^{low}(summation)

Nucleon E/M Form Factors









- ✱ Clear signal due to algorithmic advances
- ✱ Tsink chosen based on excited states and quality of fits

Nucleon charged radii

Dipole fit: motivated by vector-meson pole contributions to FFs

$$G_E(Q^2) = \frac{1}{\left(1 + Q^2/m_E^2\right)},\ G_M(Q^2) = \frac{G_M(0)}{\left(1 + Q^2/m_M^2\right)^2},\ \langle r^2_{E,M} \rangle = \frac{12}{m^2_{E,M}}$$

z-expansion: model-independent, expected to model better the low-Q^2

$$G_i(Q^2) = \sum_k a_k z(Q^2)^k,\ z(Q^2) = \frac{\sqrt{t_{cut} + Q^2} - \sqrt{t_{cut}}}{\sqrt{t_{cut} + Q^2} + \sqrt{t_{cut}}},\ t_{cut} = 4m_\pi^2,\ \langle r^2_{E,M} \rangle = -\frac{6a_1^{E,M}}{4t_{cut}a_0^{E,M}}$$

Nucleon charged radii

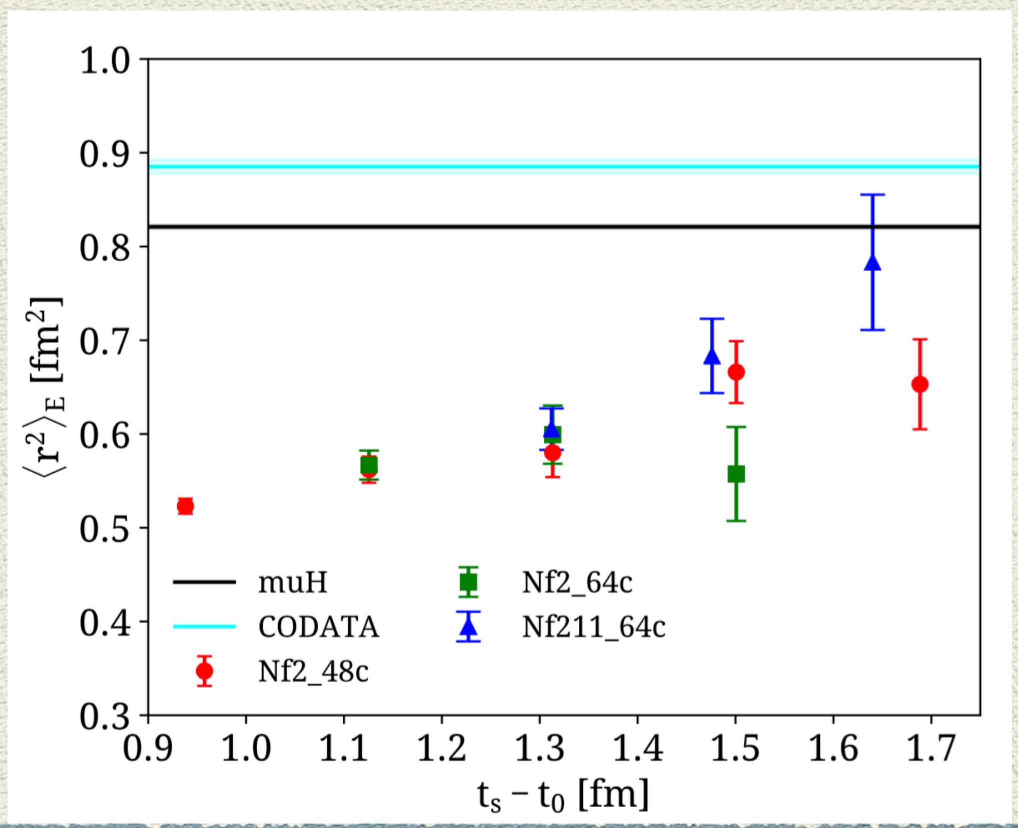
Dipole fit: motivated by vector-meson pole contributions to FFs

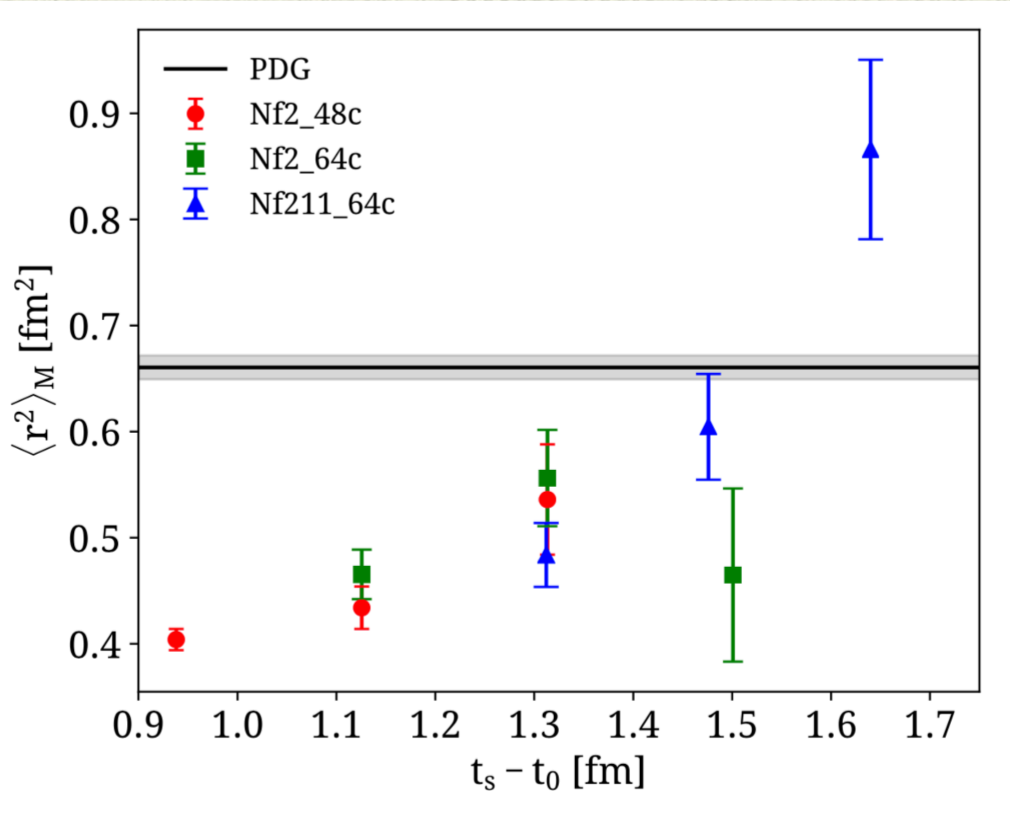
$$G_E(Q^2) = \frac{1}{\left(1+Q^2/m_E^2\right)},\ G_M(Q^2) = \frac{G_M(0)}{\left(1+Q^2/m_M^2\right)^2},\ \langle r^2_{E,M}\rangle = \frac{12}{m^2_{E,M}}$$

z-expansion: model-independent, expected to model better the low-Q^2

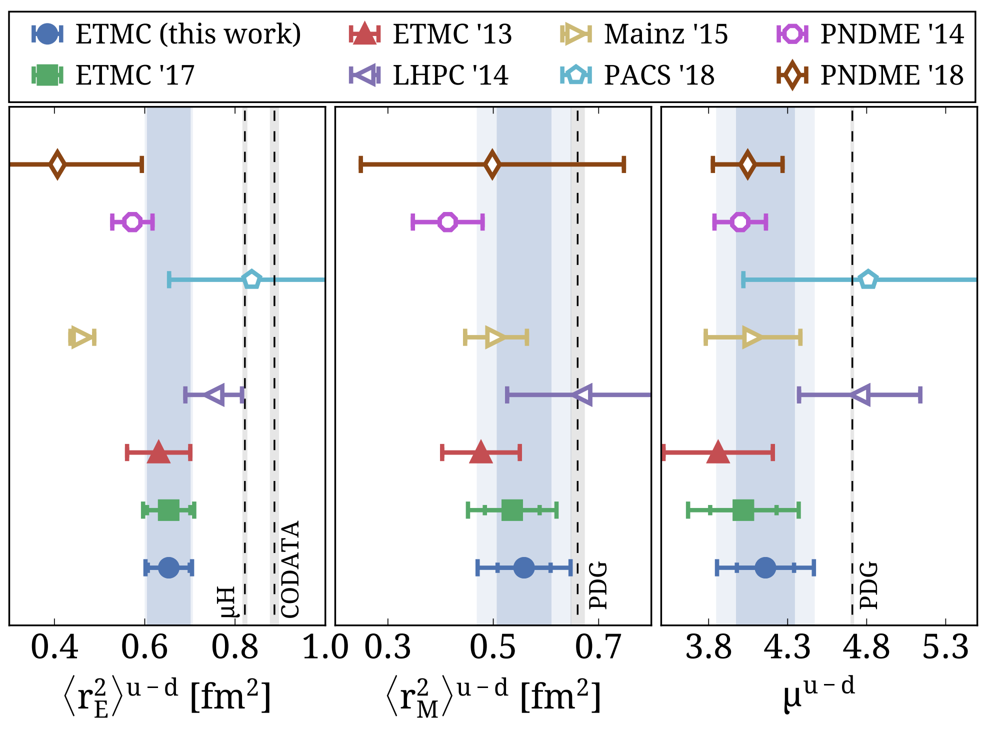
$$G_i(Q^2) = \sum_k a_k z(Q^2)^k,\ z(Q^2) = \frac{\sqrt{t_{cut}+Q^2}-\sqrt{t_{cut}}}{\sqrt{t_{cut}+Q^2}+\sqrt{t_{cut}}},\ t_{cut} = 4m_\pi^2,\ \langle r^2_{E,M}\rangle = -\frac{6a_1^{E,M}}{4t_{cut}a_0^{E,M}}$$

- Estimation of radii strongly depends on small Q^2
- Large volume: access to Q^2 close to zero



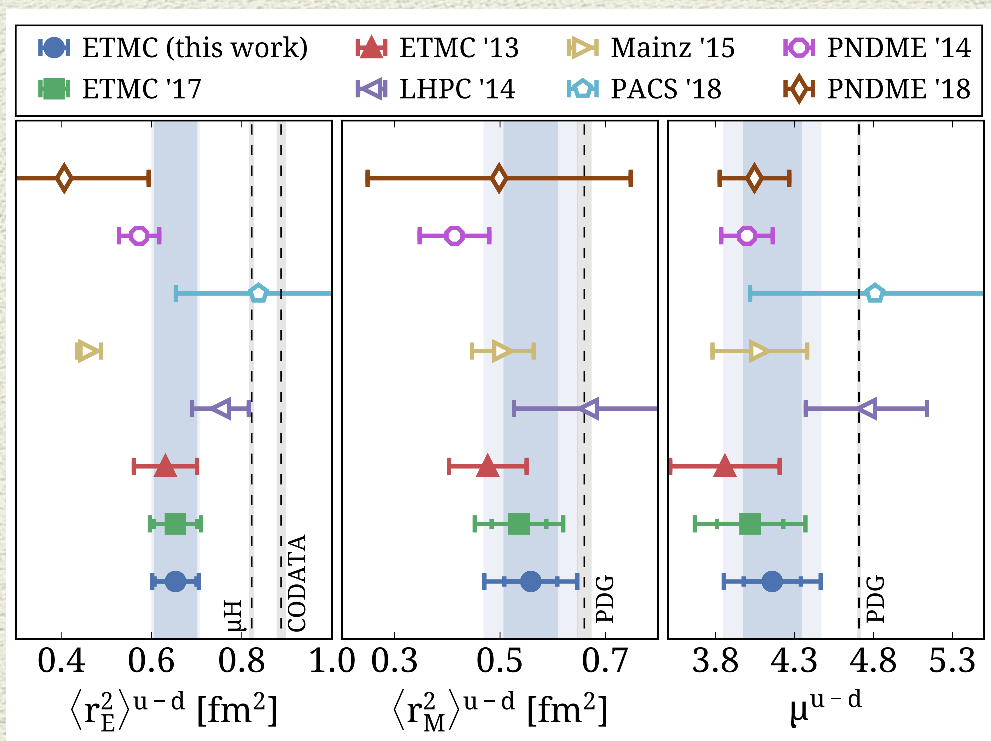


Nucleon charged radii



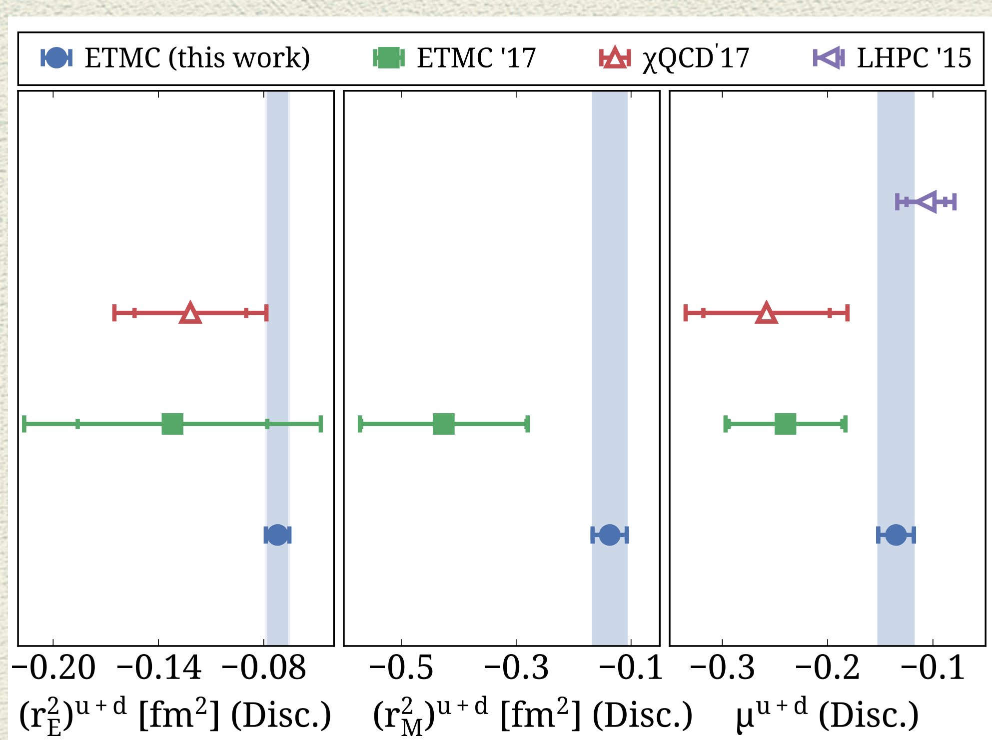
- E/M radii well-studied by several lattice groups
- Agreement among several formulations, but quantities sensitive to systematics, thus one must examine fit methods
- Magnetic moment from $G_M(0)$

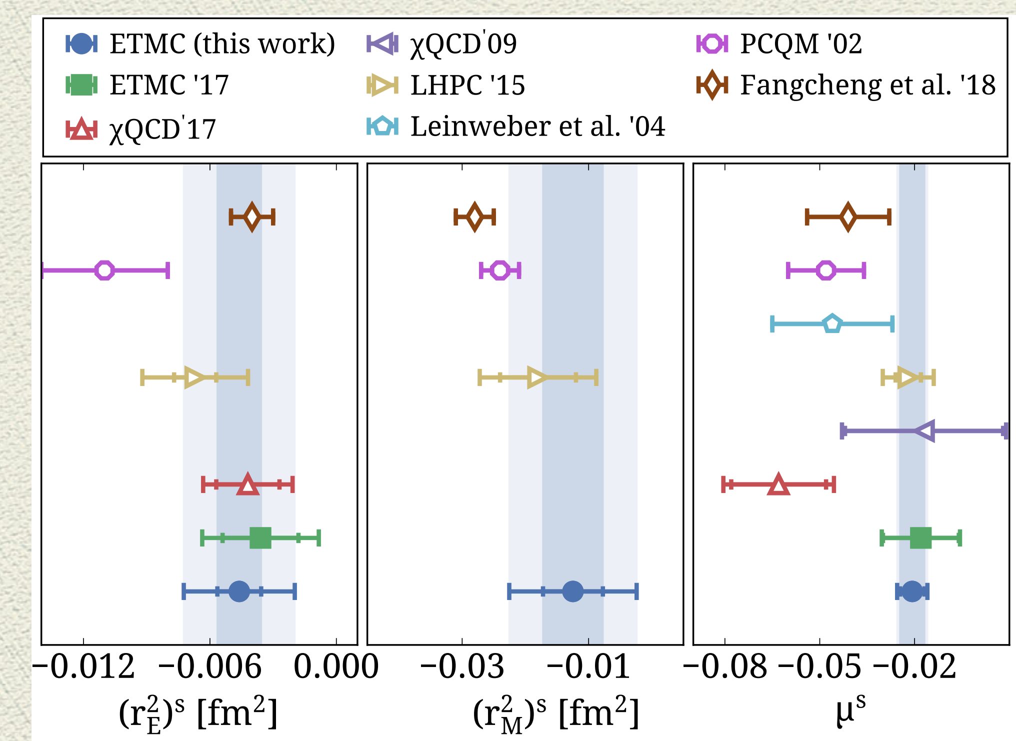
Nucleon charged radii



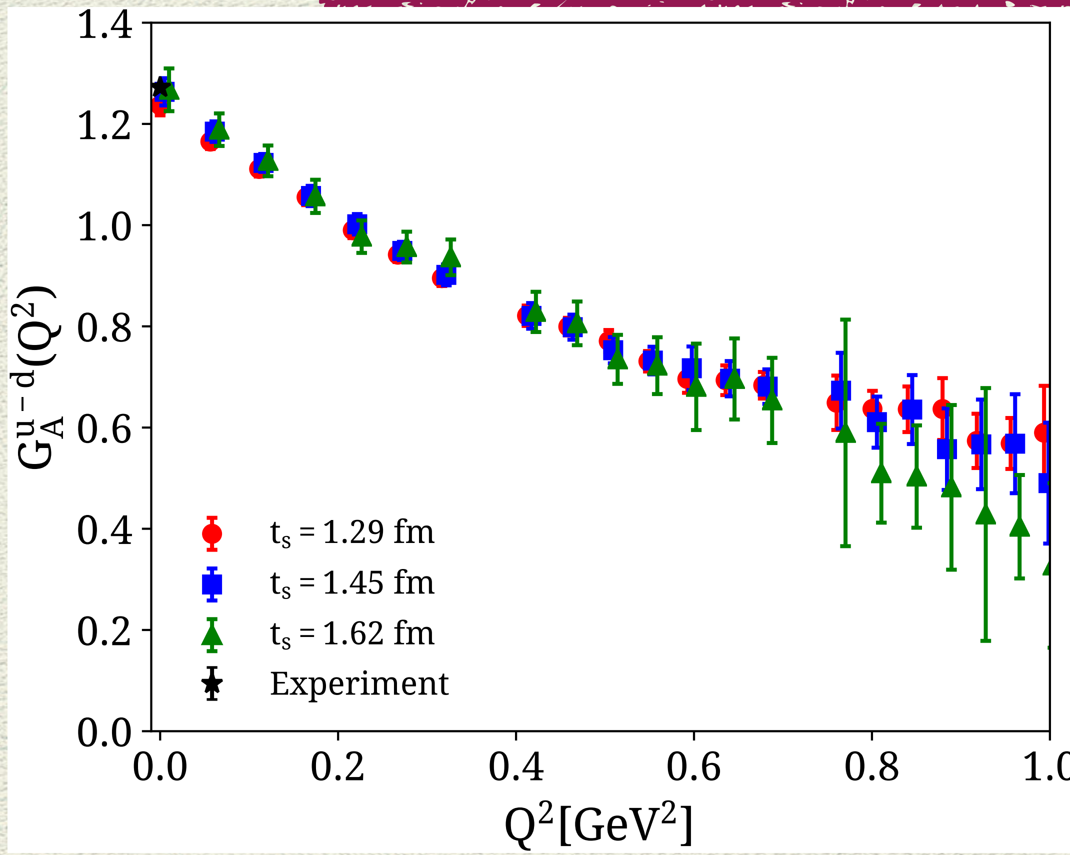
- E/M radii well-studied by several lattice groups
- Agreement among several formulations, but quantities sensitive to systematics, thus one must examine fit methods
- Magnetic moment from $G_M(0)$

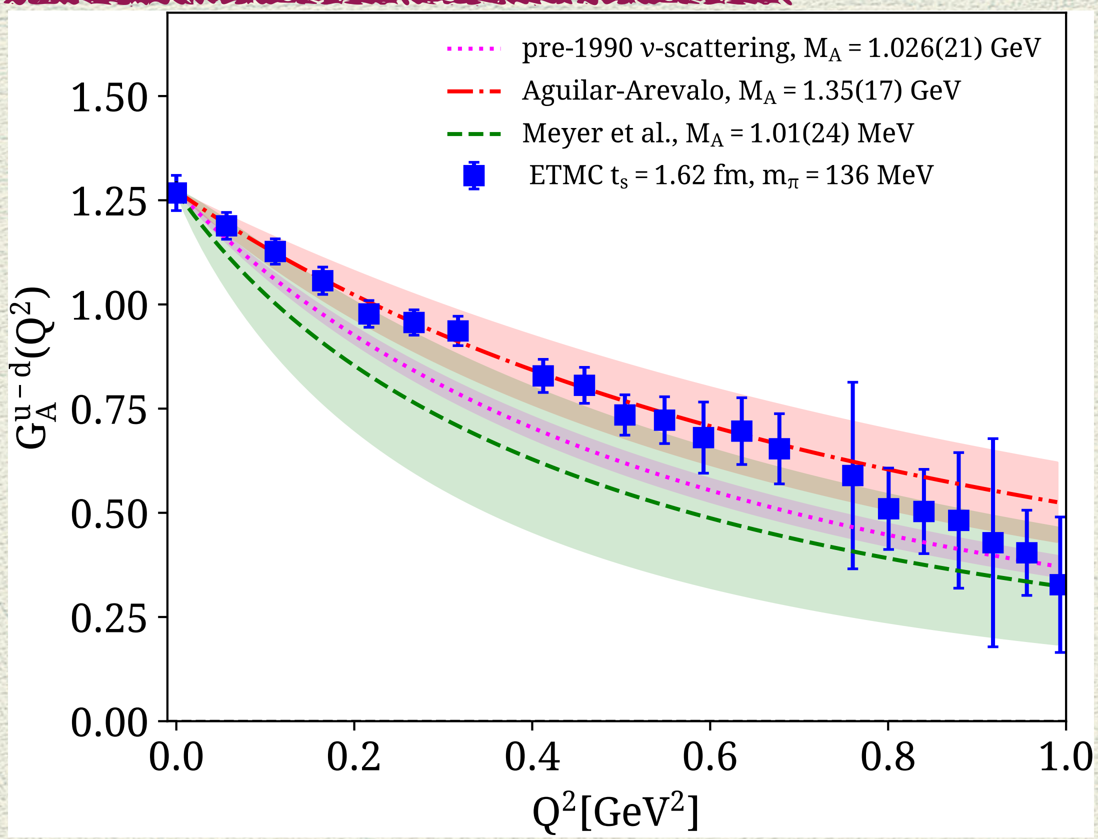
- Fewer studies for (light quark) disconnected contributions

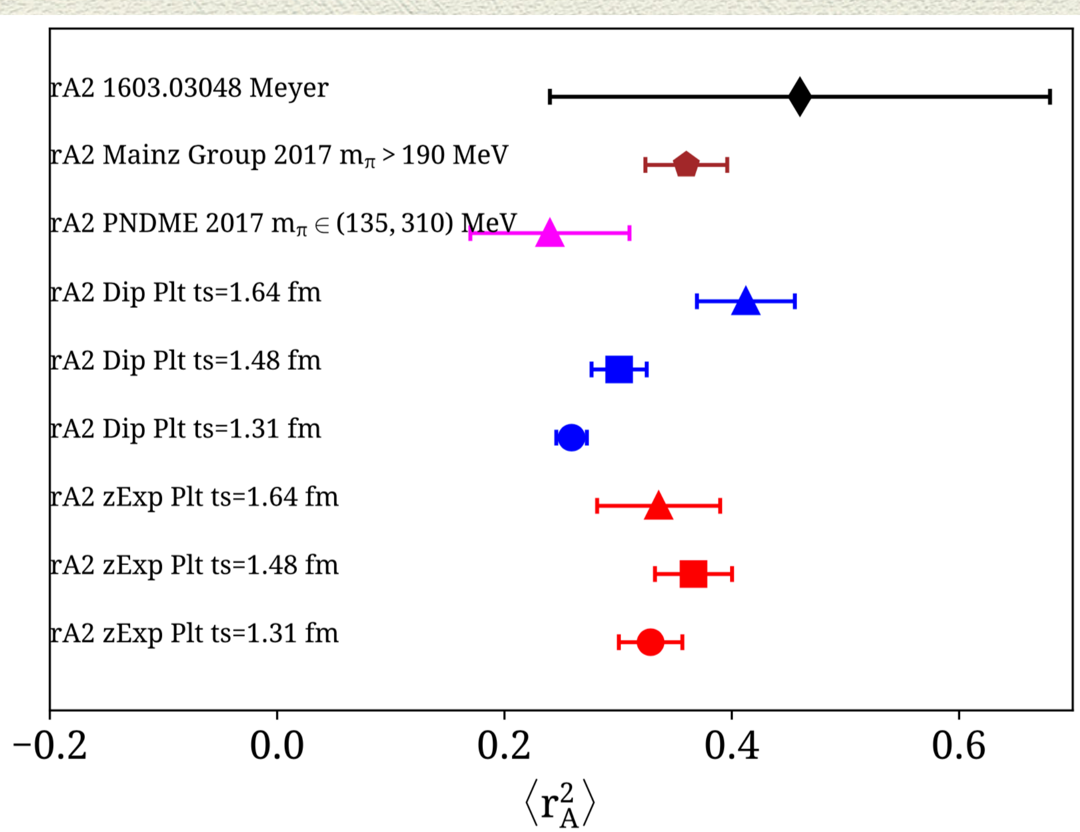




Nucleon Axial Form Factors



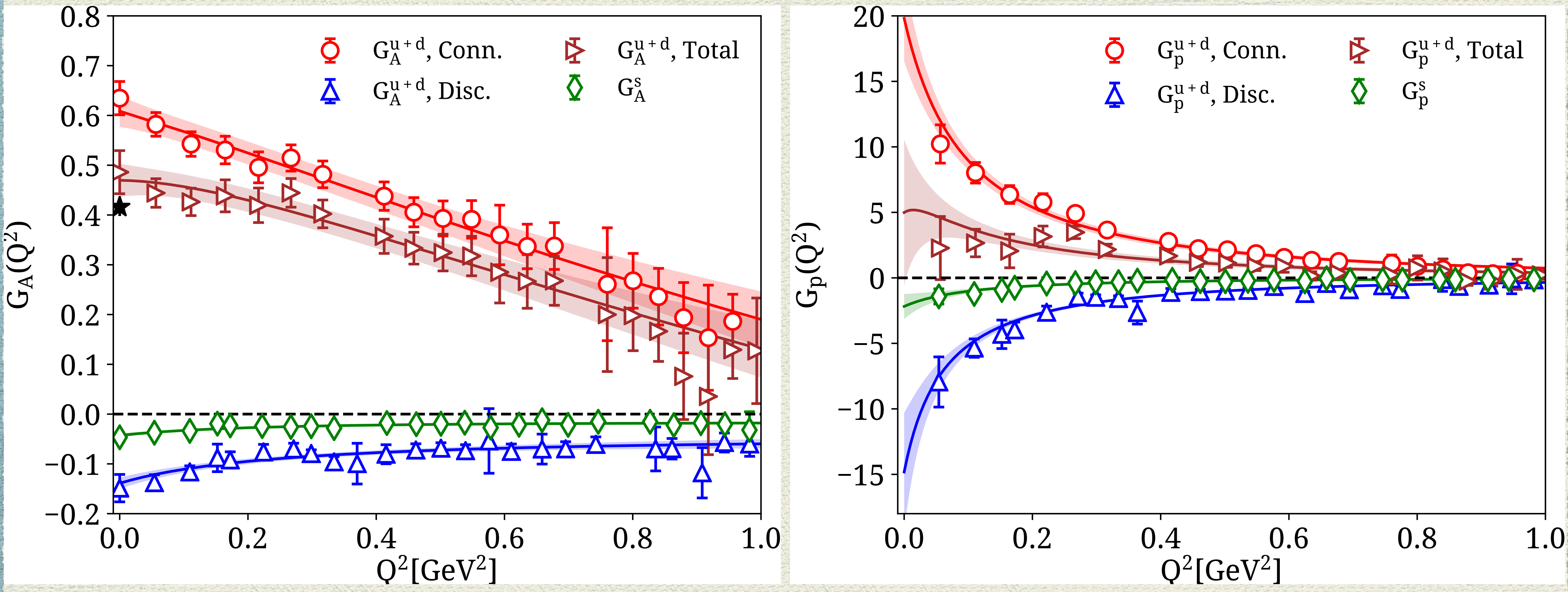




- Left: Data for Tsink > 1.2fm compatible, but slope different
- Right: Lattice data compatible with upper range of neutrino-nucleus cross sections (green band) and with MiniBooNE (red band)

- Parametrization of lattice data: Dipole fit, z-expansion
- Deviations on <r_A> from different methods

Nucleon Axial Form Factors (isoscalar)



- Light quark disconnected contributions are sizable
- G_s small but necessary in order to bring G_A^{total} close to experimental value
- $G_p^{u+d,DI}$ cancels (within uncertainties) $G_p^{u+d,CI}$
- G_p^s suppressed compared to light quark contribution

OUTLINE

Spin structure from first principles

DIS experiment (1988) shows surprising results for proton spin

"... g1(x) for the proton has been determined and its integral over x found to be 0.114±0.012±0.026, in disagreement with the Ellis-Jaffe sum rule. ... These values for the integrals of g1 lead to the conclusion that the total quark spin constitutes a rather small fraction of the spin of the nucleon." [J. Ashman et al., Phys. Lett., vol. B206 (1988) 364]

Spin structure from first principles

DIS experiment (1988) shows surprising results for proton spin

"... g1(x) for the proton has been determined and its integral over x found to be 0.114±0.012±0.026, in disagreement with the Ellis-Jaffe sum rule. ... These values for the integrals of g1 lead to the conclusion that the total quark spin constitutes a rather small fraction of the spin of the nucleon." [J. Ashman et al., Phys. Lett., vol. B206 (1988) 364]

We need a theoretical formulation to address the proton spin puzzle

Lattice QCD

Spin structure from first principles

DIS experiment (1988) shows surprising results for proton spin

"... g1(x) for the proton has been determined and its integral over x found to be 0.114±0.012±0.026, in disagreement with the Ellis-Jaffe sum rule. ... These values for the integrals of g1 lead to the conclusion that the total quark spin constitutes a rather small fraction of the spin of the nucleon." [J. Ashman et al., Phys. Lett., vol. B206 (1988) 364]

We need a theoretical formulation to address the proton spin puzzle

Lattice QCD

Spin Sum Rule (Ji)

$$\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left(L^q + \frac{1}{2}\Delta\Sigma^q \right) + J^G$$

L_q: Quark orbital angular momentum

$\Delta\Sigma_q$: Intrinsic spin

J_g: Gluon spin

Spin structure from first principles

DIS experiment (1988) shows surprising results for proton spin

"... g1(x) for the proton has been determined and its integral over x found to be 0.114±0.012±0.026, in disagreement with the Ellis-Jaffe sum rule. ... These values for the integrals of g1 lead to the conclusion that the total quark spin constitutes a rather small fraction of the spin of the nucleon." [J. Ashman et al., Phys. Lett., vol. B206 (1988) 364]

We need a theoretical formulation to address the proton spin puzzle

Lattice QCD

Spin Sum Rule (Ji)

$$\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left(L^q + \frac{1}{2}\Delta\Sigma^q \right) + J^G$$

L_q: Quark orbital angular momentum

$\Delta\Sigma_q$: Intrinsic spin

J_g: Gluon spin

Extraction from Lattice QCD

$$J^q = \frac{1}{2}\left(A_{20}^q + B_{20}^q\right), \quad L^q = J^q - \Sigma^q, \quad \Sigma^q = g_A^q$$

Spin structure from first principles

DIS experiment (1988) shows surprising results for proton spin

"... g1(x) for the proton has been determined and its integral over x found to be 0.114±0.012±0.026, in disagreement with the Ellis-Jaffe sum rule. ... These values for the integrals of g1 lead to the conclusion that the total quark spin constitutes a rather small fraction of the spin of the nucleon." [J. Ashman et al., Phys. Lett., vol. B206 (1988) 364]

We need a theoretical formulation to address the proton spin puzzle

Lattice QCD

Spin Sum Rule (Ji)

$$\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left(L^q + \frac{1}{2}\Delta\Sigma^q \right) + J^G$$

L_q: Quark orbital angular momentum

$\Delta\Sigma_q$: Intrinsic spin

J_g: Gluon spin

Extraction from Lattice QCD

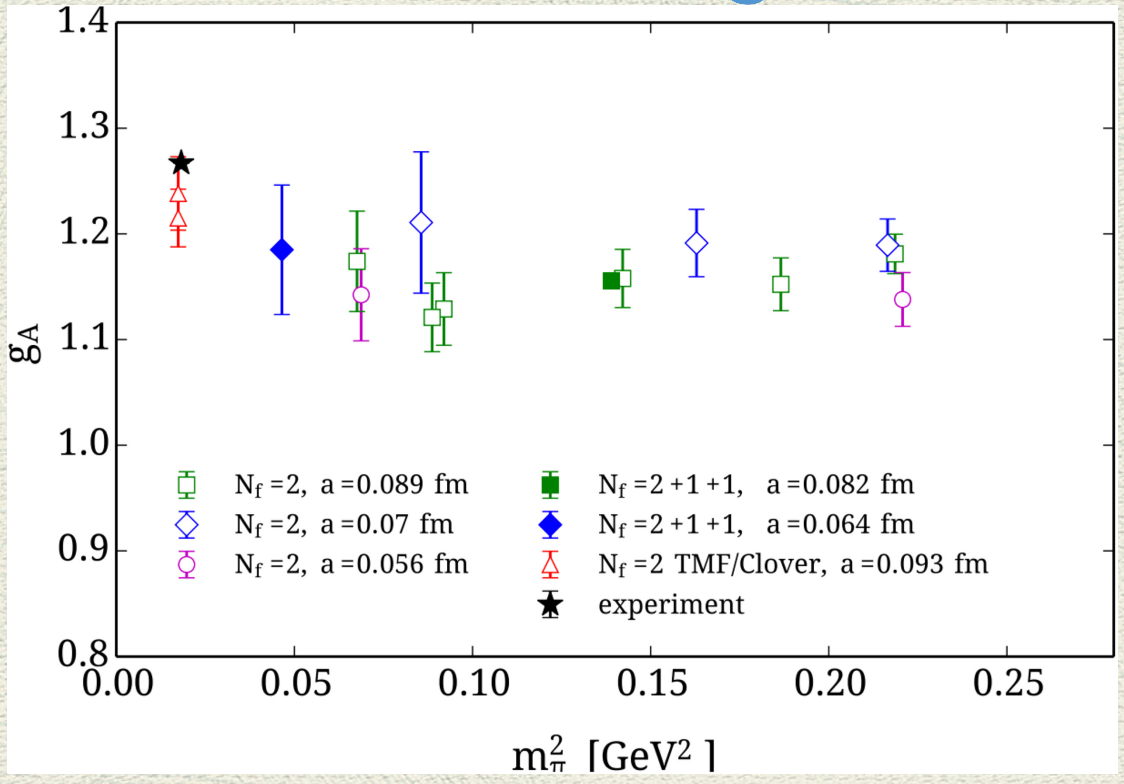
$$J^q = \frac{1}{2}\left(A_{20}^q + B_{20}^q \right), \quad L^q = J^q - \Sigma^q, \quad \Sigma^q = g_A^q$$

Necessary computations:

- Axial Charge
- Quark momentum fraction
- Gluon momentum fraction

The proton spin from LQCD

Axial charge

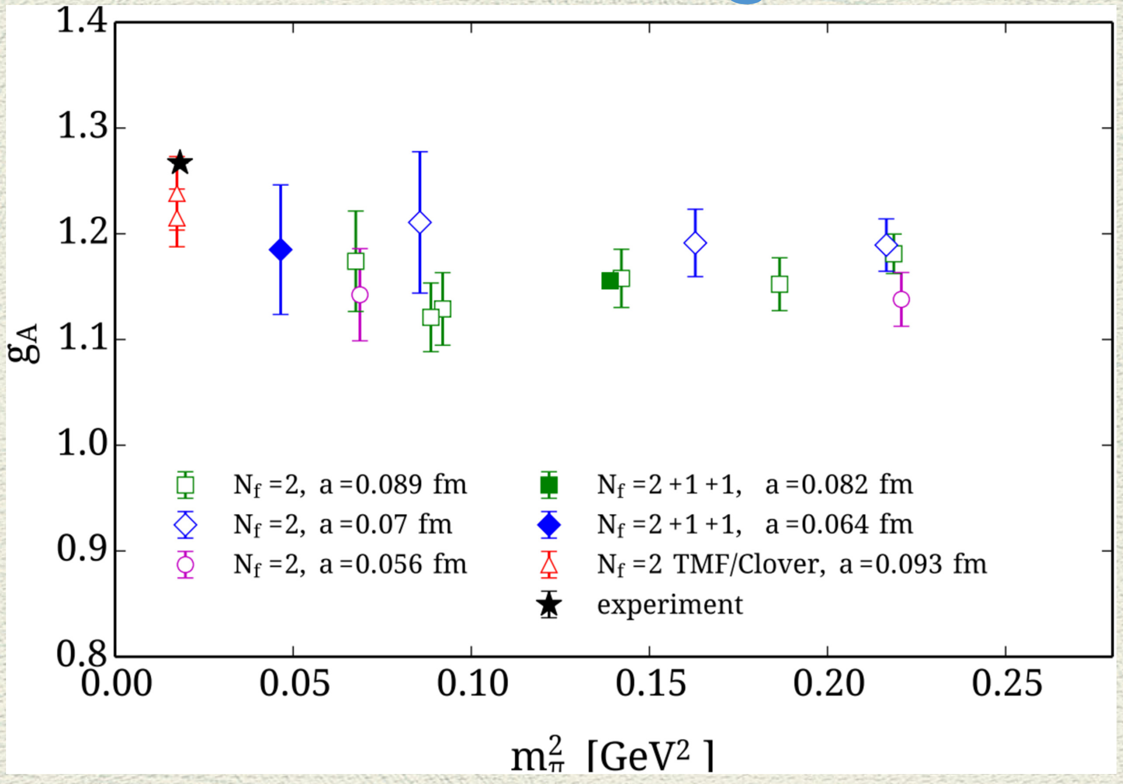


Excited states:

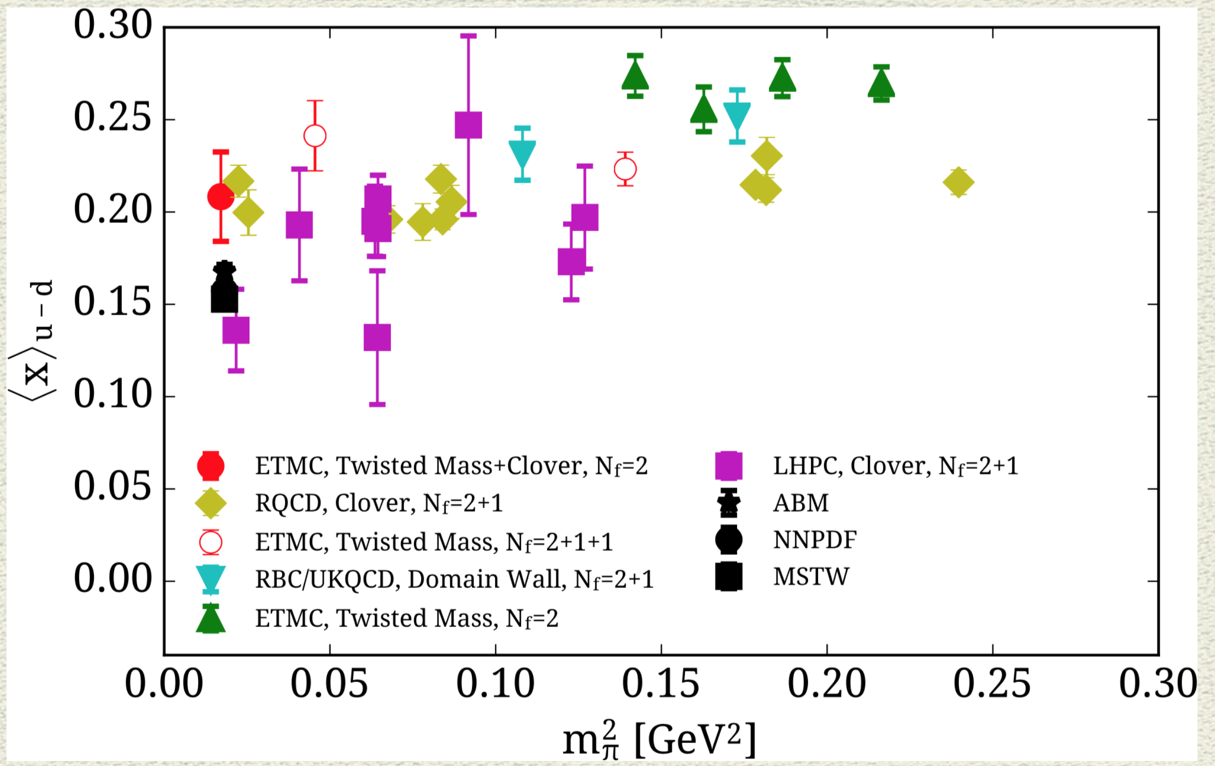
- $g_{A\ u\text{-}d}$: non-negligible
- $\langle x \rangle_{u\text{-}d}$: sizable
- $\langle x \rangle_g$: negligible

The proton spin from LQCD

Axial charge



Quark momentum fraction

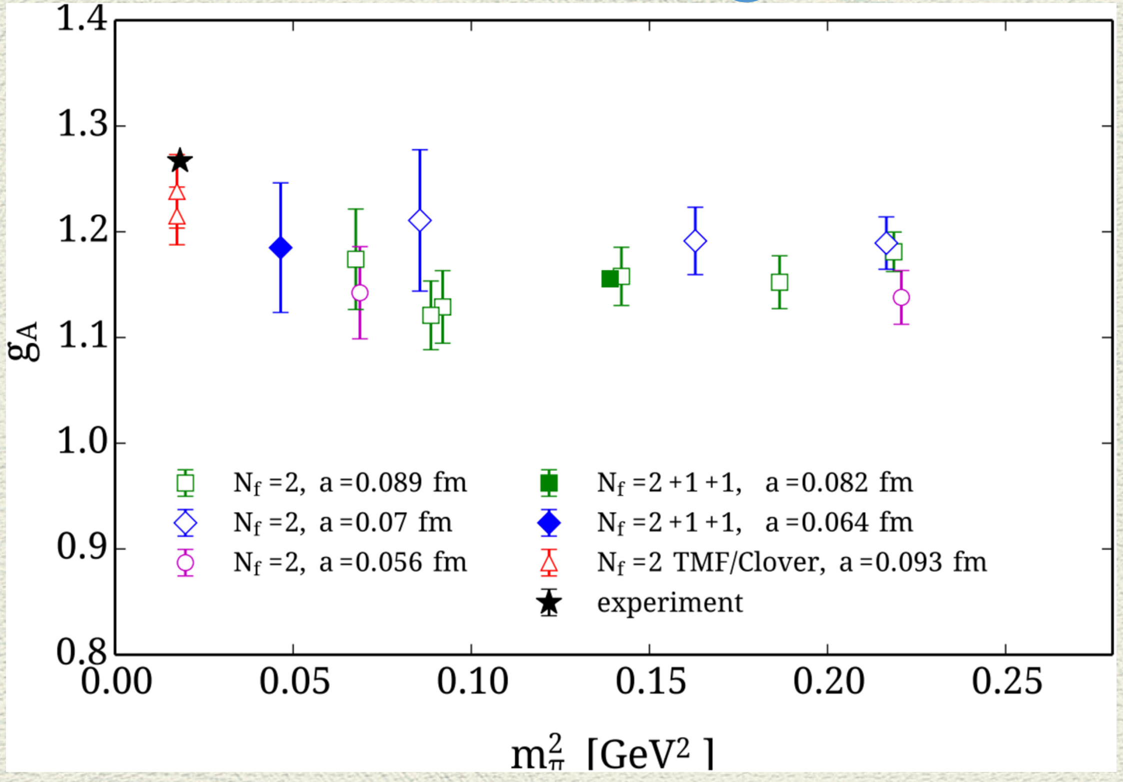


Excited states:

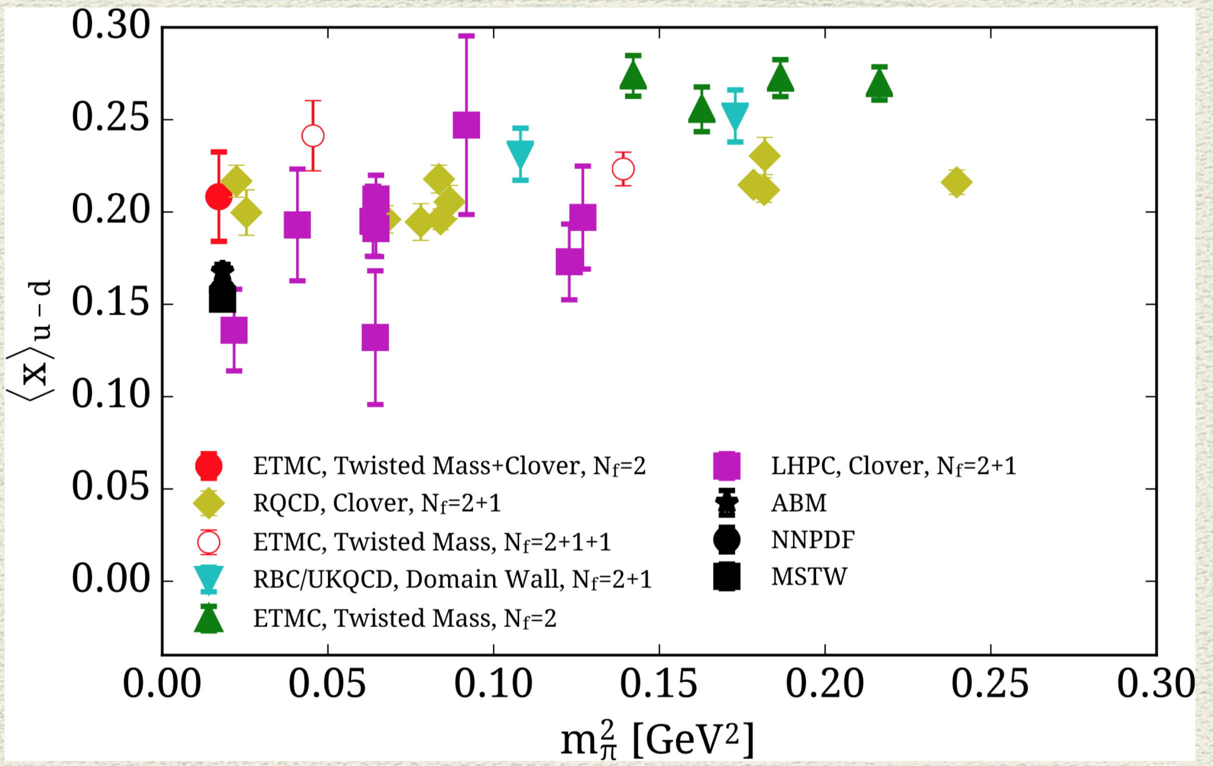
- $g_{A\ u\text{-}d}$: non-negligible
- $\langle x \rangle_{u\text{-}d}$: sizable
- $\langle x \rangle_{g}$: negligible

The proton spin from LQCD

Axial charge



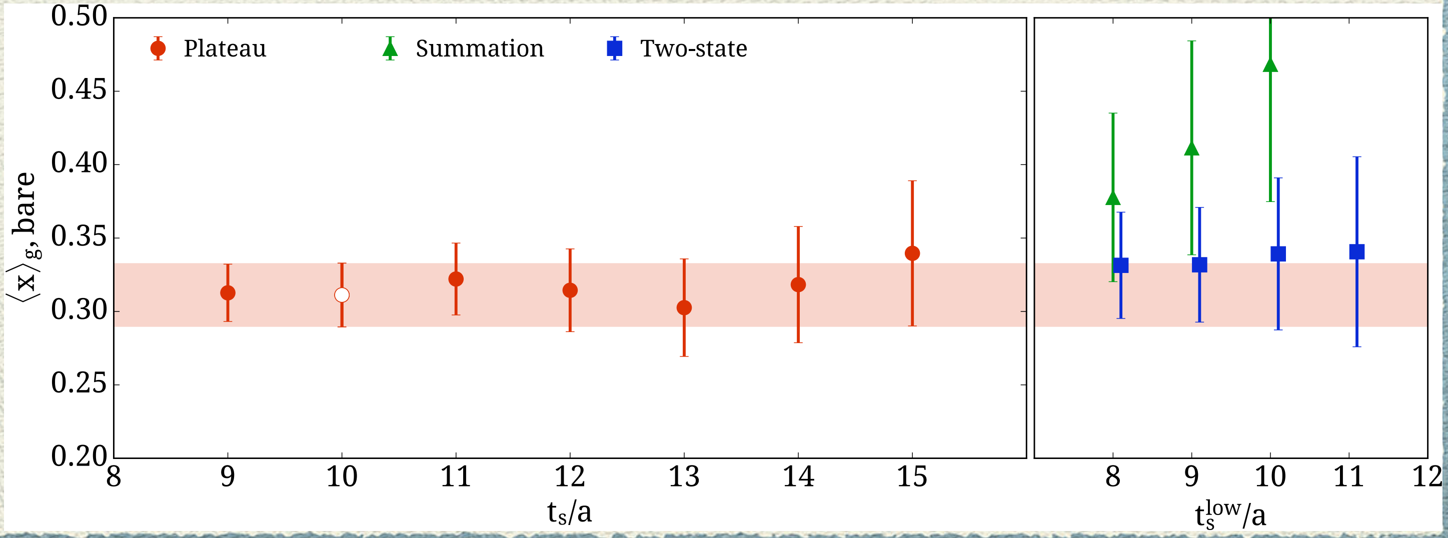
Quark momentum fraction



Excited states:

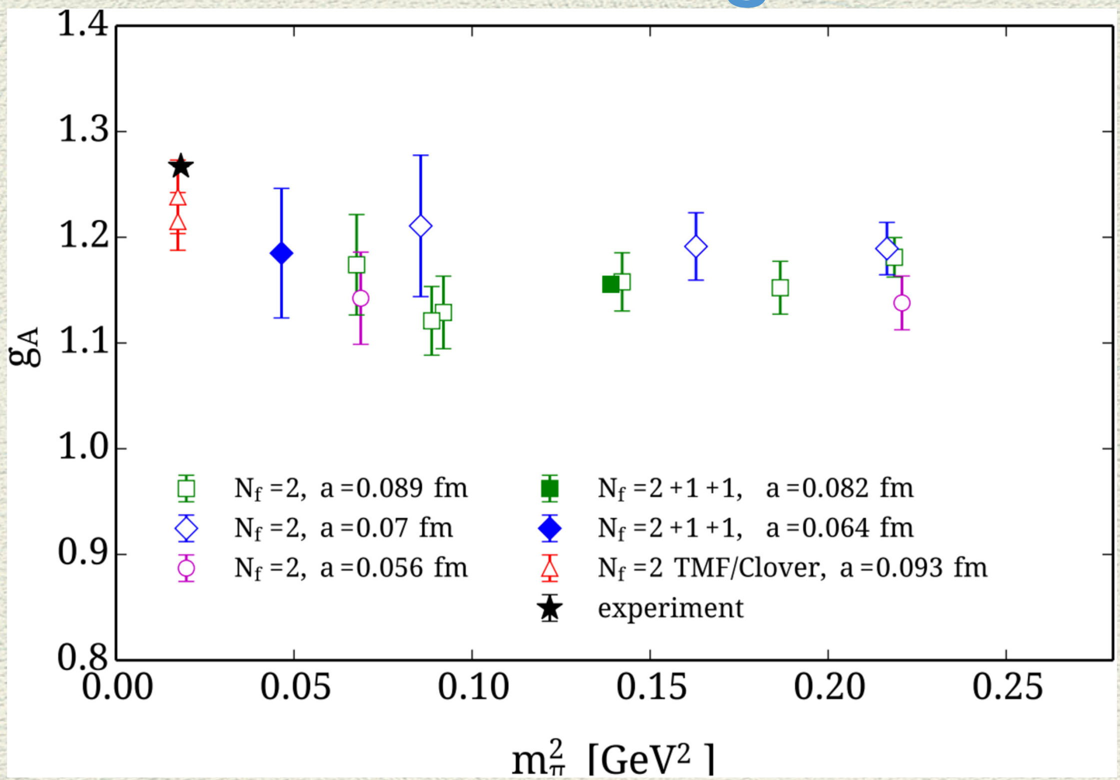
* $g_{A\ u-d}$: non-negligible
* $\langle x \rangle_{u-d}$: sizable
* $\langle x \rangle_g$: negligible

Gluon momentum fraction

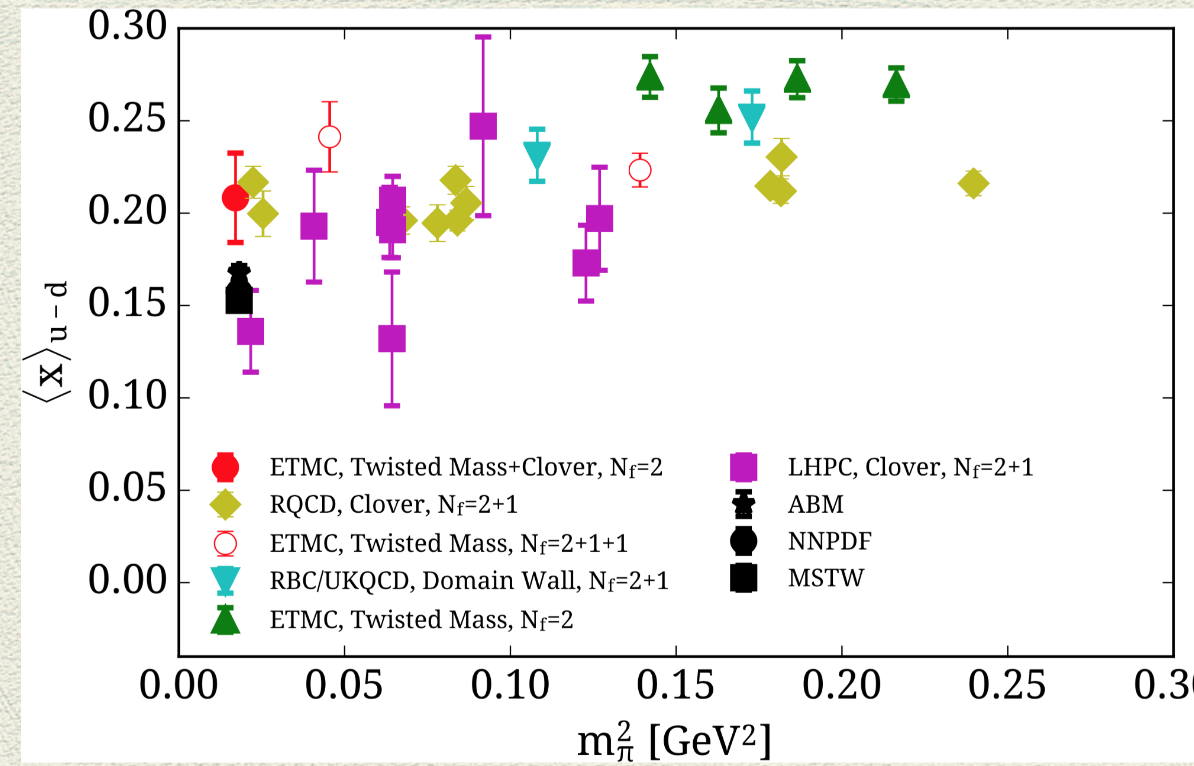


The proton spin from LQCD

Axial charge



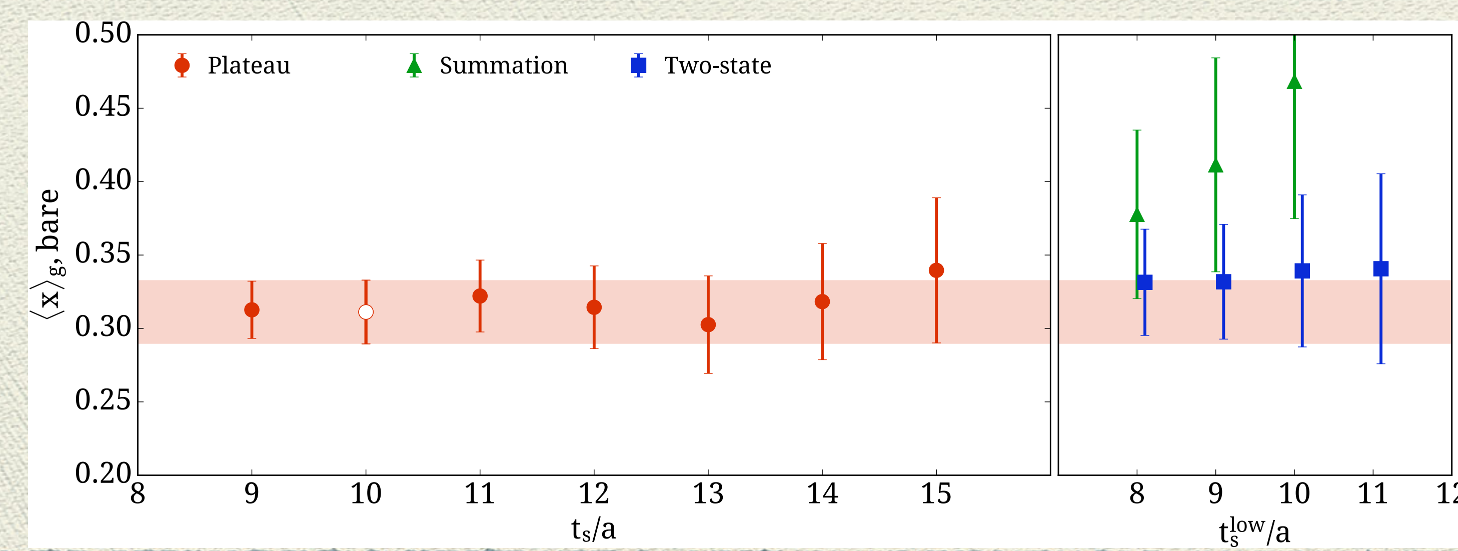
Quark momentum fraction



Excited states:

- $g_{A\,u-d}$: non-negligible
- $\langle x \rangle_{u-d}$: sizable
- $\langle x \rangle_g$: negligible

Gluon momentum fraction

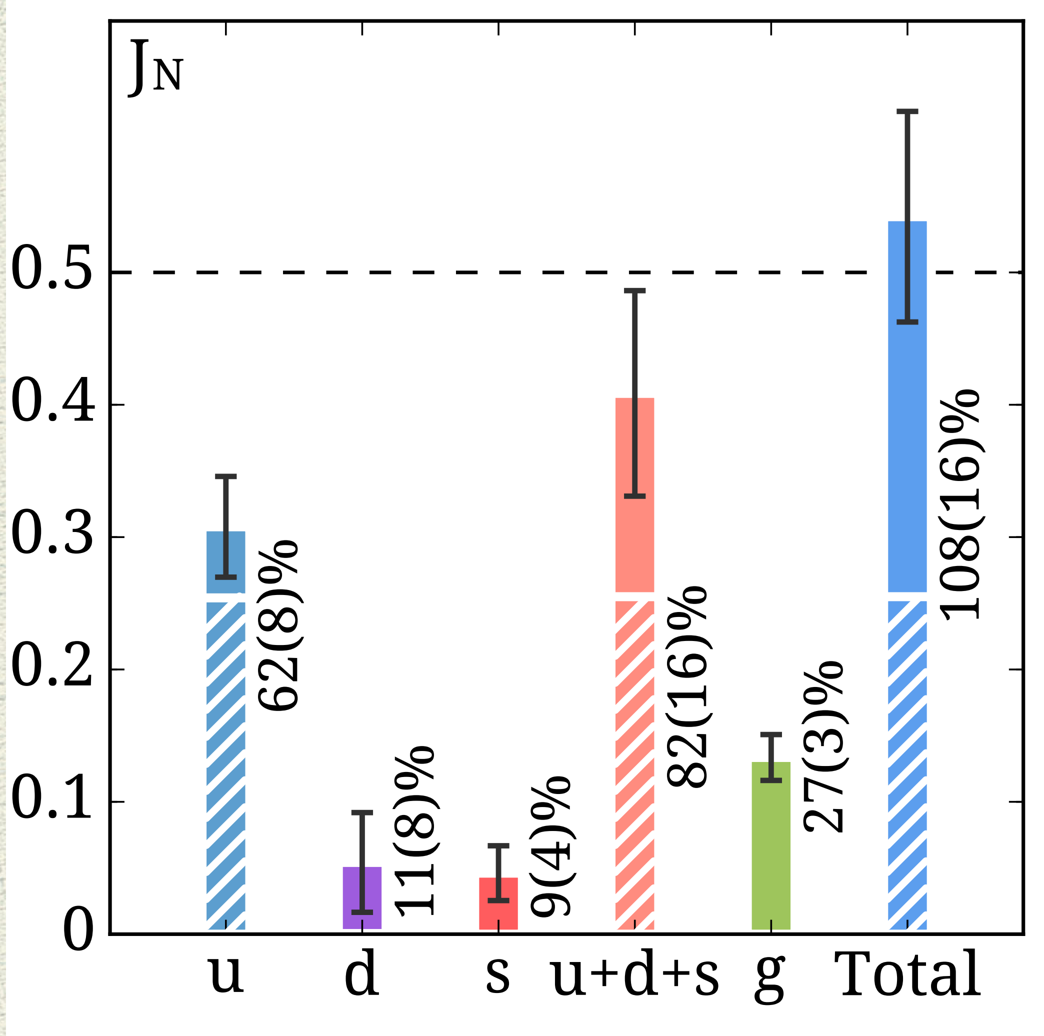


Flavor decomposition:

- Similar quality results for isoscalar and disconnected contributions

The proton spin from LQCD

[C. Alexandrou et al., Phys. Rev. Lett. 119, 142002 (2017), [arXiv:1706.02973]]

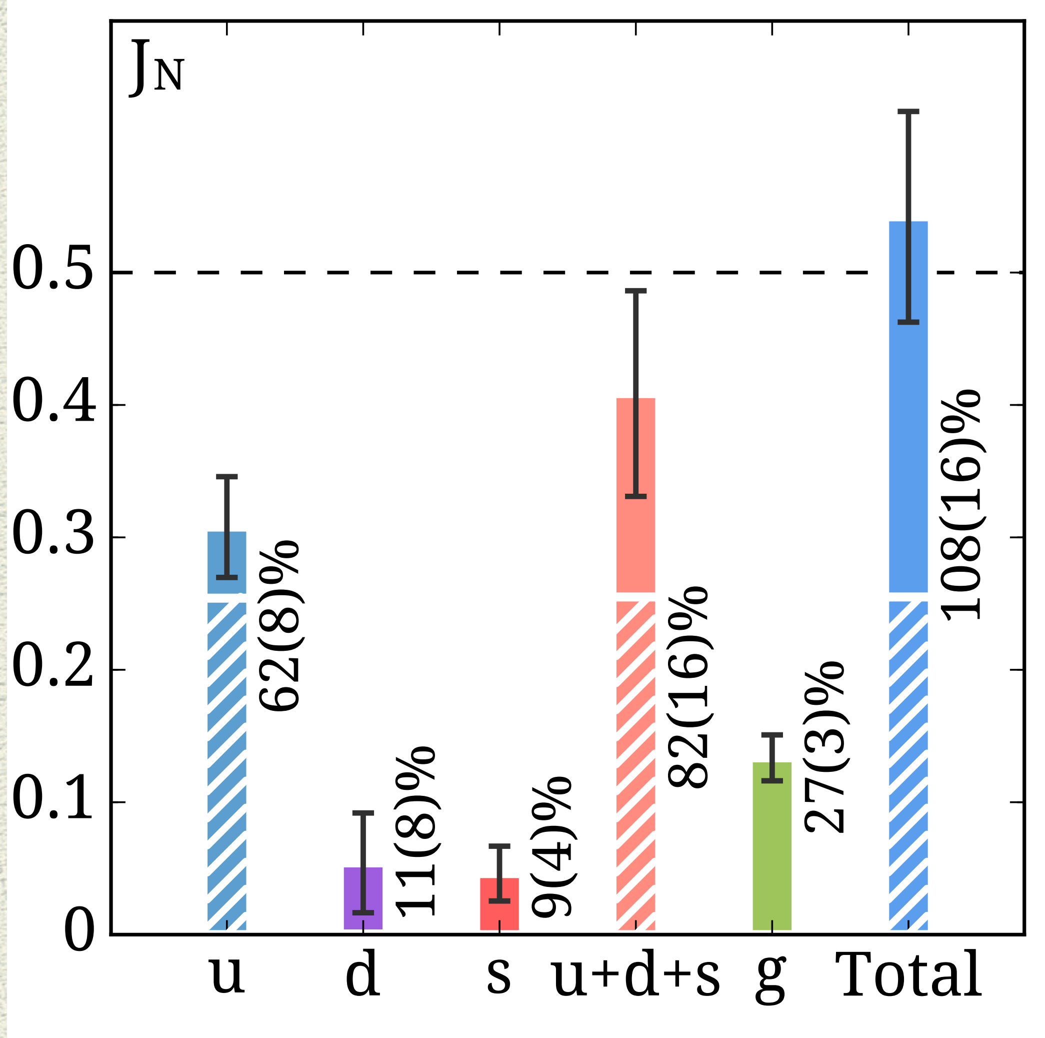


Striped segments: valence quark contributions (connected)
Solid segments: sea quark & gluon contributions (disconnected)

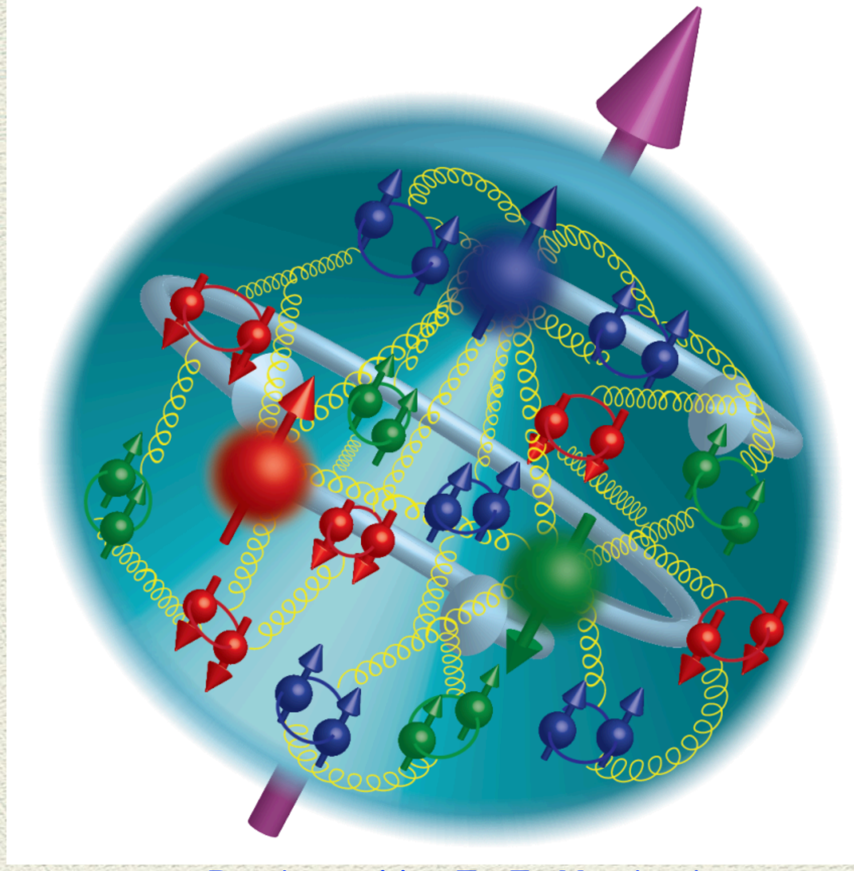
* Satisfaction of spin and momentum sum rule is not forced

The proton spin from LQCD

[C. Alexandrou et al., Phys. Rev. Lett. 119, 142002 (2017), [arXiv:1706.02973]]



Better understanding of the spin distribution



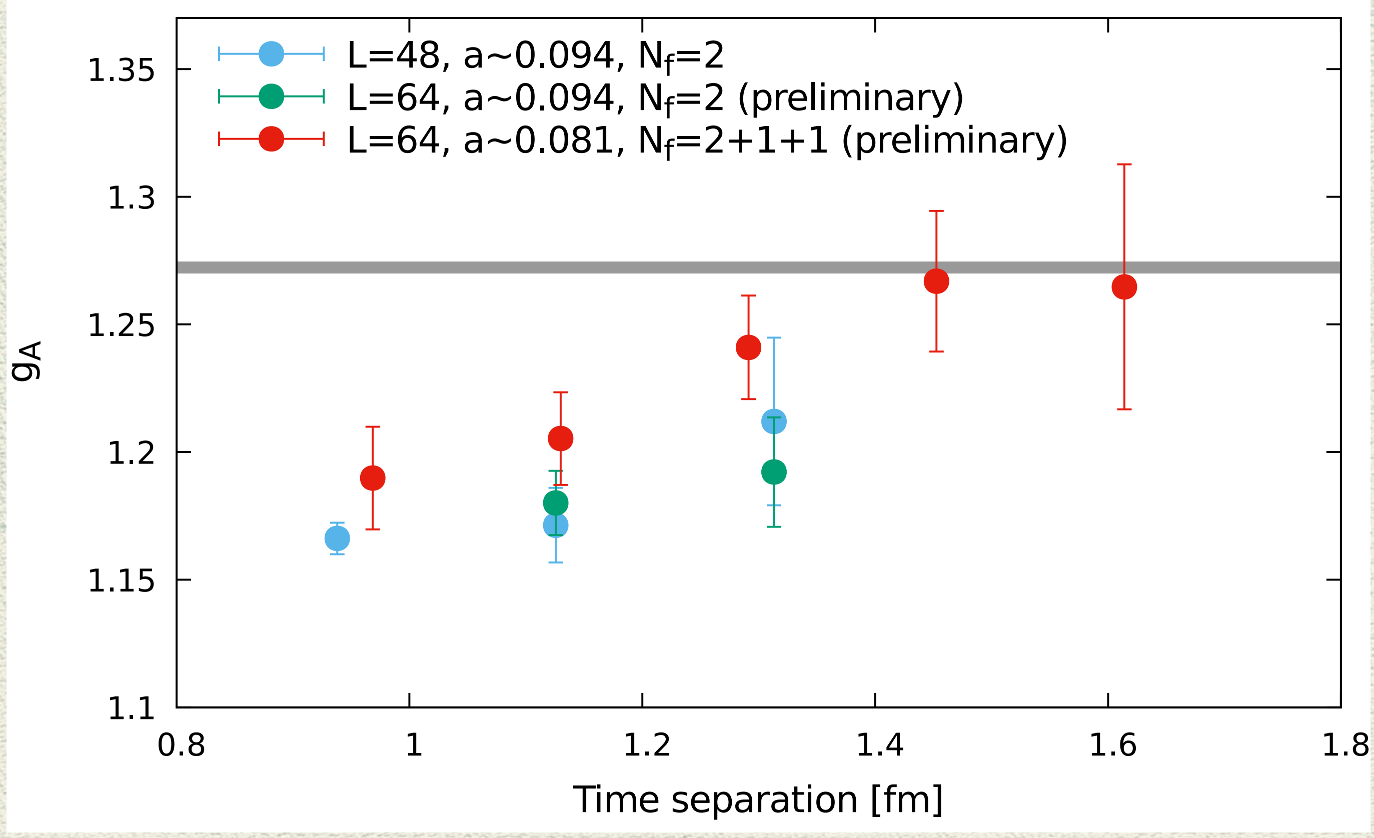
Designed by Z.-E. Meziani

Striped segments: valence quark contributions (connected)
Solid segments: sea quark & gluon contributions (disconnected)

* Satisfaction of spin and momentum sum rule is not forced

The proton spin from LQCD

We study volume and quenching effects



- Need of $T_{sink} > 1.3$fm to find agreement with experiment
- Volume effects within statistical uncertainties
- Currently increasing statistics for $T_{sink} = 1.5, 1.7$fm

Results **PRELIMINARY**:

- More statistics to collect
- Finalize analyses

OUTLINE

A. Introduction

B. Nucleon on the Lattice

C. Nucleon Form Factors

   1. Electromagnetic
   2. Axial

D. Proton Spin

E. Access to x-dependent PDFs

F. Discussion

Parton Distribution Functions

- Several ideas how to access PDFs on Lattice

★ **Hadronic tensor** [K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]

★ **Fictitious heavy quark** [W. Detmold, C. J. D, Lin, Phys. Rev. D73, 014501 (2006)]

★ **Higher moments** [Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]

★ **Compton amplitude and OPE** [A. Chambers et al. (QCDSF), arXiv:1703.01153]

★ **Quasi-PDFs** [X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539]

★ **Pseudo-PDFs** [A. Radyushkin, Phys. Rev. D 96, 034025 (2017), arXiv:1705.01488]

★ **Lattice cross-sections** [Y-Q Ma&J. Qiu, Phys. Rev. Lett. 120, 022003 (2018), arXiv:1709.03018]

Parton Distribution Functions

- Several ideas how to access PDFs on Lattice

★ **Hadronic tensor** [K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]

★ **Fictitious heavy quark** [W. Detmold, C. J. D, Lin, Phys. Rev. D73, 014501 (2006)]

★ **Higher moments** [Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]

★ **Compton amplitude and OPE** [A. Chambers et al. (QCDSF), arXiv:1703.01153]

★ **Quasi-PDFs** [X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539]

★ **Pseudo-PDFs** [A. Radyushkin, Phys. Rev. D 96, 034025 (2017), arXiv:1705.01488]

★ **Lattice cross-sections** [Y-Q Ma&J. Qiu, Phys. Rev. Lett. 120, 022003 (2018), arXiv:1709.03018]

All methods have been investigated on the lattice

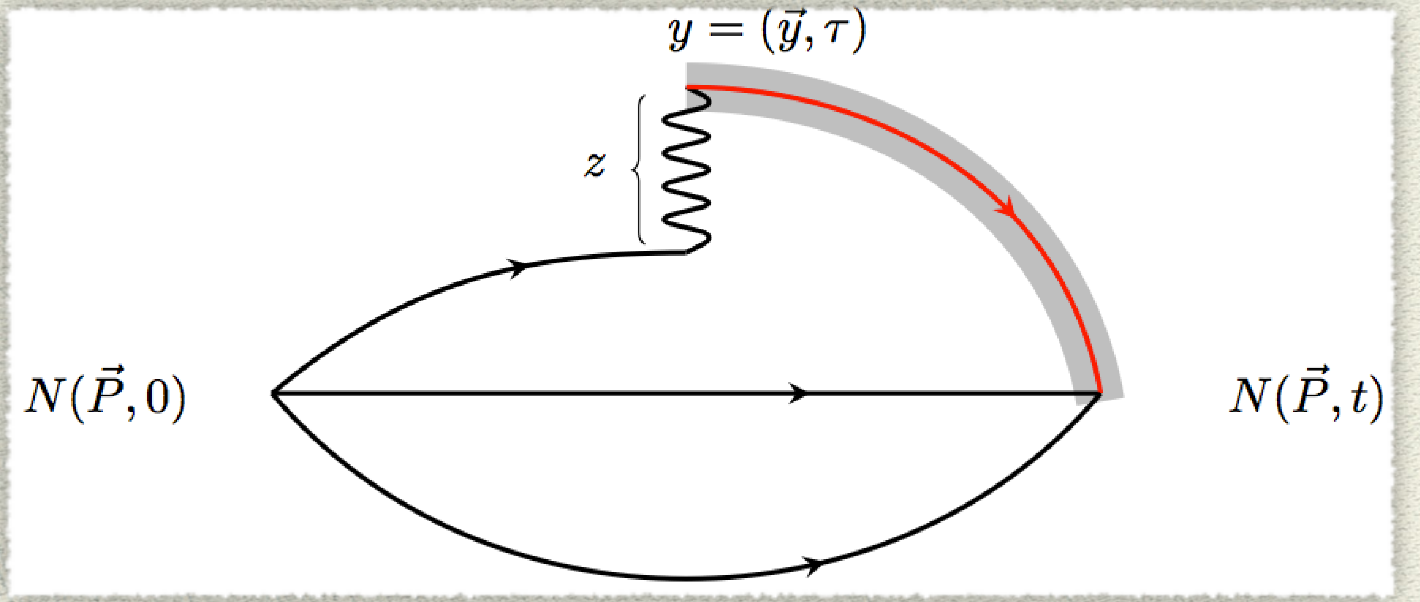
Access of PDFs on a Euclidean Lattice

quasi-PDFs

* Based on matrix elements of spatial operators

$$\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{-i x P_3 z} \langle N(P_3) | \bar{\Psi}(z) \gamma^z A(z,0) \Psi(0) | N(P_3) \rangle_{\mu^2}$$

$A(z,0)$: Wilson Line of length z



* Hadron boosted with momentum in spatial direction

Reconstruction of light-cone PDFs

Contact with light-cone PDFs feasible:

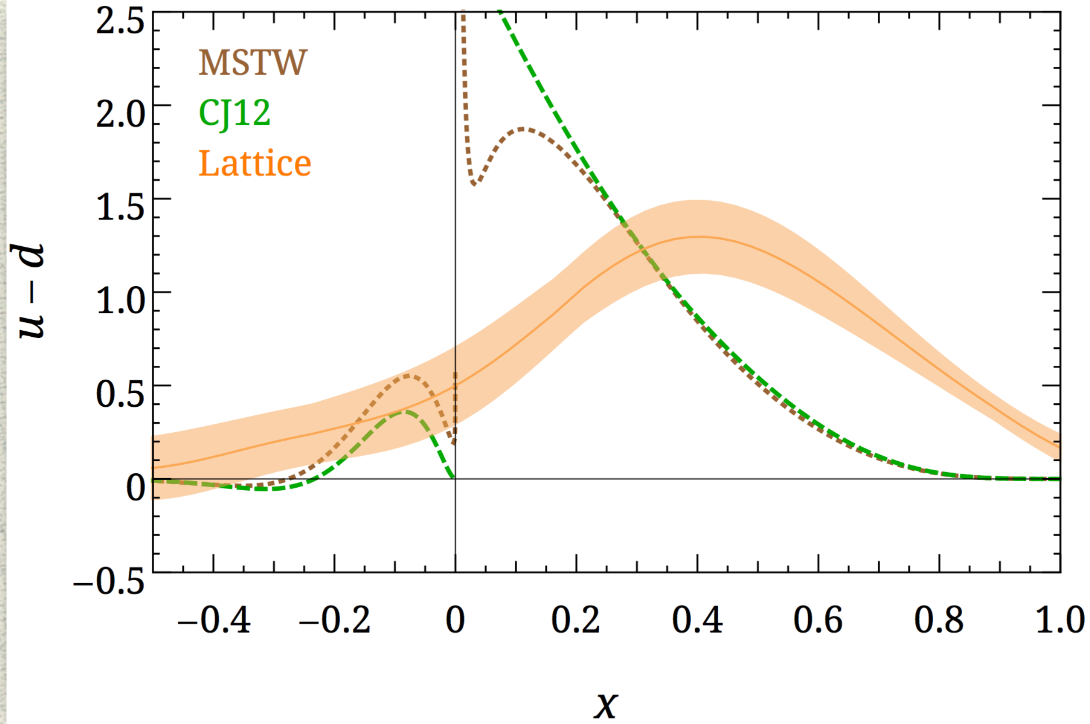
* Difference reduced as P increases $O\left(\frac{\Lambda^2_{\text{QCD}}}{P_3^2}, \frac{m_N^2}{P_3^2}\right)$
* Matching procedure in large momentum EFT (LaMET) to relate quasi-PDFs to light-cone PDF

Reconstruction of light-cone PDFs

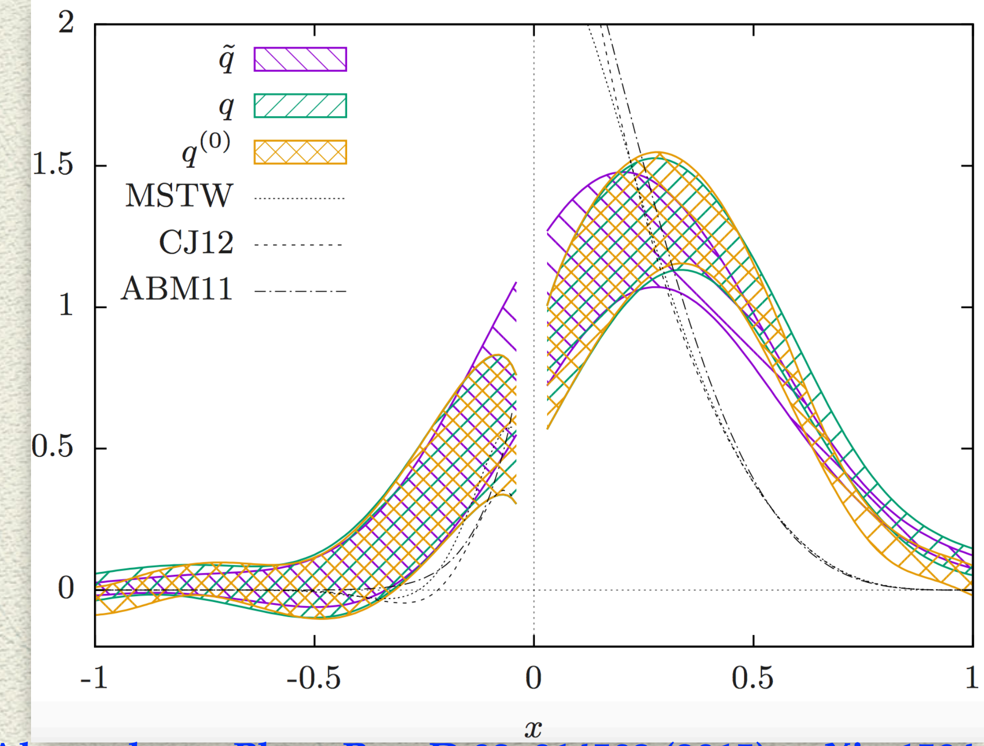
Contact with light-cone PDFs feasible:

* Difference reduced as P increases $O\left(\frac{\Lambda^2_{QCD}}{P_3^2}, \frac{m_N^2}{P_3^2}\right)$
* Matching procedure in large momentum EFT (LaMET) to relate quasi-PDFs to light-cone PDF

First exploratory (nucleon) studies feasible:

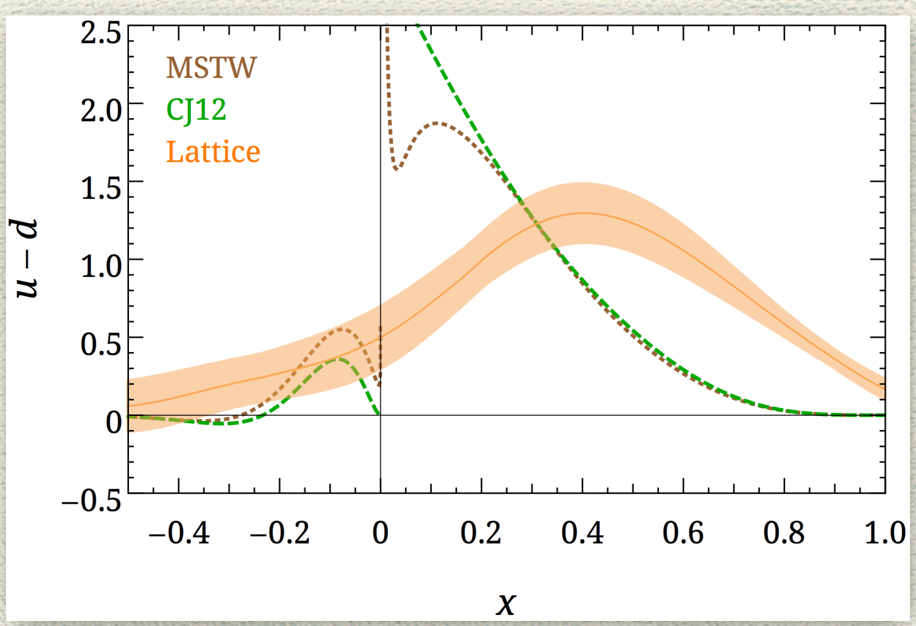


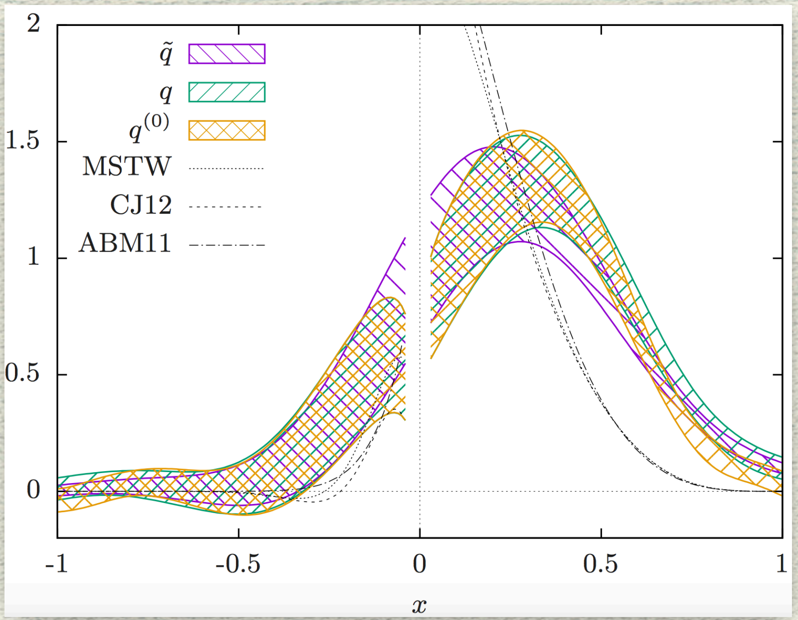
[H.W. Lin et al., Phys. Rev. D 91, 054510 (2015), arXiv:1402.1462]



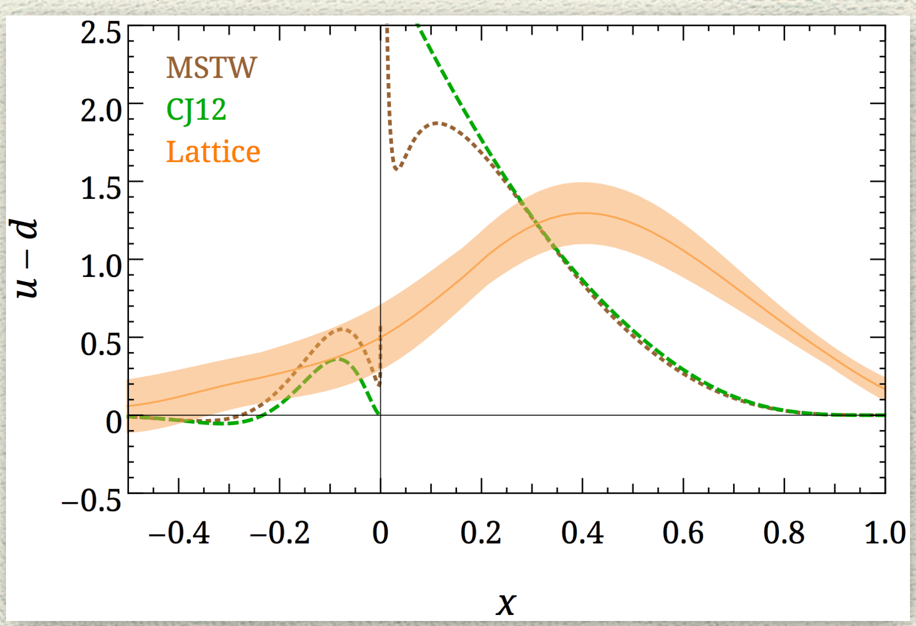
[C. Alexandrou, Phys. Rev. D 92, 014502 (2015), arXiv:1504.07455]

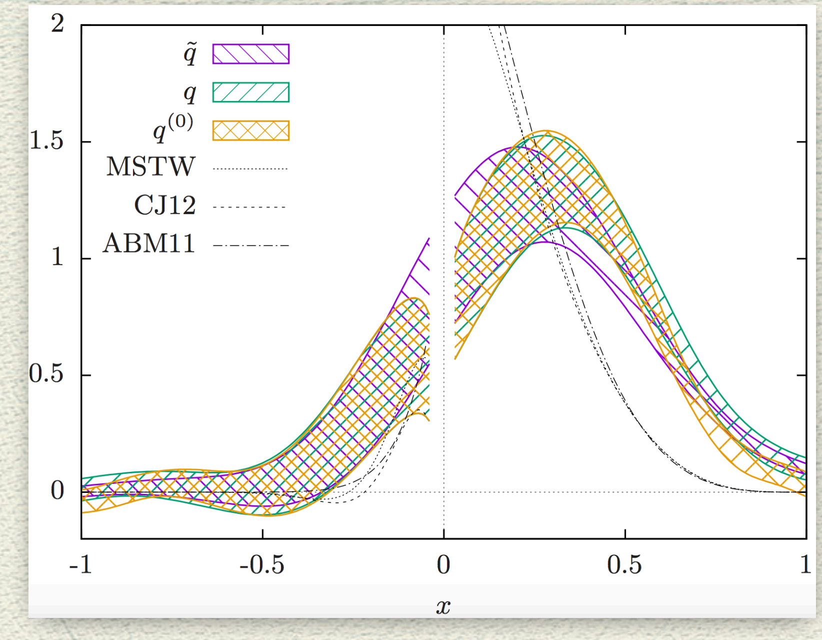
Lattice studies of quasi-PDFs





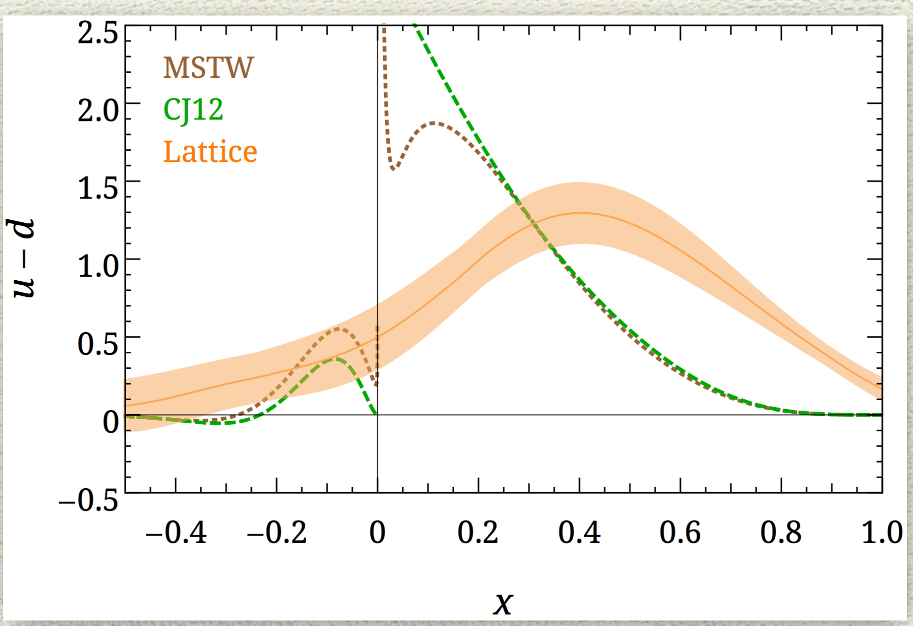
Lattice studies of quasi-PDFs

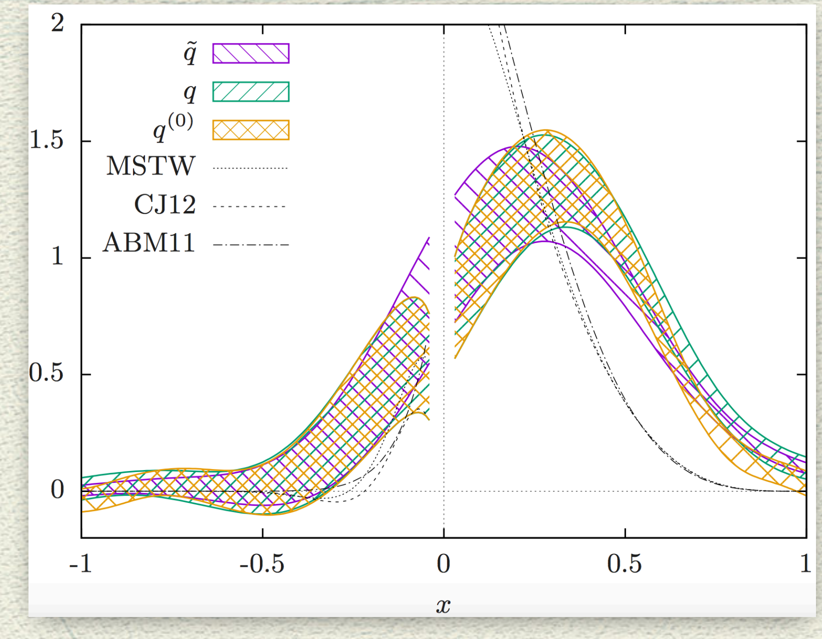




* Calculations significantly improved...

Lattice studies of quasi-PDFs

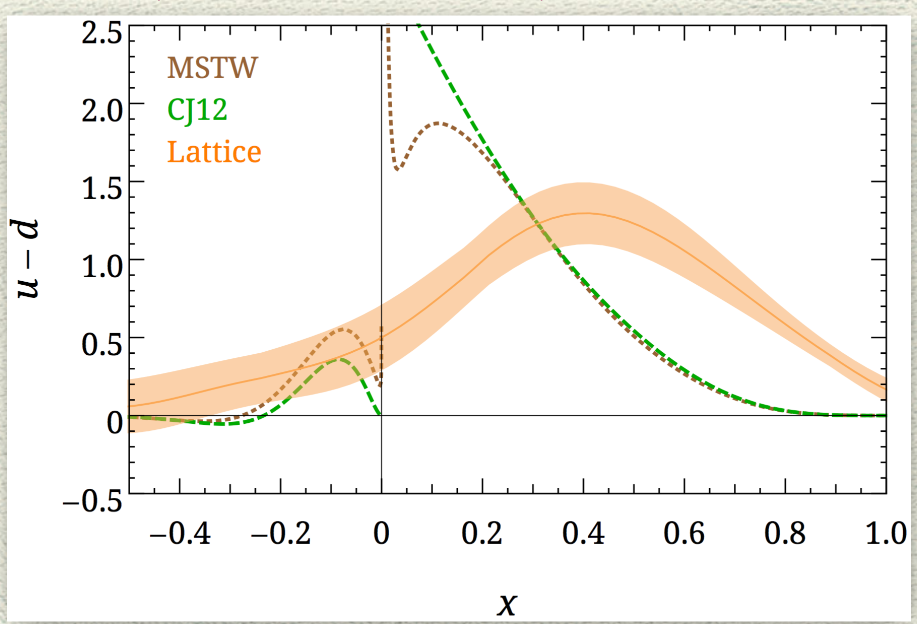


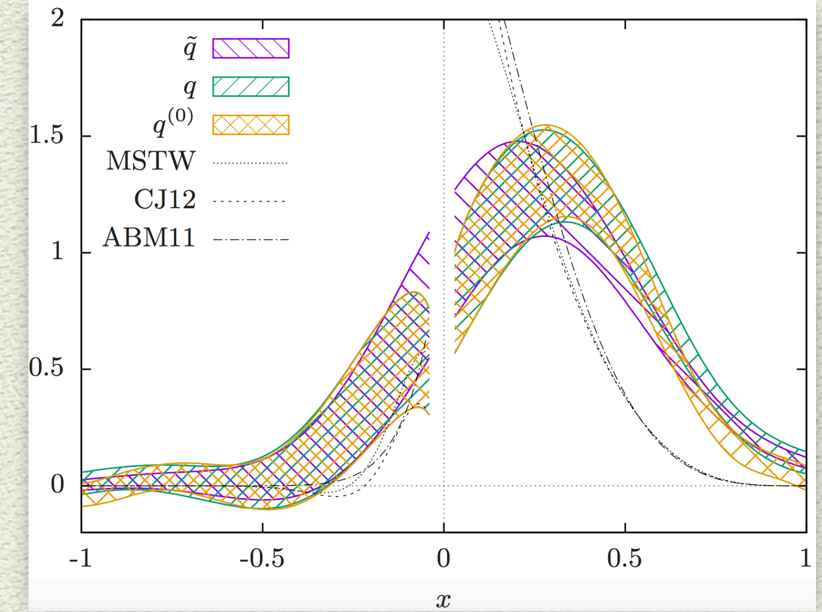


* Calculations significantly improved…

[X. Xiong et al., arXiv:1310.7471], [H-W. Lin et al., arXiv:1402.1462], [Y. Ma et al., arXiv:1404.6860], [Y.-Q. Ma et al., arXiv:1412.2688], [C. Alexandrou et al., arXiv:1504.07455], [H.-N. Li et al., arXiv:1602.07575], [J.-W. Chen et al., arXiv:1603.06664], [J.-W. Chen et al., arXiv:1609.08102], [T. Ishikawa et al., arXiv:1609.02018], [C. Alexandrou et al., arXiv:1610.03689], [C. Monahan et al., arXiv:1612.01584], [A. Radyushkin et al., arXiv:1702.01726], [C. Carlson et al., arXiv:1702.05775], [R. Briceno et al., arXiv:1703.06072], [M. Constantinou et al., arXiv:1705.11193], [C. Alexandrou et al., arXiv:1706.00265], [J-W Chen et al., arXiv:1706.01295], [X. Ji et al., arXiv:1706.08962], [K. Orginos et al., arXiv:1706.05373], [T. Ishikawa et al., arXiv:1707.03107], [J. Green et al., arXiv:1707.07152], [Y-Q Ma et al., arXiv:1709.03018], [I. Stewart et al, arXiv:1709.04933], [J. Karpie et al., arXiv:1710.08288, [J-W Chen et al., arXiv:1711.07858], [C.Alexandrou et al., arXiv:1710.06408], [T. Izubuchi et al., arXiv:1801.03917], [C.Alexandrou et al., arXiv:1803.02685], [J-W Chen et al, arXiv:1803.04393], [C.Alexandrou et al., arXiv:1807.00232], …

Lattice studies of quasi-PDFs





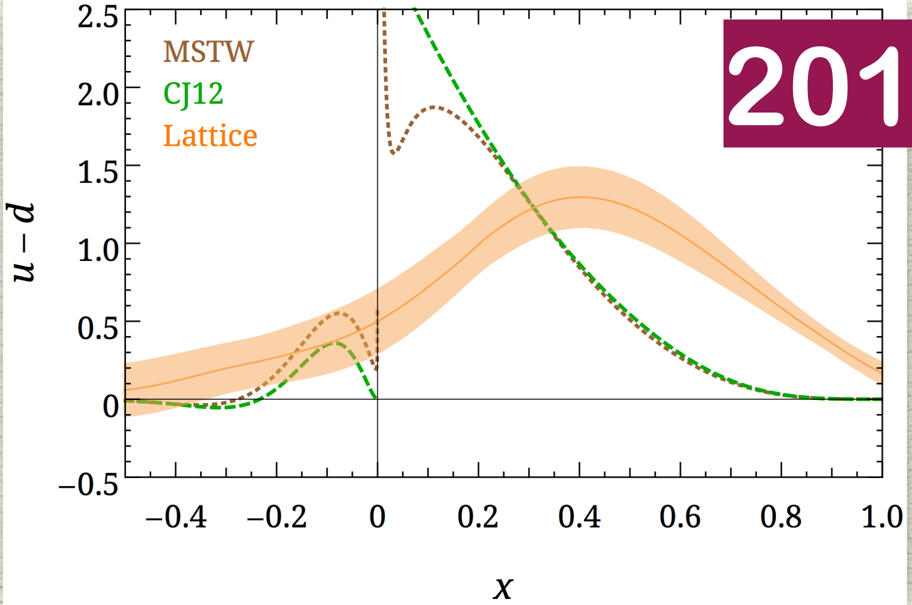
* Calculations significantly improved…

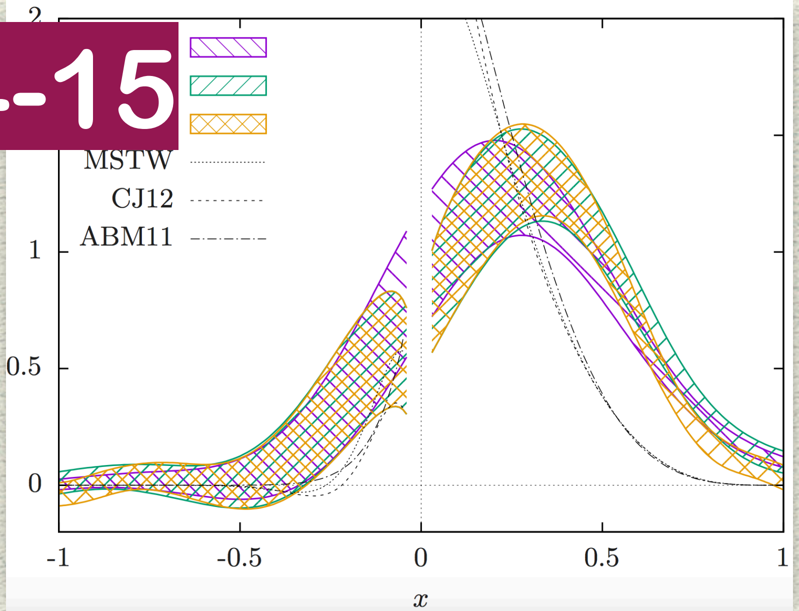
[X. Xiong et al., arXiv:1310.7471], [H-W. Lin et al., arXiv:1402.1462], [Y. Ma et al., arXiv:1404.6860], [Y.-Q. Ma et al., arXiv:1412.2688], [C. Alexandrou et al., arXiv:1504.07455], [H.-N. Li et al., arXiv:1602.07575], [J.-W. Chen et al., arXiv:1603.06664], [J.-W. Chen et al., arXiv:1609.08102], [T. Ishikawa et al., arXiv:1609.02018], [C. Alexandrou et al., arXiv:1610.03689], [C. Monahan et al., arXiv:1612.01584], [A. Radyushkin et al., arXiv:1702.01726], [C. Carlson et al., arXiv:1702.05775], [R. Briceno et al., arXiv:1703.06072], [M. Constantinou et al., arXiv:1705.11193], [C. Alexandrou et al., arXiv:1706.00265], [J-W Chen et al., arXiv:1706.01295], [X. Ji et al., arXiv:1706.08962], [K. Orginos et al., arXiv:1706.05373], [T. Ishikawa et al., arXiv:1707.03107], [J. Green et al., arXiv:1707.07152], [Y-Q Ma et al., arXiv:1709.03018], [I. Stewart et al, arXiv:1709.04933], [J. Karpie et al., arXiv:1710.08288, [J-W Chen et al., arXiv:1711.07858], [C.Alexandrou et al., arXiv:1710.06408], [T. Izubuchi et al., arXiv:1801.03917], [C.Alexandrou et al., arXiv:1803.02685], [J-W Chen et al, arXiv:1803.04393], [C.Alexandrou et al., arXiv:1807.00232], …

* … and extended to other hadrons

Recent review: C. Monahan @ Lattice 2018

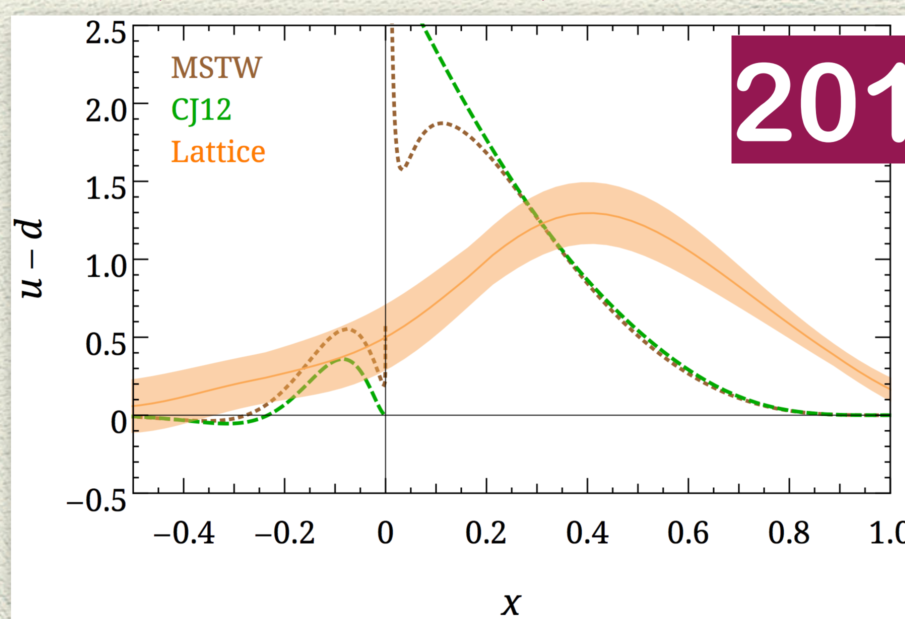
Lattice studies of quasi-PDFs



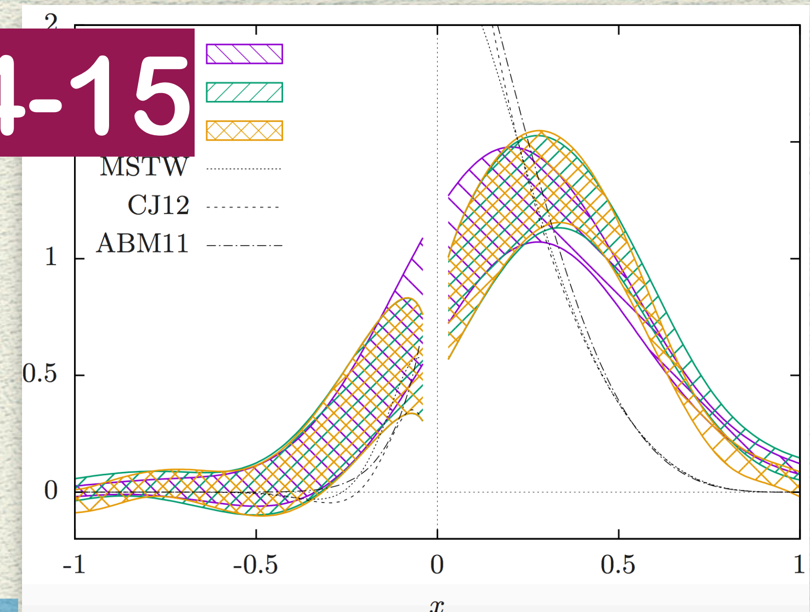


2014-15

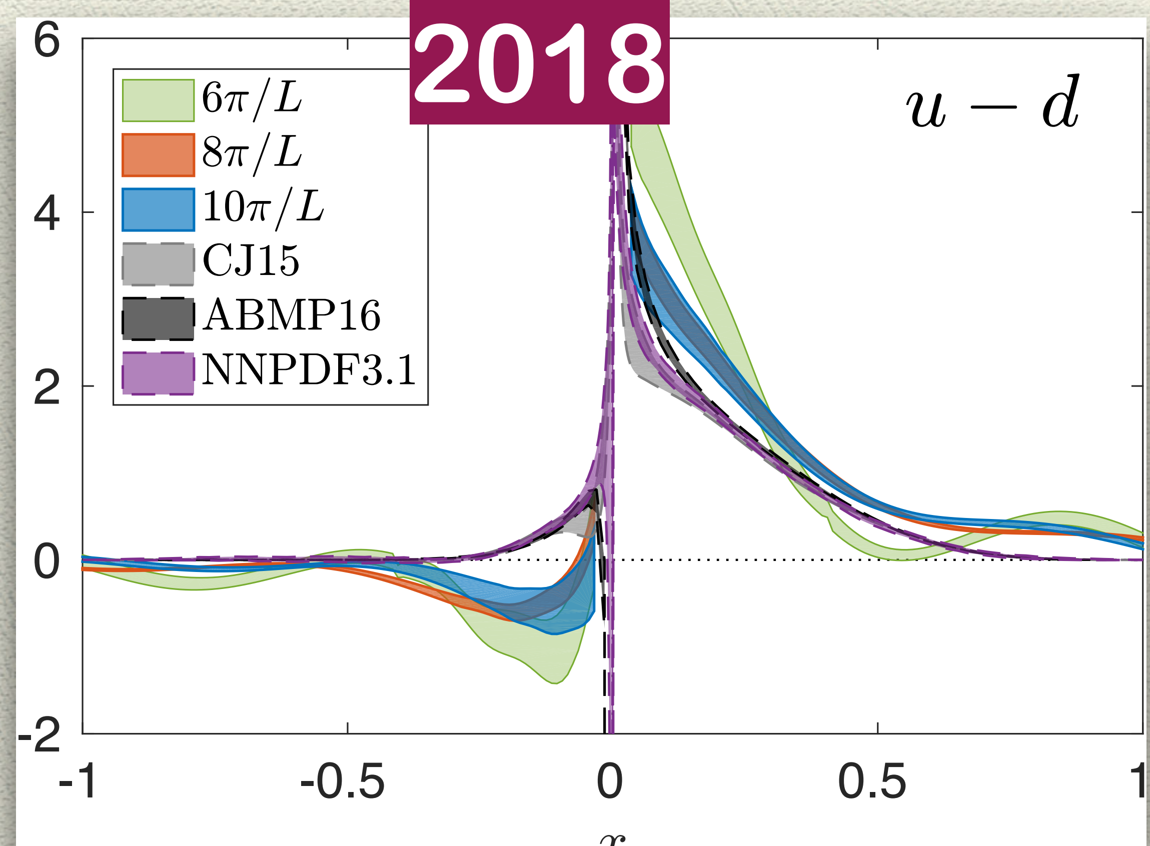
Lattice studies of quasi-PDFs



2014-15



2018



- Simulations at physical point
- Renormalization
- Matching

Parameters of Calculation

[C. Alexandrou et al., (PRL), arXiv:1803.02685], [C. Alexandrou et al., arXiv:1807.00232]

* Nf=2 twisted mass fermions & clover term
* Ensemble parameters:

β=2.10,	c_{SW}=1.57751, a=0.0938(3)(2) fm	
$48^3 \times 96$	$a\mu = 0.0009$	$m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV	$m_\pi L = 2.98(1)$

Parameters of Calculation

[C. Alexandrou et al., (PRL), arXiv:1803.02685], [C. Alexandrou et al., arXiv:1807.00232]

* Nf=2 twisted mass fermions & clover term
* Ensemble parameters:

β=2.10,	c_{SW}=1.57751, a=0.0938(3)(2) fm
$48^3 \times 96$	$a\mu = 0.0009$ $m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV $m_\pi L = 2.98(1)$

* Nucleon momentum & statistics:

$P = \frac{6\pi}{L}$ (0.83 GeV)			$P = \frac{8\pi}{L}$ (1.11 GeV)			$P = \frac{10\pi}{L}$ (1.38 GeV)		
Ins.	$N_{\rm conf}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm meas}$
γ_3	100	9600	γ_3	425	38250	γ_3	811	72990
γ_0	50	4800	γ_0	425	38250	γ_0	811	72990
$\gamma_5\gamma_3$	65	6240	$\gamma_5\gamma_3$	425	38250	$\gamma_5\gamma_3$	811	72990

Parameters of Calculation

[C. Alexandrou et al., (PRL), arXiv:1803.02685], [C. Alexandrou et al., arXiv:1807.00232]

- Nf=2 twisted mass fermions & clover term
- Ensemble parameters:

β=2.10, c_{SW}=1.57751, a=0.0938(3)(2) fm	
$48^3 \times 96$	$a\mu = 0.0009$ $m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV $m_\pi L = 2.98(1)$

- Nucleon momentum & statistics:

$P = \frac{6\pi}{L}$ (0.83 GeV)			$P = \frac{8\pi}{L}$ (1.11 GeV)			$P = \frac{10\pi}{L}$ (1.38 GeV)		
Ins.	$N_{\rm conf}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm meas}$
γ_3	100	9600	γ_3	425	38250	γ_3	811	72990
γ_0	50	4800	γ_0	425	38250	γ_0	811	72990
$\gamma_5\gamma_3$	65	6240	$\gamma_5\gamma_3$	425	38250	$\gamma_5\gamma_3$	811	72990

- Excited states investigation:

$$T_{\rm sink} = 8a, 9a, 10a, 12a \qquad (T_{\rm sink} = 0.75, 0.84, 0.094, 1.13\text{fm})$$

Challenges of calculation

Noise-to-signal ratio increases with:

- Hadron momentum boost
- Simulations at the physical point
- Source-sink separation

Challenges of calculation

Noise-to-signal ratio increases with:

- Hadron momentum boost
- Simulations at the physical point
- Source-sink separation

Noise problem must be tamed to investigate uncertainties

Challenges of calculation

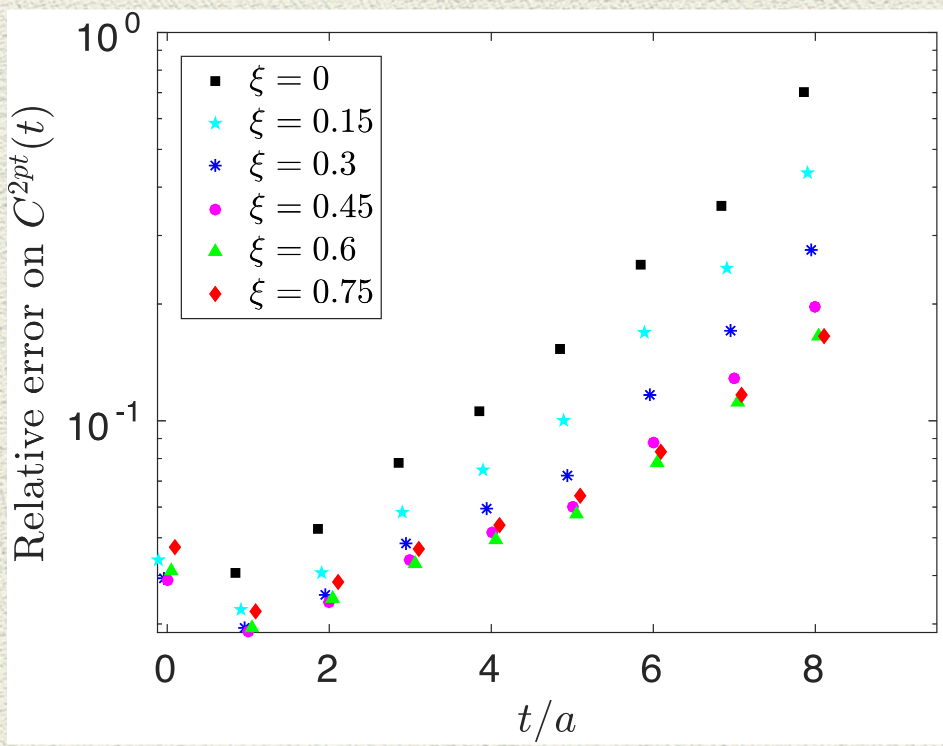
Noise-to-signal ratio increases with:

- Hadron momentum boost
- Simulations at the physical point
- Source-sink separation

Noise problem must be tamed to investigate uncertainties

Momentum smearing

[G. Bali et al., PRD93, 094515 (2016)]



- Momentum smearing helps reach higher momenta

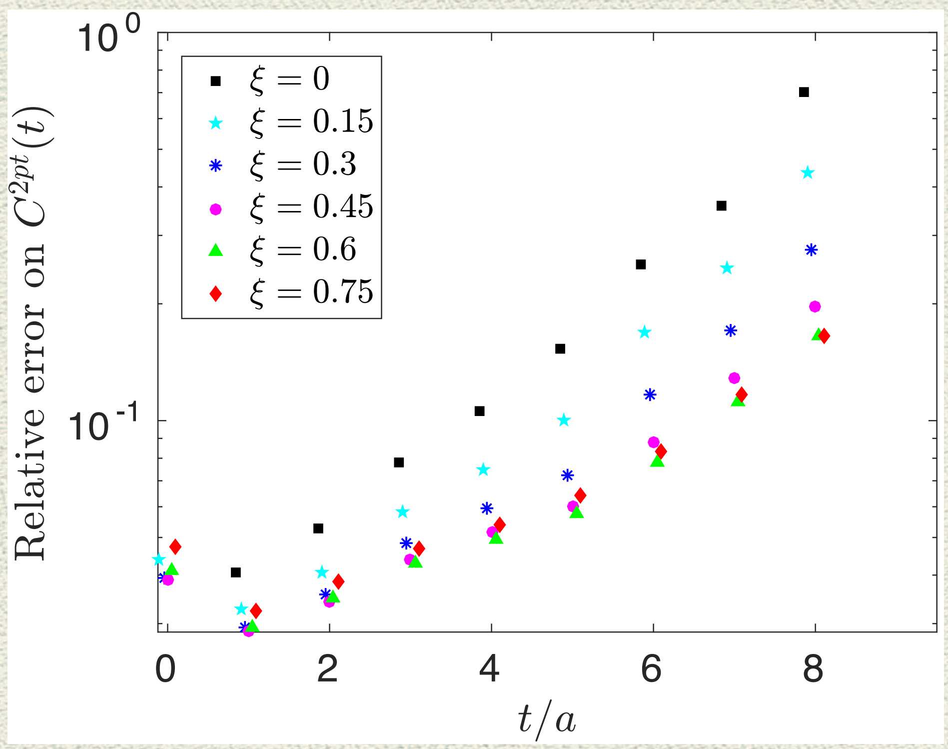
Challenges of calculation

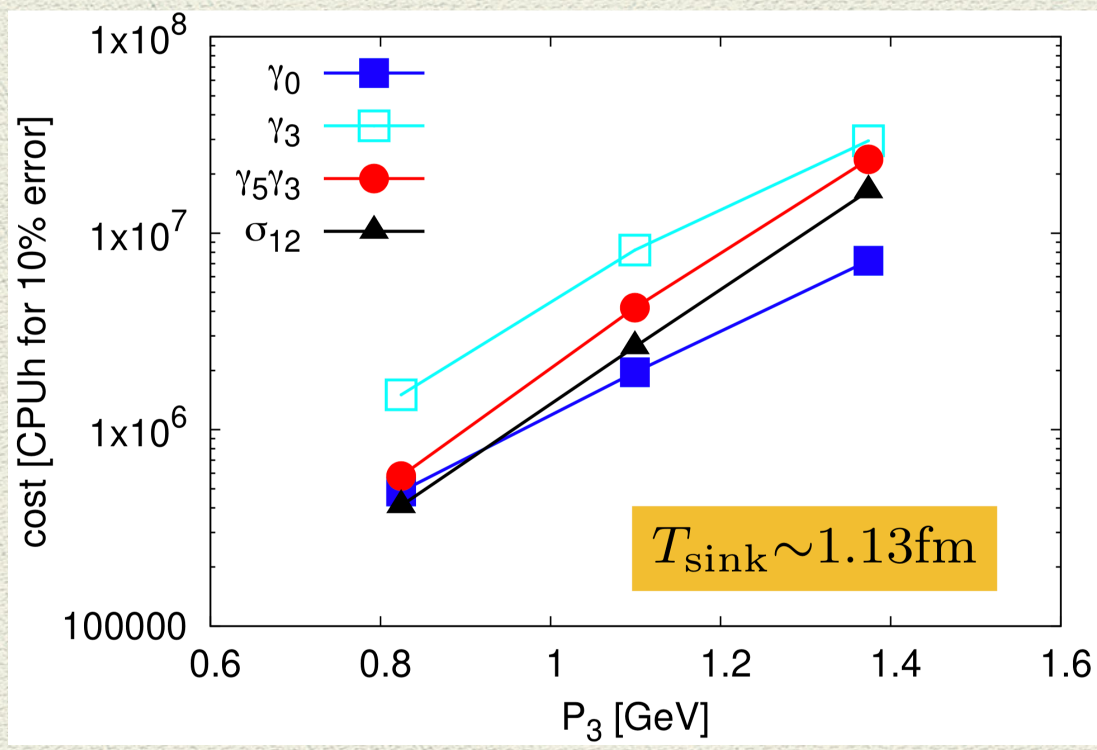
Noise-to-signal ratio increases with:

- Hadron momentum boost
- Simulations at the physical point
- Source-sink separation

Noise problem must be tamed to investigate uncertainties

Momentum smearing
[G. Bali et al., PRD93, 094515 (2016)]





- Momentum smearing helps reach higher momenta
- But limitations in max momentum due to comput. cost

Bare matrix elements

Unpolarized:

* Initial studies used γ^μ in same direction with Wilson line
* Mixing with higher twist revealed perturbatively

Bare matrix elements

Unpolarized:

- Initial studies used γ^μ in sam… …ction with Wilson line
- Mixing with higher … revealed perturbatively

Abandoned

Bare matrix elements

Unpolarized:

- Initial studies used γ^μ in sam… …ction with Wilson line
- Mixing with higher … revealed perturbatively
- No mixing for γ^0 (perpendicular to Wilson line)

Abandoned

[M. Constantinou, H. Panagopoulos, Phys. Rev. D 96 (2017) 054506, [arXiv:1705.11193]

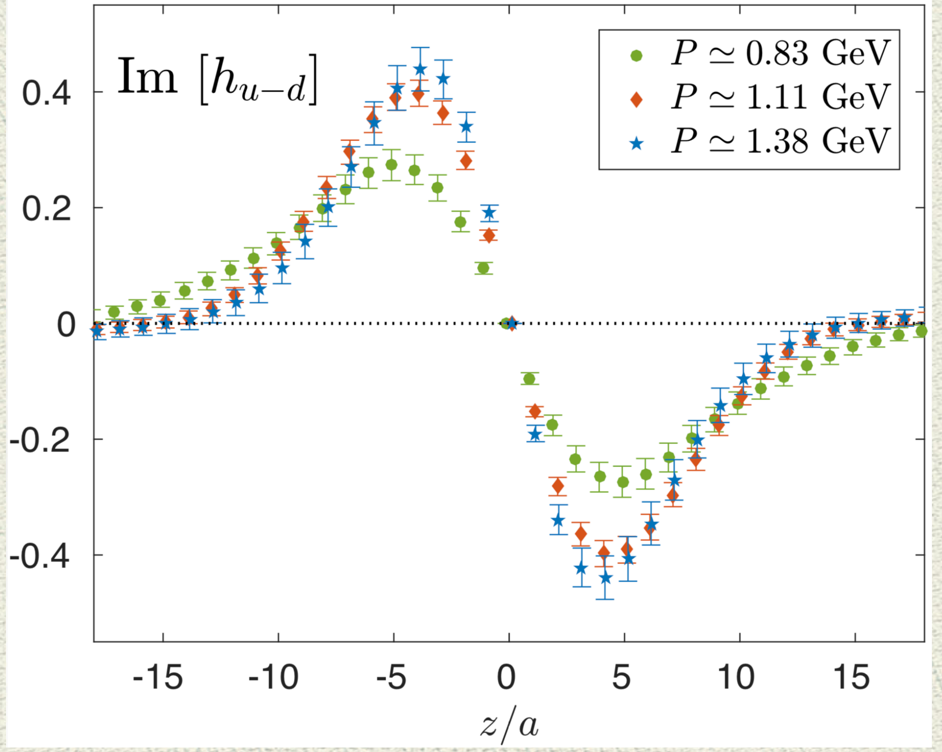
Bare matrix elements

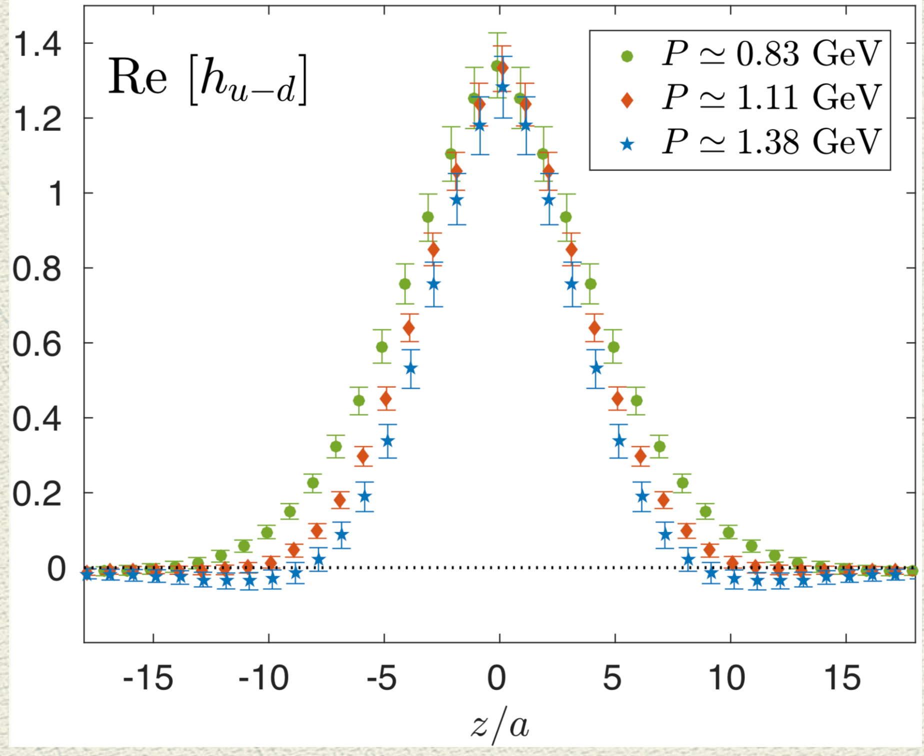
Unpolarized:

- Initial studies used γ^μ in same direction with Wilson line
- Mixing with higher ... revealed perturbatively
- No mixing for γ^0 (perpendicular to Wilson line)

Abandoned

[M. Constantinou, H. Panagopoulos, Phys. Rev. D 96 (2017) 054506, [arXiv:1705.11193]





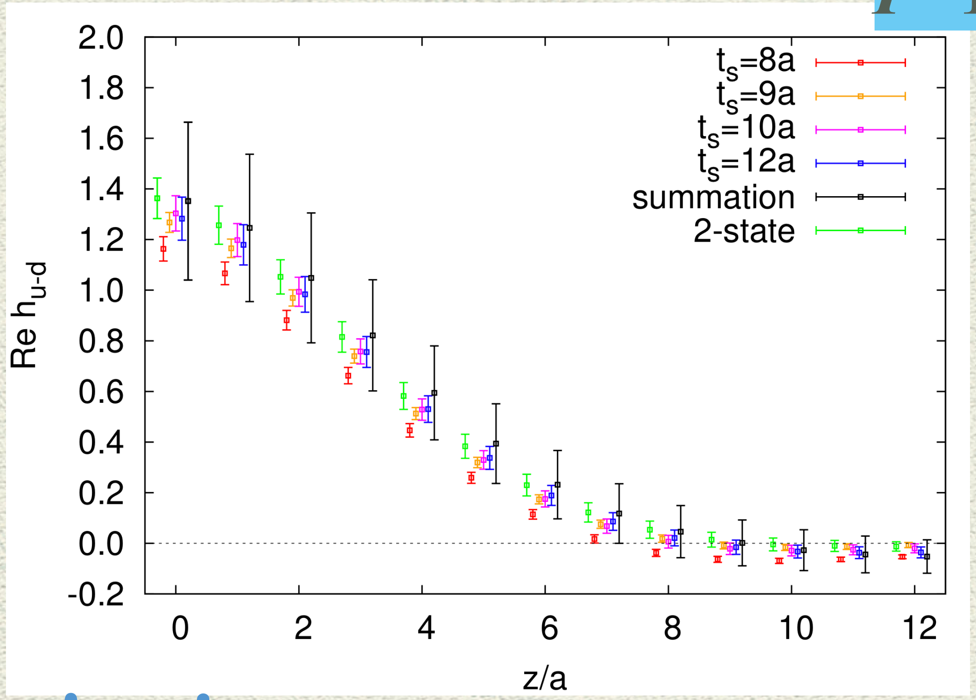
- Similar general features for polarized and transversity
- Highest priority: deliver reliable results

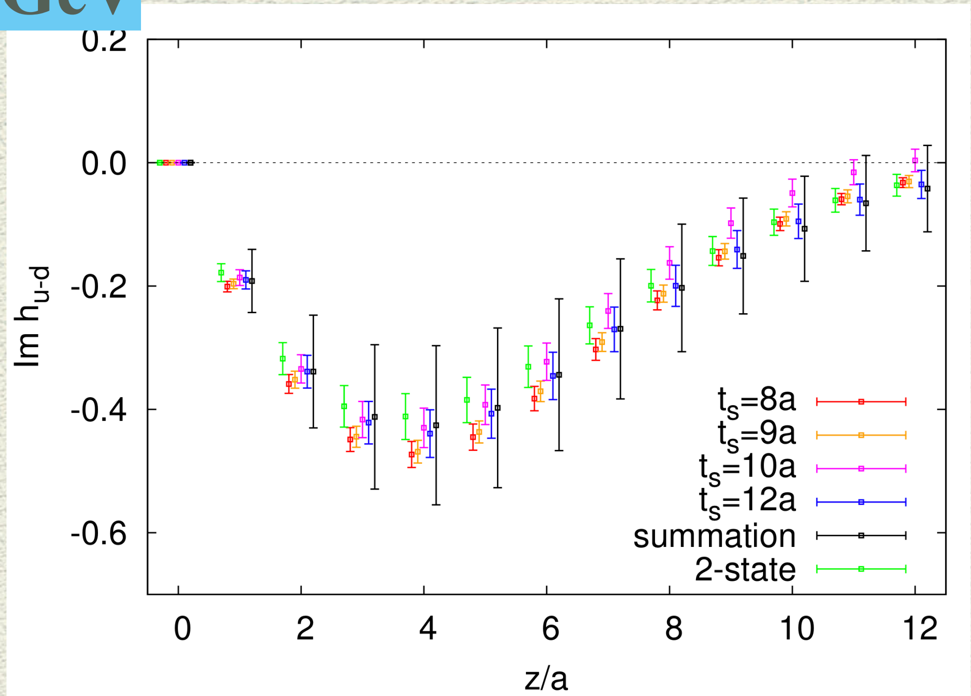
Excited states contamination

Analyses techniques:

* Single-state fit, Two-state fit, Summation method

P=1.4 GeV





Conclusions:

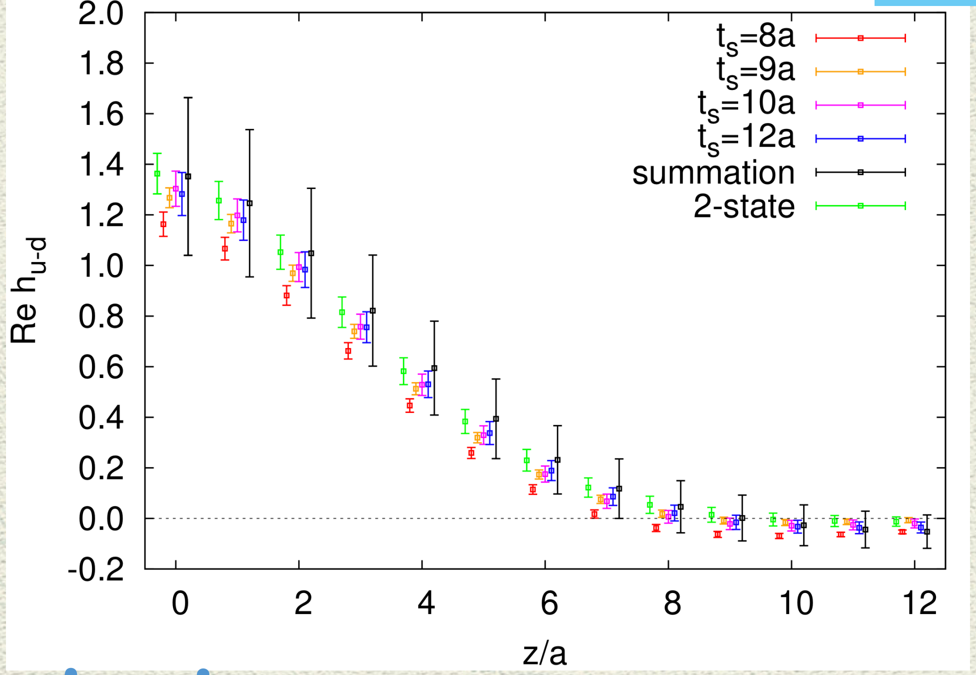
* Tsink=8a heavily contaminated by excited states
* Tsink=9a-10a not consistent within uncertainties

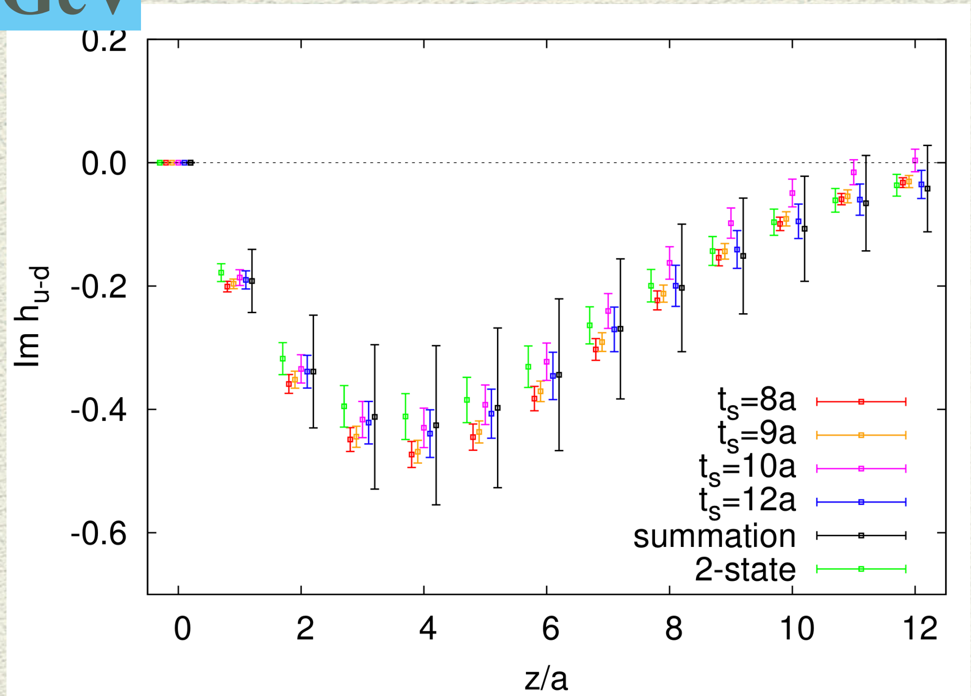
Excited states contamination

Analyses techniques:

* Single-state fit, Two-state fit, Summation method

P=1.4 GeV





Conclusions:

* Tsink=8a heavily contaminated by excited states
* Tsink=9a-10a not consistent within uncertainties

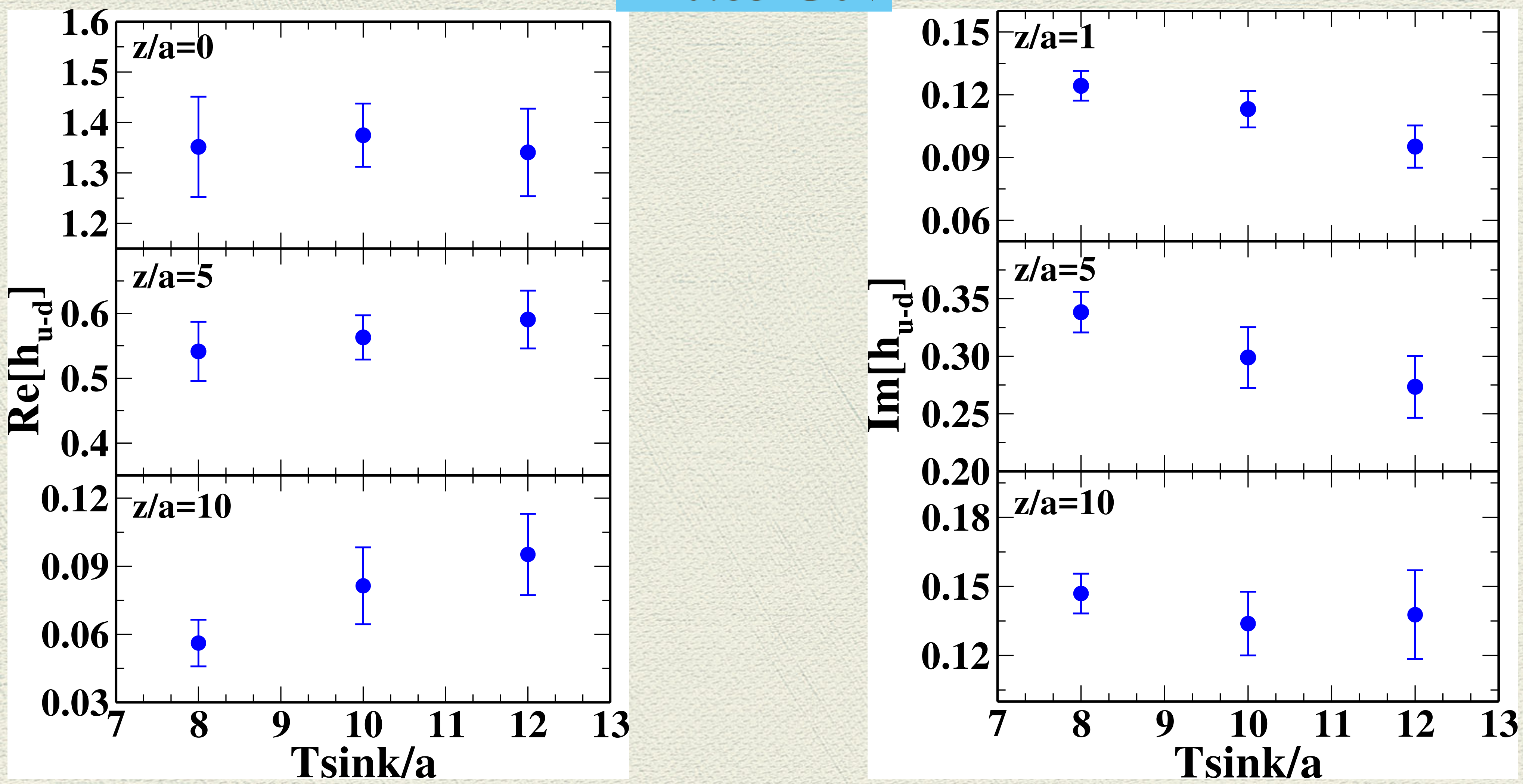
! Crucial to have same error for reliable 2-state fit

! Excited states worsen as momentum P increases

! For momenta in this work, Tsink=1fm is safe

Excited states contamination

P=0.83 GeV



- Non-predictible behavior (depends in *z* value)
- Real and imaginary part affected differently

Conclusions:

- Excited states uncontrolled for Tsink <1fm
- Multi-sink analysis <u>demands</u> same accuracy for all data

Towards light-cone PDFs

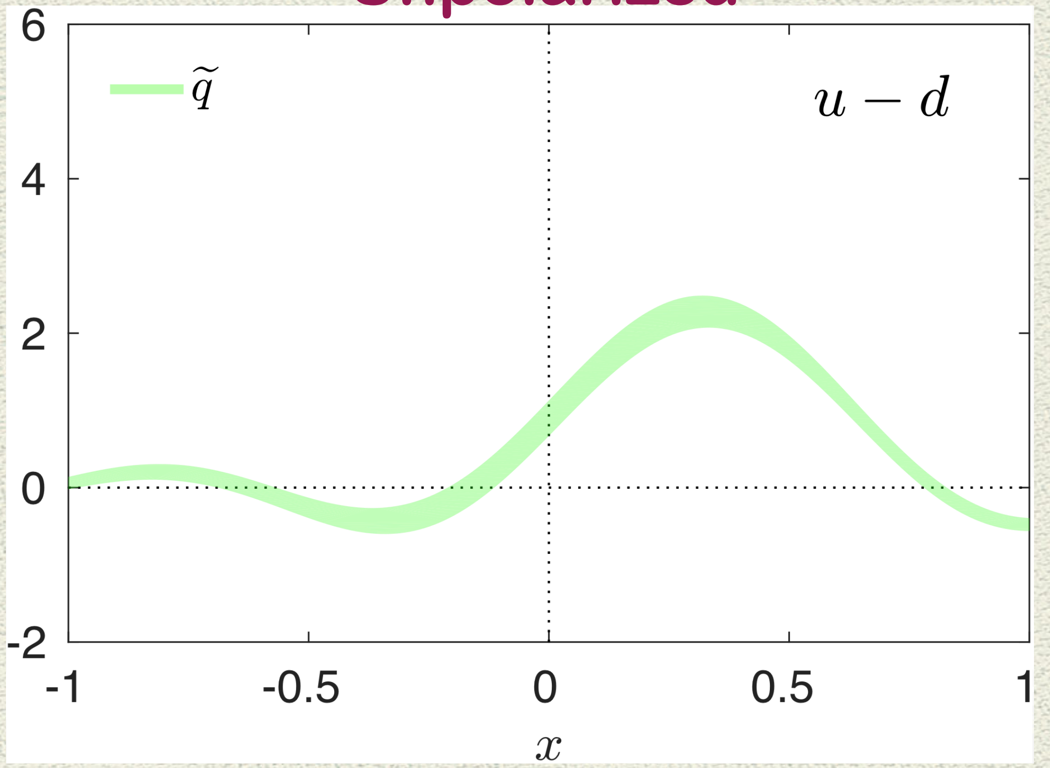
Evolution of lattice data (P=1.4GeV):

Towards light-cone PDFs

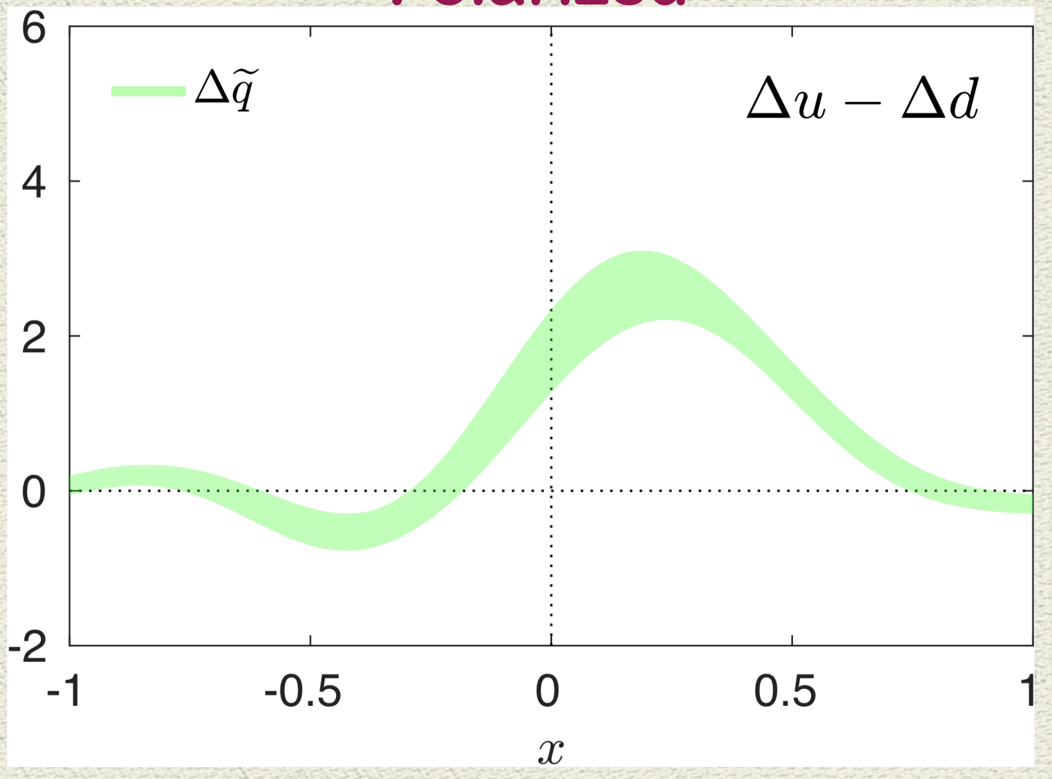
Evolution of lattice data (P=1.4GeV):

* Fourier Transform of renormarmalized matrix elements

Unpolarized



Polarized

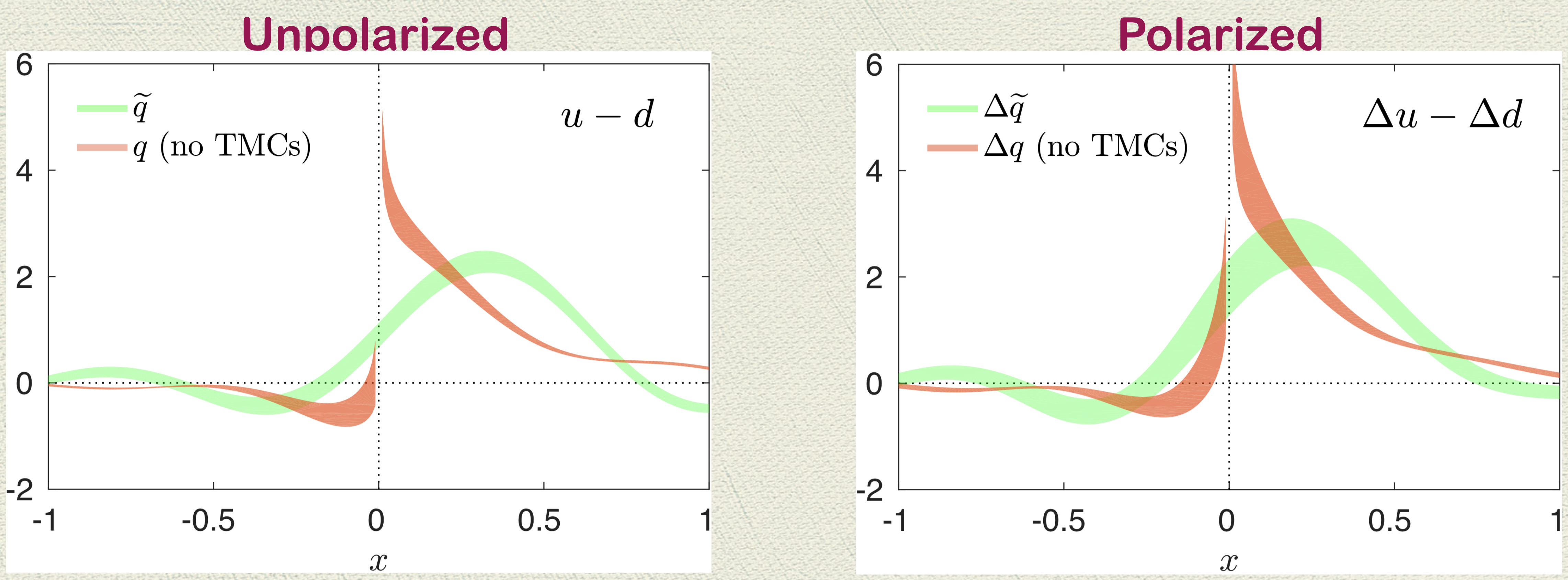


Towards light-cone PDFs

Evolution of lattice data (P=1.4GeV):

* Fourier Transform of renormarmalized matrix elements
* Matching of quasi-PDFs (LaMET)

[C. Alexandrou et al., (PRL), arXiv:1803.02685]

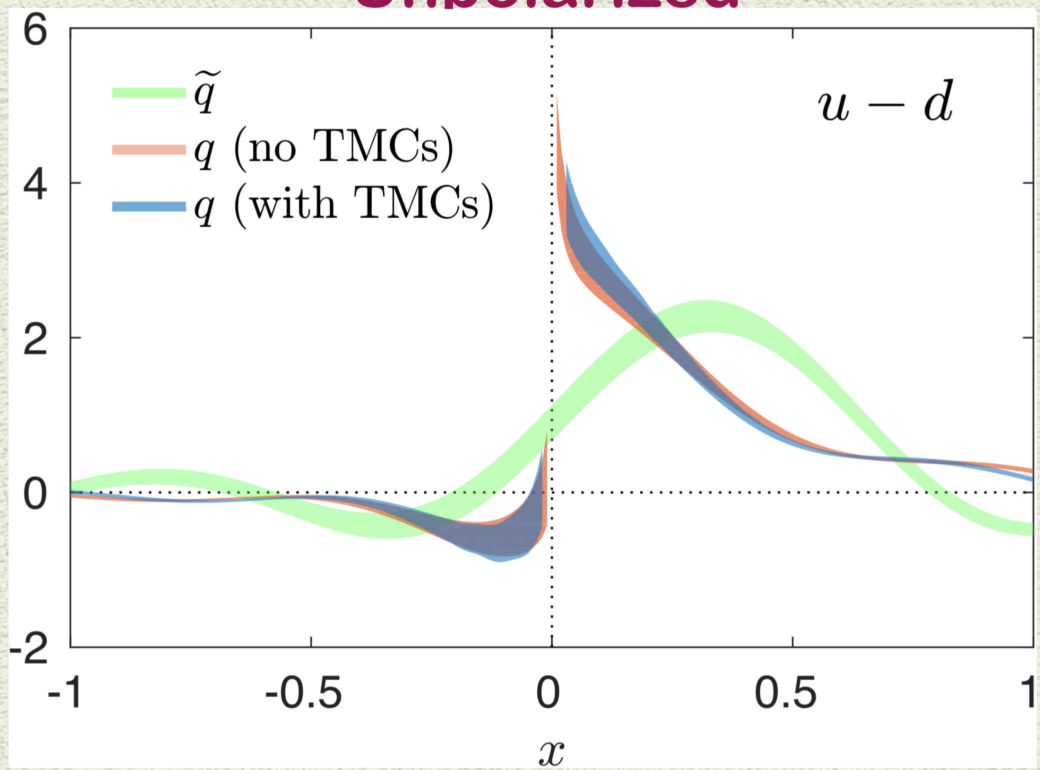


Towards light-cone PDFs

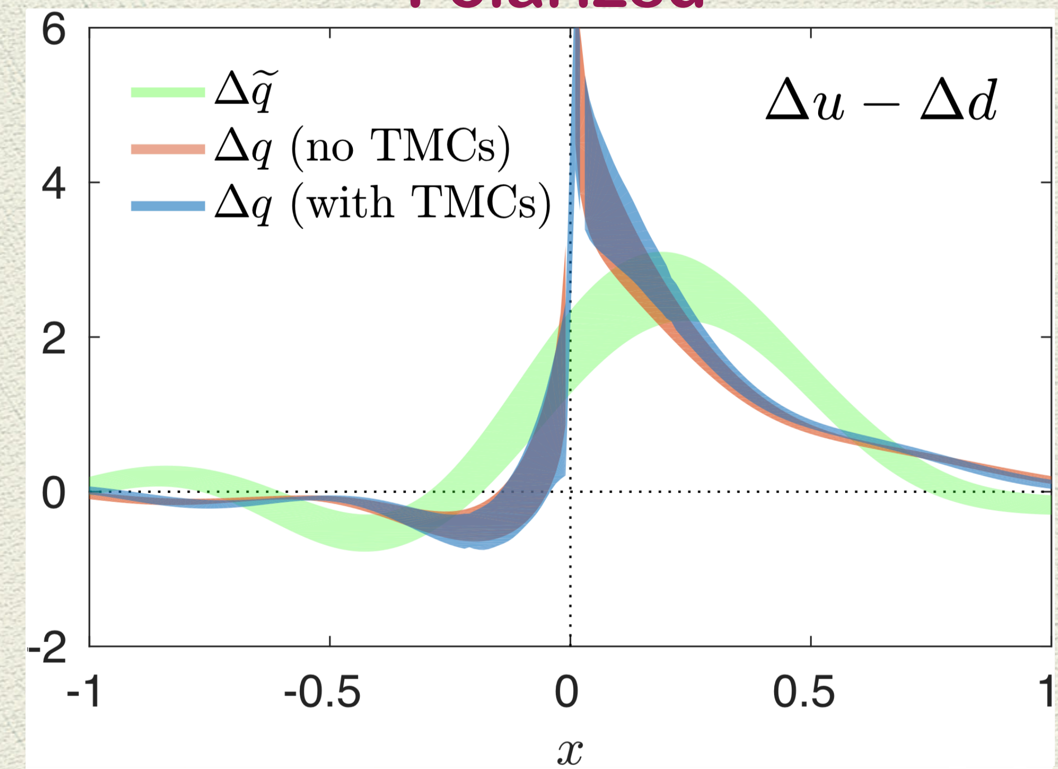
Evolution of lattice data (P=1.4GeV):

- Fourier Transform of renormarmalized matrix elements
- Matching of quasi-PDFs (LaMET)
  [C. Alexandrou et al., (PRL), arXiv:1803.02685]
- Target Mass Corrections (m_N/P: finite)
  [J.W. Chen et al., Nucl. Phys. B911 (2016) 246, arXiv:1603.06664]

Unpolarized



Polarized

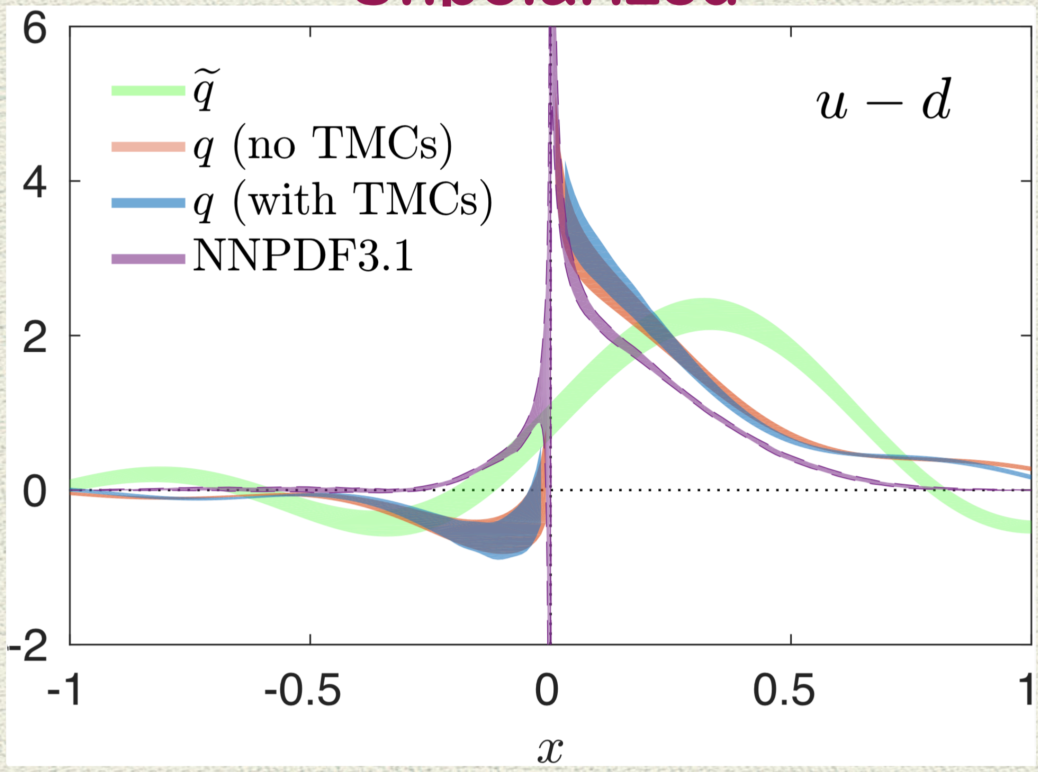


Towards light-cone PDFs

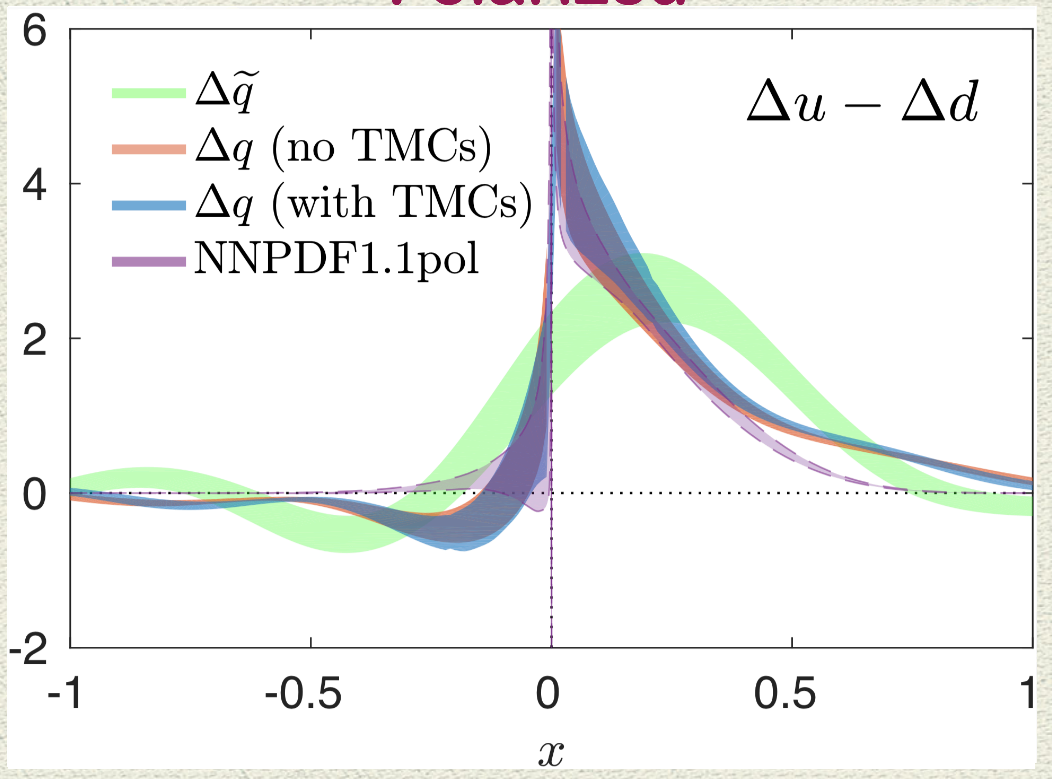
Evolution of lattice data (P=1.4GeV):

* Fourier Transform of renormarmalized matrix elements
* Matching of quasi-PDFs (LaMET)
  [C. Alexandrou et al., (PRL), arXiv:1803.02685]
* Target Mass Corrections (m_N/P: finite)
  [J.W. Chen et al., Nucl. Phys. B911 (2016) 246, arXiv:1603.06664]

Unpolarized



Polarized

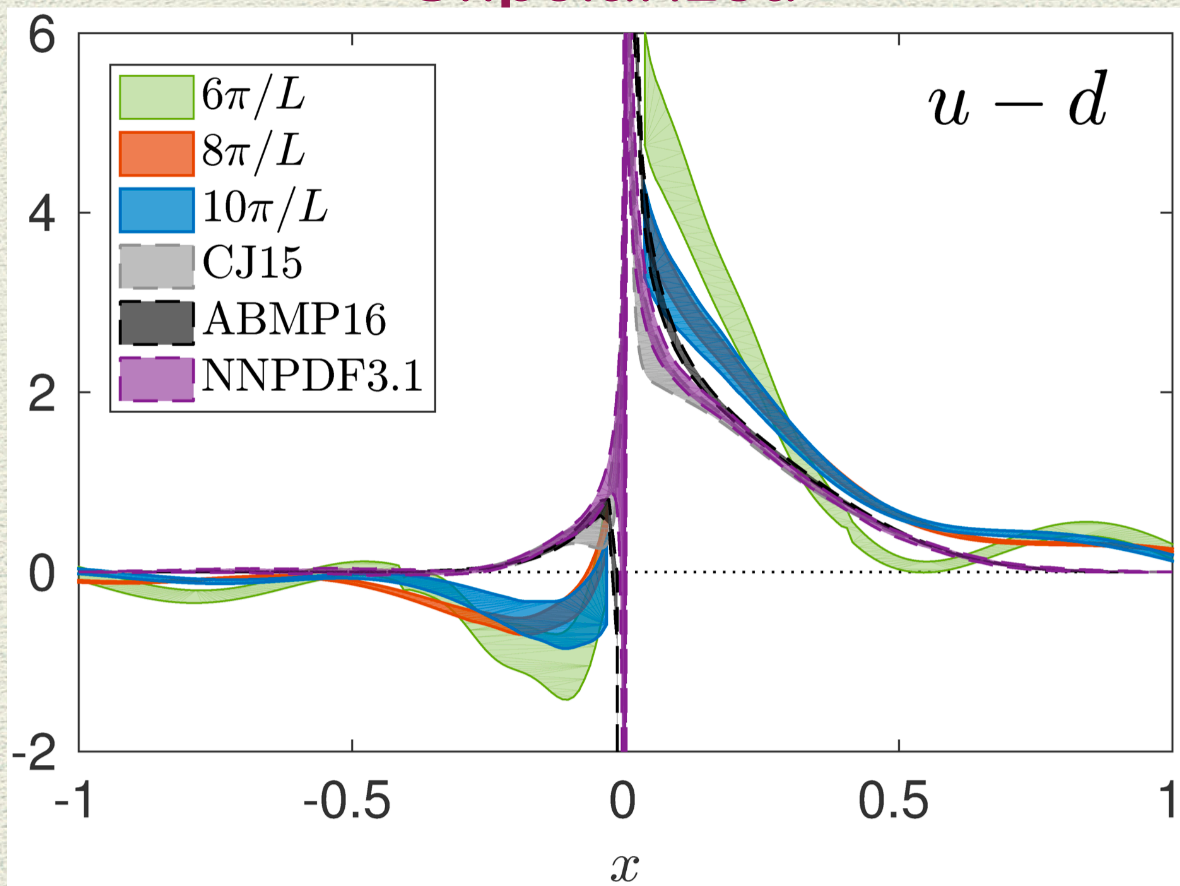


* Lattice PDFs similar behavior as phenomenological fits

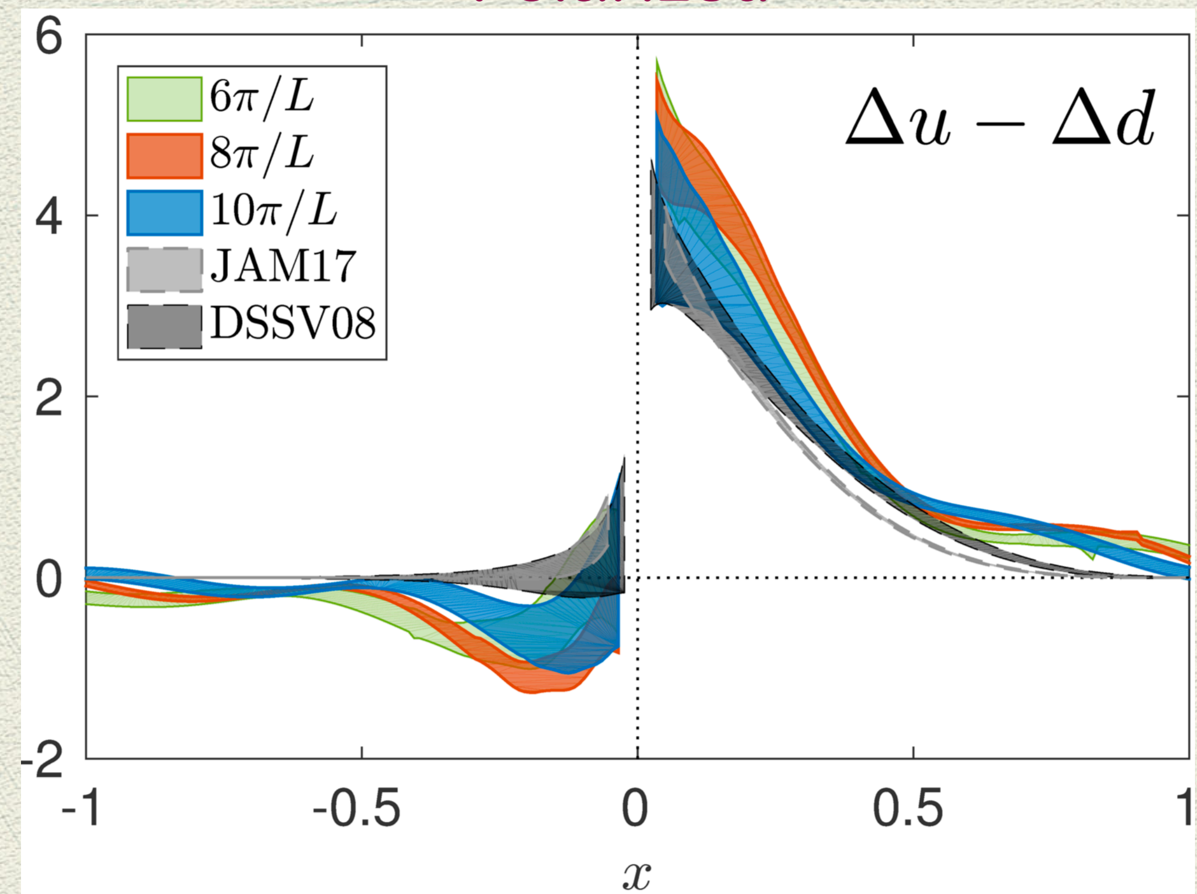
Towards light-cone PDFs

Nucleon boost dependence:

Unpolarized



Polarized

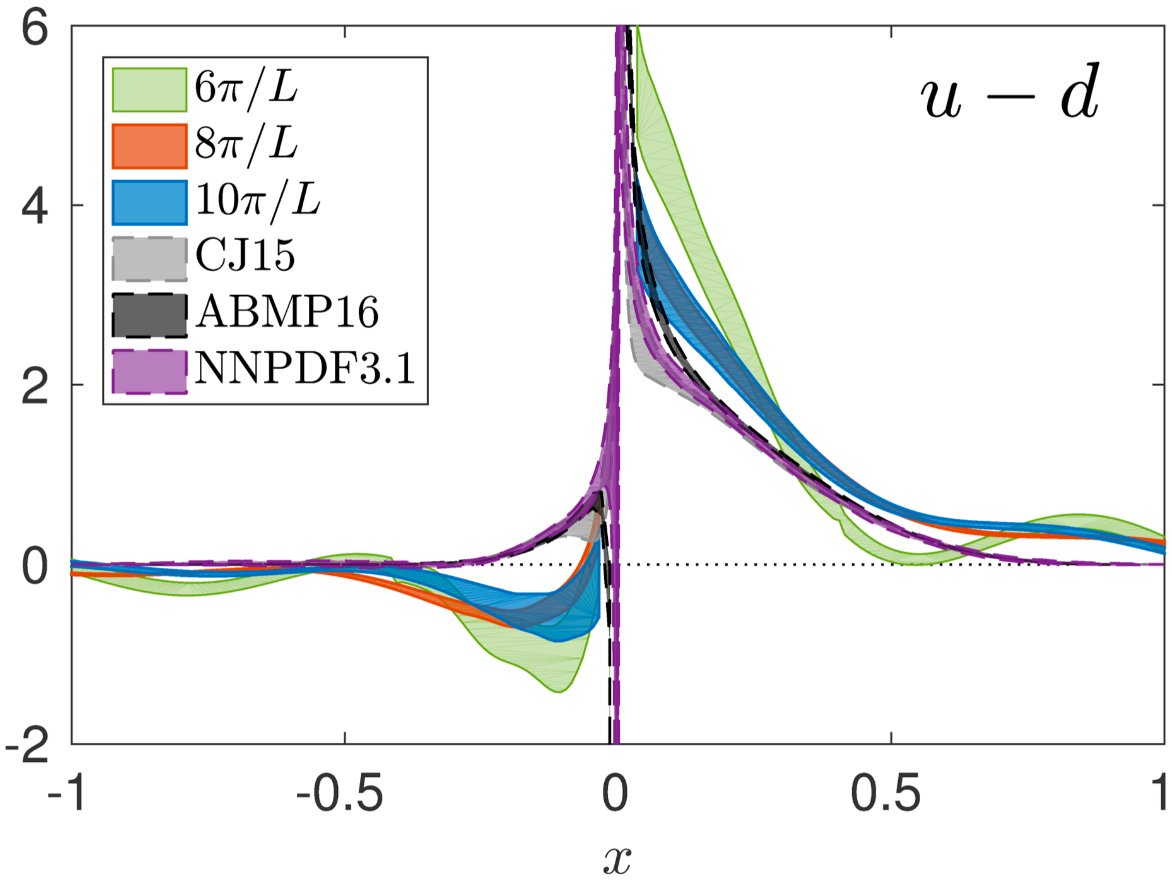


- Increasing momentum approaches the phenom. fits a saturation of PDFs for p=8π/L and p=10π/L
- 0<x<0.5 : Lattice polarized PDF overlap with phenomenology
- Negative x region: anti-quark contribution
- x~1: affected by finite nucleon momentum

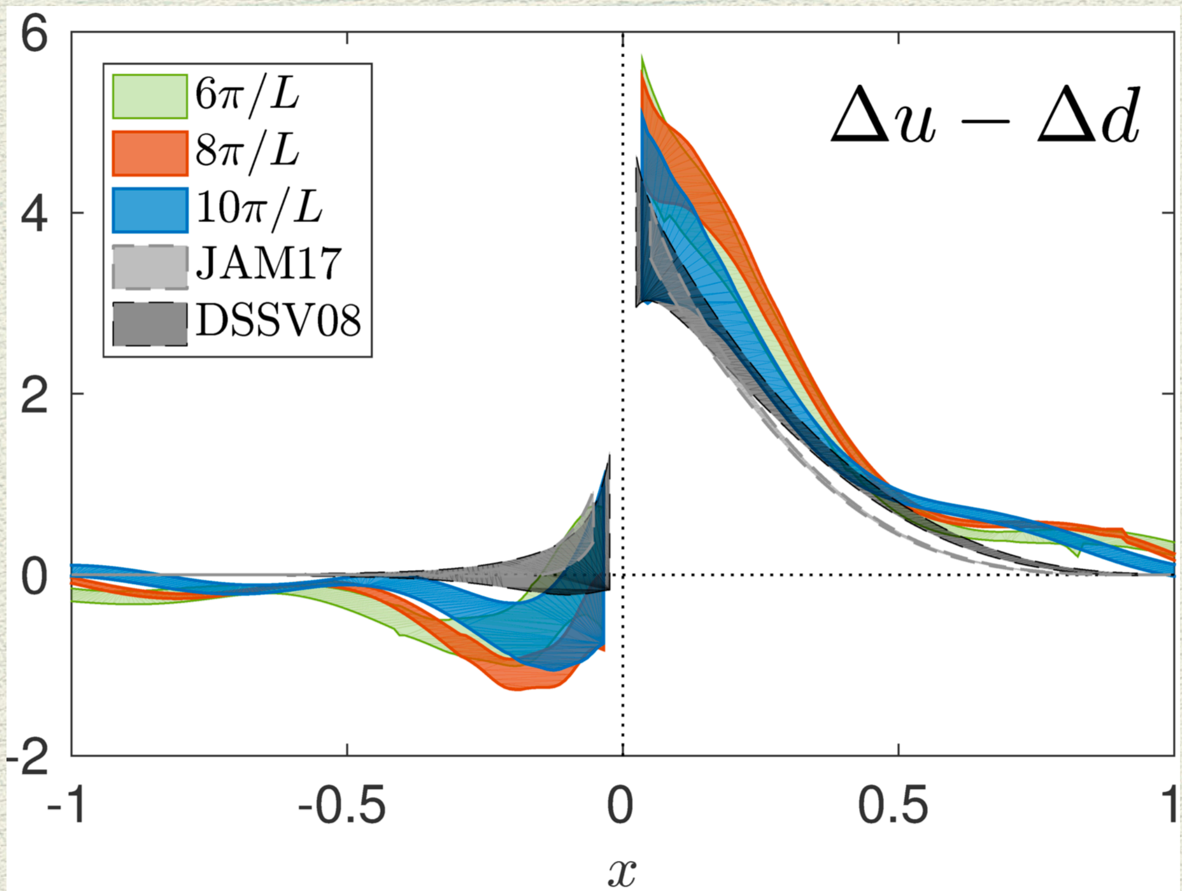
Towards light-cone PDFs

Nucleon boost dependence:

Unpolarized



Polarized

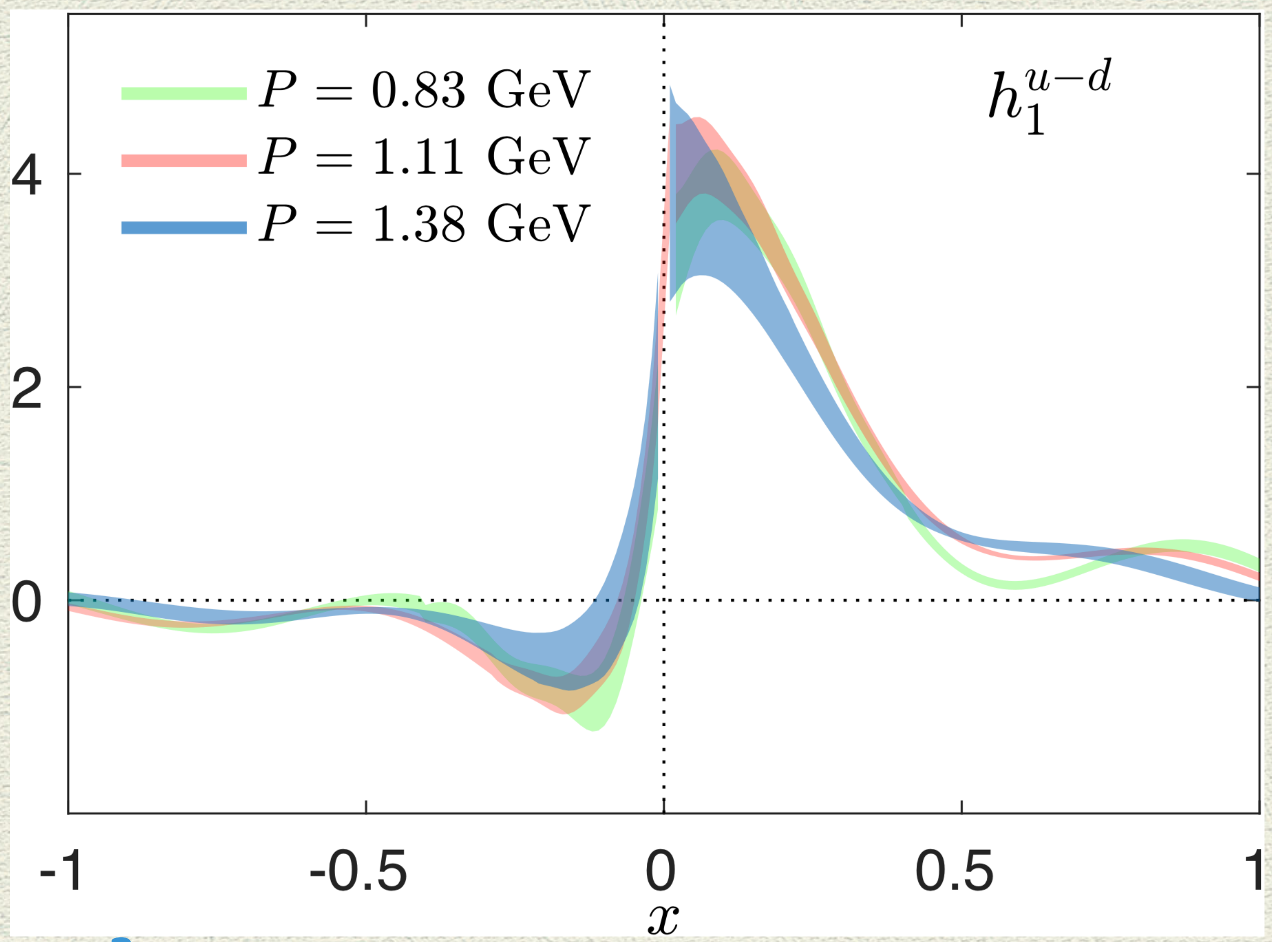


- Increasing momentum approaches the phenom. fits a saturation of PDFs for p=8π/L and p=10π/L
- 0<x<0.5 : Lattice polarized PDF overlap with phenomenology
- Negative x region: anti-quark contribution
- x~1: affected by finite nucleon momentum

BACKUP SLIDES

Transversity

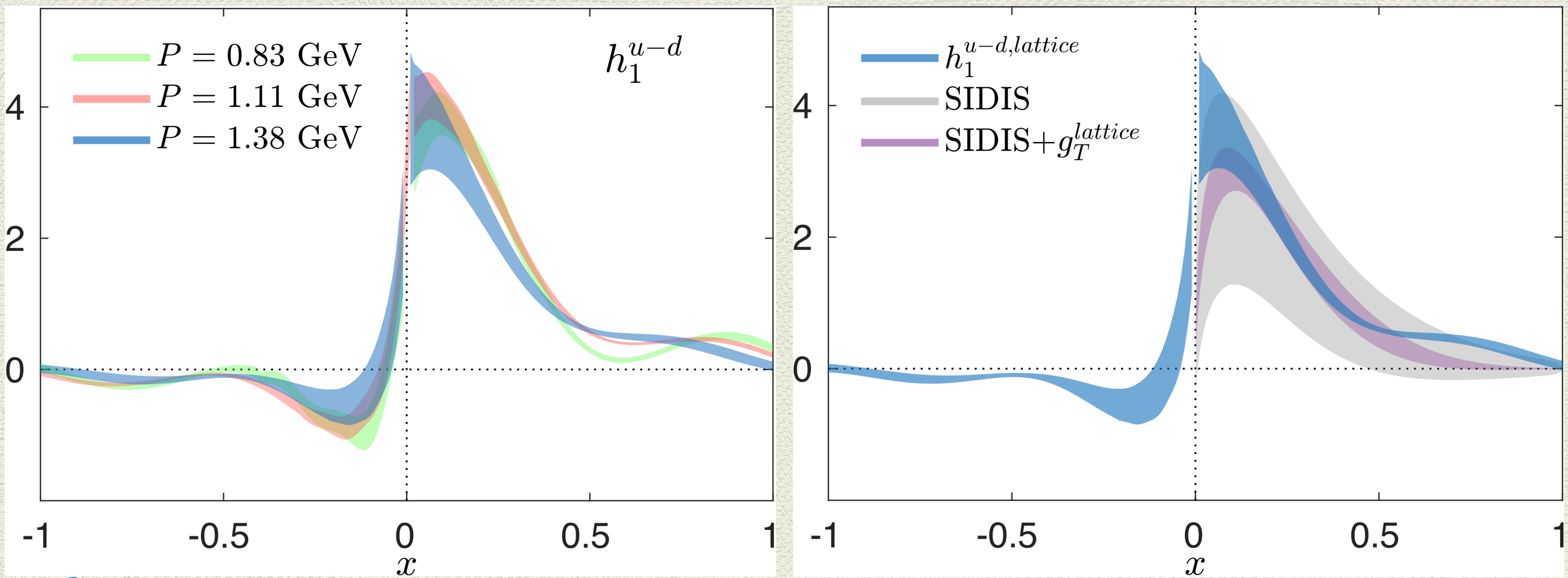
[C. Alexandrou et al., arXiv:1807.00232]



- Mild dependence on nucleon momentum
- Integral of PDF (g_T=1.09(11)) compatible with results from moments [C. Alexandrou et al., Phys. Rev. D95, 114514 (2017)]

Transversity

[C. Alexandrou et al., arXiv:1807.00232]



- Mild dependence on nucleon momentum
- Integral of PDF (g_T=1.09(11)) compatible with results from moments [C. Alexandrou et al., Phys. Rev. D95, 114514 (2017)]
- Lattice data from quasi-PDFs more accurate that SIDIS
- SIDIS improved with g_T^{Lat} constraints, but ***ab initio*** quasi-PDFs statistically more accurate

OUTLINE

A. Introduction

B. Nucleon on the Lattice

C. Nucleon Form Factors

1. Electromagnetic
2. Axial

D. Proton Spin

E. Access to x-dependence of PDFs

F. Discussion

Discussion

Summary:

* Simulations at the physical point for nucleon structure

* Investigations of systematic uncertainties:
  - gA agreement with experiment for Tsink ~1.5fm
  - Slope of Axial & E/M form factors sensitive to Tsink

* Disconnected contributions to FFs computed for u, d, s:
  - necessary to bring g^{u+d} in agreement with experiment
  - large contributions to G^{u+d} that partly cancels connected part
  - strange contributions to FFs non-negligible

* Significant progress on other direction (quasi-PDFs)

Discussion

Future Investigations:

FFs & proton spin

* Increase statistics and addition of separations larger than 1.5fm

* Two additional 2+1+1 ensembles (physical point, same physical volume)

quasi-PDFs

* Increase of momentum seems a natural next step
  BUT is a major challenge if reliable results is the goal

* Other directions should be pursued, e.g. 2-loop matching

THANK YOU

TMD Topical Collaboration

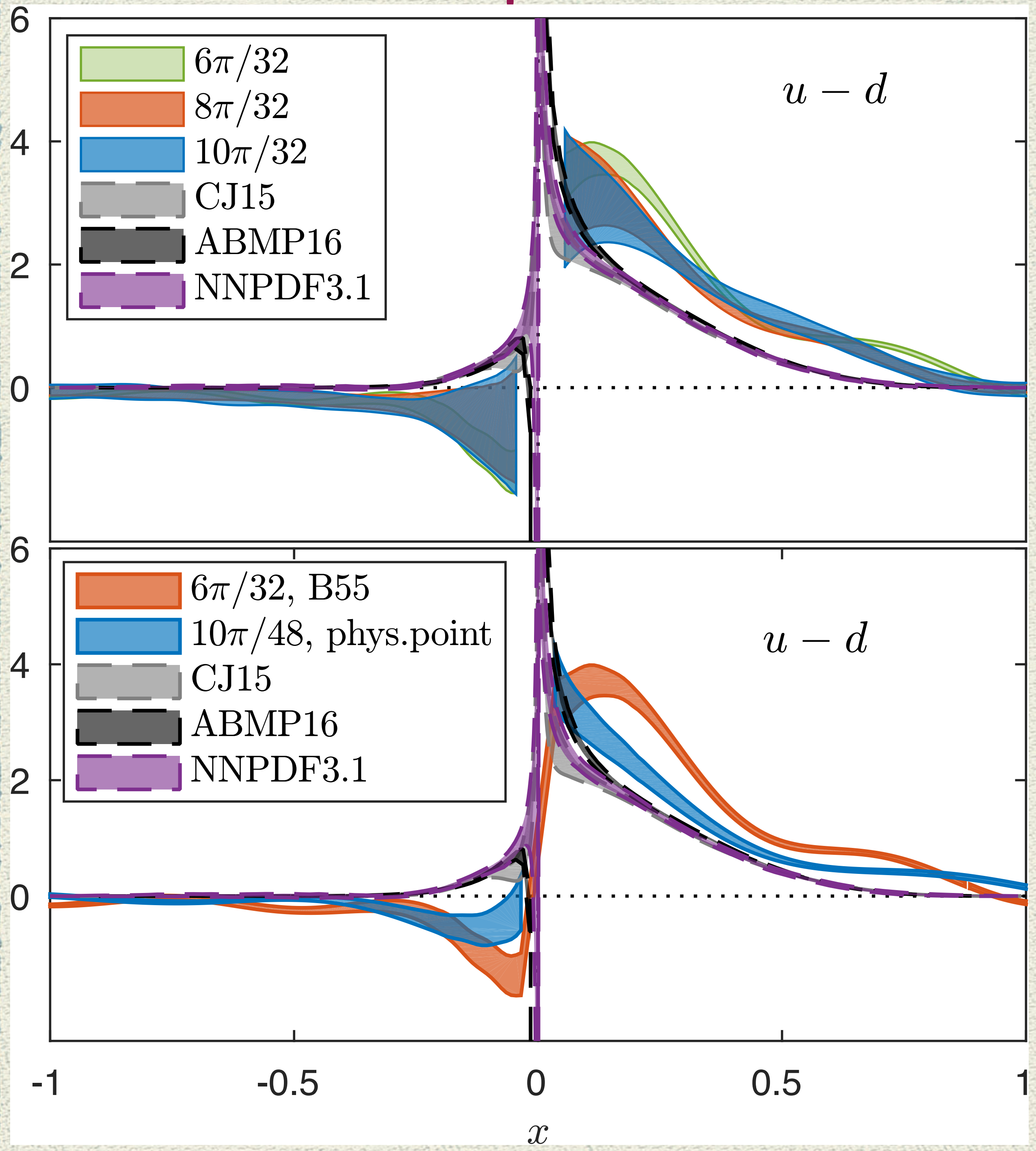
Grant No. PHY-1714407

BACKUP SLIDES

Pion mass dependence

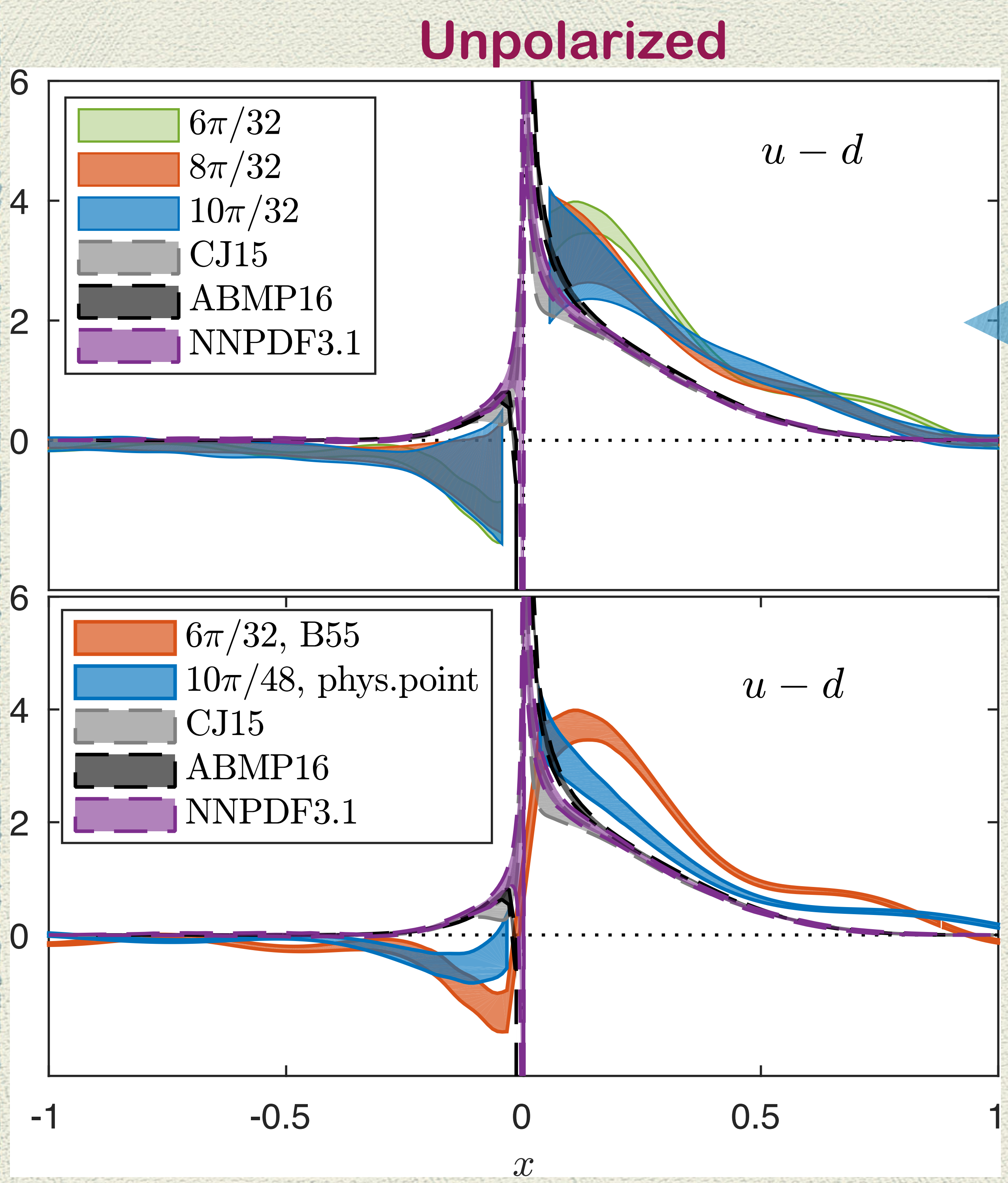
Simulations at physical m_π crucial for above conclusions

Unpolarized



Pion mass dependence

Simulations at physical m_π crucial for above conclusions

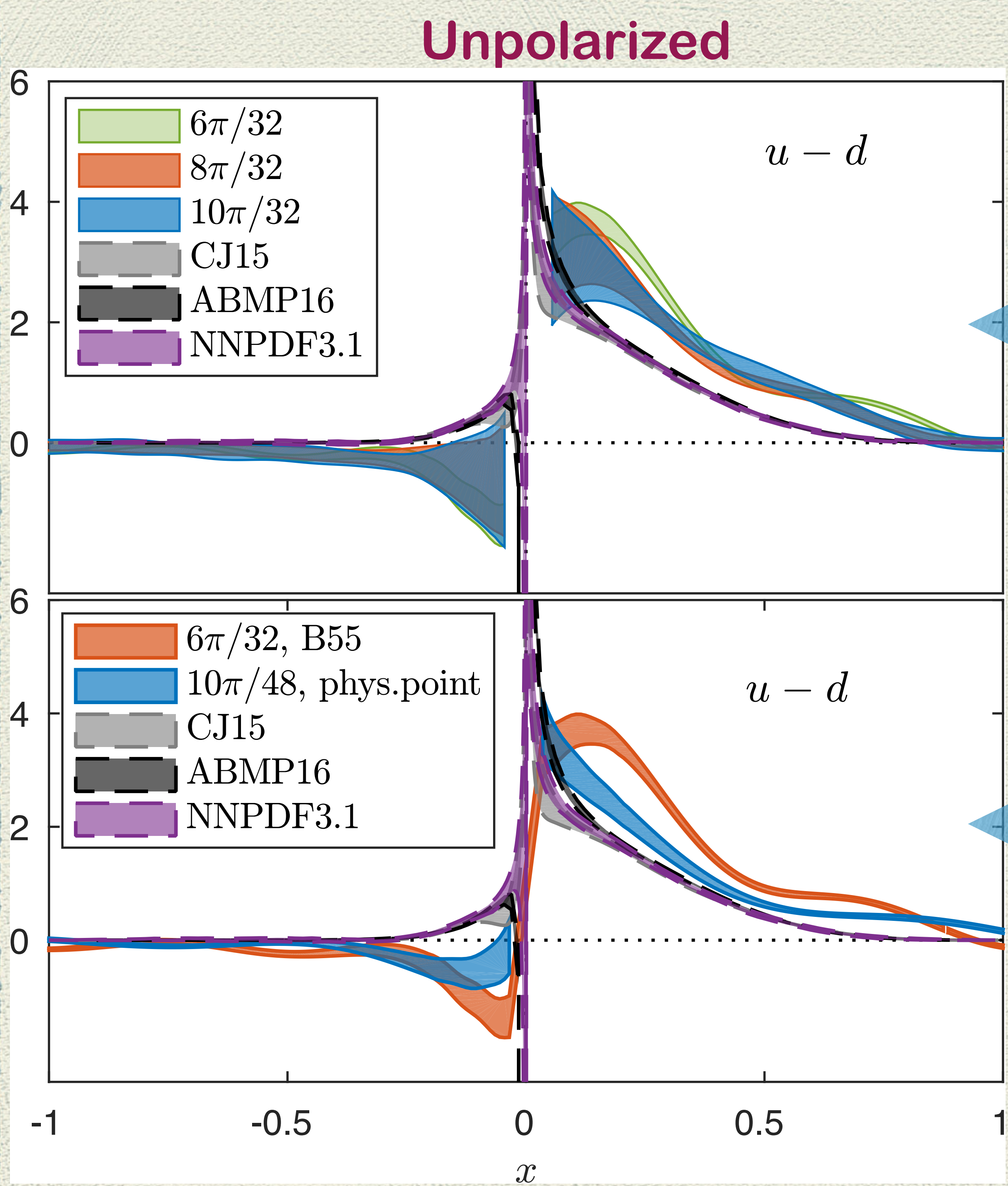


m_π = 375 MeV:

Lattice data saturate away from phenomenology

Pion mass dependence

Simulations at physical m_π crucial for above conclusions



m_π = 375 MeV:
Lattice data saturate away from phenomenology

m_π= 132, 375 MeV
Significant pion mass dependence

Alternative Fourier

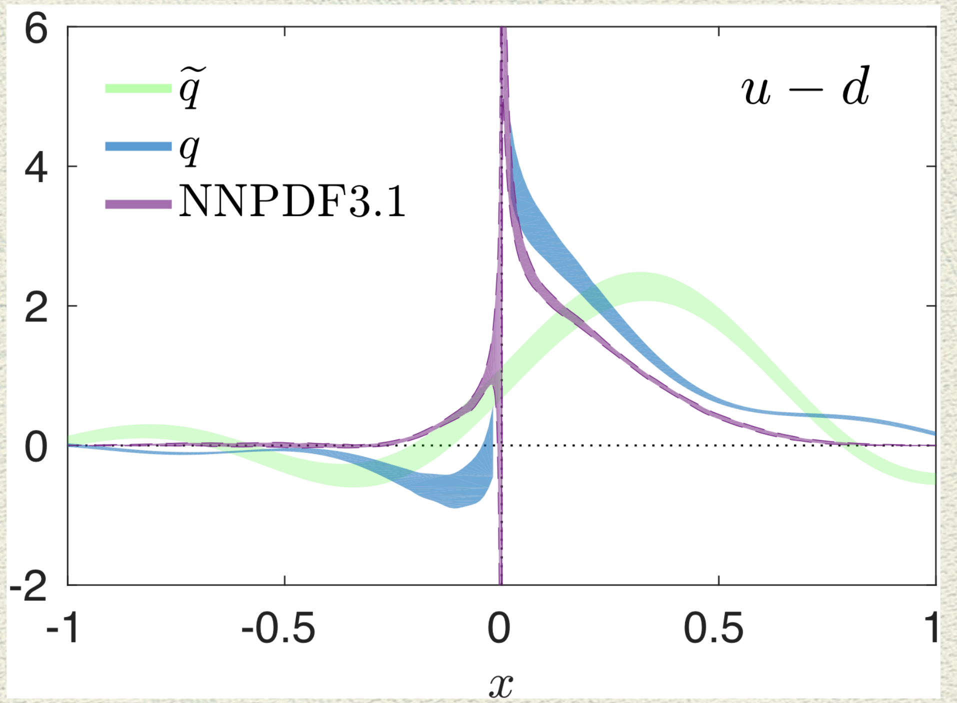
- Standard Fourier (SF) : $\tilde{q}(x) = 2P_3 \int_{-z_{\max}}^{z_{\max}} \frac{dz}{4\pi} e^{ixzP_3} h(z)$

can be written using integration by parts (DF):

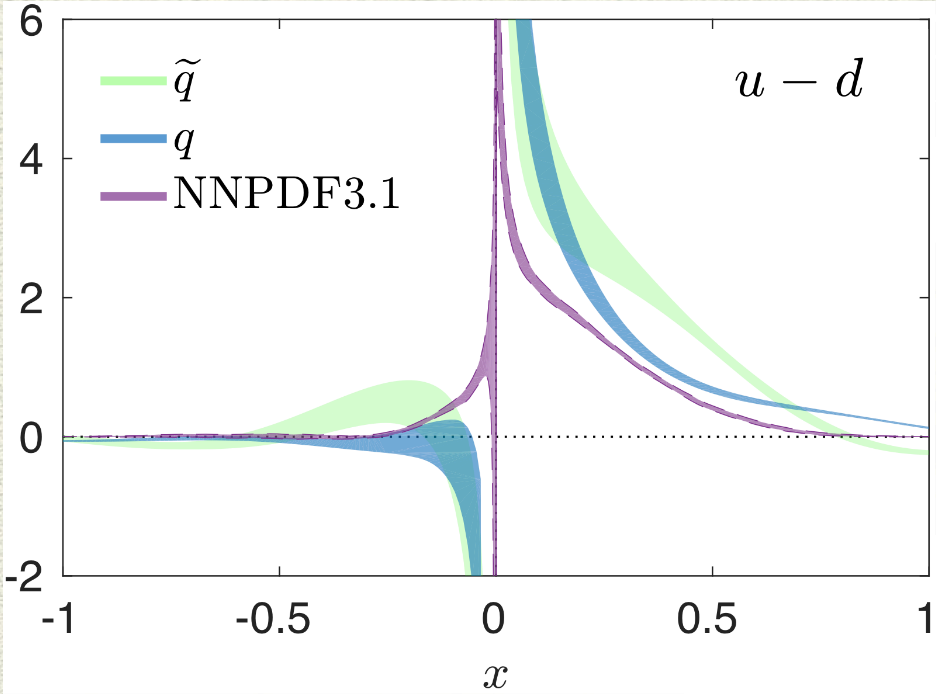
$$\tilde{q}(x) = h(z) \frac{e^{ixzP_3}}{2\pi i x} \Big|_{-z_{\max}}^{z_{\max}} - \int_{-z_{\max}}^{z_{\max}} \frac{dz}{2\pi} \frac{e^{ixzP_3}}{ix} h'(z)$$

[H.W. Lin et al., arXiv:1708.05301]

Standard FT



Derivative FT

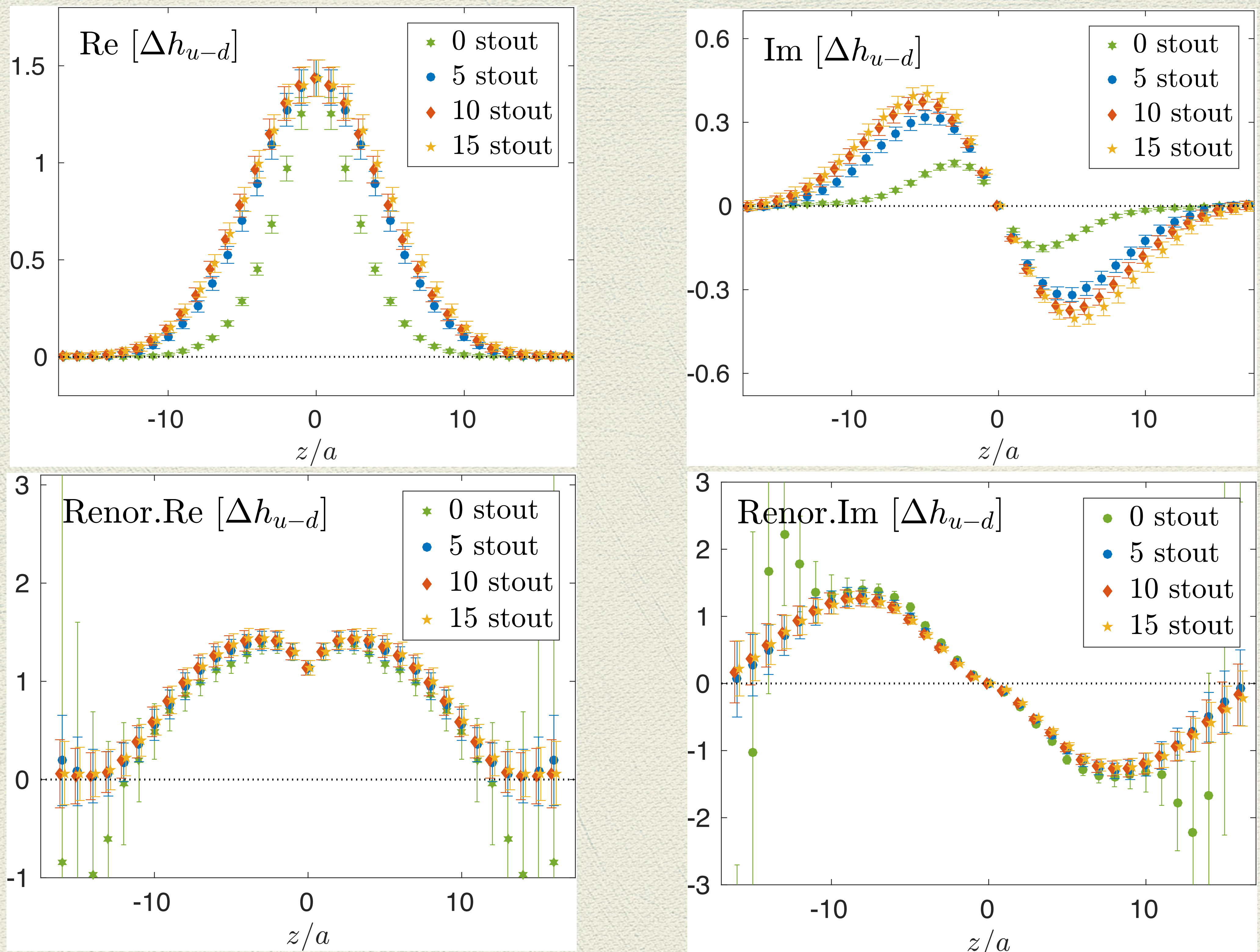


- Truncation at zmax (SF) vs neglecting surface term (DF)
  (latter non-negligible numerically)

- Oscillations reduced for DF, but small-x not well-behaved

- SF, DF different systematics, but DF uses interpolated data instead of raw ME ⇒ enhanced cut-off effects

Renormalized Matrix Elements



* Renormalized ME have no dependence on the stout smering