Aspects of Quark Orbital Angular Momentum

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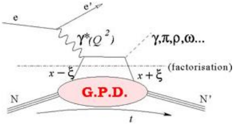
Outline

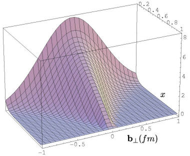


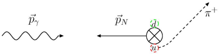
- Jaffe-Manohar vs. Ji decomposition of spin

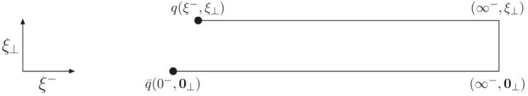
$\hookrightarrow$ $\mathcal{L}^q_{JM} - L^q_{Ji}$ = change in OAM as quark

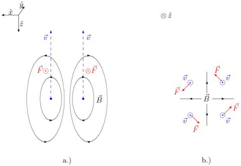
- OAM from twist-3 GPD G_2
  - model studies of sum rule
  - $\delta(x)$ in $G_2(x,0,t)$

  $\hookrightarrow$ implications for DVCS..
- Summary









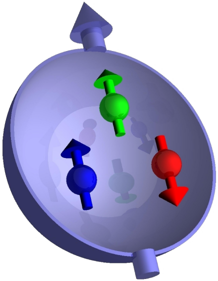
spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}$$

Longitudinally polarized DIS:

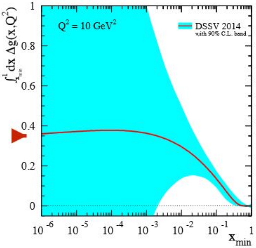
- $\Delta\Sigma = \sum_q \Delta q \equiv \sum_q \int_0^1 dx\, [q_\uparrow(x) - q_\downarrow(x)] \approx 30\%$

↪ only small fraction of proton spin due to quark spins



Gluon spin ΔG

could possibly account for remainder of nucleon spin, but still large uncertainties → EIC



Quark Orbital Angular Momentum

- how can we measure $\mathcal{L}_{q,g}$

↪ need correlation between position & momentum

- how exactly is $\mathcal{L}_{q,g}$ defined

spin sum rule

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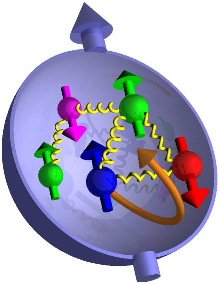
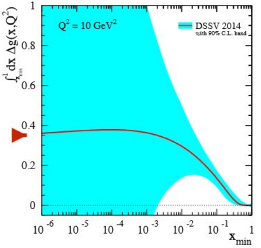
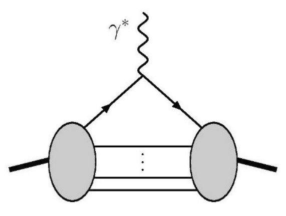




form factor



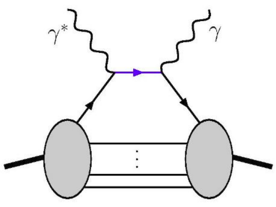
- electron hits nucleon & nucleon remains intact

↪ form factor $F(q^2)$

- position information from Fourier trafo
- no sensitivity to quark momentum
- $F(q^2) = \int dx GPD(x, q^2)$

↪ GPDs provide momentum disected form factors

Compton scattering



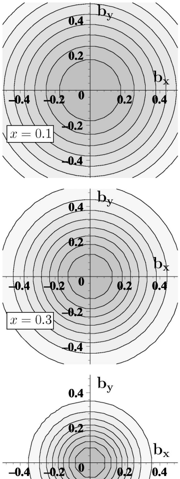
- electron hits nucleon, nucleon remains intact & photon gets emitted
- additional quark propagator

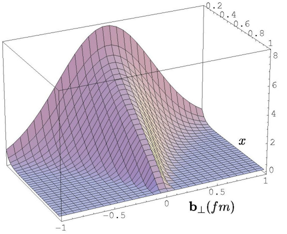
↪ additional information about momentum fraction x of active quark

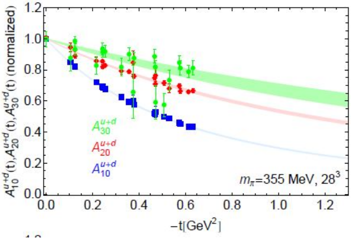
↪ generalized parton distributions $GPD(x, q^2)$

- info about both position and momentum of active quark

$q(x, \mathbf{b}_\perp)$ for unpol. p







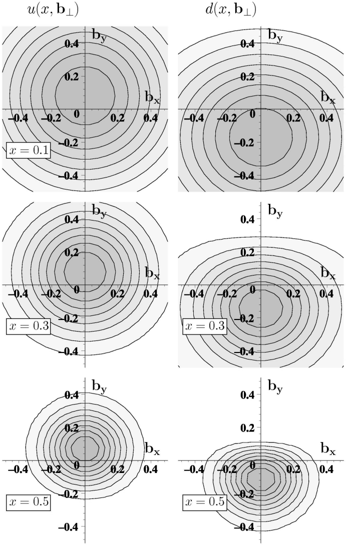
unpolarized proton

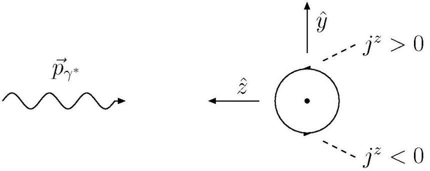
- $q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$

$\hookrightarrow$ probabilistic interpretation

- $F_1(-\mathbf{\Delta}_\perp^2) = \int dx H(x, 0, -\mathbf{\Delta}_\perp^2)$

Physics of GPDs: 3D Imaging MB, IJMPA 18, 173 (2003)





proton polarized in $+\hat{x}$ direction

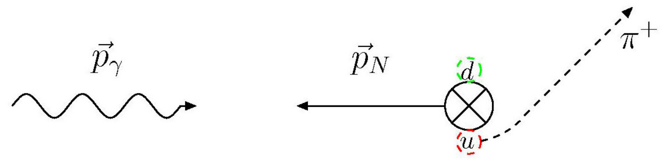
$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

$$-\frac{1}{2M}\frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

- relevant density in DIS is $j^+ \equiv j^0 + j^z$ and left-right asymmetry from j^z
- av. shift model-independently related to anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q}{2M}$$

example: $\gamma p \to \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive final state interaction (FSI) deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction $\rightarrow$ chromodynamic lensing

$\Rightarrow$ $\kappa_p\,,\kappa_n \quad \longleftrightarrow \quad$ sign of SSA!!!!!!!! (MB,2004)

- confirmed by HERMES & COMPASS data

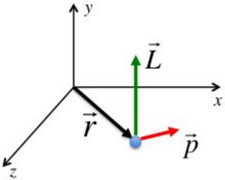
Angular Momentum Carried by Quarks

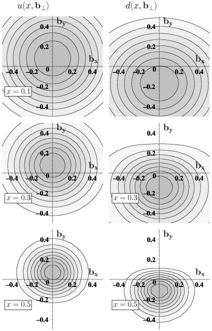
- $L_x = yp_z - zp_y$
- if state invariant under rotations about $\hat{x}$ axis then $\langle yp_z \rangle = -\langle zp_y \rangle$

$\hookrightarrow$ $\langle L_x \rangle = 2\langle yp_z \rangle$

- GPDs provide simultaneous information about longitudinal momentum and transverse position

$\hookrightarrow$ use quark GPDs to determine angular momentum carried by quarks





Ji sum rule (1996)

$$J_q^x = \frac{1}{2}\int dx\, x\, [H(x,0,0) + E(x,0,0)]$$

$\hookrightarrow$ M.Constantinou, K.F.Liu,..

- parton interpretation in terms of 3D distributions only for $\perp$ component (MB,2001,2005)

QED with electrons

$$\begin{aligned}\vec{J}_\gamma &= \int d^3r\, \vec{r} \times \left(\vec{E} \times \vec{B}\right) = \int d^3r\, \vec{r} \times \left[\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right)\right] \\ &= \int d^3r \left[E^j \left(\vec{r} \times \vec{\nabla}\right) A^j - \vec{r} \times (\vec{E} \cdot \vec{\nabla})\vec{A}\right] \\ &= \int d^3r \left[E^j \left(\vec{r} \times \vec{\nabla}\right) A^j + \left(\vec{r} \times \vec{A}\right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}\right]\end{aligned}$$

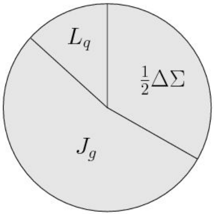
- replace 2^{nd} term (eq. of motion $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

$$\vec{J}_\gamma = \int d^3r \left[\psi^\dagger \vec{r} \times e\vec{A}\psi + E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \vec{E} \times \vec{A}\right]$$

- $\psi^\dagger \vec{r} \times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger \vec{r} \times (\vec{p} - e\vec{A})\psi$

↪ decomposing $\vec{J}_\gamma$ into spin and orbital also shuffles angular momentum from photons to electrons!

Ji decomposition



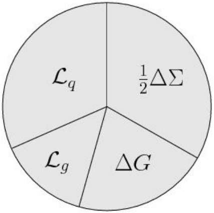
'pizza tre stagioni'

$$\frac{1}{2} = \sum_q \left(\frac{1}{2}\Delta q + L_q\right) + J_g$$

$$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$
- DVCS $\longrightarrow$ GPDs $\longrightarrow L^q$

Jaffe-Manohar decomposition



'pizza quattro stagioni'

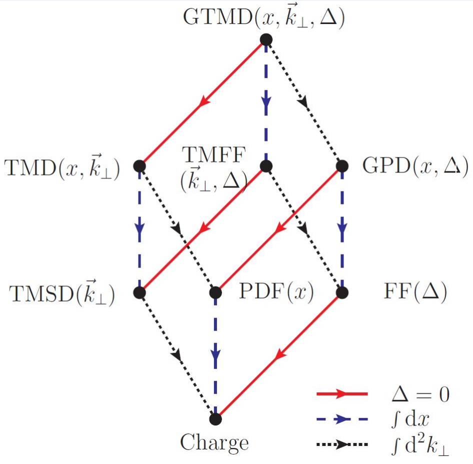
$$\frac{1}{2} = \sum_q \left(\frac{1}{2}\Delta q + \mathcal{L}_q\right) + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P,S| \bar{q}(\vec{r}) \gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) |P,S\rangle$$

- light-cone gauge $A^+ = 0$
- $\vec{p}\,\overleftarrow{p} \longrightarrow \Delta G \longrightarrow \mathcal{L} \equiv \sum_{i\in q,g} \mathcal{L}^i$
- manifestly gauge inv. def. exists

How large is difference $\mathcal{L}_q - L_q$ in QCD and what does it represent?

5-D Wigner Functions (Lorcé, Pasquini)



$$W(x, \vec{b}_\perp, \vec{k}_\perp) \equiv \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} GTMD(x, \vec{k}_\perp, \vec{\Delta}_\perp)$$

5-D Wigner Functions (Lorcé, Pasquini)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+ q(\xi)|PS\rangle.$$

- TMDs: $f(x,\mathbf{k}_\perp) = \int d^2\mathbf{b}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$
- GPDs: $q(x,\mathbf{b}_\perp) = \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$
- $L_z = \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)(b_x k_y - b_y k_x)$
- need to include Wilson-line gauge link $\mathcal{U}_{0\xi} \sim \exp\left(i\frac{g}{\hbar}\int_0^\xi \vec{A}\cdot d\vec{r}\right)$ to connect 0 and ξ

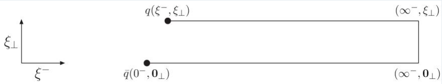
$\hookrightarrow$ 'light-cone staple' crucial for SSAs in SIDIS & DY

straight line for $\mathcal{U}_{0\xi}$

straigth Wilson line from 0 to ξ yields Ji-OAM:

$$L^q = \int d^3x \langle P,S|q^\dagger(\vec{x})\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

Light-Cone Staple for $\mathcal{U}_{0\xi}$



'light-cone staple' yields $\mathcal{L}_{Jaffe-Manohar}$

$\mathcal{L}_{\sqsupset}/\mathcal{L}_{\sqsubset}$

$\mathcal{L}$ with light-cone staple at $x^- = \pm\infty$

PT (Hatta)

- PT$\longrightarrow \mathcal{L}_{\sqsupset} = \mathcal{L}_{\sqsubset}$

(different from SSAs due to factor $\vec{x}$ in OAM)

Bashinsky-Jaffe

- $A^+ = 0$ no complete gauge fixing

$\hookrightarrow$ residual gauge inv. $A^\mu \to A^\mu + \partial^\mu \phi(\vec{x}_\perp)$

- $\vec{x} \times i\vec{\partial} \quad \rightarrow \quad \mathcal{L}_{JB} \equiv \vec{x} \times \left[i\vec{\partial} - g\vec{\mathcal{A}}(\vec{x}_\perp) \right]$
- $\vec{\mathcal{A}}_\perp(\vec{x}_\perp) = \frac{\int dx^- \vec{A}_\perp(x^-, \vec{x}_\perp)}{\int dx^-}$

Bashinsky-Jaffe ↔ light-cone staple

- $A^+ = 0$

$\hookrightarrow$ $\mathcal{L}_{\sqsupset/\sqsubset} = \vec{x} \times \left[i\vec{\partial} - g\vec{A}_\perp(\pm\infty, \vec{x}_\perp) \right]$

- $\mathcal{L}_{JB} = \vec{x} \times \left[i\vec{\partial} - g\vec{\mathcal{A}}(\vec{x}_\perp) \right]$
- $\vec{\mathcal{A}}_\perp(\vec{x}_\perp) = \frac{\int dx^- \vec{A}_\perp(x^-, \vec{x}_\perp)}{\int dx^-} = \frac{1}{2}\left(\vec{A}_\perp(\infty, \vec{x}_\perp) + \vec{A}_\perp(-\infty, \vec{x}_\perp) \right)$

$\hookrightarrow$ $\mathcal{L}_{JB} = \frac{1}{2}\left(\mathcal{L}_{\sqsupset} + \mathcal{L}_{\sqsubset} \right) = \mathcal{L}_{\sqsupset} = \mathcal{L}_{\sqsubset}$

straight line (→Ji)

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple (→ Jaffe-Manohar)

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$$i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$$

difference $\mathcal{L}^q - L^q$

$$\mathcal{L}^q - L^q = -g\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left[\vec{x}\times\int_{x^-}^{\infty} dr^- F^{+\perp}(r^-, \mathbf{x}_\perp)\right]^z q(\vec{x})|P,S\rangle$$

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x$$

Quark OAM from Wigner Distributions

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color Lorentz Force on ejected quark (MB, PRD 88 (2013) 014014)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v}\times\vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

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light-cone staple ($\rightarrow$ Jaffe-Manohar)

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difference $\mathcal{L}^q - L^q$

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Torque along the trajectory of q

$$T^z = \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

Change in OAM

$$\Delta L^z = \int_{x^-}^\infty dr^- \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

(Ji et al., 2016)

- for e^-: $\mathcal{L}_{JM} - L_{Ji} = 0$ to $\mathcal{O}(\alpha)$
  - earlier paper by M.B. & H.BC overlooked $h = 0$ component of Pauli-Villars γ
- $\mathcal{L}_{JM} - L_{Ji} \stackrel{?}{=} 0$ in general?
- how significant is $\mathcal{L}_{JM} - L_{Ji}$?

why scalar diquark model?

- Lorentz invariant
- 1^{st} to illustrate: FSI→SSAs (Brodsky,Hwang,Schmidt 2002)

↪ Sivers $\neq 0$

$\mathcal{L}_{JM} - L_{Ji} = \langle \bar{q}\gamma^+ \left(\vec{r} \times \vec{A}\right)^z q\rangle$ in scalar diquark model

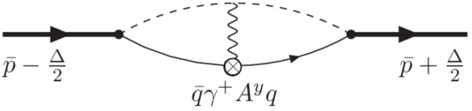
- pert. evaluation of $\langle \bar{q}\gamma^+ \left(\vec{r} \times \vec{A}\right)^z q\rangle$

↪ $\mathcal{L}_{JM} - L_{Ji} = \mathcal{O}(\alpha)$

- same order as Sivers

↪ $\mathcal{L}_{JM} - L_{Ji}$ as significant as SSAs

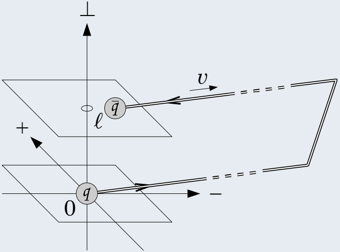
calculation



- nonforward matrix elem. of $\bar{q}\gamma^+ A^y q$
- $\left.\frac{d}{d\Delta^x}\right|_{\Delta=0}$

↪ $\langle k_\perp^q \rangle = \frac{3m_q + M}{12}\pi \langle \bar{q}\gamma^+ \left(\vec{r} \times \vec{A}\right)^z q\rangle$

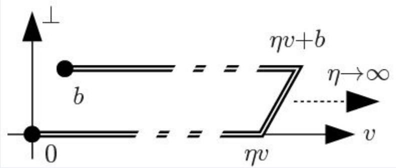
challenge



- TMDs/Wigner functions relevant for SIDIS require (near) light-like Wilson lines
- on Euclidean lattice, all distances are space-like

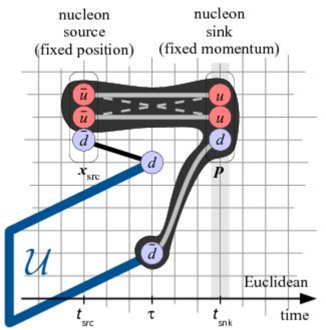
TMDs in lattice QCD

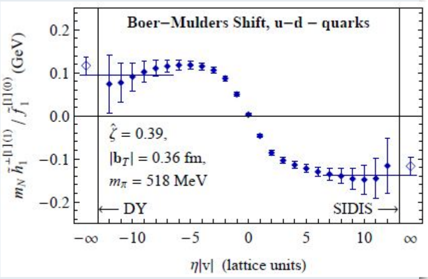
M.Engelhardt, P.Hägler, B.Musch, J.Negele, A.Schäfer



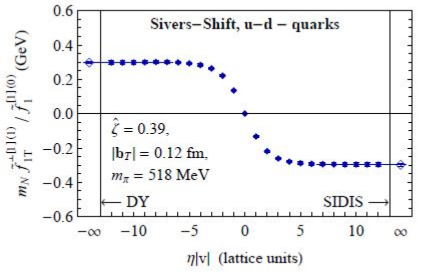
- calculate space-like staple-shaped Wilson line pointing in $\hat{z}$ direction; length $L \to \infty$
- momentum projected nucleon sources/sinks
- remove IR divergences by considering appropriate ratios

$\hookrightarrow$ extrapolate/evolve to $P_z \to \infty$

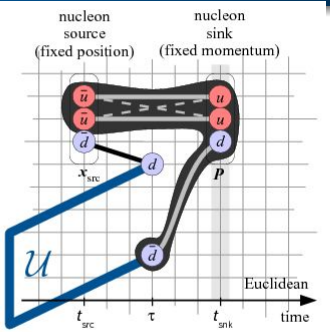


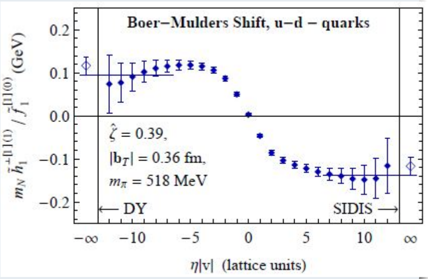


$f^{\perp}_{1T,SIDIS} = -f^{\perp}_{1T,DY}$ (Collins)



$f^{\perp}_{1T}(x, \mathbf{k}_{\perp})$ is $\mathbf{k}_{\perp}$-odd term in quark-spin averaged momentum distribution in $\perp$ polarized target





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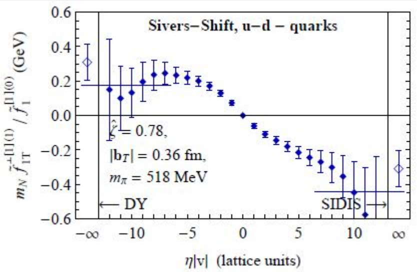

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difference $\mathcal{L}^q - L^q$

$\mathcal{L}^q_{JM} - L^q_{Ji} = \Delta L^q_{FSI}$ = change in OAM as quark leaves nucleon

$$\mathcal{L}^q_{JM} - L^q_{Ji} = -g \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left[\vec{x}\times\int_{x^-}^{\infty} dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{x})|P,S\rangle$$

e^+ moving through dipole field of e^-

- consider e^- polarized in $+\hat{z}$ direction

↪ $\vec{\mu}$ in $-\hat{z}$ direction (Figure)

- e^+ moves in $-\hat{z}$ direction

↪ net torque negative

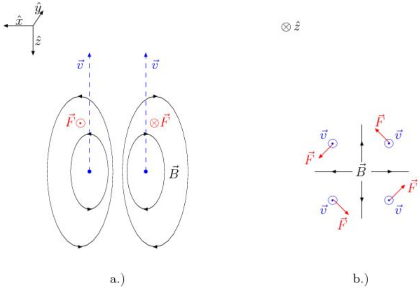
sign of $\mathcal{L}^q - L^q$ in QCD

- color electric force between two q in nucleon attractive

↪ same as in positronium

- spectator spins positively correlated with nucleon spin

↪ expect $\mathcal{L}^q - L^q < 0$ in nucleon



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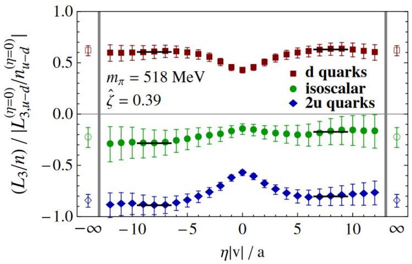
lattice QCD M.Engelhardt et al.

- L_{staple} vs. staple length

↪ L^q_{Ji} for length $= 0$

↪ $\mathcal{L}^q_{JM}$ for length $\to \infty$



Alternative OAM sum rule Polyakov & Kitpily

$$\int dx\, x G_2^q(x,0,0) = -L_q$$

transverse force imaging/tomography F.Aslan, MB, M.Schlegel

2D Fourier transform of x^2 moment of appropriate twist-3 GPDs yields position resolved transverse force

looked at this in spectator models and found some weird stuff...

twist-3 GPDs

Polyakov & Kitpily

$$\int dz^- e^{ixz^-\bar{p}^+}\langle p'|\bar{q}(z^-/2)\gamma^x q(-z^-/2)|p\rangle$$

$$= \frac{1}{2\bar{p}^+}\bar{u}(p')\left[\frac{\Delta^x}{2M}G_1 + \gamma^x(H+E+G_2) + \frac{\Delta^x\gamma^+}{\bar{p}^+}G_3 + \frac{i\Delta^y\gamma^+\gamma_5}{\bar{p}^+}G_4\right]u(p)$$

Lorentz invariance relations

- $\int dx G_1^q(x,\xi,t) = 0$
- $\int dx G_2^q(x,\xi,t) = 0$
- $\int dx G_3^q(x,\xi,t) = 0$
- $\int dx G_4^q(x,\xi,t) = 0$

QCD Eqs. of motion

Polyakov & Kitpily

- $\int dx\, x G_2^q(x,0,0) = -L^q$
- same relation also derived in scalar Yukawa

Tests

- test above relations in scalar diquark model & QCD
- $\mathcal{L}(x) \overset{?}{=} -\int_x^1 dy G_2(y)$

$\hookrightarrow$ answer: SDM yes, QTM no

issues

- $\delta(x)$ in G_2^q?
- G_2^q from DVCS?

$d_2 \leftrightarrow$ average $\perp$ force on quark in DIS from $\perp$ pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

$\hookrightarrow$ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0)\gamma^+ gF^{+y}(0)q(0) \right| P,S \right\rangle$$

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 114502)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

matrix element defining $d_2 \leftrightarrow 1^{st}$ integration point in QS-integral
$d_2 \Rightarrow \perp$ force $\leftrightarrow$ QS-integral $\Rightarrow \perp$ impulse

sign of d_2

- $\perp$ deformation of $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ sign of d_2^q: opposite Sivers

magnitude of d_2

- $\langle F^y \rangle = -2M^2 d_2 = -10 \frac{GeV}{fm} d_2$
- $|\langle F^y \rangle| \ll \sigma \approx 1 \frac{GeV}{fm} \Rightarrow d_2 = \mathcal{O}(0.01)$

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consitent with experiment (JLab,SLAC), model calculations (Weiss), and lattice QCD calculations (Göckeler et al., 2005)

- take x^2 moment of twist-3 GPDs ($\xi = 0$)
- subtract twist-2 parts
- take 2D Fourier transform

$$\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} \int dx\, x^2 \tilde{G}_2^{tw\,3}(x, 0, -\Delta_\perp^2)$$

$\hookrightarrow$ $\langle R_\perp = 0, S_\perp \,|\bar{q}(b_\perp)\gamma^+ gF^{+y}(b_\perp)q(b_\perp)|\, R_\perp = 0, S_\perp\rangle$

$\hookrightarrow$ distribution of $\perp$ force in $\perp$ plane

- x^2 moments of different twist-3 GPDs provide info about $\perp$ force tomography for different spin combinations

$\hookrightarrow$ twist-3 GPDs $\Rightarrow$ 2D $\perp$ force maps

- could be done immediately in lattice QCD
- need to address some issues regarding experimental access...

motivation

- wanted to test Polyakov sum rule
- definitions for PDFs/GPDs based on Lorentz invariance

$\hookrightarrow$ need Lorentz invariant model...

findings: singularities in twist 3 GPDs/PDFs

- Polyakov sum rule worked, but
- twist 3 GPDs discontinuous at $x = \pm\xi$
- $\delta(x)$ in forward limit
- $\delta(x)$ essential for various Lorentz invariance relations, such as
  - $\int_{-1}^{1} dx G_2(x, 0, t) = 0$
  - $\int_{-1}^{1} dx h_L(x) = \int_{-1}^{1} dx h_1(x)$

happy ending

discontinuities cancel in linear combinations that enter DVCS amplitude

example: scalar diquark

$$q_\Gamma(x,k_\perp) = \int dk^- \bar{u}(P,S) \frac{\not{k}+m}{k^2-m^2+i\varepsilon} \Gamma \frac{\not{k}+m}{k^2-m^2+i\varepsilon} u(P,S) \frac{1}{(P-k)^2-\lambda^2+i\varepsilon}$$

- similar for quark target (QCD)
- $k^+ = xp^+$

denominator integral

$$I_{den} \equiv \int dk^- \frac{1}{(k^2-m^2+i\varepsilon)^2} \frac{1}{(P-k)^2-\lambda^2+i\varepsilon}$$

- $k^2 = 2k^+k^- - k_\perp^2$, $(P-k)^2 = 2(P^+-k^+)(P^- - k^-) - k_\perp^2$
- $I_{den} = 0$ for $k^+ < 0$: all k^- poles in UHP
- $I_{den} = 0$ for $k^+ > P^+$: all k^- poles in LHP
- $I_{den} = \frac{-\pi i}{P^+(1-x)x^2} \frac{1}{\left[2P^+P^- - \frac{k_\perp^2+m^2}{x} - \frac{k_\perp^2+\lambda^2}{1-x}\right]^2}$
- twist-2: Γ contains γ^+; $\not{k} = k^-\gamma^+$

$\hookrightarrow$ numerator only function of x, $k_\perp$ as $\gamma^+\gamma^+ = 0$ $\Rightarrow$ straightforward!

example: scalar diquark

$$q_\Gamma(x,k_\perp) = \int dk^- \bar{u}(P,S) \frac{\not{k}+m}{k^2-m^2+i\varepsilon} \Gamma \frac{\not{k}+m}{k^2-m^2+i\varepsilon} u(P,S) \frac{1}{(P-k)^2-\lambda^2+i\varepsilon}$$

- similar for quark target (QCD)
- similar for 1-loop corrections

twist-3: example $\Gamma = 1\!\!1$

- numerator $(\not{k}+m)^2 = k^2+m^2+2m\not{k}$
- $\bar{u}(P,S)\not{k}u(P,S) = 2P^+k^- + ...$
- $2k^- = \frac{(P-k)^2-\lambda^2}{P^+-k^+} - \left[P^- - \frac{k_\perp^2+\lambda^2}{P^+-k^+}\right]$
- 2^{nd} term canonical (from LF Hamiltonian pert. theory → SJB)
- 1^{st} term cancels spectator propagator

↪ $I_\delta = \int dk^- \frac{1}{(k^2-m^2+i\varepsilon)^2} = \int dk^- \frac{1}{(2k^+k^- - k_\perp^2 - m^2 + i\varepsilon)^2} = ?$

- $I_\delta = 0$ for $k^+ = 0$ as pole can be avoided
- $\int d^2k_L \frac{1}{(k^2-m^2+i\varepsilon)^2} \equiv \int dk^+dk^- \frac{1}{(k^2-m^2+i\varepsilon)^2} = \frac{\pi i}{k_\perp^2+\lambda^2} \Rightarrow I_\delta = \frac{\pi i}{k_\perp^2+\lambda^2}\delta(k^+)$

Relevance of $\delta(x)$ for QCD

sum rules for twist-3 PDFs

MB, PRD **52**, 3841 (1995)

- $\int_{-1}^{1} dx g_T(x) = \int_{-1}^{1} dx g_1(x)$
- $\int_{-1}^{1} dx h_L(x) = \int_{-1}^{1} dx h_1(x)$
- $\int_{-1}^{1} dx e(x) = \frac{1}{2M}\langle P|\bar{q}q|P\rangle$ (σ-term sum rule)
- first two are Lorentz invariance (LI) relations

If sum rule is tested by evaluating e.g. $\lim_{\varepsilon\to 0}\int_{\varepsilon}^{1} dx\,[h_L(x) + h_L(-x)]$ then presence of $\delta(x)$ in h_L would result in violation of LI relation!

violation of twist-3 sum rules in QCD

MB & Y. Koike, NPB **632**, 311 (2002)

Using moment relations based on QCD eqs. of motion one finds

- $h_L^\delta(x)$ contains $\delta(x)$ at 1 loop: LI relation 'violated'!
- $g_T^\delta(x)$ no $\delta(x)$ (LI relation o.k.!)
- $e(x)$ contains $\delta(x)$ at 1 loop: σ-term sum rule 'violated'

implications for twist-3 GPDs

what does presence of $\delta(x)$ in twist-3 PDFs imply for twist-3 GPDs?

- relevant energy denominators:

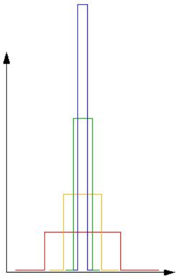
$$\int dk^- \frac{1}{\left(k-\frac{\Delta}{2}\right)^2 - m^2 + i\varepsilon} \frac{1}{\left(k+\frac{\Delta}{2}\right)^2 - m^2 + i\varepsilon} \frac{1}{(P-k)^2 - \lambda^2 + i\varepsilon}$$

- twist-3: k^- from Dirac numerator can cancel $(P-k)^2 - \lambda^2 + i\varepsilon$

$\hookrightarrow$ $\int dk^- \frac{1}{\left(k-\frac{\Delta}{2}\right)^2 - m^2 + i\varepsilon} \frac{1}{\left(k+\frac{\Delta}{2}\right)^2 - m^2 + i\varepsilon} \sim \frac{\Theta\left(\frac{-\Delta^+}{2} < k^+ < \frac{\Delta^+}{2}\right)}{\Delta^+} \frac{1}{k_\perp^2 + m^2}$

- contribution to ERBL region only!
  - nonzero only for $-\xi < x < \xi$
  - discontinuous at $x \pm \xi$
  - $\propto \frac{1}{\xi}$ for $-\xi < x < \xi$

  $\hookrightarrow$ representation of δ function as $\xi \to 0$
- big issue: convergence of $\int \frac{dx}{x-\xi} GPD(x,\xi,t)$ when $GPD(x,\xi,t)$ discontinuous at $x \pm \xi$
- presence of such terms ‘normal’ for twist-3 GPDs

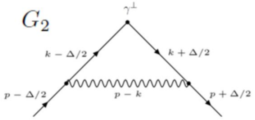


$G_2, \widetilde{G}_2$ in QCD (1 loop)

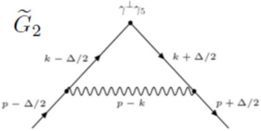
How do the discontinuities behave as $\xi \to 0$

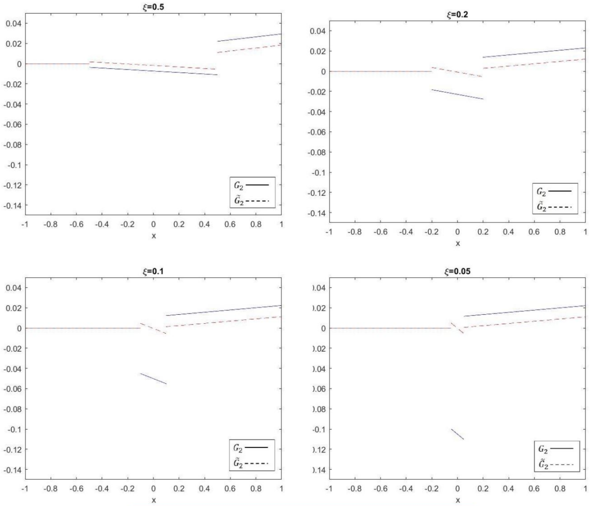
G_2 and $\widetilde{G}_2$ in Quark Target Model

G_2



$\widetilde{G}_2$

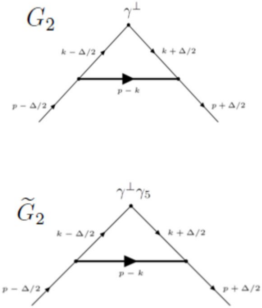


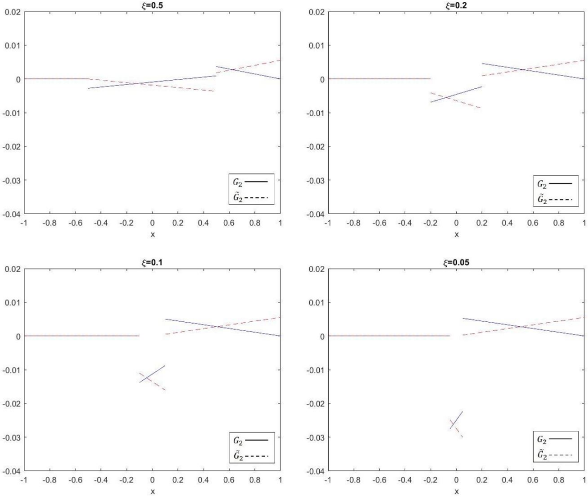


Twist-3 GPD	Discontinuities as $\xi \to 0$
G_2	Divergent
$\widetilde{G}_2$	Finite

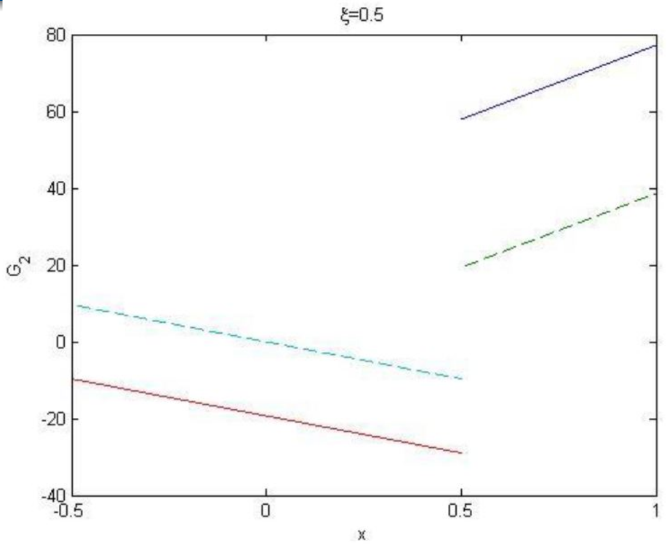
What happens in different models?

G_2 and $\widetilde{G}_2$ in Scalar Diquark Model

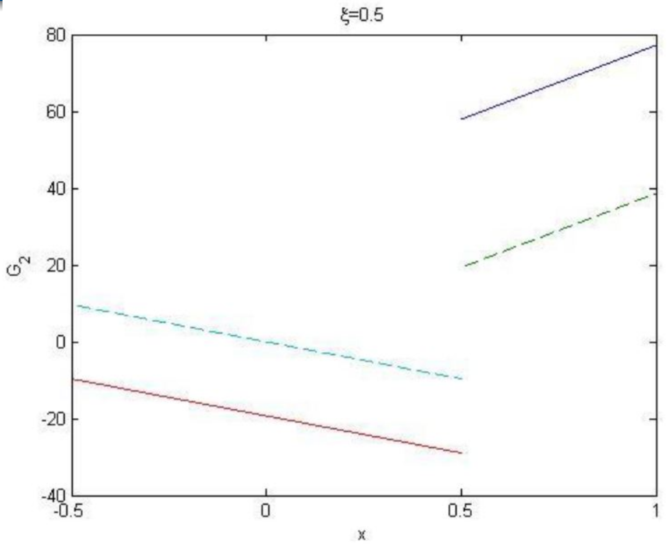




Twist-3 GPD	Discontinuities as $\xi \to 0$
G_2	Divergent
$\widetilde{G}_2$	Divergent



- $G_2(\Gamma = \gamma_\perp)$, $\tilde{G}_2(\Gamma = \gamma_\perp\gamma_5)$ discontinuous at $x = -\xi$
- $\int \frac{dx}{x \pm \xi} G_2(x, \xi, t)$ divergent



- $G_2(\Gamma = \gamma_\perp)$, $\tilde{G}_2(\Gamma = \gamma_\perp \gamma_5)$ discontinuous at $x = -\xi$
- $\int \frac{dx}{x \pm \xi} G_2(x, \xi, t)$ divergent — oops!

$\hookrightarrow$ factorization?

terms in DVCS amplitude involving twist 3

$$\int_{-1}^{1} dx[F_\perp^\nu C^+(x,\xi) - i\varepsilon^\nu_{\perp\alpha}\tilde{F}^\alpha_\perp C^-(x,\xi)]$$
$$= \int_{-1}^{1} dx\Bigg[\Delta_\perp^\nu \frac{b}{2m}(G_1 C^+ + (\tilde{E}+\tilde{G}_1)C^-) + h_\perp^\nu\Bigg((H+E+G_2)C^+ - \frac{\Delta_\perp^2}{4\xi m^2}(\tilde{E}+\tilde{G}_1)C^- - \frac{1}{\xi}(\tilde{H}+\tilde{G}_2)C^-\Bigg)$$
$$+ \Delta_\perp^\nu \frac{h^+}{P^+}\Bigg(G_3 C^+ - \frac{\bar{m}^2}{2m^2}(\tilde{E}+\tilde{G}_1)C^- - \tilde{G}_4 C^-\Bigg)$$
$$+ \tilde{\Delta}_\perp^\nu \frac{\tilde{h}^+}{P^+}\Bigg(G_4 C^+ + \frac{t}{8\xi m^2}(\tilde{E}+\tilde{G}_1)C^- + \frac{1}{2\xi}(\tilde{H}+\tilde{G}_2)C^- - \tilde{G}_3 C^-\Bigg)\Bigg].$$

- $G_i/\tilde{G}_i$ twist 3 vector/axialvector GPDs
- $C^{\pm} = \frac{1}{x-\xi+i\varepsilon} \pm \frac{1}{x+\xi-i\varepsilon}$
- $\tilde{G}_2 \stackrel{forward}{\longrightarrow} g_2$
- $\int dx x G_2^q(x,0,0) = -L_q$

factorization?

- both in quark target model and scalar diquark model G_i & $\tilde{G}_i$ discontinuous at $x = \pm\xi$
- GPDs in factorized DVCS amplitude convoluted with $C^{\pm} = \frac{1}{x-\xi+i\varepsilon} \pm \frac{1}{x+\xi-i\varepsilon} \ldots$

good news

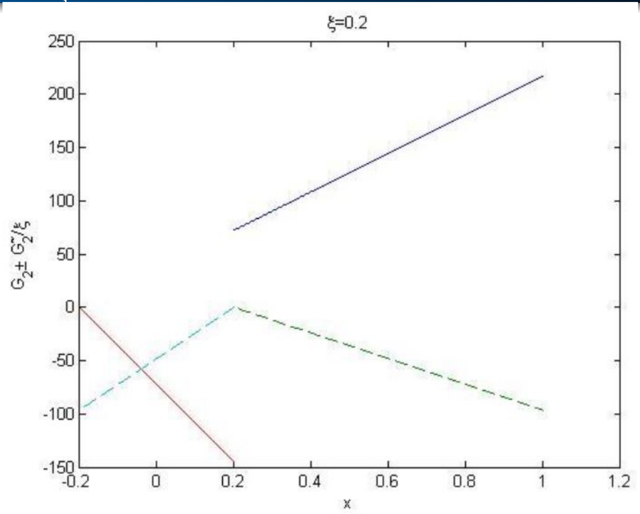
in quark target model (not in scalar diquark model) discontinuities cancel in linear combinations of twist 3 GPDs that enter $\mathcal{A}_{DVCS}$

$$\mathcal{A}_1 \sim G_1 C^+ + \widetilde{G}_1' C^-,$$

$$\mathcal{A}_2 \sim G_2' C^+ - \frac{1}{\xi}\left(\widetilde{G}_2' + \frac{\vec{\Delta}_\perp^2}{4m^2}\widetilde{G}_1'\right) C^-,$$

$$\mathcal{A}_3 \sim G_3 C^+ - \left(\widetilde{G}_4 + \frac{P^2}{2m^2}\widetilde{G}_1'\right) C^-,$$

$$\mathcal{A}_4 \sim G_4 C^+ - \left[\widetilde{G}_3 - \frac{1}{2\xi}\left(\widetilde{G}_2' + \frac{\Delta^2}{4m^2}\widetilde{G}_1'\right)\right] C^-$$



- $G_2 + \frac{1}{\xi}\tilde{G}_2$ continuous at $x = -\xi$
- $G_2 - \frac{1}{\xi}\tilde{G}_2$ continuous at $x = \xi$

$\hookrightarrow$ makes world a lot safer for twist-3 factorization!

quasi PDFs/TMDs

- Let $\rho_P^\Gamma(k_z, k_\perp)$ be momentum distribution of quarks
- P momentum of nucleon (in $\hat{z}$-direction)
- Γ: Dirac structure of quark bilinear ($\Gamma = \gamma^z$ for twist 2, unpol.)

$\hookrightarrow$ $q_\Gamma(x, k_\perp) \equiv \lim_{P\to\infty} P\rho_P^\Gamma(xP, k_\perp)$ 'quasi-PDF' X.Ji++

twist-3 quasi PDFs $\Gamma = \mathbb{1}$ (quark target model)

$\rho_P^{\mathbb{1}}(k_z, k_\perp) \sim \int dk_0 \frac{k^2+m^2+2p\cdot k}{[k^2-m^2]^2[(p-k)^2-\lambda^2]}$

- only den: $P\rho_P^{\mathbb{1}}(k_z, k_\perp) \stackrel{P\to\infty}{\longrightarrow} \frac{1}{(1-x)x^2}\frac{1}{\left[M^2-\frac{k_\perp^2+m^2}{x}-\frac{k_\perp^2+\lambda^2}{1-x}\right]^2}$ $\quad x = \frac{k_z}{P}$
- $2p\cdot k = p^2 + k^2 - (p-k)^2 = p^2 + k^2 - \lambda^2 - \left[(p-k)^2 - \lambda^2\right]$

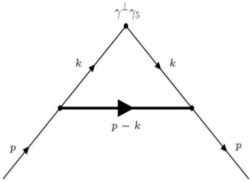
$\hookrightarrow$ contribution to $\rho_P^\Gamma(k_z, k_\perp)$ that is independent of P

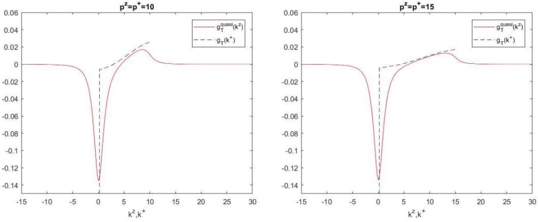
$$\rho_P^{\mathbb{1},\delta}(k_z, k_\perp) \sim \int dk^0 \frac{1}{[k^2 - m^2]^2}$$

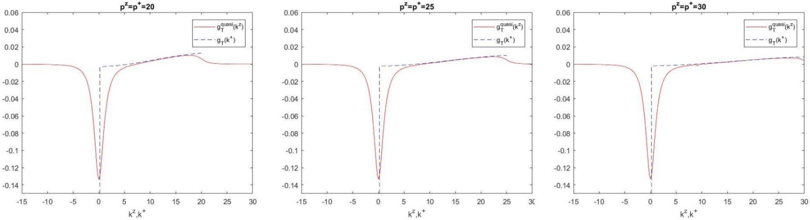
$\hookrightarrow$ corresponding quasi PDF is representation of δ function!!!!!

- some quarks 'left behind' when 'hadron' gets boosted

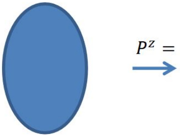
Twist -3 pdf g_T & Twist -3 quasi-pdf $g_T{}^{quasi}$ in scalar diquark model





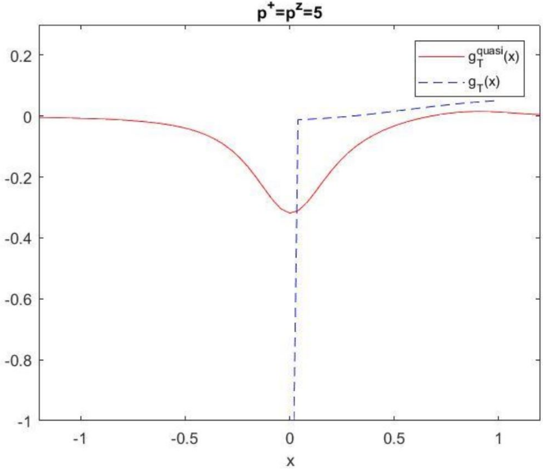


There is a momentum component in the nucleon state which does not scale as the nucleon is boosted to the infinite momentum frame.



$P^z = P^+ = 5$

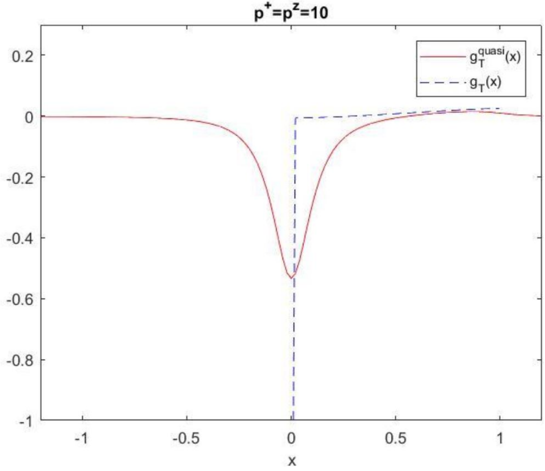
$g_T(\boldsymbol{x}),\ \ g_T{}^{quasi}(\boldsymbol{x})$

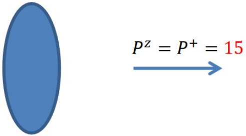


Quasi/Pseudo PDFs when $\delta(x)$ is Present

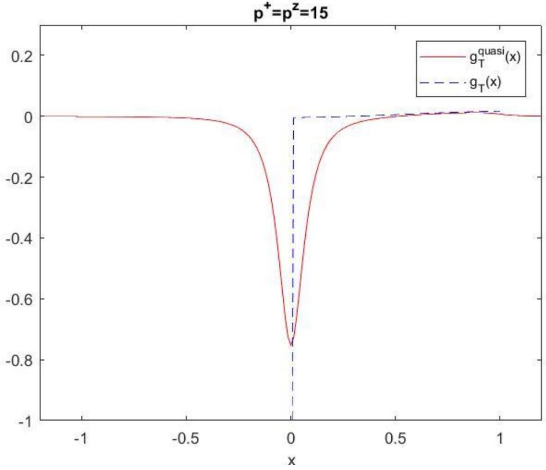
$P^z = P^+ = 10$

$g_T(\boldsymbol{x}),\ g_T{}^{quasi}(\boldsymbol{x})$

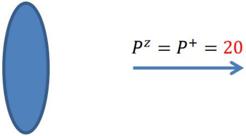




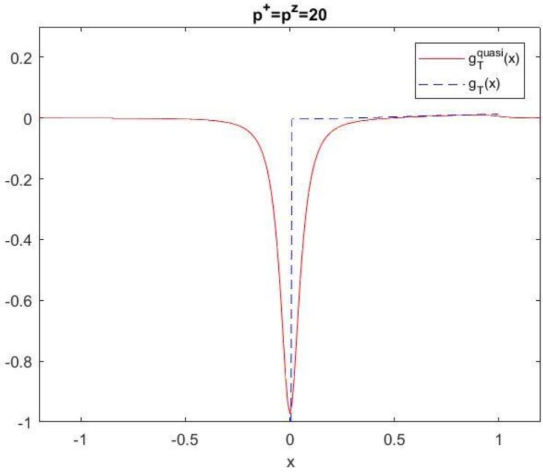
$g_T(x)$, $g_T^{quasi}(x)$

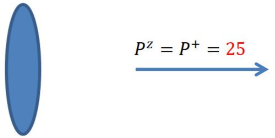


Quasi/Pseudo PDFs when $\delta(x)$ is Present



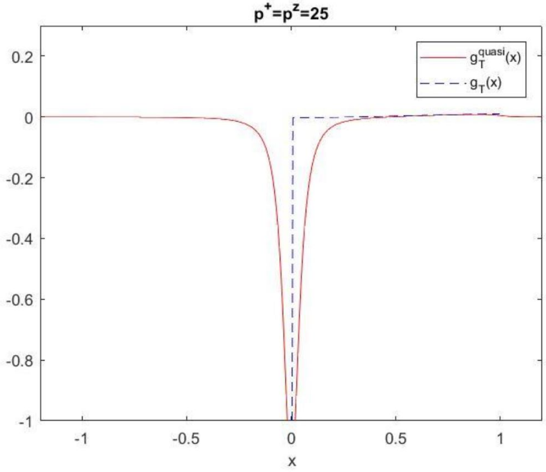
$g_T(x)$, $g_T{}^{quasi}(x)$





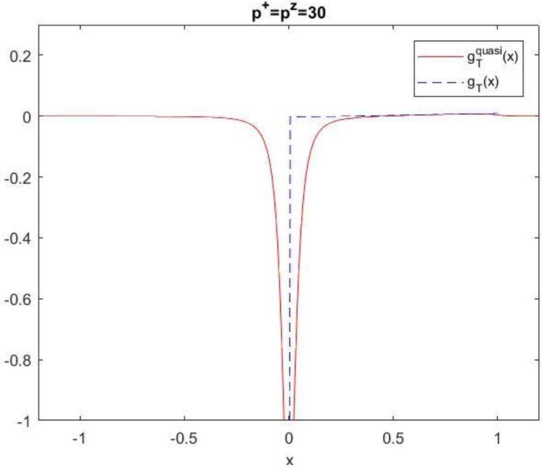
$P^z = P^+ = 25$

$g_T(x),\ g_T{}^{quasi}(x)$

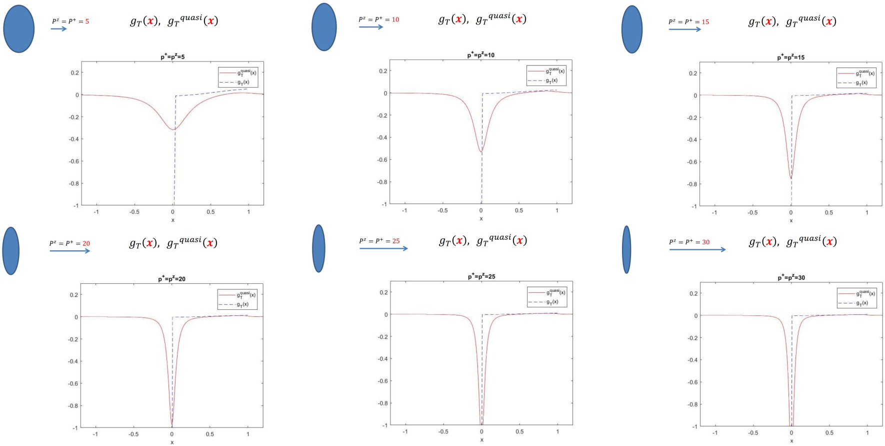


$P^z = P^+ = 30$

$g_T(x),\ \ g_T{}^{quasi}(x)$



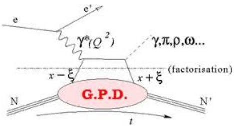
Quasi/Pseudo PDFs when $\delta(x)$ is Present



lattice:

this should be visible for all twist 3 PDFs containing potential $\delta(x)$

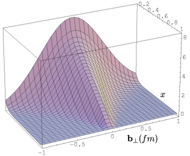
Summary



- GPDs $\xrightarrow{FT} q(x, \mathbf{b}_\perp)$ '3d imaging'
- x^2 moment of twist-3 GPDs

$\hookrightarrow$ $\bar{q}\gamma^+ F^{+\perp} q$ distribution

$\hookrightarrow$ $\perp$ force tomography



- $\delta(x)$ in twist-3 PDF

$\hookrightarrow$ discontinuities in twist-3 GPDs

- rep. of $\delta(x)$ as $\xi \to 0$
- cancel in DVCS amplitude $\sim G_2 \pm \frac{1}{\xi}\tilde{G}_2$
- individual extraction of G_2 & $\tilde{G}_2$ questionable
- some quarks 'left behind' in IMF at twist 3