Transverse Force

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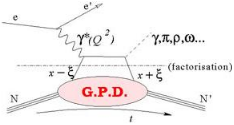
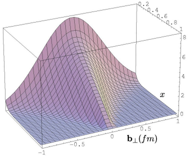
- GPDs ⟶ 3D imaging of the nucleon
- twist-3 PDFs $g_2(x)$ ⟶ ⊥ force

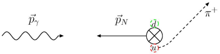
↪ twist-3 GPDs ⟶ ⊥ force tomography

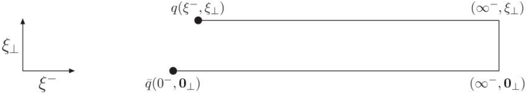
  - twist 2 GPDs ⟶⊥ imaging (of quark densities)

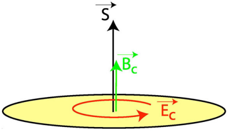
  ↪ twist 3 GPDs ⟶⊥ imaging of ⊥ forces
- Summary











MB, PRD62, 071503 (2000)

- form factors: $\stackrel{FT}{\longleftrightarrow} \rho(\vec{r})$
- $GPDs(x, \vec{\Delta})$: form factor for quarks with momentum fraction x

$\hookrightarrow$ suitable FT of $GPDs$ should provide spatial distribution of quarks with momentum fraction x

Impact Parameter Dependent Quark Distributions

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} GPD(x, 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$$

$q(x, \mathbf{b}_\perp)$ = parton distribution as a function of the separation $\mathbf{b}_\perp$ from the transverse center of momentum $\mathbf{R}_\perp \equiv \sum_{i \in q,g} \mathbf{r}_{\perp,i} x_i$

- probabilistic interpretation!
- no relativistic corrections: Galilean subgroup! (MB,2000)

$\hookrightarrow$ corollary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free from relativistic corrections (MB,2003;G.A.Miller, 2007)

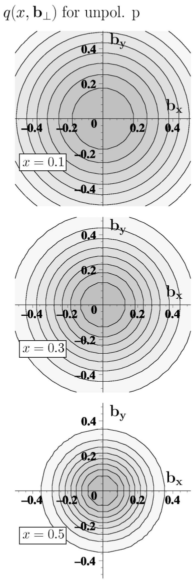
$\perp$ localized state

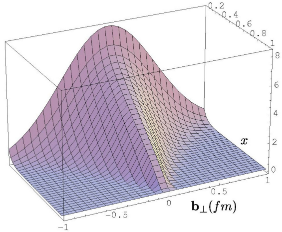
$$|\mathbf{R}_\perp = 0, p^+, \Lambda\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp |\mathbf{p}_\perp, p^+, \Lambda\rangle$$

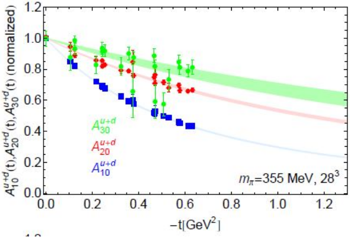
$\perp$ charge distribution (unpolarized quarks)

$$\begin{aligned}
\rho_{\Lambda'\Lambda}(\mathbf{b}_\perp) &\equiv \langle \mathbf{R}_\perp = 0, p^+, \Lambda' | \bar{q}(\mathbf{b}_\perp)\gamma^+ q(\mathbf{b}_\perp) | \mathbf{R}_\perp = 0, p^+, \Lambda\rangle \\
&= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \langle \mathbf{p}'_\perp, p^+, \Lambda' | \bar{q}(0)\gamma^+ q(0) | \mathbf{p}_\perp, p^+, \Lambda\rangle e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp - \mathbf{p}'_\perp)} \\
&= \int d^2\boldsymbol{\Delta}_\perp F_{\Lambda'\Lambda}(-\boldsymbol{\Delta}^2_\perp) e^{-i\mathbf{b}_\perp\cdot\boldsymbol{\Delta}_\perp}
\end{aligned}$$

- crucial: $\langle \mathbf{p}'_\perp, p^+, \Lambda' | \bar{q}(0)\gamma^+ q(0) | \mathbf{p}_\perp, p^+, \Lambda\rangle$ depends only on $\boldsymbol{\Delta}_\perp$
- $F_{\Lambda'\Lambda}(-\boldsymbol{\Delta}^2_\perp)$ some linear combination of F_1 & F_2 - depending on Λ, Λ'
- similar for various polarized quark densities
- similar for x-dependent densities $\longrightarrow$ GPDs



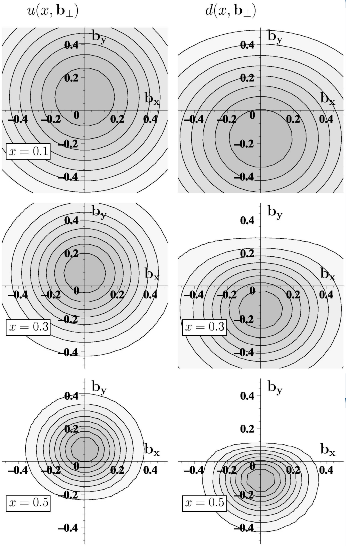




unpolarized proton

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$
- $F_1(-\mathbf{\Delta}_\perp^2) = \int dx H(x, 0, -\mathbf{\Delta}_\perp^2)$
- x = momentum fraction of the quark
- $\mathbf{b}_\perp$ relative to $\perp$ center of momentum
- small x: large 'meson cloud'
- larger x: compact 'valence core'
- $x \to 1$: active quark becomes center of momentum

$\hookrightarrow$ $\vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$



proton polarized in $+\hat{x}$ direction

no axial symmetry!

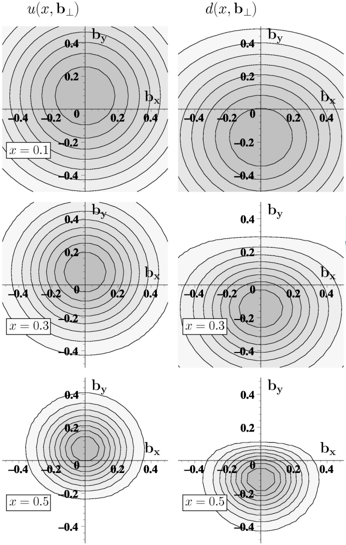
$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$
$$- \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

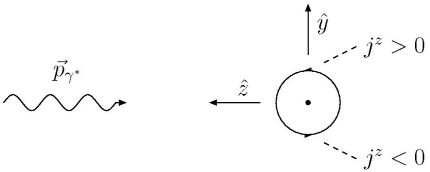
Physics: relevant density in DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from j^3

intuitive explanation

- moving Dirac particle with anomalous magnetic moment has electric dipole moment $\perp$ to $\vec{p}$ and $\perp$ magnetic moment

$\hookrightarrow$ γ^* 'sees' flavor dipole moment of oncoming nucleon





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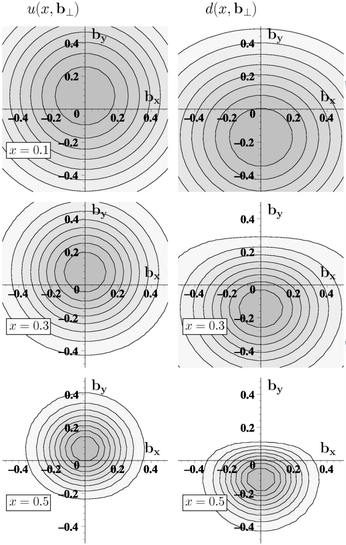

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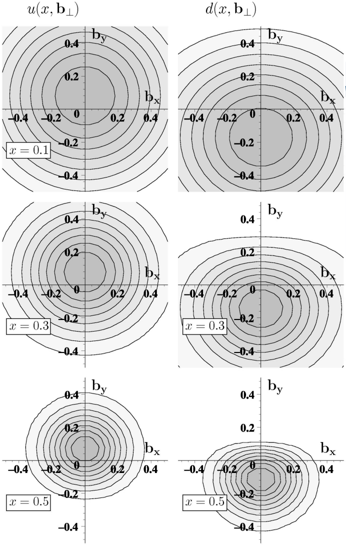
$$-\frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

sign & magnitude of the average shift

model-independently related to p/n anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x,\mathbf{b}_\perp) b_y$$

$$= \frac{1}{2M}\int dx E_q(x,0) = \frac{\kappa_q}{2M}$$



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$\kappa^p = 1.913 = \frac{2}{3}\kappa_u^p - \frac{1}{3}\kappa_d^p + \dots$

- u-quarks: $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$

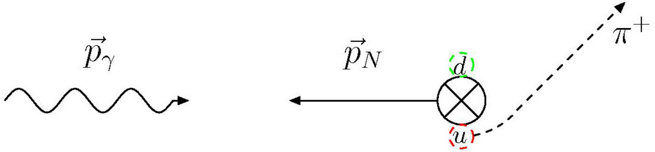
$\hookrightarrow$ shift in $+\hat{y}$ direction

- d-quarks: $\kappa_u^p = 2\kappa_n + \kappa_p = -2.033$

$\hookrightarrow$ shift in $-\hat{y}$ direction

- $\langle b_y^q \rangle = \mathcal{O}(\pm 0.2 fm)$!!!!

example: $\gamma p \to \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive final state interaction (FSI) deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction$\to$ chromodynamic lensing

$\Rightarrow$ $\kappa_p\,, \kappa_n \quad \longleftrightarrow \quad$ sign of SSA!!!!!!!! (MB,2004)

- confirmed by HERMES & COMPASS data

$d_2 \leftrightarrow$ average $\perp$ force on quark in DIS from $\perp$ pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

$\hookrightarrow$ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2} S^x} \left\langle P,S \left| \bar{q}(0)\gamma^+ g F^{+y}(0) q(0) \right| P,S \right\rangle$$

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 114502)

$$\sqrt{2} F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

matrix element defining $d_2 \leftrightarrow 1^{st}$ integration point in QS-integral

$d_2 \Rightarrow \perp$ force $\leftrightarrow$ QS-integral $\Rightarrow \perp$ impulse

sign of d_2

- $\perp$ deformation of $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ sign of d_2^q: opposite Sivers

magnitude of d_2

- $\langle F^y \rangle = -M^2 d_2 = -5 \frac{GeV}{fm} d_2$
- $|\langle F^y \rangle| \ll \sigma \approx 1 \frac{GeV}{fm} \Rightarrow d_2 = \mathcal{O}(0.01)$

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consitent with experiment (JLab,SLAC), model calculations (Weiss), and lattice QCD calculations (Göckeler et al., 2005)

Twist-3 PDFs $\longrightarrow \perp$ Force on Quarks in DIS

chirally even spin-dependent twist-3 PDF $g_2(x)$

MB, PRD 88 (2013) 114502

- $\int dx\, x^2 g_2(x) \Rightarrow \perp$ force on unpolarized quark in $\perp$ polarized target

$\hookrightarrow$ 'Sivers force'

scalar twist-3 PDF $e(x)$

MB, PRD 88 (2013) 114502

- $\int dx\, x^2 e(x) \Rightarrow \perp$ force on $\perp$ polarized quark in unpolarized target

$\hookrightarrow$ 'Boer-Mulders force'

chirally odd spin-dependent twist-3 PDF $h_2(x)$

M.Abdallah & MB, PRD94 (2016) 094040

- $\int dx\, x^2 h_2(x) = 0$

$\hookrightarrow$ $\perp$ force on $\perp$ pol. quark in long. pol. target vanishes due to parity

- $\int dx\, x^3 h_2(x) \Rightarrow$ long. gradient of $\perp$ force on $\perp$ polarized quark in long. polarized target

$\hookrightarrow$ chirally odd 'wormgear force'

force distributions

F.Aslan & MB

- use FT of twist-3 GPDs to map these forces in the $\perp$ plane (later...)

Wigner Functions (Belitsky, Ji, Yuan; Lorcé, Pasquini; Metz et al.)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+ q(\xi)|PS\rangle.$$

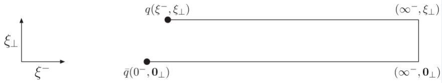
- TMDs: $f(x,\mathbf{k}_\perp) = \int d^2\mathbf{b}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$
- GPDs: $q(x,\mathbf{b}_\perp) = \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$
- $L_z = \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)(b_x k_y - b_y k_x)$
- need to include Wilson-line gauge link $\mathcal{U}_{0\xi} \sim \exp\left(i\frac{g}{\hbar}\int_0^\xi \vec{A}\cdot d\vec{r}\right)$ to connect 0 and ξ

$\hookrightarrow$ crucial for SSAs in SIDIS et al.

gauge invariance

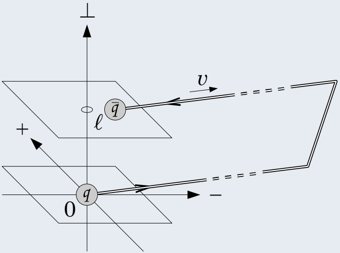
- need Wilson line gauge link
- light-like staple for FSI in DIS

Light-Cone Staple for $\mathcal{U}_{0\xi}$



'light-cone staple' yields $\mathcal{L}_{Jaffe-Manohar}$

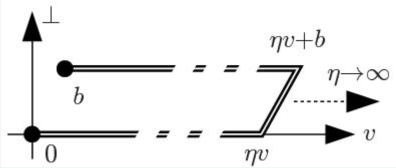
challenge



- TMDs/Wigner functions relevant for SIDIS require (near) light-like Wilson lines
- on Euclidean lattice, all distances are space-like

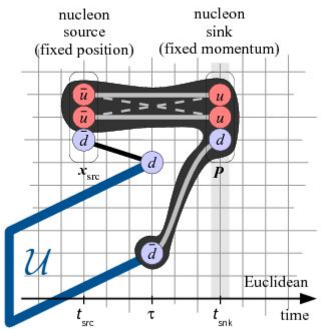
TMDs in lattice QCD

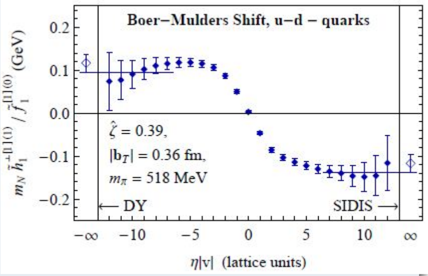
M.Engelhardt, B.Musch, P.Hägler, A.Schäfer,..



- calculate space-like staple-shaped Wilson line pointing in $\hat{z}$ direction; length $L \to \infty$
- momentum projected nucleon sources/sinks
- remove IR divergences by considering appropriate ratios

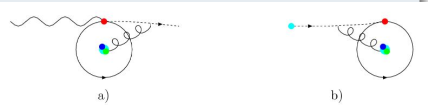
↪ extrapolate/evolve to $P_z \to \infty$

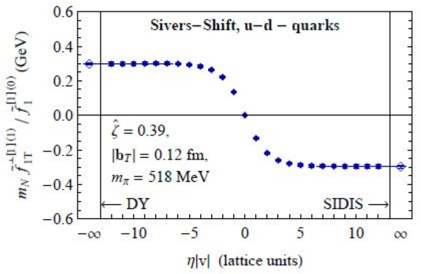




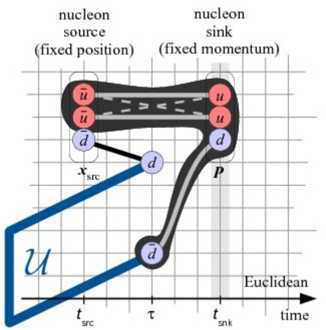
$f_{1T,SIDIS}^{\perp} = -f_{1T,DY}^{\perp}$ (Collins)

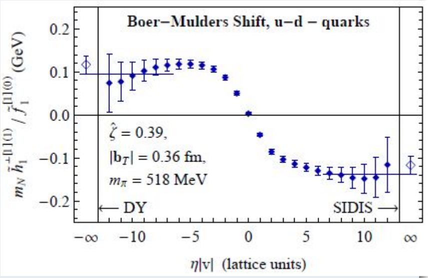
M.Engelhardt,P.Hägler, B.Musc, A.Schäfer,..





$f_{1T}^{\perp}(x, \mathbf{k}_\perp)$ is $\mathbf{k}_\perp$-odd term in quark-spin averaged momentum distribution in $\perp$ polarized target





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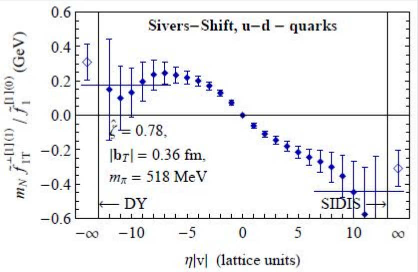

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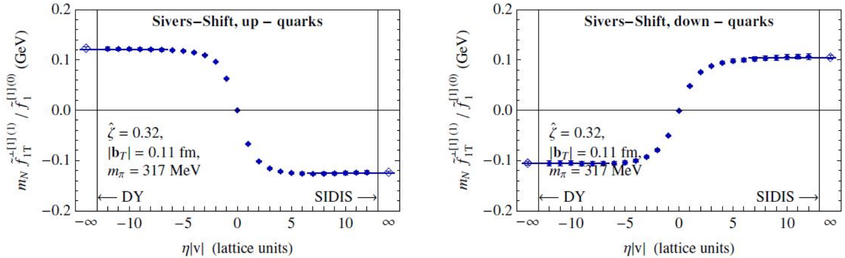
Flavor Decomposition (no disconnected diagrams!)



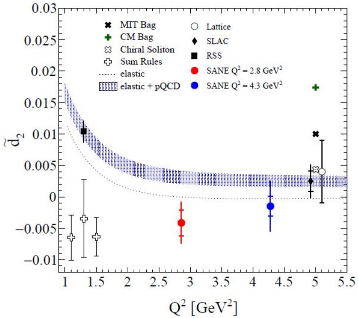
NOTE: Up-quark data are normalized to give the contribution from 1 up quark

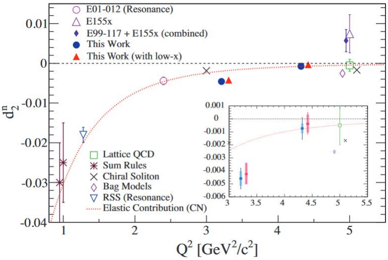
- clearly opposite sign Sivers
- 'Force' = slope at origin (in IMF..)

↪ several hundred $\frac{MeV}{\text{fm}}$
also opposite sign u vs. d: $d_2^d \approx -\frac{1}{2} d_2^u$

- $d_2^p - d_2^n = \frac{1}{3}\left(d_2^u - d_2^d\right) \approx \frac{1}{2} d_2^u$
- $d_2^p + d_2^n = \frac{5}{9}\left(d_2^u + d_2^d\right) \approx \frac{5}{18} d_2^u$

↪ d_2^n smaller than d_2^p and opposite sign





- d_2^p (SANE): sign inconsistent with sign of Sivers!
- d_2 matrix element of $\bar{q}(0)\gamma^+F^{+\perp}(0)q(0)$ (local)
- SSA (Sivers) $\sim$ matrix element of QS operator
  $\int_0^\infty d\zeta^- \bar{q}(0)\gamma^+F^{+\perp}(\zeta^-)q(0)$ (nonlocal)

$\hookrightarrow$ different signs (Sivers vs. d_2) requires sign change as function of ζ^- in QS matrix element

⊥ force distribution (unpolarized quarks)

$$F^i_{\Lambda'\Lambda}(\mathbf{b}_\perp) \equiv \langle \mathbf{R}_\perp = 0, p^+, \Lambda' | \bar{q}(\mathbf{b}_\perp)\gamma^+ g F^{+i}(\mathbf{b}_\perp) q(\mathbf{b}_\perp) | \mathbf{R}_\perp = 0, p^+, \Lambda\rangle$$
$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \langle \mathbf{p}'_\perp, p^+, \Lambda | \bar{q}(0)\gamma^+ g F^{+i}(0) q(0) | \mathbf{p}_\perp, p^+, \Lambda\rangle e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$

Form factors of qgq correlator

$$\langle p', \lambda' | \bar{q}(0)\gamma^+ g F^{+i}(0) q(0) | p, \lambda\rangle = \bar{u}(p', \lambda') \left[\frac{P^+}{M}\gamma^+ \frac{\Delta^i}{M} F_{FT,1}(t) + \frac{P^+}{M} i\sigma^{+i} F_{FT,2}(t) + \frac{P^+}{M}\frac{\Delta^i}{M}\frac{i\sigma^{+\Delta}}{M} F_{FT,3}(t) + \frac{P^+}{M}\frac{\Delta^+}{M}\frac{i\sigma^{i\Delta}}{M} F_{FT,4}(t) \right] u(p, \lambda)$$

crucial:

- for ${p^+}' = p^+$, $\langle p', \lambda' | \bar{q}(0)\gamma^+ g F^{+i}(0) q(0) | p, \lambda\rangle$ only depends on $\mathbf{\Delta}_\perp$

↪ similar to ⊥ charge density ...

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$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \langle \mathbf{p}'_\perp, p^+, \Lambda | \bar{q}(0)\gamma^+ g F^{+i}(0) q(0) | \mathbf{p}_\perp, p^+, \Lambda\rangle e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$

Form factors of qgq correlator

$$\langle p', \lambda' | \bar{q}(0)\gamma^+ g F^{+i}(0) q(0) | p, \lambda\rangle = \bar{u}(p', \lambda') \Big[\frac{P^+}{M}\gamma^+ \frac{\Delta^i}{M} F_{FT,1}(t) + \frac{P^+}{M} i\sigma^{+i} F_{FT,2}(t)$$
$$+ \frac{P^+}{M}\frac{\Delta^i}{M}\frac{i\sigma^{+\Delta}}{M} F_{FT,3}(t) + \frac{P^+}{M}\frac{\Delta^+}{M}\frac{i\sigma^{i\Delta}}{M} F_{FT,4}(t)\Big] u(p, \lambda$$

$F_{FT,1}$

- unpolarized target
- axially symmetric 'radial' force

$F_{FT,2}$

- $\perp$ polarized target; force $\perp$ to target spin

$\hookrightarrow$ spatially resolved Sivers force

$\perp$ force distribution (unpolarized quarks)

$$F^i_{\Lambda'\Lambda}(\mathbf{b}_\perp) \equiv \langle \mathbf{R}_\perp = 0, p^+, \Lambda' | \bar{q}(\mathbf{b}_\perp)\gamma^+ g F^{+i}(\mathbf{b}_\perp) q(\mathbf{b}_\perp) | \mathbf{R}_\perp = 0, p^+, \Lambda\rangle$$
$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \langle \mathbf{p}'_\perp, p^+, \Lambda | \bar{q}(0)\gamma^+ g F^{+i}(0) q(0) | \mathbf{p}_\perp, p^+, \Lambda\rangle e^{i\mathbf{b}_\perp \cdot (\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$

Form factors of qgq correlator

$$\langle p', \lambda' | \bar{q}(0)\gamma^+ g F^{+i}(0) q(0) | p, \lambda\rangle = \bar{u}(p', \lambda') \left[\frac{P^+}{M}\gamma^+ \frac{\Delta^i}{M} F_{FT,1}(t) + \frac{P^+}{M} i\sigma^{+i} F_{FT,2}(t) + \frac{P^+}{M}\frac{\Delta^i}{M}\frac{i\sigma^{+\Delta}}{M} F_{FT,3}(t) + \frac{P^+}{M}\frac{\Delta^+}{M}\frac{i\sigma^{i\Delta}}{M} F_{FT,4}(t) \right] u(p, \lambda)$$

$F_{FT,3}$

- tensor type force
- similar to charged particle flying through magnetic dipole field

$F_{FT,4}$

- no conmtribution for $\Delta^+ = 0$

form factors of $\bar{q}\gamma^\rho F^{\mu\nu} q$

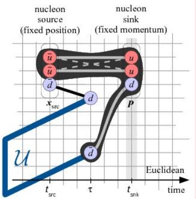
$$
\begin{aligned}
\langle p'|\bar{q}\gamma^\rho F^{\mu\nu} q|p\rangle \;=\; & \bar{u}(p')\left[\frac{P^\rho\left(P^\mu\Delta^\nu - P^\nu\Delta^\mu\right)}{M^3}F_{FT,1} + \frac{g^{\mu\rho}\Delta^\nu - g^{\nu\rho}\Delta^\mu}{M}F_{FT,2}\right. \\
+ \; & \frac{\varepsilon^{\rho\mu\nu\Delta}}{M}\gamma_5 F_{FT,3} + \frac{P^\rho}{M}i\sigma^{\mu\nu}F_{FT,4} + \frac{P^\mu i\sigma^{\nu\rho} - P^\nu i\sigma^{\mu\rho}}{M}F_{FT,5} \\
+ \; & \frac{\left(P^\mu\Delta^\nu - P^\nu\Delta^\mu\right)i\sigma^{\rho\Delta}}{M^3}F_{FT,6} + \Delta^\rho\frac{P^\mu i\sigma^{\nu\Delta} - P^\nu i\sigma^{\mu\Delta}}{M^3}F_{FT,7} \\
+ \; & \left. P^\rho\frac{\Delta^\mu i\sigma^{\nu\Delta} - \Delta^\nu i\sigma^{\mu\Delta}}{M^3}F_{FT,8}\right]u(p)
\end{aligned}
$$

(notation slightly different from above)

determining $F_{FT,i}$

- match with x^2 moments of twist-3 GPDs (minus WW parts) *in progress*
- lattice QCD: fit to nonforward matrix elements of 'the force'-operator *planned by both NMSU and Adelaide groups*

the force operator (modulo operator mixing..)



- form factor with quark denstity involving Wilson line staple
- take derivative w.r.t. staple length at length =0

$\hookrightarrow$ $\langle p'|\bar{q}\Gamma F^{zy} q|p\rangle$

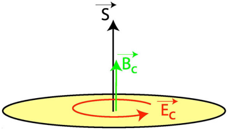
- $\langle p'|\bar{q}\gamma^+ F^{zy} q|p\rangle =$
  $\langle p'|\bar{q}\gamma^+ F^{+y} q|p\rangle - \langle p'|\bar{q}\gamma^+ F^{-y} q|p\rangle$

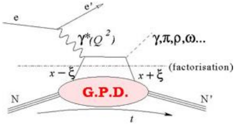
$\hookrightarrow$ both color-electric and magnetic force

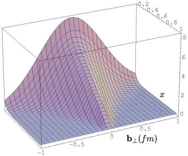
- GPDs $\xrightarrow{FT} q(x, \mathbf{b}_\perp)$ '3d imaging'
- x^2 moment of twist-3 PDFs $\rightarrow$ force
- x^2 moment of twist-3 GPDs

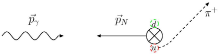
$\hookrightarrow$ $\bar{q}\gamma^+ F^{+\perp} q$ distribution

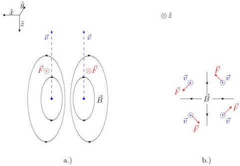
$\hookrightarrow$ $\perp$ force tomography











Summary

- GPDs $\xrightarrow{FT} q(x, \mathbf{b}_\perp)$ '3d imaging'
- x^2 moment of twist-3 PDFs $\rightarrow$ force
- x^2 moment of twist-3 GPDs

$\hookrightarrow$ $\bar{q}\gamma^+ F^{+\perp} q$ distribution

$\hookrightarrow$ $\perp$ force tomography