Towards lattice calculations of parton distributions

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and A. Vladimirov

Outline

- Second moment of the pion distribution amplitude

  RQCD, work in progress

- Pion distribution amplitude from Euclidean correlation functions

  RQCD, 1709.04325 and 1807.06671

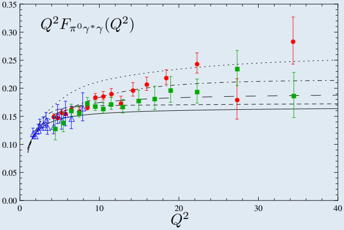
- Renormalons and power corrections to parton quasi-distributions

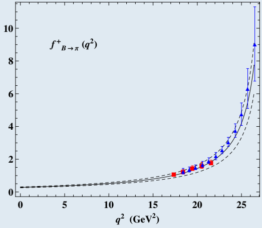
  VB, A. Vladimirov, J.-H. Zhang, in preparation

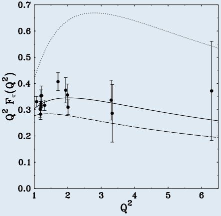
Pion distribution amplitude

A. Radyushkin '77, hep-ph/0410276

- Simplest example of a parton distribution
- Many applications: $\gamma^* \to \gamma\pi(\eta,\eta')$, pion electroproduction, $B \to \pi\ell\nu$, $B \to \pi\pi$ etc.







V.B. *et al.* (RQCD Collaboration), PRD **92** (2015) 014504

$$\langle \xi^2 \rangle^{\overline{\rm MS}} = \int_0^1 du\,(2u-1)^2 \phi_\pi(u,\mu) = 0.2361(41)(39)(?)$$

$$a_2^{\overline{\rm MS}} = \frac{7}{18} \int_0^1 du\, C_2^{3/2}(2u-1)\,\phi_\pi(u,\mu) = 0.1364(154)(145)(?)$$

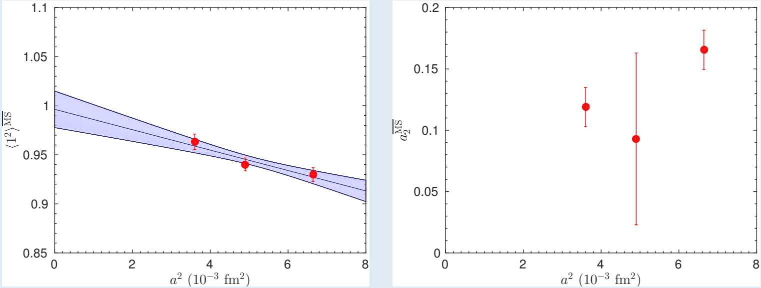
$\mu = 2$ GeV

(?) Continuum extrapolation

$$O^-_{\rho\mu\nu}(x) = \bar{d}(x)\left[\overleftarrow{D}_{(\mu}\overleftarrow{D}_{\nu} - 2\overleftarrow{D}_{(\mu}\overrightarrow{D}_{\nu} + \overrightarrow{D}_{(\mu}\overrightarrow{D}_{\nu}\right]\gamma_{\rho)}\gamma_5\, u(x)\,,$$

$$O^+_{\rho\mu\nu}(x) = \bar{d}(x)\left[\overleftarrow{D}_{(\mu}\overleftarrow{D}_{\nu} + 2\overleftarrow{D}_{(\mu}\overrightarrow{D}_{\nu} + \overrightarrow{D}_{(\mu}\overrightarrow{D}_{\nu}\right]\gamma_{\rho)}\gamma_5\, u(x)$$

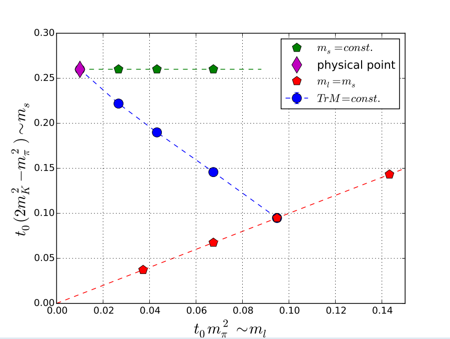
RQCD, 2015:



2015: $N_f = 2$, $\quad a = 0.06 - 0.08$ fm
2018: $N_f = 2+1$, $a = 0.04 - 0.08$ fm

CLS/RQCD simulation strategy

Simulate along several trajectories in the $m_s, m_{u,d}$ plane enabling Gell-Mann–Okubo/SU(3) and SU(2) χPT extrapolations

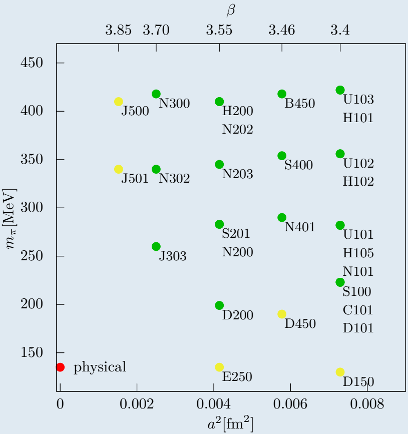


here:

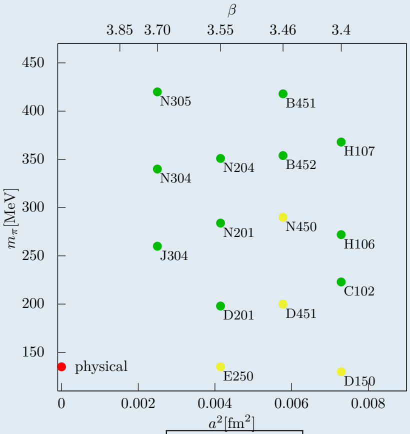
$$\langle \xi^2 \rangle_X = [\xi_0^3 + \bar{A}\bar{m} + A_x \delta m^2] \times [1 + a(c_0 + \bar{c}\bar{m} + c_x \delta m^2)] \,, \qquad X = \pi, K$$

$SU(3)$ NLO ChPT and $\mathcal{O}(a)$ discretization effects (8 parameters)

CLS ensemble overview



$m_s + 2m_\ell =$ const



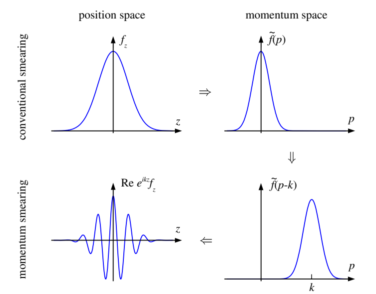
$\widehat{m}_s \approx$ const

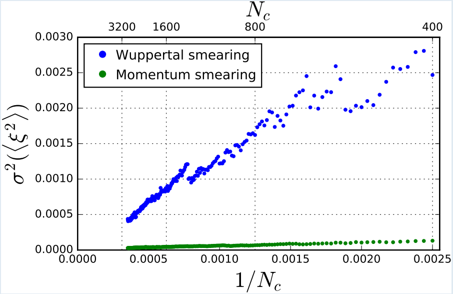
E: $192 \cdot 96^3$, J: $192 \cdot 64^3$, D: $128 \cdot 64^3$, N: $128 \cdot 48^3$, C: $96 \cdot 48^3$,
S: $128 \cdot 32^3$, H: $96 \cdot 32^3$, B: $64 \cdot 32^3$, U: $128 \cdot 24^3$.

$\exists$ additional ensembles with $m_s = m_\ell$.

Momentum smearing

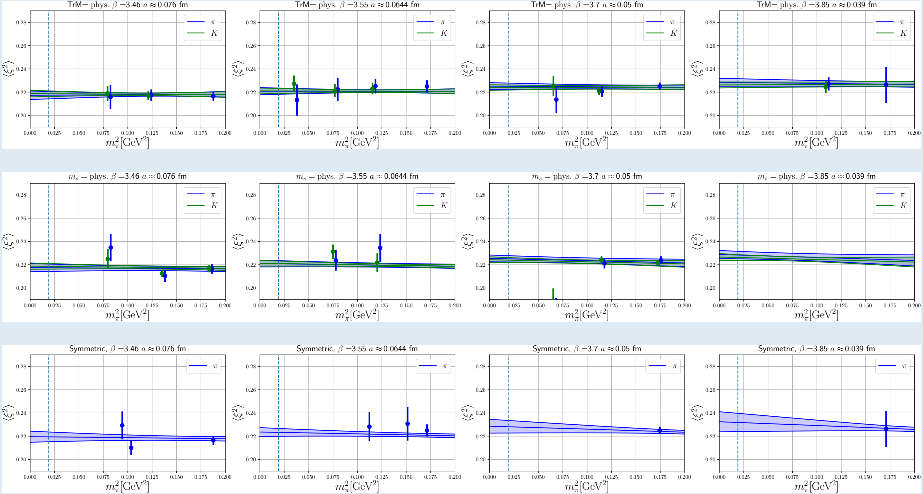
G. Bali, B. Lang, B. Musch, A. Schäfer,1602.05525





Right: Squared error as a function of the statistics;
Ensemble "H105" for $\vec{n}_{\vec{p}} = (110), (101), (011)$

- Momentum smearing: 2 inversions for each momentum (6 inversions)
- Wuppertal smearing: 1 inversion + additional Fourier sums


TrM= phys. β =3.46 a ≈ 0.076 fm
TrM= phys. β =3.55 a ≈ 0.0644 fm
TrM= phys. β =3.7 a ≈ 0.05 fm
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Symmetric, β =3.46 a ≈ 0.076 fm
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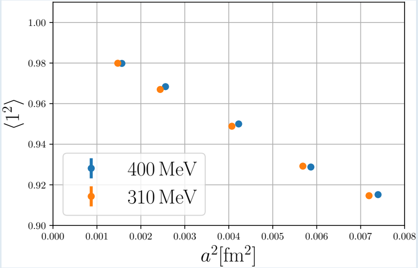
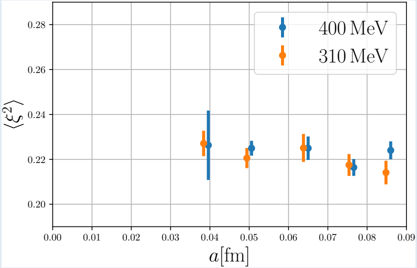
Symmetric, β =3.7 a ≈ 0.05 fm
Symmetric, β =3.85 a ≈ 0.039 fm
π
K
⟨ξ²⟩
m_π²[GeV²]

RQCD, 2018: (preliminary)





- preliminary results (at 2 GeV):

$$\langle \xi^2 \rangle_\pi^{\overline{\mathrm{MS}}} = 0.2399(64)\,, \qquad a_{2\pi}^{\overline{\mathrm{MS}}} = 0.116(19)$$

$$\langle \xi^2 \rangle_K^{\overline{\mathrm{MS}}} = 0.2343(43)\,, \qquad a_{2K}^{\overline{\mathrm{MS}}} = 0.100(13)$$

- a few configuration sets still running
- renormalization not finalized, error not included

Summary: second moment

- work in progress
  - — Accuracy goal: 3% for $\langle \xi^2 \rangle^{\overline{\rm MS}}$; 15% for $a_2^{\overline{\rm MS}}$
  - — Will also have K-meson first moment
  - — Hope to complete before the end of the year

DAs/PDFs/GPDs from custom-made lattice (Euclidean) correlation functions

- Going beyond the second moment is not feasible:
  - — mixing with lower-dimensional operators
  - — adding more derivatives deteriorates signal-to-noise ratio

- General idea: perturbative factorization of Euclidean correlation functions

$$\langle H(p)|J_1(z)J_2(-z)|H(p)\rangle = C(z^2, p\cdot z;\mu_F)\otimes P(p\cdot z;\mu_F) \qquad z^2<0$$

  - —factorization in terms of PDFs is done in continuum (in $\overline{MS}$)
  - — small z necessary for factorization (and suppresses higher-twists)
  - — large $p\cdot z$ is necessary as a lever-arm on accessible momentum fractions
  - — additional *renormalization* factors may occur

- (Collinear) factorization in position space: light-ray operator product expansion

Zavialov '76; Balitsky, Braun '89-'91

- Example: *Braun, Müller, EPJC, **55**, 349 (2008)*

$$\langle 0|T\{\bar q(z)\gamma_\mu q(z)\bar q(-z)\gamma_\nu q(-z)\}\pi^0(p)\rangle \;=\; -\frac{5i}{9}f_\pi\epsilon_{\mu\nu\rho\sigma}\frac{z^\rho p^\sigma}{8\pi^2 z^4}T(p\cdot z,z^2)$$

$$T(p\cdot z,z^2) \;=\; \int_0^1 du\, e^{i(2u-1)p\cdot z} H(u,\mu_F^2 z^2,\alpha_s(\mu_F))\,\phi_\pi(u,\mu_F) \;+\; \mathcal{O}(z^2)$$

- Many alternatives possible, e.g. *Ji, PRL **110**, 262002 (2013); Ma, Qiu, PRL **120**, 022003 (2018)*

$$\langle 0|T\{\bar q(z)\not z\gamma_5[z,-z]q(-z)\}\pi^0(p)\rangle = if_\pi(pz)\int_0^1 du\, e^{i(2u-1)pz}\tilde H(u,\mu_F^2 z^2,\alpha_s(\mu_F))\phi_\pi(u,\mu_F) + \mathcal{O}(z^2)$$

- In all cases $H = 1 + c\alpha_s \ln z^2\mu^2$
- Need large hadron momentum for kinematic coverage

What is better?

- Propagator: Physical observable, $\vec z$ direction arbitrary, extra handle from spinor structure
- Wilson line: cheaper, $\vec z = (0,0,z)$, no extra spinor structure, nonlocal RG factors

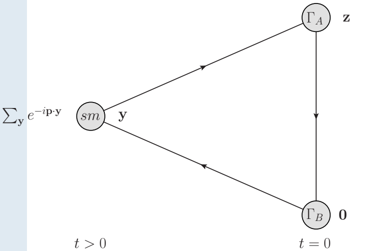
Can one keep systematic errors in such studies under control?

RQCD exploratory study: 1709.04325 and 1807.06671

- We follow *Braun, Müller, EPJC,* **55**, *349 (2008)* with generic currents

$$\langle 0|T\{\bar{q}(z/2)\Gamma_A q(z/2)\bar{q}(-z/2)\Gamma_B q(-z/2)\}\pi^0(p)\rangle \sim T_{AB}(p\cdot z, z^2)$$

- $\Gamma_A, \Gamma_B \to \gamma_\mu, \gamma_\mu\gamma_5, \gamma_5, \mathbb{I}$



Tree level result:

$$T(p\cdot z, z^2) = F_\pi \frac{p\cdot z}{2\pi^2 z^4}\Phi_\pi(p\cdot z),$$

with the position-space DA

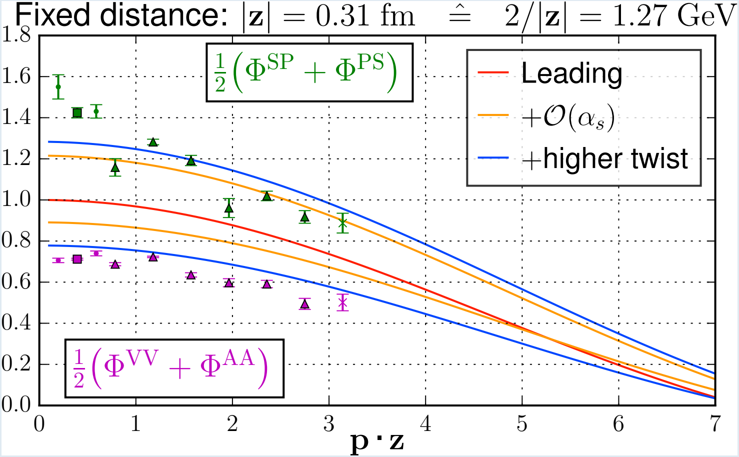
$$\Phi_\pi(p\cdot z) = \int_0^1 du\, e^{i(u-1/2)p\cdot z}\phi_\pi(u).$$

- QCD factorization

$$T_{AB}(p\cdot z, z^2) \; = \; \int_0^1 du\, e^{i(2u-1)p\cdot z} H_{AB}(u, \mu_F^2 z^2, \alpha_s(\mu_F))\, \phi_\pi(u, \mu_F) \; + \; \mathcal{O}(z^2)$$

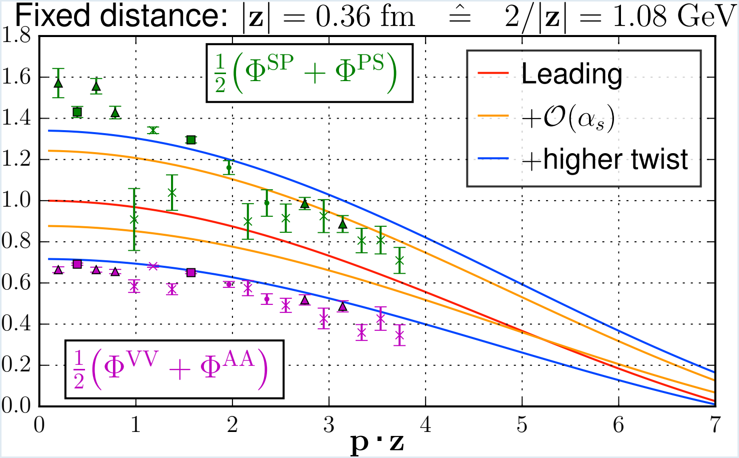
- Take into account $H_{AB}(u, \ldots)$ to NLO and twist-4 corrections $\mathcal{O}(z^2)$

Exploring universality and higher-twist effects, 1807.06671



$N_f = 2$, $32^3 \times 64$, $a \simeq 0.071$ fm, $m_\pi \simeq 295$ MeV

Exploring universality and higher-twist effects, 1807.06671



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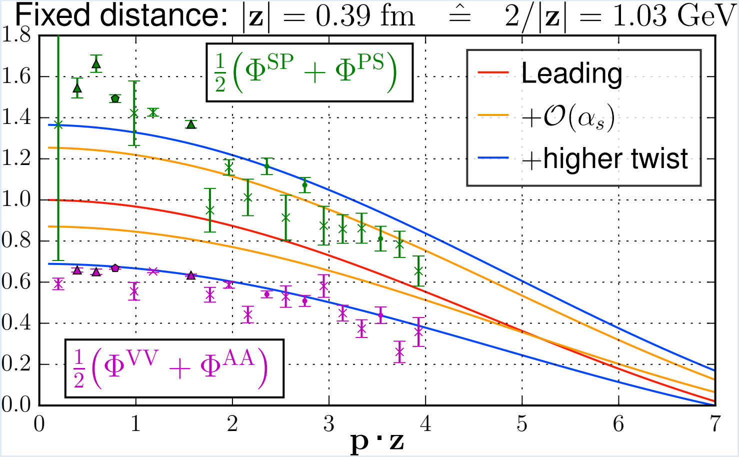

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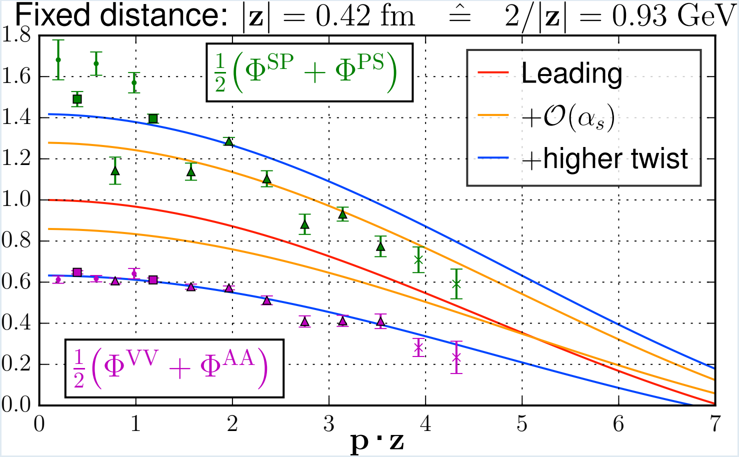

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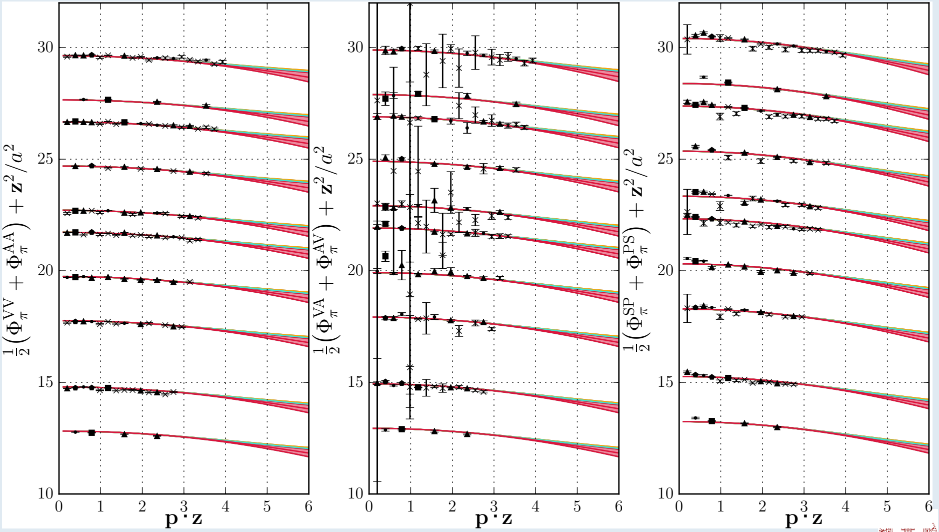


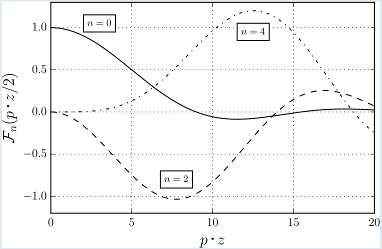
$N_f = 2, \quad 32^3 \times 64, \quad a \simeq 0.071$ fm, $\quad m_\pi \simeq 295$ MeV

What can be learnt about the pion DA from these data?

Gegenbauer expansion

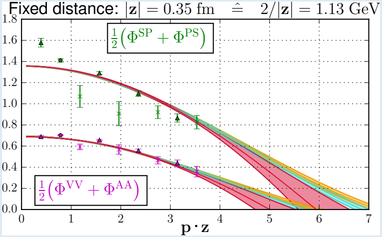
$$\phi_\pi(u,\mu) = 6u(1-u)\sum_{n=0}^{\infty} a_n^\pi(\mu) C_n^{3/2}(2u-1)$$

$$\Phi_\pi(pz,\mu) = \sum_{n=0}^{\infty} a_n^\pi(\mu)\mathcal{F}_n(pz/2)$$

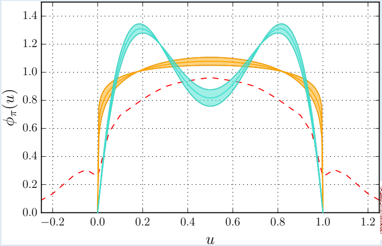


Dashes: pion DA by the quasi-distribution method, Chen et.al., 1712.10025 ⟶

with data for $pz \lesssim 3$, e.g.



cannot distinquish between



Higher-twist effects

- The fit returns "reasonable" results for $a_2(2\text{ GeV}) \sim 0.2 - 0.3$
  uncorrelated with the higher-twist parameter δ_π^2

$$\langle 0|\bar{u}(0)\gamma^\rho ig\widetilde{G}_{\rho\mu}u(0)|\pi^0(p)\rangle = p_\mu F_\pi \delta_2^\pi; \quad 0.20\text{ GeV}^2 \le \delta_2^\pi \le 0.25\text{ GeV}^2$$

- First determination in lattice QCD

Summary BM approach

- DAs from customized Euclidean correlation functions
  - — We presented a proof of concept
  - — In 1807.06671 a new algorithm implemented that enables smaller statistical errors
  - — For $2/|z| \gtrsim 1\,\text{GeV}$ need $|\vec{p}| \gtrsim 4\,\text{GeV}$ to reach "Ioffe times" $|p \cdot z|$ large enough to discriminate between different shapes
  - — Study of several current combinations allows to quantify higher-twist effects
- In future
  - — Repeat on a sample of "fine" CLS configurations to push for higher pz
  - — Full NNLO factorization in continuum
  - — In a long term, expand to PDFs
  - — ? discretisation errors $\sim ap$

Power corrections to non-standard observables

Lattice QCD can simulate a large variety of Euclidean correlation functions at large momenta. Whereas BM strategy employed by RQCD may be viewed as a "standard candle", there are many more options:

- parton quasi-distributions — *Ji, PRL* **110***, 262002 (2013)*
- parton pseudo-distributions — *Radyushkin, PRD* **96***, 034025 (2017)*
- generic — "lattice cross sections" — *Ma, Qiu, PRL* **120***, 022003 (2018)*

Exploratory studies demonstrate that statistical error in such calulations can be made sufficiently small, but what about systematic errors, both lattice and continuum?

— Power corrections to non-standard observables? —

Renormalons: Concept

M. Beneke, Phys.Rept. 317 (1999)
M. Beneke, V. Braun, hep-ph/0010208

- Leading twist calculation "knows" about the necessity to add a power correction

  Example:

$$F_2(x,Q^2) = 2x\int_x^1 \frac{dy}{y} C_2(y, Q^2/\mu^2) q(\tfrac{x}{y},\mu^2) + \frac{1}{Q^2} D_2(x)$$

$$C_2(y) = \delta(1-y) + \sum_{n=0}^{\infty} c_n \alpha_s^{n+1}, \qquad \alpha_s = \alpha_s(\mu)$$

Cut-off scheme

Imagine the separation between CFs and MEs is done using exlicit cutoff at $|k| = \mu$.
CFs will be modified compared to usual calculation by terms $\sim \mu^2/Q^2$

$$C_2(y)|^{\rm cut} = \delta(1-y) + \sum_{n=0}^{\infty} c_n \alpha_s^{n+1} - \frac{\mu^2}{Q^2} d_2(x) + \mathcal{O}\left(\frac{\mu^4}{Q^4}\right)$$

The dependence on μ must cancel:

- Logarithmic terms $\ln Q^2/\mu^2$ in CFs against μ-dependence in PDFs
- Power-terms μ^2/Q^2 against the higher-twist contributions

This means that $D_2(x)$ *in the cutoff scheme* must have the form

$$D_2(x) = \mu^2 2x \int_x^1 \frac{dy}{y} d_2(x) q(\tfrac{x}{y}) + \delta D_2(x)$$

— related to quadratic UV divergences in matrix elements of twist-4 operators (in this scheme!)

Dimensional regularization

- In dim.reg. power-like terms in the CFs do not appear. Instead, the coefficients c_k (e.g., in $\overline{\text{MS}}$) diverge factorially with the order k

— The factorial divergence implies that the sum of the pert. series is only defined to a power accuracy and this ambiguity (renormalon ambiguity) must be compensated by adding a non-perturbative higher–twist correction

— Detailed analysis [Beneke:2000kc]: the asymptotic large-order behavior of the coefficients (the renormalons) is in one-to-one correspondence with the sensitivity to extreme (small or large) loop momenta

— Infrared renormalons in the l.t. CF are compensated by ultraviolet renormalons in the MEs of twist-four operators. At the end the same picture re-appears: only the details depend on the factorization method

Deep inelastic scattering

Quadratic term in μ is spurious since its sole purpose is to cancel the similar contribution to the CF $\Rightarrow$ does not contribute to any physical observable.

— Assume that the "true" twist four term is of the same order, get a **renormalon model**

$$D_2(x) = \varkappa \Lambda^2_{\rm QCD} 2x \int_x^1 \frac{dy}{y} d_2(x) q(\tfrac{x}{y})\,, \qquad \varkappa = \mathcal{O}(1)$$

One-loop result:

$$d_2^{(q)} = -\frac{4}{[1-x]_+} + 4 + 2x + 12x^2 - 9\delta(1-x) - \delta'(1-x)$$

$$d_L^{(q)} = 8x^2 - 4\delta(1-x)$$

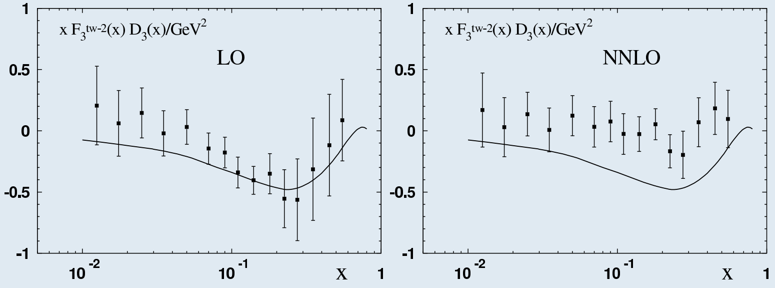
Main conclusions:

- ratio twist-4/twist-2 is target-independent (if assumption correct)
- enhancement at $x \to 1$:

$$\left[\frac{\Lambda^2}{Q^2(1-x)}\right]^n$$

Is $\varkappa$ universal? E.g. the same for F_L and F_2? Dokshitzer, Webber: $\varkappa \to \bar{\alpha}_0$ universal nonperturbative coupling

Example: twist-4 correction to $F_3(x,Q^2)$ from CCFR data:



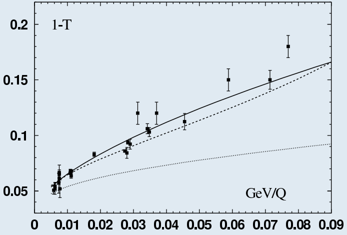
Kataev et al., hep-ph/9706534

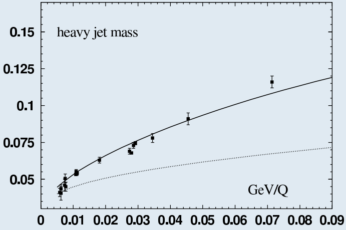
Hadronic event shape variables in e^+e^- annihilation

$$\langle S\rangle = \int d\mathsf{PS}[p_i]|\mathcal{M}_{q\bar{q}g}|^2 S(p_i)$$

e.g. thrust $S = 1 - T$

$$\langle 1-T\rangle = t_1\alpha_s(\mu) + \left[t_2^{(0)} + t_2^{(1)}\ln\frac{\mu^2}{Q^2}\right]\alpha_s^2(\mu) + \varkappa_T\frac{\Lambda_{\mathrm{T}}}{Q} + \mathcal{O}(1/Q^2)$$





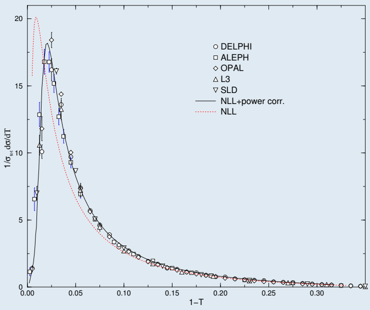
Dotted lines: NLO, solid: with $\frac{\Lambda}{Q}$, dashed: NLO with $\mu = 0.07Q$.
Universality $\Lambda_{\mathrm{thrust}} \simeq \Lambda_{\mathrm{jet\,mass}}$ works to 20%.

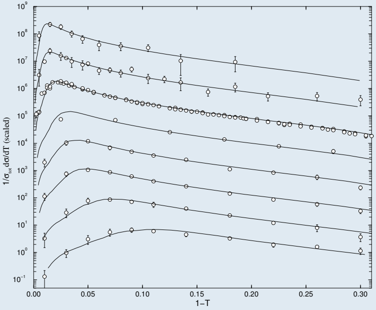
[The JADE Collaboration] EPJC 1, 461 (1998)

Event shape distributions

$$\frac{d\sigma}{dt}(t) \mapsto \frac{d\sigma}{dt}(t - \Lambda/Q) + \mathcal{O}\left(1/(tQ)^2\right)$$

If $t \sim \Lambda/Q$ all terms in $1/(tQ)^k$ have to be resummed — "shape function"





— fitted for $Q = 91.2$ GeV, used for $Q = 14, 22, 35, 44, 55, 91, 133, 161$ GeV:

G.P. Korchemsky, hep-ph/9806537

Quasi-distributions (1)

$z^\mu \mapsto z v^\mu, \; z \in \mathbb{R}$

- Ioffe-time quasi-distributions

$$\langle N(p)|\bar{q}(zv)[zv,0]\not{z}q(0)|N(p)\rangle = 2(pz)\mathcal{I}^{\parallel}(z^2v^2, pzv)$$
$$\langle N(p)|\bar{q}(zv)[zz,0]\not{\epsilon}q(0)|N(p)\rangle = 2(p\epsilon)\mathcal{I}^{\perp}(z^2v^2, pzv)\,, \qquad (\epsilon\cdot v) = 0$$

QCD factorization (light-ray OPE)

$$\mathcal{I}^{\parallel(\perp)}(v^2z^2, pvz) = C^{\parallel(\perp)}(\mu_F) \otimes \int_{-1}^{1} dx\, e^{ixpvz} q(x,\mu_F) \quad \longleftarrow \quad \text{Ioffe-time distribution}$$
$$+\, \mathcal{O}(v^2z^2)$$

Quasi-distributions (2)

Let $z^\mu \mapsto z v^\mu, \quad z \in \mathbb{R}$

- parton quasi-distributions [Ji:2013dva]

$$\mathcal{Q}^{\|(\perp)}(x,p) = (pv)\int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{-ixz(pv)}\, \mathcal{I}^{\|(\perp)}(z^2v^2, pvz)$$

- parton pseudo-distributions [Radyushkin:2017cyf]

$$\mathcal{P}(x,z) = z\int_{-\infty}^{\infty} \frac{d(pv)}{2\pi} e^{-ixz(pv)}\, \mathcal{I}^{\perp}(z^2v^2, pvz)$$

Normalized (scale-invariant) quasi-distributions

Quasidistributions are scale-dependent (Z_q in axial gauge) and in renormalization schemes not based on dim. reg. also suffer from a linear UV divergence of the Wilson line. Problem can be avoided by considering a scale-independent ratio:

- diving out the value at zero proton momentum [Orginos:2017kos]

$$\mathbf{I}(z,pv) = \mathcal{I}(z,pv,\mu)/\mathcal{I}(z,0,\mu)$$

- or, alternatively, normalizing the qITD to the vacuum correlator

$$\widehat{\mathbf{I}}(z,pv) = \mathcal{I}(z,pv,\mu)/\mathcal{N}(z,\mu)\,, \qquad \mathcal{N}(z,\mu) = \left(\frac{2iN_c}{\pi^2 z^2}\right)^{-1} \langle 0|\bar{q}(z)\not{z}[z,0]q(0)|0\rangle$$

In this way one can define the scale-independent qPDF/pPDF

$$\mathbf{Q}(x,p) = (pv)\int_{-\infty}^{\infty}\frac{dz}{2\pi}e^{-ixz(pv)}\,\mathbf{I}(z,pv)$$

$$\mathbf{P}(x,z) = z\int_{-\infty}^{\infty}\frac{d(pv)}{2\pi}e^{-ixz(pv)}\,\mathbf{I}(z,pv)$$

and similarly for $\widehat{\mathbf{Q}}(x,p)$ and $\widehat{\mathbf{P}}(x,z)$

- normalization to the vacuum correlator does not affect leading $\mathcal{O}(z^2)$ power corrections, whereas the normalization to zero momentum, as we will see, has a nontrivial effect.

Borel transform and renormalons

- light-ray OPE

$$\bar{q}(zv)\not{v}[zv,0]q(0) = \int_0^1 d\alpha\, H^{\parallel}(z,\alpha,\mu,\mu_F)\Pi^{\mu_F}_{\rm l.t.}[\bar{q}(\alpha z v)\not{z}q(0)] + \ldots$$

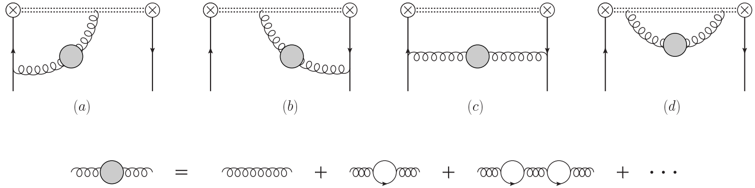
$$H = \delta(1-\alpha) + \sum_{k=0}^{\infty} h_k a_s^{k+1}\,, \qquad a_s = \frac{\alpha_s(\mu)}{4\pi}\,, \qquad h_k \propto k!$$

- A convenient way to handle such a series is to consider the Borel transform

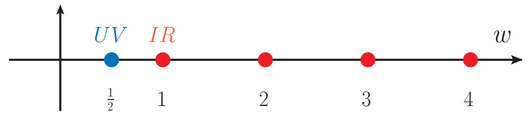
$$B[H](w) = \sum_{k=0}^{\infty}\frac{h_k}{k!}\left(\frac{w}{\beta_0}\right)^k \qquad H = \delta(1-\alpha) + \frac{1}{\beta_0}\int_0^\infty dw\, e^{-w/(\beta_0 a_s)}B[H](w)$$

However, integration is obscured by singularites on the integration path — renormalons

- Bubble-chain approximation 't Hooft, '77



Singularities of the Borel transform



- Ultraviolet renormalon at $w = 1/2$ M. Beneke, VB, '94

$$B[H^{\parallel(\perp)}] \stackrel{w\to 1/2}{=} \frac{4C_F}{w-1/2}\left(-\frac{v^2z^2\mu^2e^{5/3}}{4e^{-2\gamma_E}}\right)^{1/2}$$

— removed by normalization or by explicit subtraction

- Infrared renormalon at $w = 1$

$$B[H^{\parallel}](w) \stackrel{w\to 1}{=} \frac{-4C_F}{1-w}\left[\alpha + \bar\alpha\ln\bar\alpha\right]\left(-\frac{v^2z^2\mu^2e^{5/3}}{4e^{-2\gamma_E}}\right)$$

$$B[H^{\perp}](w) \stackrel{w\to 1}{=} \frac{-4C_F}{1-w}\left[\alpha + \bar\alpha\ln\bar\alpha + \alpha\bar\alpha\right]\left(-\frac{v^2z^2\mu^2e^{5/3}}{4e^{-2\gamma_E}}\right)$$

Leading power corrections to quasi-PDFs

$$\mathcal{Q}^{\parallel}(x,p) = q(x) - \frac{v^2\Lambda^2}{x^2(pv)^2}\int_{|x|}^{1}\frac{dy}{y}\left[\frac{y^2}{[1-y]_+} + 2\delta(\bar{y}) + \delta'(\bar{y})\right]q(\tfrac{x}{y})$$

$$\mathcal{Q}^{\perp}(x,p) = q(x) - \frac{v^2\Lambda^2}{x^2(pv)^2}\int_{|x|}^{1}\frac{dy}{y}\left[\frac{y^2}{[1-y]_+} + 3\delta(\bar{y}) + \delta'(\bar{y}) - 2y^2\right]q(\tfrac{x}{y})$$

where $\Lambda = \mathcal{O}(\Lambda_{\rm QCD})$

For a numerical study, present the result in the form

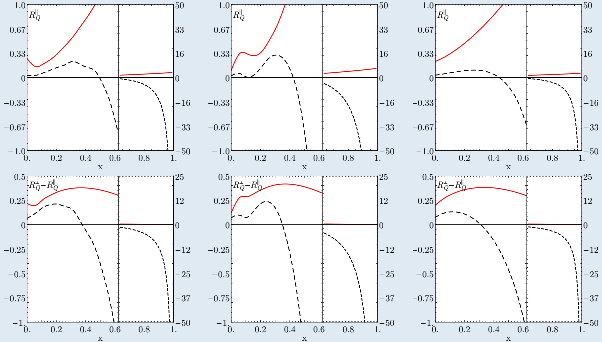
$$\mathcal{Q}^{\parallel(\perp)}(x,p) = q(x)\left\{1 - \frac{v^2\Lambda^2}{x^2(1-x)(pv)^2}\mathcal{R}_{\mathcal{Q}}^{\parallel(\perp)}(x)\right\}$$

Leading power corrections to quasi-PDFs

u-quark | d-quark | $q(x) = x^{-1/2}(1-x)^3$

$\mathcal{R}_\mathcal{Q}^{\parallel}$

$\mathcal{R}_\mathcal{Q}^{\perp} - \mathcal{R}_\mathcal{Q}^{\parallel}$



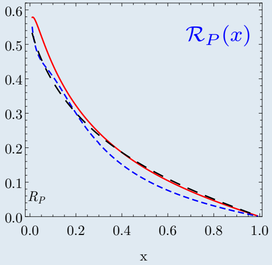
red: non-normalized qPDFs (original Ji's definition)
black: normalized qPDFs (after subtraction at zero momentum)

used: MSTW NLO valence quark PDFs at 2 GeV

Leading power corrections to pseudo-PDFs

$$\mathcal{P}(x,z,\mu) = q(x) + (v^2z^2\Lambda^2)\int_{|x|}^1 \frac{dy}{y}\,(y+\bar y\ln\bar y + y\bar y)q(\tfrac{x}{y}) \;\equiv\; q(x)\Big\{1+(v^2z^2\Lambda^2)\mathcal{R}_P(x)\Big\}$$

$$\mathbf{P}(x,z,\mu) = q(x) + (v^2z^2\Lambda^2)\int_{|x|}^1 \frac{dy}{y}\,[y+\bar y\ln\bar y + y\bar y]_+q(\tfrac{x}{y}) \;\equiv\; q(x)\Big\{1+(v^2z^2\Lambda^2)\mathbf{R}_P(x)\Big\}$$



MSTW u-quarks (black)
MSTW d-quarks (blue)
$q(x) = x^{-1/2}(1-x)^3$ (red)

$$\mathcal{R}_P(x) \overset{x\to 1}{=} \mathcal{O}(1-x)\,,$$

$$\mathbf{R}_P(x) = \mathcal{R}_P(x) - \frac{5}{12}\,,$$

Summary: power corrections to qPDFs

- Power corrections for qPDFs have a generic behavior

$$\mathcal{Q}(x,p) = q(x)\left\{1 + \mathcal{O}\left(\frac{\Lambda^2}{p^2}\cdot\frac{1}{x^2(1-x)}\right)\right\}$$

Normalization to zero momentum considerably reduces the correction at $0.1 < x < 0.6$ at the cost of a strong enhancement at $x > 0.6$

- Power corrections for pPDFs have a generic behavior

$$\mathcal{P}(x,z) = q(x)\left\{1 + \mathcal{O}\left(z^2\Lambda^2(1-x)\right)\right\}$$

but the suppression at $x \to 1$ is lifted by the normalization to zero momentum

- Position space PDFs (qITDs) have flat power corrections at large Ioffe times