# Exclusive dijet or $\rho$ meson production in $eA$ collisions
# CGC at 1 loop, collinear factorization and the CGC

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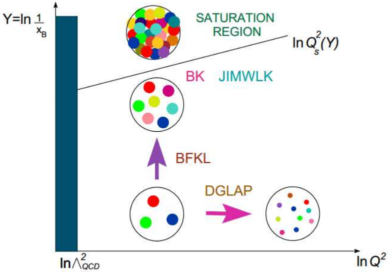
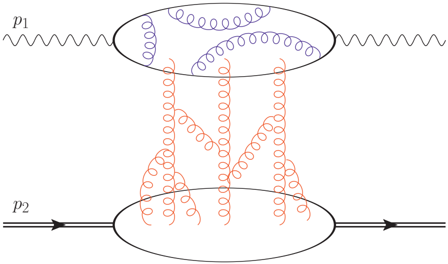
INT workshop
November 2018

# The shockwave description of the Color Glass Condensate



# Kinematics




$$p_1 = p^+ n_1 - \frac{Q^2}{2s} n_2$$

$$p_2 = \frac{m_t^2}{2p_2^-} n_1 + p_2^- n_2$$

$$p^+ \sim p_2^- \sim \sqrt{\frac{s}{2}}$$

Lightcone (Sudakov) vectors

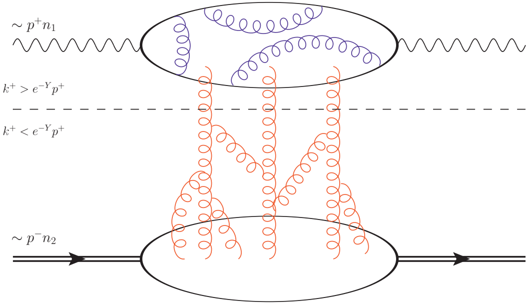
$$n_1 = \sqrt{\frac{1}{2}}(1, 0_\perp, 1), \quad n_2 = \sqrt{\frac{1}{2}}(1, 0_\perp, -1), \quad (n_1 \cdot n_2) = 1$$

Lightcone coordinates:

$$x = (x^0, x^1, x^2, x^3) \to (x^+, x^-, \vec{x})$$

$$x^+ = x_- = (x \cdot n_2) \quad x^- = x_+ = (x \cdot n_1)$$

# Rapidity separation

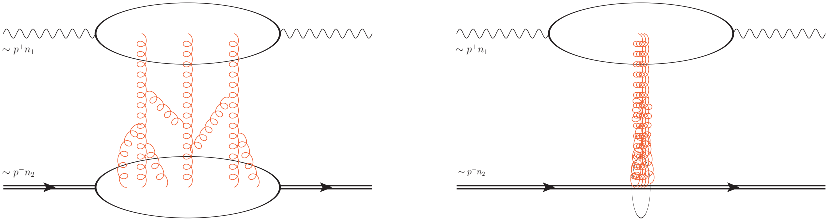



Let us split the gluonic field between "fast" and "slow" gluons

$$\begin{aligned} \mathcal{A}^{\mu a}(k^+, k^-, \vec{k}) &= A_\eta^{\mu a}(|k^+| > e^\eta p^+, k^-, \vec{k}) \\ &+ b_\eta^{\mu a}(|k^+| < e^\eta p^+, k^-, \vec{k}) \end{aligned}$$

$e^\eta = e^{-Y} \ll 1$

# Large longitudinal boost to the projectile frame




$$b^+(x^+, x^-, \vec{x}) \longrightarrow \frac{1}{\Lambda} b^+(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$b^-(x^+, x^-, \vec{x}) \longrightarrow \Lambda b^-(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$b^k(x^+, x^-, \vec{x}) \longrightarrow b^k(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$\Lambda \sim \sqrt{\frac{s}{m_t^2}}$$

$$b^\mu(x) \to b^-(x)\, n_2^\mu = \delta(x^+)\, \mathbf{B}(\vec{x})\, n_2^\mu + O(\sqrt{\frac{m_t^2}{s}})$$

*Shockwave* approximation

# Effective Feynman rules in the external shockwave field

Wilson lines :

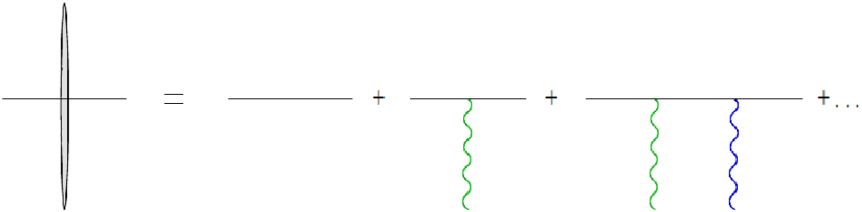
$$U_i^\eta = U_{\vec{z}_i}^\eta = P \exp \left[ ig \int_{-\infty}^{+\infty} b_\eta^- (z_i^+, \vec{z}_i) \, dz_i^+ \right]$$

Fourier transform of a Wilson line

$$\tilde{U}^\eta(\vec{p}) = \int d^{D-2}\vec{z} \; e^{-i(\vec{p}\cdot\vec{z})} U_{\vec{z}}^\eta$$

$$U_i^\eta = 1 + ig \int_{-\infty}^{+\infty} b_\eta^- (z_i^+, \vec{z}_i) dz_i^+ + (ig)^2 \int_{-\infty}^{+\infty} b_\eta^- (z_i^+, \vec{z}_i) b_\eta^- (z_j^+, \vec{z}_j) \theta(z_{ji}^+) dz_i^+ dz_j^+$$

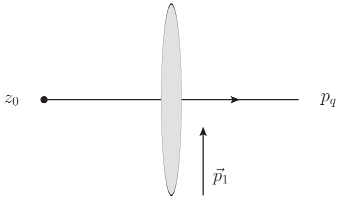
...



# Quark line in the external field in momentum space

$$\bar{u}\left(p_q,\, z_0\right) \;=\; \int \frac{d^d \vec{p}_1}{(2\pi)^d} e^{i p_q^+ z_0^- + i z_0^+ \frac{(\vec{p}_q - \vec{p}_1)^2 - i0}{2p_q^+} - i\left(\vec{p}_q - \vec{p}_1\right)\cdot \vec{z}_0}$$

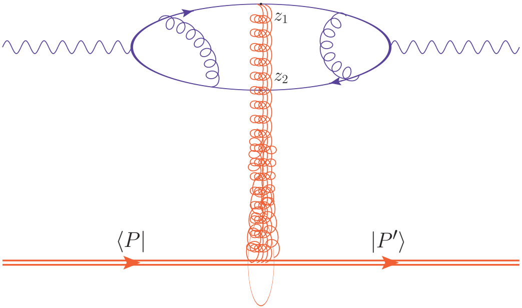
$$\times \bar{u}_{p_q} \gamma^+ \left[ \tilde{U}_{\vec{p}_1} \theta\left(-z_0^+\right) + (2\pi)^d \delta\left(\vec{p}_1\right) \theta\left(z_0^+\right) \right] \frac{p_q^+ \gamma^- + \hat{p}_{q\perp} - \hat{p}_{1\perp}}{2p_q^+}$$




Exchange in $t$-channel of an effective off-shell particle with 0 momentum along $n_1$

$$\bar{u}\left(p_q,\, z_0\right)\big|_{U=1} \;=\; e^{i\left(p_q \cdot z_0\right)} \bar{u}_{p_q} \left(1 - \frac{\hat{p}_q \gamma^+}{2p_q^+}\right)$$

# Factorized picture




Factorized amplitude

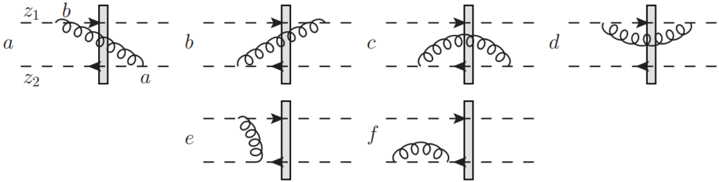
$$\mathcal{A}^\eta = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2\, \Phi^\eta(\vec{z}_1, \vec{z}_2)\, \langle P'|[\mathrm{Tr}(U^\eta_{\vec{z}_1} U^{\eta\dagger}_{\vec{z}_2}) - N_c]|P\rangle$$

Dipole operator $\mathcal{U}^\eta_{ij} = \frac{1}{N_c}\mathrm{Tr}(U^\eta_{\vec{z}_i} U^{\eta\dagger}_{\vec{z}_j}) - 1$

Written similarly for any number of Wilson lines in any color representation!

# Evolution for the dipole operator

$$\mathcal{U}_{12}^{\eta+\delta\eta} - \mathcal{U}_{12}^{\eta}$$




B-JIMWLK hierarchy of equations

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

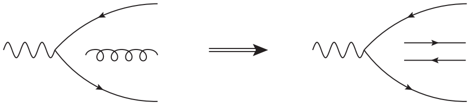
$$\frac{\partial \mathcal{U}_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\mathcal{U}_{13}^{\eta} + \mathcal{U}_{32}^{\eta} - \mathcal{U}_{12}^{\eta} + \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta}\right]$$

$$\frac{\partial \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta}}{\partial \eta} = \ldots$$

Evolves a dipole into a double dipole

# The BK equation

Mean field approximation, or 't Hooft planar limit $N_c \to \infty$ in the dipole B-JIMWLK equation



[Mueller]

$\Rightarrow$ BK equation [Balitsky, 1995] [Kovchegov, 1999]

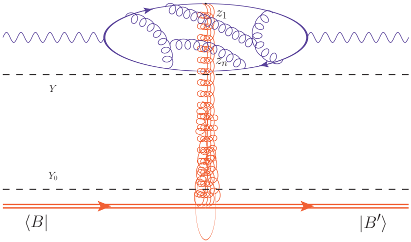
$$\frac{\partial \langle \mathcal{U}_{12}^{\eta} \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \, \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2} \vec{z}_{23}^{\,2}} \left[ \langle \mathcal{U}_{13}^{\eta} \rangle + \langle \mathcal{U}_{32}^{\eta} \rangle - \langle \mathcal{U}_{12}^{\eta} \rangle + \langle \mathcal{U}_{13}^{\eta} \rangle \, \langle \mathcal{U}_{32}^{\eta} \rangle \right]$$

BFKL/BKP part — Triple pomeron vertex

Non-linear term : saturation

# Practical use of the formalism

- Compute the upper impact factor using the effective Feynman rules
- Build non-perturbative models for the matrix elements of the Wilson line operators acting on the target states
- Solve the B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity $\eta = Y_0$
- Evaluate the solution at a typical projectile rapidity $\eta = Y$, or at the rapidity of the slowest gluon
- Convolute the solution and the impact factor




$$\mathcal{A} = \int d\vec{z}_1 ... d\vec{z}_n \ \Phi(\vec{z}_1, ..., \vec{z}_n)$$

$$\times \langle P' | U_{\vec{z}_1} ... U_{\vec{z}_n} | P \rangle$$

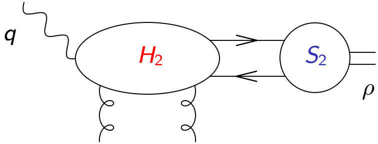
Exclusive diffraction allows one to probe the $b_\perp$-dependence of the non-perturbative scattering amplitude

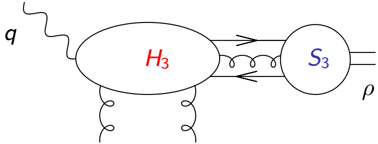
# Collinear factorization for light vector meson production

Light Cone Collinear Factorization (LCCF)
vs
Covariant Collinear Factorization (CCF)

# Collinear factorization: basic principle

The impact factor is the convolution of a hard part and the vacuum-to-meson matrix element of an operator



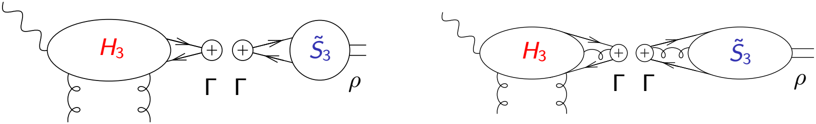




$$\int_x (H_2(x))_{ij}^{\alpha\beta} \left\langle \rho \left| \bar{\psi}_i^\alpha(x)\psi_j^\beta(0) \right| 0 \right\rangle \qquad \int_{x_1,x_2} (H_3^\mu(x_1,x_2))_{ij,c}^{\alpha\beta} \left\langle \rho \left| \bar{\psi}_i^\alpha(x_1) A_\mu^c(x_2) \psi_j^\beta(0) \right| 0 \right\rangle$$

$H$ and $S$ are by convolution and by summation over spinor and color indices

# Spinorial and color factorization

Applying a simple Fierz decomposition in color space and in spinor space




$$\frac{1}{4N_c}\mathrm{Tr}_{c,D}\left[H_2\,\Gamma^\lambda\right]\left\langle\rho\left|\bar{\psi}\,\Gamma_\lambda\,\psi\right|0\right\rangle \qquad \frac{1}{4}\mathrm{Tr}_{c,D}\left[H_3^{\mu,c}\,t^c\Gamma^\lambda\right]\left\langle\rho\left|\bar{\psi}\,A_\mu\Gamma_\lambda\,\psi\right|0\right\rangle$$

We thus need to study the following matrix elements:

$$\left\langle\rho\left|\bar{\psi}\,\Gamma_\lambda\,\psi\right|0\right\rangle \quad \text{and} \quad \left\langle\rho\left|\bar{\psi}\,A_\mu\Gamma_\lambda\,\psi\right|0\right\rangle$$

# Light Cone Collinear Factorization

# Collinear factorization

## Light Cone Collinear approach

The Light Cone Collinear Factorization approach

Momentum factorization

- Define a Sudakov vector $n$ such that $p \cdot n = 1$ and write $d^4 p_q = \int dx \, d^4 p_q \; \delta(x - p_q \cdot n)$.
- Taylor expansion of the hard part $H(p_q)$ along the collinear direction $xp$:

$$H(p_q) e^{-ip_q \cdot z} S(z)$$

$$= H(xp) e^{-ip_q \cdot z} S(z) \quad + \quad \left. \frac{\partial H(p_q)}{\partial p_q^{\mu}} \right|_{p_q = xp} (p_q - x\,p)^{\mu} e^{-ip_q \cdot z} S(z) + \ldots$$

- $p_q^{\mu} \overset{IbP}{\longrightarrow}$ derivative of the soft term: $\int d^4 z \; e^{-i\ell \cdot z} \langle \rho(p) | \psi(0) \, i \; \overleftrightarrow{\partial_{\mu}} \, \bar{\psi}(z) | 0 \rangle$
- Standard derivative $\Rightarrow$ need for 3-body contributions to combine into a covariant derivative.

# Twist 2

## LCCF at twist 2

- Leading twist DA for a longitudinally polarized light vector meson

$$\left\langle \rho \left| \bar{\psi}(z)\gamma^{\mu}\psi(0) \right| 0 \right\rangle \rightarrow p^{\mu} f_{\rho} \int_0^1 dx e^{ix(p\cdot z)} \varphi_1(x)$$

- Leading twist DA for a transversely polarized light vector meson

$$\left\langle \rho \left| \bar{\psi}(z)\sigma^{\mu\nu}\psi(0) \right| 0 \right\rangle \rightarrow i(p^{\mu}\varepsilon_{\rho}^{\nu} - p^{\nu}\varepsilon_{\rho}^{\mu}) f_{\rho}^{T} \int_0^1 dx e^{ix(p\cdot z)} \varphi_{\perp}(x)$$

The twist 2 DA for a transverse meson is chiral odd, thus $\gamma^* A \rightarrow \rho_T A$ starts at twist 3

# Twist 3 Distribution Amplitudes

Required DAs for $\rho_T$ production at twist 3 in LCCF

- 2-body DAs

$$\left\langle \rho(p) \left| \bar{\psi}(z) \gamma_\mu \psi(0) \right| 0 \right\rangle \to m_\rho f_\rho \left[ \varphi_1(x) \left( \varepsilon_\rho^* \cdot n \right) p_\mu + \varphi_3(x) \varepsilon_{\rho T \mu}^* \right]$$

$$\left\langle \rho(p) \left| \bar{\psi}(z) \gamma_5 \gamma_\mu \psi(0) \right| 0 \right\rangle \to m_\rho f_\rho i \varphi_A(x) \epsilon_{\mu\lambda\beta\delta} \varepsilon_{\rho T}^{\lambda *} p^\beta n^\delta$$

$$\left\langle \rho(p) \left| \bar{\psi}(z) \gamma_\mu i \overleftrightarrow{\partial}_\alpha \psi(0) \right| 0 \right\rangle \to m_\rho f_\rho \varphi_{1T}(x) p_\mu \varepsilon_{\rho T \alpha}^*$$

$$\left\langle \rho(p) \left| \bar{\psi}(z) \gamma_5 \gamma_\mu i \overleftrightarrow{\partial}_\alpha \psi(0) \right| 0 \right\rangle \to m_\rho f_\rho i \varphi_{AT}(x) p_\mu \epsilon_{\alpha\lambda\beta\delta} \varepsilon_{\rho T}^{*\lambda} p^\beta n^\delta$$

- 3-body DAs

$$\left\langle \rho(p) \left| \bar{\psi}(z_1) \gamma_\mu g A_\alpha(z_2) \psi(0) \right| 0 \right\rangle \to m_\rho f_3^V B(x_1, x_2) p_\mu \varepsilon_{\rho T \alpha}^*$$

$$\left\langle \rho(p) \left| \bar{\psi}(z_1) \gamma_5 \gamma_\mu g A_\alpha(z_2) \psi(0) \right| 0 \right\rangle \to m_\rho f_3^A i D(x_1, x_2) p_\mu \epsilon_{\alpha\lambda\beta\delta} \varepsilon_{\rho T}^{*\lambda} p^\beta n^\delta$$

# Minimal set of DAs

7 required DAs

- Equations of motion: Dirac equation

$$\left\langle \left(i\hat{D}\psi_\alpha\right)(0)\,\bar{\psi}_\beta\left(z\right)\right\rangle = 0, \quad \left\langle \psi_\alpha\left(0\right)\left(i\hat{D}\bar{\psi}_\beta\right)\left(z\right)\right\rangle = 0$$

- Leads to two equations

$$x_1\varphi_3\left(x_1\right) + \bar{x}_1\varphi_A\left(x_1\right) + \varphi_{1T}\left(x_1\right) + \varphi_{AT}\left(x_1\right) + \int dx_2\left[\zeta_3^V B\left(x_1, x_2\right) + \zeta_3^A D\left(x_1, x_2\right)\right] = 0$$

$$\bar{x}_1\varphi_3\left(x_1\right) - x_1\varphi_A\left(x_1\right) - \varphi_{1T}\left(x_1\right) + \varphi_{AT}\left(x_1\right) - \int dx_2\left[\zeta_3^V B\left(x_2, x_1\right) - \zeta_3^A D\left(x_2, x_1\right)\right] = 0$$

7-2 required DAs

# Minimal set of DAs

## 7-2 required DAs

- *n*-independence. *n* appeared in three constraints:
  - Lighcone direction of the separation $z$: $z = \lambda n$
  - Definition of the transverse polarization $\varepsilon_\rho \cdot n = 0$
  - Chosen gauge $n \cdot A = 0$
- Leads to 2 additional constraints for the DAs, plus the gauge invariance condition.

## 7-4 required DAs

$\varphi(x)$ ← 2-body twist 2 correlator
$B(x_1, x_2)$ ← 3-body genuine twist 3 vector correlator
$D(x_1, x_2)$ ← 3-body genuine twist 3 axial correlator

# Covariant Collinear Factorization

# Collinear factorization

## Covariant Collinear approach

- Work directly on the operators, with gauge invariant light ray operators
- 2-body correlators

$$\left\langle \rho(p) \left| \bar{\psi}(z)[z,0]\gamma^{\mu}\psi(0)\right| 0\right\rangle \to f_{\rho} m_{\rho}\left[-ip^{\mu}\left(\varepsilon_{\rho}^{*}\cdot z\right) h(x) + \varepsilon_{\rho}^{\mu *} g_{\perp}^{(v)}(x)\right]$$

$$\left\langle \rho(p) \left| \bar{\psi}(z)[z,0]\gamma_5\gamma_{\mu}\psi(0)\right| 0\right\rangle \to \frac{1}{4} f_{\rho} m_{\rho}\epsilon_{\mu\alpha\beta\delta}\varepsilon_{\rho\perp}^{\alpha} p^{\beta} z^{\delta} g_{\perp}^{(a)}(x)$$

- 3-body correlators

$$\left\langle \rho(p) \left| \bar{\psi}(z)[z,tz]\gamma_{\alpha} g G_{\mu\nu}(tz)[tz,0]\psi(0)\right| 0\right\rangle$$
$$\to -i m_{\rho} f_{3\rho}^{V} p_{\alpha}\left(p_{\mu}\varepsilon_{\rho\perp\nu}^{*} - p_{\nu}\varepsilon_{\rho\perp\mu}^{*}\right) V(x_1, x_2)$$

$$\left\langle \rho(p) \left| \bar{\psi}(z)[z,tz]\gamma_{\alpha}\gamma_5 g \tilde{G}_{\mu\nu}(tz)[tz,0]\psi(0)\right| 0\right\rangle$$
$$\to -m_{\rho} f_{3\rho}^{A} p_{\alpha}\left(p_{\mu}\varepsilon_{\rho\perp\nu}^{*} - p_{\nu}\varepsilon_{\rho\perp\mu}^{*}\right) A(x_1, x_2)$$

- Equations of motions $\Rightarrow$ only 3 DAs are required

# Matching

## Matching at twist 3 accuracy

| LCCF | CCF |
|---|---|
| $\varphi_3(x)$ | $g_\perp^{(v)}(x)$ |
| $\varphi_1^T(x)$ | $\tilde{h}(x) - h(x)$ |
| $\varphi_A(x)$ | $-\frac{1}{4}\frac{\partial g_\perp^{(a)}}{\partial x}(x)$ |
| $\varphi_A^T(x)$ | $-\frac{1}{4}g_\perp^{(a)}(x)$ |
| $B(x_q, x_{\bar{q}}; x_g)$ | $\frac{-V\left(x_q, x_{\bar{q}}; x_g\right)}{1-x_q-x_{\bar{q}}}$ |
| $D(x_q, x_{\bar{q}})$ | $\frac{-A\left(x_q, x_{\bar{q}}; x_g\right)}{1-x_q-x_{\bar{q}}}$ |

A process-specific comparison was done previously [Anikin, Ivanov, Pire, Szymanowski, Wallon]

A completely generic proof exists [RB *et al*, to be published].

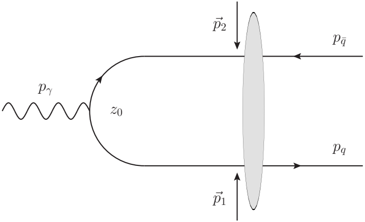
# Open parton production at NLO

# First step: open parton production

- Regge-Gribov limit : $s \gg Q^2 \gg \Lambda_{QCD}$
- Otherwise completely general kinematics
- Shockwave (CGC) Wilson line approach
- Transverse dimensional regularization $d = 2 + 2\varepsilon$, longitudinal cutoff

$$|p_g^+| > \alpha p_\gamma^+$$

# LO diagram



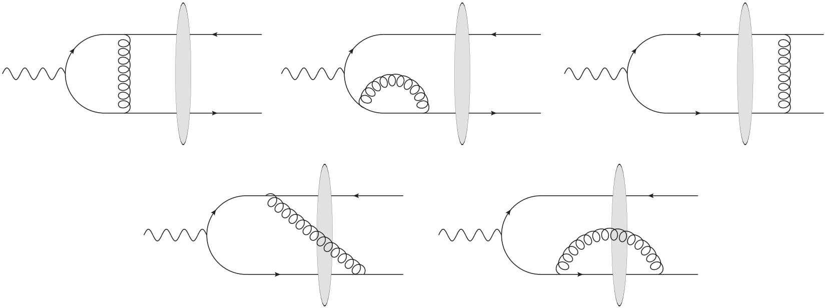

$$\mathcal{A} = \frac{\delta^{ik}}{\sqrt{N_c}} \int d^D z_0 [\bar{u}(p_q, z_0)]_{ij} (-ie_q) \hat{\varepsilon}_\gamma e^{-i(p_\gamma \cdot z_0)} [v(p_{\bar{q}}, z_0)]_{jk} \theta(-z_0^+)$$

$$= \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2) \Phi_0(\vec{p}_1, \vec{p}_2)$$

$$\times \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

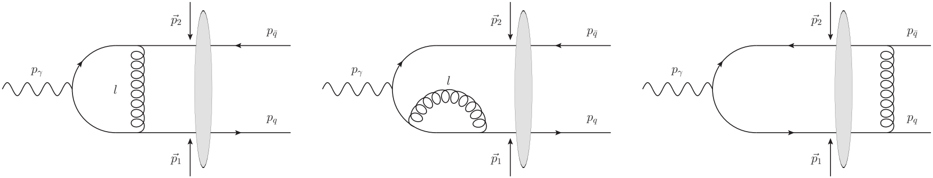
$$\tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) = \int d^d \vec{z}_1 d^d \vec{z}_2 \, e^{-i(\vec{p}_1 \cdot \vec{z}_1) - i(\vec{p}_2 \cdot \vec{z}_2)} [\tfrac{1}{N_c} \mathrm{Tr}(U^\alpha_{\vec{z}_1} U^{\alpha\dagger}_{\vec{z}_2}) - 1]$$

# NLO open $q\bar{q}$ production



Diagrams contributing to the NLO correction

# First kind of virtual corrections



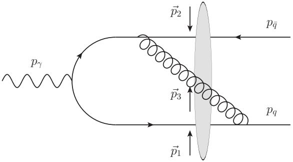

Color factor

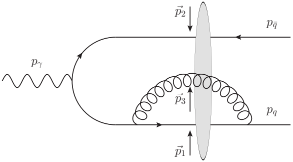
$$\frac{C_F}{\sqrt{N_c}}\tilde{\mathcal{U}}^{\alpha}(\vec{p}_1,\vec{p}_2)$$

Impact factor

$$\mathcal{A}_{NLO}^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+)\int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)\,\Phi_{V1}(\vec{p}_1,\vec{p}_2)$$
$$\times C_F \left\langle P' \middle| \tilde{\mathcal{U}}^{\alpha}(\vec{p}_1,\vec{p}_2) \middle| P \right\rangle$$

# Second kind of virtual corrections




Color factor

$$\frac{\delta^{ik}}{\sqrt{N_c}}(t^a U_1 t^b U_2^\dagger)_{ik}(U_3)^{ab}$$

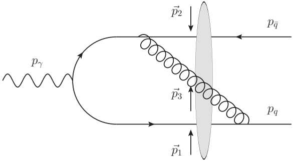
Action of the Wilson line in the adjoint representation

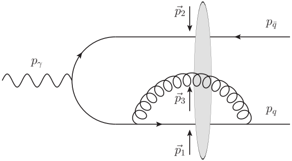
$$(U_3)^{ab} t^b = U_3 t^a U_3^\dagger \quad \Rightarrow \quad (U_3)^{ab} = 2\mathrm{Tr}(t^a U_3 t^b U_3^\dagger)$$

$+$ Fierz identity

$$C_F \mathcal{U}_{12} + \frac{1}{2}[\mathcal{U}_{13} + \mathcal{U}_{32} - \mathcal{U}_{12} + \mathcal{U}_{13}\mathcal{U}_{32}] \quad = C_F \mathcal{U}_{12} + \mathcal{W}_{123}$$

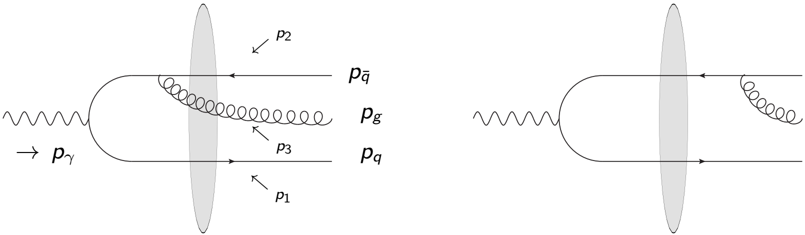
# Second kind of virtual corrections




$$\mathcal{A}_{NLO}^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 d^d \vec{p}_3 \, \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$
$$\times [\Phi'_{V1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \right| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \left| P \right\rangle (2\pi)^d \delta(\vec{p}_3)$$
$$+ \Phi_{V2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \right| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left| P \right\rangle ]$$

# LO open $q\bar{q}g$ production




$$\mathcal{A}_R^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

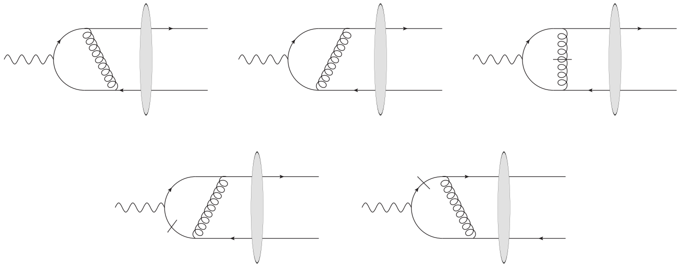
$$\times [\Phi'_{R1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \right| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \left| P \right\rangle (2\pi)^d \delta(\vec{p}_3)$$

$$+ \Phi_{R2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \right| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left| P \right\rangle ]$$

$$\mathcal{A}_R^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)$$

$$\times \Phi_{R1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \right| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \left| P \right\rangle$$

# Generic computation method



- Perform the $k_\perp$ integration with the usual *d*-dimensional regularization methods
- Perform the $k^+$ integration with the longitudinal cutoff $\alpha p_\gamma^+$ when possible, or isolate the divergent term by + prescription

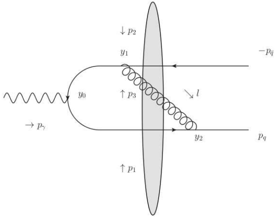
$$\int_{\alpha p_\gamma^+}^{p^+} dk^+ \frac{F(k^+)}{k^+} = \int_{\alpha p_\gamma^+}^{p^+} dk^+ \frac{F(0)}{k^+} + \int_0^{p^+} dk^+ \left[\frac{F(k^+)}{k^+}\right]_+$$

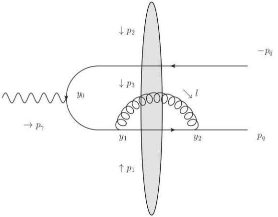
# Divergences

## Divergences

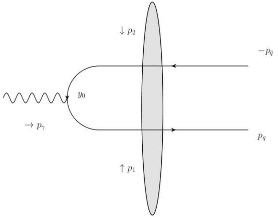
- Rapidity divergence $p_g^+ \to 0$ $\Phi_{V2}\Phi_0^* + \Phi_0\Phi_{V2}^*$
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

# Rapidity divergence




Double dipole virtual correction $\Phi_{V2}$




B-JIMWLK evolution of the LO term : $\Phi_0 \otimes \mathcal{K}_{BK}$

# Rapidity divergence

## B-JIMWLK equation for the dipole operator

$$\frac{\partial \tilde{\mathcal{U}}_{12}^{\alpha}}{\partial \log \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d\vec{k}_1 d^d\vec{k}_2 d^d\vec{k}_3}{(2\pi)^{2d}} \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3-\vec{p}_1-\vec{p}_2)\left(\tilde{\mathcal{U}}_{13}^{\alpha}\tilde{\mathcal{U}}_{32}^{\alpha}+\tilde{\mathcal{U}}_{13}^{\alpha}+\tilde{\mathcal{U}}_{32}^{\alpha}-\tilde{\mathcal{U}}_{12}^{\alpha}\right)$$

$$\times\left[2\frac{(\vec{k}_1-\vec{p}_1)\cdot(\vec{k}_2-\vec{p}_2)}{(\vec{k}_1-\vec{p}_1)^2(\vec{k}_2-\vec{p}_2)^2}+\frac{\pi^{\frac{d}{2}}\Gamma(1-\frac{d}{2})\Gamma^2(\frac{d}{2})}{\Gamma(d-1)}\left(\frac{\delta(\vec{k}_2-\vec{p}_2)}{\left[(\vec{k}_1-\vec{p}_1)^2\right]^{1-\frac{d}{2}}}+\frac{\delta(\vec{k}_1-\vec{p}_1)}{\left[(\vec{k}_2-\vec{p}_2)^2\right]^{1-\frac{d}{2}}}\right)\right]$$

$\eta$ rapidity divide, which separates the upper and the lower impact factors

$$\Phi_0\,\tilde{\mathcal{U}}_{12}^{\alpha} \to \Phi_0\,\tilde{\mathcal{U}}_{12}^{\eta} + 2\log\left(\frac{e^{\eta}}{\alpha}\right)\mathcal{K}_{BK}\,\Phi_0\,\tilde{\mathcal{W}}_{123}$$

# Rapidity divergence

Virtual contribution

$$(\Phi^{\mu}_{V2})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{x\bar{x}}{\alpha^2}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$

BK contribution

$$(\Phi^{\mu}_{BK})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{\alpha^2}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] \right\}$$

Sum : the $\alpha$ dependence cancels

$$(\Phi'^{\mu}_{V2})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{x\bar{x}}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$

# Rapidity divergence

Cancellation of the remaining $1/\epsilon$ divergence

Convolution

$$(\Phi'^{\mu}_{V2} \otimes \mathcal{W}) = 2\int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \left\{ 4\ln\left(\frac{x\bar{x}}{e^{2\eta}}\right)\left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon}\right\}$$
$$\times \quad \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2} - \vec{p}_3)\left[\tilde{\mathcal{U}}_{13} + \tilde{\mathcal{U}}_{32} - \tilde{\mathcal{U}}_{12} - \tilde{\mathcal{U}}_{13}\tilde{\mathcal{U}}_{32}\right]\Phi_0^{\mu}(\vec{p}_1, \vec{p}_2)$$

Rq :

- $\Phi_0(\vec{p}_1, \vec{p}_2)$ only depends on one of the $t$-channel momenta.
- The double-dipole operators cancels when $\vec{z}_3 = \vec{z}_1$ or $\vec{z}_3 = \vec{z}_2$.

This permits one to show that the convolution cancels the remaining $\frac{1}{\varepsilon}$ divergence.

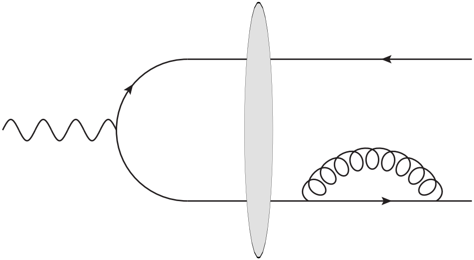
Then $\tilde{\mathcal{U}}^{\alpha}_{12}\Phi_0 + \Phi_{V2}$ is finite

# Divergences

- Rapidity divergence
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ $\qquad \Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ $\qquad \Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\qquad \Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ $\qquad \Phi_{R1}\Phi_{R1}^*$

# UV divergence

Tadpole diagrams



Some null diagrams just contribute to turning UV divergences into IR divergences

$$\Phi = 0 \propto \left( \frac{1}{2\epsilon_{IR}} - \frac{1}{2\epsilon_{UV}} \right)$$

In the massless limit, renormalization of the external quark lines is absent in dimensional regularization.

# Divergences

- Rapidity divergence
- UV divergence
- Soft divergence $p_g \to 0$ — $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$, $\Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ — $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ — $\Phi_{R1}\Phi_{R1}^*$

Constructing a finite cross section

Exclusive diffractive production of a forward dijet

From partons to jets

[R.B.,A.V.Grabovsky,L.Szymanowski,S.Wallon]

JHEP 1611 (2016) 149

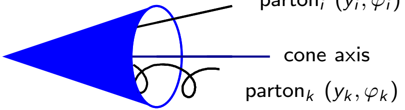
# Soft and collinear divergence

## Jet cone algorithm

We define a cone width for each pair of particles with momenta $p_i$ and $p_k$, rapidity difference $\Delta Y_{ik}$ and relative azimuthal angle $\Delta\varphi_{ik}$

$$(\Delta Y_{ik})^2 + (\Delta\varphi_{ik})^2 = R_{ik}^2$$

If $R_{ik}^2 < R^2$, then the two particles together define a single jet of momentum $p_i + p_k$.




Applying this in the small $R^2$ limit cancels our soft and collinear divergence.

# Remaining divergence

Soft real emission

$$(\Phi_{R1}\Phi^*_{R1})_{soft} \propto (\Phi_0\Phi^*_0)\int_{\text{outside the cones}} \left|\frac{p_q^\mu}{(p_q.p_g)} - \frac{p_{\bar{q}}^\mu}{(p_{\bar{q}}.p_g)}\right|^2 \frac{dp_g^+}{p_g^+}\frac{d^d p_g}{(2\pi)^d}$$

Collinear real emission

$$(\Phi_{R1}\Phi^*_{R1})_{col} \propto (\Phi_0\Phi^*_0)\left(\mathcal{N}_q + \mathcal{N}_{\bar{q}}\right)$$

Where $\mathcal{N}$ is the number of jets in the quark or the antiquark

$$\mathcal{N}_k = \frac{(4\pi)^{\frac{d}{2}}}{\Gamma(2-\frac{d}{2})}\int_{\alpha p_\gamma^+}^{p_{jet}^+}\frac{dp_g^+ dp_k^+}{2p_g^+ 2p_k^+}\int_{\text{in cone k}}\frac{d^d\vec{p}_g d^d\vec{p}_k}{(2\pi)^d\,\mu^{d-2}}\frac{\mathrm{Tr}\left(\hat{p}_k\gamma^\mu\hat{p}_{jet}\gamma^\nu\right)d_{\mu\nu}(p_g)}{2p_{jet}^+\left(p_k^- + p_g^- - p_{jet}^-\right)^2}$$

Those two contributions cancel exactly the virtual divergences

# Cancellation of divergences

Total divergence

$$(d\sigma_1)_{div} = \alpha_s \frac{\Gamma(1-\varepsilon)}{(4\pi)^{1+\varepsilon}} \left(\frac{N_c^2-1}{2N_c}\right) \left(S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2}\right) d\sigma_0$$

Virtual contribution

$$S_V = \left[2\ln\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) - 3\right]\left[\ln\left(\frac{x_j x_{\bar{j}} \mu^2}{(x_j \vec{p}_{\bar{j}} - x_{\bar{j}} \vec{p}_j)^2}\right) - \frac{1}{\epsilon}\right] + 2i\pi \ln\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) + \ln^2\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) - \frac{\pi^2}{3} + 6$$

Real contribution

$$S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} = 2\left[\ln\left(\frac{(x_{\bar{j}} \vec{p}_j - x_j \vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^{\,2} \vec{p}_j^{\,2}}\right) \ln\left(\frac{4E^2}{x_{\bar{j}} x_j (p_\gamma^+)^2}\right) + 2\ln\left(\frac{x_{\bar{j}} x_j}{\alpha^2}\right)\left(\frac{1}{\epsilon} - \ln\left(\frac{x_{\bar{j}} x_j \mu^2}{(x_{\bar{j}} \vec{p}_j - x_j \vec{p}_{\bar{j}})^2}\right)\right) - \ln^2\left(\frac{x_{\bar{j}} x_j}{\alpha^2}\right) + \frac{3}{2}\ln\left(\frac{16\mu^4}{R^4 \vec{p}_j^{\,2} \vec{p}_{\bar{j}}^{\,2}}\right) - \ln\left(\frac{x_j}{x_{\bar{j}}}\right)\ln\left(\frac{x_j \vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}} \vec{p}_j^{\,2}}\right) - \frac{3}{\epsilon} - \frac{2\pi^2}{3} + 7\right]$$

## Cancellation of divergences

Total "divergence"

$$
\begin{aligned}
div &= S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} \\
&= 4\left[\frac{1}{2}\ln\left(\frac{(x_{\bar{j}}\vec{p}_j - x_j\vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^{\,2}\vec{p}_j^{\,2}}\right)\left(\ln\left(\frac{4E^2}{x_{\bar{j}}x_j(p_\gamma^+)^2}\right)+\frac{3}{2}\right)\right. \\
&+ \left.\ln(8) - \frac{1}{2}\ln\left(\frac{x_j}{x_{\bar{j}}}\right)\ln\left(\frac{x_j\vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}}\vec{p}_j^{\,2}}\right) + \frac{13-\pi^2}{2}\right]
\end{aligned}
$$

Our cross section is thus finite

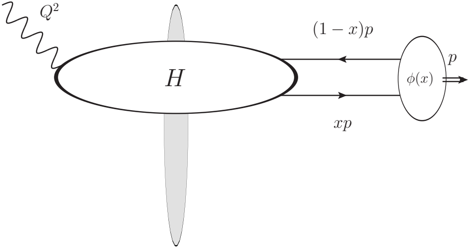
# Constructing a finite amplitude

## Exclusive diffractive $\rho_L$ production

[RB, Grabovsky, Ivanov, Szymanowski, Wallon]
Phys.Rev.Lett. 119 (2017) ; arXiv:1612.08026
Non-forward and non-dilute extension of [Ivanov, Kotsky, Papa]

# $s$-channel collinear factorization

## Twist 2: $\rho_L$ production

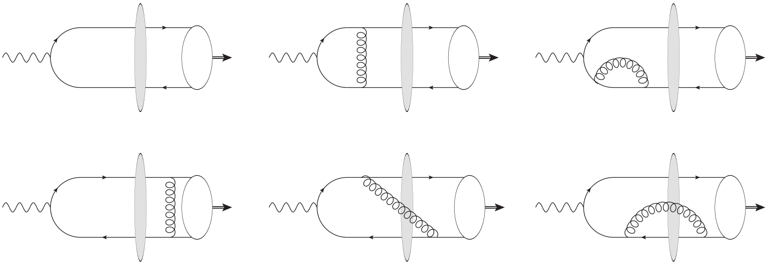



Singlet transition $\Rightarrow$ only virtual diagrams contribute.
Leading twist matrix element:

$$\left\langle \rho_L(p) \left| \bar{\psi}(z)\gamma^\lambda \psi(0) \right| 0 \right\rangle \quad \rightarrow \quad f_\rho m_\rho p^\lambda \int_0^1 dx \; e^{-ixp\cdot z} \varphi_\parallel(x)$$

Take the NLO open parton production result with collinear kinematics $(p_q, p_{\bar{q}}) = (xp, \bar{x}p)$, project on the leading twist Fierz matrix $\gamma^-$ and convolute with the twist 2 DA $\varphi_1$

# Exclusive diffractive production of a light neutral vector meson



$$\mathcal{A}_0 = -\frac{e_V\, f_V\, \varepsilon_\beta}{N_c} \int_0^1 dx \varphi_\parallel (x) \int \frac{d^d \vec{p}_1}{(2\pi)^d} \frac{d^d \vec{p}_2}{(2\pi)^d}$$
$$\times \; (2\pi)^{d+1} \delta\left(p_V^+ - p_\gamma^+\right) \delta\left(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2\right)$$
$$\times \; \Phi_0^\beta\left(x,\, \vec{p}_1,\, \vec{p}_2\right) \tilde{\mathcal{U}}_{12}^\eta .$$

Leading twist for a longitudinally polarized meson

Otherwise general kinematics, including transverse virtual photon (twist 3) contributions, and the photoproduction limit (for large $t-$channel momentum transfer)

# ERBL evolution equation

Efremov, Radyushkin, Brodsky, Lepage evolution equation for a DA

Renormalization of the bilocal operator

$$\bar{\psi}(z)\gamma^{\mu}\psi(0)$$

$\Rightarrow$ Evolution equation for the distribution amplitude in the $\overline{MS}$ scheme

$$\frac{\partial\varphi(x,\mu_F^2)}{\partial\ln\mu_F^2} = \frac{\alpha_s C_F}{2\pi}\frac{\Gamma(1-\epsilon)}{(4\pi)^{\epsilon}}\left(\frac{\mu_F^2}{\mu^2}\right)^{\epsilon}\int_0^1 dz\varphi(z,\mu_F^2)\mathcal{K}(x,z),$$

$\mathcal{K}$ = ERBL kernel

# ERBL evolution equation

Evolution equation for the distribution amplitude in the $\overline{MS}$ scheme

$$\frac{\partial \varphi(x,\mu_F^2)}{\partial \ln \mu_F^2} = \frac{\alpha_s C_F}{2\pi} \frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon} \left(\frac{\mu_F^2}{\mu^2}\right)^\epsilon \int_0^1 dz \varphi(z,\mu_F^2)\mathcal{K}(x,z),$$

where we parameterize the ERBL kernel for consistency as

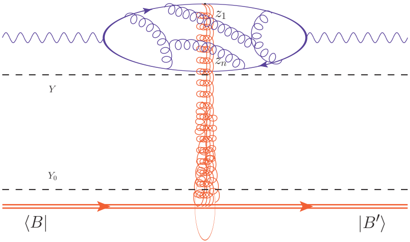
$$\begin{aligned}\mathcal{K}(x,\, z) &= \frac{x}{z}\left[1+\frac{1}{z-x}\right]\theta(z-x-\alpha) \\ &+ \frac{1-x}{1-z}\left[1+\frac{1}{x-z}\right]\theta(x-z-\alpha) \\ &+ \left[\frac{3}{2}-\ln\left(\frac{x(1-x)}{\alpha^2}\right)\right]\delta(z-x).\end{aligned}$$

It is equivalent to the usual ERBL kernel
It provides the right counterterm to obtain a finite amplitude

Practical use of such results for phenomenology

# Practical use of such results

- Compute the upper impact factor using the effective Feynman rules
- Build non-perturbative models for the matrix elements of the Wilson line operators acting on the target states
- Solve the B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity $\eta = Y_0$
- Evaluate the solution at a typical projectile rapidity $\eta = Y$, or at the rapidity of the slowest gluon (cf. Bertrand's talk)
- Convolute the solution and the impact factor




$$\mathcal{A} = \int d\vec{z}_1 ... d\vec{z}_n \; \Phi(\vec{z}_1, ..., \vec{z}_n)$$
$$\times \langle P' | U_{\vec{z}_1} ... U_{\vec{z}_n} | P \rangle$$

## Residual parameter dependence

**Required parameters**

- Renormalization scale $\mu_R$
- Factorization scale $\mu_F$ if assumed that $\mu_F \neq \mu_R$
- Typical target rapidity $Y_0$
- Typical projectile rapidity $Y$

In the linear BFKL limit, the cross section only depends on $Y - Y_0$, so one only needs one arbirary parameter $s_0$ defined by

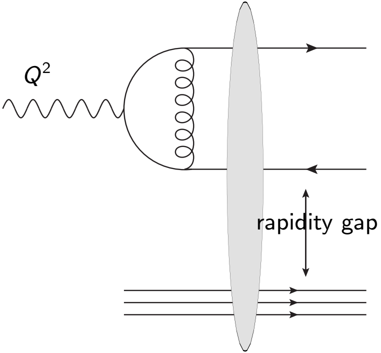
$$Y - Y_0 = \ln\left(\frac{s}{s_0}\right).$$

Modifying any of these parameter results in a higher order (NNLO) contribution

## General amplitude

Very general result

- The hard scale can be $Q^2$ or $t$
- The target can be either a proton or an ion, or another impact factor
- Finite results for $Q^2 = 0$ at large $t$
- One can study ultraperipheral collision in the limit $Q^2 \to 0$ at large $t$.
- Thus suited for HERA fitting, LHC UPC predictions, and perfectly suited for EIC studies




The general amplitude

# Theoretical issues

Two theoretical questions

- How to get to the BFKL limit at NLL?
- What about end-point singularities for the power-suppressed $\gamma_T \to \rho_L$ contribution?

# Comparison with previous results:JIMWLK/BFKL equivalence

In the forward $t = 0$ limit and in the linear BFKL limit, the $\gamma_L \to \rho_L$ impact factor was computed at NLO [Ivanov, Kotsky, Papa].

JIMWLK convolution

$$\int d^d p_1 d^d p_2 \Phi_{CGC}(p_1, p_2) \tilde{\mathcal{U}}(p_1, p_2)$$

$\tilde{\mathcal{U}}(p_1, p_2)$ dipole scattering operator

BFKL convolution

$$\int d^d q_1 d^d q_2 \Phi_{BFKL}(q_1, q_2)\, R(q_1) R(q_2)$$

$R(q)$ Reggeon field

Defining the Reggeon field in the CGC as the logarithm of a Wilson line [Caron-Huot]

$$R^a(x) \equiv \frac{f^{abc}}{gC_A} \left( \ln U_x^{adj} \right)^{bc}$$

$$U_x = 1 + igt^a R^a(x) - \frac{g^2}{2} t^a t^b R^a(x) R^b(x) + O(g^3)$$

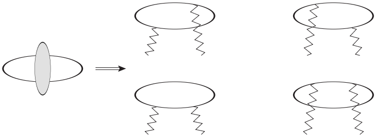
Such fields are Reggeized by the JIMWLK Hamiltonian, satisfy the BFKL equation and satisfy bootstrap equations.

# JIMWLK/BFKL equivalence

Linear limit of diffractive CGC impact factors

$$\int d^2 p_1 d^2 p_2 \, \varphi\left(p_1, p_2\right) \tilde{\mathcal{U}}\left(p_1, p_2\right)$$

$$= \frac{g^2}{4N_c} \int d^2 q_1 d^2 q_2 R^a\left(q_1\right) R^a\left(q_2\right) \left[2\varphi\left(q_1, q_2\right) - \varphi\left(q_1 + q_2, 0\right) - \varphi\left(0, q_1 + q_2\right)\right]$$



This matches our result to the leading order BFKL result.

At NLL accuracy, things are interestingly worse due to the ambiguity of distribution of radiative corrections between impact factors and kernels.

# Equivalence with BFKL at NLL accuracy

Linear limit: usual $k_t$-factorization (BFKL framework)

$s$-channel discontinuity of $A + B \to A' + B'$ scattering amplitudes

$$\delta(p_{A'} + p_{B'} - p_A - p_B) Disc_s \mathcal{A}_{AB}^{A'B'} \propto \Phi(A', A) \otimes \mathcal{K} \otimes \Phi(B', B)$$

For any non-singular operator $\mathcal{O}$ this discontinuity is invariant under

$$\Phi(A', A) \to \Phi(A', A)\, \mathcal{O}, \quad \mathcal{K} \to \mathcal{O}^{-1} \mathcal{K} \mathcal{O}, \quad \Phi(B', B) \to \mathcal{O}^{-1} \Phi(B', B)$$

i.e. there is an ambiguity of distribution of corrections between the impact factors and the kernel. In the linear approximation of BK there exists an operator $\mathcal{O}$ such that

$$\Phi_{BK} \otimes \mathcal{K}_{BK} \otimes \Phi_{BK} = (\Phi_{BFKL} \otimes \mathcal{O}) \otimes (\mathcal{O}^{-1} \otimes \mathcal{K}_{BFKL} \otimes \mathcal{O}) \otimes (\mathcal{O}^{-1} \otimes \Phi_{BFKL})$$

The expression for $\mathcal{O}$ to make the kernels explicitly equivalent at NLO accuracy under such a change of variables is known [Fadin, Fiore, Grabovsky, Papa]
Comparing our NLL CGC impact factor with the NLL BFKL impact factor should confirm this expression.

# End point singularities and factorization

## End point singularities?

Leading order impact factor for, respectively, $\gamma_L^* \to V_L$ and $\gamma_T^* \to V_L$ transitions:

$$\Phi_L^{(0)} = \frac{2x\bar{x}p_V^+ Q}{(\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2},$$

$$\Phi_T^{(0)} = -\frac{(x-\bar{x})p_V^+(\bar{x}\vec{p}_{1\perp} - x\vec{p}_{2\perp})\cdot\vec{\varepsilon}_{\gamma_T}}{(\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2}$$

No end point singularity, even for a transverse photon and even in the photoproduction limit and even at NLO.

With null transverse momenta in the $t$ channel, one could encounter $x \in \{0, 1\}$ end point singularities as $\frac{1}{x\bar{x}Q^2}$ thus breaking collinear factorization.

## Twist 3 production

# Production of a transverse light vector meson

Non-forward and non-dilute extension of
[Anikin, Besse, Ivanov, Pire, Szymanowski, Wallon]

## What changes in the CGC rules

Effective CGC Feynman rules for fields

The recursion to exponentiate slow gluon scatterings into a Wilson line only starts at order $g_s$

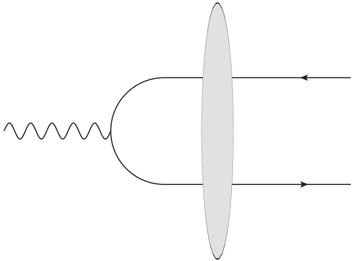
$$A^{\mu}_{eff}(z_0)|_{z_0^+<0} = A^{\mu}(z_0) - 2i\int d^D z_3\, \delta\left(z_3^+\right) G^{\mu}_{\sigma_\perp}(z_{30}) \left(U^{ba}_{\vec{z}_3} - \delta^{ba}\right) F^{+\sigma_\perp}(z_3)$$

$$\overline{\psi}_{eff}(z_0)|_{z_0^+<0} = \bar{\psi}(z_0) + \int d^D z_1\, \delta\left(z_1^+\right) \overline{\psi}(z_1)(U_{\vec{z}_1} - 1)\gamma^+ G(z_{10})$$

$$\psi_{eff}(z_0)|_{z_0^+<0} = \psi(z_0) - \int d^D z_2 \delta\left(z_2^+\right) G(z_{02})\gamma^+ \psi(z_2)\left(U^{\dagger}_{\vec{z}_2} - 1\right)$$

## 2-body diagrams

### Natural 2-body CGC diagram

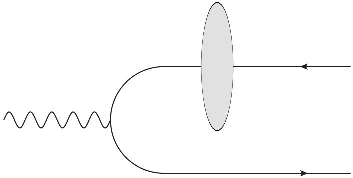


$$\mathcal{A}_{q\bar{q}}^{2b} = \int d^2\vec{z}_1 d^2\vec{z}_2 \; \Phi_{q\bar{q}}^{2b}(\vec{z}_1, \vec{z}_2)\, \mathrm{Tr}[(U_1 - 1)(U_2^\dagger - 1)]$$

Contains monopole contributions

# 2-body diagrams

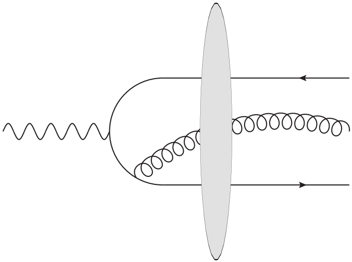
## Antiquark monopole 2-body diagram



$$\mathcal{A}_{\bar{q}}^{2b} = \int d^2\vec{z}_2 \; \Phi_{\bar{q}}^{2b}(\vec{z}_2)\,\mathrm{Tr}[(U_2^\dagger - 1)]$$

## 3-body diagrams

### Natural 3-body CGC diagram



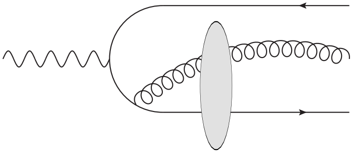
$$\mathcal{A}^{3b}_{q\bar{q}g} = \int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3\ \Phi^{3b}_{q\bar{q}g}(\vec{z}_1, \vec{z}_2, \vec{z}_3)\, \mathrm{Tr}[(U_1 - 1)t^b(U_2^\dagger - 1)t^a](U_3^{ab} - \delta^{ab})$$

Contains dipole and monopole contributions

Double-dipole term even at tree level ⇒ Great sensitivity to saturation

# 3-body diagrams

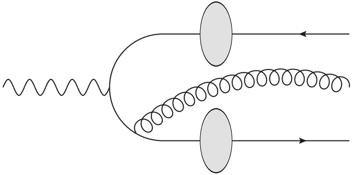
## 3-body ($\bar{q}g$)-dipole diagram



$$\mathcal{A}_{\bar{q}g}^{3b} = \int d^2\vec{z}_2 d^2\vec{z}_3 \; \Phi_{\bar{q}g}^{3b}(\vec{z}_2, \vec{z}_3) \, \mathrm{Tr}[t^b(U_2^\dagger - 1)t^a](U_3^{ab} - \delta^{ab})$$

# 3-body diagrams

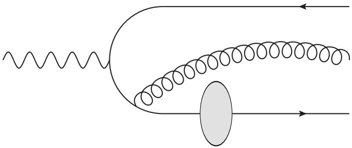
## 3-body $(q\bar{q})$-dipole diagram



$$\mathcal{A}_{q\bar{q}}^{3b} = \int d^2\vec{z}_1 d^2\vec{z}_2 \ \Phi_{q\bar{q}}^{3b}\left(\vec{z}_1, \vec{z}_2\right) \mathrm{Tr}[(U_1 - 1)t^b(U_2^\dagger - 1)t^a]\delta^{ab}$$

# 3-body diagrams

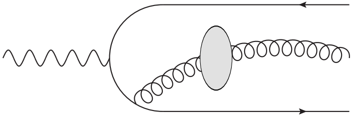
## 3-body $(q)$-monopole diagram



$$\mathcal{A}_q^{3b} = \int d^2\vec{z}_1 \Phi_q^{3b}(\vec{z}_1)\,\mathrm{Tr}[(U_1 - 1)t^b t^a]\delta^{ab}$$

# 3-body diagrams

## 3-body ($g$)-monopole diagram



$$\mathcal{A}_g^{3b} = \int d^2\vec{z}_3\, \Phi_g^{3b}(\vec{z}_3)\, \mathrm{Tr}[t^b t^a](U_3^{ab} - \delta^{ab})$$

# Cancelling the 2-body monopoles

## 2-body diagrams

### Antiquark monopole part of the natural CGC diagram

- Monopole part of the quark line

$$\overline{\psi}_{eff}\left(z_0\right)|_{z_0^+<0} = \bar{\psi}\left(z_0\right) + \int d^D z_1\, \delta\left(z_1^+\right) \overline{\psi}\left(z_1\right)\left(U_{\vec{z}_1} - 1\right)\gamma^+ G\left(z_{10}\right)$$

- Simple algebra allows one to get

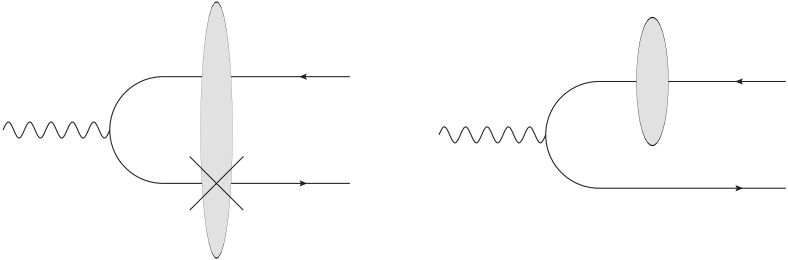
$$\int d^D z_1 \int \frac{d^D q}{(2\pi)^D}\delta\left(z_1^+\right)\left(\frac{-i\bar{\psi}\left(z_1\right)}{\left(q^- - \frac{\vec{q}^2 - i0}{2q^+}\right)} + \frac{\bar{\psi}\left(z_1\right)\overleftarrow{\partial}\gamma^\mu\gamma^+}{2q^+\left(q^- - \frac{\vec{q}^2 - i0}{2q^+}\right)}\right) e^{-i(q\cdot z_{10})}$$

- Thus one term contributes to a 2-body monopole contribution, and (Dirac equation) the other term contributes to a 3-body monopole contribution.

## 2-body diagrams

### Sum of the 2-body antiquark monopoles

Using the fact that the non-perturbative collinear matrix elements do not depend on $z^+$ variables at twist 3 accuracy ...[censored technicalities]... we get the sum between the natural 2-body antiquark monopole diagram and the 2-body antiquark monopole part of the natural CGC diagram



$$\frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)} - \frac{\vec{q}^2}{2q^+}} - \frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)} - 0}$$

## 2-body diagrams

### Sum of the 2-body antiquark monopoles

Using the fact that the non-perturbative collinear matrix elements are at most linear in $z_\perp$, the sum cancels iff

$$\left.\frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)} - \frac{\vec{q}^2}{2q^+}} - \frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)}}\right|_{\vec{q}=\vec{0}} = 0$$

$$\frac{\partial}{\partial q_\perp^\mu}\left.\left(\frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)} - \frac{\vec{q}^2}{2q^+}} - \frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)}}\right)\right|_{\vec{q}=\vec{0}} = 0$$

# Cancelling the 3-body unnatural dipoles and monopoles

# 3-body diagrams

"Unnatural" 3-body diagrams

$$\Phi_{qg}\left(\vec{z}_1,\vec{z}_3\right)=\int d^2\vec{z}_2\,\Phi_{q\bar{q}g}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right)+\mathrm{Twist}\,4$$

$$\Phi_{\bar{q}g}\left(\vec{z}_2,\vec{z}_3\right)=\int d^2\vec{z}_1\,\Phi_{q\bar{q}g}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right)+\mathrm{Twist}\,4$$

$$\Phi_{q\bar{q}}\left(\vec{z}_1,\vec{z}_2\right)=\int d^2\vec{z}_3\,\Phi_{q\bar{q}g}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right)+\mathrm{Twist}\,4$$

$$\Phi_{g}\left(\vec{z}_3\right)=\int d^2\vec{z}_1 d^2\vec{z}_2\,\Phi_{q\bar{q}g}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right)+\mathrm{Twist}\,4$$

$$\Phi_{q}\left(\vec{z}_1\right)=\int d^2\vec{z}_2 d^2\vec{z}_3\,\Phi_{q\bar{q}g}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right)+\mathrm{Twist}\,4$$

Hence the 3-body total from 3-body diagrams

$$\mathcal{A}_3^{3b}=\int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3\,\Phi_{q\bar{q}g}^{3b}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right)$$
$$\times\left[\mathrm{Tr}(U_1 t^b U_2^\dagger t^a)U_3^{ab}-\mathrm{Tr}(t^b U_2^\dagger t^a \delta^{ab})\right]$$

# 3-body diagrams

Total from 3-body diagrams

$$\mathcal{A}^{3b} = \int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3 \, \Phi^{3b}_{q\bar{q}g}\left(\vec{z}_1, \vec{z}_2, \vec{z}_3\right) \times [\mathrm{Tr}(U_1 t^b U_2^\dagger t^a) U_3^{ab} - \mathrm{Tr}(t^b U_2^\dagger t^a \delta^{ab})]$$

"3-body" antiquark monopole from the natural 2-body diagram

$$\Phi_2^{3b}\left(\vec{z}_2\right) = \int d^2\vec{z}_1 d^2\vec{z}_3 \, \Phi_{q\bar{q}g}\left(\vec{z}_1, \vec{z}_2, \vec{z}_3\right) + \mathrm{Twist}\ 4$$

Sums up to a gauge invariant amplitude

$$\mathcal{A}^{3b} = \int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3 \, \Phi^{3b}_{q\bar{q}g}\left(\vec{z}_1, \vec{z}_2, \vec{z}_3\right) \times [\mathrm{Tr}(U_1 t^b U_2^\dagger t^a) U_3^{ab} - C_F]$$

# QCD gauge invariance

## Final amplitude

$$\mathcal{A} = \int d^2\vec{z}_1 d^2\vec{z}_2\, \Phi^{2b}_{q\bar{q}}(\vec{z}_1,\vec{z}_2) \left[\mathrm{Tr}\left(U_1 U_2^\dagger\right) - N_c\right]$$
$$+ \int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3\, \Phi^{3b}_{q\bar{q}g}(\vec{z}_1,\vec{z}_2,\vec{z}_3) \left[\mathrm{Tr}\left(U_1 t^b U_2^\dagger t^a\right) U_3^{ab} - C_F\right]$$

Expansion in Reggeons in the dilute limit: (Reggeon momenta $q_1, q_2$)

$$\Phi_{BFKL} = \int d^2\vec{z}_1 d^2\vec{z}_2\, \Phi^{2b}_{q\bar{q}}(\vec{z}_1,\vec{z}_2) \left(e^{i(\vec{q}_1\cdot\vec{z}_2)} - e^{i(\vec{q}_1\cdot\vec{z}_1)}\right)\left(e^{i(\vec{q}_2\cdot\vec{z}_1)} - e^{i(\vec{q}_2\cdot\vec{z}_2)}\right)$$
$$- \int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3 \Phi^{3b}_{q\bar{q}g}(\vec{z}_1,\vec{z}_2,\vec{z}_3) \left[N_c \left(e^{i(\vec{q}_1\cdot\vec{z}_3)} - e^{i(\vec{q}_1\cdot\vec{z}_1)}\right)\left(e^{i(\vec{q}_2\cdot\vec{z}_3)} - e^{i(\vec{q}_2\cdot\vec{z}_2)}\right)\right.$$
$$\left. - \left(\frac{N_c^2-1}{2N_c}\right)\left(e^{i(\vec{q}_1\cdot\vec{z}_2)} - e^{i(\vec{q}_1\cdot\vec{z}_1)}\right)\left(e^{i(\vec{q}_2\cdot\vec{z}_1)} - e^{i(\vec{q}_2\cdot\vec{z}_2)}\right)\right]$$

Obviously gauge invariant in the BFKL sense: $\Phi_{BFKL} = 0$ for $q_1 = 0$ or $q_2 = 0$.

In the dilute, forward limit, our result matches the previous BFKL results

# Conclusion

- We provided the full computation of the impact factors for two exclusive diffractive processes at NLO accuracy, as well as the impact factor for a twist 3 process.
- In the linear limit, our NLO result will provide the consistency check of the JIMWLK/BFKL correspondence
- The computation can be adapted for twist 3 NLO production in the Wandzura-Wilczek approximation, removing factorization breaking end-point singularities even at NLO
- Exclusive diffractive processes are perfectly suited for precision saturation physics and gluon tomography with $b_\perp$ dependence at the EIC. Dijets probe the dipole Wigner distribution at small $x$ [Hatta, Xiao, Yuan], $\rho$ meson probes gluon GPDs at small $x$.