TMD Phenomenology:

Recent developments on polarized TMD global analyses

Mariaelena Boglione

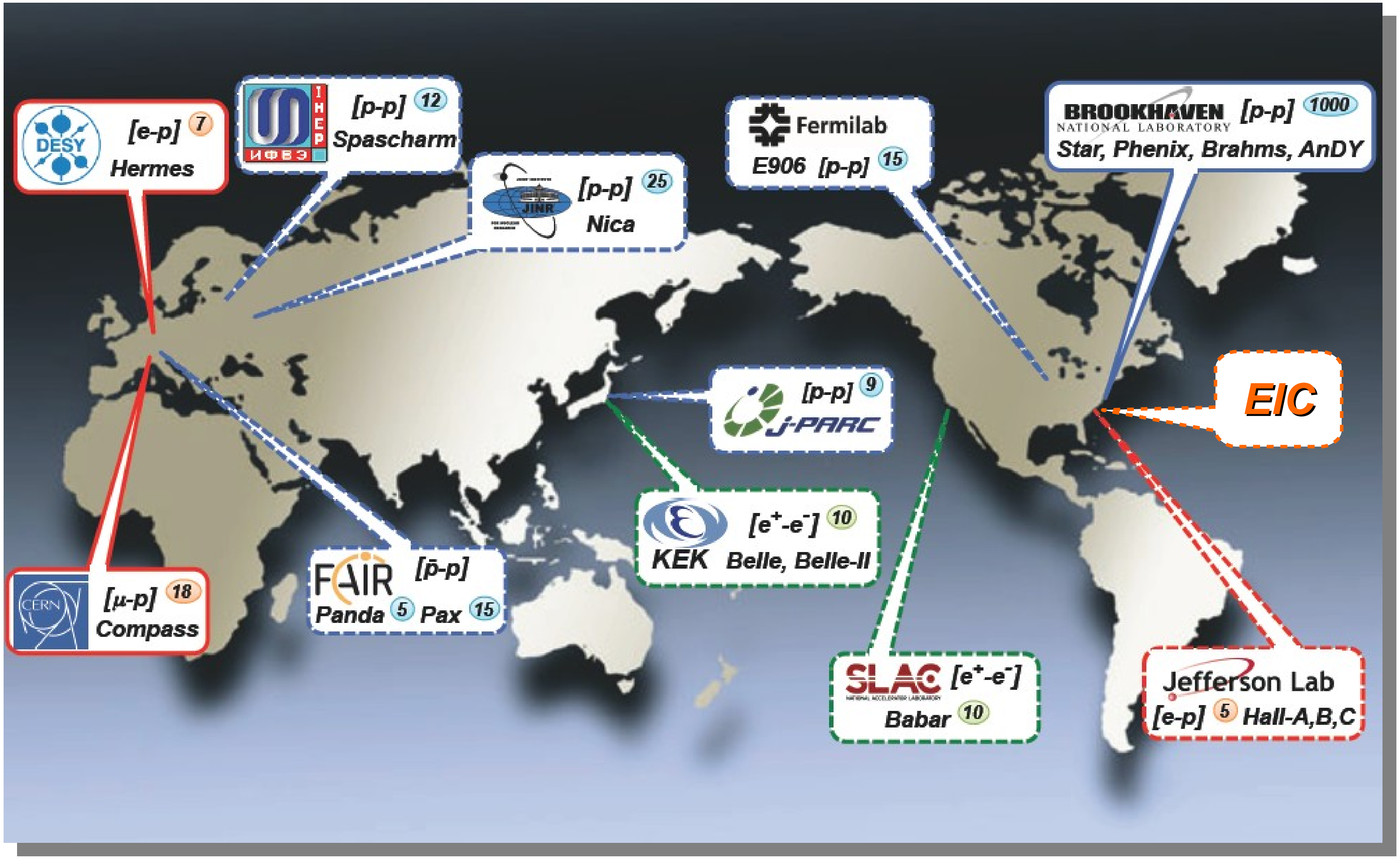
INFN Torino

UNIVERSITÀ DEGLI STUDI DI TORINO
ALMA UNIVERSITAS TAURINENSIS

Scuola di Scienze della Natura

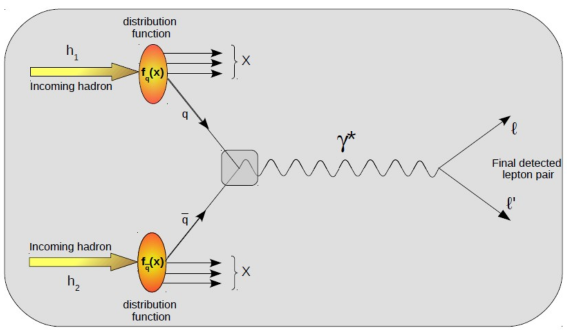
Where can we learn about the 3D structure of matter ?

Where can we learn about the 3D structure of matter ?



Experimental data for TMD studies

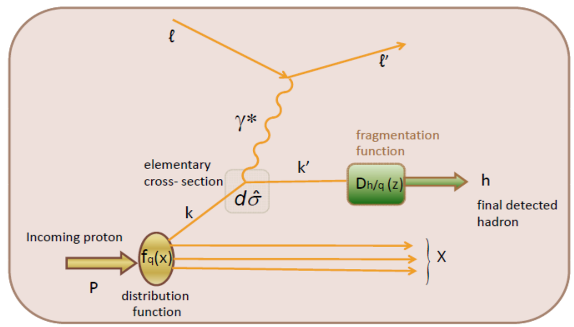
Unpolarized and Polarized Drell-Yan scattering



$$\sigma_{Drell-Yan} = f_q(x,k_\perp) \otimes f_{\bar{q}}(x,k_\perp) \otimes \hat{\sigma}^{q\bar{q}\to\ell\ell'}$$

Allows extraction of distribution functions

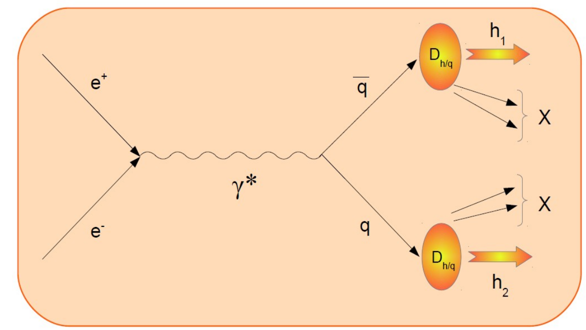
Unpolarized and Polarized SIDIS scattering



$$\sigma_{SIDIS} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

Allows extraction of distribution and fragmentation functions

$e^+ e^- \to h_1 h_2 X$

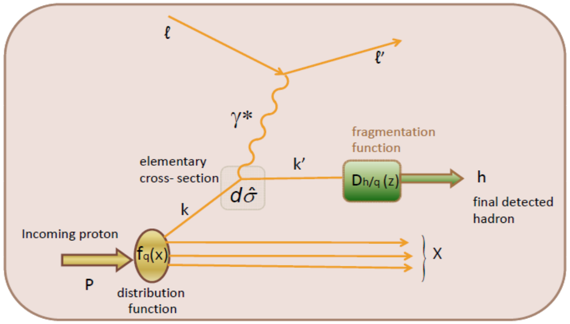


$$\sigma_{h1\,h2} \propto D(z_1) \otimes D(z_2) \otimes \hat{\sigma}$$

Allows extraction of fragmentation functions

Experimental data for TMD studies

Unpolarized and Polarized SIDIS scattering



$$\sigma_{SIDIS} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

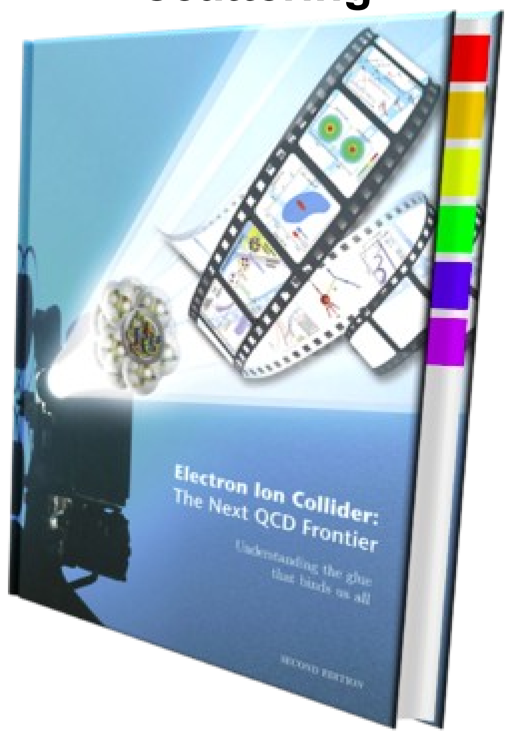
Allows extraction of distribution and fragmentation functions

Experimental data for TMD studies

Jefferson Lab
EXPLORING THE NATURE OF MATTER

e- JLab12

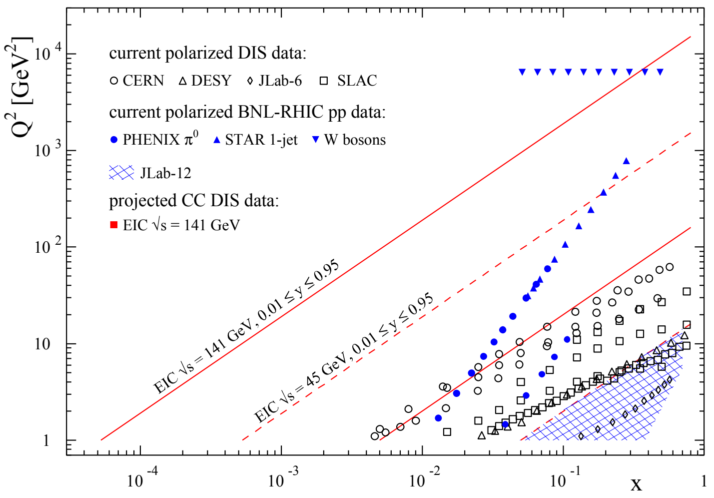
Unpolarized and Polarized SIDIS scattering



hermes

COMPASS

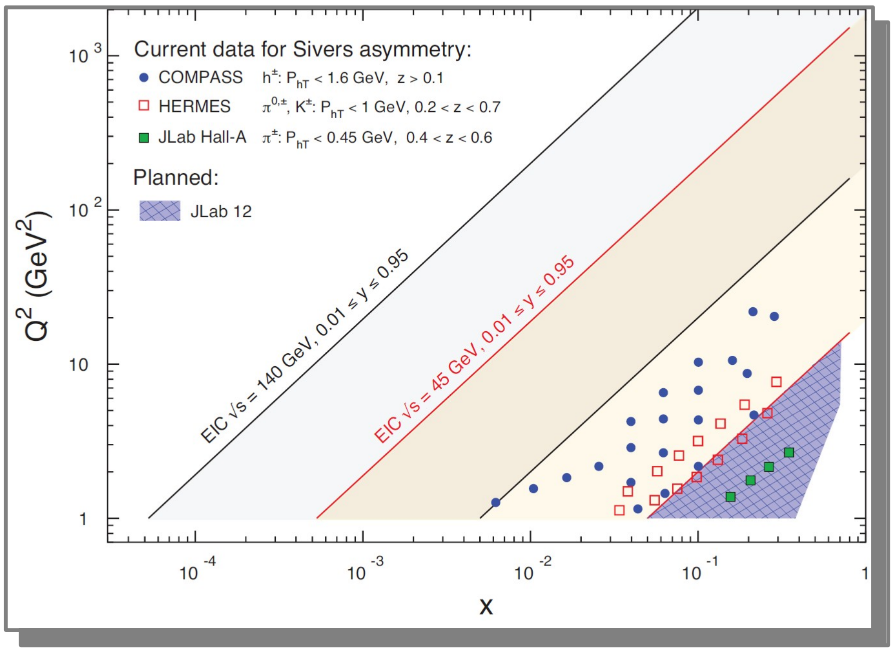
EIC kinematic coverage



EIC will extend Q^2 coverage

EIC will extend x coverage

Plot from E. Aschenauer @ SPIN 2016

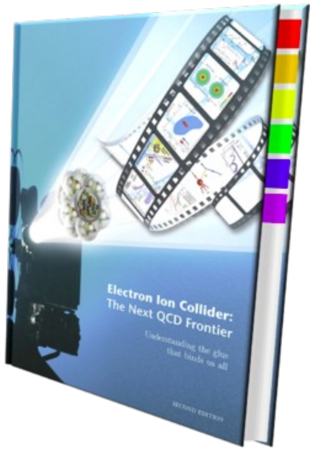


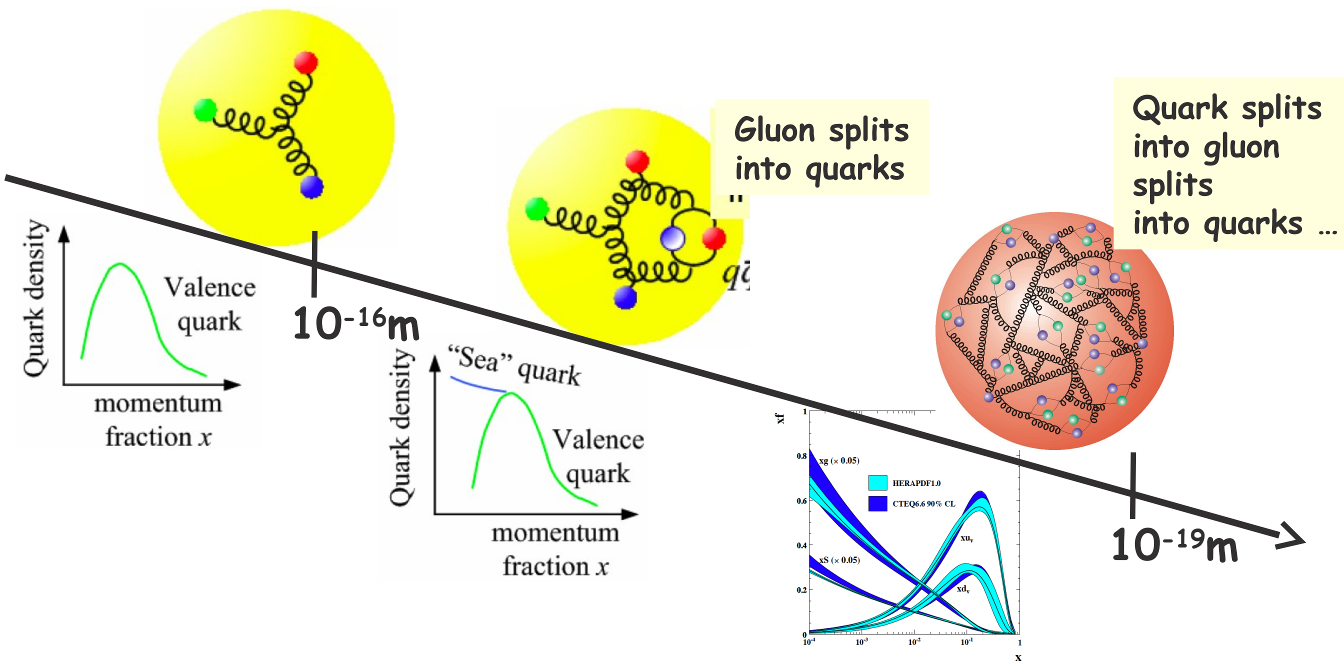
EIC will extend Q^2 coverage

EIC will extend x coverage

Plot from the EIC white book

EIC kinematics coverage



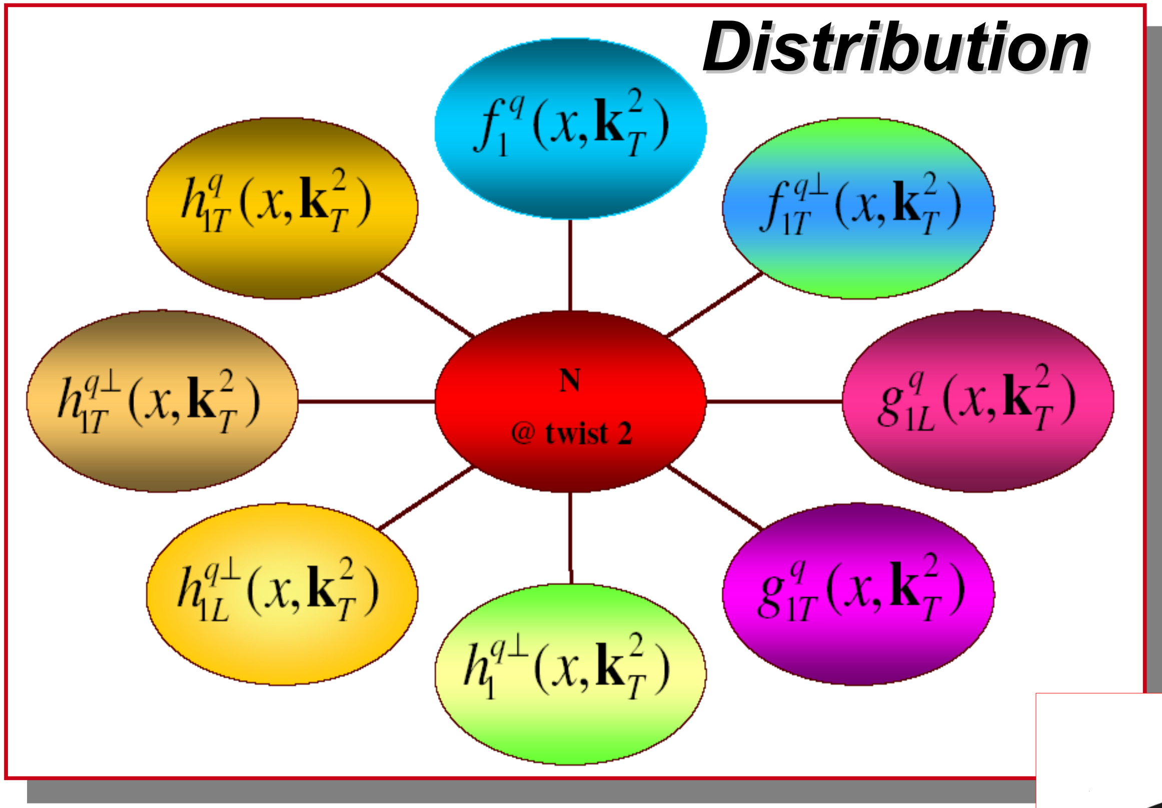


Higher √s and Q^2 values will increase resolution

Plot from E. Aschenauer @ SPIN 2016

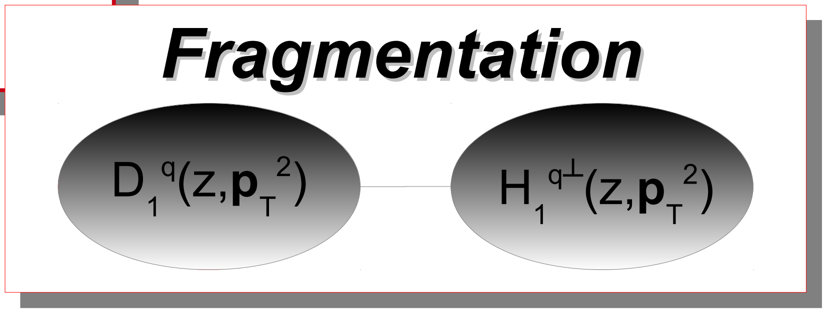
Transverse momentum dependent parton distribution and fragmentation functions

TMD distribution and fragmentation functions



Correlations between spin and transverse momentum

(orbital motion)

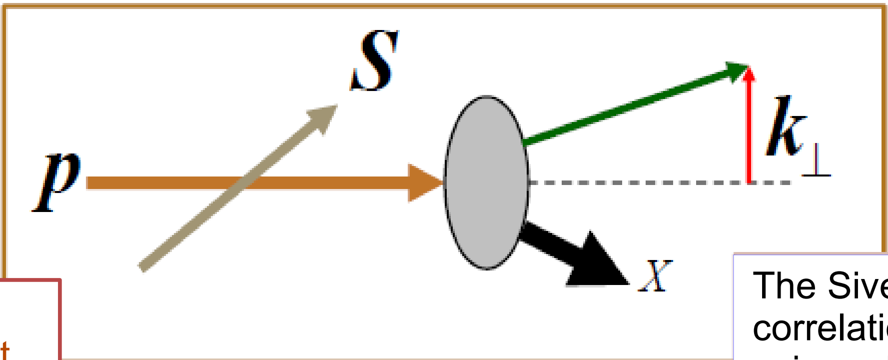


Extracting polarized TMDs from SIDIS data: the Sivers function

The Sivers Distribution Function

$$f_{q/p,S}(x,k_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$
$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

The Sivers function is related to the probability of finding an unpolarized quark inside a transversely polarized proton

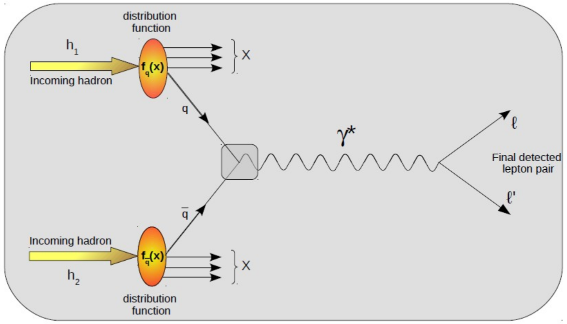


The Sivers function, is particularly interesting, as it provides information on the partonic orbital angular momentum

The Sivers function embeds correlations between proton spin and quark transverse momentum

Where do we learn about the Sivers function?

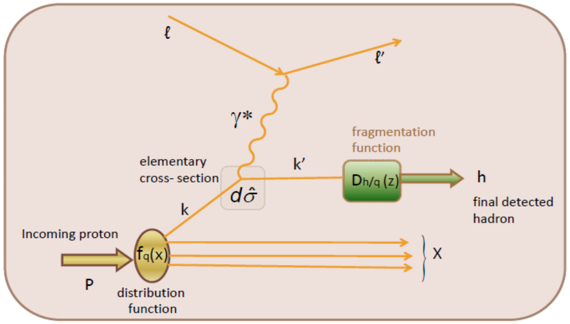
Polarized Drell-Yan scattering



$$\sigma_{Drell-Yan} = f_q(x,k_\perp) \otimes f_{\bar{q}}(x,k_\perp) \otimes \hat{\sigma}^{q\bar{q}\to\ell\ell'}$$

Allows extraction of distribution functions

Polarized SIDIS scattering



$$\sigma_{SIDIS} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

Allows extraction of distribution and fragmentation functions

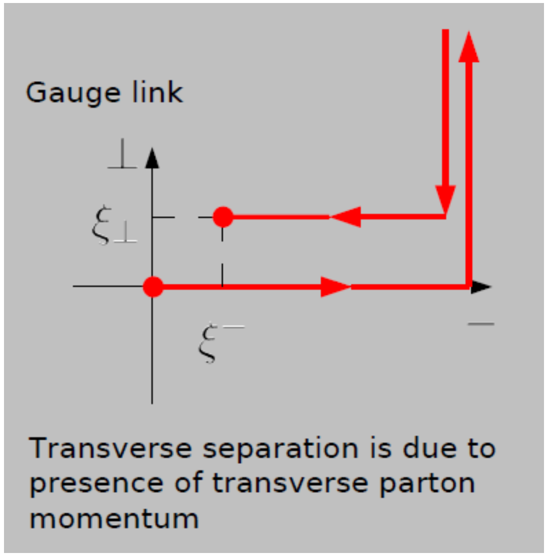
Fermilab BROOKHAVEN NATIONAL LABORATORY COMPASS hermes Jefferson Lab COMPASS

Sivers function sign change

- TMDs have to be defined in a color-gauge invariant way

$$\Phi_{ij}(x,\mathbf{k}_\perp) = \int \frac{\mathbf{d}\xi^-}{(\mathbf{2}\pi)} \frac{\mathbf{d^2}\xi_\perp}{(\mathbf{2}\pi)^\mathbf{2}} \mathbf{e}^{\mathbf{ixP}^+\xi^-} \, \mathbf{e}^{-\mathbf{ik}_\perp\xi_\perp} \langle \mathbf{P},\mathbf{S_P}|\bar{\psi}_\mathbf{j}(\mathbf{0})\,\mathcal{U}(\mathbf{0},\xi)\,\psi_\mathbf{i}(\xi)|\mathbf{P},\mathbf{S_P}\rangle\Big|_{\xi^+=\mathbf{0}}$$

- The struck quark propagates in the gauge field of the remnant and forms gauge links

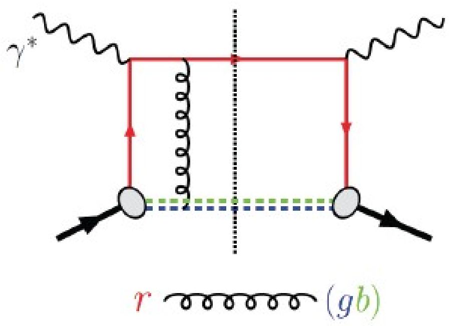


- Gauge links generate initial and final state interactions

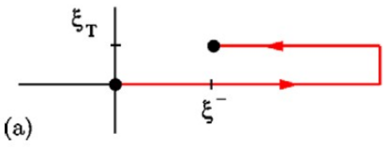
Sivers function sign change

SIDIS

- The gluon couples to the proton remnant after the quark is scattered
- Attractive final state interaction

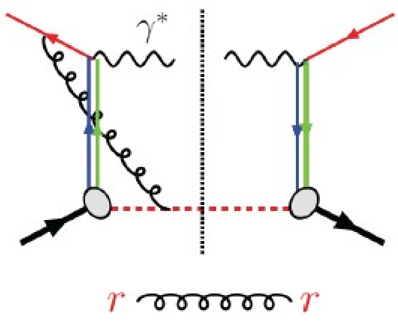


Attractive

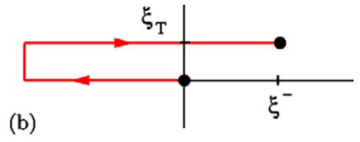


DRELL YAN

- The gluon couples before the quark annihilates
- Repulsive initial state interaction



Repulsive



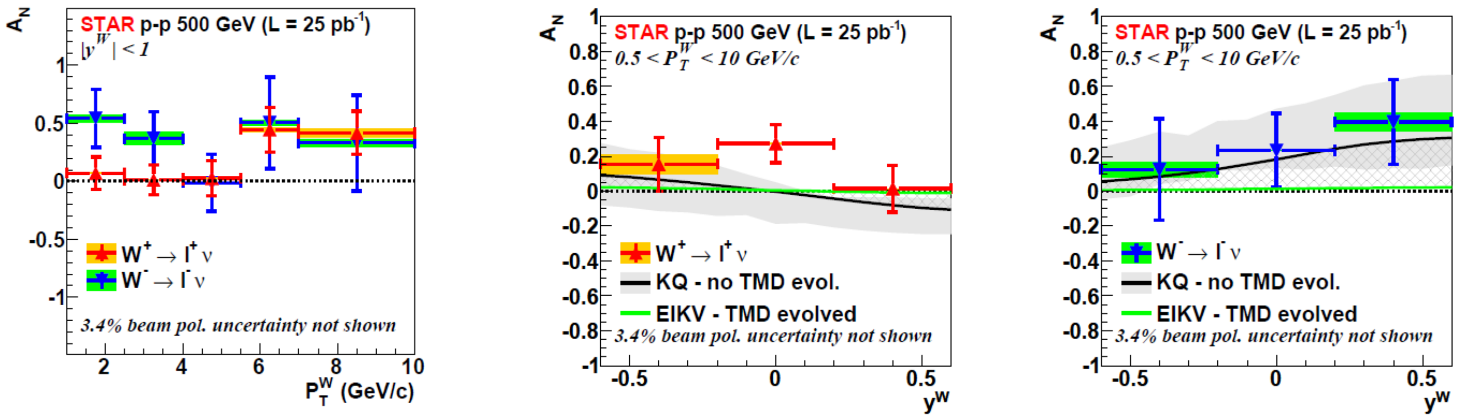
The Sivers function is process dependent: it reverses its sign when measured in SIDIS w.r.t Drell Yan processes

$$[f_{1T}^{q\perp}]_{\rm SIDIS} = -[f_{1T}^{q\perp}]_{\rm DY}$$

First hints of sign change

Sivers function in $p^{\uparrow} + p \to W^{\pm}/Z$ @ RHIC

STAR Collaboration, Phys. Rev. Lett. 116 132301 (2016)



$$A_N^W = \frac{d\sigma^{p\to WX} - d\sigma^{p\to WX}}{d\sigma^{p\to WX} + d\sigma^{p\to WX}} \equiv \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

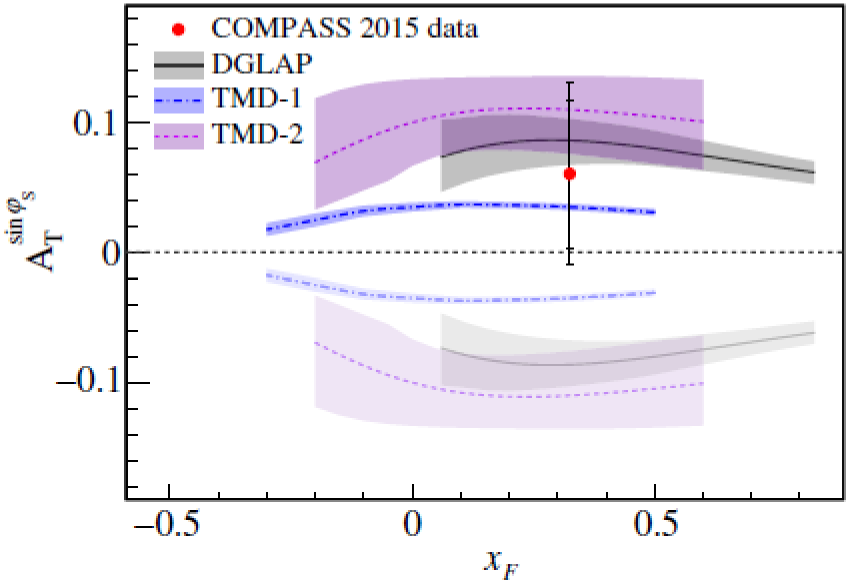
Sivers function

$$= \frac{\sum_{q_1,q_2} |V_{q_1,q_2}|^2 \int d^2\boldsymbol{k}_{\perp 1}\, d^2\boldsymbol{k}_{\perp 2}\, \delta^2(\boldsymbol{k}_{\perp 1} + \boldsymbol{k}_{\perp 2} - \boldsymbol{q}_T)\, \boldsymbol{S}\cdot(\hat{\boldsymbol{p}}_1 \times \hat{\boldsymbol{k}}_{\perp 1})\, \Delta^N f_{q_1/p^{\uparrow}}(x_1, k_{\perp 1})\, f_{q_2/p}(x_2, k_{\perp 2})}{2\sum_{q_1,q_2} |V_{q_1,q_2}|^2 \int d^2\boldsymbol{k}_{\perp 1}\, d^2\boldsymbol{k}_{\perp 2}\, \delta^2(\boldsymbol{k}_{\perp 1} + \boldsymbol{k}_{\perp 2} - \boldsymbol{q}_T)\, f_{q_1/p}(x_1, k_{\perp 1})\, f_{q_2/p}(x_2, k_{\perp 2})}$$

Sivers single spin asymmetry in pion induced Drell Yan @ COMPASS

COMPASS Collaboration, Phys. Rev. Lett. 119, 112002 (2017)

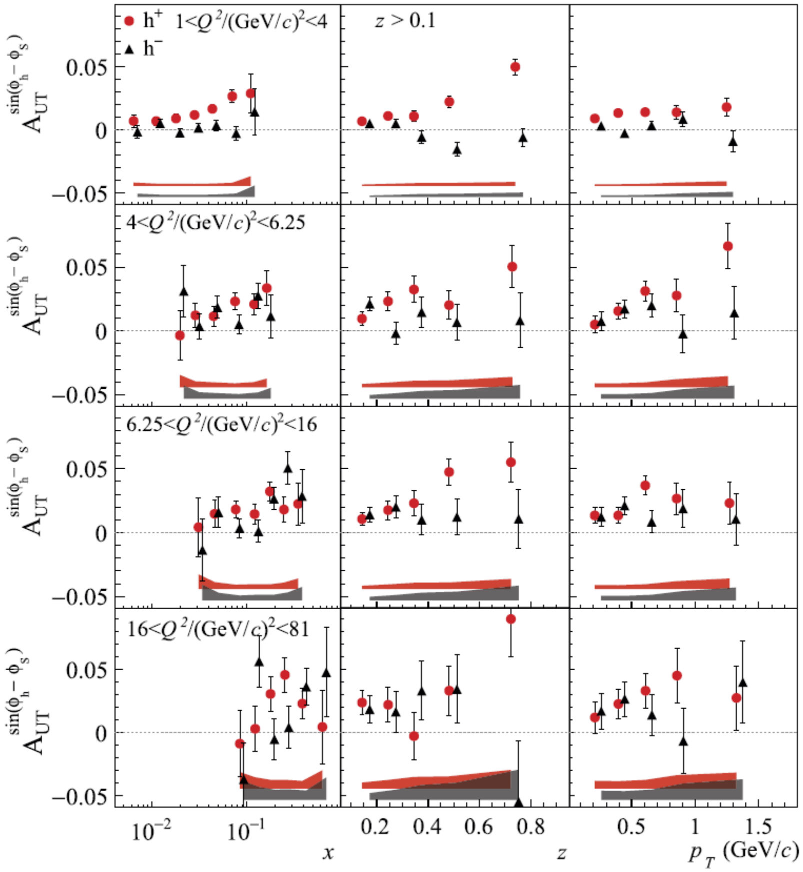
190GeV/c π- beam scattered off a transversely polarized NH3 target (polarized proton)



Sivers single spin asymmetry in SIDIS at the hard scales of Drell Yan @ COMPASS

New COMPASS Sivers data (higher statistics, higher precision, multidimensional binning) require a **new phenomenological extraction of the Sivers function** (more detailed estimation of uncertainties, evaluation of the bias induced by parametric form, study of Q^2 scale dependence)

COMPASS Collaboration, Phys. Lett. B 770, 138 (2017)



New, comprehensive study of the Sivers effect

Extraction of Sivers functions from SIDIS data

Anselmino, Boglione, D'Alesio, Murgia, Prokudin, JHEP 1704 (2017) 046

Unpolarized TMD PDF

$$f_{q/p}(x,k_\perp) = f_q(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

Unpolarized TMD FF

$$D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{1}{\pi\langle p_\perp^2\rangle}\,e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

OLD MODEL

Sivers function

$$\begin{cases}
\Delta^N f_{q/}(x,k_\perp) = 2\,\mathcal{N}_q(x)\,h(k_\perp)\,f_{q/p}(x,k_\perp) \\
h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1}\,e^{-k_\perp^2/M_1^2} \\
\mathcal{N}_q(x) = N_q\;x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}} \\
\mathcal{N}_{\bar q}(x) = N_{\bar q}
\end{cases}$$

Sivers function parametrized in terms of the unpolarized PDF

Sivers width parametrized starting from unpolarized width

New extraction of the Sivers function

Boglione, Gonzalez, Flore, D'Alesio, JHEP 1807 (2018) 148

New parametrization of the Sivers function

$$\Delta^N f_{q/p^\uparrow} = 4 N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{M_p}{\langle k_\perp^2 \rangle_S} k_\perp \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S}$$

First moment of the Sivers fn.
Flavour dependent (u_v, d_v)

$k_\perp$ dependence of the Sivers fn.
Flavour independent

Sivers functions not proportional to TMD PDFs

No direct control on the positivity bound

M_p is a fixed parameter to give the right dimensions. It is fixed to 0.938 GeV

- In perspective: parametrization in terms of momentum better suited for the study of TMD evolution
- It makes the expression of the actual Sivers asymmetry as simple as possible (within this model)

Sivers Asymmetry (numerator)

$$F_{UT}^{\sin(\phi_S-\phi_h)} = 2 \frac{z P_T M_p}{\langle P_T^2 \rangle_S} \frac{e^{-P_T^2/\langle P_T^2 \rangle_S}}{\pi \langle P_T^2 \rangle_S} \sum_q e_q^2 \Big(N_q x^{\alpha_q} (1-x)^{\beta_q} \Big) D_{h/q}(z)$$

New extraction of the Sivers function

Boglione, Gonzalez, Flore, D'Alesio, JHEP 1807 (2018) 148

New parametrization of the Sivers function

Sivers functions not proportional to TMD PDFs

$$\Delta^N f_{q/p^\uparrow} = 4 N_q x^{\alpha_q}(1-x)^{\beta_q} \frac{M_p}{\langle k_\perp^2 \rangle_S} k_\perp \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S}$$

No direct control on the positivity bound

First moment of the Sivers fn.
Flavour dependent (u_v, d_v)

$k_\perp$ dependence of the Sivers fn.
Flavour independent

M_p is a fixed parameter to give the right dimensions.
It is fixed to 0.938 GeV

- In perspective: parametrization in terms of momentum better suited for the study of TMD evolution
- It makes the expression of the actual Sivers asymmetry as simple as possible (within this model)

First moment of the Sivers function

$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) = \int d^2 k_\perp \frac{k_\perp}{4 M_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp)$$

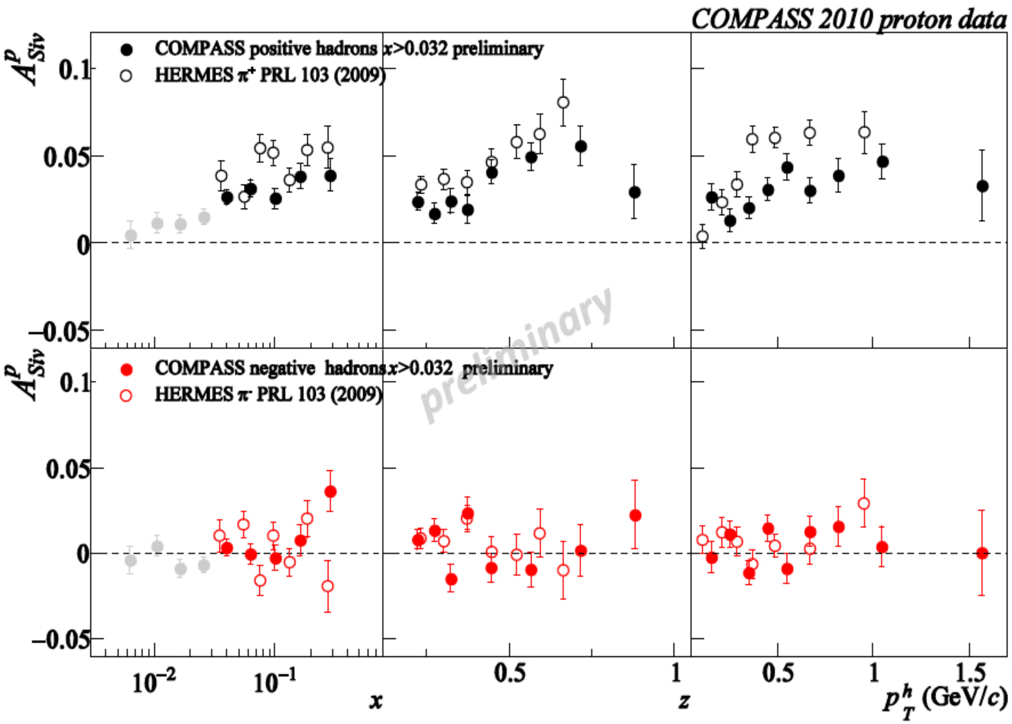
$$\Delta^N f^{(1)}_{q/p^\uparrow} = N_q x^{\alpha_q}(1-x)^{\beta_q}$$

Before attempting any "global fitting" we have to check data for compatibility

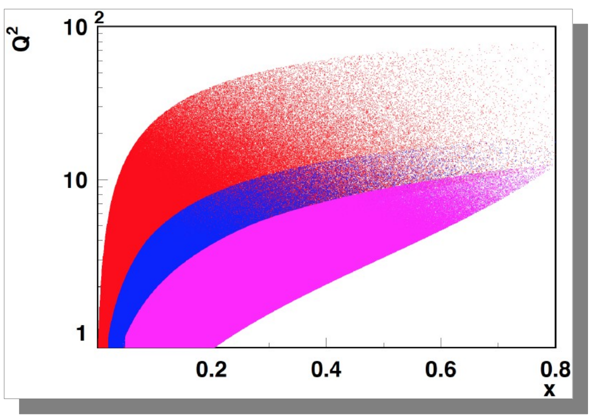
Sivers effect: COMPASS vs. HERMES

Boglione, Gonzalez, Flore, D'Alesio, JHEP 1807 (2018) 148

Apparently …
some tension between
COMPASS and HERMES data



However, COMPASS and HERMES span different ranges in Q^2 and have different $< Q^2 >$.



Kinematics effects
Possible signal of TMD evolution?

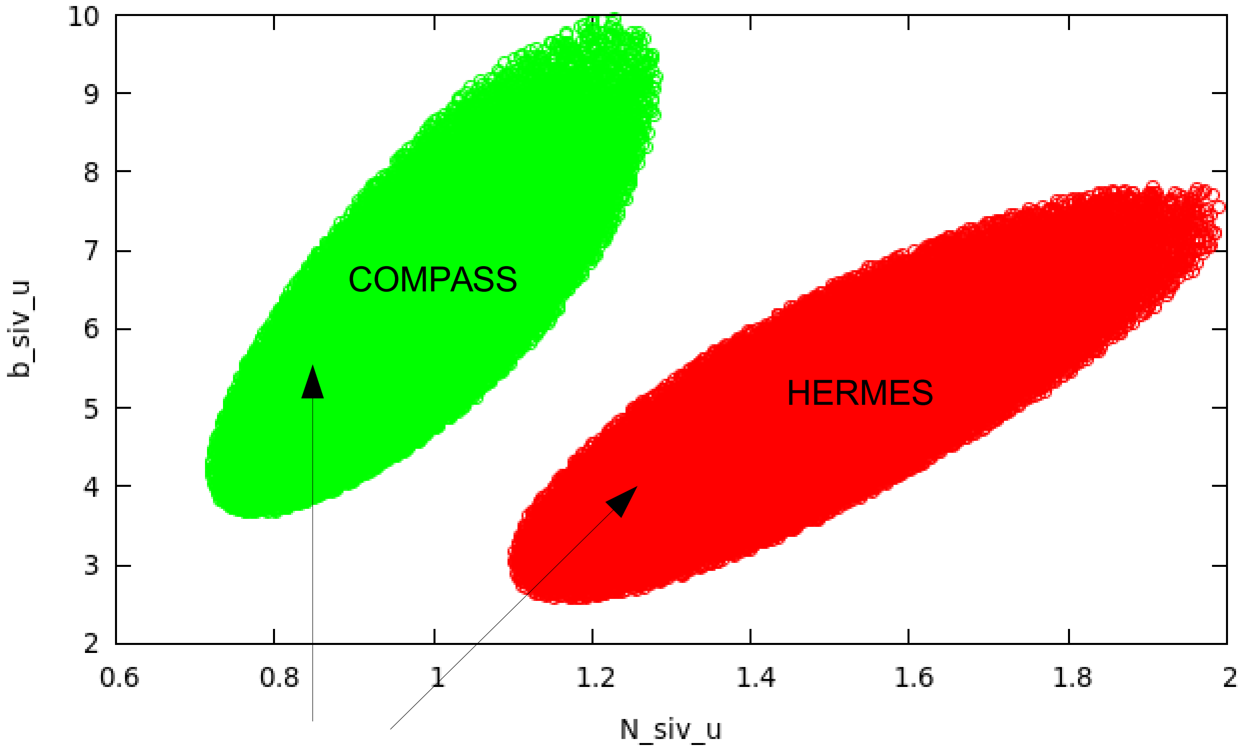
About unpolarized TMDs ...

Boglione, Gonzalez, Flore, D'Alesio, JHEP 1807 (2018) 148

Signal of some tension between independent fit solutions for COMPASS and HERMES data

Start by using a very simple model only 5 parameters and no Q evolution

Use only u flavour and only π+ data



Independent fits

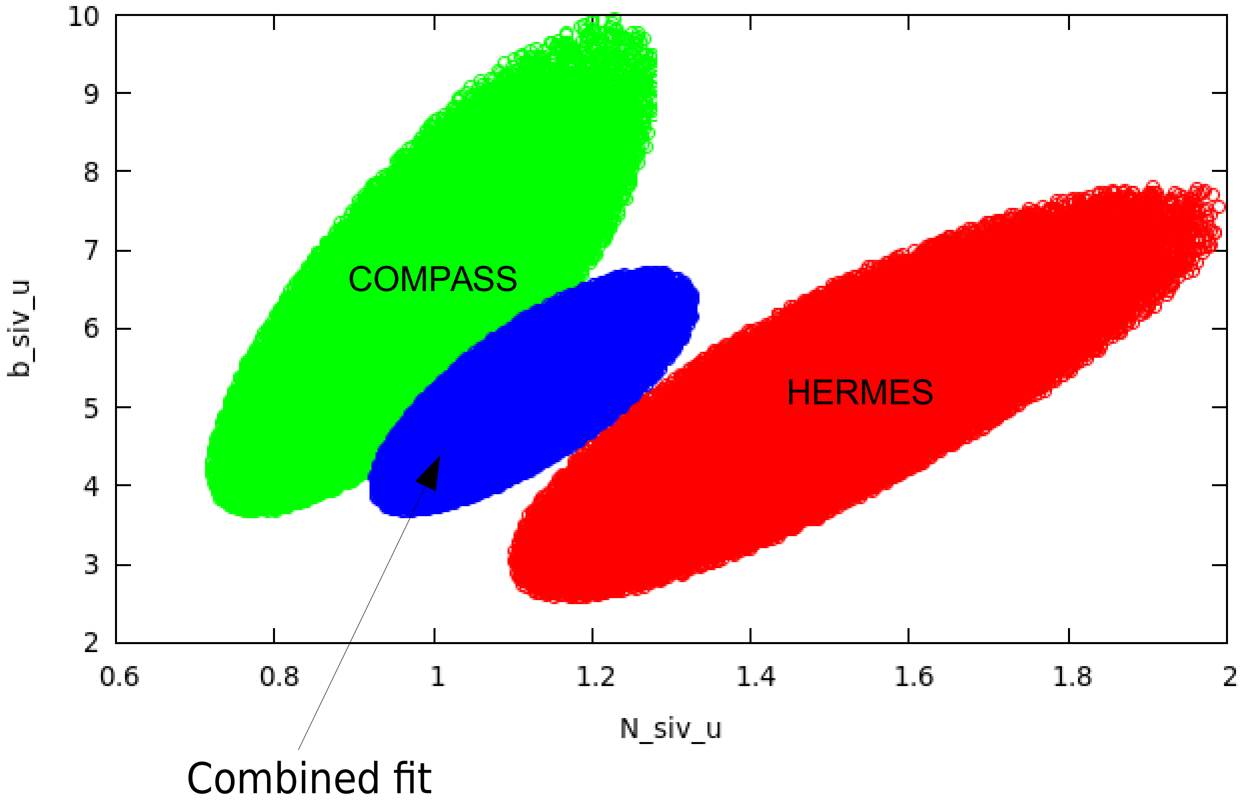
New extraction of the Sivers function

Boglione, Gonzalez, Flore, D'Alesio, JHEP 1807 (2018) 148

Signal of some tension between independent fit solutions for COMPASS and HERMES data

Start by using a very simple model only 5 parameters and no Q evolution

Use only u flavour and only π+ data



Before attempting any "global fitting" we have to check data for compatibility …

… and we have to check that the unpolarized cross sections are computed consistently and reproduce data successfully

Relevance of unpolarized p_T distributions

To calculate any spin asymmetry it is crucial to use the appropriate denominator, i.e. the appropriate unpolarized cross section

$$F_{UU} = \sum_q e_q^2 \, f_{q/p}(x_B) \, D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

with $\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$

See talks by A. Signori and N. Sato

- It is very important to measure **p_T distributions of unpolarized cross sections** in SIDIS, Drell-Yan, e+e- processes
- These measurements will allow us to

Perturbative QCD

- **TEST THEORY**, and assess whether or not theory errors are under control (large q_T corrections, factorization errors, kinematics ...)

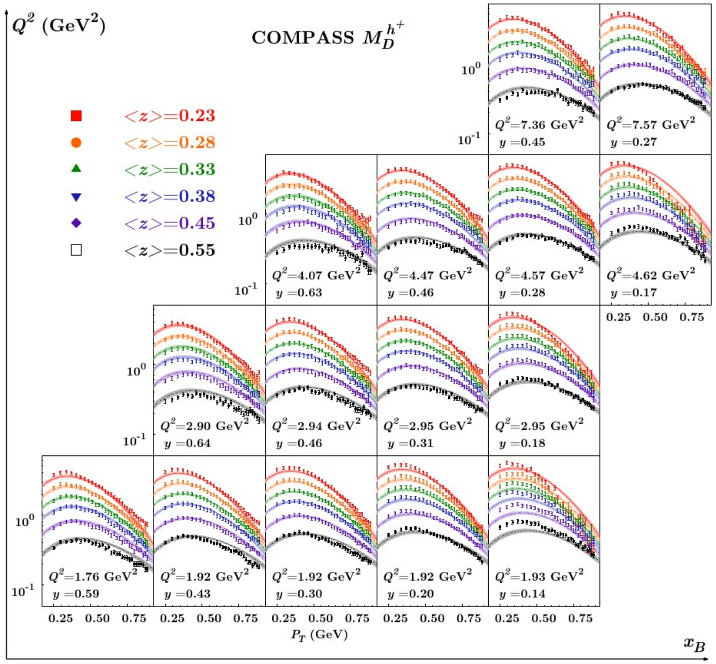
NON Perturbative QCD

- **HAVE BETTER MODELS for TMDs**

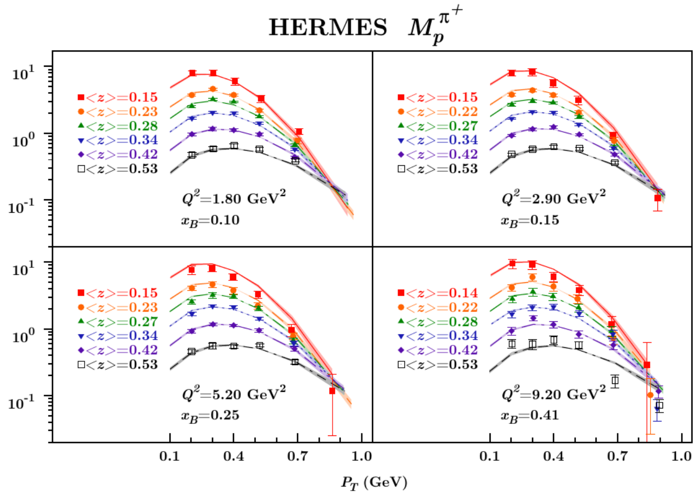
Relevance of unpolarized p_T distributions

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005

$$F_{UU} = \sum_q e_q^2 \, f_{q/p}(x_B) \, D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle} \qquad \text{with} \qquad \langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle$$



COMPASS, Adolph et al., Eur. Phys. J. C 73 (2013) 2531



Airapetian et al, Phys. Rev. D 87 (2013) 074029

$$\langle k_\perp^2\rangle = 0.60 \pm 0.14 \text{ GeV}^2$$
$$\langle p_\perp^2\rangle = 0.20 \pm 0.02 \text{ GeV}^2$$
$$\chi^2_{\text{dof}} = 3.42$$

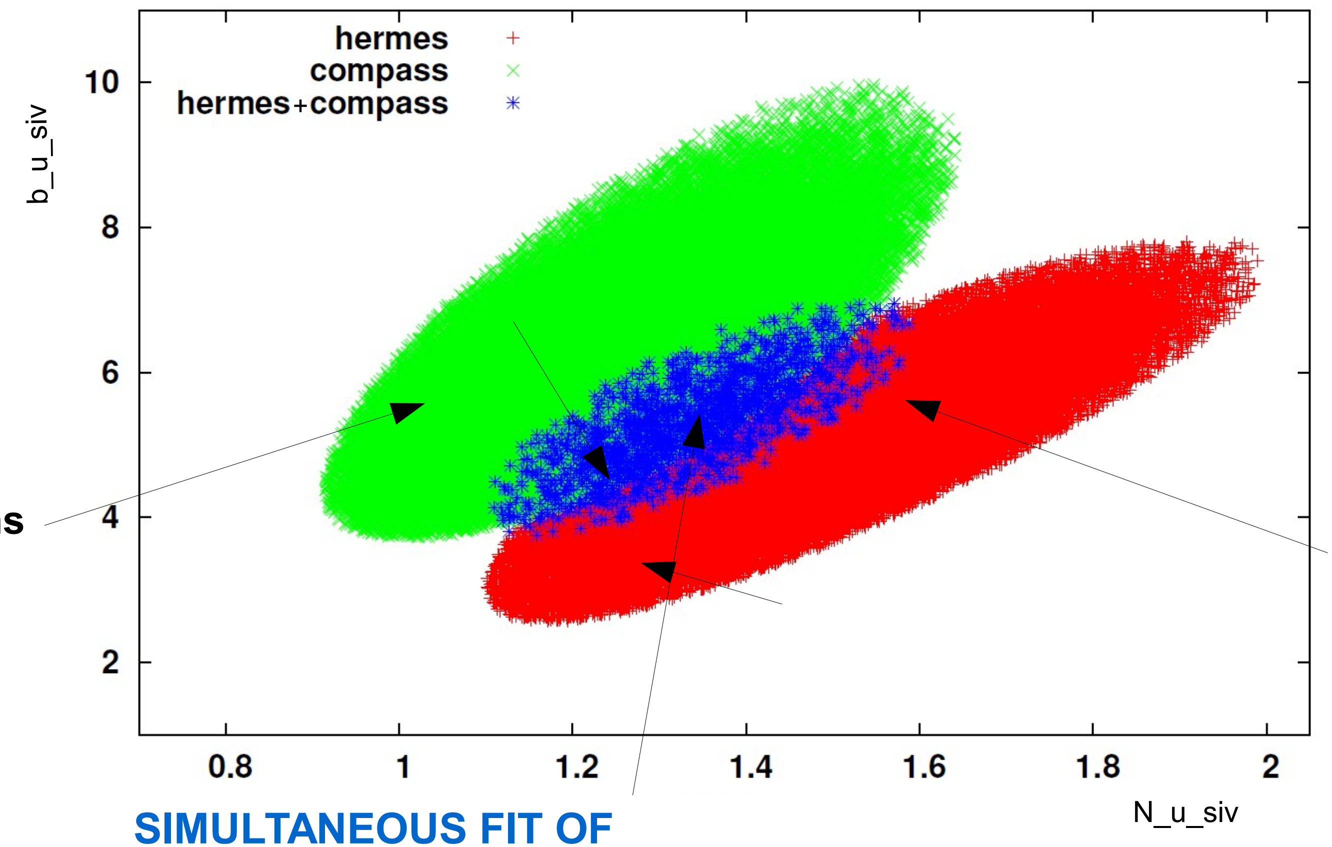
Simple models seems to work well, but cannot describe both data sets simultaneously

$$\langle k_\perp^2\rangle = 0.57 \pm 0.08 \text{ GeV}^2$$
$$\langle p_\perp^2\rangle = 0.12 \pm 0.01 \text{ GeV}^2$$
$$\chi^2_{\text{dof}} = 1.69$$

New extraction of the Sivers function

Tension relaxes when the asymmetry is computed using the appropriate unpolarized widths for each data set

Reference Fit



COMPASS-2015 (COMPASS widths no-evolution)

HERMES-2009 (HERMES widths no-evolution)

SIMULTANEOUS FIT OF
HERMES-2009 (HERMES widths - no evolution)
COMPASS-2015 (COMPASS widths – no evolution)

New extraction of the Sivers function

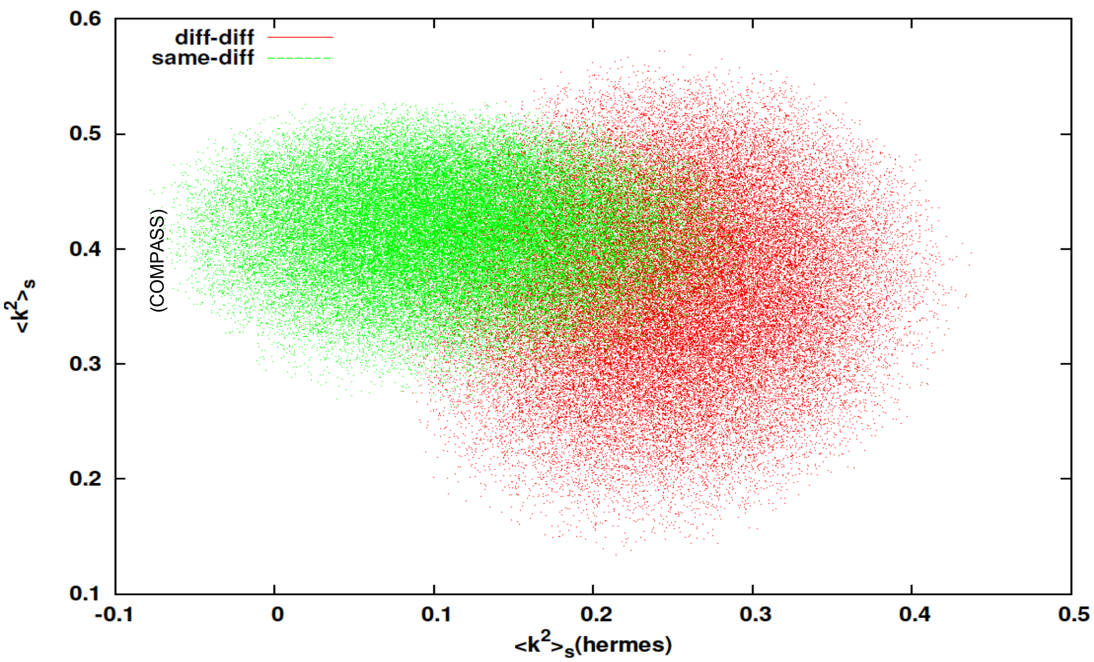
Boglione Gonzalez Flore D'Alesio,
in preparation

If we use different unpolarized widths for HERMES and COMPASS data, do we have to use different **Sivers** widths as well?

Sivers widths: HERMES vs. COMPASS

Allowing for different **Sivers** widths for each experiments, does not improve the quality of the fit, and the extracted values are very similar

No-evolution



Simple models seem to work well, but cannot describe both data sets simultaneously …

However, more refined calculations seem to be presenting serious difficulties

See talks by A. Signori and N. Sato

Global fits

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori, JHEP06 (2017) 081

Extraction of partonic transverse momentum distributions from semi-inclusive deep-inelastic scattering, Drell-Yan and Z-boson production

Alessandro Bacchetta,[a,b] Filippo Delcarro,[a,b] Cristian Pis...
and Andrea Signori[c]

[a] Dipartimento di Fisica, Università di Pavia, via Bassi 6, I-27100 Pavia, Italy
[b] INFN Sezione di Pavia, via Bassi 6, I-27100 Pavia, Italy
[c] Theory Center, Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, U.S.A.

E-mail: alessandro.bacchetta@unipv.it, filippo.delcarro@pv.infn.it, cristian.pisano@unipv.it, marco.radici@pv.infn.it, asignori@jlab.org

JHEP ... 081

Abstract: We present an extraction of unpolarized partonic transverse momentum distributions (TMDs) from a simultaneous fit of available data measured in semi-inclusive deep-inelastic scattering, Drell-Yan and Z boson production. To connect data at different scales, we use TMD evolution at next-to-leading logarithmic accuracy. The analysis is restricted to the low-transverse-momentum region, with no matching to fixed-order calculations at high transverse momentum. We introduce specific choices to deal with TMD evolution at low scales, of the order of 1 GeV^2. This could be considered as a first attempt at a global fit of TMDs.

Although the shape in transverse momentum space is well described, normalization is very problematic

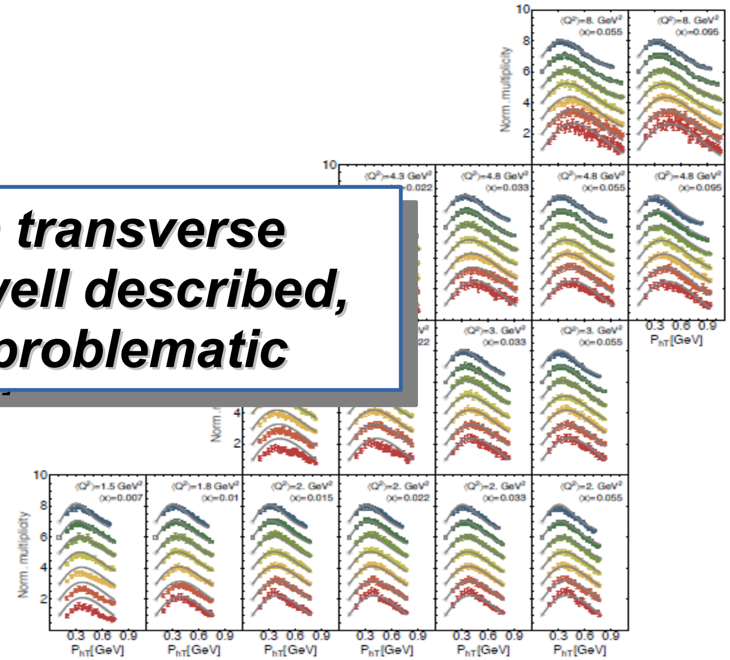


Figure 5. COMPASS multiplicities for production of negative hadrons (π^-) off a deuteron for different $\langle x\rangle$, $\langle z\rangle$, and $\langle Q^2\rangle$ bins as a function of the transverse momentum of the detected hadron P_{hT}. Multiplicities are normalized to the first bin in P_{hT} for each $\langle z\rangle$ value (see (3.1)). For clarity, each $\langle z\rangle$ bin has been shifted by an offset indicated in the legend.

$\chi^2_{tot} = 1.55$

- Y-term is neglected
- Sum of two Gaussian k_T distributions is introduced

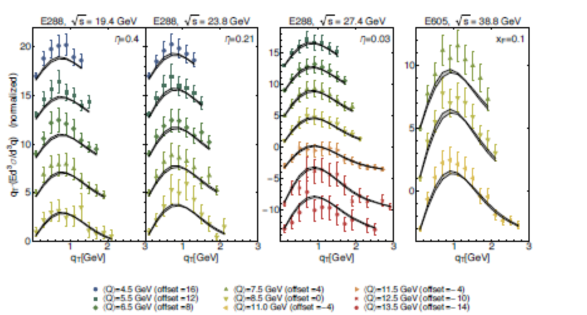


Figure 7. Drell-Yan differential cross section for different experiments and different values of $\sqrt{s}$ and for different $\langle Q\rangle$ bins. For clarity, each $\langle Q\rangle$ bin has been normalized (the first data point has been set always equal to 1) and then shifted by an offset indicated in the legend.

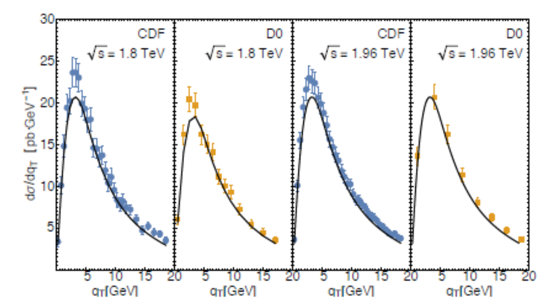
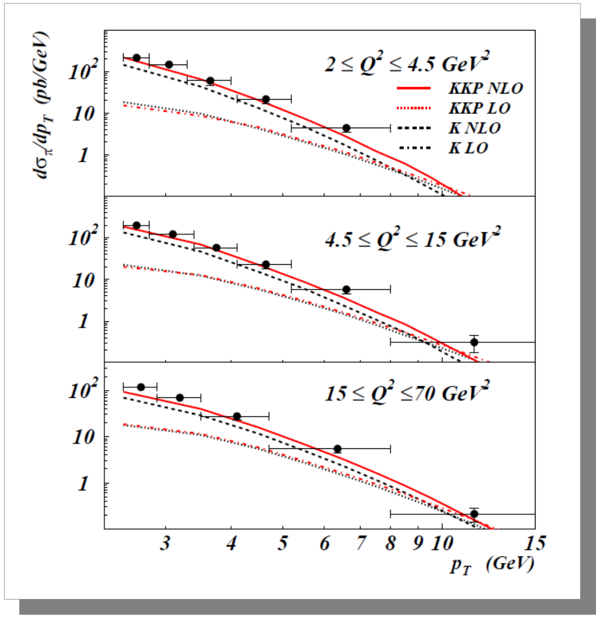
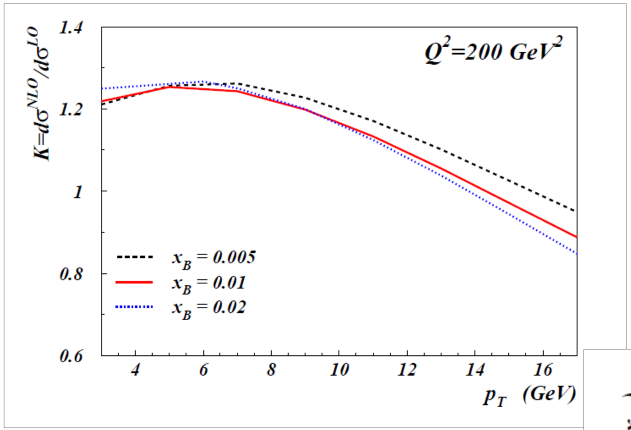


Figure 8. Cross section differential with respect to the transverse momentum q_T of a Z boson produced from $p\bar{p}$ collisions at Tevatron. The four panels refer to different experiments (CDF and D0) with two different values for the center-of-mass energy ($\sqrt{s} = 1.8$ TeV and $\sqrt{s} = 1.96$ TeV). In this case the band is narrow due to the narrow range for the best-fit values of g_2.

Normalization and K factor

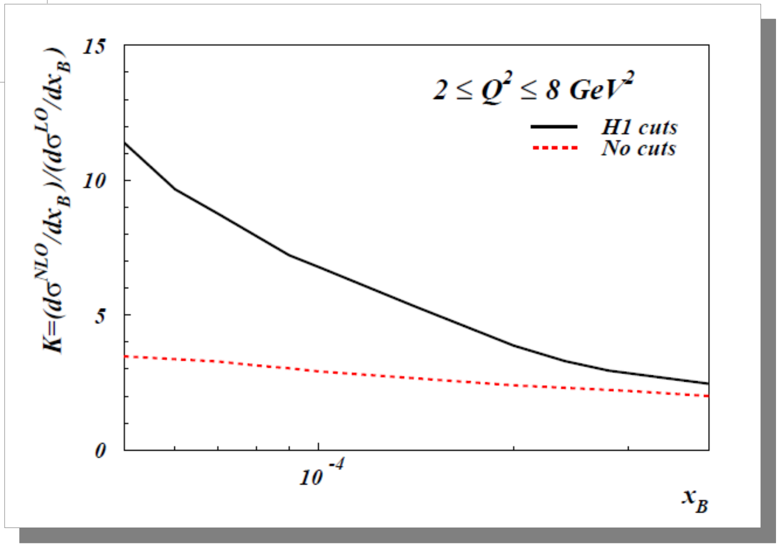


How can we address the normalization problem ???



- K factor depends on p_T
- Kinematics cuts can affect the size of K factors … up to a factor 10 !

Stringent cuts on the pion production angle in H1 data suppresses LO and NLO contributions in a different way



Daleo, De Florian, Sassot, Phys.Rev. D71 (2005) 034013

Daleo, De Florian, Sassot, Braz.J.Phys. 37 (2007) 585-590

Aktas et al., H1 Collaboration, Eur. Phys. J. C36 (2004) 441

"The rather large size of the K-factor can be understood as a consequence of the opening of a new dominant ('leading-order') channel, and not to the 'genuine' increase in the partonic cross section [...]. The dominance of the new channel is due to the size of the gluon distribution at small x_B and to the fact that the H1 selection cuts highlight the kinematical region dominated by the $\gamma + g \to g + q + \bar{q}$ partonic process. In particular, without the experimental cuts for the final state hadrons, the gg component represents less than 25% of the total NLO contribution at small x_B."

Daleo, De Florian, Sassot, Phys.Rev. D71 (2005) 034013

Daleo, De Florian, Sassot, Braz.J.Phys. 37 (2007) 585-590

Large transverse momentum behaviour in SIDIS

J.O. Gonzalez-Hernandez, T.C. Rogers, N. Sato, B. Wang, arXiv:1808.04396

Challenges with Large Transverse Momentum in Semi-Inclusive Deeply Inelastic Scattering

J. O. Gonzalez-Hernandez,[1,2,3,*] T. C. Rogers,[1,4,†] N. Sato,[4,‡] and B. Wang[1,4,5,§]

[1]*Department of Physics, Old Dominion University, Norfolk, VA 23529, USA*
[2]*Dipartimento di Fisica, Università di Torino, Via P. Giuria 1, 10125 Torino, Italy*
[3]*INFN-Sezione Torino, Via P. Giuria 1, 10125 Torino, Italy*
[4]*Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA*
[5]*Zhejiang Institute of Modern Physics, Department of Physics, Zhejiang University, Hangzhou 310027, China*

(Dated: 13 August 2018)

We survey the current phenomenological status of semi-inclusive deep inelastic scattering at moderate hard scales and in the limit of very large transverse momentum. As the transverse momentum becomes comparable to or larger than the overall hard scale, the differential cross sections should be calculable with fixed order pQCD methods, while small transverse momentum (TMD factorization) approximations should eventually break down. We find large disagreement between HERMES and COMPASS data and fixed order calculations done with modern parton densities, even in regions of kinematics where such calculations should be expected to be very accurate. Possible interpretations are suggested.

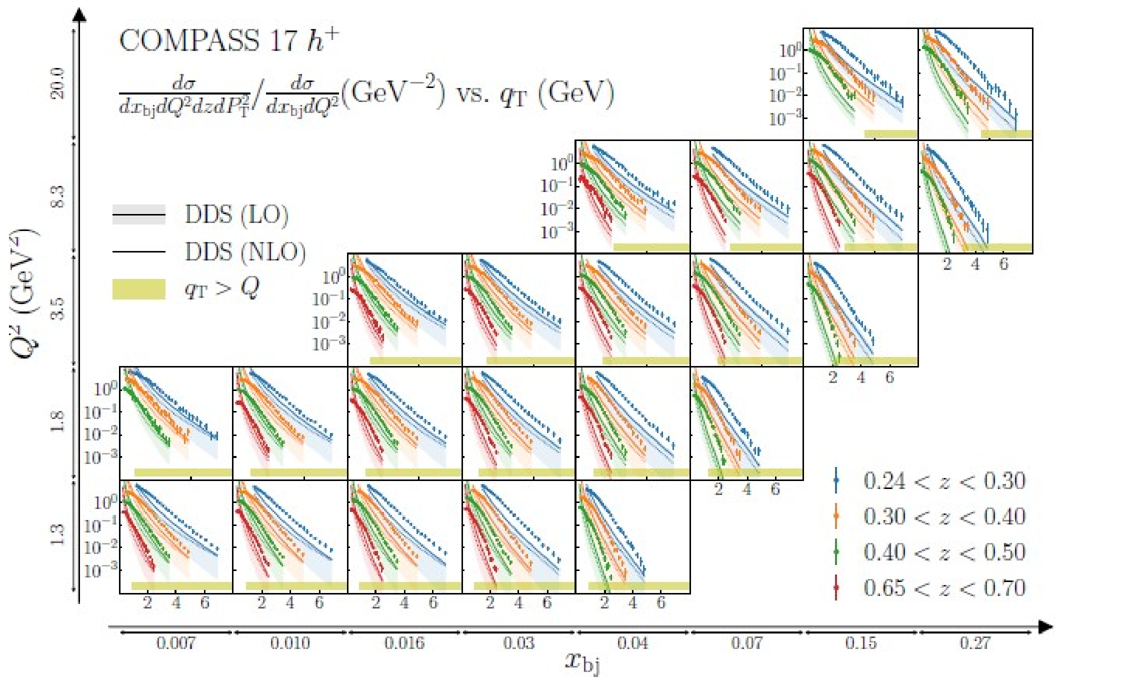


FIG. 4. Calculation of $O(\alpha_s)$ and $O(\alpha_s^2)$ transversely differential multiplicity using code from [22], shown as the curves labeled DDS. The bar at the bottom marks the region where $q_T > Q$. The PDF set used is CJNLO [25] and the FFs are from [26]. Scale dependence is estimated using $\mu = ((\zeta_Q Q)^2 + (\zeta_{q_T} q_T)^2)^{1/2}$ where the band is constructed point-by-point in q_T by taking the min and max of the cross section evaluated across the grid $\zeta_Q \times \zeta_{q_T} = [1/2, 1, 3/2, 2] \times [0, 1/2, 1, 3/2, 2]$ except $\zeta_Q = \zeta_{q_T} = 0$. The red band is generated with $\zeta_Q = 1$ and $\zeta_{q_T} = 0$. A lower bound of 1 GeV is place on μ when $Q/2$ would be less than 1 GeV.

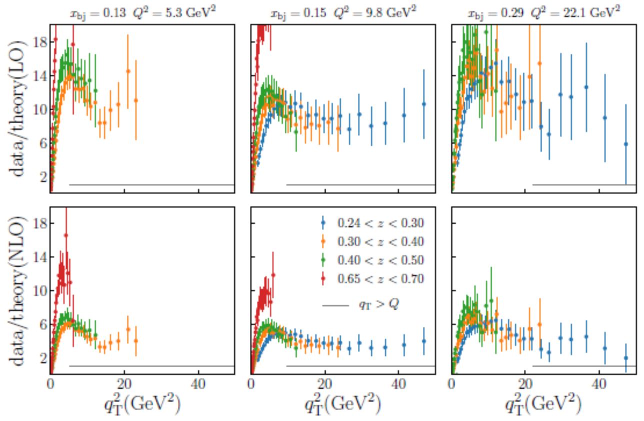


FIG. 5. Ratio of data to theory for several near-valence region panels in Fig. 4. The grey bar at the bottom is at 1 on the vertical axis and marks the region where $q_T > Q$.

There are large discrepancies between data and fixed order calculations. They seem to be generated by collinear PDFs and FFs

Now, back to Sivers ...

New extraction of the Sivers function

Boglione, Gonzalez, Flore, D'Alesio, JHEP 1807 (2018) 148

Main indications directly inferred from data:

- u seems well constrained
- d is **not** constrained: it can be replaced by sea contributions with equally good fits - hard to distinguish where this contribution comes from.
- Sivers sea is totally unconstrained

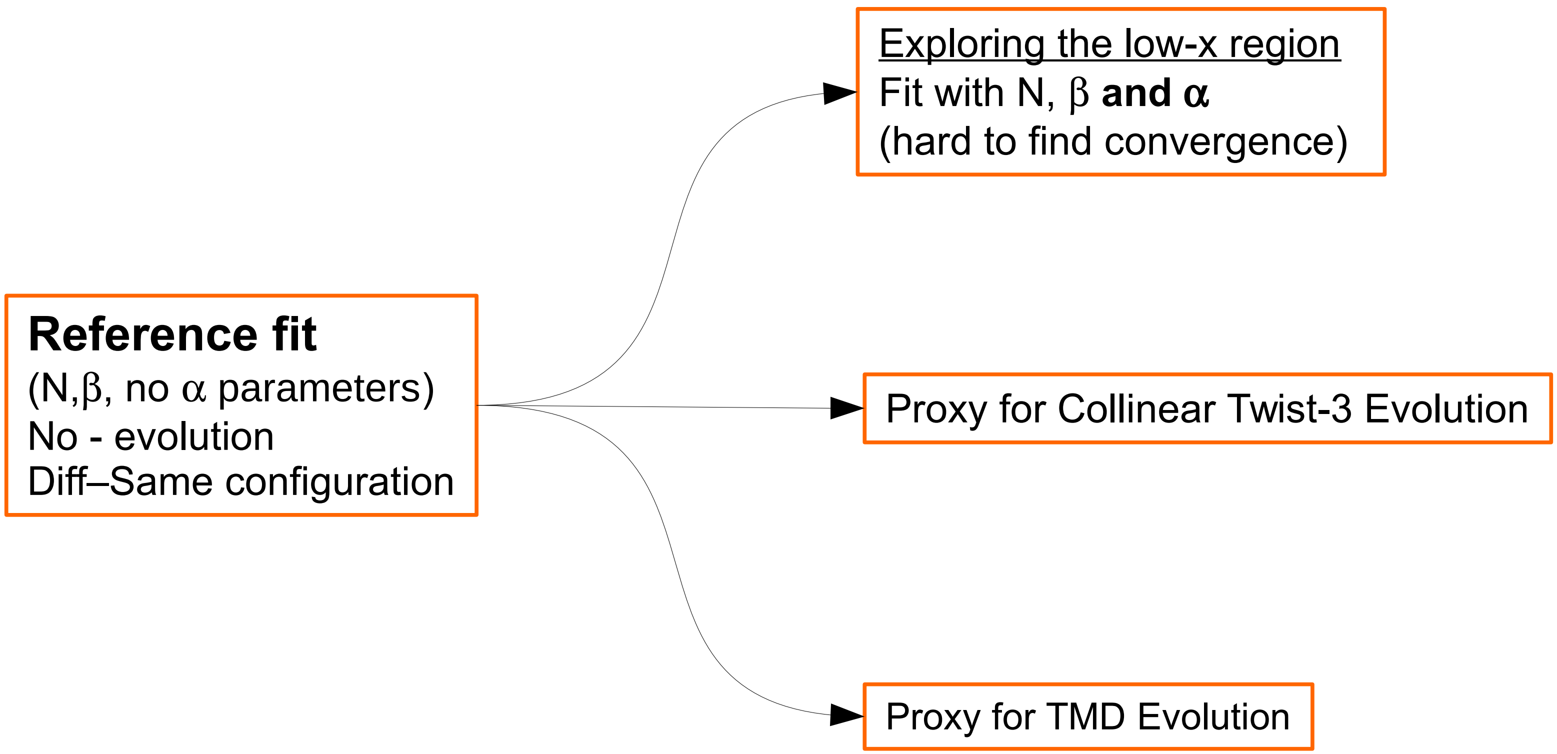
n. of data points = 220		
One flavour fits (3 parameters)		
	χ^2_{tot}	χ^2_{dof}
u	408	1.88
d	914	4.21
Two flavour fits (5 parameters)		
	χ^2_{tot}	χ^2_{dof}
$u, \bar{u}$	266	1.24
$u, \bar{d}$	228	1.06
$\boldsymbol{u, d}$	**213**	0.99

It is of vital importance to gain information on the d content of the Sivers function

We strongly rely on SIDIS measurements of the Sivers asymmetry on deuterium target @ COMPASS, as well as @ the future EIC

Study of the uncertainties in the extraction of the Sivers function

Boglione, Gonzalez, Flore, D'Alesio, JHEP 1807 (2018) 148



Study of the uncertainties in the extraction of the Sivers function

Boglione, Gonzalez, Flore, D'Alesio, JHEP 1807 (2018) 148

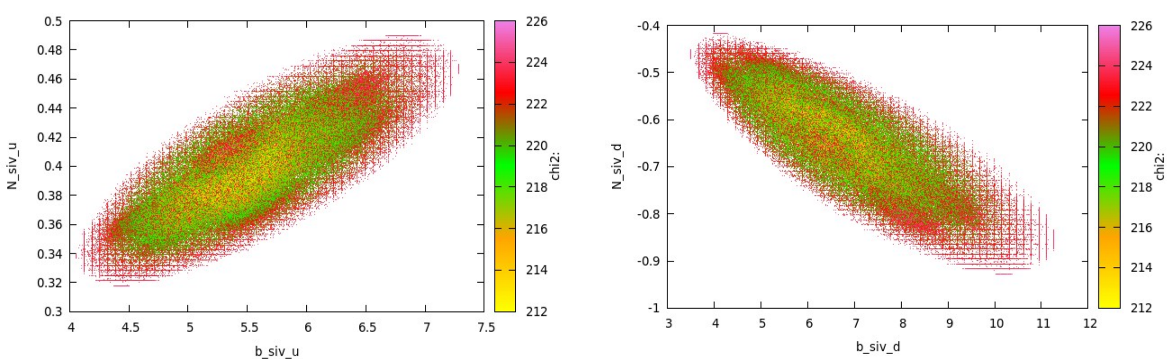
Reference fit - no evolution		
$\chi^2_{\rm tot} = 212.8$	n. of points = 220	
$\chi^2_{\rm dof} = 0.99$	n. of free parameters = 5	
$\Delta\chi^2 = 11.3$		
HERMES	$\langle k_\perp^2\rangle = 0.57$ GeV^2	$\langle p_\perp^2\rangle = 0.12$ GeV^2
COMPASS	$\langle k_\perp^2\rangle = 0.60$ GeV^2	$\langle p_\perp^2\rangle = 0.20$ GeV^2
$N_u = 0.40 \pm 0.09$	$\beta_u = 5.43 \pm 1.59$	
$N_d = -0.63 \pm 0.23$	$\beta_d = 6.45 \pm 3.64$	
$\langle k_\perp^2\rangle_S = 0.30 \pm 0.15$ GeV^2		

Reference Fit

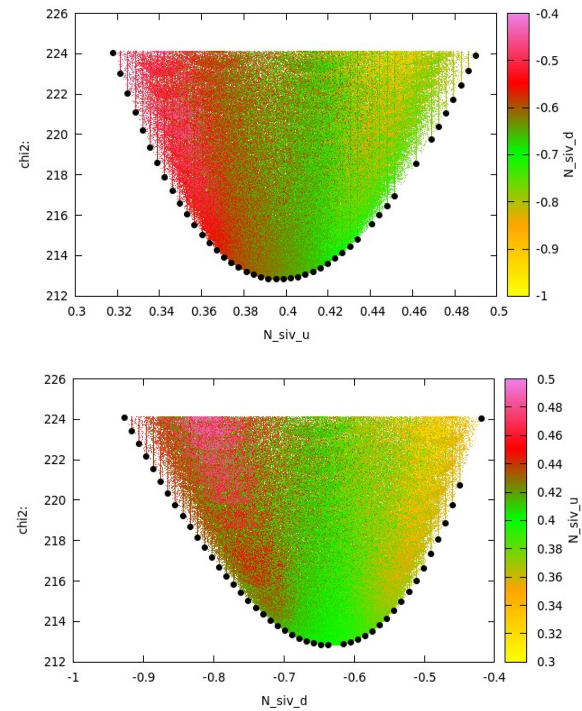
220 data points
5 free-parameters
$\Delta\chi^2$ = 11.31

$$P = \int_0^{\Delta\chi^2} \frac{1}{2\Gamma(M/2)} \left(\frac{\chi^2}{2}\right)^{(M/2)-1} \exp\left(-\frac{\chi^2}{2}\right) d\chi^2 \,.$$

Parameter correlations



χ^2 scans: some examples



The χ^2 profile can reasonably be approximated with a quadratic, Hessian approx. works well, MINUIT errors give reliable estimates of the uncertainty on the free parameters.

Study of the uncertainties in the extraction of the Sivers function

Study of Low-x Uncertainties

(include α_u and α_d in the parametrization of the Sivers function)

Attempt to minimize bias Induced by the choice of parametric form

Reference fit – no evolution
5 params.

α fit – no evolution
7 params.

χ^2 does not improve !

Reference fit - no evolution		
$\chi^2_{tot} = 212.8$	n. of points = 220	
$\chi^2_{dof} = 0.99$	n. of free parameters = 5	
$\Delta\chi^2 = 11.3$		
HERMES	$\langle k_\perp^2 \rangle = 0.57$ GeV^2	$\langle p_\perp^2 \rangle = 0.12$ GeV^2
COMPASS	$\langle k_\perp^2 \rangle = 0.60$ GeV^2	$\langle p_\perp^2 \rangle = 0.20$ GeV^2
$N_u = 0.40 \pm 0.09$	$\beta_u = 5.43 \pm 1.59$	
$N_d = -0.63 \pm 0.23$	$\beta_d = 6.45 \pm 3.64$	
$\langle k_\perp^2 \rangle_S = 0.30 \pm 0.15$ GeV^2		

α fit - no evolution			
$\chi^2_{tot} = 211.5$		n. of points = 220	
$\chi^2_{dof} = 0.99$		n. of free parameters = 7	
$\Delta\chi^2 = 14.3$			
HERMES	$\langle k_\perp^2 \rangle = 0.57$ GeV^2	$\langle p_\perp^2 \rangle = 0.12$ GeV^2	
COMPASS	$\langle k_\perp^2 \rangle = 0.60$ GeV^2	$\langle p_\perp^2 \rangle = 0.20$ GeV^2	
$N_u = 0.40 \pm 0.09$		$\beta_u = 5.93 \pm 3.86$	$\alpha_u = 0.073 \pm 0.46$
$N_d = -0.63 \pm 0.23$		$\beta_d = 5.71 \pm 7.43$	$\alpha_d = -0.075 \pm 0.83$
$\langle k_\perp^2 \rangle_S = 0.30 \pm 0.15$ GeV^2			

Alpha and N parameters are totally correlated

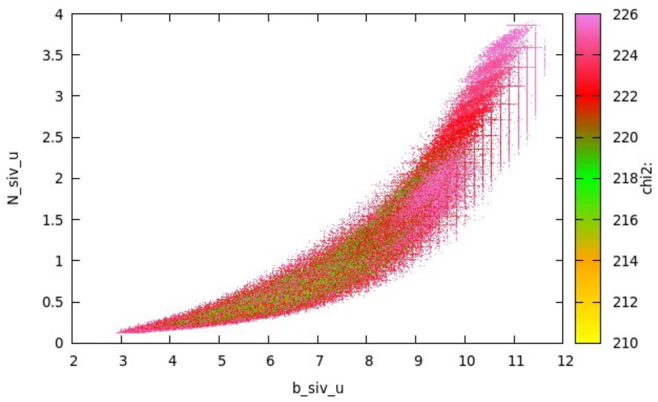
For the alpha-fit, the χ^2 profile is NOT quadratic, Hessian approx. does not work, MINUIT errors do NOT give reliable estimates of the uncertainty on the parameters, especially on the N parameters.

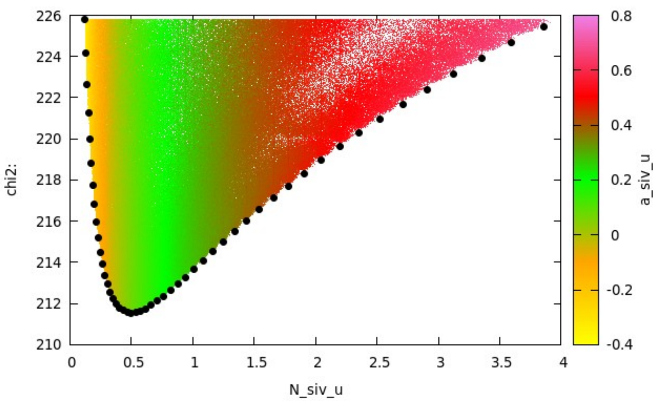
Study of the uncertainties in the extraction of the Sivers function

Boglione, Gonzalez, Flore, D'Alesio, JHEP 1807 (2018) 148

Parameter correlations

χ^2 scans

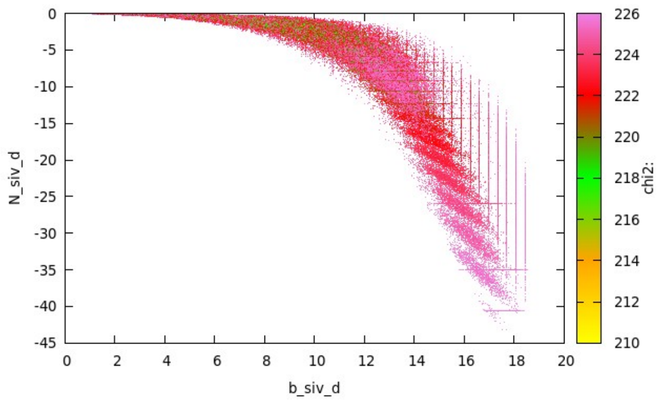




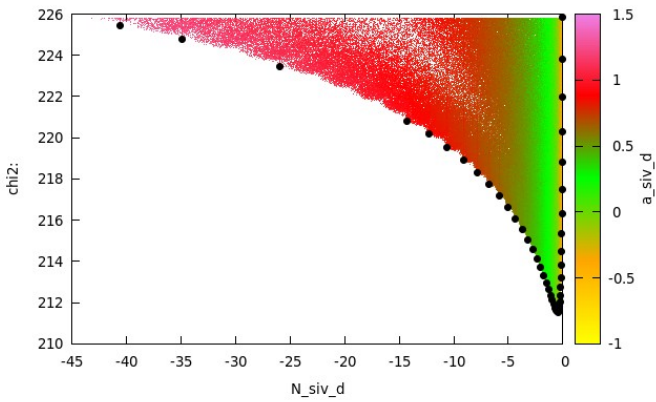
Study of Low-x Uncertainties

(include α_u and α_d in the parametrization of the Sivers function)

no-evolution



220 data points
7 free-parameters
$\Delta\chi^2$ = 14.34



For the alpha-fit, the χ^2 profile is NOT quadratic, Hessian approx. does not work, MINUIT errors do NOT give reliable estimates of the uncertainty on the parameters, especially on the N parameters.

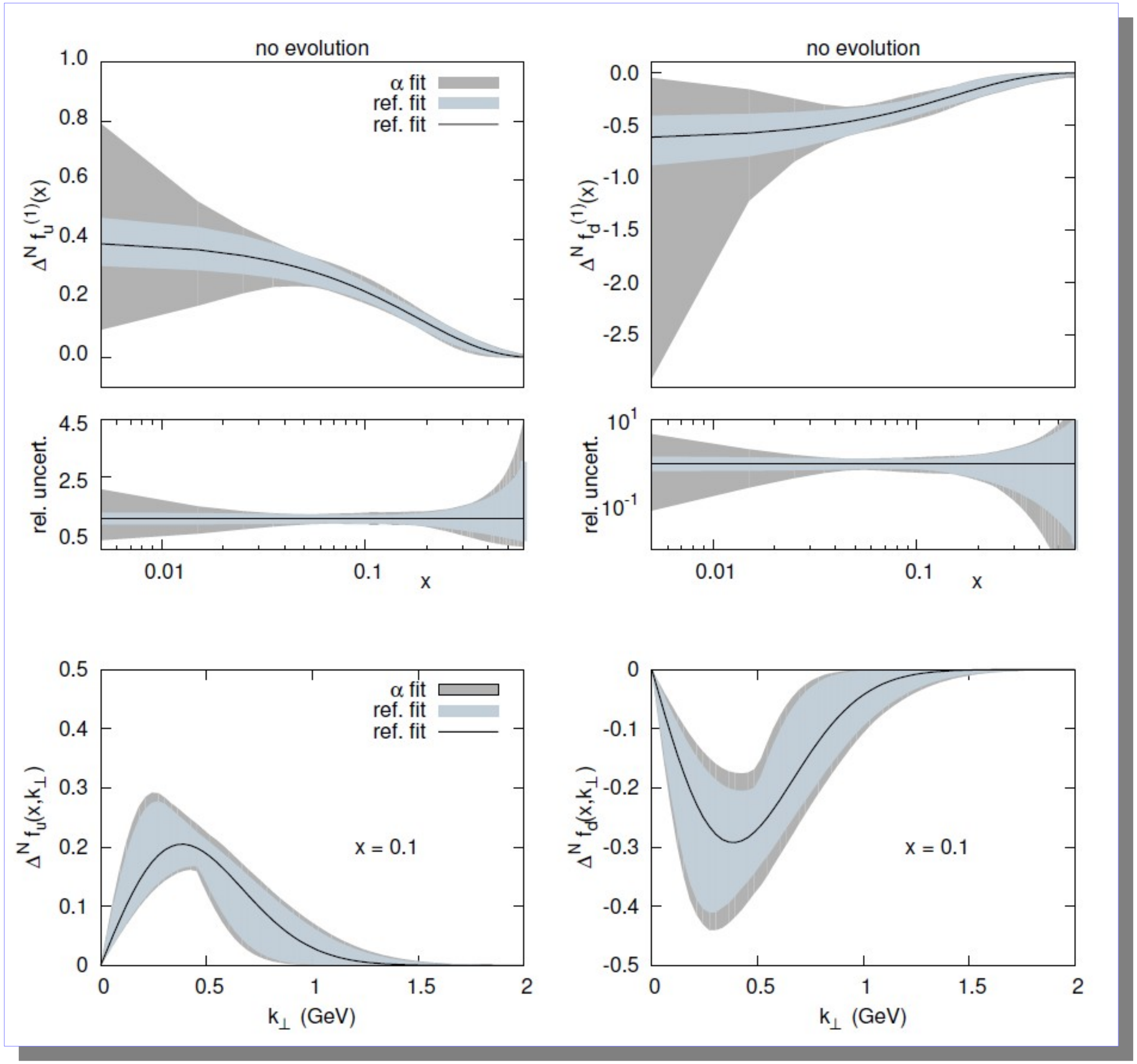
Uncertainty bands – Sivers first moment

Boglione, Gonzalez, Flore, D'Alesio, JHEP 1807 (2018) 148

Reference Fit
(no-evolution)

Study of Low-x Uncertainties
(include α_u and α_d in the parametrization of the Sivers function)

Sivers First moment u-contribution



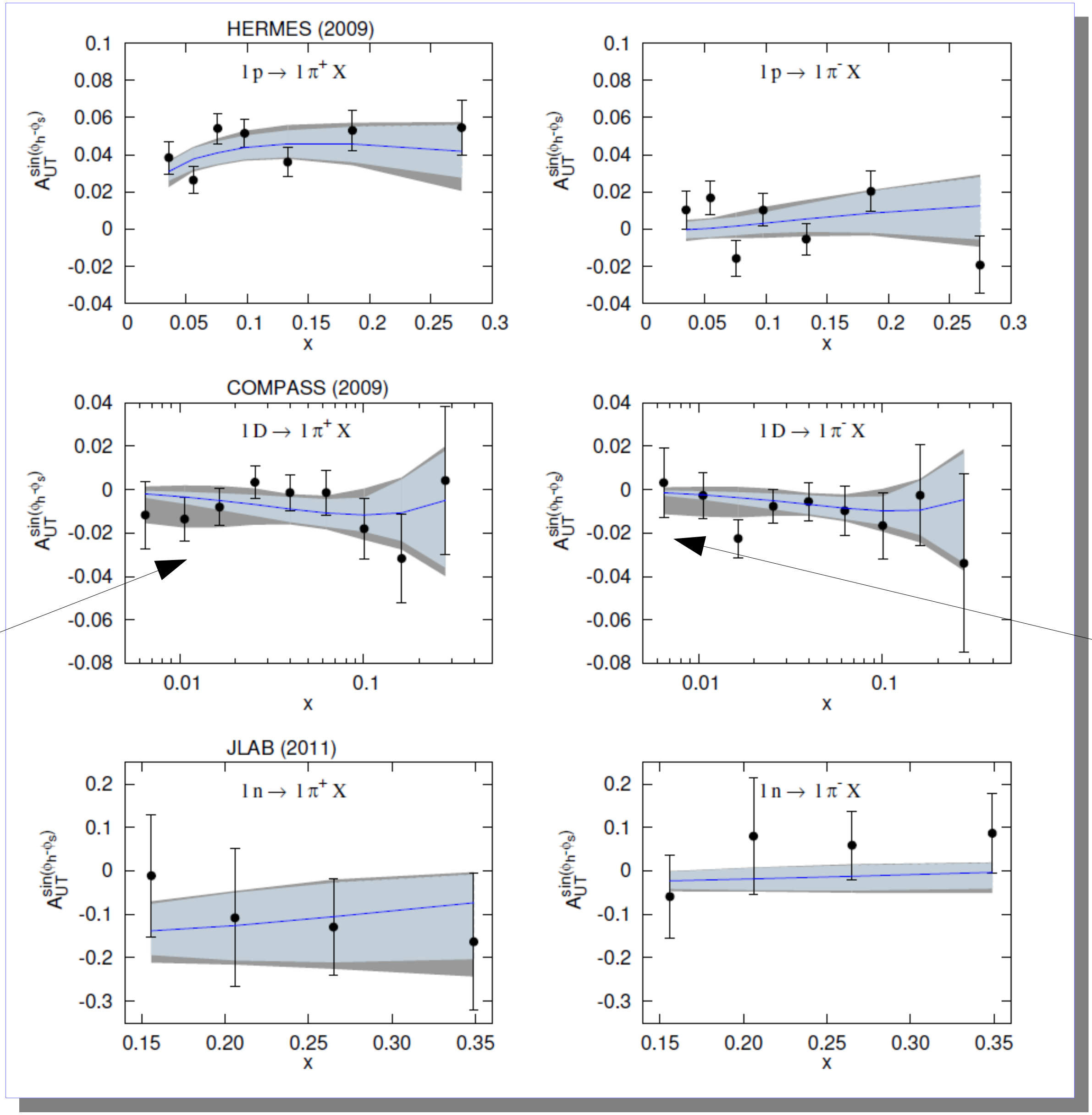
Sivers First moment d-contribution

Uncertainty bands – Sivers Asymmetries

Boglione, Gonzalez, Flore, D'Alesio, JHEP 1807 (2018) 148

Reference Fit
(no-evolution)

Uncertainty bands from the reference fit (light-blue) become artificially small at small x.

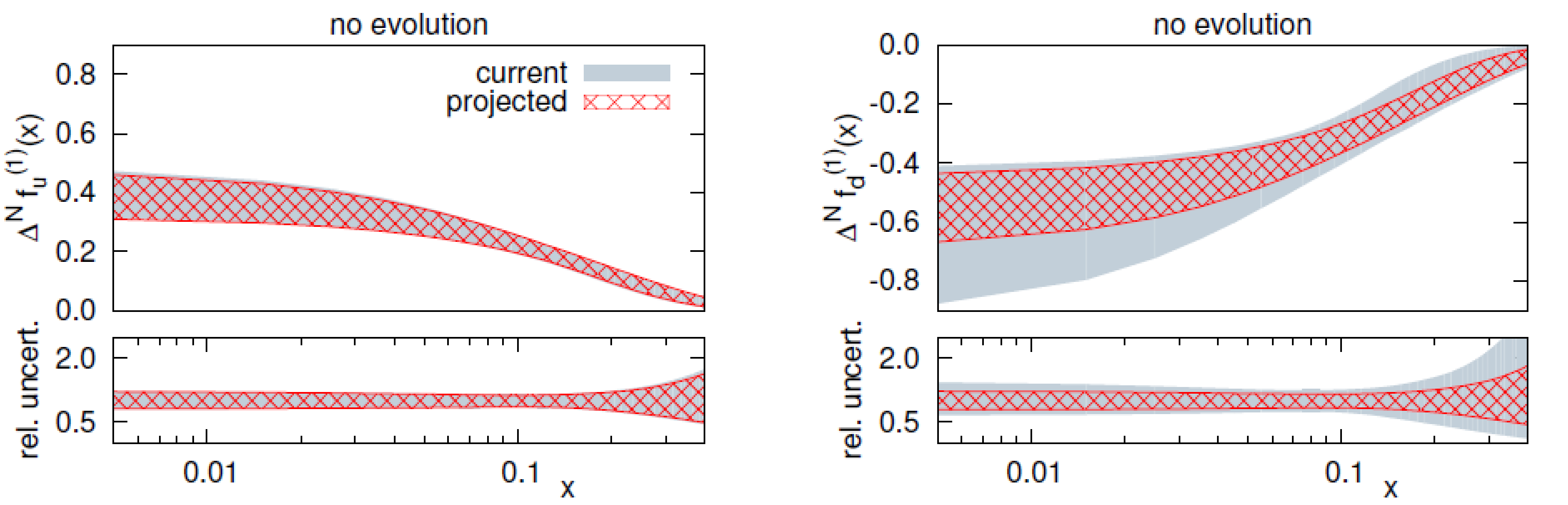


Study of Low-x Uncertainties
(include α_u and α_d in the parametrization of the Sivers function)

Alpha fit gives better estimates of uncertainties at small x (gray bands).

Impact of the precision of SIDIS deuteron data

Boglione Gonzalez Flore D'Alesio,
in preparation



- Light-blue bands represent the uncertainties corresponding to the reference fit
- Red meshed bands correspond to the uncertainties estimated by using the same model, with the projected experimental errors of the future COMPASS run on deuteron target.

COMPASS Collaboration, d-Quark Transversity and. Proton Radius. Addendum to the COMPASS-II Proposal. CERN–SPSC–2017–034. SPSC-P-340-ADD-1. January 2018

Signals of Q scale dependence

TMD Factorization approach and Collinear twist-three factorization approach

TMD factorization approach

- Spin asymmetries are generated by spin and transverse momentum correlations between the identified hadron and the active parton.
- This correlations are embedded in the TMD parton distribution or fragmentation functions, which can be interpreted as probability densities.

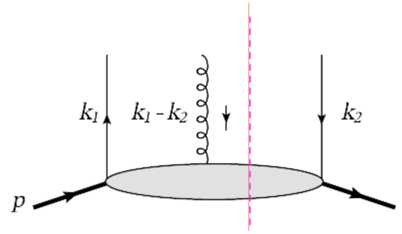
Collinear twist-three factorization approach

- The correlation between spin and transverse momentum is included into the **high twist** collinear parton distributions or fragmentation functions.
- Twist-3 collinear parton distributions or fragmentation functions have no probabilistic interpretation. They are interpreted as the quantum interference between a collinear active quark state in the scattering amplitude and a collinear quark–gluon composite state in its complex conjugate amplitude.

Q^2 evolution affects both x and $k_\perp$

- TMDs and quark–gluon correlation functions are closely related to each other.
- The first $k_\perp$-moment of the Sivers function is equal to the twist-3 quark–gluon correlation functions $T_{q,F}(x, x)$
- **Evolution kernels for $T_{q,F}(x, x)$ are known; we can exploit them in our study (off diagonal terms are not included)**

Q^2 evolution occurs only through x



$$\frac{\partial T_{q,F}(x,x,\mu)}{\partial \ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Big\{P_{qq}(z)\,T_{q,F}(\xi,\xi,\mu) + \frac{N_c}{2}\left[\frac{1+z^2}{1-z}\,(T_{q,F}(\xi,x,\mu)-T_{q,F}(\xi,\xi,\mu)) + z\,T_{q,F}(\xi,x,\mu)+T_{\Delta q,F}(x,\xi,\mu)\right] - N_c\,\delta(1-z)\,T_{q,F}(x,x,\mu)+\frac{1}{2N_c}\left[(1-2z)T_{q,F}(x,x-\xi,\mu)+T_{\Delta q,F}(x,x-\xi,\mu)\right]\Big\}$$

J.B. Kang and J.W. Qiu, Phys. Rev. D 79 (2009) 016003

W. Vogelsang and F. Yuan, Phys. Rev. D 79 (2009) 094010

V.M. Braun, A.N. Manashov, B. Pirnay, Phys. Rev. D 80 (2009) 114002

Z.B Kang and J.W. Qiu, Phys. Lett. B713 (2012) 273-276

TMD evolution of the Sivers function

Aybat, Collins, Qiu, Rogers, Phys. Rev. D 85 (2012) 034043

Configuration space

Input function

Perturbative evolution

$$\tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F) = \tilde{F}'^{\perp f}_{1T}(x, b_T; \mu_0, Q_0^2) \exp\left\{ \ln \frac{\sqrt{\zeta_F}}{Q_0} \tilde{K}(b_*; \mu_b) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right.$$
$$\left. + \int_{\mu_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{\sqrt{\zeta_F}}{Q_0} \gamma_K(g(\mu')) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}.$$

Non perturbative evolution

A proxy for TMD evolution: $\langle k_\perp^2 \rangle_S = g_1 + g_2 \ln \frac{Q^2}{Q_0^2}$

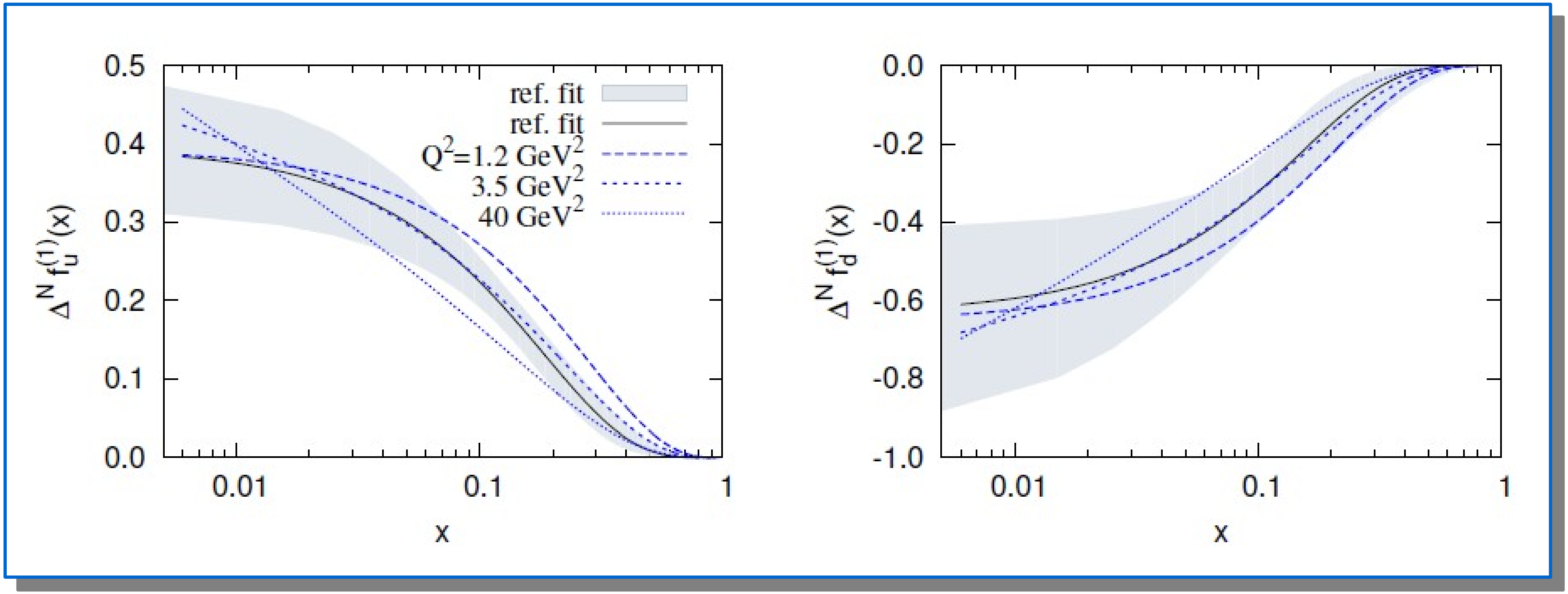
Free parameters

Signals of scale dependence

Boglione, Gonzalez, Flore, D'Alesio, JHEP 1807 (2018) 148

Collinear twist-3 evolution

Collinear twist-3 evolution		
$\chi^2_{\rm tot} = 201.5$	n. of points $= 220$	
$\chi^2_{\rm dof} = 0.94$	n. of free parameters $= 5$	
$\Delta\chi^2 = 11.3$		
HERMES	$\langle k_\perp^2\rangle = 0.57$ GeV^2	$\langle p_\perp^2\rangle = 0.12$ GeV^2
COMPASS	$\langle k_\perp^2\rangle = 0.60$ GeV^2	$\langle p_\perp^2\rangle = 0.20$ GeV^2
$N_u = 0.39 \pm 0.08$	$\beta_u = 3.55 \pm 1.26$	
$N_d = -0.65 \pm 0.27$	$\beta_d = 4.77 \pm 3.41$	
$\langle k_\perp^2\rangle_S = 0.33 \pm 0.14$ GeV^2		

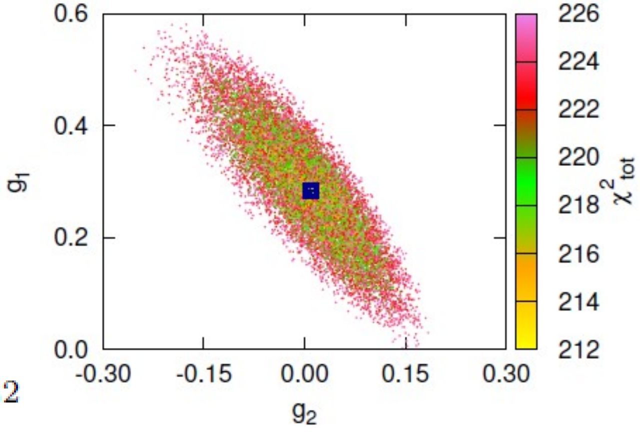


Signals of scale dependence

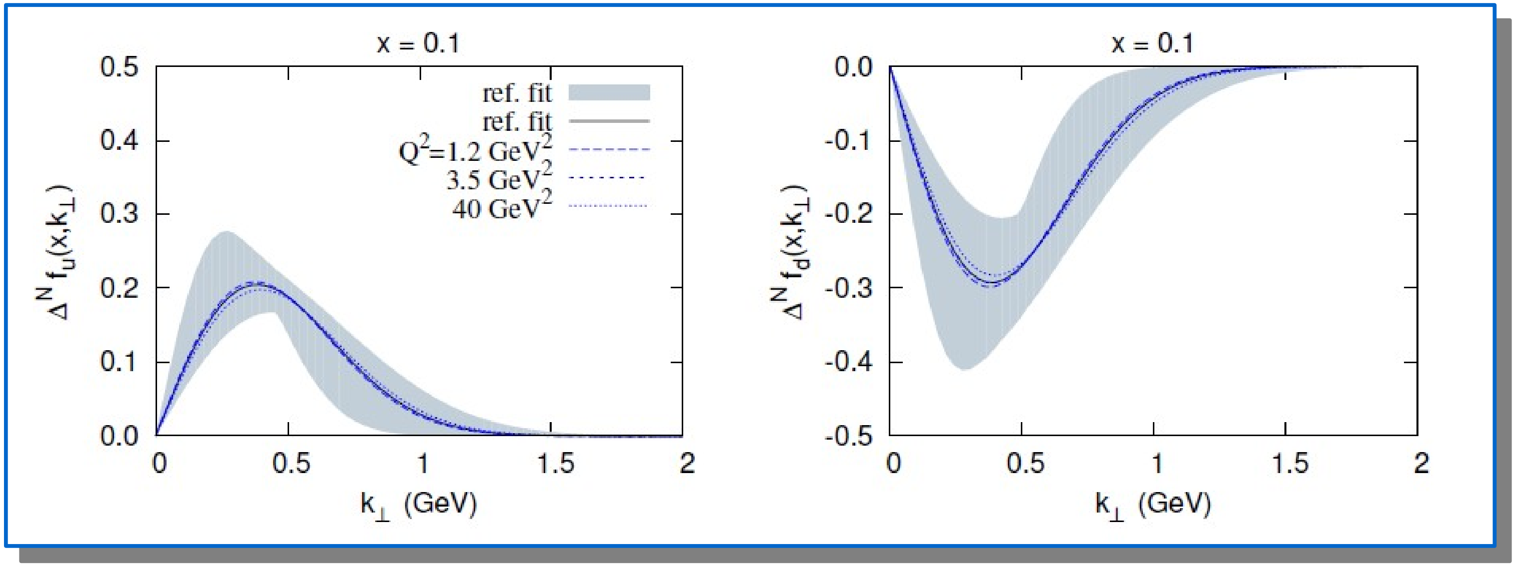
Boglione, Gonzalez, Flore, D'Alesio, JHEP 1807 (2018) 148

TMD evolution proxy

Q^2-dependent $\langle k_\perp^2 \rangle_S$ fit		
$\chi^2_{\rm tot} = 212.8$		n. of points = 220
$\chi^2_{\rm dof} = 0.99$		n. of free parameters = 6
$\Delta\chi^2 = 12.9$		
HERMES	$\langle k_\perp^2 \rangle = 0.57$ GeV^2	$\langle p_\perp^2 \rangle = 0.12$ GeV^2
COMPASS	$\langle k_\perp^2 \rangle = 0.60$ GeV^2	$\langle p_\perp^2 \rangle = 0.20$ GeV^2
$N_u = 0.40 \pm 0.09$		$\beta_u = 5.42 \pm 1.70$
$N_d = -0.63 \pm 0.26$		$\beta_d = 6.45 \pm 3.89$
$\langle k_\perp^2 \rangle_S = g_1 + g_2 \log(Q^2/Q_0^2)$		
$g_1 = 0.28 \pm 0.29$ GeV^2		$g_2 = 0.01 \pm 0.20$ GeV^2



$$\langle k_\perp^2 \rangle_S = g_1 + g_2 \ln \frac{Q^2}{Q_0^2}$$



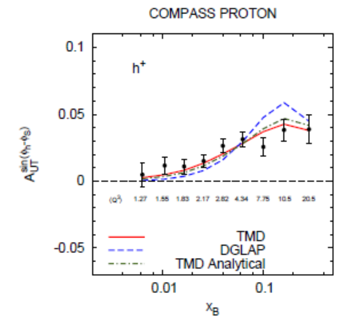
TMD evolution of the Sivers function

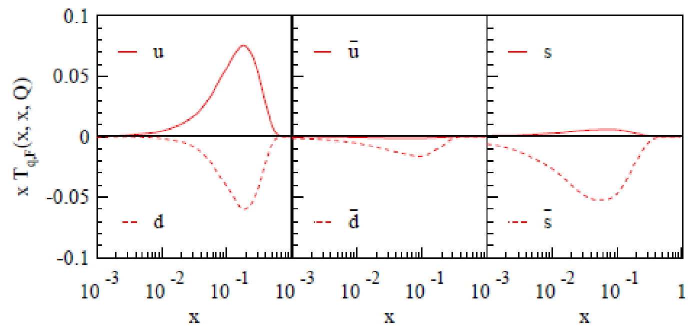
- Aybat, Collins, Qiu, Rogers, Phys. Rev. D85 (2012) 034043
- Aybat, Prokudin, Rogers, Phys. Rev. Lett. 108 (2012) 242003
- Anselmino, Boglione, Melis, Phys. Rev. D86 (2012) 014028
- Echevarria, Idilbi, Schafer, Scimemi, arXiv:1208.1281
- Echevarria, Idilbi, Scimemi, arXiv:1208.1281
- Godbole, Misra, Mukherjee, Raswoot, Phys. Rev. D89 (2014) 074013
- Sun, Yuan, Phys. Rev. D88 (2013)
- Boer, Nucl. Phys. B874 (2013) 217
- Echevarria, Idilbi, Kang, Phys. Rev. D89 (2014) 074013

  ...

  ...

… and many others ...





- *...until the most recent studies on the Sivers function in J/ψ production*
  A. Mukherjee et al.

- *and on the gluon contribution to the Sivers functions*
  Zheng, Aschenauer,Lee, Xiao,Yin, Phys. Rev. D98, 034011 (2018)

See talks by E. Aschenawer, C. Pisano, A. Mukherjee

EIC will give important contribution!

Simultaneous extraction of transversity and the Collins function

What about Q^2 evolution ?

$e^+e^- \to h_1h_2X$
BELLE - BaBar
$Q^2 \sim 100$ GeV^2

SIDIS
HERMES - COMPASS
$Q^2 \sim 3$ GeV^2

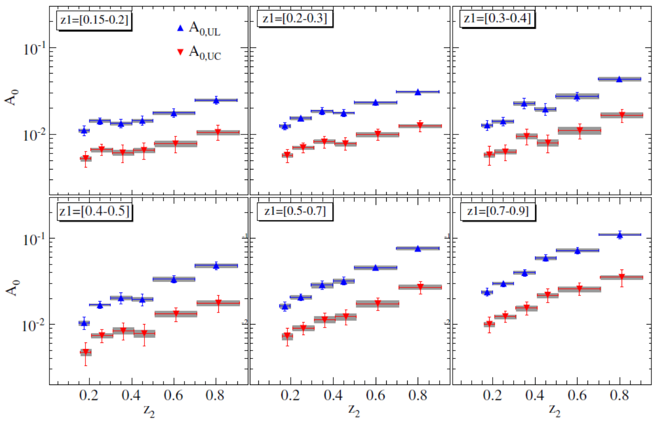
Simultaneous fits of SIDIS and $e^+e^- \to h_1h_2X$ involve data sets at very different Q^2 scales

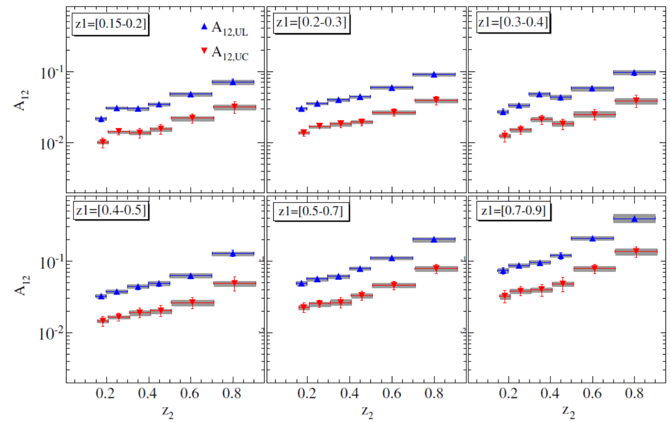
In our computation the Collins TMD function evolves according to DGLAP evolution equations, through its $D_{h/q}(z,p_t,Q^2)$ component

- Could TMD evolution be an issue ?
- Could TMD evolution affect our results ?

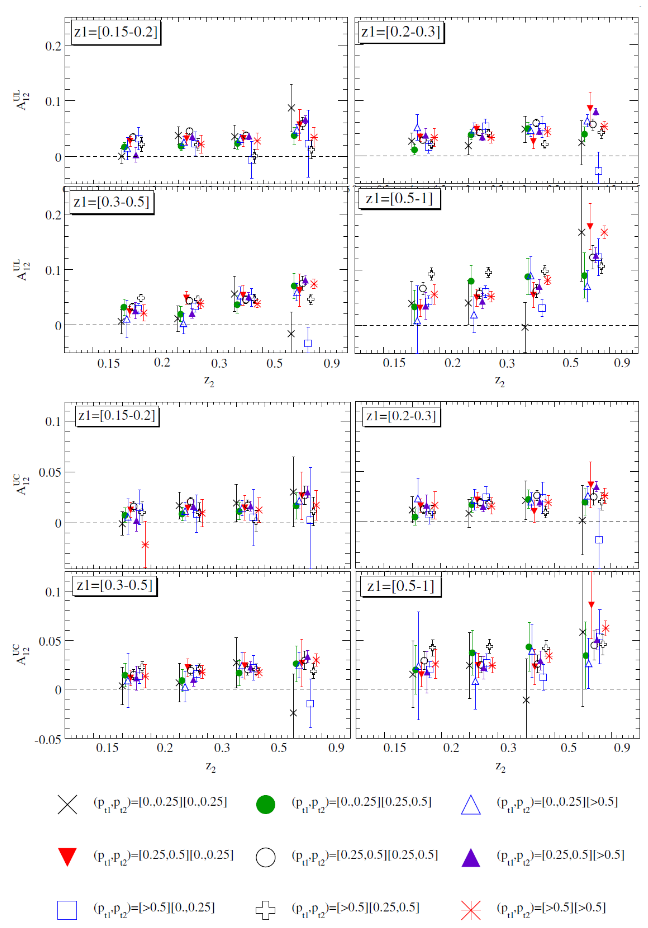
New BaBar data

BaBar measurements of A_0 and A_{12} as a function of z_1 and z_2

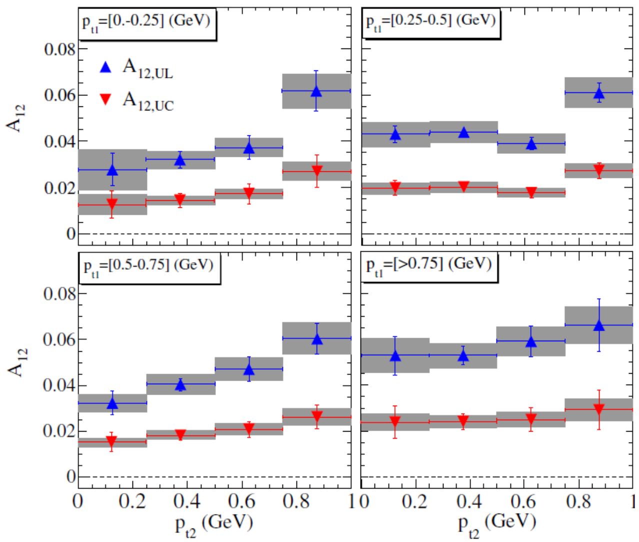


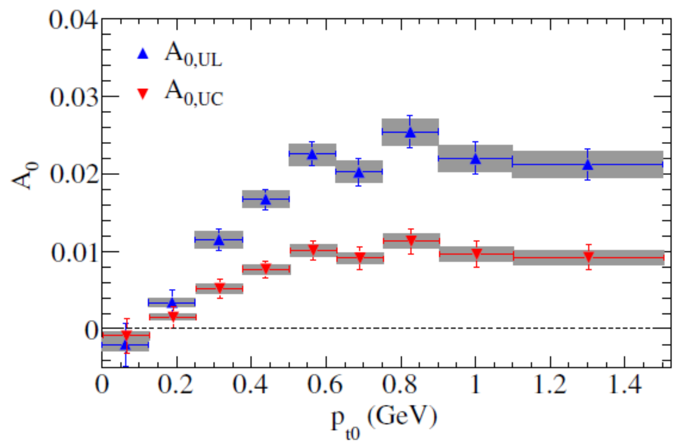


BaBar multidimensional data on A_{12} in bins of (z_1, z_2, p_{t1}, p_{t2})



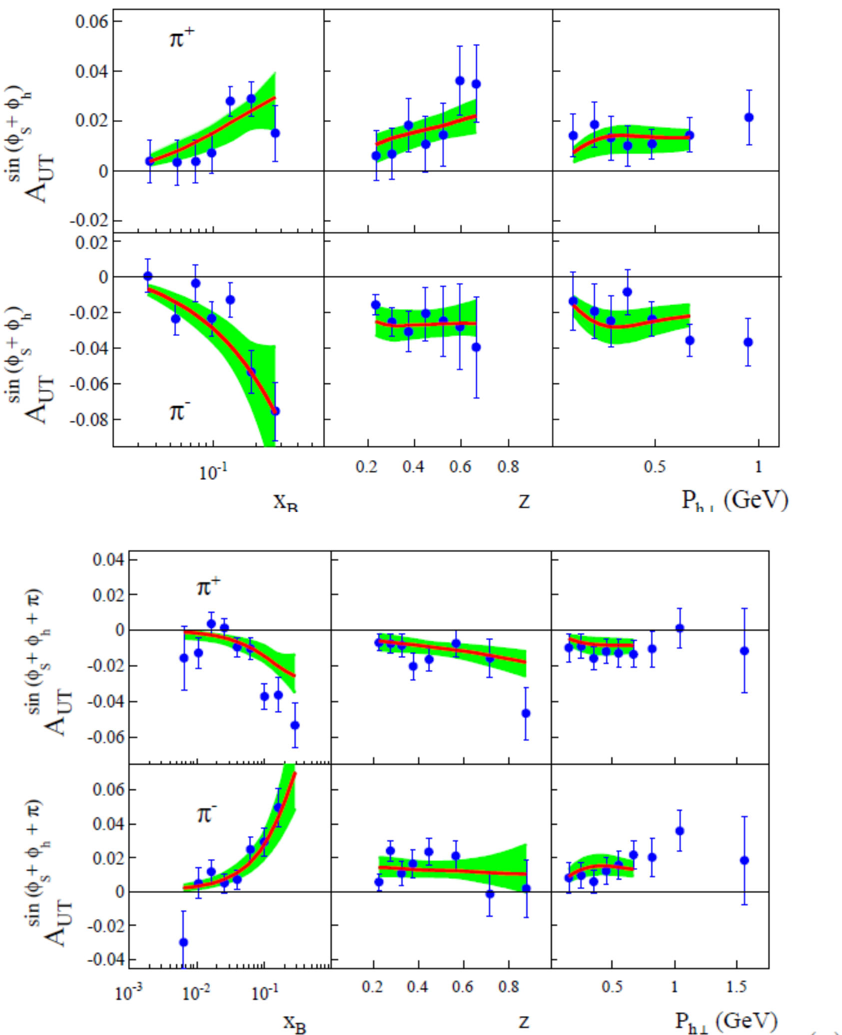
BaBar measurements of A_0 and A_{12} as a function of p_{t0} ,p_{t1} and p_{t2}





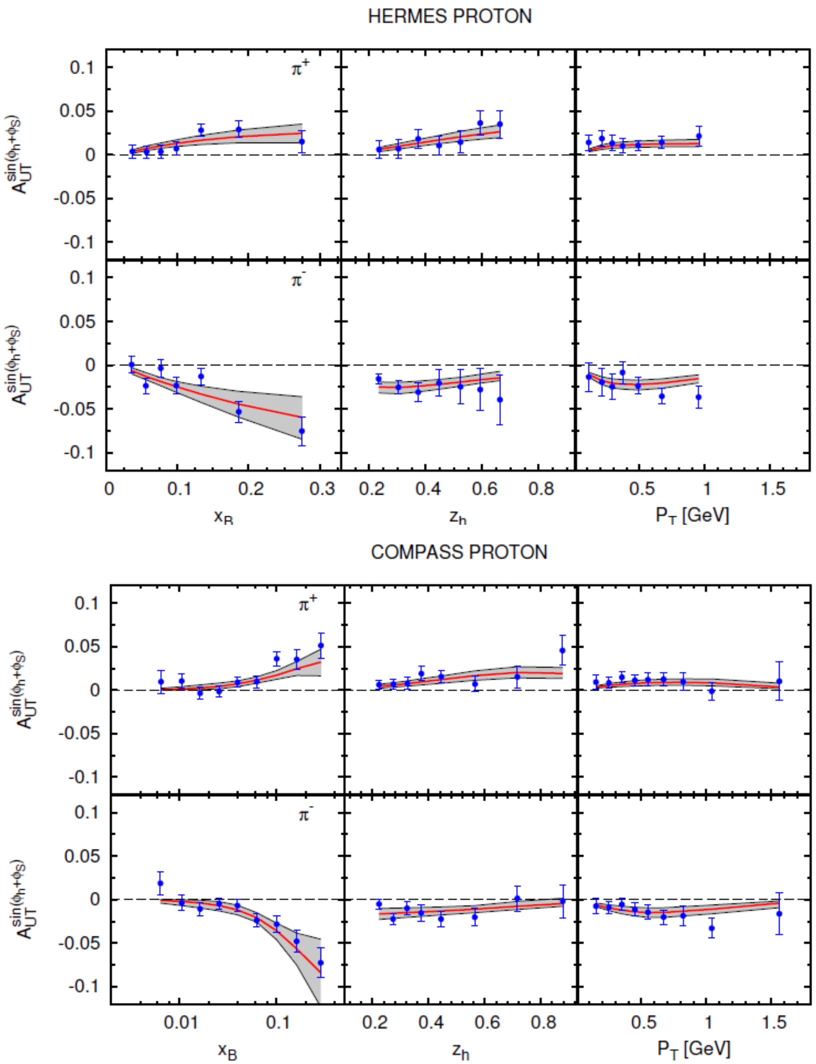
CSS/TMD evolution and Collins/Transversity

- TMD evolution



Kang et al: Phys. Rev. D93 (2016) 014009

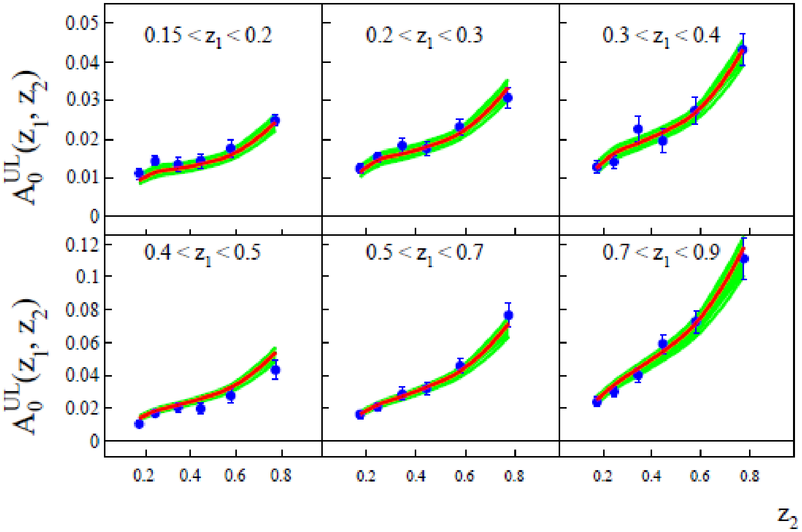
- Naive TMD

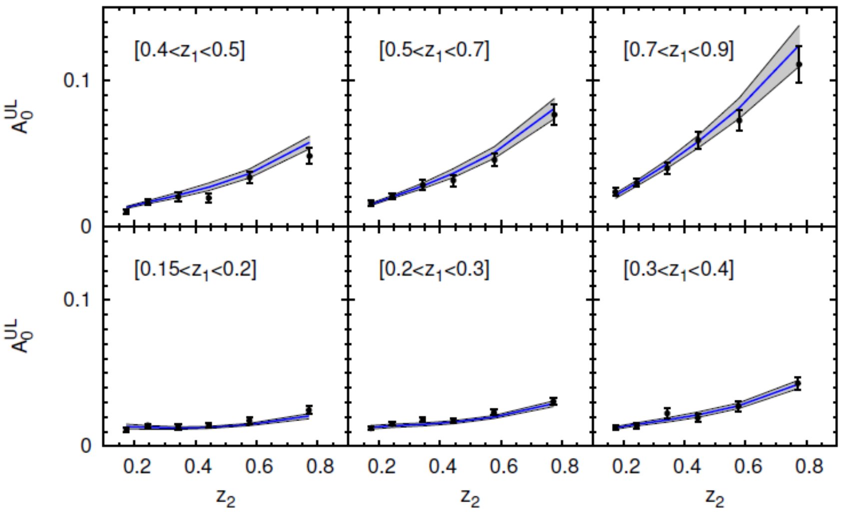


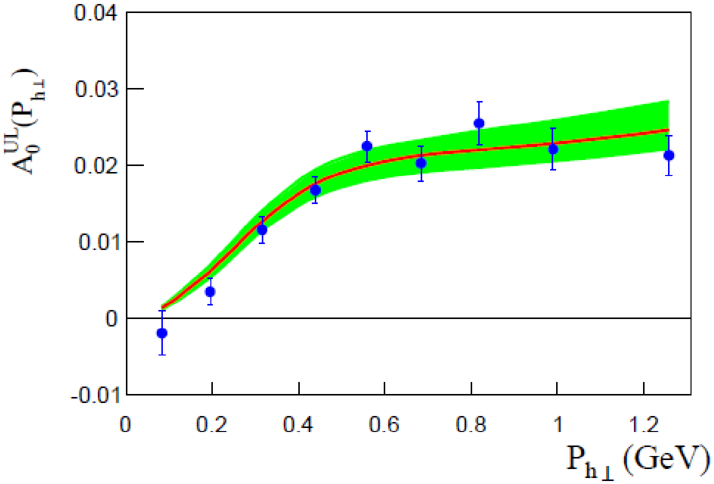
Anselmino et al., Phys. Rev. D92 (2015), 114023

CSS/TMD evolution and Collins/Transversity

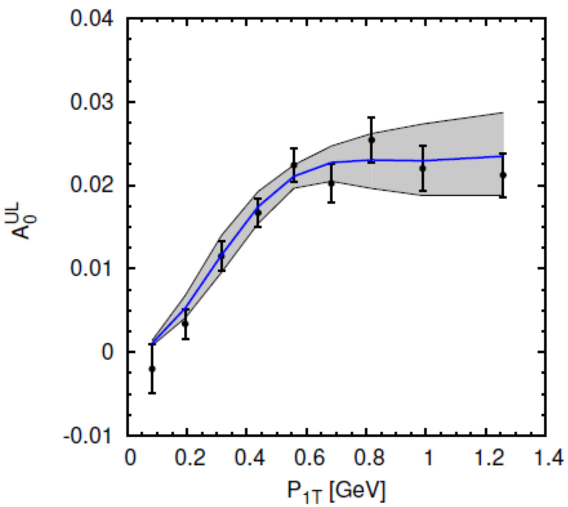
- TMD evolution
- Naive TMD







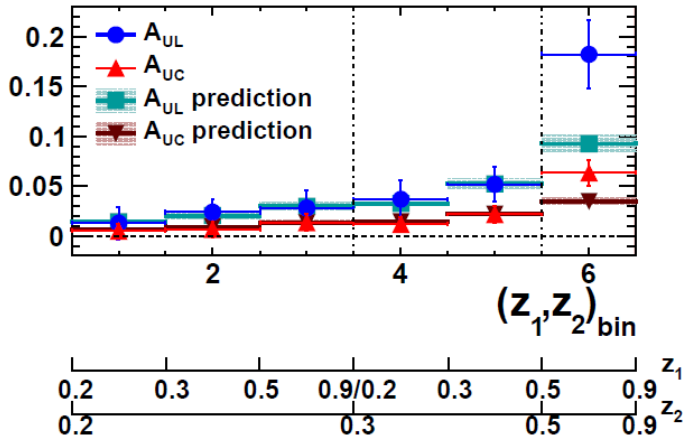
Kang et al: Phys. Rev. D93 (2016) 014009

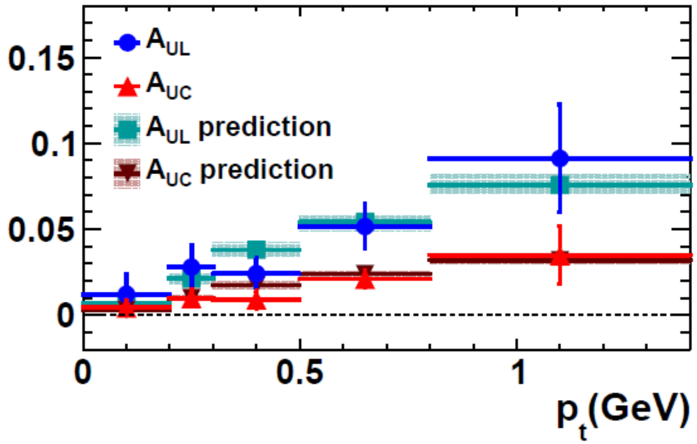


Anselmino et al., Phys. Rev. D92 (2015), 114023

CSS/TMD evolution and Collins/Transversity

- TMD evolution effects at BESIII ?

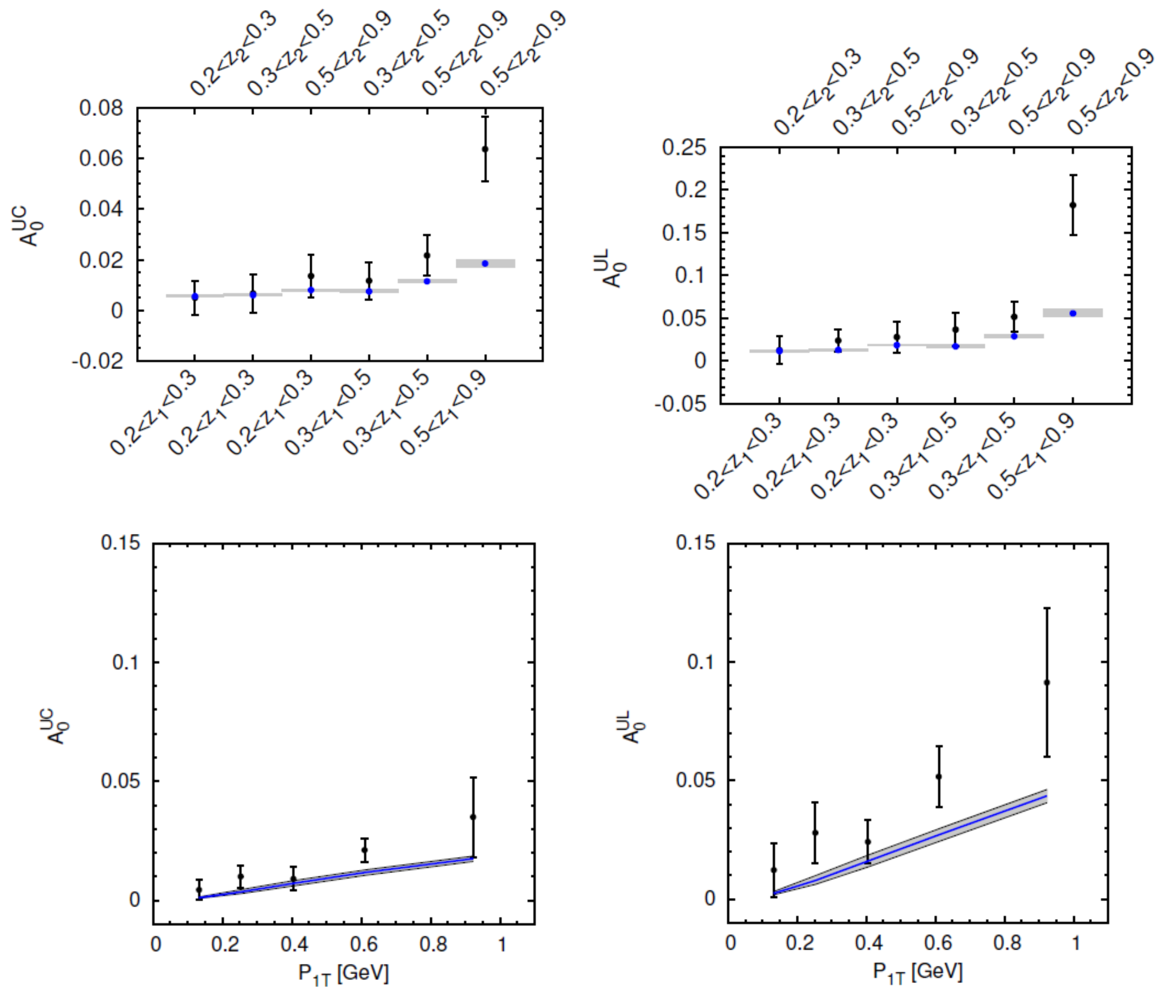




BESIII, Ablikim et al.,PRL116 (2016) 042001

Kang et al: Phys. Rev. D93 (2016) 014009

- Naive TMD



BESIII, Ablikim et al.,PRL116 (2016) 042001

Anselmino et al., Phys. Rev. D92 (2015), 114023

Other phenomenological analyses for the extraction of the Collins function

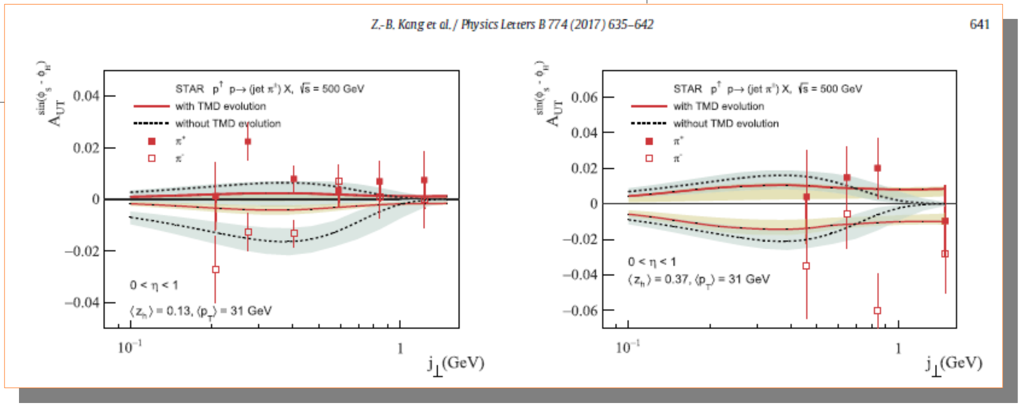
There are other ways to study the Collins function …

… for example studying the transverse momentum distribution of single hadron pp production in **jets**

Don't miss the TMD/Jet Session on Tuesday; X. Liu, Y. Makris, F. Ringer, D. Pitoniak

- *Kang, Liu, Ringer, Xing, JHEP (2017) 11:068*
- *Kang, Prokudin, Ringer, Yuan, Phys. Lett. B774 (2017) 635-642*
- *D'Alesio, Murgia, Pisano, Phys. Lett. B773 (2017) 300-306*

These analyses, performed on independent processes, provide evidence that the Collins function extracted in these processes is well consistent with that extracted by fitting e+e- and SIDIS data simultaneously. Moreover, they confirm that the experimental data presently available do not show signals of strong evolution effects, and cannot resolve calculations done with or without TMD evolution



EIC will give important contribution!

Other phenomenological analyses for the extraction of the Collins function

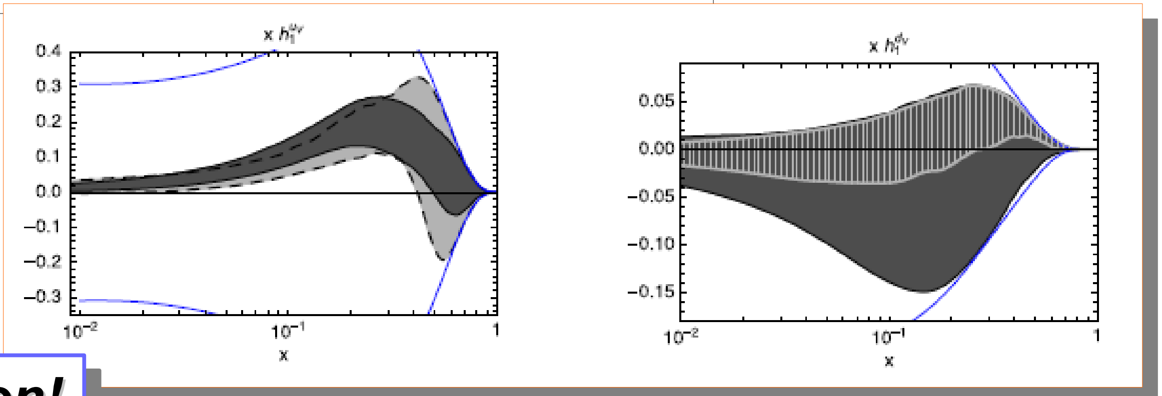
There are other ways to study the Collins function …

… Using collinear twist-three formalism

Don't miss the Collinear/ Twist-3 sessions on Wednesday and Thursday
W. Vogelsang, M. Schlegel, L. Gamberg

… Or studying closely related phenomena like the di-hadron fragmentation functions coupled to transversity

Dedicated talks on Thursday by
A. Vossen, M. Radici



EIC will give important contribution!

Outlooks and perspectives

- Phenomenological studies of TMDs, TMD factorization and TMD extraction have come a long way. Some issues remain open and need further investigation

- P_T distributions of unpolarized SIDIS cross sections need to be measured (over the largest possible P_T range) and further investigated on the phenomenological point of view.

- Simultaneous fits of SIDIS, Drell-Yan and e^+e^- annihilation data are highly recommended, but they should be performed within a consistent and solid framework where they can be implemented.

- New data allow for
  - Much more reliable extraction of the Sivers function
  - Detailed study of the uncertainties
  - Reduce the bias introduce by the choice of a specific parametrization on the final results

- Data selection is crucial in global fitting:
  - not too many
    (only data within the ranges where the TMD scheme works should be considered)
  - not too few
    (too strict a selection can bias the fit results and neglect important information from experimental data)

As discussed by T. Rogers this morning