Gluon TMDs in eA collisions

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Outline

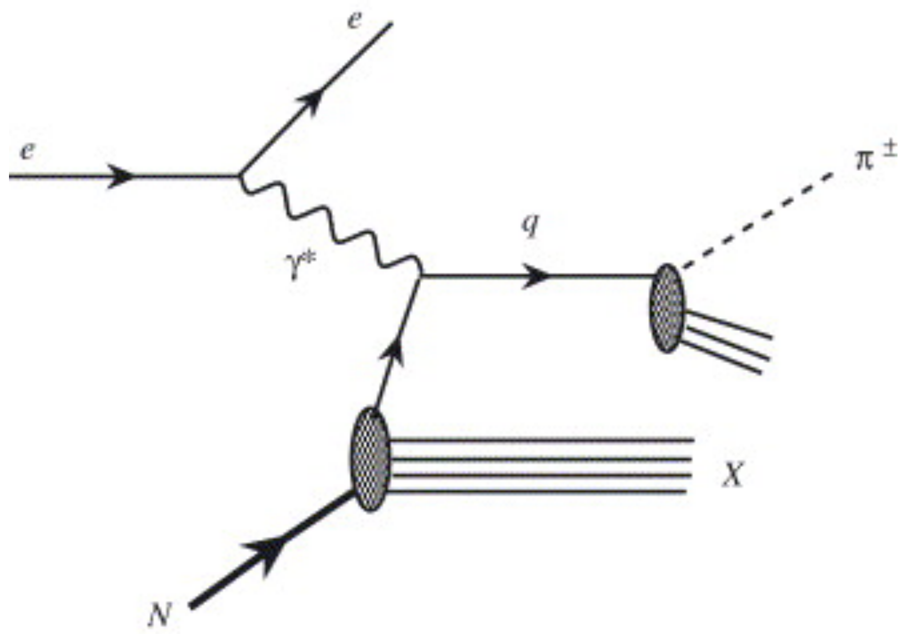
- Gluon TMDs and their process dependence
- Sivers effect example
- Small x limit - Wilson loop matrix elements
- Unpolarized gluon TMDs at small x
- Linear polarization of gluons (& the CGC) in eA
- GTMDs and odderon effects (odd harmonics)

Gluon TMDs
&
process dependence

Typical TMD processes

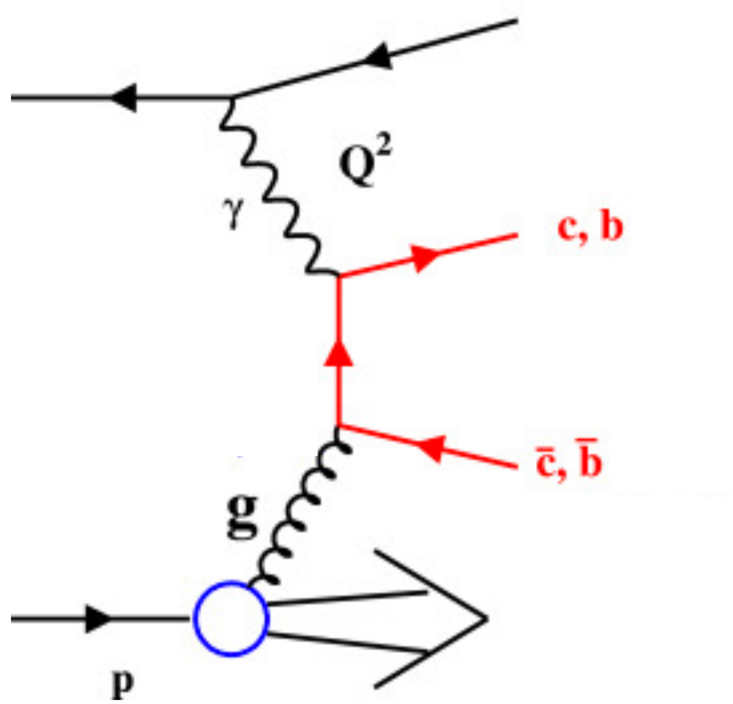
Semi-inclusive DIS is a process sensitive to the transverse momentum of quarks

$$e\,p \to e'\,h\,X$$



D-meson pair production is sensitive to transverse momentum of gluons

$$e\,p \to e'\,D\,\bar{D}\,X$$



Gluons TMDs

The gluon correlator:

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \mathrm{F.T.}\langle P|\mathrm{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{-g_T^{\mu\nu} f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right) h_1^{\perp\,g}(x,\boldsymbol{p}_T^2)\right\}$$

unpolarized gluon TMD

linearly polarized gluon TMD

[Mulders, Rodrigues '01]

Gluons inside *unpolarized* protons can be polarized!

For transversely polarized protons:

gluon Sivers TMD

$$\Gamma_T^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{g_T^{\mu\nu}\frac{\epsilon_T^{\rho\sigma}\,p_{T\rho}\,S_{T\sigma}}{M_p} f_{1T}^{\perp\,g}(x,\boldsymbol{p}_T^2) + \ldots\right\}$$

Process dependence of gluon TMDs

The color flow in a process may lead to different correlators in different processes

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \text{F.T.}\langle P|\text{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

$$\mathcal{U}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right) \qquad \xi = [0^+, \xi^-, \boldsymbol{\xi}_T]$$

Gauge links arise from the initial and/or final state interactions (ISI/FSI) in a process

[Collins & Soper, 1983; D.B. & Mulders, 2000; Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, X. Ji & F. Yuan, 2003; D.B., Mulders & Pijlman, 2003]

This has observable effects, as was first shown for quark Sivers effect asymmetries

[Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003]

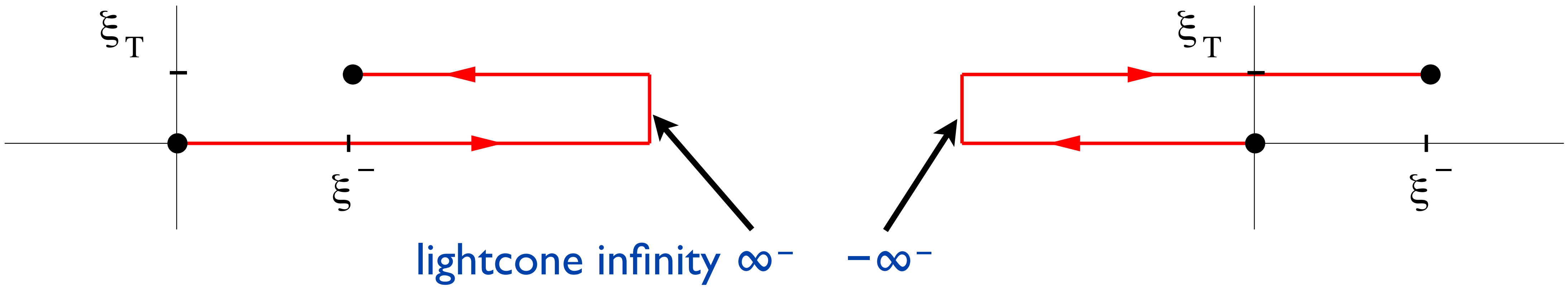
This even affects unpolarized gluon TMDs, as was first realized in a small-x context

[Dominguez, Marquet, Xiao, Yuan, 2011]

Process dependence of Sivers TMDs

SIDIS DY

FSI lead to a future pointing Wilson line (+ link), whereas ISI to past pointing (− link)



One can use parity and time reversal invariance to relate the quark Sivers TMDs

$$f_{1T}^{\perp q[\mathrm{SIDIS}]}(x, k_T^2) = -f_{1T}^{\perp q[\mathrm{DY}]}(x, k_T^2)$$

[Collins '02]

A similar sign change relation for gluon Sivers functions holds, but due to the appearance of two gauge links, there are more possibilities

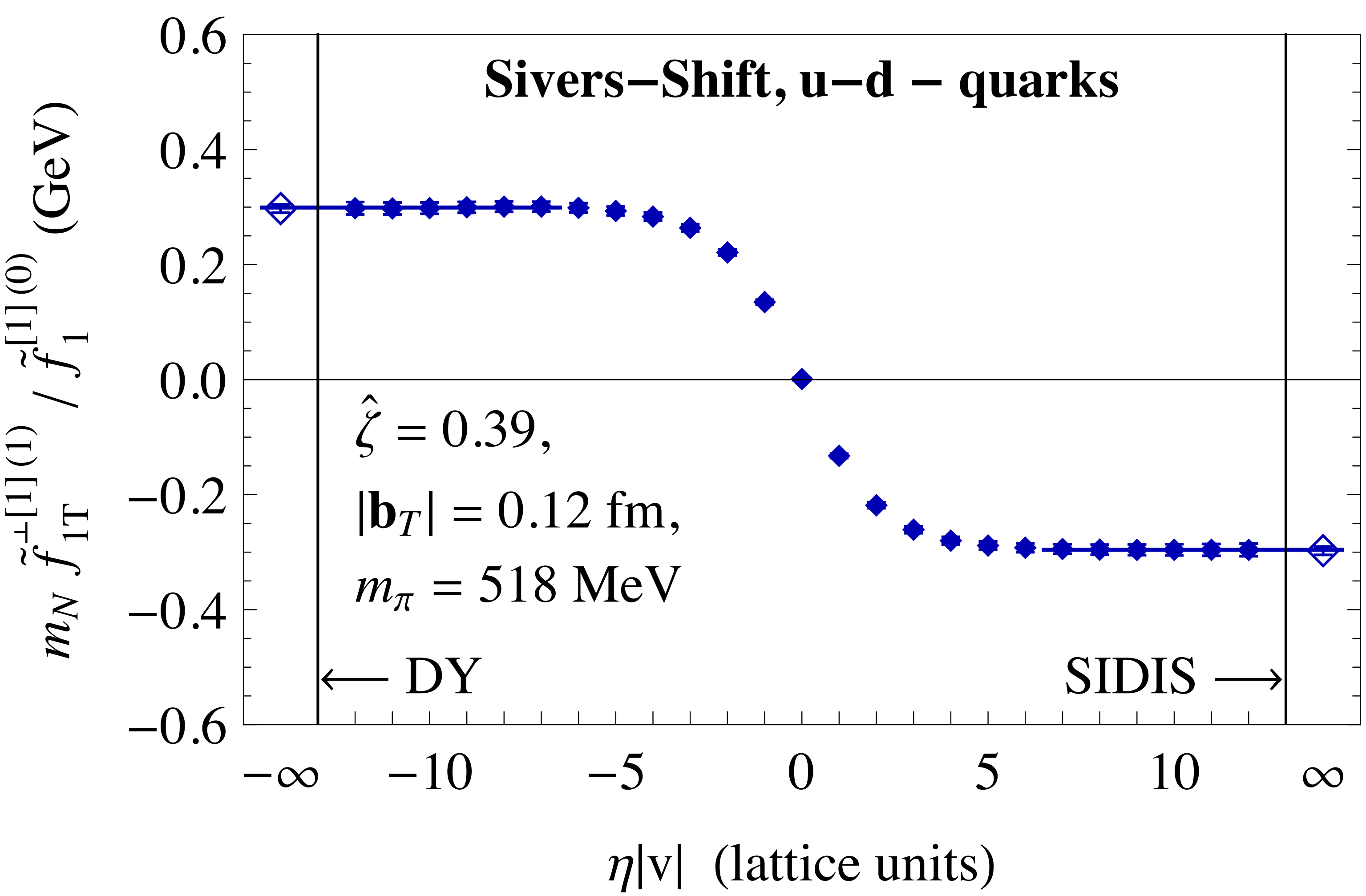
For most processes of interest there are 2 link combinations to consider: [+,+] and [+,−], because [−,−] and [−,+] are related to them by P and T

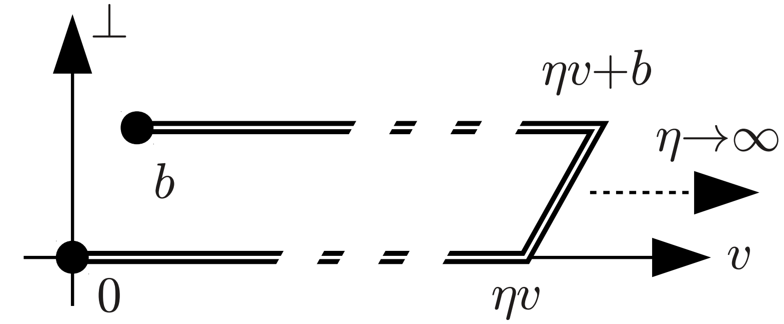
More complicated structures often only enter in processes where TMD factorization is questionable anyway

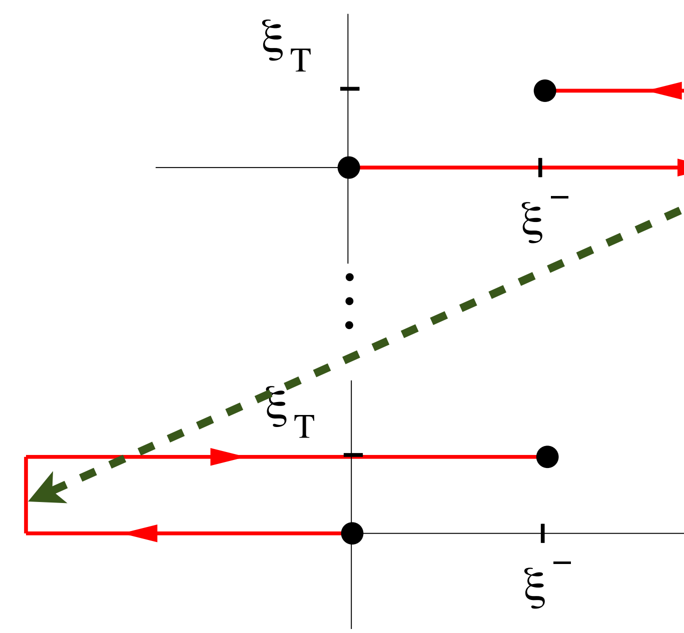
Quark Sivers function on the lattice

By taking specific x and k_T integrals one can define the "Sivers shift" $\langle k_T \times S_T \rangle(n, b_T)$: the average transverse momentum shift orthogonal to transverse spin S_T
[D.B., Gamberg, Musch, Prokudin, 2011]

This well-defined quantity can be evaluated on the lattice
[Musch, Hägler, Engelhardt, Negele & Schäfer, 2012]







This is the first `first-principle' demonstration that the Sivers function is nonzero for staple-like links. It clearly displays the sign change relation

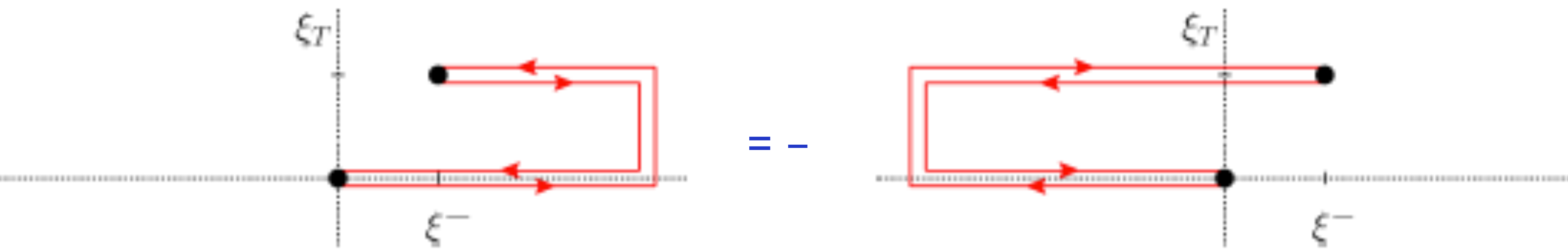
Sign change relation for gluon Sivers TMD

$e\,p^{\uparrow} \to e'\,Q\bar{Q}\,X$ $\qquad \gamma^*\,g \to Q\bar{Q}$ probes [+,+]

$p^{\uparrow}\,p \to \gamma\,\gamma\,X$ Qiu, Schlegel, Vogelsang, 2011

In the kinematic regime where the pair rapidity is central, one effectively selects the subprocess:

$g\,g \to \gamma\,\gamma$ probes [−,−]



$$f_{1T}^{\perp\,g\,[e\,p^{\uparrow}\to e'\,Q\,\overline{Q}\,X]}(x,p_T^2) = -f_{1T}^{\perp\,g\,[p^{\uparrow}\,p\to\gamma\,\gamma\,X]}(x,p_T^2)$$

EIC $\qquad$ RHIC

D.B., Mulders, Pisano, J. Zhou, 2016

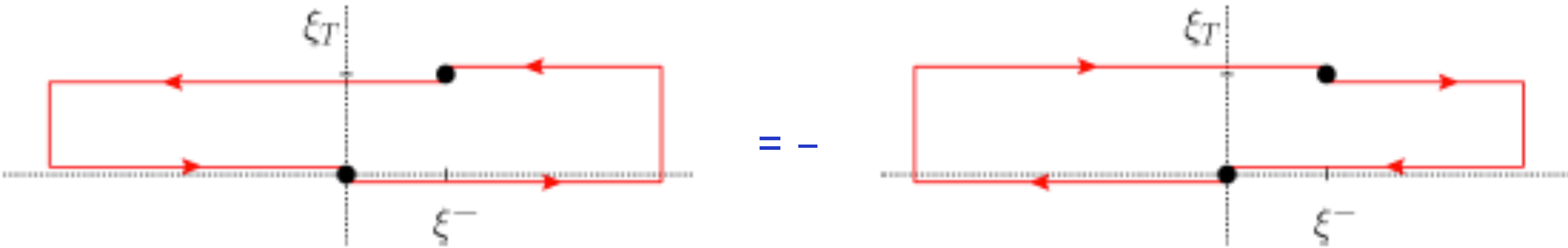
Challenging to measure, dijets are more promising, but theoretically less clean

Zheng, Aschenauer, Lee, Xiao, Yin, PRD 98 (2018) 034011

f and d type gluon Sivers TMD

$p^\uparrow\, p \to \gamma\, \mathrm{jet}\, X$

In the kinematic regime where gluons in the polarized proton dominate, one effectively selects the subprocess: $g\, q \to \gamma\, q$ probes [+,−]



This process probes a distinct, *independent* gluon Sivers function

Gluon Sivers TMDs for [+,+] & [+,−] are related to the f^{abc} & d^{abc} color structures

Bomhof, Mulders, 2007; Buffing, Mukherjee, Mulders, 2013

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \xrightarrow{x\to 0} \frac{k_T^i k_T^j}{2\pi L}\Gamma_0^{[\Box]}(\boldsymbol{k}_T)$$

a single Wilson loop matrix element

$$U^{[\Box]} = U^{[+]}_{[0,y]} U^{[-]}_{[y,0]}$$

D.B., Cotogno, van Daal, Mulders, Signori & Ya-Jin Zhou, JHEP 2016

d-type gluon Sivers effect

The d-type gluon Sivers function $f_{1T}^{\perp\, g\, [+,-]}$ at small x is part of:

$$\Gamma_{(d)}^{(T-\mathrm{odd})} \equiv \left(\Gamma^{[+,-]} - \Gamma^{[-,+]}\right) \propto \mathrm{F.T.}\, \langle P, S_T | \mathrm{Tr} \left[U^{[\Box]}(0_T, y_T) - U^{[\Box]\dagger}(0_T, y_T) \right] | P, S_T \rangle$$

D.B., Echevarria, Mulders, J. Zhou, PRL 2016

At small x it can be identified with the *spin-dependent odderon* [J. Zhou, 2013]

In $p^{\uparrow}A$ collisions this can be probed in $\gamma^{(*)}$-jet production in the back-to-back correlation limit, but also in backward charged hadron production for moderate p_T (as the odderon is C-odd, for gg-dominated scattering one should select final states that are not C-even)

It is the only relevant contribution to A_N in backward ($x_F < 0$) charged hadron production in $p^{\uparrow}p$ or $p^{\uparrow}A$ (in contrast to the many contributions at $x_F > 0$)

A_N is not a TMD factorizing process, but at small x one can apply a hybrid factorization (at least at one-loop order)

Chirilli, Xiao, Yuan, PRL & PRD 2012

Importance of the loop in quark TMDs

In the small-x limit of the leading twist TMD formalism there is no odderon contribution for an unpolarized proton (nor a helicity distribution)

At small x and/or for large A the Wilson loop matrix element is important
For large x and for quarks we do not know, but a lattice test is possible

$$f_1^{[+]}(x, p_T^2) = f_1^{[-]}(x, p_T^2)$$

$$f_1^{[\Box+]}(x, p_T^2) \neq f_1^{[+]}(x, p_T^2)$$

[D.B., Buffing, Mulders, JHEP 2015]

Irrespective of whether one can isolate the function with an additional loop from experiment, one can study particular Mellin-Bessel moments of it on the lattice:

$$\frac{\tilde{f}_1^{[1](1)[\Box+]}(\boldsymbol{b}_T^2; \mu, \zeta)}{\tilde{f}_1^{[1](1)[+]}(\boldsymbol{b}_T^2; \mu, \zeta)} = \frac{\langle P|\overline{\psi}(0, 0_T)\gamma^+ \, U_{[0,b]}^{[+]} U_{[b,0]}^{[-]} U_{[0,b]}^{[+]} \, \psi(0, b_T)|P\rangle}{\langle P|\overline{\psi}(0, 0_T)\gamma^+ \, U_{[0,b]}^{[+]} \, \psi(0, b_T)|P\rangle}$$

At b_T=0 this ratio should be **1**, for large b_T it should become a flat curve

This will give us information on how important the flux of $F^{\mu\nu}$ through the loop is and hence how important the process dependence effects are or can be

Unpolarized gluon TMDs

WW vs DP

For most processes of interest there are 2 relevant unpolarized gluon distributions

Dominguez, Marquet, Xiao, Yuan, 2011

$$xG^{(1)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle \quad [+,+]$$

$$xG^{(2)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle \quad [+,-]$$

For unpolarized gluons [+,+] = [−,−] and [+,−] = [−,+]

At small x the two correspond to the Weizsäcker-Williams (WW) and dipole (DP) distributions, which are generally different in magnitude and width:

$$xG^{(1)}(x,k_\perp) = -\frac{2}{\alpha_S}\int \frac{d^2v}{(2\pi)^2}\frac{d^2v'}{(2\pi)^2}\, e^{-ik_\perp\cdot(v-v')}\left\langle \mathrm{Tr}\left[\partial_i U(v)\right]U^\dagger(v')\left[\partial_i U(v')\right]U^\dagger(v)\right\rangle_{x_g} \quad \text{WW}$$

$$xG^{(2)}(x,q_\perp) = \frac{q_\perp^2 N_c}{2\pi^2\alpha_s} S_\perp \int \frac{d^2r_\perp}{(2\pi)^2} e^{-iq_\perp\cdot r_\perp}\frac{1}{N_c}\left\langle \mathrm{Tr}U(0)U^\dagger(r_\perp)\right\rangle_{x_g} \quad \text{DP}$$

Explains Kharzeev, Kovchegov & Tuchin's "tale of two gluon distributions" (2003)

MV model

In the MV model one may not notice the difference between WW and DP:

$$xG_g^{(2)}(x,q_\perp) \overset{\text{MV}}{\propto} q_\perp^2 \nabla_{q_\perp}^2 xG_g^{(1)}(x,q_\perp)$$

Processes involving $G^{(1)}$ (WW) [+,+] in the MV model can be expressed in terms of $G^{(2)} \sim C(k_\perp)$, e.g.

$$\gamma\, A \to Q\bar{Q}\, X$$

Gelis, Peshier, 2002

$$\frac{d\sigma_T}{dy\, dk_\perp} = \pi R^2 \frac{2N_c(Z\alpha)^2}{3\pi^3} \ln\!\left(\frac{\gamma}{2mR}\right) k_\perp C(k_\perp) \times \left\{ 1 + \frac{4(k_\perp^2 - m^2)}{k_\perp\sqrt{k_\perp^2+4m^2}} \operatorname{arcth} \frac{k_\perp}{\sqrt{k_\perp^2+4m^2}} \right\}$$

$$C(k_\perp) = \int d^2x_\perp\, e^{ik_\perp \cdot x_\perp} \langle U(0) U^\dagger(x_\perp) \rangle$$

Heavy quark pair production in DIS probes the WW distribution

For general x expressions, see Pisano, D.B., Brodsky, Buffing, Mulders, 2013

WW vs DP

Different processes probe one or the other (or a mixture)

	DIS	DY	SIDIS	$pA \to \gamma\,\text{jet}\,X$	$e\,p \to e'\,Q\,\overline{Q}\,X$ $e\,p \to e'\,j_1\,j_2\,X$	$pp \to \eta_{c,b}\,X$ $pp \to H\,X$	$pp \to J/\psi\,\gamma\,X$ $pp \to \Upsilon\,\gamma\,X$
$f_1^{g\,[+,+]}$ (WW)	$\times$	$\times$	$\times$	$\times$	$\surd$	$\surd$	$\surd$
$f_1^{g\,[+,-]}$ (DP)	$\surd$	$\surd$	$\surd$	$\surd$	$\times$	$\times$	$\times$

↑ γ^*g fusion probes WW

There are sufficient processes in ep and pp collisions to test the expectations

Q: How different can the two unpolarized gluon distributions be?

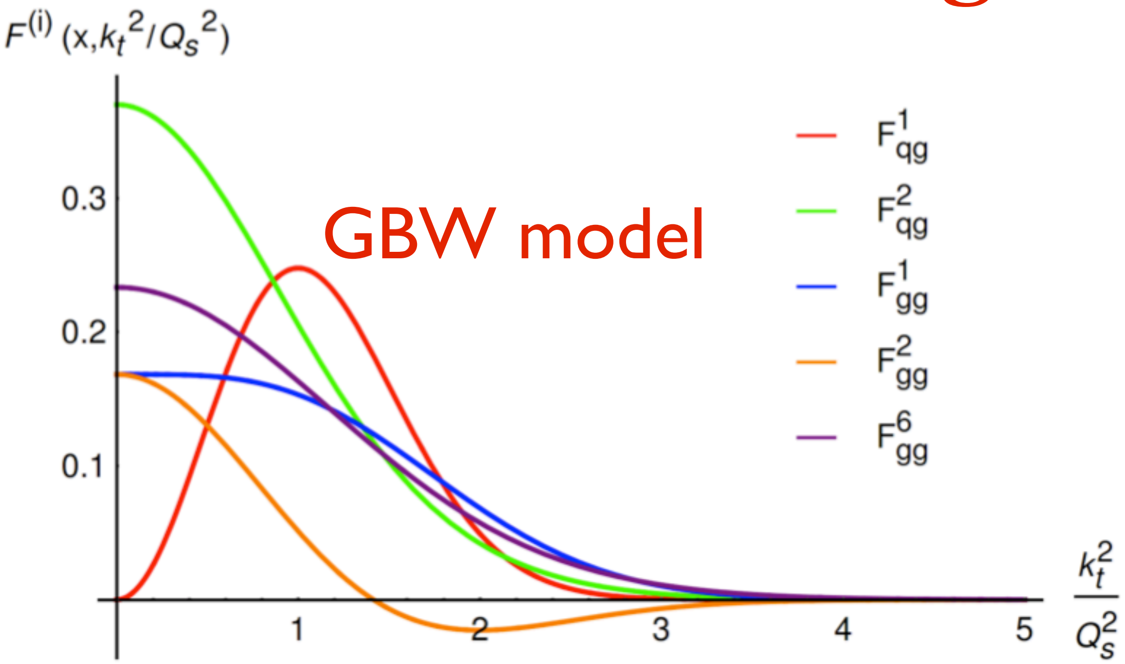
The first transverse moment must coincide

$$\int d\boldsymbol{k}_T\, f_1^{g\,[+,+]}(x,\boldsymbol{k}_T^2) = \int d\boldsymbol{k}_T\, f_1^{g\,[+,-]}(x,\boldsymbol{k}_T^2)$$

Also the large-k_T tail of the functions might be expected to coincide

But other than that the functions can have rather different shapes and magnitudes

More general link structures



$$\mathcal{F}_{qg}^{(1)} = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\xi} \left\langle \mathrm{Tr}\left[F(\xi)\,\mathcal{U}^{[-]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle = x_2 G^{(2)}(x_2,k_t)\,,$$

$$\mathcal{F}_{qg}^{(2)} = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\xi} \left\langle \mathrm{Tr}\left[F(\xi)\,\frac{\mathrm{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c}\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle\,.$$

$$\mathcal{F}_{gg}^{(1)} = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\xi} \left\langle \mathrm{Tr}\left[F(\xi)\,\frac{\mathrm{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c}\mathcal{U}^{[-]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle\,,$$

$$\mathcal{F}_{gg}^{(2)} = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\xi} \frac{1}{N_c}\left\langle \mathrm{Tr}\left[F(\xi)\,\mathcal{U}^{[\Box]\dagger}\right]\mathrm{Tr}\left[F(0)\,\mathcal{U}^{[\Box]}\right]\right\rangle\,,$$

$$\mathcal{F}_{gg}^{(3)} = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\xi} \left\langle \mathrm{Tr}\left[F(\xi)\,\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle = x_2 G^{(1)}(x_2,k_t)\,.$$

The large-k_T tails indeed coincide, except for $\mathcal{F}_{gg}^{(2)}$ that vanishes

$$\mathcal{F}_{qg}^{(1)}\,,\mathcal{F}_{qg}^{(2)}\,,\mathcal{F}_{gg}^{(1)}\,,\mathcal{F}_{gg}^{(6)} \simeq \frac{N_c S_\perp Q_s^2}{4\pi^3 \alpha_s k_t^2} + \mathcal{O}\left(\frac{Q_s^4}{k_t^4}\log\frac{k_t^2}{\Lambda^2}\right)\,,$$

$$\mathcal{F}_{gg}^{(2)} \simeq \mathcal{O}\left(\frac{Q_s^4}{k_t^4}\log\frac{k_t^2}{\Lambda^2}\right)\,.$$

Kotko, Kutak, Marquet, Petreska, Sapeta, van Hameren, 2015 & 2016

Although there are in principle infinitely many gauge link structures possible, the p_T^2 average consists of 5 independent terms in a *calculable* combination (similar to the f and d pieces for the first p_T-moment of the Sivers TMD)

$$\int d^2p_T\,\frac{p_T^\alpha p_T^\beta}{M^2}\,\Gamma^{\mu\nu[U,U']}(x,p_T) = \Gamma^{\mu\nu;\alpha\beta}_{\partial\partial}(x) + \sum_{c=1}^{4} C^{[U,U']}_{GG,c}\,\Gamma^{\mu\nu;\,\alpha\beta}_{GG,c}(x)$$

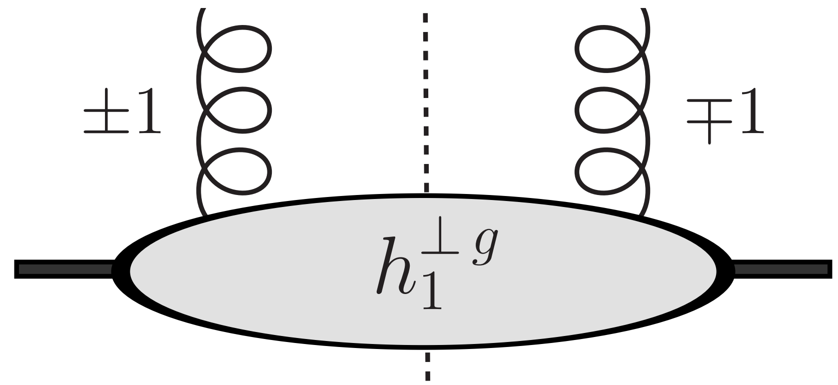
D.B., Buffing, Mulders, 2015

Linearly polarized gluon TMDs

Gluon polarization inside unpolarized protons

Linearly polarized gluons can exist in **unpolarized** hadrons

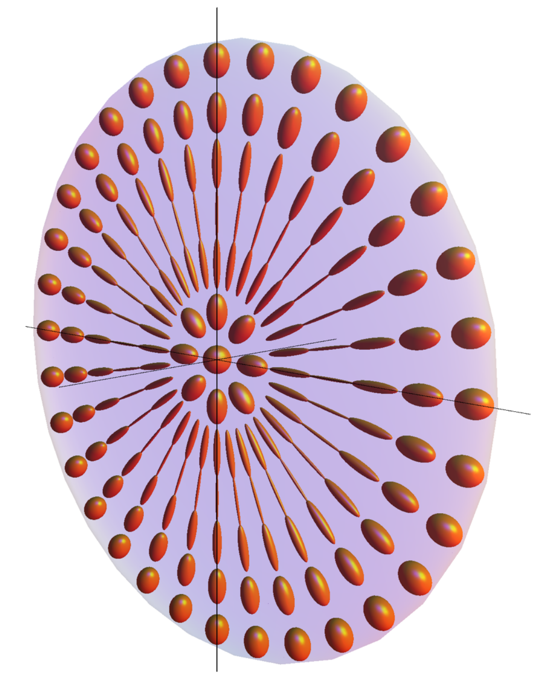
[Mulders, Rodrigues, 2001]



an interference between ±1 helicity gluon states

It requires nonzero transverse momentum: TMD

For $h_1^{\perp\, g} > 0$ gluons prefer to be polarized along k_T, with a *cos* 2ϕ distribution of linear polarization around it, where $\phi = \angle(k_T, \varepsilon_T)$



$$\Gamma_U^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{x}{2} \left\{ -\, g_T^{\mu\nu}\, f_1^g(x, \boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu} \frac{\boldsymbol{p}_T^2}{2M_p^2} \right) h_1^{\perp\, g}(x, \boldsymbol{p}_T^2) \right\}$$

For linearly polarized gluons also [+,+] = [−,−] and [+,−] = [−,+]

Probes of linear gluon polarization

$h_1^{\perp g}$ is more difficult to extract, as it cannot be probed in DIS, DY, SIDIS, nor in inclusive hadron or γ+jet production in pp or pA collisions

Processes that probe the WW or DP linearly polarized gluon TMD:

	$pp \to \gamma\,\gamma\,X$	$pA \to \gamma^*\,\text{jet}\,X$	$e\,p \to e'\,Q\,\overline{Q}\,X$ $e\,p \to e'\,j_1\,j_2\,X$	$pp \to \eta_{c,b}\,X$ $pp \to H\,X$	$pp \to J/\psi\,\gamma\,X$ $pp \to \Upsilon\,\gamma\,X$
$h_1^{\perp\,g\,[+,+]}$ (WW)	$\surd$	$\times$	$\surd$	$\surd$	$\surd$
$h_1^{\perp\,g\,[+,-]}$ (DP)	$\times$	$\surd$	$\times$	$\times$	$\times$

% level or less at RHIC
Qiu, Schlegel, Vogelsang, 2011

5% level at RHIC
D.B., Mulders, Zhou, Zhou, 2017

10% level or less at EIC
D.B., Brodsky, Pisano, Mulders, 2011; Dumitru, Lappi, Skokov, 2015; D.B., Pisano, Mulders, J. Zhou, 2016

10% level for η_Q and % level for Higgs at LHC
D.B. & den Dunnen, 2014; Echevarria, Kasemets, Mulders, Pisano, 2015

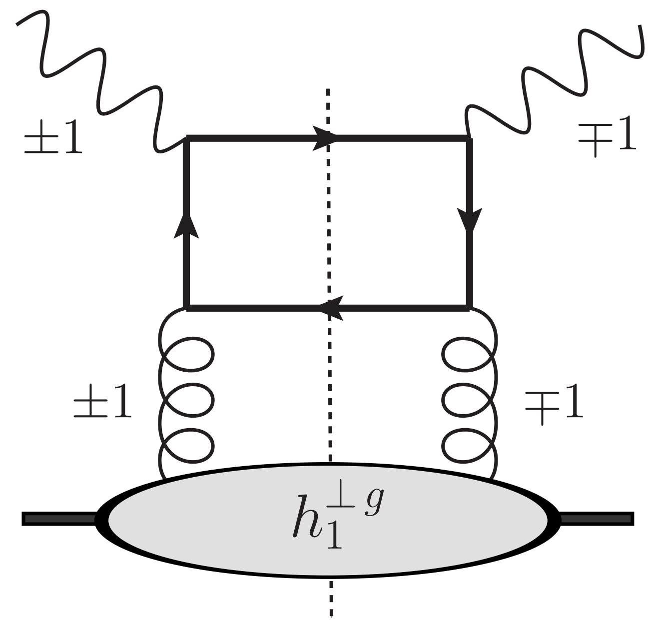
Higgs and $0^{\pm +}$ quarkonium production uses the angular *independent* p_T distribution

All other suggestions use angular modulations

Open heavy quark electro-production

Unpolarized open heavy quark production allows to probe linearly polarized gluons in *unpolarized* hadrons

$$ep \to e' Q\bar{Q} X$$



[D.B., Brodsky, Mulders & Pisano, 2010]

It gives rise to an angular distributions: a $\cos 2(\phi_T - \phi_\perp)$ asymmetry, where $\phi_{T/\perp}$ are the angles of $K_\perp^Q \pm K_\perp^{\bar{Q}}$

$h_1^{\perp g}$ appears by itself, so effects could be significant, especially towards smaller x

It is expected to keep up with the growth of the unpolarized gluons as $x \to 0$

Linear gluon polarization at small x

There is no theoretical reason why $h_1^{\perp g}$ should be small, especially at small x

DGLAP evolution: $h_1^{\perp g}$ has the same 1/x growth as f_1

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b) C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

The small-x limit of the DP correlator in the TMD formalism:

$$\Gamma^{[+,-]\,ij}(x, \boldsymbol{k}_T) \xrightarrow{x \to 0} \frac{k_T^i k_T^j}{2\pi L} \Gamma_0^{[\Box]}(\boldsymbol{k}_T) \qquad U^{[\Box]} = U^{[+]}_{[0,y]} U^{[-]}_{[y,0]}$$

$$\Gamma_U^{ij}(x, \boldsymbol{k}_T) = \frac{x}{2} \left[-g_T^{ij} f_1(x, \boldsymbol{k}_T^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \boldsymbol{k}_T^2) \right] \xrightarrow{x \to 0} \frac{k_T^i k_T^j}{2M^2} e(\boldsymbol{k}_T^2)$$

$$\lim_{x \to 0} x f_1(x, \boldsymbol{k}_T^2) = \frac{\boldsymbol{k}_T^2}{2M^2} \lim_{x \to 0} x h_1^\perp(x, \boldsymbol{k}_T^2) = \frac{\boldsymbol{k}_T^2}{2M^2} e(\boldsymbol{k}_T^2)$$

In the TMD formalism the DP $h_1^{\perp g}$ becomes maximal when $x \to 0$

D.B., Cotogno, van Daal, Mulders, Signori, Zhou, 2016

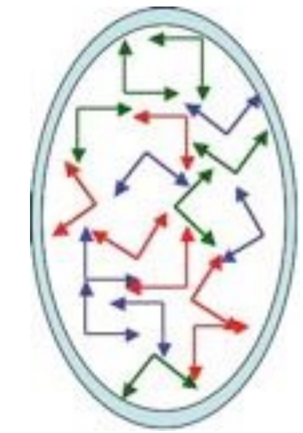
Polarization of the CGC

MV model calculations show the CGC gluons are in fact linearly polarized

$$h_{1,WW}^{\perp g} \ll f_{1,WW}^{\perp g} \quad \text{for } k_\perp \ll Q_s, \qquad h_{1,WW}^{\perp g} = 2 f_{1,WW}^{\perp g} \quad \text{for } k_\perp \gg Q_s$$

$$x h_{1,DP}^{\perp g}(x, k_\perp) = 2 x f_{1,DP}^{g}(x, k_\perp)$$

Metz, Zhou '11



The WW $h_1^{\perp g}$ is (moderately) suppressed for small transverse momenta:

$$\frac{h_{1\,WW}^{\perp g}}{f_{1\,WW}} \propto \frac{1}{\ln Q_s^2/k_\perp^2}$$

The CGC can be 100% polarized, but its observable effects depend on the process

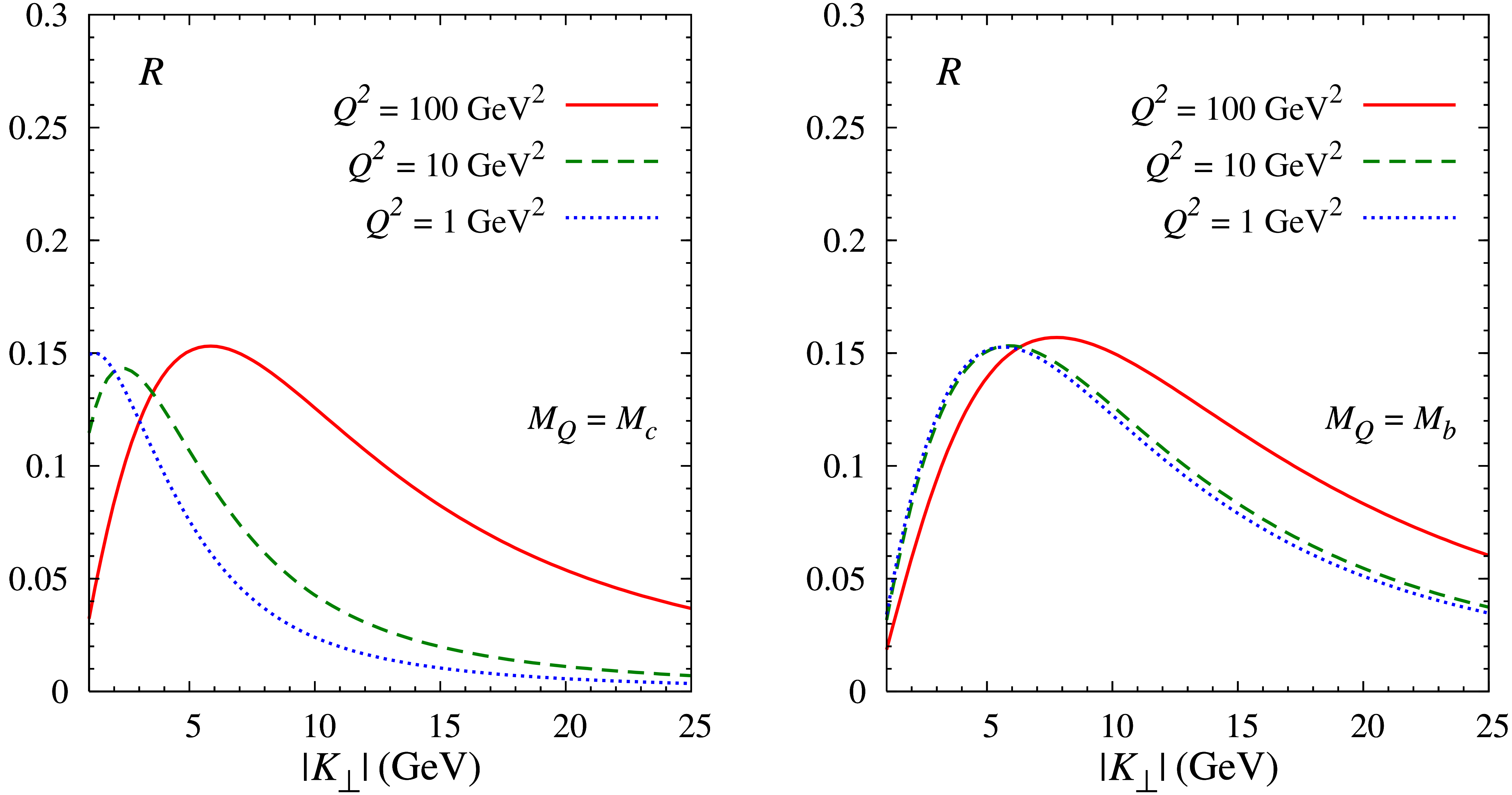
The "k_T-factorization" approach (CCFM) yields maximum polarization too (but no process dependence):

$$\Gamma_g^{\mu\nu}(x, \boldsymbol{p}_T)_{\text{max pol}} = \frac{p_T^\mu p_T^\nu}{\boldsymbol{p}_T^2} \, x \, f_1^g$$

Catani, Ciafaloni, Hautmann, 1991

Maximum asymmetries in heavy quark pair production

$ep \to e' Q\bar{Q} X$ $\quad$ $R =$ bound on $|\langle \cos 2(\phi_T - \phi_\perp) \rangle|$



[Pisano, D.B., Brodsky, Buffing & Mulders, JHEP 10 (2013) 024]

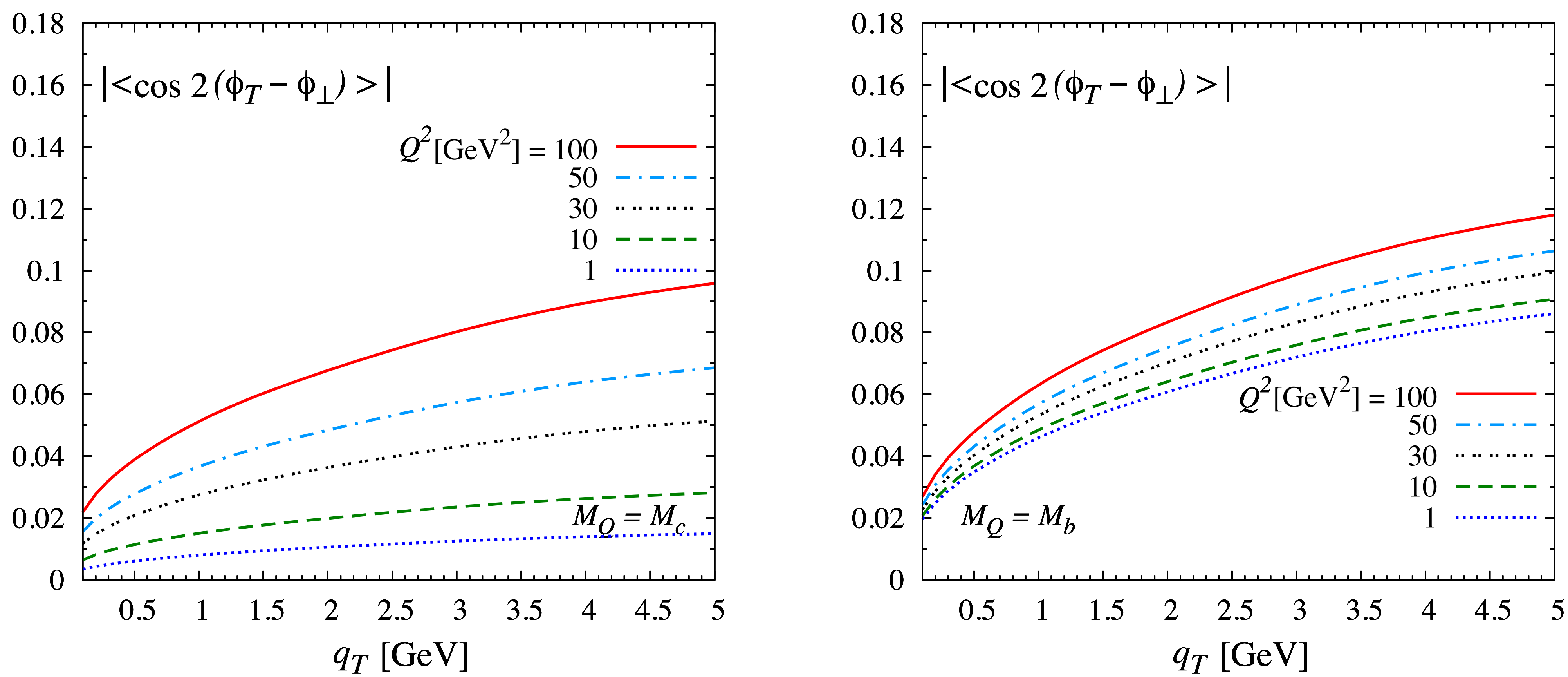
Maximal asymmetries can be substantial (for any Q^2 and for both charm & bottom)

Asymmetries in heavy quark pair production

But this process probes the WW distributions, which is not maximal at small x

The WW $h_1^{\perp g}$ is (moderately) suppressed for small transverse momenta:

$$\frac{h_{1\,WW}^{\perp\,g}}{f_{1\,WW}} \propto \frac{1}{\ln Q_s^2/k_\perp^2}$$



small x
MV model

$|\boldsymbol{K}_\perp| = 10\,\mathrm{GeV}$

$z = 0.5$

$y = 0.3$

D.B., Pisano, Mulders, Zhou, 2016

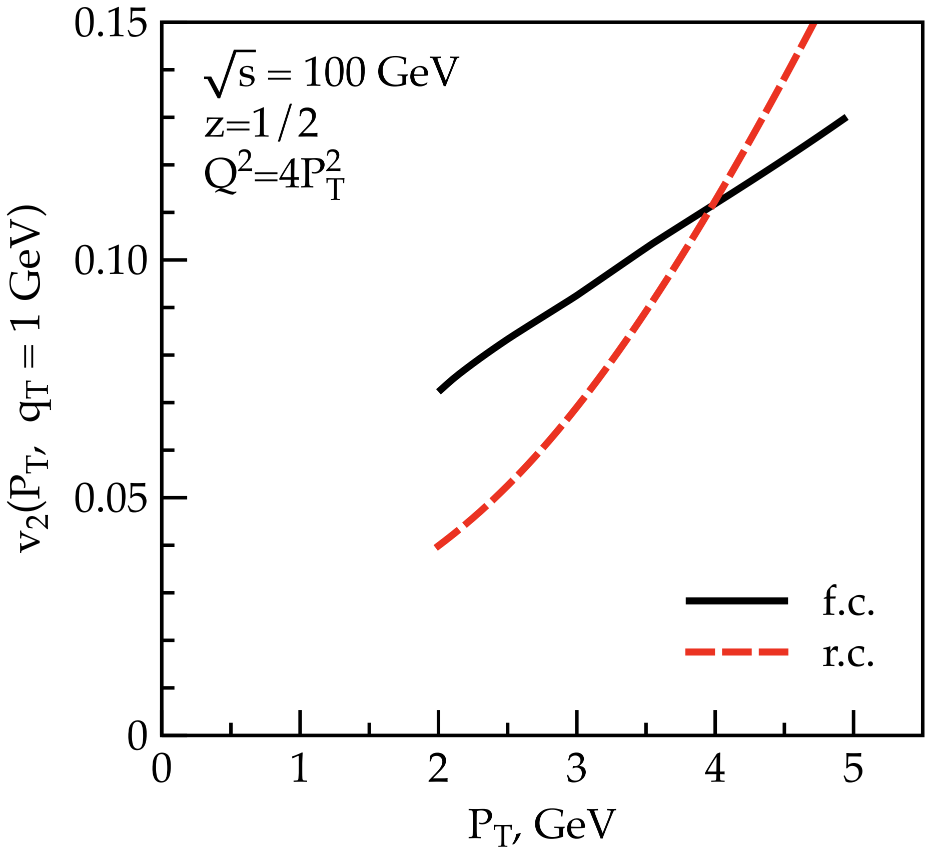
Still sizeable asymmetries result

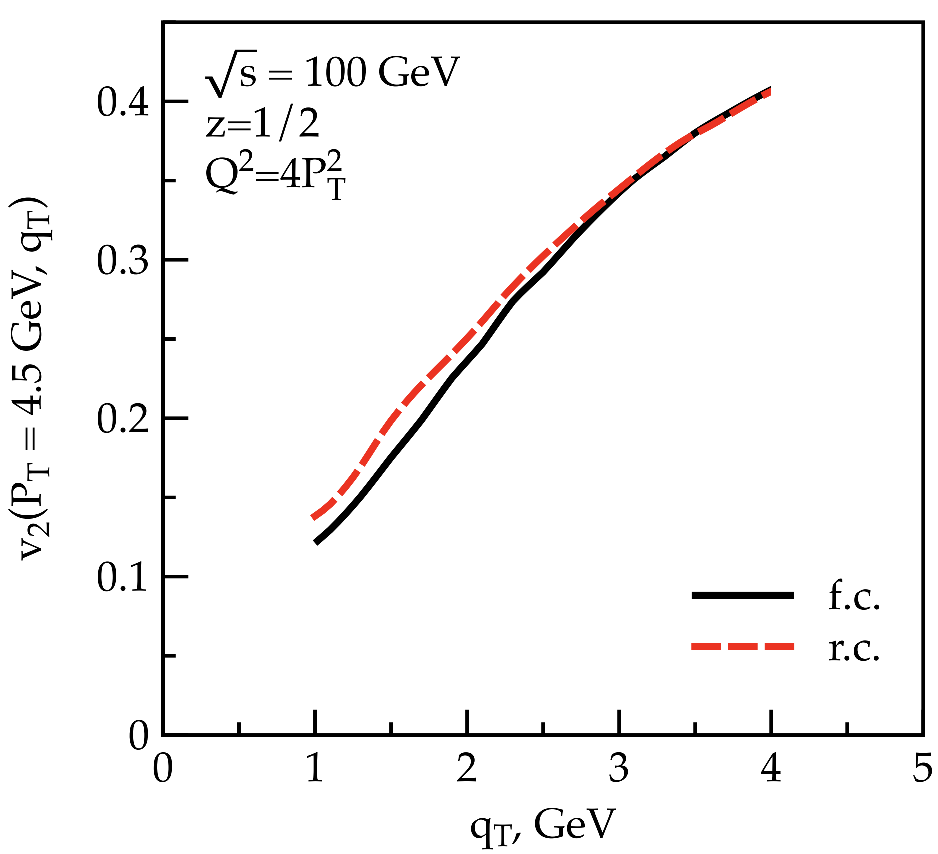
Dijet production at EIC

$h_1^{\perp g}$ (WW) is accessible in dijet production in eA collisions at a high-energy EIC

[Metz, Zhou 2011; Pisano, D.B., Brodsky, Buffing, Mulders, 2013; D.B., Pisano, Mulders, Zhou, 2016]

Polarization shows itself through a cos2ϕ distribution



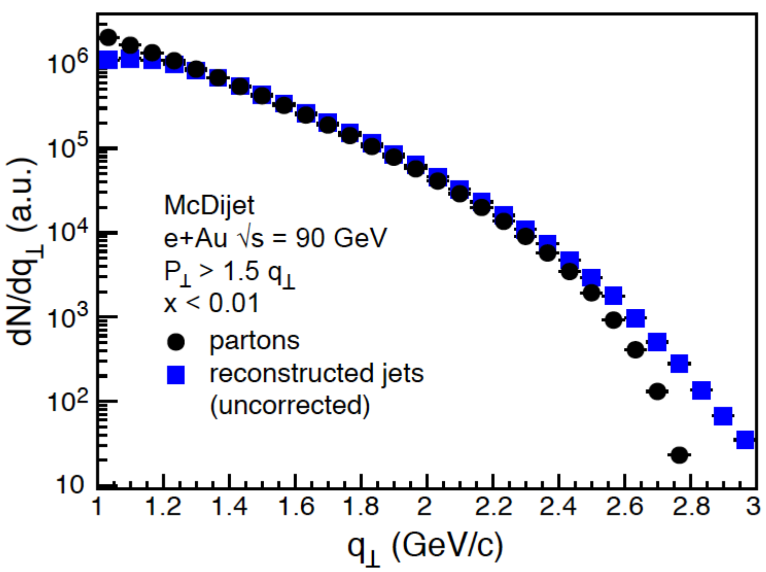


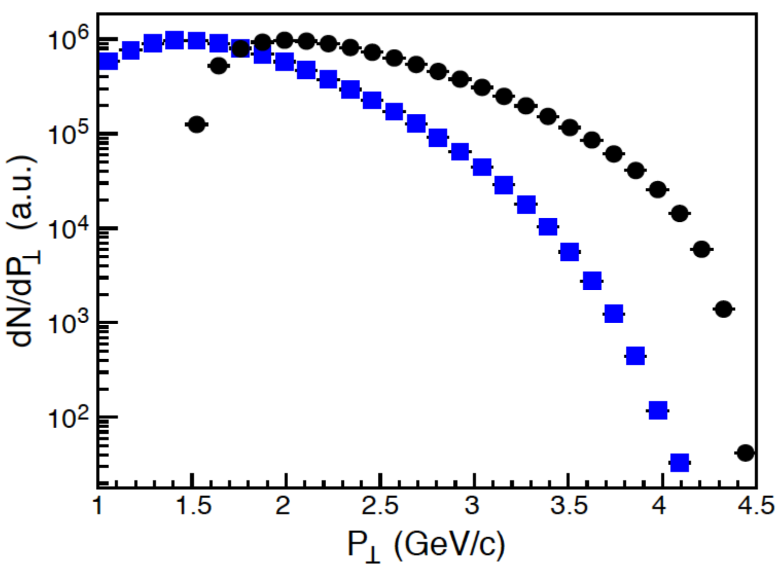
Large effects are found

Dumitru, Lappi, Skokov, 2015

cos2ϕ has opposite signs for L and T γ^* polarization

Dumitru, Skokov, Ullrich, 2018

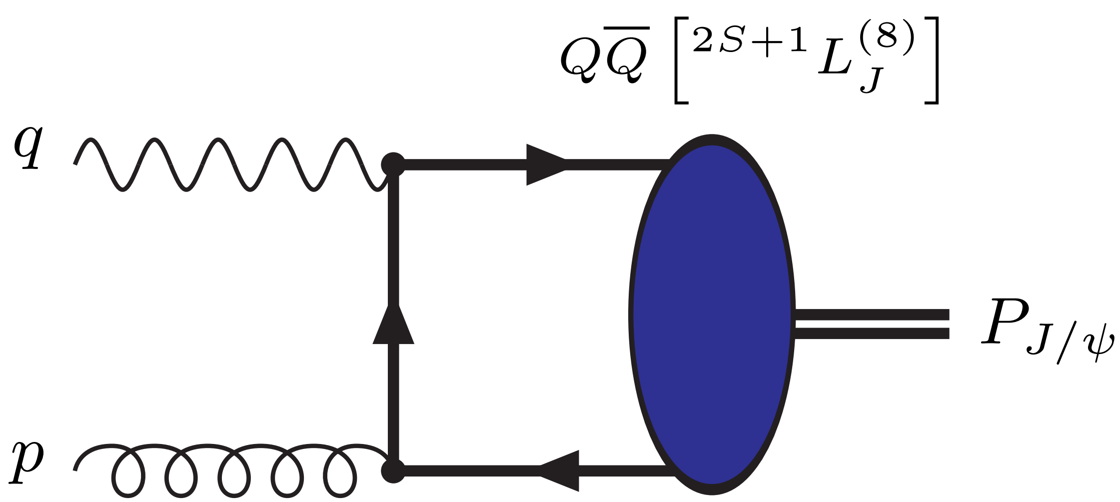




Quarkonia

$$e\, p^{\uparrow} \to e'\, \mathcal{Q}\, X \text{ with } \mathcal{Q} \text{ either a } J/\psi \text{ or a } \Upsilon \text{ meson}$$

[Godbole, Misra, Mukherjee, Rawoot, 2012/3; Godbole, Kaushik, Misra, Rawoot, 2015; Mukherjee, Rajesh, 2017; Rajesh, Kishore, Mukherjee, 2018]



One either uses the Color Evaporation Model or NRQCD for Color Octet (CO) states

$$A^{\sin(\phi_S-\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p} \frac{f_{1T}^{\perp\, g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

Other asymmetries depend on the quite uncertain CO NRQCD LDMEs, but one can consider ratios of asymmetries to cancel them out

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S+\phi_T)}} = \frac{\boldsymbol{q}_T^2}{M_p^2} \frac{h_1^{\perp\, g}(x,\boldsymbol{q}_T^2)}{h_1^g(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S-3\phi_T)}} = -\frac{1}{2} \frac{h_1^{\perp\, g}(x,\boldsymbol{q}_T^2)}{h_{1T}^{\perp\, g}(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\sin(\phi_S-3\phi_T)}}{A^{\sin(\phi_S+\phi_T)}} = -\frac{\boldsymbol{q}_T^2}{2M_p^2} \frac{h_{1T}^{\perp\, g}(x,\boldsymbol{q}_T^2)}{h_1^g(x,\boldsymbol{q}_T^2)}$$

[Bacchetta, D.B., Pisano, Taels, arXiv:1809.02056]

CO NRQCD LDMEs @ EIC

But one can also consider ratios where the TMDs cancel out
and one can obtain new experimental information on the CO NRQCD LDMEs

This requires a comparison of $e\,p \to e'\,\mathcal{Q}\,X$ and $ep \to e'Q\bar{Q}X$

$$\mathcal{R}^{\cos 2\phi} = \frac{\int d\phi_T \cos 2\phi_T \, d\sigma^{\mathcal{Q}}(\phi_S, \phi_T)}{\int d\phi_T \, d\phi_\perp \cos 2\phi_T \, d\sigma^{Q\overline{Q}}(\phi_S, \phi_T, \phi_\perp)}$$

$$\mathcal{R} = \frac{\int d\phi_T \, d\sigma^{\mathcal{Q}}(\phi_S, \phi_T)}{\int d\phi_T \, d\phi_\perp \, d\sigma^{Q\overline{Q}}(\phi_S, \phi_T, \phi_\perp)}$$

Two observables depending on two unknowns:

$$\mathcal{O}_8^S \equiv \langle 0|\mathcal{O}_8^{\mathcal{Q}}(^1S_0)|0\rangle$$

$$\mathcal{O}_8^P \equiv \langle 0|\mathcal{O}_8^{\mathcal{Q}}(^3P_0)|0\rangle$$

$$\mathcal{R}^{\cos 2\phi_T} = \frac{27\pi^2}{4}\,\frac{1}{M_Q}\left[\mathcal{O}_8^S - \frac{1}{M_Q^2}\,\mathcal{O}_8^P\right]$$

$$\mathcal{R} = \frac{27\,\pi^2}{4}\,\frac{1}{M_Q}\,\frac{[1+(1-y)^2]\,\mathcal{O}_8^S + (10-10y+3y^2)\,\mathcal{O}_8^P/M_Q^2}{26-26y+9y^2}$$

[Bacchetta, D.B., Pisano, Taels, arXiv:1809.02056]

Plus similar (but different) equations for *polarized* quarkonium production

GTMDs

GTMDs

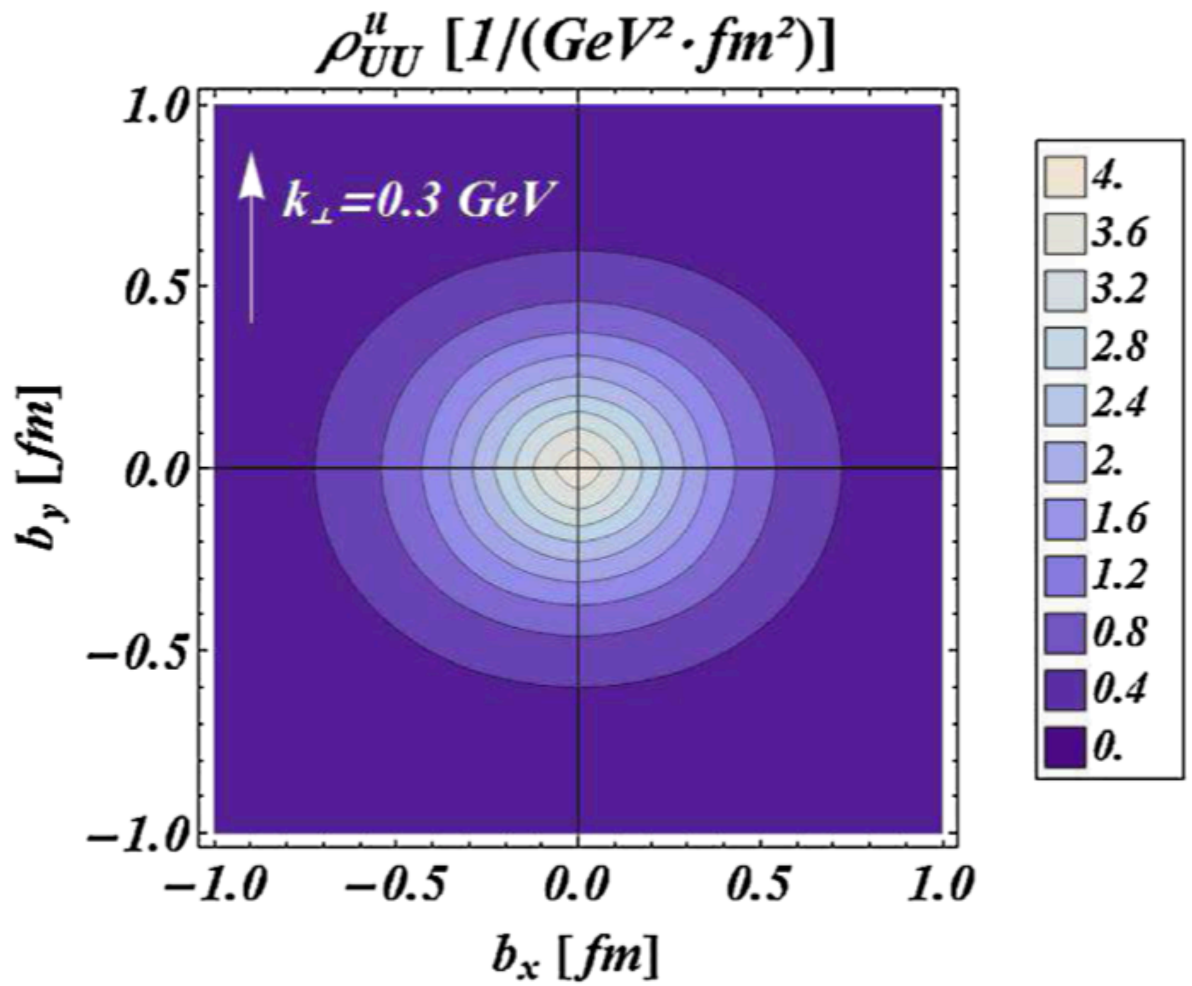
GTMD = off-forward TMD = Fourier transform of a Wigner distribution

$$G(x, \boldsymbol{k}_T, \boldsymbol{\Delta}_T) \xleftrightarrow{FT} W(x, \boldsymbol{k}_T, \boldsymbol{b}_T)$$

Meißner, Metz, Schlegel, 2009 — Ji, 2003; Belitsky, Ji & Yuan, 2004

Wigner distributions can display distortions in the b_T plane depending on k_T and vice versa, that vanish upon b_T or k_T integration

Lorce & Pasquini, 2011



Quark orbital angular momentum can be expressed as integrals over a Wigner distribution

Lorce, Pasquini, Xiong, Yuan, 2012

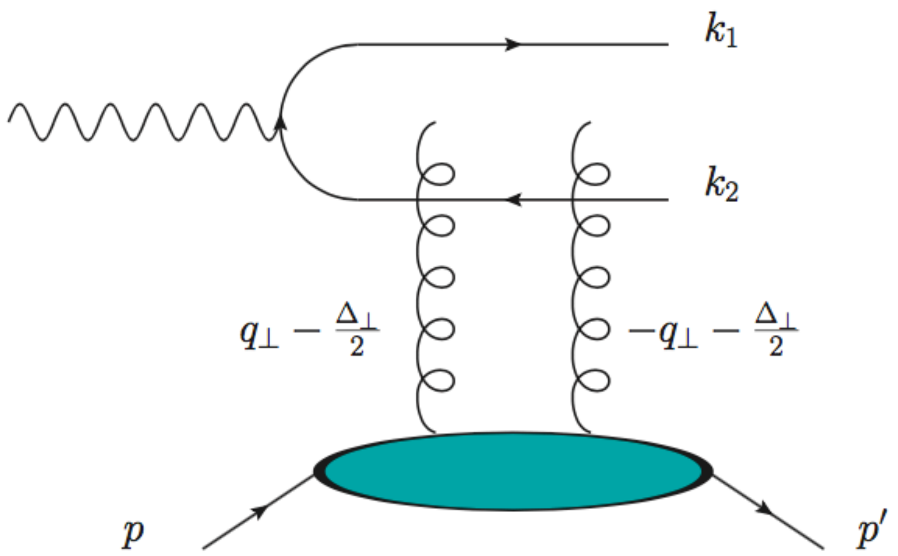
Analogously, gluon Wigner distributions and gluon GTMDs can be defined

See recent review: More, Mukherjee, Nair, Eur.Phys.J. C78 (2018)

Gluon GTMDs

First suggestion to measure gluon GTMDs: hard diffractive dijet production in eA

Altinoluk, Armesto, Beuf, Rezaeian, 2016; Hatta, Xiao, Yuan, 2016



Extension of an earlier suggestion
to probe gluon GPDs

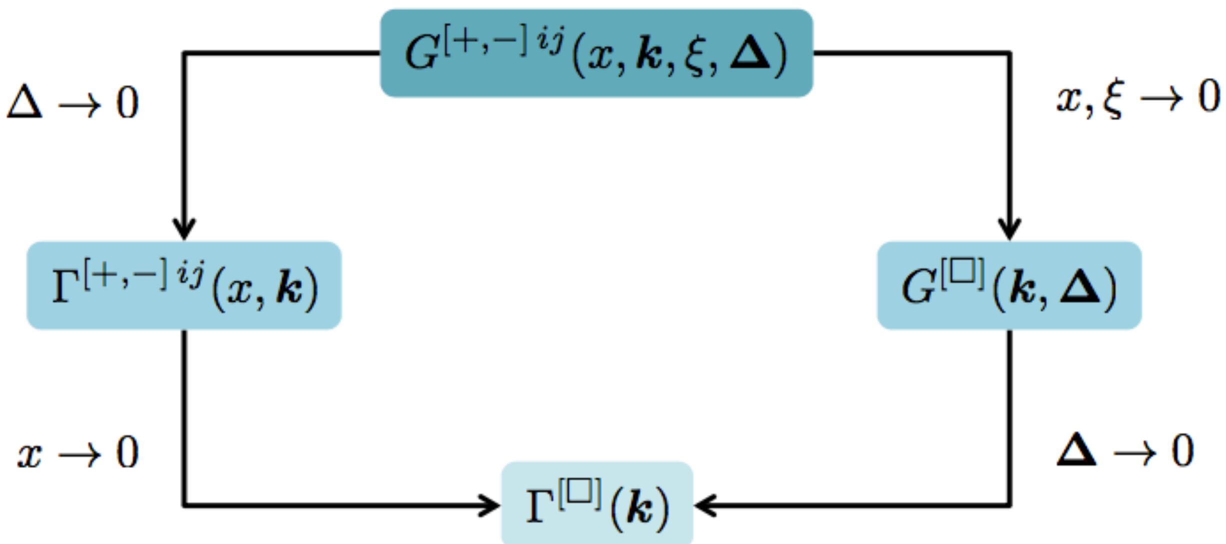
Braun, Ivanov, 2005

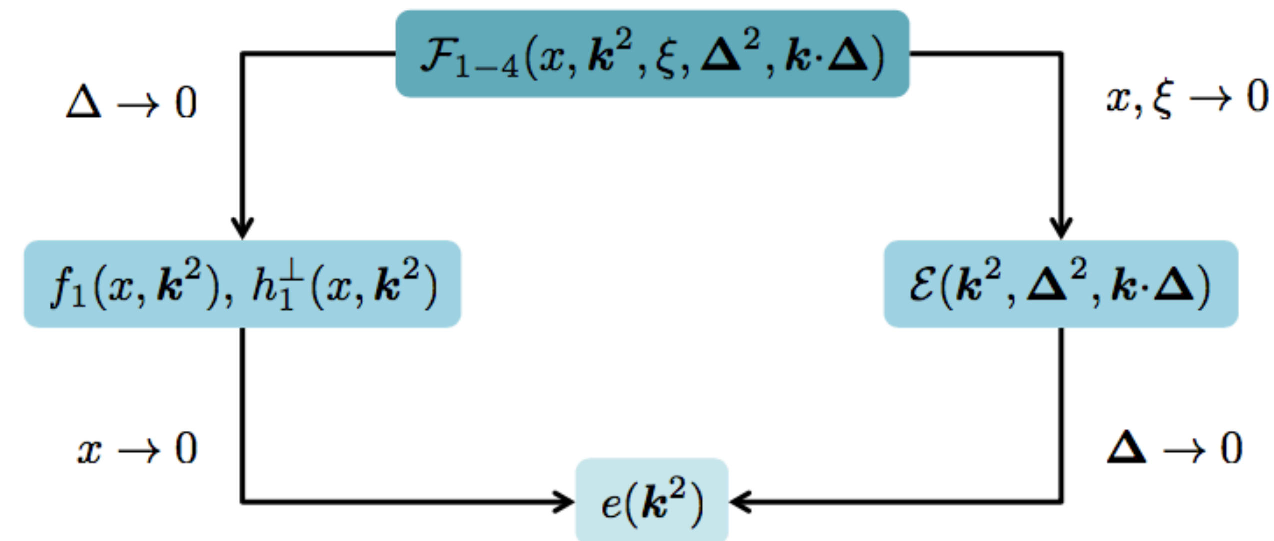
For unpolarized hadrons there are 4 independent (complex valued) gluon GTMDs
For [+,−] there is only one gluon GTMD in the limit $x \to 0$ (at leading twist)

This is again a Wilson loop matrix element, with the imaginary part corresponding to the odderon operator, which for an unpolarized proton vanishes in the forward limit

D.B., van Daal, Mulders, Petreska, 2018

Gluon GTMDs





Ph.D, thesis by Tom van Daal, 2018

$$W^{[\Box]}(\boldsymbol{b},\boldsymbol{k}) = \int d^2\boldsymbol{\Delta}\; e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}}\; G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta})$$

The cos 2(ϕ_b-ϕ_k) part (of $\mathcal{F}_1$ or $\mathcal{E}$) is called the "elliptic" Wigner distribution

Hatta, Xiao, Yuan, 2016; J. Zhou, 2016

At finite x there can be such an "elliptic" piece in any of the 4 Wigner functions

$$xW(x,\boldsymbol{b},\boldsymbol{k}) = x\mathcal{W}_0(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + 2\cos(\phi_b-\phi_k)\,x\mathcal{W}_1(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + 2\cos 2(\phi_b-\phi_k)\,x\mathcal{W}_2(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + \ldots$$

Gluon GTMDs in a nucleus

$$G(x, \boldsymbol{k}_T, \boldsymbol{\Delta}_T) \overset{FT}{\longleftrightarrow} W(x, \boldsymbol{k}_T, \boldsymbol{b}_T)$$

In a nucleus there are two sources of Δ_T:

- the position of the operator w.r.t. the "center"
- the nuclear profile

$$\mathbf{R}_\perp^{CM} \equiv \sum_i x_i \mathbf{r}_{\perp i}$$

$$T(\boldsymbol{b}) = \int_{-\infty}^{\infty} dz\, \rho_A\left(\sqrt{\boldsymbol{b}^2 + z^2}\right)$$

The latter leads to an odderon contribution, present even in the forward limit
This can lead to odd harmonics in forward two-particle production in pA

D.B., van Daal, Mulders, Petreska, 2018

Similar to elliptic flow arising from the color-dipole orientation in pp/pA collisions
Iancu, Rezaeian, 2017

The contribution comes primarily from the edge of the nucleus
No azimuthal anisotropy needed, just inhomogeneity in $|\mathbf{b}|$

Azimuthal anisotropy can also lead to odd harmonics

Dumitru, Giannini, 2015; Lappi, Schenke, Schlichting, Venugopalan, 2016

Conclusions

Conclusions

- All TMDs are process dependent, with observable and testable effects
- At small x the unpolarized WW and DP gluon TMDs both matter and there are sufficient processes in ep/eA and pp collisions to test the expectations
- Same applies to the linear polarization of gluons inside unpolarized hadrons, which can lead to sizeable effects for $\cos 2\phi$ asymmetries in eA processes
- At small x the dipole gluon TMD correlator becomes a Wilson loop correlator leading to relations among the TMDs: maximal linear gluon polarization
- J/ψ or Υ production in ep/eA collisions allows to probe gluon TMDs, but also two LO CO NRQCD LDMEs that are still poorly known
- The four complex valued [+,−] gluon GTMDs for unpolarized hadrons reduce to one in the small-x limit. This allows for a nuclear odderon contribution, which only has odd harmonic contributions (elliptic is $\cos 2\varphi$)

Back-up slides

Wilson loop correlator

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \ \xrightarrow{x\to 0}\ \frac{k_T^i k_T^j}{2\pi L}\Gamma_0^{[\Box]}(\boldsymbol{k}_T)$$

a single Wilson loop matrix element

D.B., Cotogno, van Daal, Mulders, Signori & Ya-Jin Zhou, JHEP 2016

$$U^{[\Box]} = U^{[+]}_{[0,y]} U^{[-]}_{[y,0]}$$

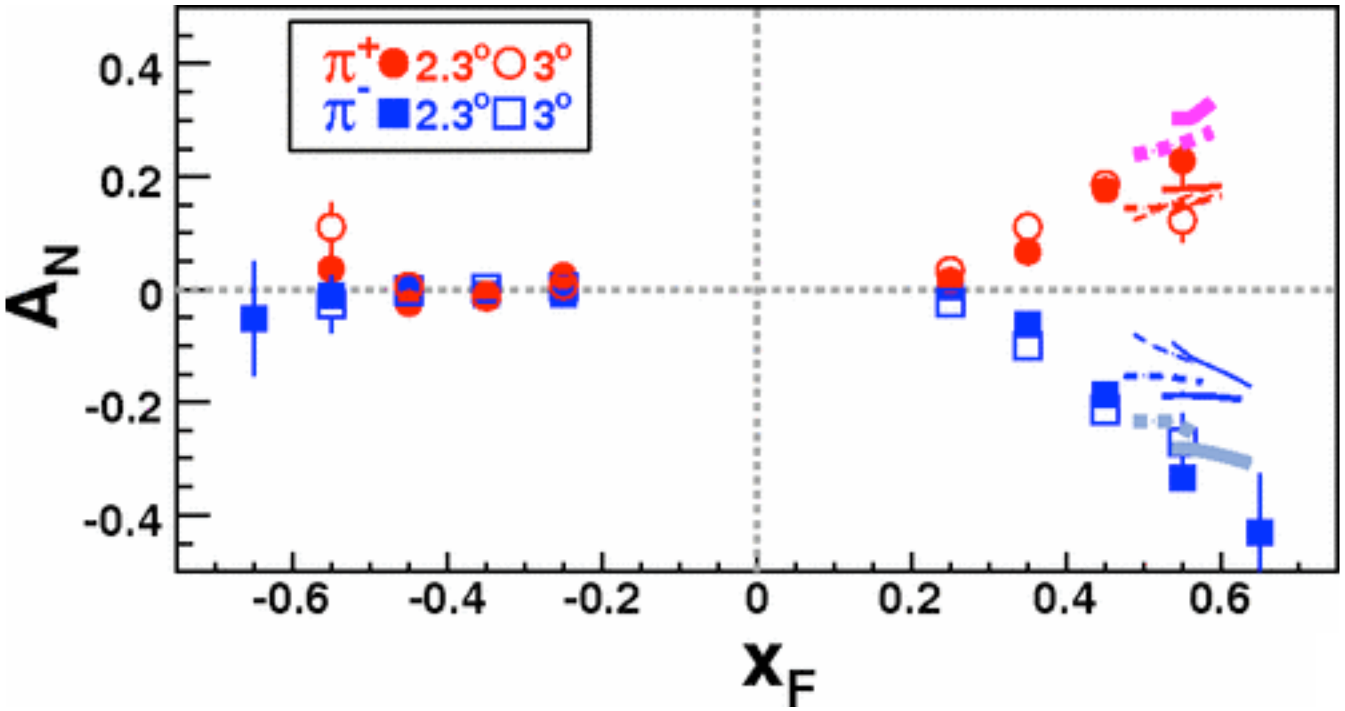
$$\begin{aligned}
k_T^i k_T^j\,\Gamma_0^{[\Box]}(\boldsymbol{k}_T) &= 4\int \frac{d^2\xi_T}{(2\pi)^2}\, e^{ik_T\cdot\xi_T}\,\langle P|\,G_T^i(0)\,U^{[+]}_{[0,\xi]}\,G_T^j(\xi)\,U^{[-]}_{[\xi,0]}\,|P\rangle\Big|_{\xi\cdot n=0} \\
&= \int \frac{d\eta{\cdot}P\,d\eta'{\cdot}P\,d^2\xi_T}{(2\pi)^2}\, e^{ik_T\cdot\xi_T}\,\langle P|\,F^{ni}(\eta')\,U^{[+]}_{[\eta',\eta]}\,F^{nj}(\eta)\,U^{[-]}_{[\eta,\eta']}\,|P\rangle\Big|_{\substack{\eta'\cdot n=\eta\cdot n=0,\\ \eta'_T=0_T,\,\eta_T=\xi_T}} \\
&= 2\pi L\int \frac{d\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\,\langle P|\,F^{ni}(0)\,U^{[+]}_{[0,\xi]}\,F^{nj}(\xi)\,U^{[-]}_{[\xi,0]}\,|P\rangle\Big|_{\xi\cdot n=k\cdot n=0} \\
&= 2\pi L\,\Gamma^{[+,-]\,ij}(0,\boldsymbol{k}_T),
\end{aligned} \tag{3.1}$$

$$\left[i\partial_z^k, U^{[\pm]}_{[a,z]}\right] = \mp g U^{[\pm]}_{[a,z]}\, G_T^k(z),$$

$$G_T^k(z) \equiv \frac{1}{2}\int_{-\infty}^{\infty} d\eta^-\, U^n_{[z^-,\eta^-;z]}\, F^{+k}(z^+,\eta^-,z)\, U^n_{[\eta^-,z^-;z]}\,.$$

$$\begin{aligned}
G^{[+,-]\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) &= \frac{16}{\langle P|P\rangle}\int \frac{d^2\boldsymbol{x}\,d^2\boldsymbol{y}}{(2\pi)^3}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}} \\
&\quad\times \langle p'|\,\mathrm{Tr}\left(G_T^j(x)\,U^{[-]}_{[x,y]}\,G_T^i(y)\,U^{[+]}_{[y,x]}\right)|p\rangle\Big|_{x^+=y^+=0} \\
&= \frac{4}{g^2\,\langle P|P\rangle}\int \frac{d^2\boldsymbol{x}\,d^2\boldsymbol{y}}{(2\pi)^3}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}} \\
&\quad\times \langle p'|\,\partial_x^j\partial_y^i\,\mathrm{Tr}\left(U^{[\Box]}_{[y,x]}\right)|p\rangle\Big|_{x^+=y^+=0},
\end{aligned}$$

$p^{\uparrow}p \rightarrow h^{\pm}\, X$ at $x_F < 0$

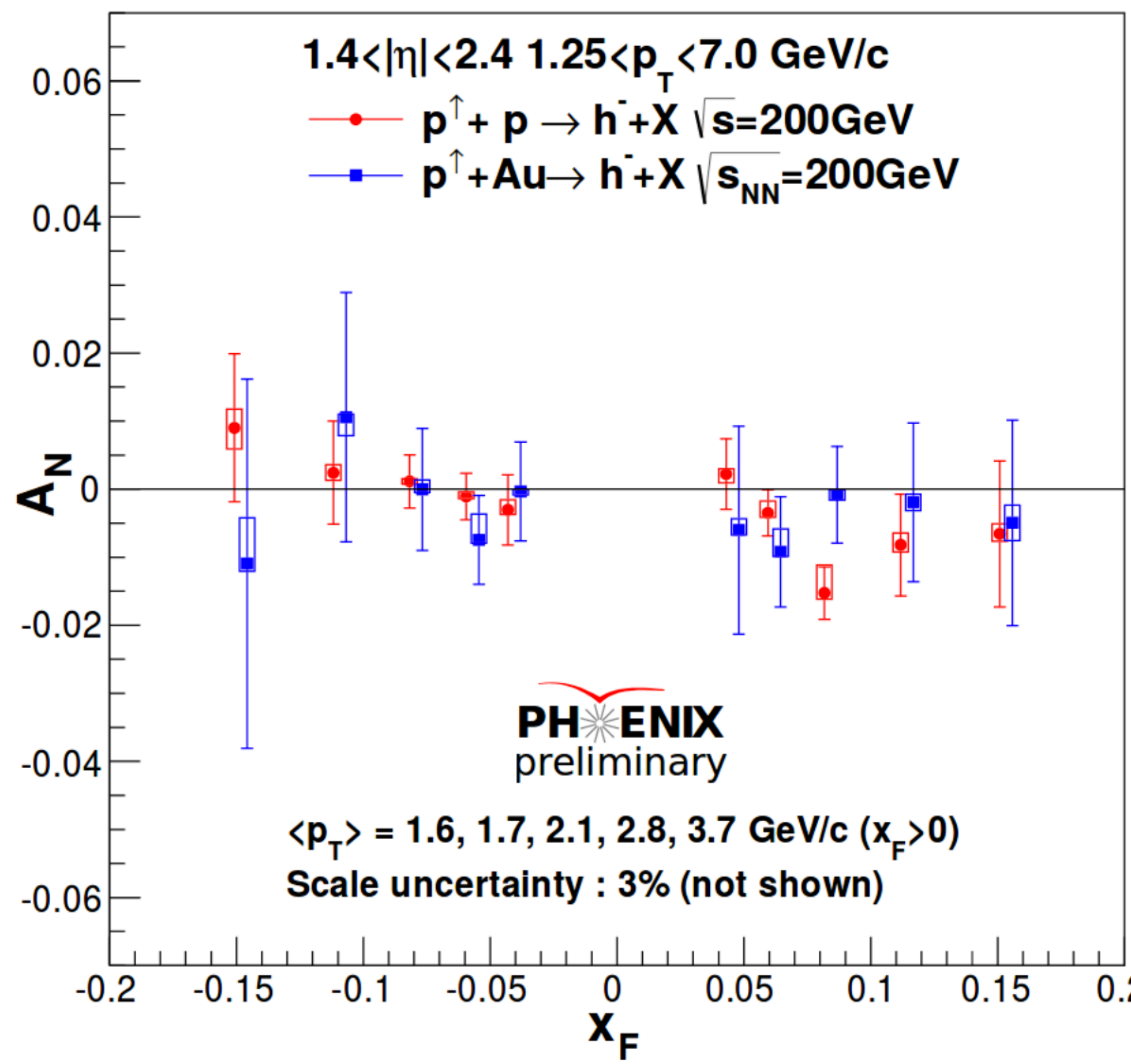


BRAHMS, 2008 $\sqrt{s}$ = 62.4 GeV
low p_T, up to roughly 1.2 GeV
where gg channel dominates

spin-dependent odderon is C-odd,
whereas gg in the CS state is C-even

expect smaller asymmetries
in neutral pion and jet production

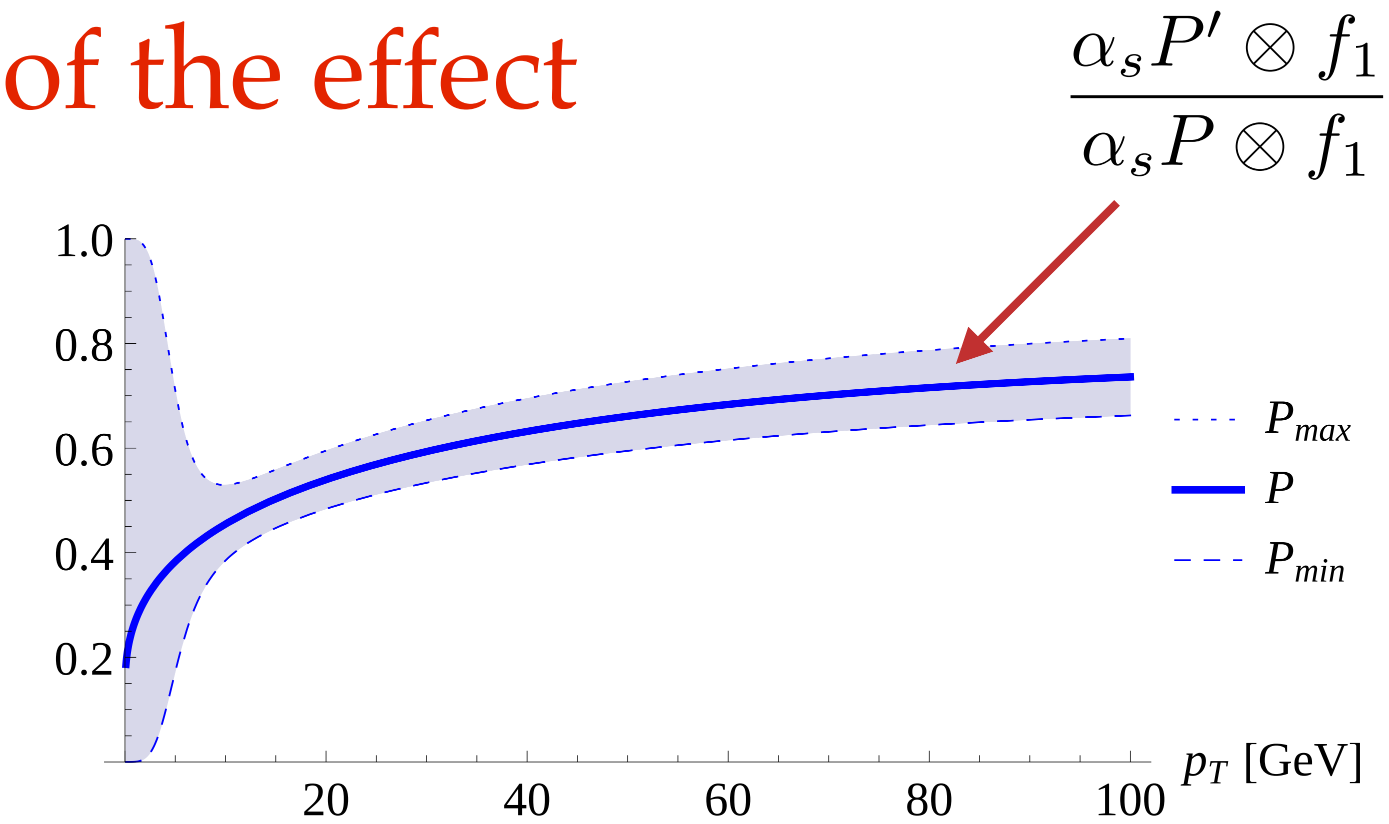
PHENIX, 2017
$\sqrt{s}$ = 200 GeV
p_T between 1.25 and 7 GeV



Size of the effect

Amount of linear gluon polarization:

D.B., Den Dunnen, Pisano, Schlegel '13



Ratio of large-k_T tails of $h_1^\perp$ and f_1 is large, does ***not*** mean large effects at large Q_T (observables involve ***integrals*** over all partonic k_T)

What matters is the small-b behavior of the Fourier transformed TMD:

$$\tilde{f}_1^g(x, b^2; \mu_b^2, \mu_b) = f_{g/P}(x; \mu_b) + \mathcal{O}(\alpha_s)$$

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b) C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

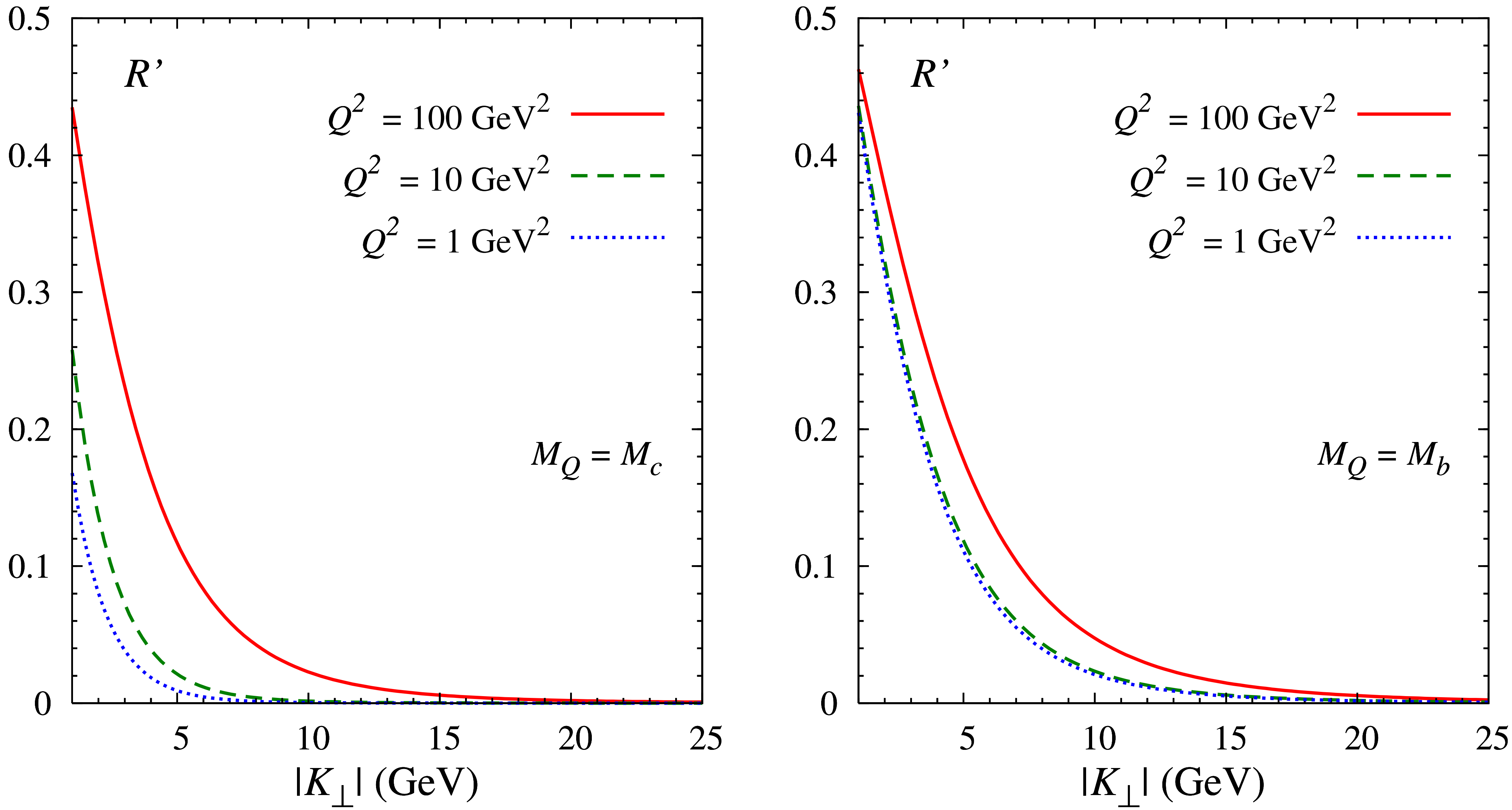
[Nadolsky, Balazs, Berger, C.-P. Yuan, 2007; Catani, Grazzini, 2010; P. Sun, B.-W. Xiao, F. Yuan, 2011]

The linear polarization starts at order α_s, leading to a **suppression** w.r.t. f_1

Maximum asymmetries in heavy quark production

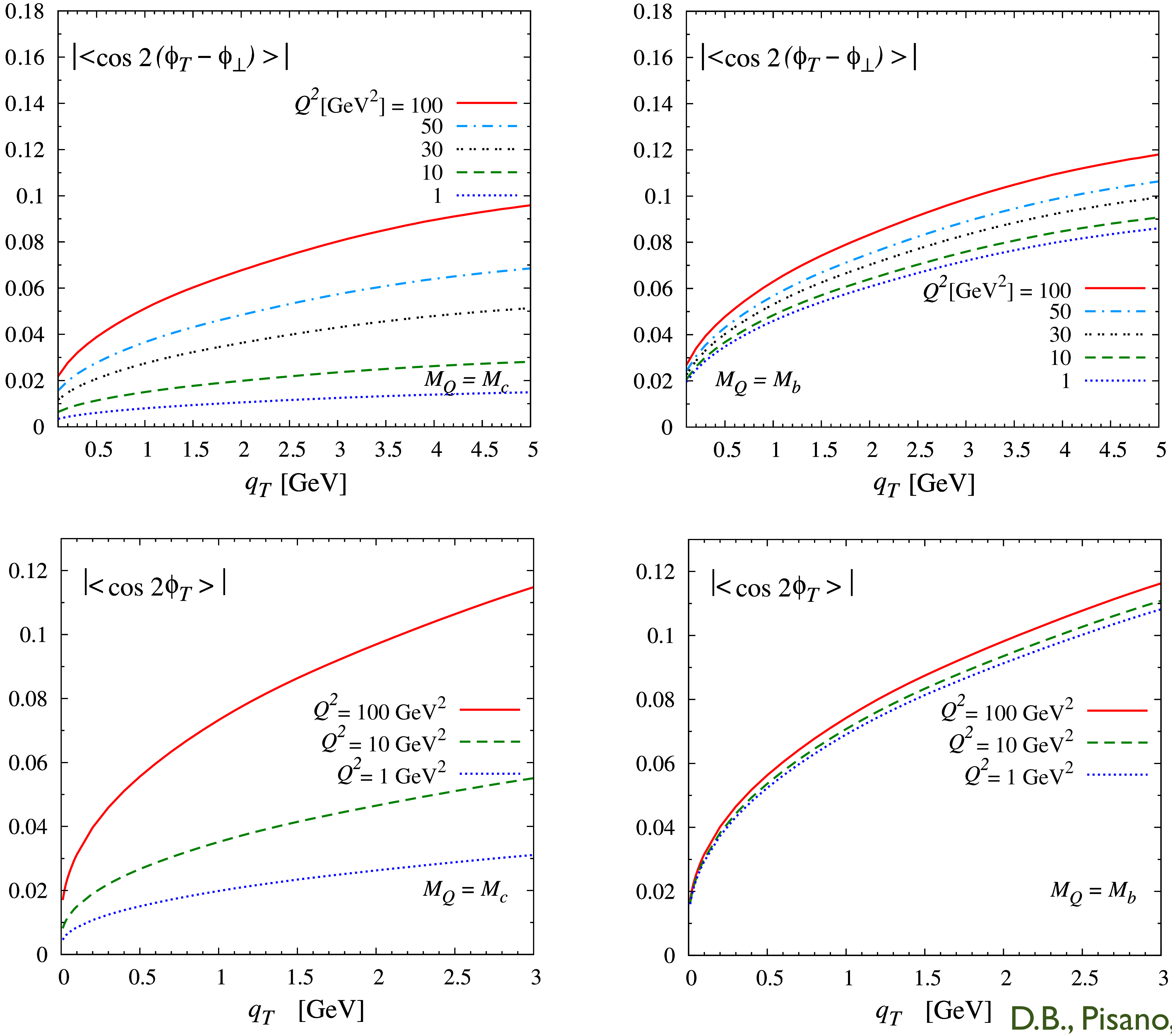
There are also angular asymmetries w.r.t. the lepton scattering plane, whose maxima are large at smaller $|K_\perp|$

$$ep \to e' Q \bar{Q} X \qquad R' = \text{bound on } |\langle \cos 2(\phi_\ell - \phi_T) \rangle|$$



[Pisano, D.B., Brodsky, Buffing & Mulders, 2013]

Heavy quark pair production at EIC



small x
MV model

$|\boldsymbol{K}_\perp| = 10\,\mathrm{GeV}$

$z = 0.5$

$y = 0.3$

$|\boldsymbol{K}_\perp| = 6\,\mathrm{GeV}$

$z = 0.5$

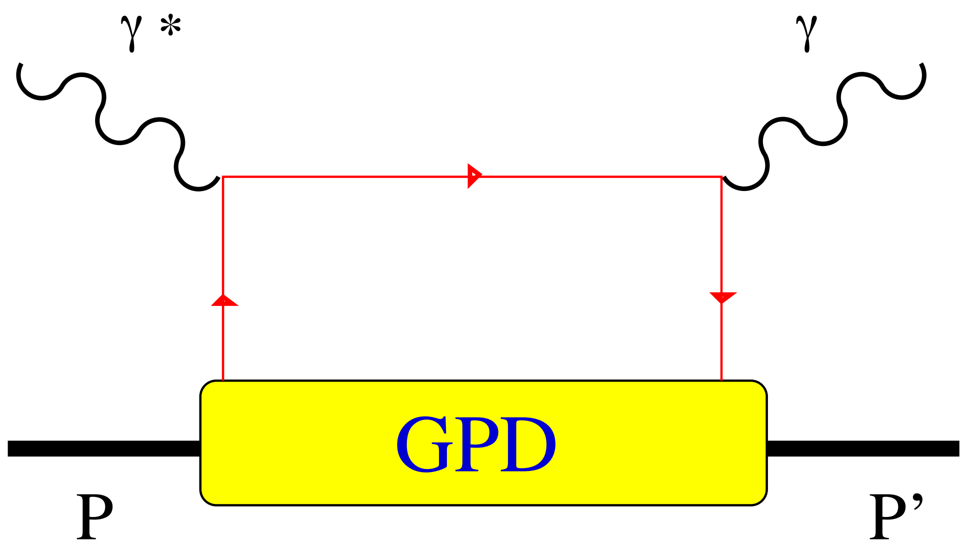
$y = 0.1$

D.B., Pisano, Mulders, Zhou, 2016

GPDs

Deeply Virtual Compton Scattering (DVCS):

$$\gamma^* + p \to \gamma + p'$$



Theoretical description involves Generalized Parton Distributions (GPDs)

GPDs are off-forward matrix elements (P’ ≠ P)

GPDs provide information about the spatial distribution of quarks inside nucleons

Functions of b_T, which is *not* the Fourier conjugate of k_T (which gives the transverse "size" distribution of quarks), rather it gives the transverse spatial distance of quarks w.r.t. the "center" of the proton

$$\mathbf{R}_\perp^{CM} \equiv \sum_i x_i \mathbf{r}_{\perp i}$$

The transverse center of longitudinal momentum [Burkardt 2000; Soper 1977]

Gluon GTMDs

$$G^{[U,U']\,ij}(x,\boldsymbol{k},\xi,\boldsymbol{\Delta}) = x\left(\delta_T^{ij}\,\mathcal{F}_1 + \frac{k_T^{ij}}{M^2}\,\mathcal{F}_2 + \frac{\Delta_T^{ij}}{M^2}\,\mathcal{F}_3 + \frac{k_T^{[i}\Delta_T^{j]}}{M^2}\,\mathcal{F}_4\right)$$

$$G^{[+,-]\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) = \frac{2N_c}{\alpha_s}\left[\frac{1}{2}\left(\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}\right)\delta_T^{ij} + k_T^{ij} - \frac{\Delta_T^{ij}}{4} - \frac{k_T^{[i}\Delta_T^{j]}}{2}\right] G^{[\square]}(\boldsymbol{k},\boldsymbol{\Delta})$$

where

$$G^{[\square]}(\boldsymbol{k},\boldsymbol{\Delta}) \equiv \int \frac{d^2\boldsymbol{x}\,d^2\boldsymbol{y}}{(2\pi)^4}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}}\, \frac{\langle p'|\, S^{[\square]}(\boldsymbol{x},\boldsymbol{y})\,|p\rangle\big|_{\mathrm{LF}}}{\langle P|P\rangle},$$

$$\lim_{x,\xi\to 0} x\mathcal{F}_1 = \lim_{x,\xi\to 0} x\mathcal{F}_2^{(1)} = -4\lim_{x,\xi\to 0} x\mathcal{F}_3^{(1)} = -2\lim_{x,\xi\to 0} x\mathcal{F}_4^{(1)} = \mathcal{E}^{(1)}$$

$$\mathcal{F}_i^{(n)} \equiv [(\boldsymbol{k}^2 - \boldsymbol{\Delta}^2/4)/(2M^2)]^n\,\mathcal{F}_i$$

D.B., van Daal, Mulders, Petreska, 2018

$$xW_g^T(x,\vec{q}_\perp;\vec{b}_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{-ixP^+\xi^- - iq_\perp\cdot\xi_\perp} \times \left\langle P+\frac{\Delta_\perp}{2}\right| F^{+i}\left(\vec{b}_\perp + \frac{\xi}{2}\right) F^{+i}\left(\vec{b}_\perp - \frac{\xi}{2}\right)\left|P-\frac{\Delta_\perp}{2}\right\rangle$$

Hatta, Xiao, Yuan, 2016