SINGULARITIES

IN TWIST-3 GPDS & PDFS

Fatma Aslan, Matthias Burkardt

OUTLINE

- What is and why study twist-3 GPDs
- Discontinuities in twist-3 GPDs
- Discontinuities and DVCS factorization
- Singularities in Twist-3 PDFs and quasi-PDFs
- Conclusions
- Outlook

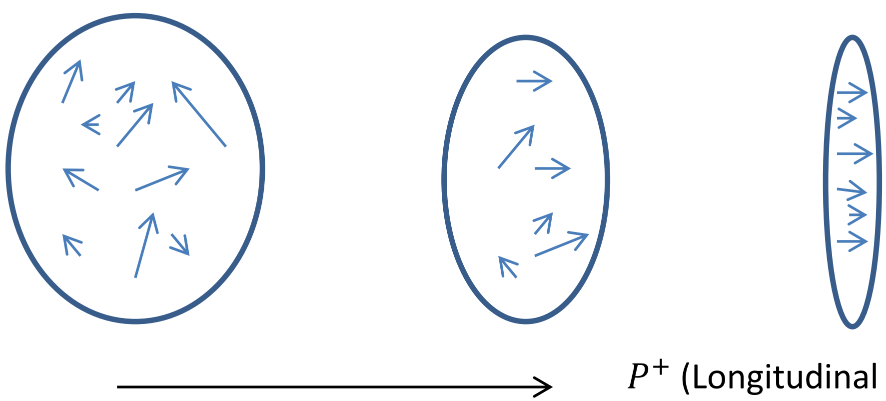
What is TWIST-3 ?

Twist→ The order in Q^2 at which a matrix element contributes to the physical amplitude.

Leading order → Twist 2
Next to leading order → Twist 3

2 -particle correlations → Twist 2
3-particle correlations (such as quark-gluon-quark) → Twist 3

Twist → Behavior under longitudinal momentum boost in the IMF



P^+ (Longitudinal nucleon momentum)

Behavior of twist-2 and twist-3 distributions under longitudinal momentum boost, P^+.

Twist-2	Twist-3
Independent of P^+	$1/P^+$

Why study TWIST-3 GPDs?

- Twist-3 effects may not be negligible in the measurement of DVCS amplitude at 12 GeV

- Quark-gluon-quark correlations ➡ (average) transverse force acting on a quark in a polarized nucleon

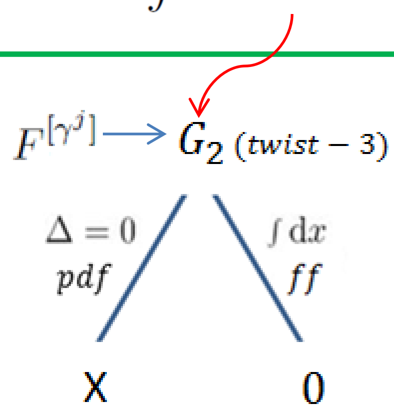
$$\int dx\, x^2 g_2(x)\,, \int dx\, x^2\, e(x) \;\rightarrow \perp force$$

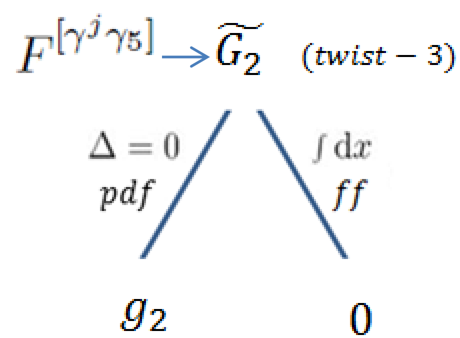
M. Burkardt, Transverse Force on Quarks in DIS (2008).

- There is a relation between one particular twist-3 GPD and the orbital angular momentum of quarks.

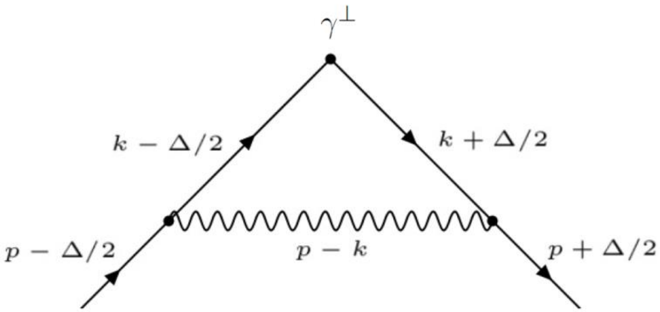
$$L^q_{kin} = -\int dx x G^q_2(x, \xi = 0, t = 0)$$

Penttinen, Polyakov, Shuvaev and Strikman, DVCS amplitude in the parton model (2000).





G_2 in quark target model



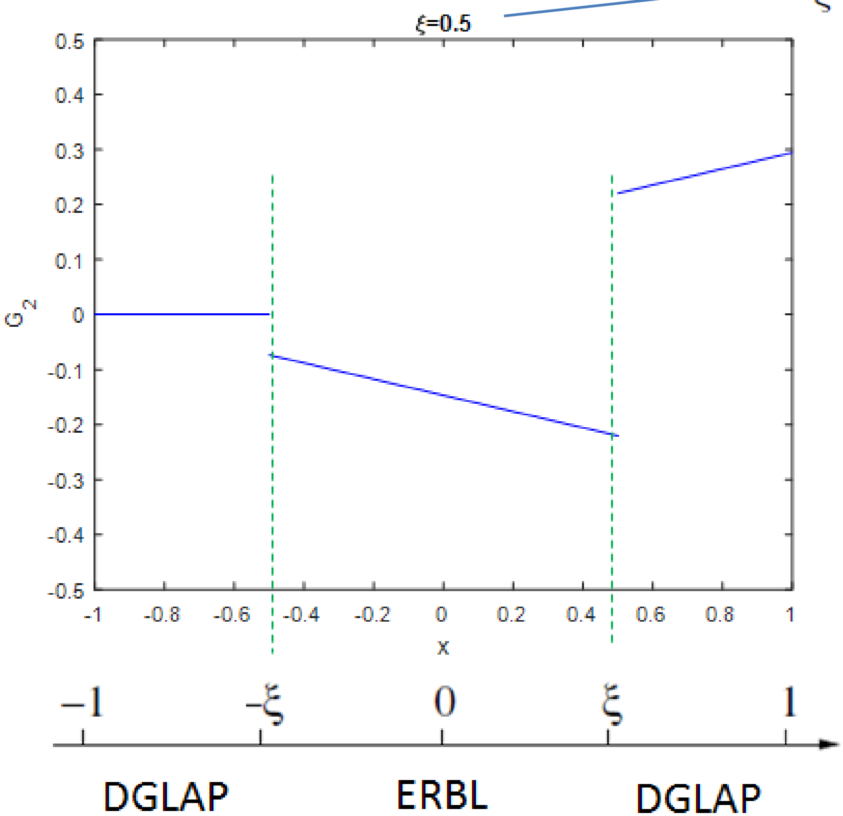
Quark target model in a symmetric frame

$$\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ixp^+z^-}\langle P',S'|\,\bar{q}(-\frac{z^-}{2})\gamma^j q(\frac{z^-}{2})\,|P,S\rangle$$

$$=\frac{1}{2p^+}\bar{u}(P',S')\Big[\frac{\Delta_\perp^j}{2M}G_1+\gamma^j\underbrace{(H+E+G_2)}+\frac{\Delta_\perp^j}{p^+}\gamma^+G_3+\frac{i\epsilon_T^{jk}\Delta_\perp^k}{p^+}\gamma^+\gamma_5G_4\Big]\bar{u}(P,S)$$

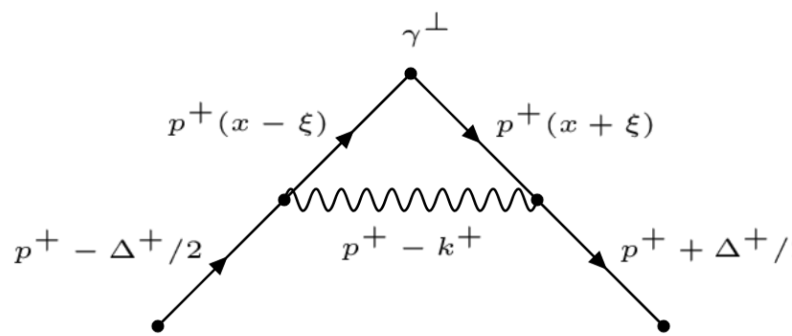
D. V. Kiptily, M. V. Polyakov , Genuine twist-3 contributions to the generalized parton distributions from instantons (2003)

$$\xi=\frac{\Delta^+}{2p^+}$$

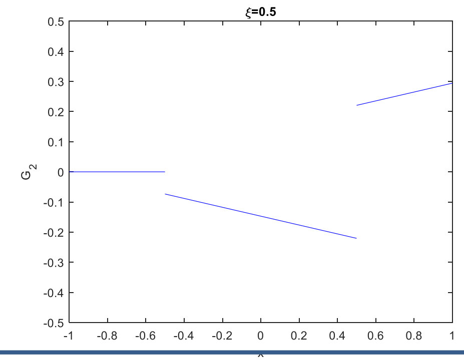


$$x=\frac{k^+}{p^+}$$

G_2 has discontinuities



- *There are discontinuities in* G_2.



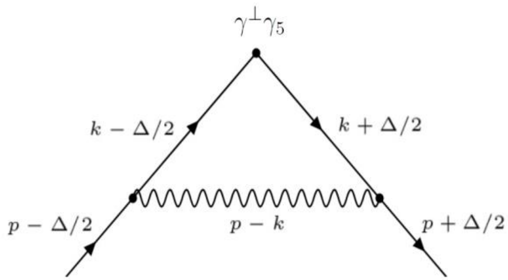
- *Factorization?*

$$\int_{-1}^{1} dx \frac{GPD}{x \pm \xi + i\varepsilon}$$

- *The relevant DVCS amplitude involves* $G_2 \pm \frac{\tilde{G}_2}{\xi}$

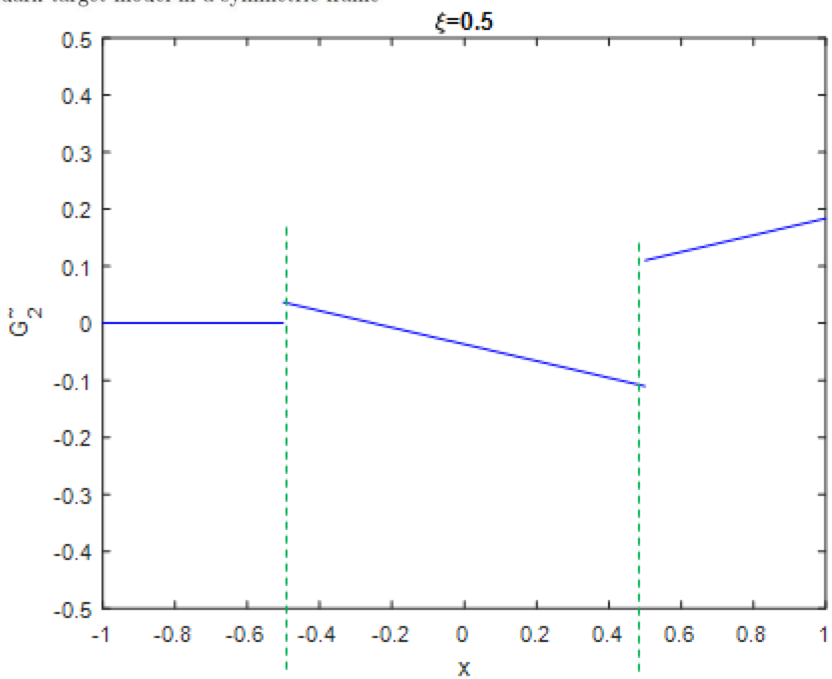
$$\int_{-1}^{1} dx \; \frac{G_2 + \frac{1}{\xi}\tilde{G}_2}{x + \xi + i\varepsilon} \qquad \int_{-1}^{1} dx \; \frac{G_2 - \frac{1}{\xi}\tilde{G}_2}{x - \xi + i\varepsilon}$$

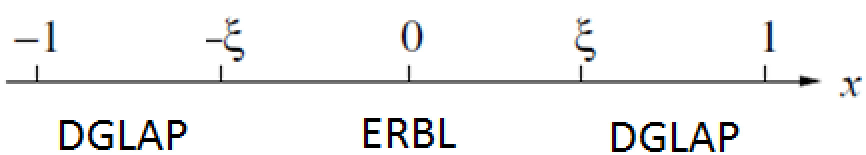
$\widetilde{G}_2$ in quark target model



Quark target model in a symmetric frame

$$\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ixp^+z^-}\langle P',S'|\,\overline{q}(-\frac{z^-}{2})\gamma^j\gamma_5 q(\frac{z^-}{2})\,|P,S\rangle$$
$$=\frac{1}{2p^+}\overline{u}(P',S')\Big[\frac{\Delta^j_\perp}{2M}\gamma_5(\widetilde{E}+\widetilde{G}_1)+\gamma^j\gamma_5\underbrace{(\widetilde{H}+\widetilde{G}_2)}+\frac{\Delta^j_\perp}{p^+}\gamma^+\gamma_5\widetilde{G}_3+\frac{i\epsilon_T^{jk}\Delta^k_\perp}{p^+}\gamma^+\widetilde{G}_4\Big]\overline{u}(P,S).$$



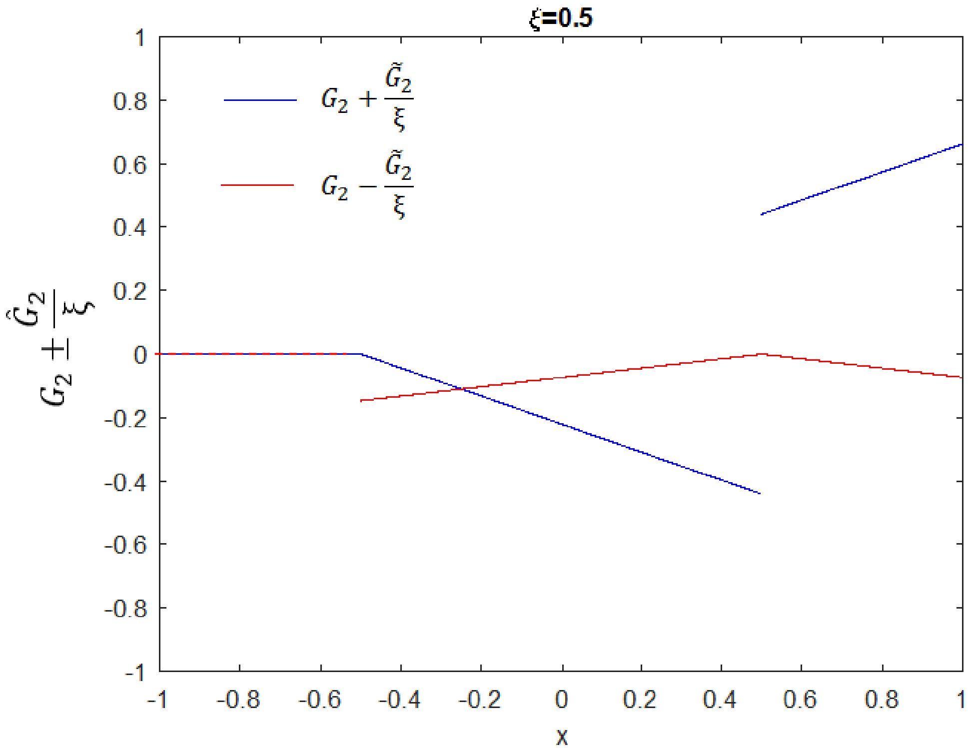


$\widetilde{G}_2$ **too has discontinuities**

FACTORIZATION

$$\int_{-1}^{1} dx\, \frac{G_2 + \frac{1}{\xi}\tilde{G}_2}{x + \xi + i\varepsilon} \qquad\qquad \int_{-1}^{1} dx\, \frac{G_2 - \frac{1}{\xi}\tilde{G}_2}{x - \xi + i\varepsilon}$$

- $G_2 + \frac{1}{\xi}\tilde{G}_2$ continuous at $x = -\xi$
- $G_2 - \frac{1}{\xi}\tilde{G}_2$ continuous at $x = \xi$



Twist-3 DVCS factorization is safe.

Fatma Aslan, Matthias Burkardt, Cédric Lorcé, Andreas Metz, Barbara Pasquini, Twist-3 GPDs in Deeply Virtual Compton Scattering (*2018)*

Factorization is fine,
but what about the discontinuities?

How do they behave?
What do they represent?
What happens in different models?
What about the forward limit?
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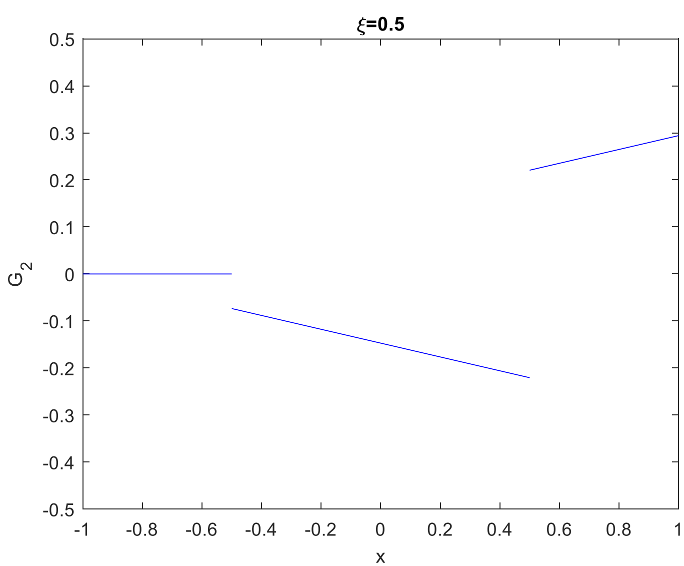
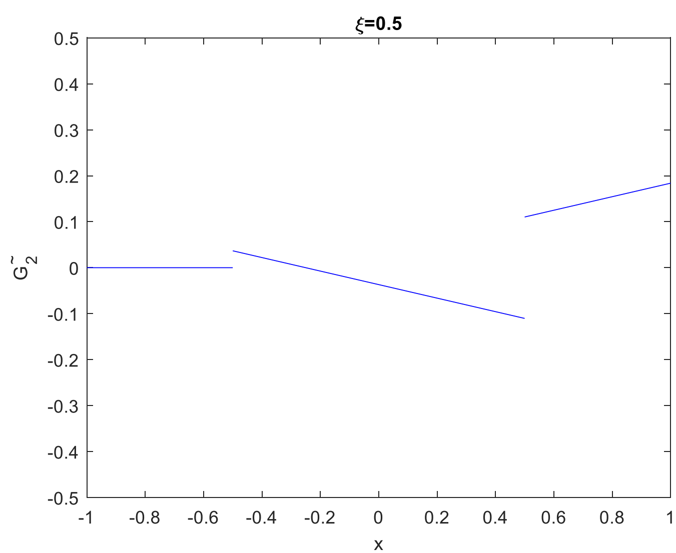
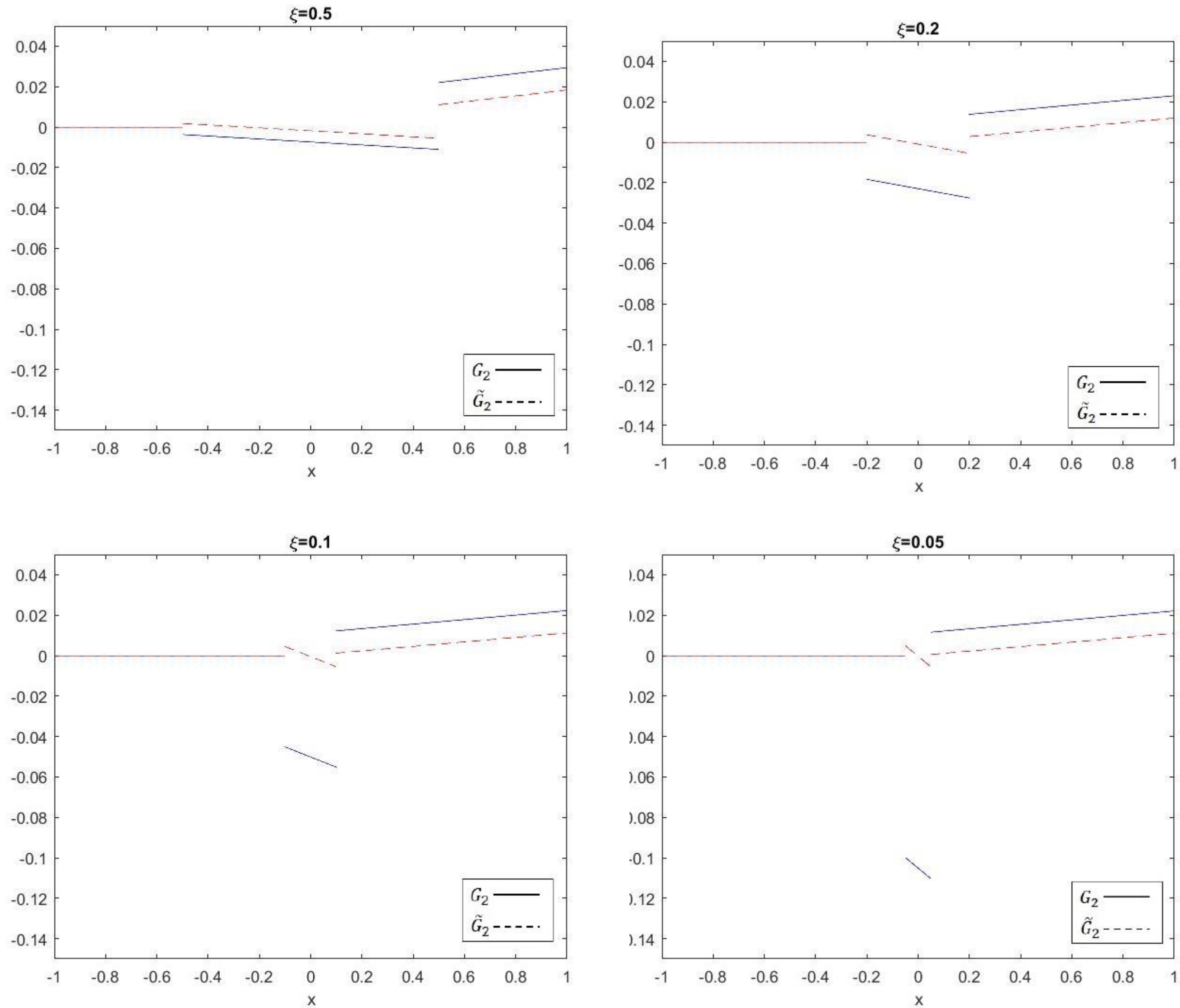


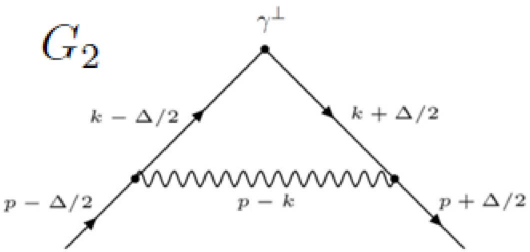


How do the discontinuities behave as ξ→0

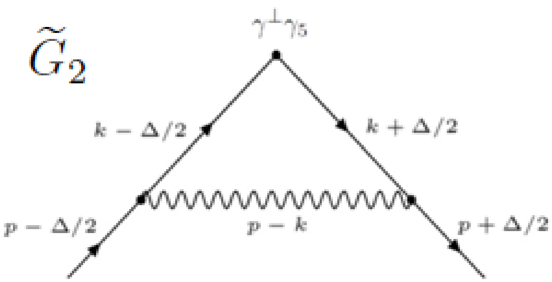
G_2 and $\widetilde{G}_2$ in Quark Target Model



G_2



$\widetilde{G}_2$

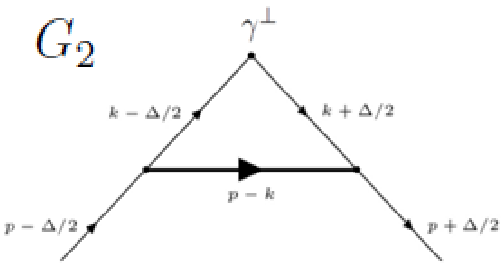


Twist-3 GPD	Discontinuities as $\xi \to 0$
G_2	Divergent
$\widetilde{G}_2$	Finite

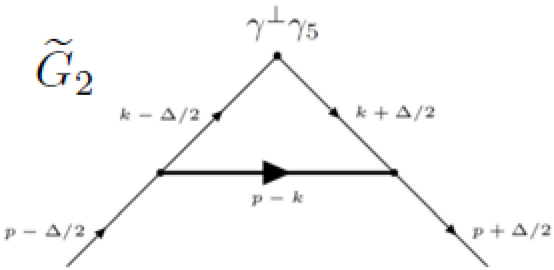
What happens in different models?

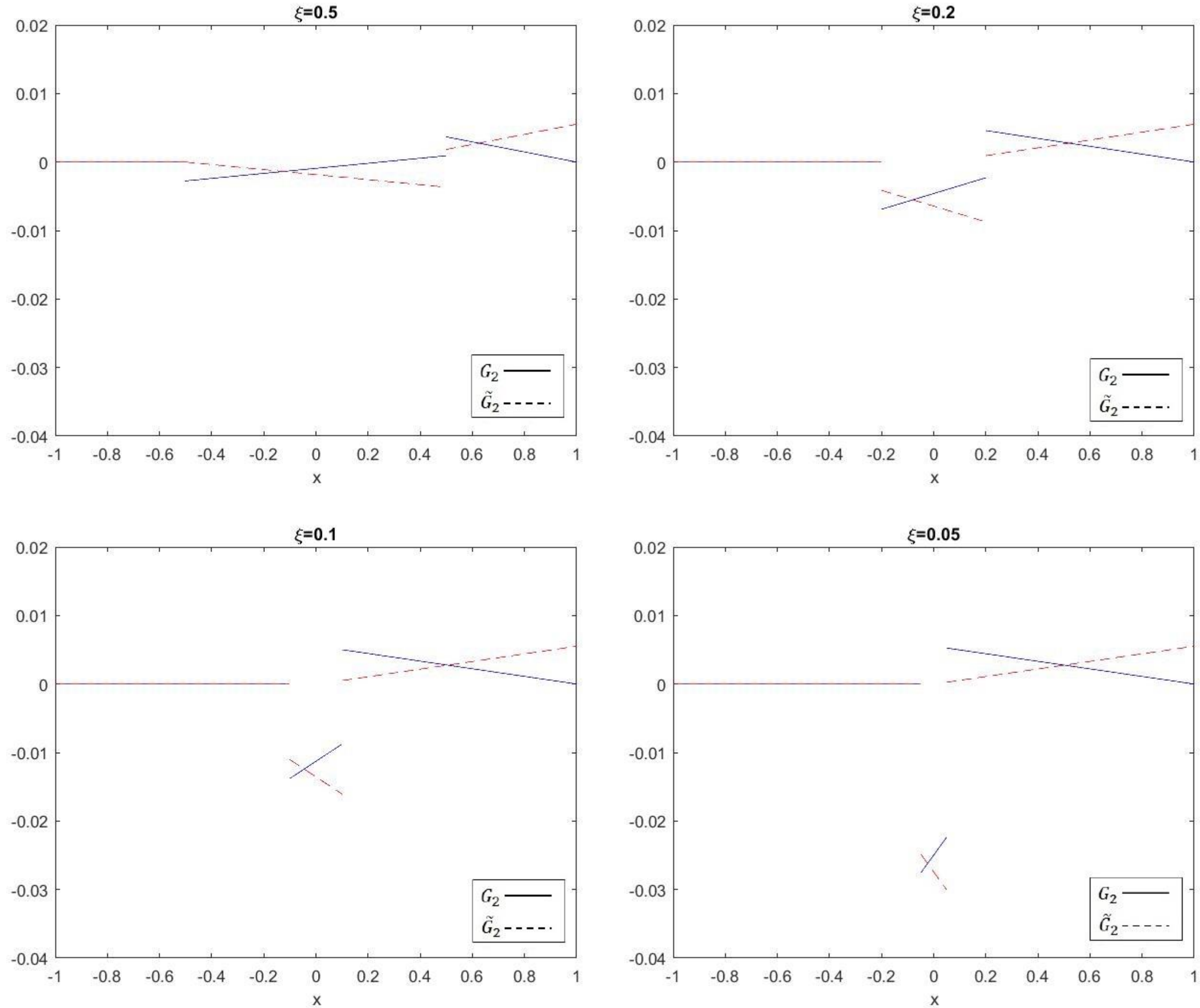
G_2 and $\widetilde{G}_2$ in Scalar Diquark Model

G_2



$\widetilde{G}_2$





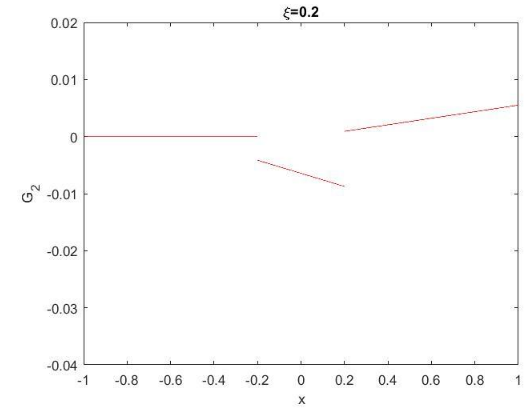
Twist-3 GPD	Discontinuities as $\xi \to 0$
G_2	Divergent
$\widetilde{G}_2$	Divergent

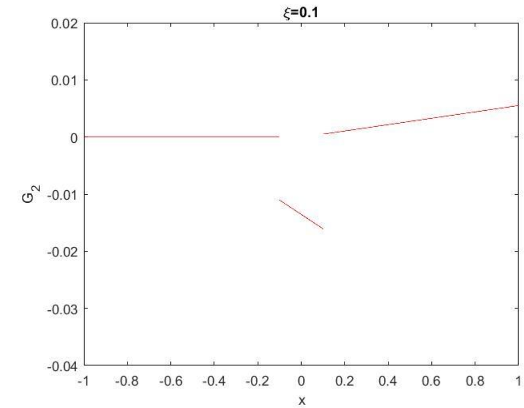
The behavior of the discontinuties of the twist-3 GPDs, $\widetilde{G}_2$ and G_2 as $\xi \to 0$ in quark target model (QTM) and scalar diquark model (SDM).

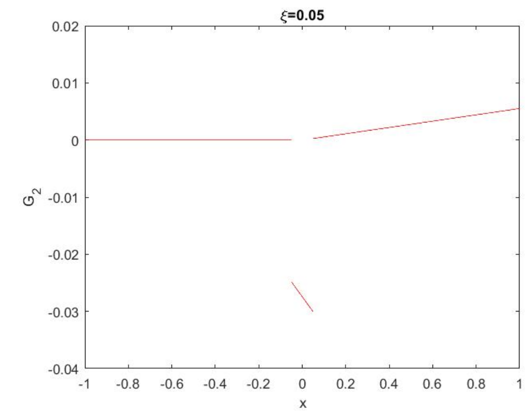
Twist-3 GPD	QTM	SDM
G_2	Divergent	Divergent
$\widetilde{G}_2$	Finite	Divergent

The ERBL region of $\widetilde{G}_2$ behaves like a $\delta(x)$ in SDM

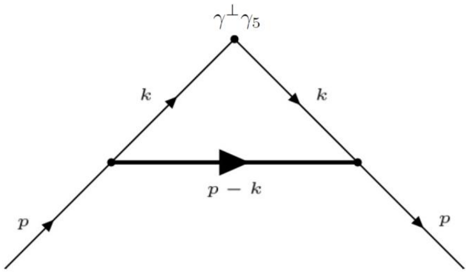
Now let's check the forward limit $\tilde{G}_2 \rightarrow g_2$







Twist -3 pdf g_T & *Twist -3 quasi-pdf* g_T^{quasi} *in scalar diquark model*



$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P,S|\overline{q}(0)\gamma^\mu\gamma_5 q(\lambda n)|P,S\rangle$$
$$= 2\{g_1(x)\hat{p}^\mu(S\cdot\hat{n}) + g_T(x)S_\perp^\mu + M^2 g_3(x)\hat{n}^\mu(S\cdot\hat{n})\}$$

$$g_T(x) = g_1(x) + g_2(x)$$

Twist -3 pdf g_T is calculated using LF coordinates , $(+,-,\perp)$

$$2g_T S^\perp = \int \frac{d^2k_\perp}{(2\pi)^2}\frac{dk^-}{2\pi}\overline{u}(p)\frac{i(\not{k}+m)}{(k^2-m^2+i\epsilon)}\gamma^\perp\gamma_5\frac{i(\not{k}+m)}{(k^2-m^2+i\epsilon)}u(p)\frac{i}{[(p-k)^2-\lambda^2+i\epsilon]}$$

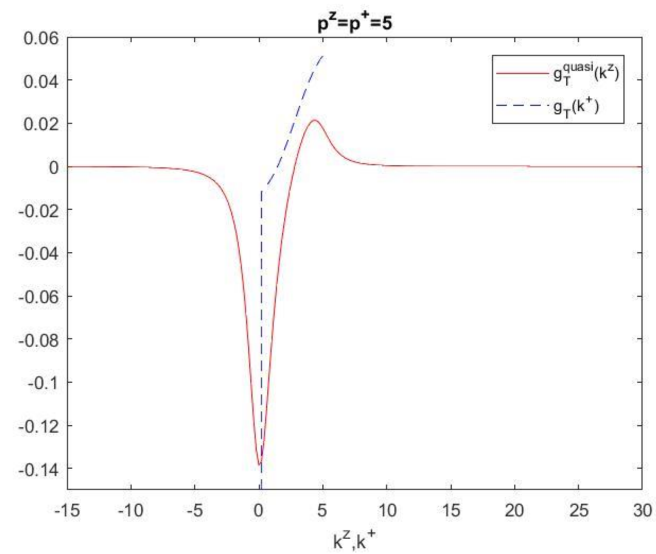
Twist -3 quasi-pdf g_T^{quasi} is calculated using normal coordinates, $(0, \perp, z)$

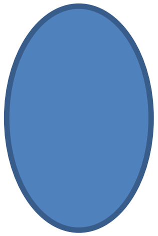
$$2g_T^{quasi} S^\perp = \int \frac{d^2k_\perp}{(2\pi)^2}\frac{dk^0}{2\pi}\overline{u}(p)\frac{i(\not{k}+m)}{(k^2-m^2+i\epsilon)}\gamma^\perp\gamma_5\frac{i(\not{k}+m)}{(k^2-m^2+i\epsilon)}u(p)\frac{i}{[(p-k)^2-\lambda^2+i\epsilon]}$$

$$g_T^{quasi} \xrightarrow{P^z\to\infty} g_T$$

Behavior of twist-2 and twist-3 distributions under longitudinal momentum boost, P^+.

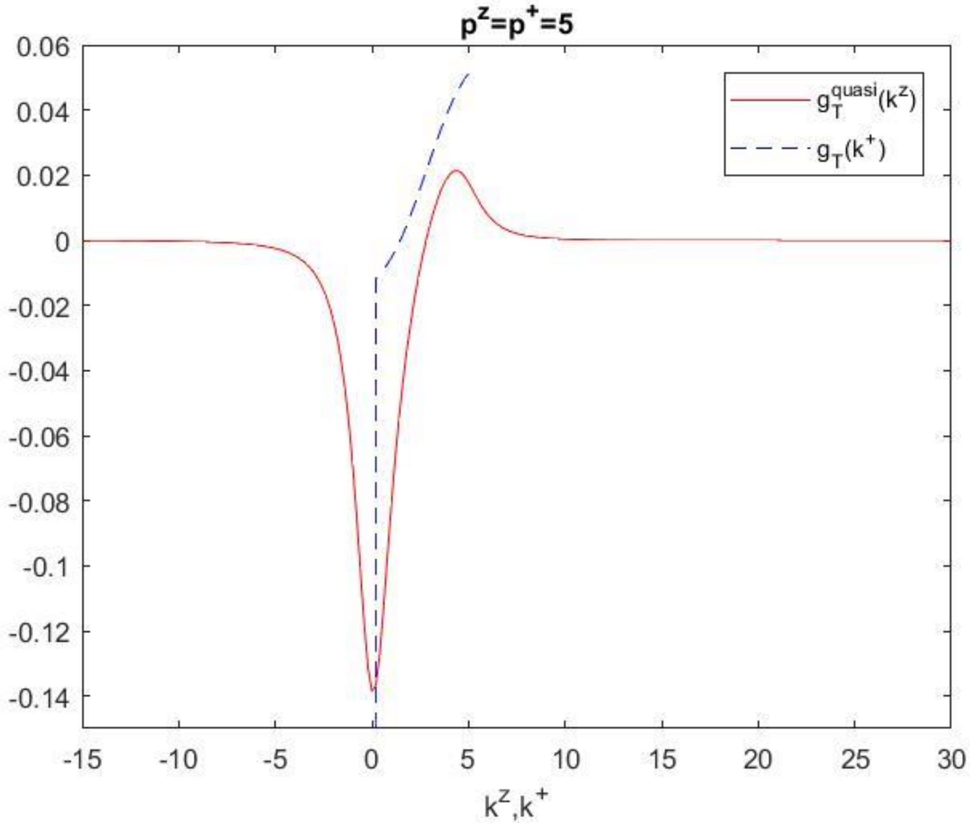
Twist-2	Twist-3
Independent of P^+	$1/P^+$

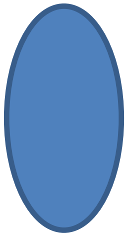




$P^z = P^+ = 5$

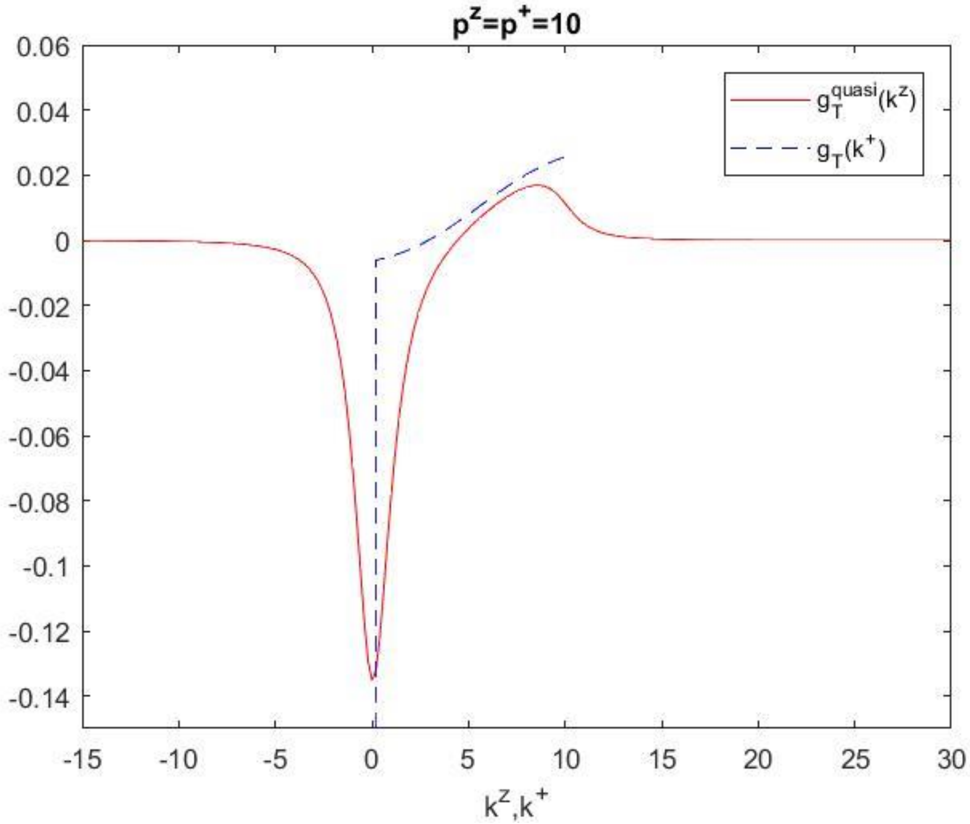
$g_T(\boldsymbol{k^+}),\ \ g_T{}^{quasi}(\boldsymbol{k^z})$





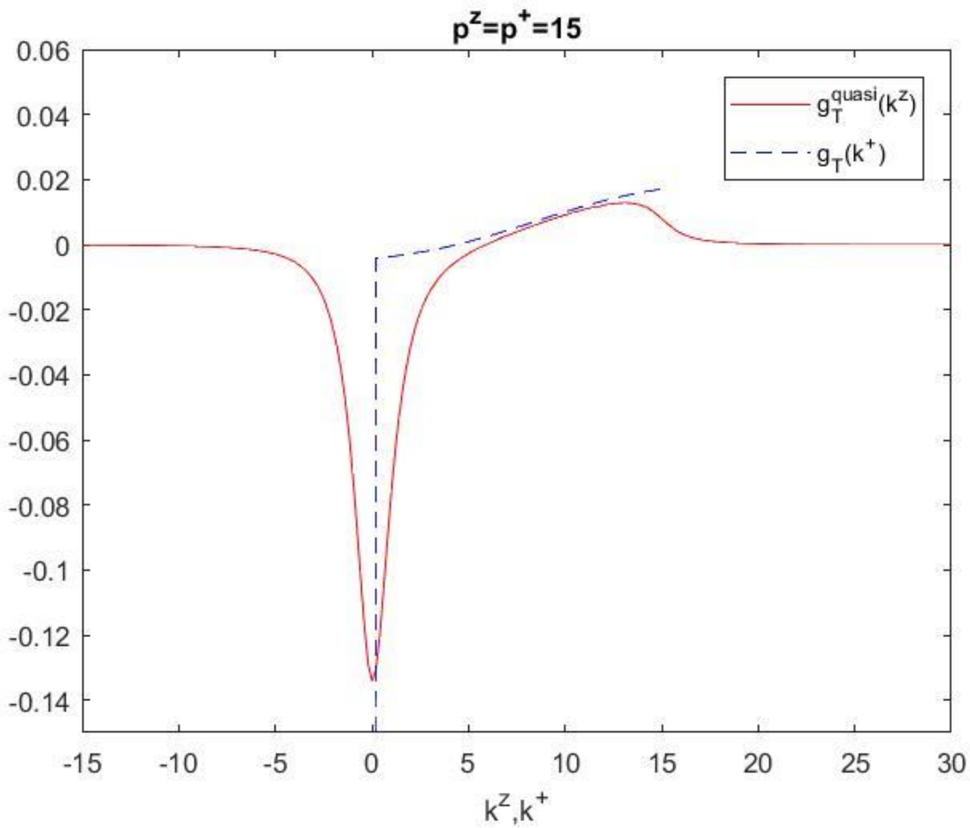
$P^z = P^+ = 10$

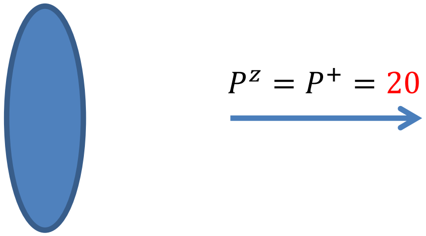
$g_T(\boldsymbol{k^+}),\ \ g_T{}^{quasi}(\boldsymbol{k^z})$



$P^z = P^+ = 15$

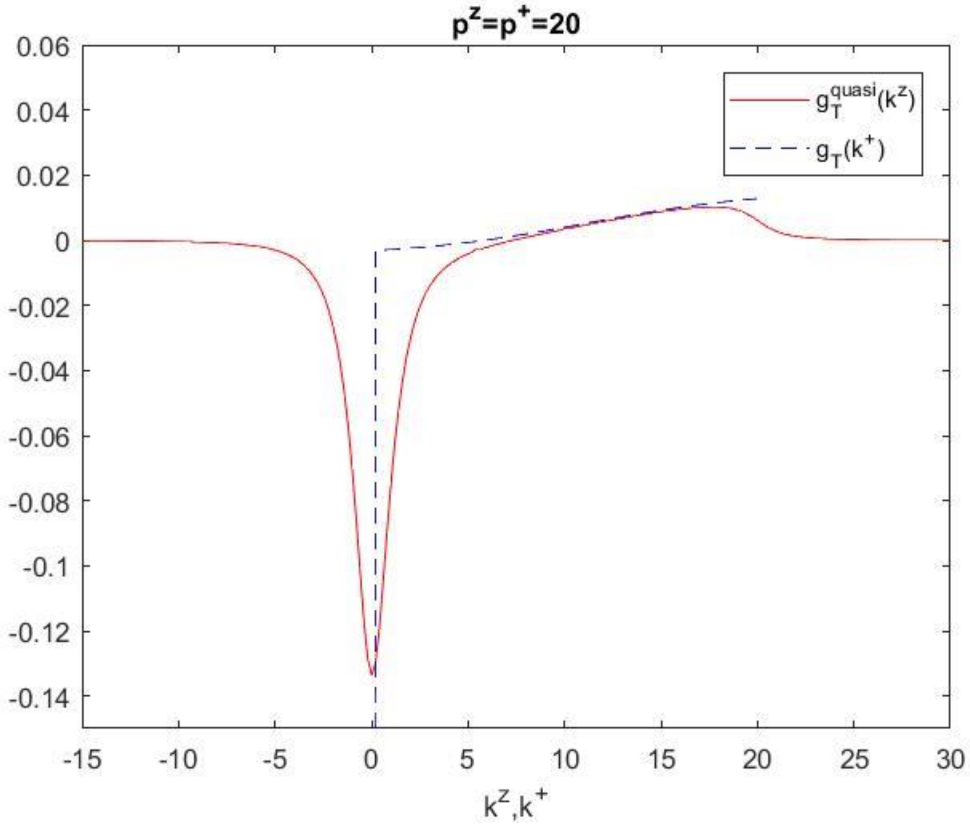
$g_T(\boldsymbol{k^+}),\ \ g_T{}^{quasi}(\boldsymbol{k^z})$





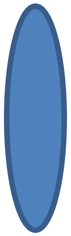
$P^z = P^+ = 20$

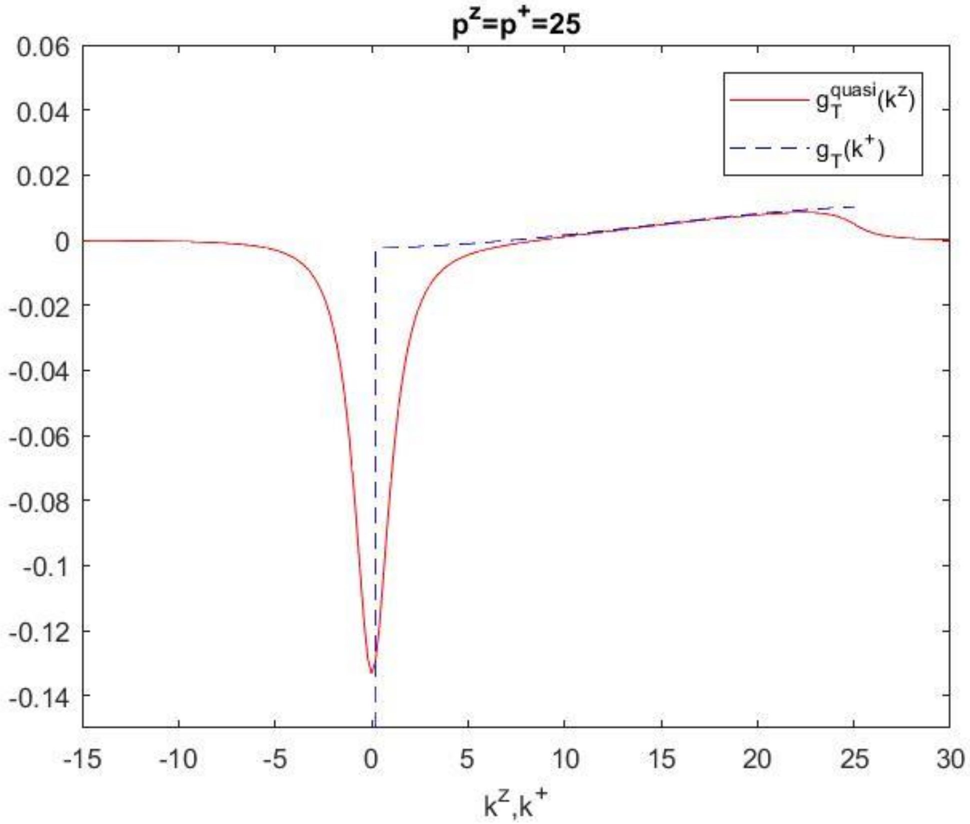
$g_T(\boldsymbol{k^+}),\ \ g_T{}^{quasi}(\boldsymbol{k^z})$



$P^z = P^+ = 25$

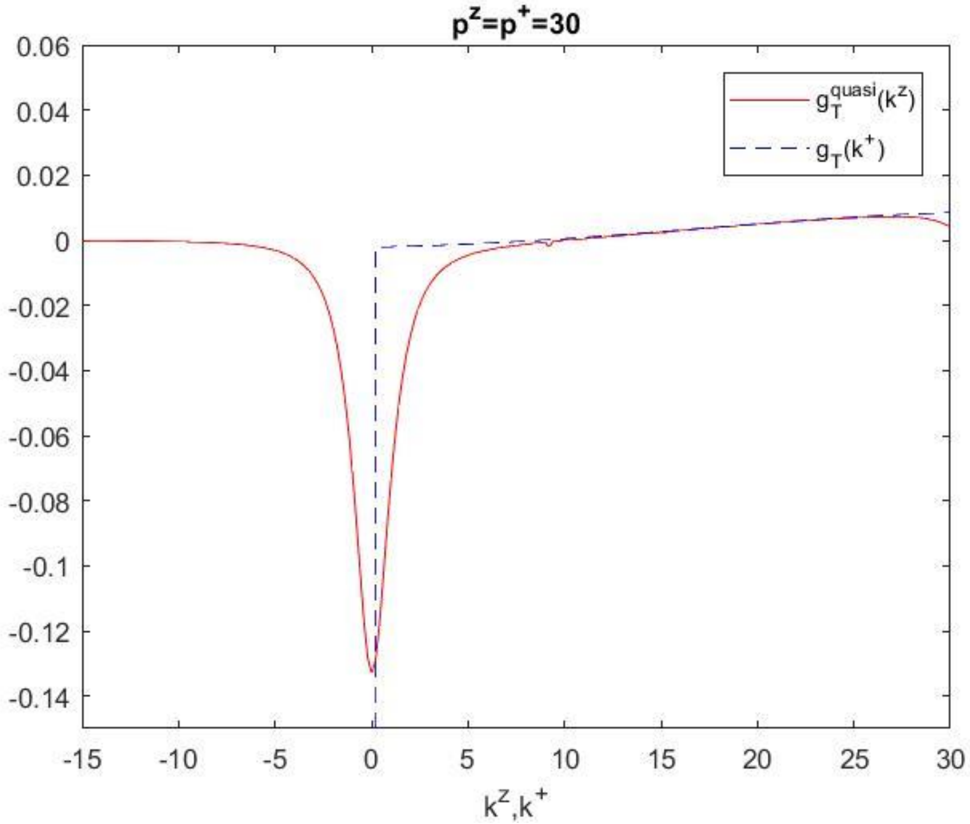
$g_T(\boldsymbol{k^+}),\ \ g_T{}^{quasi}(\boldsymbol{k^z})$



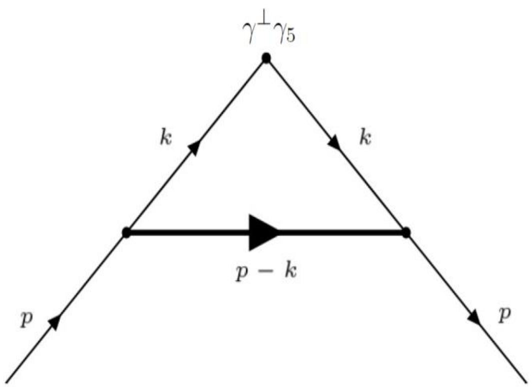


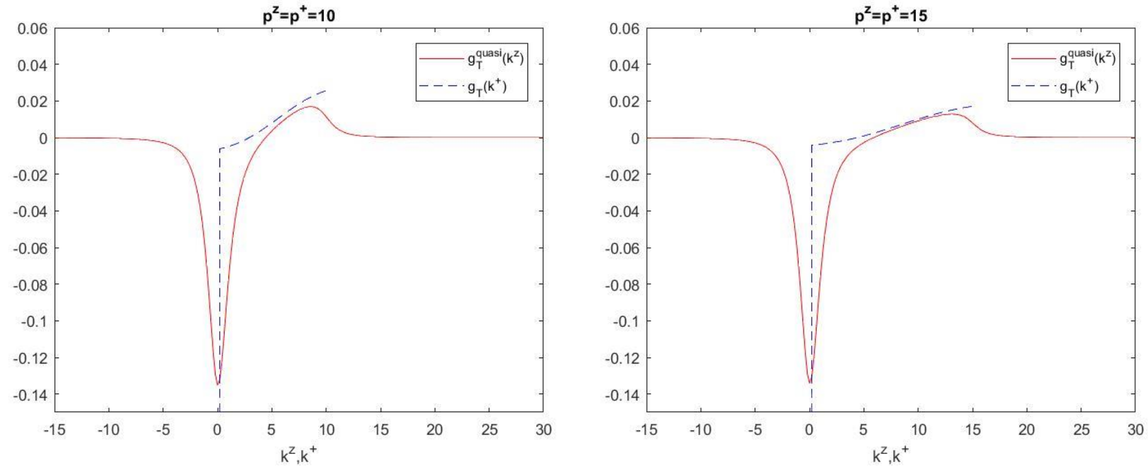
$P^z = P^+ = 30$

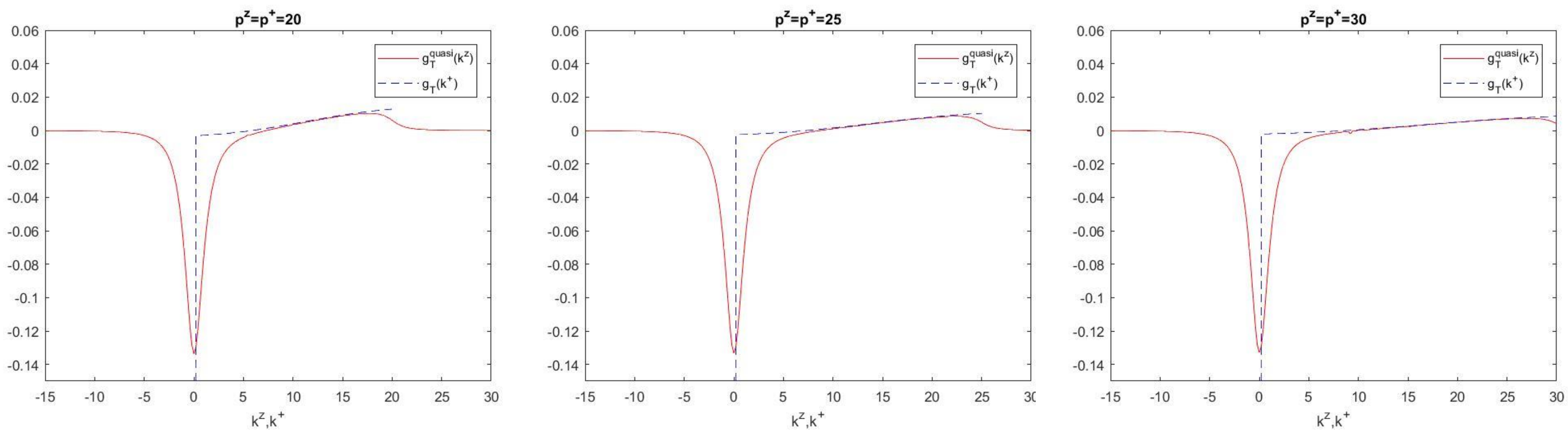
$g_T(\boldsymbol{k^+}),\ \ g_T{}^{quasi}(\boldsymbol{k^z})$



Twist-3 pdf g_T & Twist-3 quasi-pdf g_T^{quasi} in scalar diquark model



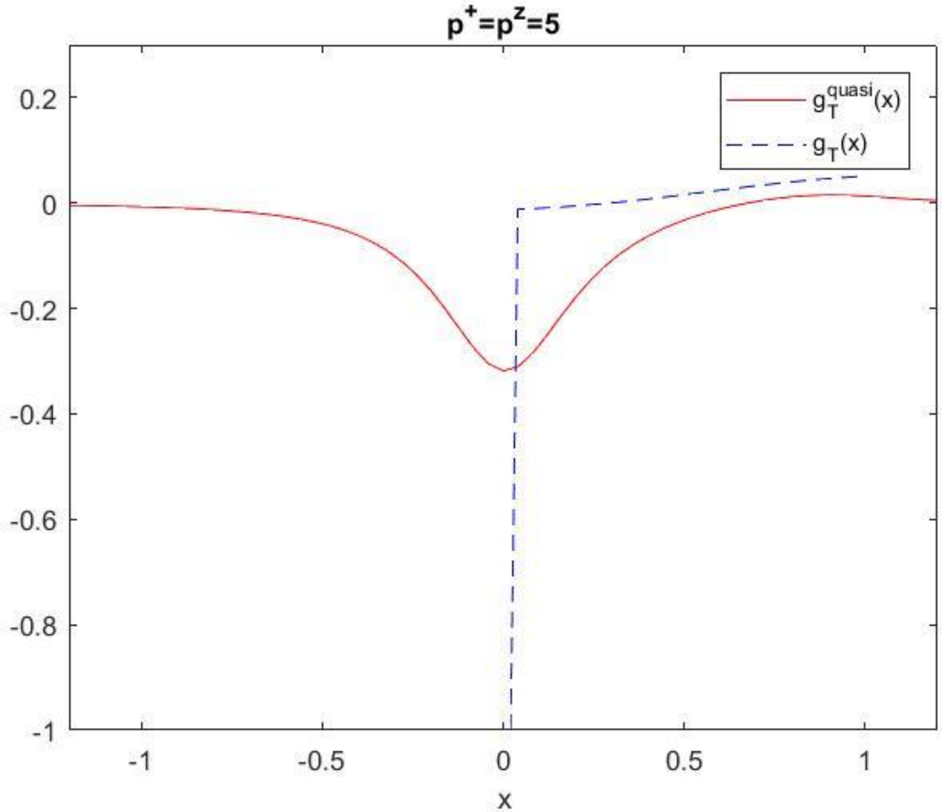


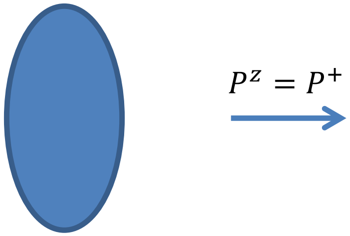


There is a momentum component in the nucleon state which does not scale as the nucleon is boosted to the infinite momentum frame.

$P^z = P^+ = 5$

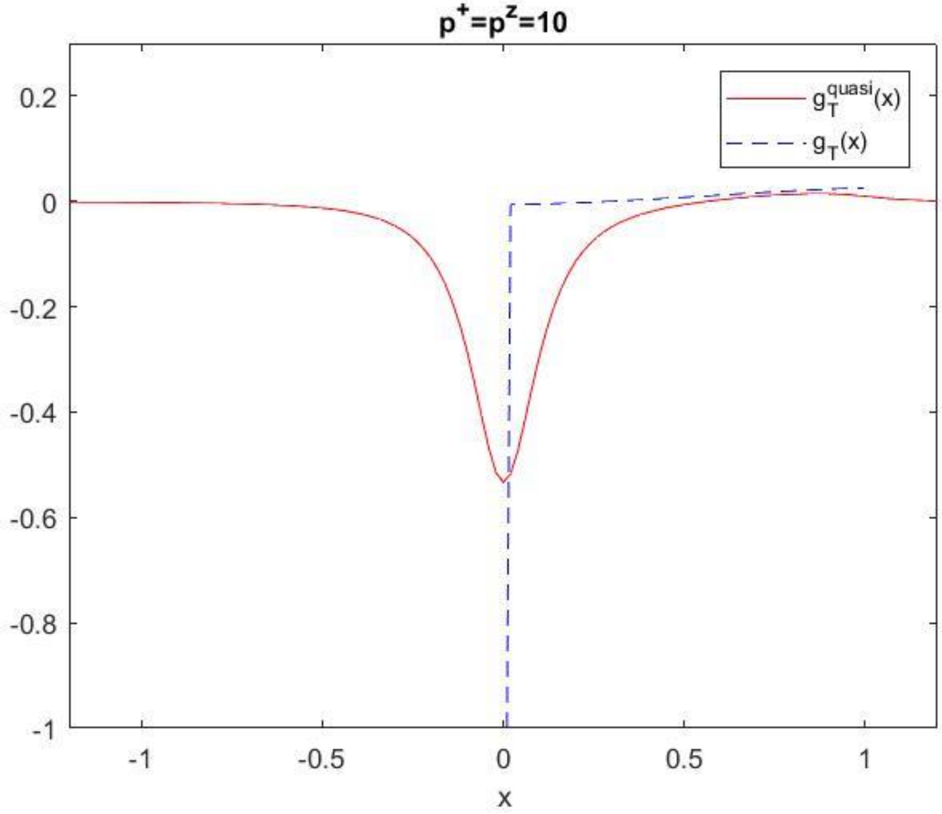
$g_T(x),\ g_T{}^{quasi}(x)$





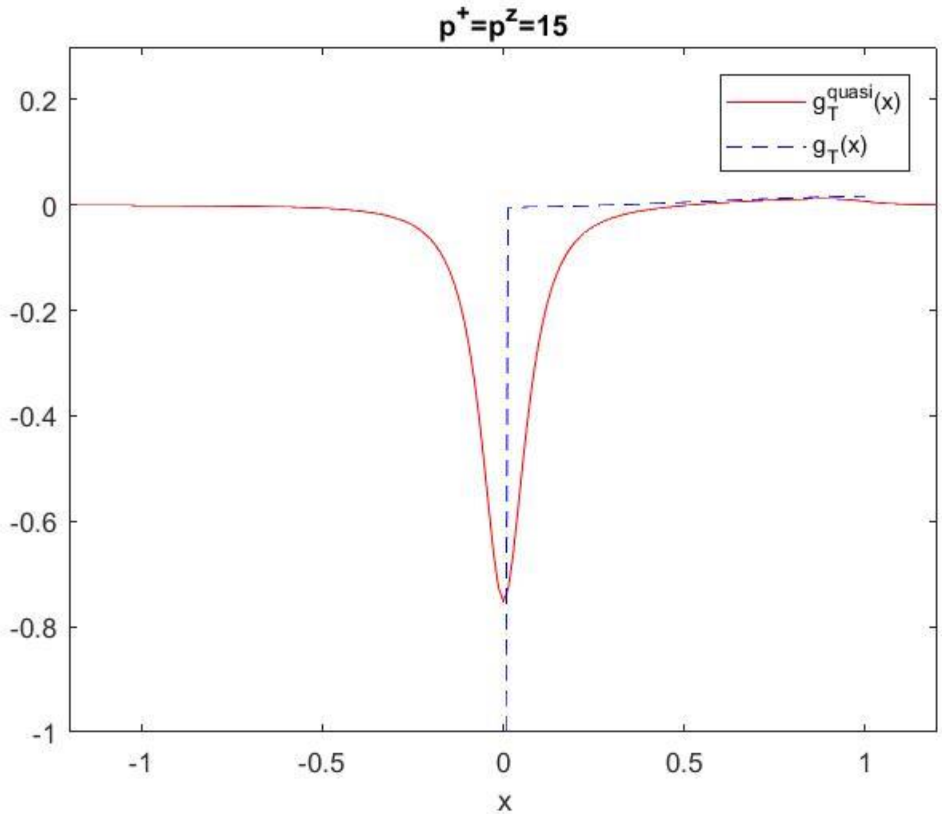
$P^z = P^+ = 10$

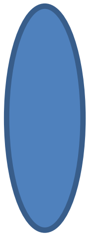
$g_T(\boldsymbol{x}),\ \ g_T{}^{quasi}(\boldsymbol{x})$



$P^z = P^+ = 15$

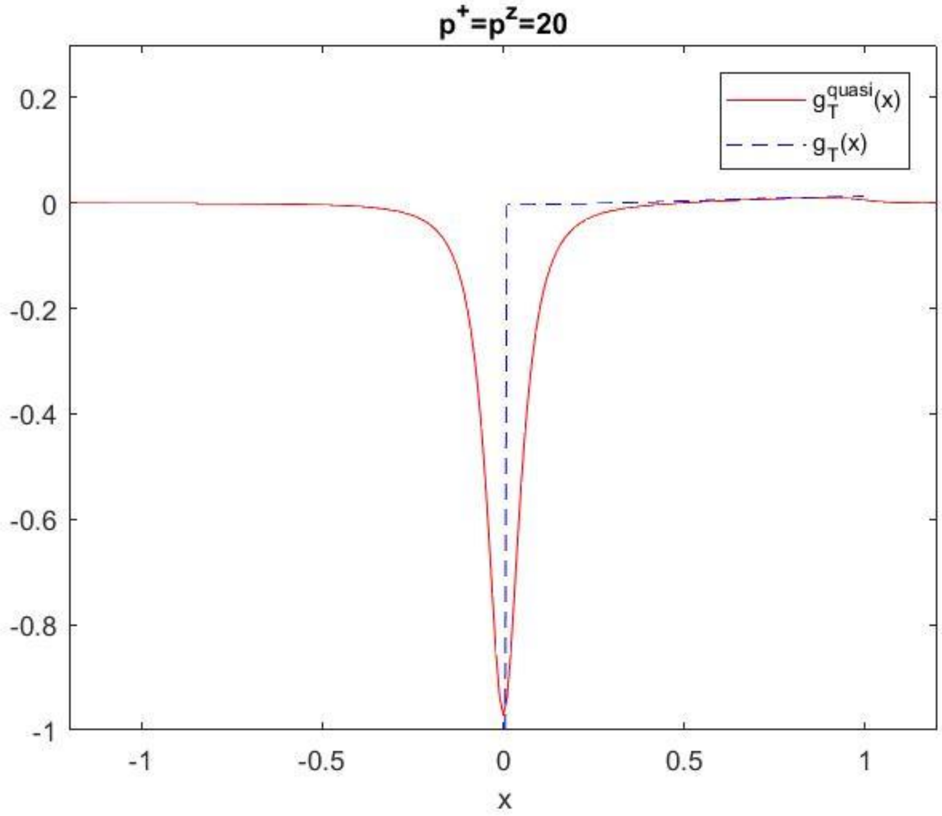
$g_T(\boldsymbol{x}),\;\; g_T{}^{quasi}(\boldsymbol{x})$

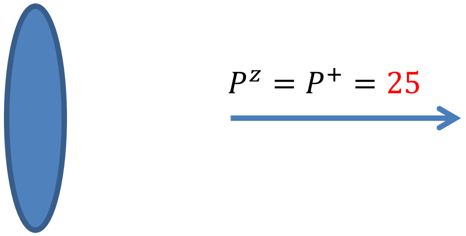




$P^z = P^+ = 20$

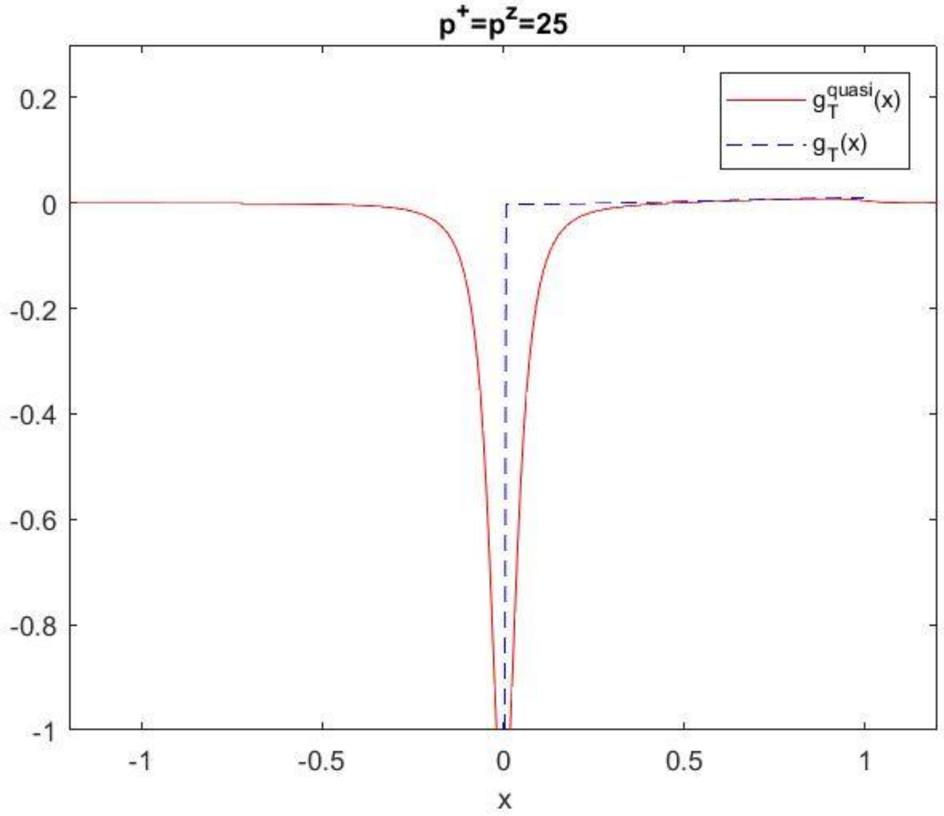
$g_T(\boldsymbol{x}),\ \ g_T{}^{quasi}(\boldsymbol{x})$





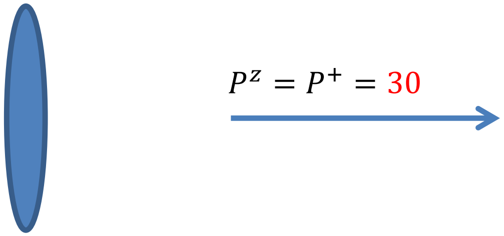
$P^z = P^+ = 25$

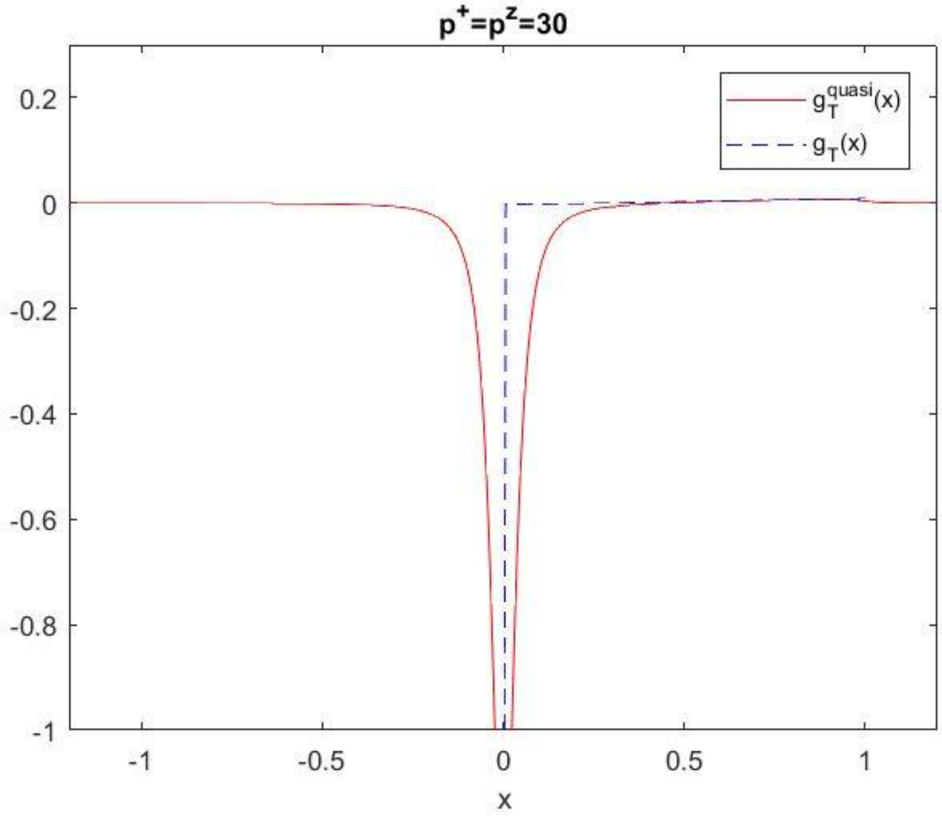
$g_T(\boldsymbol{x}),\ \ g_T{}^{quasi}(\boldsymbol{x})$



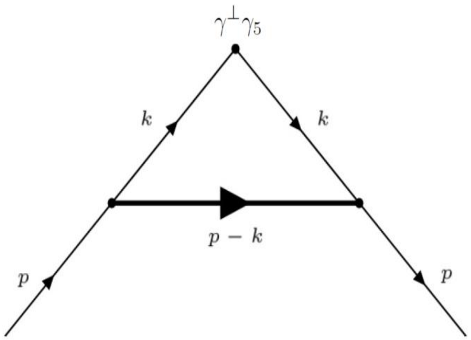
$P^z = P^+ = 30$

$g_T(\boldsymbol{x}),\ \ g_T{}^{quasi}(\boldsymbol{x})$





The origin of the singularities



$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P,S|\overline{q}(0)\gamma^\mu\gamma_5 q(\lambda n)|P,S\rangle$$
$$= 2\{g_1(x)\hat{p}^\mu(S\cdot\hat{n}) + g_T(x)S_\perp^\mu + M^2 g_3(x)\hat{n}^\mu(S\cdot\hat{n})\}$$

$$g_T(x) = g_1(x) + g_2(x)$$

$$2g_T S^\perp = \int \frac{d^2k_\perp}{(2\pi)^2}\frac{dk^-}{2\pi}\overline{u}(p)\frac{i(\not k + m)}{(k^2 - m^2 + i\epsilon)}\gamma^\perp\gamma_5\frac{i(\not k + m)}{(k^2 - m^2 + i\epsilon)}u(p)\frac{i}{[(p-k)^2 - \lambda^2 + i\epsilon]}$$

$$k^- = \frac{M^2}{2p^+} - \frac{[(p-k)^2 - \lambda^2]}{2(p^+ - k^+)} - \frac{(k_\perp^2 + \lambda^2)}{2(p^+ - k^+)}$$

Whenever k^- appears in the numerator the propagator is cancelled

$$\int \frac{dk^-}{(k^2 - m^2 + i\epsilon)^2}$$

$$\text{for } k^+ \neq 0, \int \frac{dk^-}{(k^2 - m^2 + i\epsilon)^2} = \int \frac{dk^-}{\left[2k^+\left(k^- - \frac{(k_\perp^2 + m^2)}{2k^+} + \frac{i\epsilon}{2k^+}\right)\right]^2} = 0$$

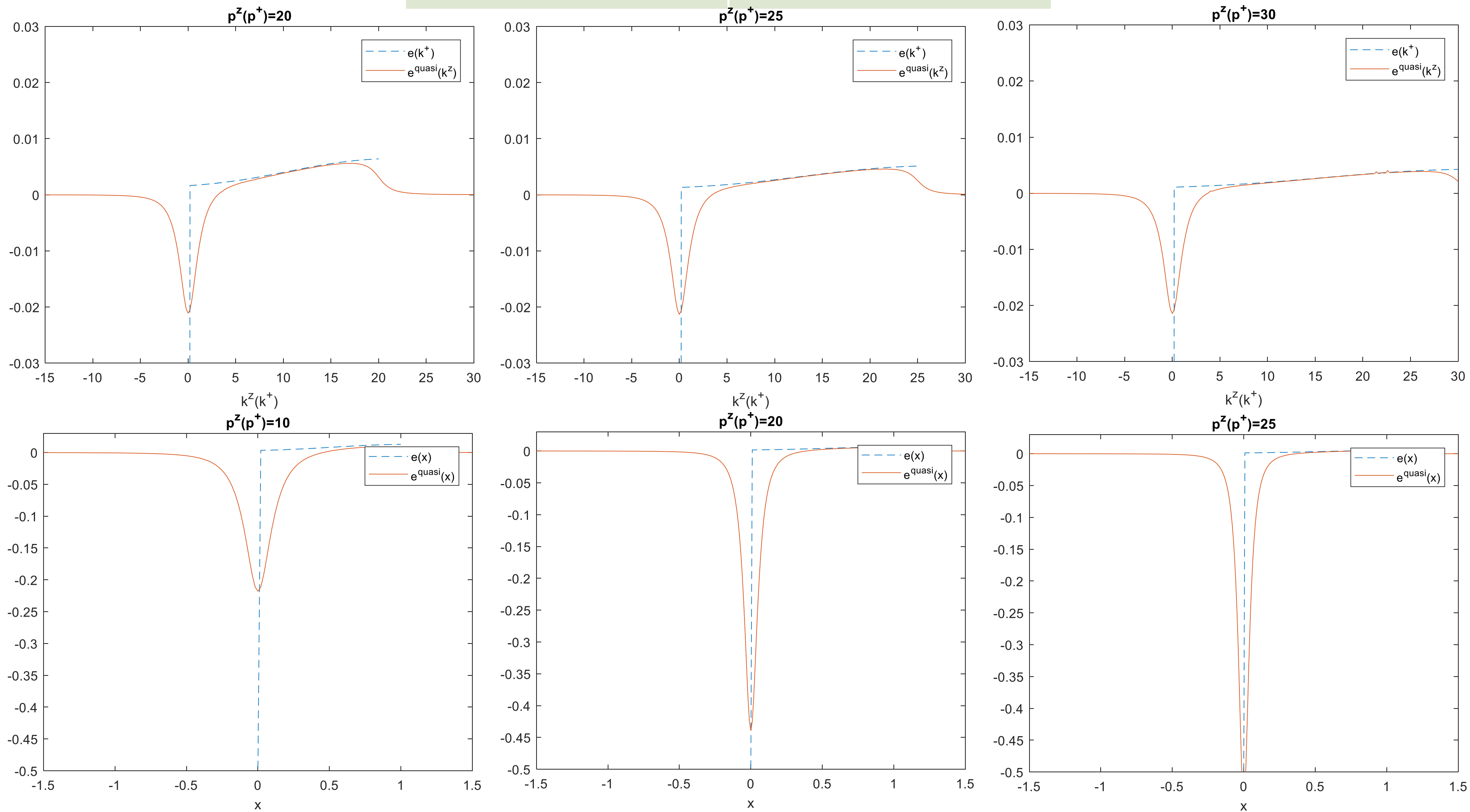
$$\int dk^+ dk^- \frac{1}{(k^2 - m^2 + i\epsilon)^2} = \int dk^+ dk^- \frac{1}{(2k^+k^- - k_\perp^2 - m^2 + i\epsilon)^2}$$
$$= \int d^2k_L \frac{1}{(k_L^2 - k_\perp^2 - m^2 + i\epsilon)^2} = \frac{i\pi}{k_\perp^2 + m^2}$$

$$\int \frac{dk^-}{(k^2 - m^2 + i\epsilon)^2} = \frac{i\pi}{k_\perp^2 + m^2}\delta(k^+).$$

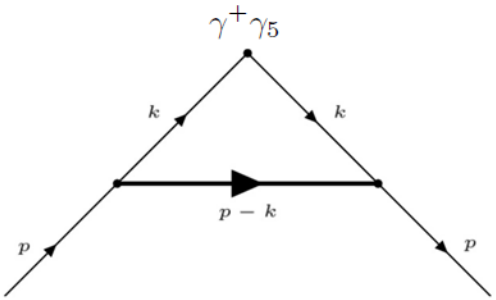
Twist-3 pdf	*Scalar diquark model*
e	✓
h_L	✓
g_T (g_2)	✓

✓ : There is $\delta(x)$

✗ : There is no $\delta(x)$



Twist -2 pdf g_1 & *Twist -2 quasi-pdf* g_1^{quasi} *in scalar diquark model*



$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P,S|\overline{q}(0)\gamma^\mu\gamma_5 q(\lambda n)|P,S\rangle$$
$$= 2\{g_1(x)\hat{p}^\mu(S\cdot\hat{n}) + g_T(x)S_\perp^\mu + M^2 g_3(x)\hat{n}^\mu(S\cdot\hat{n})\}$$

Twist -2 pdf g_1 is calculated using LF coordinates , $(+,-,\perp)$

$$2g_1S^+ = \int \frac{d^2k_\perp}{(2\pi)^2}\frac{dk^-}{2\pi}\overline{u}(p)\frac{i(\not{k}+m)}{(k^2-m^2+i\epsilon)}\gamma^+\gamma_5\frac{i(\not{k}+m)}{(k^2-m^2+i\epsilon)}u(p)\frac{i}{[(p-k)^2-\lambda^2+i\epsilon]}$$

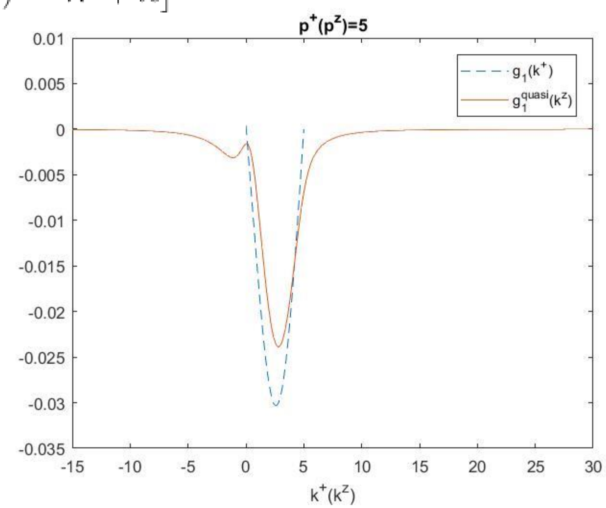
Twist -2 quasi-pdf g_1^{quasi} is calculated using normal coordinates, $(0, \perp, z)$

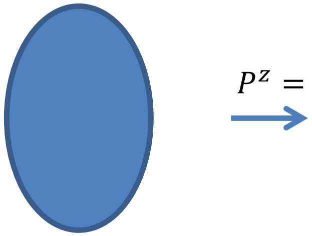
$$(\frac{P^z}{P^0}-1)g_1^{quasi}S^z = \int \frac{d^2k_\perp}{(2\pi)^2}\frac{dk^0}{2\pi}\overline{u}(p)\frac{i(\not{k}+m)}{(k^2-m^2+i\epsilon)}\gamma^z\gamma_5\frac{i(\not{k}+m)}{(k^2-m^2+i\epsilon)}u(p)\frac{i}{[(p-k)^2-\lambda^2+i\epsilon]}$$

$$g_1^{quasi} \xrightarrow{P^z\to\infty} g_1$$

Behavior of twist-2 and twist-3 distributions under longitudinal momentum boost, P^+.

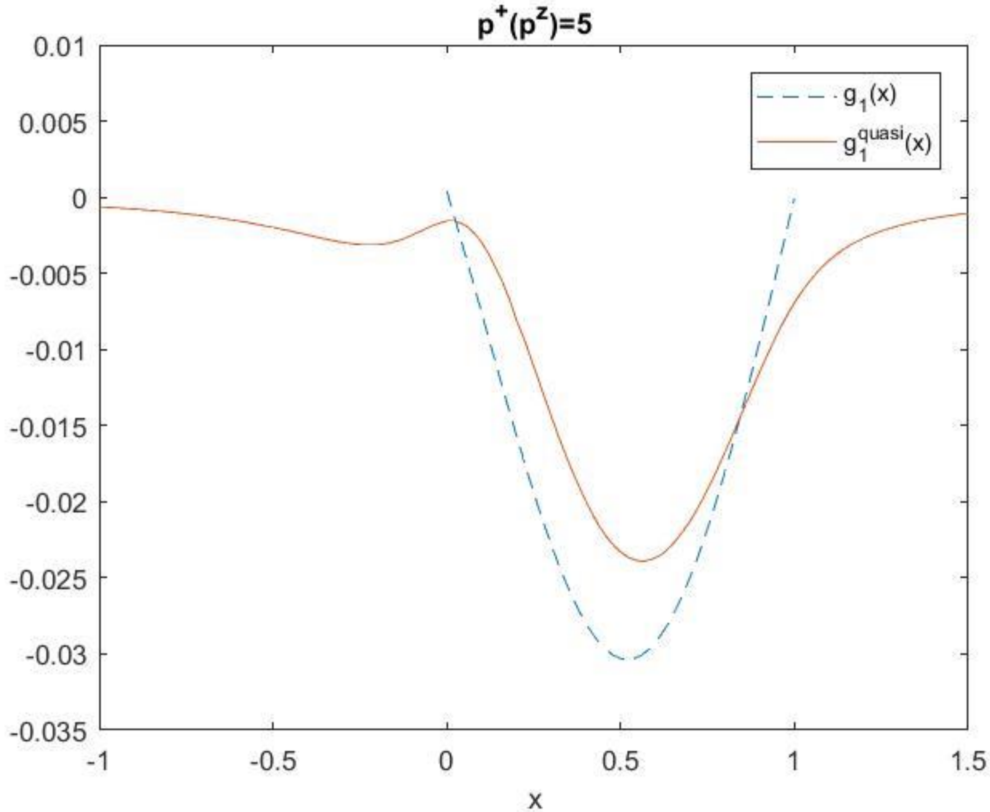
Twist-2	Twist-3
Independent of P^+	$1/P^+$





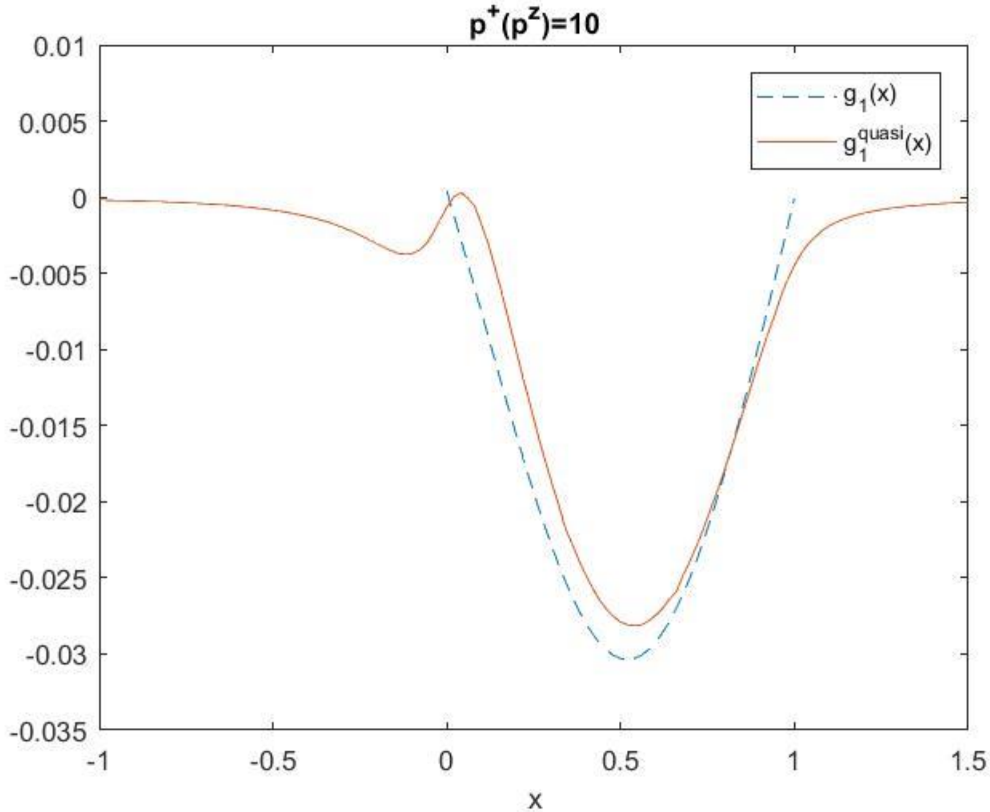
$P^z = P^+ = 5$

$g_1(x),\ g_1{}^{quasi}(x)$



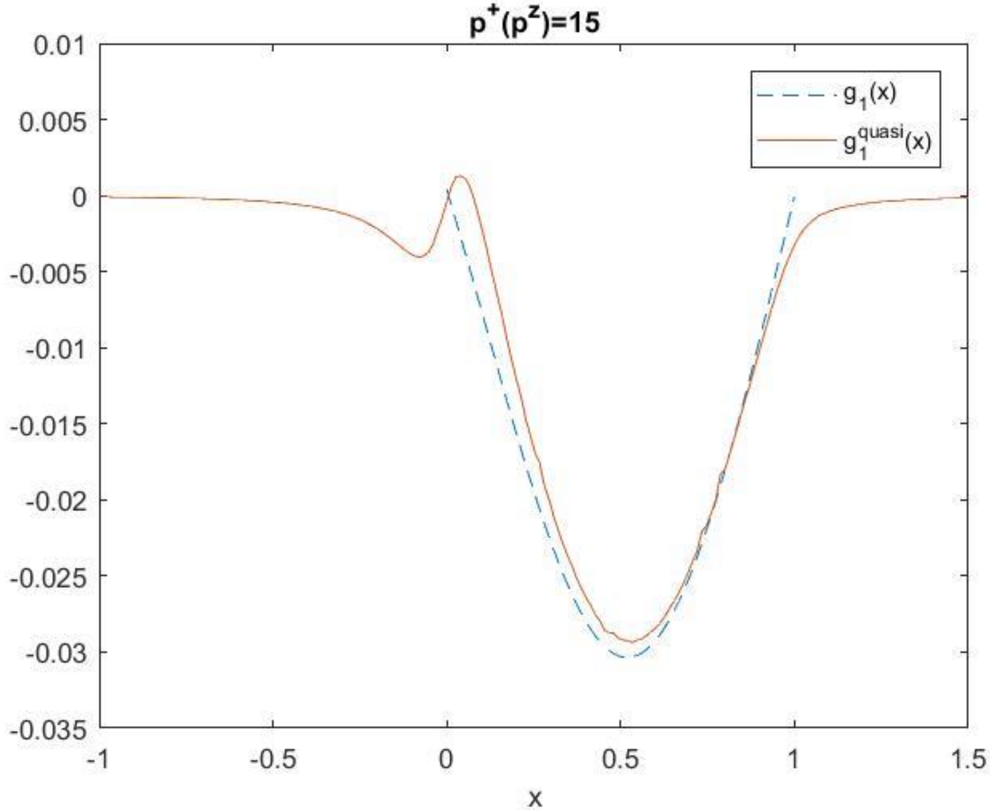
$P^z = P^+ = 10$

$g_1(x),\ \ g_1{}^{quasi}(x)$



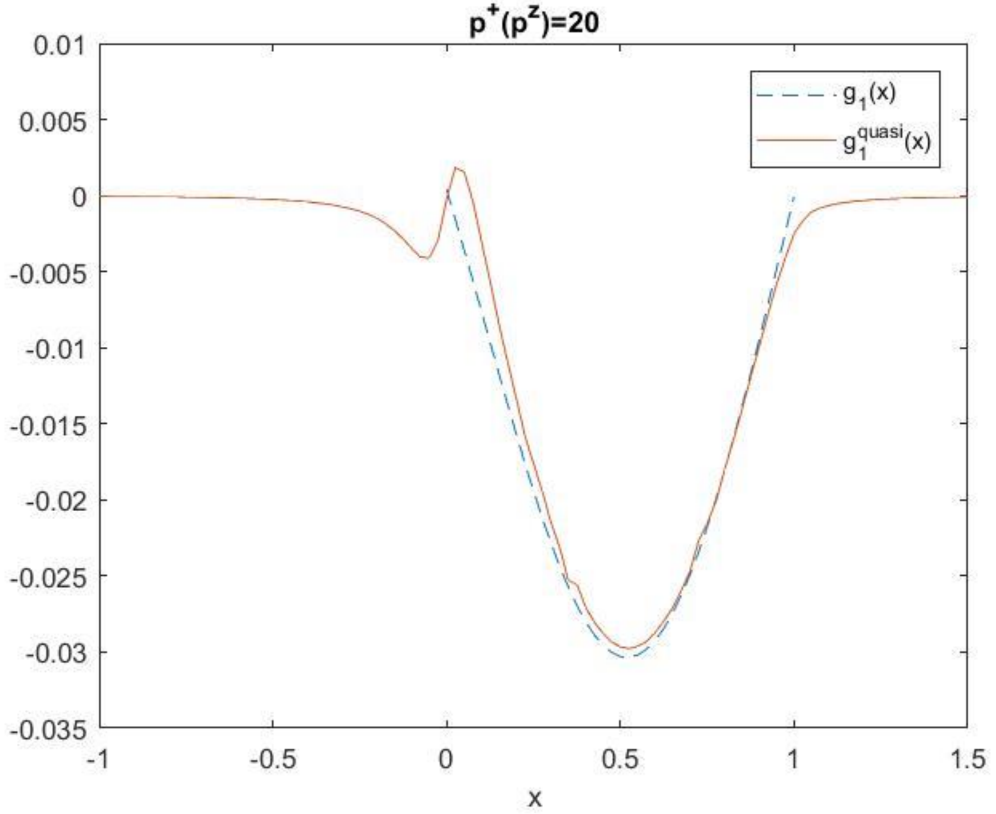
$P^z = P^+ = 15$

$g_1(x), \ g_1{}^{quasi}(x)$



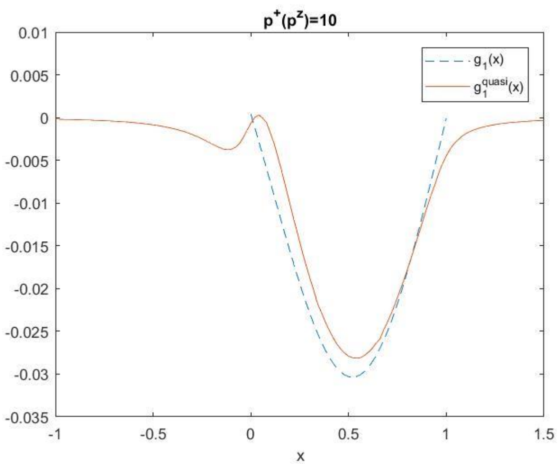
$P^z = P^+ = 20$

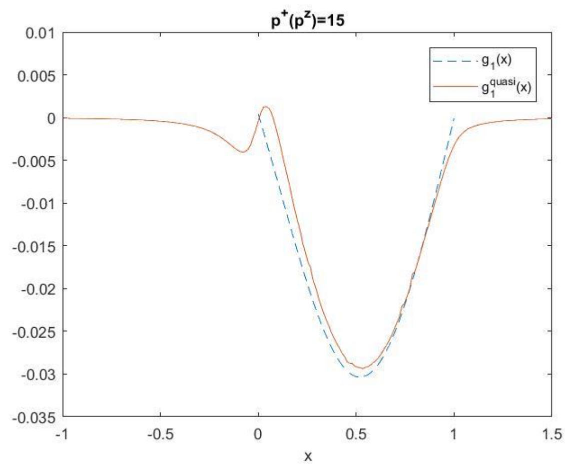
$g_1(x),\ \ g_1{}^{quasi}(x)$

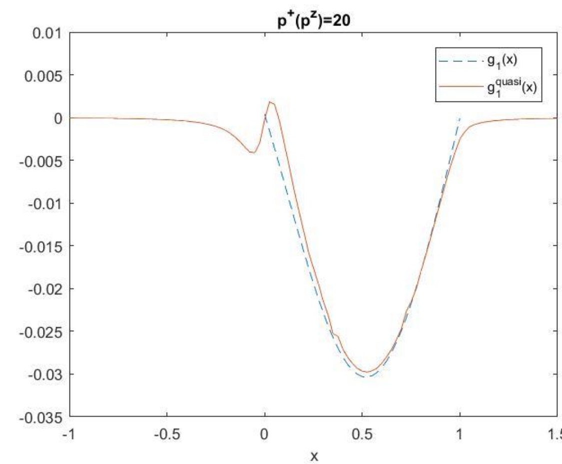


✓: *There is δ(x)*
✗: *There is no δ(x)*

Twist-2 pdf	*Scalar diquark model*
f_1	✗
g_1	✗
h_1	✗







✓ : *There is $\delta(x)$*
✗ : *There is no $\delta(x)$*

Twist-2 pdf	*Scalar diquark model*	*Quark target model*
f_1	✗	✗
g_1	✗	✗
h_1	✗	✗

Twist-3 pdf	*Scalar diquark model*	*Quark target model*
e	✓	✓
h_L	✓	✓
g_T (g_2)	✓	✗

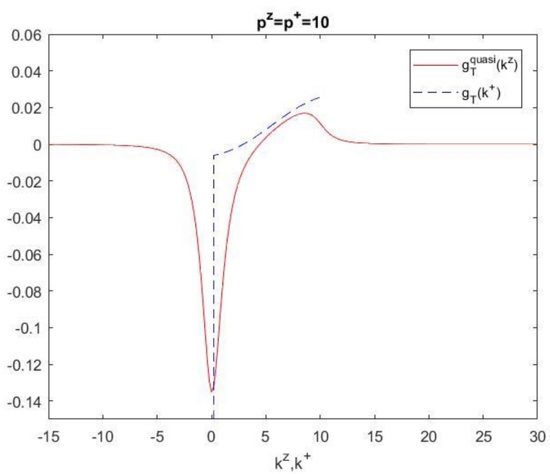
AT TWIST-3 THERE IS SOMETHING THAT DOES NOT EXIST IN TWIST-2: THERE ARE DELTA FUNCTIONS

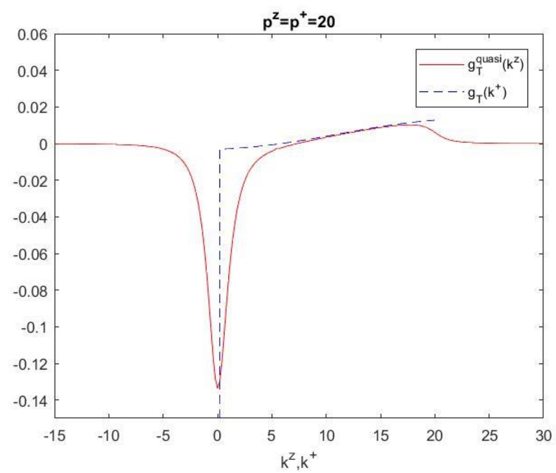
✓: *There is $\delta(x)$*

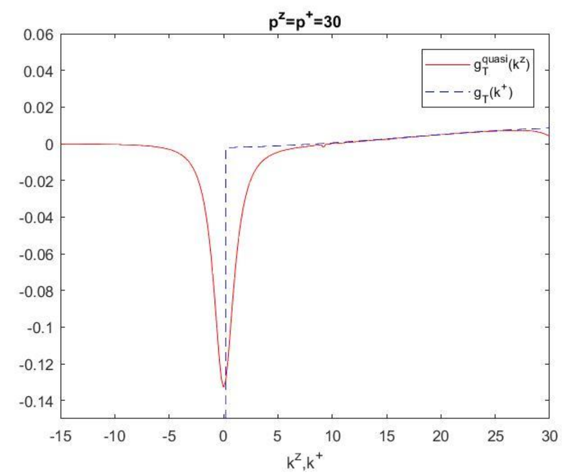
✗: *There is no $\delta(x)$*

Twist-3 pdf	*Scalar diquark model*	*Quark target model*
e	✓	✓
h_L	✓	✓
g_T (g_2)	✓	✗

We identify these delta functions with momentum components in the nucleon state that do not scale as the nucleon is boosted to the infinite momentum.







PAULI VILLARS (PV) REGULARIZATION

Without PV Regularization

$$2g_T S^{\perp} = \int \frac{d^2k_{\perp}}{(2\pi)^2}\frac{dk^-}{2\pi}\overline{u}(p)\frac{i(\not{k}+m)}{(k^2-m^2+i\epsilon)}\gamma^{\perp}\gamma_5\frac{i(\not{k}+m)}{(k^2-m^2+i\epsilon)}u(p)\frac{i}{[(p-k)^2-\lambda^2+i\epsilon]}$$

$$2g_T(x) = \frac{1}{4\pi p^+}(x+\frac{m}{M})\Big\{M(m+M)(1-x)-\lambda^2\Big\}\frac{1}{(k_{\perp}^2+\omega)}\Big|_0^{K^2} + \frac{1}{4\pi p^+}(2x-1+\frac{m}{M})\Big\{\frac{\omega}{k_{\perp}^2+\omega}+\ln(k_{\perp}^2+\omega)\Big\}\Big|_0^{K^2} - \frac{1}{4\pi}(x+\frac{m}{M})\frac{1}{(1-x)}\ln\big(\frac{K^2+m^2}{m^2}\big)\delta(x)$$

$$\omega = -x(1-x)M^2+(1-x)m^2+x\lambda^2$$

With PV Regularization

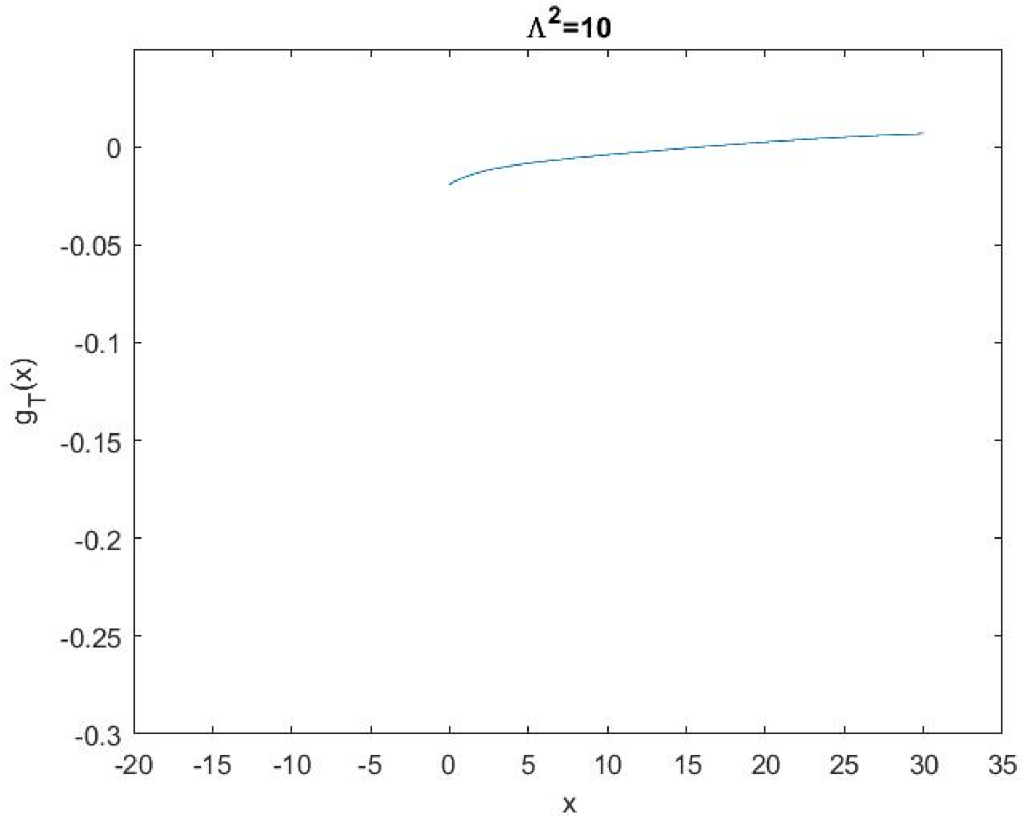
$$2g_T S^{\perp} = \int \frac{d^2k_{\perp}}{(2\pi)^2}\frac{dk^-}{2\pi}\overline{u}(p)\frac{i(\not{k}+m)}{(k^2-m^2+i\epsilon)}\gamma^{\perp}\gamma_5\frac{i(\not{k}+m)}{(k^2-m^2+i\epsilon)}u(p)\Big\{\frac{i}{[(p-k)^2-\lambda^2+i\epsilon]}-\frac{i}{[(p-k)^2-\Lambda^2+i\epsilon]}\Big\}$$

$$2g_T(x) = \frac{1}{4\pi p^+}(x+\frac{m}{M})\Big\{M(m+M)(1-x)-\lambda^2\Big\}\frac{1}{k_{\perp}^2+\omega(\lambda)}\Big|_0^{K^2} + \frac{1}{4\pi p^+}(2x-1+\frac{m}{M})\Big\{\frac{\omega(\lambda)}{k_{\perp}^2+\omega(\lambda)}+\ln(k_{\perp}^2+\omega(\lambda))\Big\}\Big|_0^{K^2}$$
$$-\frac{1}{4\pi p^+}(x+\frac{m}{M})\Big\{M(m+M)(1-x)-\Lambda^2\Big\}\frac{1}{k_{\perp}^2+\omega(\Lambda)}\Big|_0^{K^2} - \frac{1}{4\pi p^+}(2x-1+\frac{m}{M})\Big\{\frac{\omega(\Lambda)}{k_{\perp}^2+\omega(\Lambda)}+\ln(k_{\perp}^2+\omega(\Lambda))\Big\}\Big|_0^{K^2}$$

PAULI VILLARS (PV) REGULARIZATION

$$2g_T(x) = \frac{1}{4\pi p^+}(x+1)(1-2x)\frac{1}{k_\perp^2+\omega(\lambda=1)}\Big|_0^{K^2} + \frac{1}{4\pi p^+}(2x)\left\{\frac{\omega(\lambda=1)}{k_\perp^2+\omega(\lambda=1)} + \ln(k_\perp^2+\omega(\lambda=1))\right\}\Big|_0^{K^2}$$

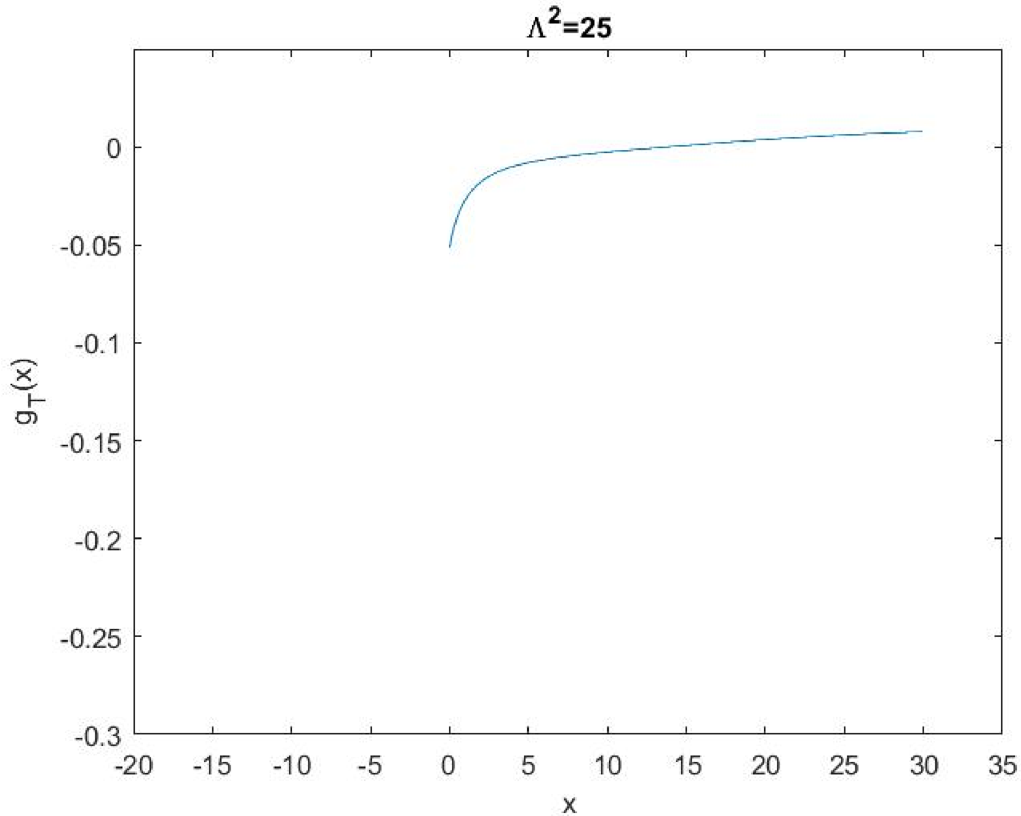
$$-\frac{1}{4\pi p^+}(x+1)\left\{2(1-x)-\Lambda^2\right\}\frac{1}{k_\perp^2+\omega(\Lambda)}\Big|_0^{K^2} - \frac{1}{4\pi p^+}(2x)\left\{\frac{\omega(\Lambda)}{k_\perp^2+\omega(\Lambda)} + \ln(k_\perp^2+\omega(\Lambda))\right\}\Big|_0^{K^2}$$



PAULI VILLARS (PV) REGULARIZATION

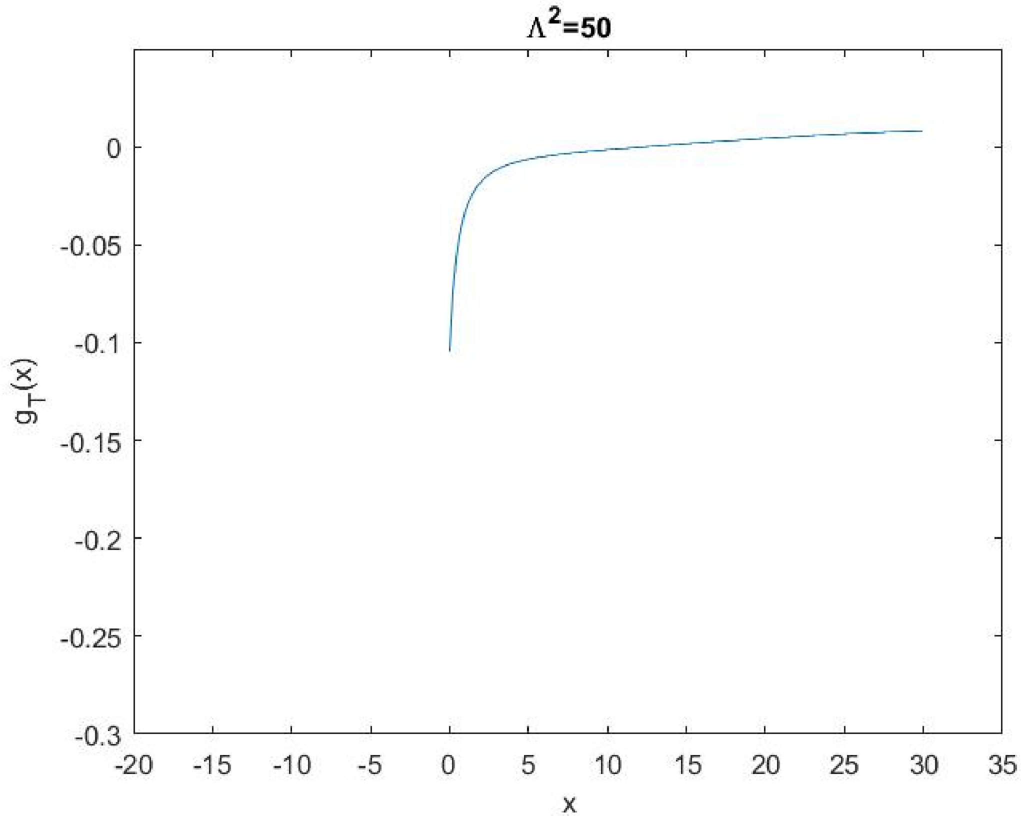
$$2g_T(x) = \frac{1}{4\pi p^+}(x+1)(1-2x)\frac{1}{k_\perp^2+\omega(\lambda=1)}|_0^{K^2} + \frac{1}{4\pi p^+}(2x)\left\{\frac{\omega(\lambda=1)}{k_\perp^2+\omega(\lambda=1)} + \ln(k_\perp^2+\omega(\lambda=1))\right\}|_0^{K^2}$$

$$-\frac{1}{4\pi p^+}(x+1)\left\{2(1-x)-\Lambda^2\right\}\frac{1}{k_\perp^2+\omega(\Lambda)}|_0^{K^2} - \frac{1}{4\pi p^+}(2x)\left\{\frac{\omega(\Lambda)}{k_\perp^2+\omega(\Lambda)} + \ln(k_\perp^2+\omega(\Lambda))\right\}|_0^{K^2}$$



PAULI VILLARS (PV) REGULARIZATION

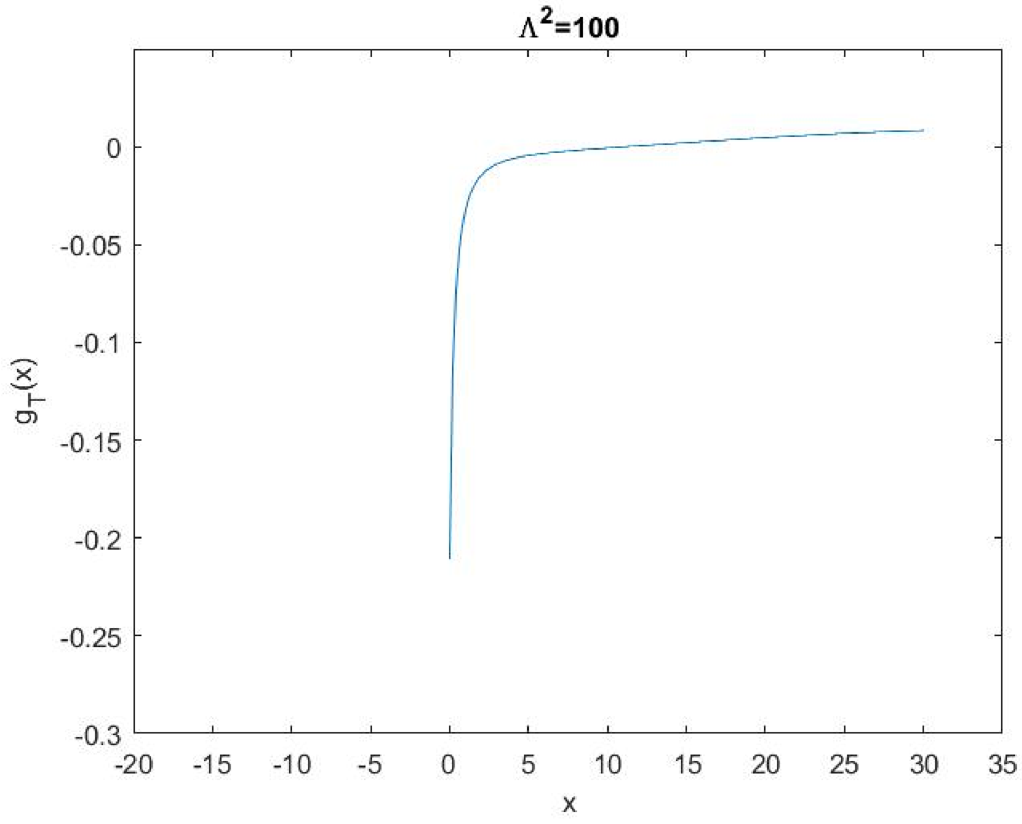
$$2g_T(x) = \frac{1}{4\pi p^+}(x+1)(1-2x)\frac{1}{k_\perp^2+\omega(\lambda=1)}|_0^{K^2} + \frac{1}{4\pi p^+}(2x)\left\{\frac{\omega(\lambda=1)}{k_\perp^2+\omega(\lambda=1)} + \ln(k_\perp^2+\omega(\lambda=1))\right\}|_0^{K^2}$$
$$-\frac{1}{4\pi p^+}(x+1)\left\{2(1-x)-\Lambda^2\right\}\frac{1}{k_\perp^2+\omega(\Lambda)}|_0^{K^2} - \frac{1}{4\pi p^+}(2x)\left\{\frac{\omega(\Lambda)}{k_\perp^2+\omega(\Lambda)} + \ln(k_\perp^2+\omega(\Lambda))\right\}|_0^{K^2}$$



PAULI VILLARS (PV) REGULARIZATION

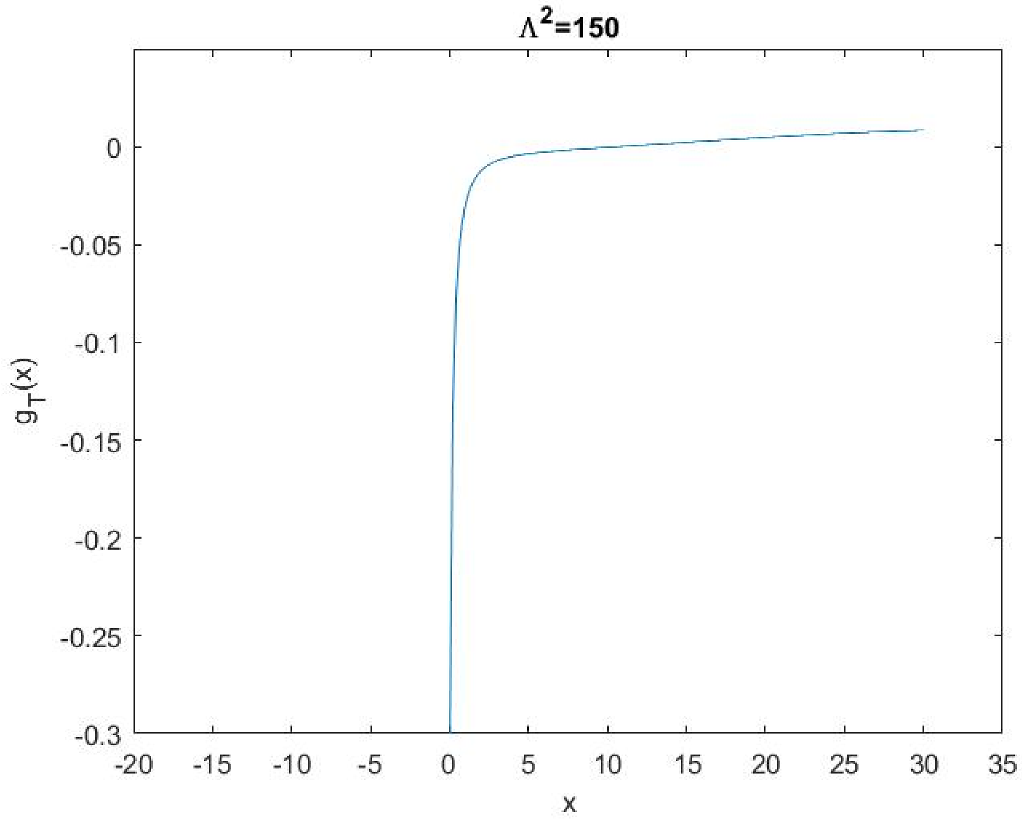
$$2g_T(x) = \frac{1}{4\pi p^+}(x+1)(1-2x)\frac{1}{k_\perp^2+\omega(\lambda=1)}\Big|_0^{K^2} + \frac{1}{4\pi p^+}(2x)\left\{\frac{\omega(\lambda=1)}{k_\perp^2+\omega(\lambda=1)} + \ln(k_\perp^2+\omega(\lambda=1))\right\}\Big|_0^{K^2}$$

$$-\frac{1}{4\pi p^+}(x+1)\left\{2(1-x)-\Lambda^2\right\}\frac{1}{k_\perp^2+\omega(\Lambda)}\Big|_0^{K^2} - \frac{1}{4\pi p^+}(2x)\left\{\frac{\omega(\Lambda)}{k_\perp^2+\omega(\Lambda)} + \ln(k_\perp^2+\omega(\Lambda))\right\}\Big|_0^{K^2}$$



PAULI VILLARS (PV) REGULARIZATION

$$2g_T(x) = \frac{1}{4\pi p^+}(x+1)(1-2x)\frac{1}{k_\perp^2+\omega(\lambda=1)}\Big|_0^{K^2} + \frac{1}{4\pi p^+}(2x)\left\{\frac{\omega(\lambda=1)}{k_\perp^2+\omega(\lambda=1)} + \ln(k_\perp^2+\omega(\lambda=1))\right\}\Big|_0^{K^2}$$
$$-\frac{1}{4\pi p^+}(x+1)\left\{2(1-x)-\Lambda^2\right\}\frac{1}{k_\perp^2+\omega(\Lambda)}\Big|_0^{K^2} - \frac{1}{4\pi p^+}(2x)\left\{\frac{\omega(\Lambda)}{k_\perp^2+\omega(\Lambda)} + \ln(k_\perp^2+\omega(\Lambda))\right\}\Big|_0^{K^2}$$



Sum rules involving twist-3 distributions are violated if we do not take the $\delta(x)$ into account.

Lorentz invariance of twist-3 GPDs

If there is a δ(x) and if it is not included:

$$\int_{-1}^{1} dx G_i(x,\xi,\Delta) = 0 \Longrightarrow \lim_{\epsilon\to 0}\int_{-1}^{\epsilon} dx G_i(x,\xi=0,\Delta) + \lim_{\epsilon\to 0}\int_{\epsilon}^{1} dx G_i(x,\xi=0,\Delta) \neq 0,$$

$$\int_{-1}^{1} dx \widetilde{G}_i(x,\xi,\Delta) = 0 \Longrightarrow \lim_{\epsilon\to 0}\int_{-1}^{\epsilon} dx \widetilde{G}_i(x,\xi=0,\Delta) + \lim_{\epsilon\to 0}\int_{\epsilon}^{1} dx \widetilde{G}_i(x,\xi=0,\Delta) \neq 0$$

$$\int_{-1}^{1} dx g_1(x) = \int_{-1}^{1} dx g_T(x)$$

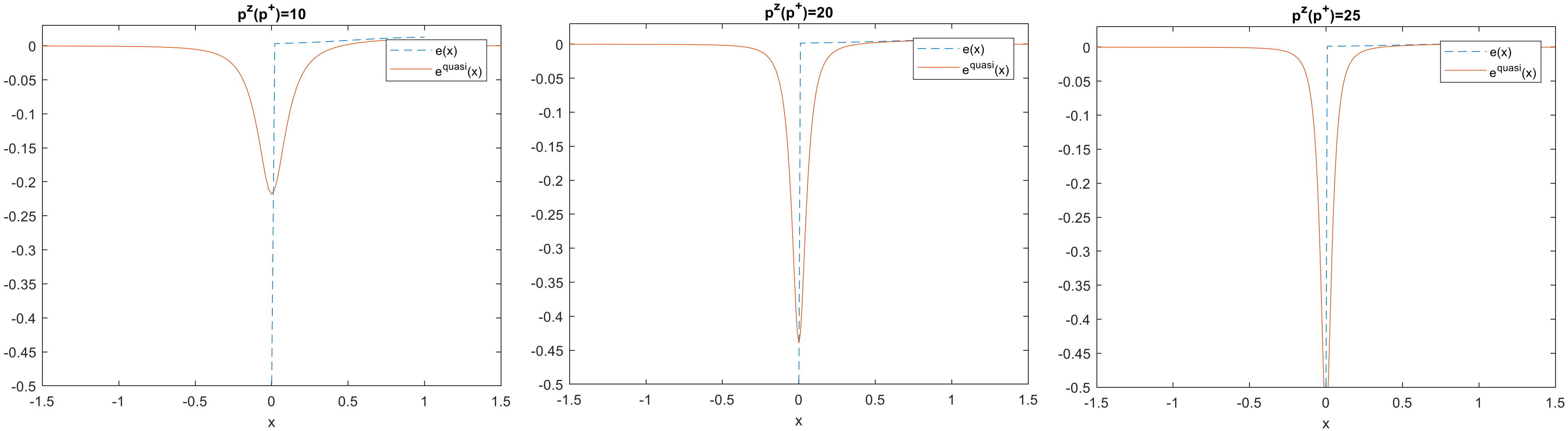
$$\int_{-1}^{1} dx h_1(x) = \int_{-1}^{1} dx h_L(x)$$

$$\int_{-1}^{1} dx e(x) = \frac{1}{2M}\langle p|\overline{\psi}(0)\psi(0)|p\rangle = \frac{d}{dm}M \Rightarrow \lim_{\epsilon\to 0}\int_{-1}^{\epsilon} dx e(x) + \lim_{\epsilon\to 0}\int_{\epsilon}^{1} dx e(x) \neq \frac{d}{dm}M$$

If one tries to confirm these sum rules experimentally by drawing conclusions from the smooth behavior <u>near x=0</u> about the behavior <u>at x=0</u> they would claim that the sum rules are violated.

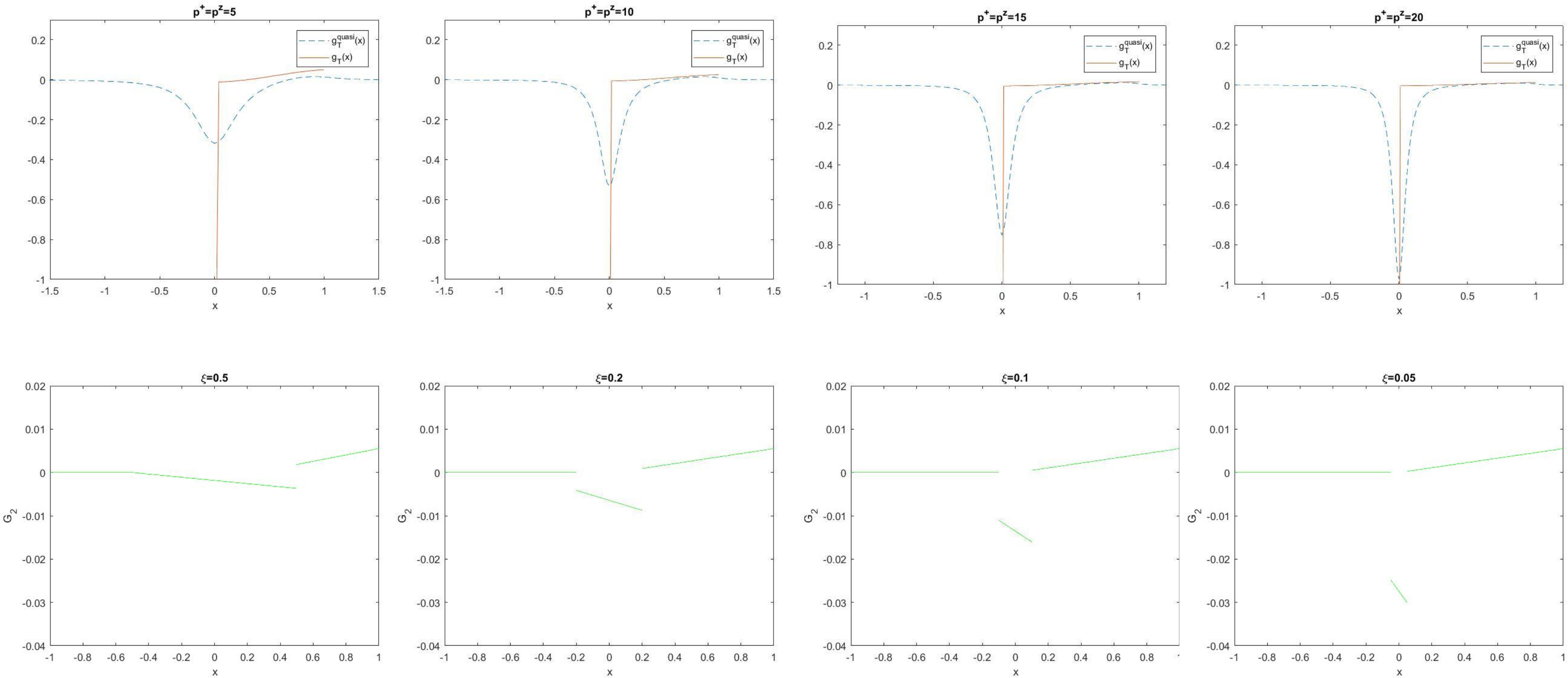
CONCLUSIONS

- *There is a $\delta(x)$ in twist -3 PDFs.*



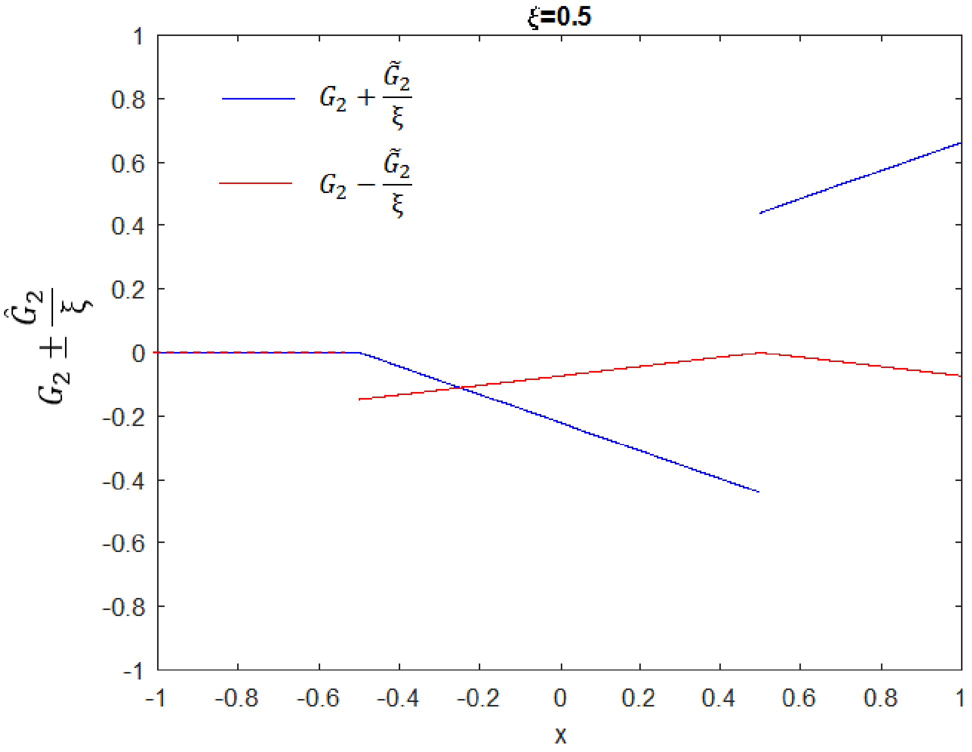
CONCLUSIONS

➢ *Corresponding twist -3 GPDs have discontinuities at* $x = \pm\xi$.



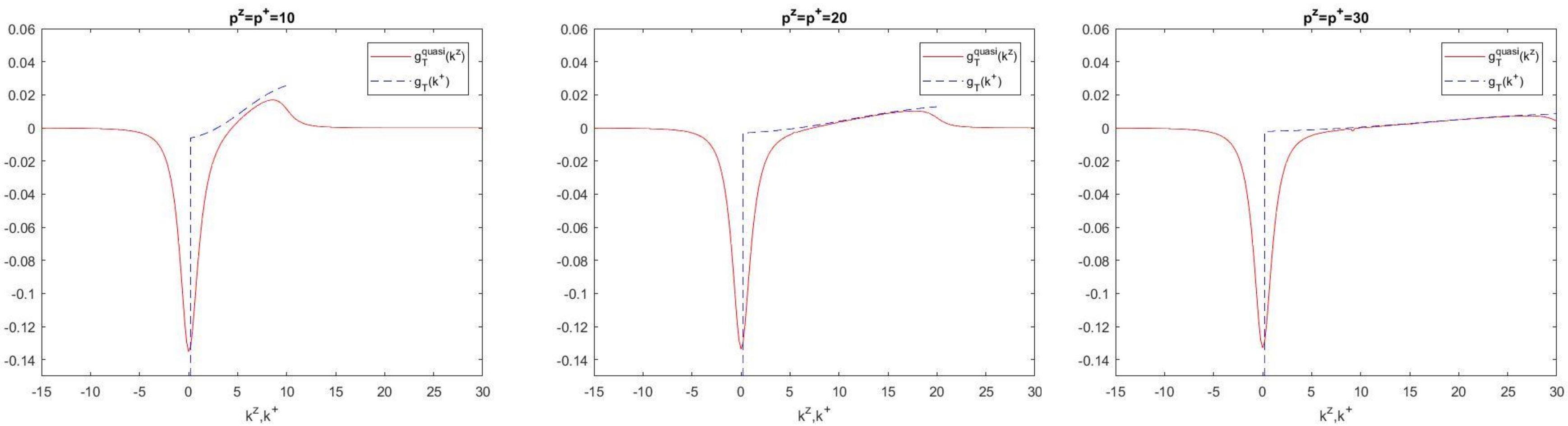
CONCLUSIONS

- *No issues with DVCS factorization for twist – 3.*



CONCLUSIONS

- $\delta(x)$ *functions are related to the wave function components that do not scale in when the nucleon is boosted to the infinite momentum frame.*



CONCLUSIONS

- *Not taking the $\delta(x)$ functions into account imply apparent violations of sum rules.*

Lorentz invariance of twist-3 GPDs

If there is a δ(x) and if it is not included:

$$\int_{-1}^{1} dx G_i(x,\xi,\Delta) = 0 \Longrightarrow \lim_{\epsilon\to 0}\int_{-1}^{\epsilon} dx G_i(x,\xi=0,\Delta) + \lim_{\epsilon\to 0}\int_{\epsilon}^{1} dx G_i(x,\xi=0,\Delta) \neq 0,$$

$$\int_{-1}^{1} dx \widetilde{G}_i(x,\xi,\Delta) = 0 \Longrightarrow \lim_{\epsilon\to 0}\int_{-1}^{\epsilon} dx \widetilde{G}_i(x,\xi=0,\Delta) + \lim_{\epsilon\to 0}\int_{\epsilon}^{1} dx \widetilde{G}_i(x,\xi=0,\Delta) \neq 0$$

OUTLOOK

- Calculation of twist-3 GPDs in models with dynamical chiral symmetry breaking.
  Current quark→ $\delta(x)$
  Dressed quark→ ?

- Decomposition of twist 3. $g_2(x) = g_2^{WW}(x) + g_2{}^m(x) + g_2{}^3(x)$

$$g_2^{WW}(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y).$$

Quark mass term potentially contains a $\delta(x)$ function

Genuine twist-3 term potentially contains a $\delta(x)$ function

Previously

$$h_L(x) = h_L^{WW}(x) + h_L^m(x) + h_L^3(x)$$

We remark that the above calculation indicates that the $\delta(x)$ term appears not only in h_L^m but also in h_L^3. Furthermore they do not cancel but add up to give rise to $-\delta(x)$ in $h_L(x, Q^2)$ itself.

Burkardt & Koike, Violation of Sum Rules for Twist-3 Parton Distributions in QCD, 2001

- Using the x^2 moments of genuine twist-3 GPDs we can map out the transverse force acting on a quark in a polarized nucleon. (Aslan, Burkardt, Schlegel)

OUTLOOK

.... Upon boosting the system to infinite momentum the partons would all become very far from $\eta = 0$, where η is the fraction of the particle's longitudinal momentum carried by the parton. Since all the vacuum activity takes place at $\eta = 0$, it seems very curious how these partons (at finite η) could "feel" what is going on at $\eta = 0$.

The right way to think about spontaneous breaking of chiral symmetry on the LC is that it somehow manifests itself through interactions between partons at finite η and $\eta = 0$ (the vacuum). The problem or puzzle with this is that matrixelements connecting states which are separated by a large distance in rapidity[1] are suppressed. So ho how could the valence quarks possibly feel what is going on at $\eta = 0$? .

Leonard Susskind, Matthias Burkardt

A Model of Mesons based on χSB in the Light-Front Frame (1994)

There is a momentum component in the nucleon state which does not scale as the nucleon is boosted to the infinite momentum frame.

Thank you for listening