Dynamic critical phenomena from real-time spectral functions on the lattice

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Based on
J. Berges, SS, D. Sexty, Nucl. Phys. B832 (2010) 228-240
L.v. Smekal, D. Smith, SS, D.Schweitzer, (work in progress)

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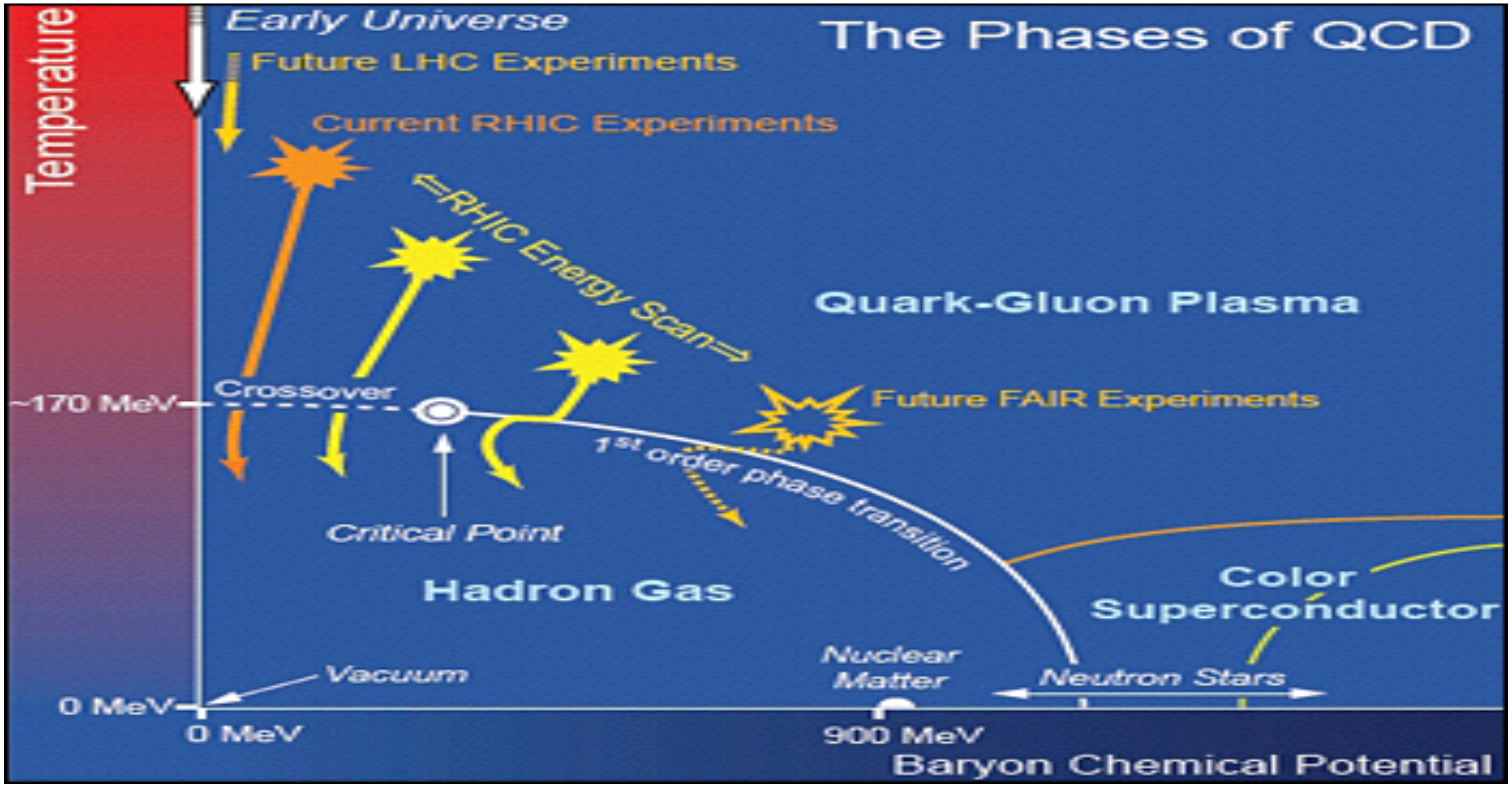
Outline

- Introduction & Motivation
- Computation of real-time spectral functions
- Critical dynamics & Spectral functions of scalar fields in (2+1)D and (3+1)D
- Summary & Conclusions

Motivation



Q: What are observable signatures of the critical point in heavy-ion experiments (RHIC BES, FAIR)?

Answer requires an understanding of the real-time dynamics near the critical point

Critical phenomena

Static critical phenomena

- Divergence of the correlation length ξ_s near the critical point of a second order phase transition

- Long distance properties near the critical point are insensitive to the microscopic physics

- Characterized in terms of critical scaling exponents

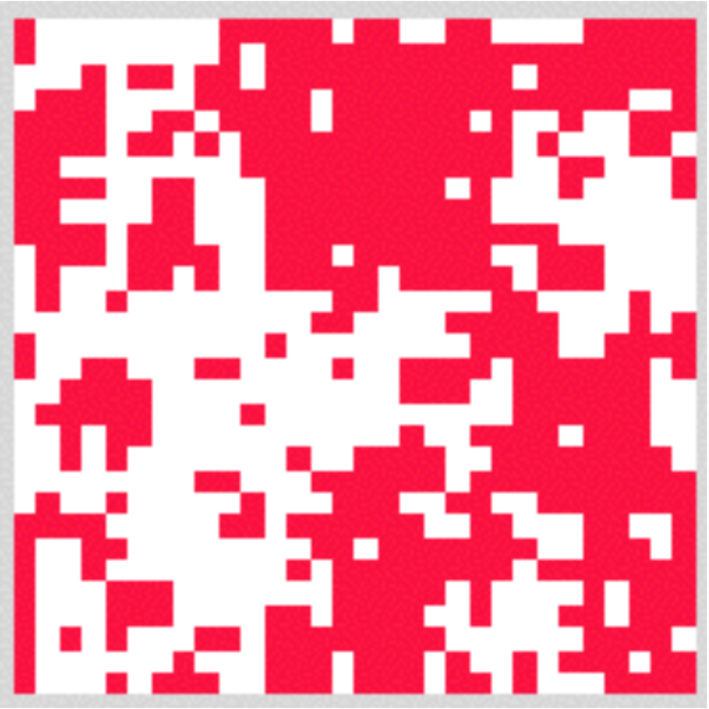
  $\alpha, \beta, \gamma, \delta, \nu, \eta$ $\qquad$ $\xi_s \sim |T-Tc|^{-\nu}$

- Universality quantities only depend on

  dimensionality, symmetry breaking pattern

Critical dynamics

Dynamic critical phenomena



- Dynamics near the critical point subject to critical slowing down *-> Divergence of the temporal correlation length* ξ_t

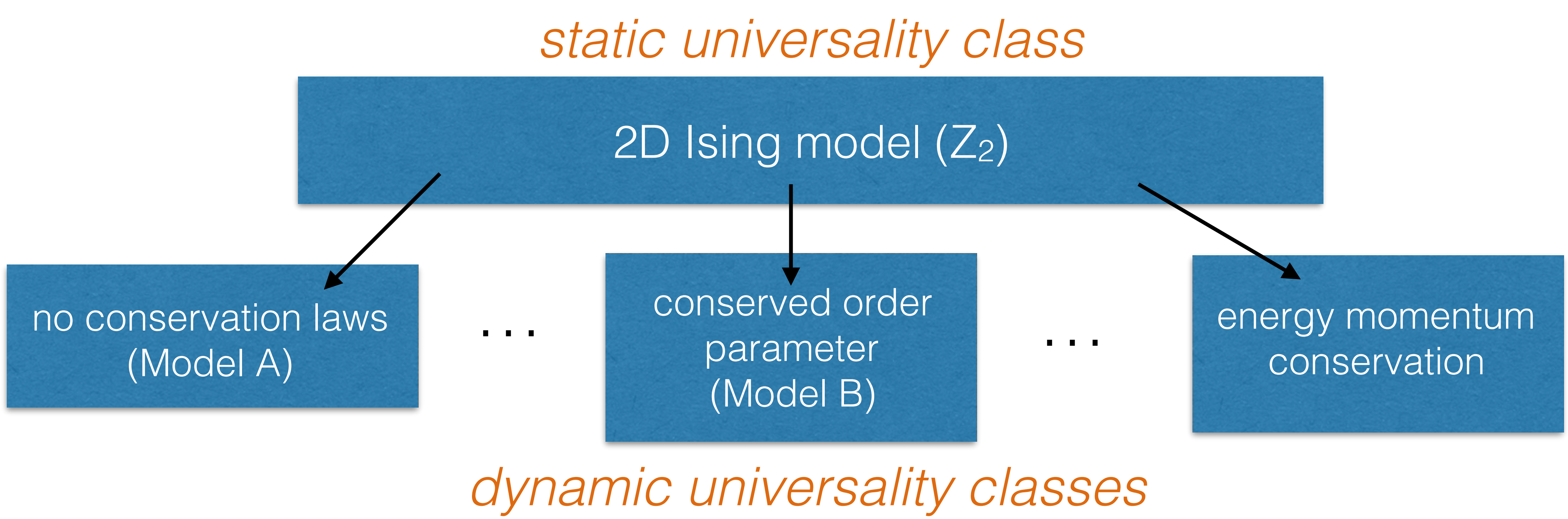
- Characterized in terms of dynamical critical exponent *z*

$$\xi_t \sim \xi^z \sim |T-Tc|^{-\nu z}$$

Critical dynamics

Dynamic critical phenomena

- Dynamical constraints (e.g conservational laws) affect the long time dynamics of the system
- Classification scheme for non-relativistic systems *(Halperin & Hohenberg '77)*



Critical dynamics

Dynamic critical phenomena

- Dynamical constraints (e.g conservational laws) affect the long time dynamics of the system
- C … s
  (I

How does a given relativistic field theory fit into this classification scheme?

What are the relevant degrees of freedom near the critical point?

no c…
(Model A)

(Model B)

…m
conservation

dynamic universality classes

First principle study of simplest possible example

Nucl. Phys. B832 (2010) 228-240 (arXiv:0912.3135 [hep-lat])

- Consider single component scalar field theory in 2+1 D

$$H = \int \mathrm{d}^2x \left(\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4!}\varphi^4 \right)$$

- Second order phase transition at $T_C>0$ for $m^2 <0$ with order parameter $\langle \phi(t,x) \rangle$

- Static universality class 2D Ising (Z_2)

->Static critical properties known exactly (Onsager solution)

Critical dynamics of relativistic scalar theory

- Effective degrees of freedom away from T_c are massive quasi-particles (with finite life-time)
- Spectral function $\rho(t-t',x-x',T) = i\langle[\phi(t,x),\phi(t',x')]\rangle_T$
- Mean-field approximation

$$\rho_0(\omega,p,T) = 2\pi i\ \mathrm{sgn}(\omega)\ \delta(\omega^2 - p^2 - M^2(T))$$

- Critical behavior $\rho(s^z\omega, sp, s^{1/\nu}T_r) = s^{-(2-\eta)}\rho(\omega,p,T_r)$

(mean field $\eta=0$ and $z=1$ for relativistic scalar)

- Classification in Halperin-Hohenberg schemes

(Model C) $z=2+\alpha/\nu$ —> $z=2$ (2D Ising)

Basic idea of the method

First principles calculation of the spectral function requires real-time simulation

- Generally not possible in the *quantum field theory*, since real-time *sign problem* (~e^{iS}) prevents use of importance sampling techniques

- However, the critical dynamics of a second order phase transition ($T_c>0$) is classical-statistical in nature

-> Quantum and classical theory are in the same (static and dynamic) universality class

- No sign problem in classical-statistical field theory.

-> Dynamic critical behavior can be studied using real-time classical lattice simulations

Quantum vs. classical-statistical dynamics

Quantum theory

$\rho(t-t',x-x',T) = i\langle[\phi(t,x),\phi(t',x')]\rangle$

$F(t-t',x-x',T) = 1/2 \langle\{\phi(t,x),\phi(t',x')\}\rangle$

Classical-statistical theory

$\rho_{cl}(t-t',x-x',T) = \langle\{\phi(t,x),\phi(t',x')\}_{PB}\rangle$

$F_{cl}(t-t',x-x',T) = \langle\phi(t,x),\phi(t',x')\rangle$

in equilibrium fluctuation dissipation relation implies
only one independent two-point function

$F(\omega,p) = (n_{BE}(\omega)+1/2)\, \rho(\omega,p)$
$\sim (T/\omega+O(w/T))\, \rho(\omega,p)$

$F(\omega,p) = T/\omega\, \rho(\omega,p)$

-> No difference for low momentum modes ($\omega/T << 1$)

Classical effective theory defined with cut-off scale Λ, such that ($\omega/T << 1$) for all modes will accurately reproduce quantum theory.

Integrating out ‘quantum modes’ above Λ leads to renormalization of model parameters, but universal critical dynamics is unaffected.

Calculation of spectral function in real-time

Computation in *classical-statistical field theory*

1) Generate ensemble of 2D thermal field configurations using standard importance sampling techniques

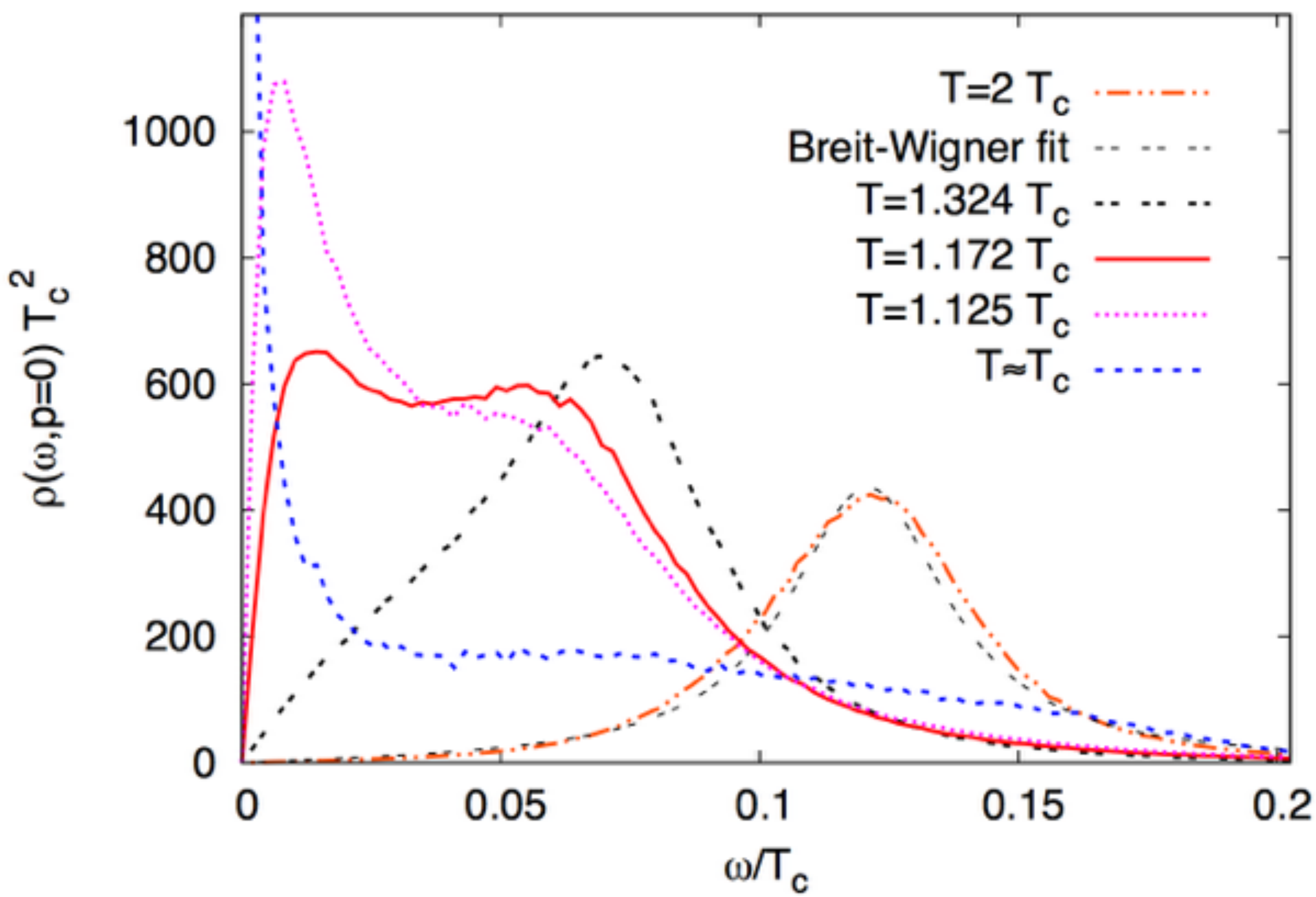
2) Solve classical-equations of motion in real-time

3) Compute spectral function from unequal time correlation function $\rho_{cl}(t-t',x-x',T) = \langle \{\phi(t,x),\phi(t',x')\}_{PB} \rangle$

classical KMS $\rho_{cl}(t-t',x-x',T) = -1/T\, \partial_{t-t'} \langle \phi(t,x),\phi(t',x') \rangle$

Critical dynamics of relativistic scalar theory

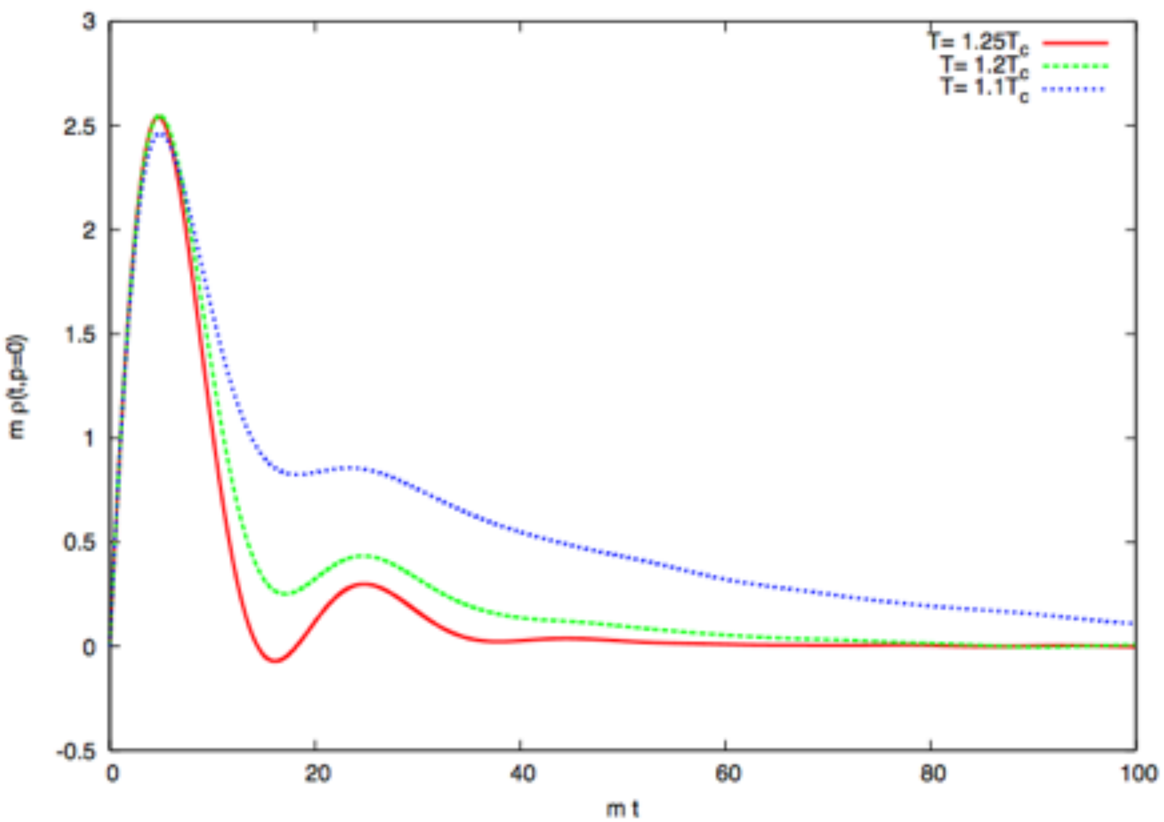
Spectral function $\rho(\omega, p=0, T_r)$ at finite temperature
from real-time lattice simulation



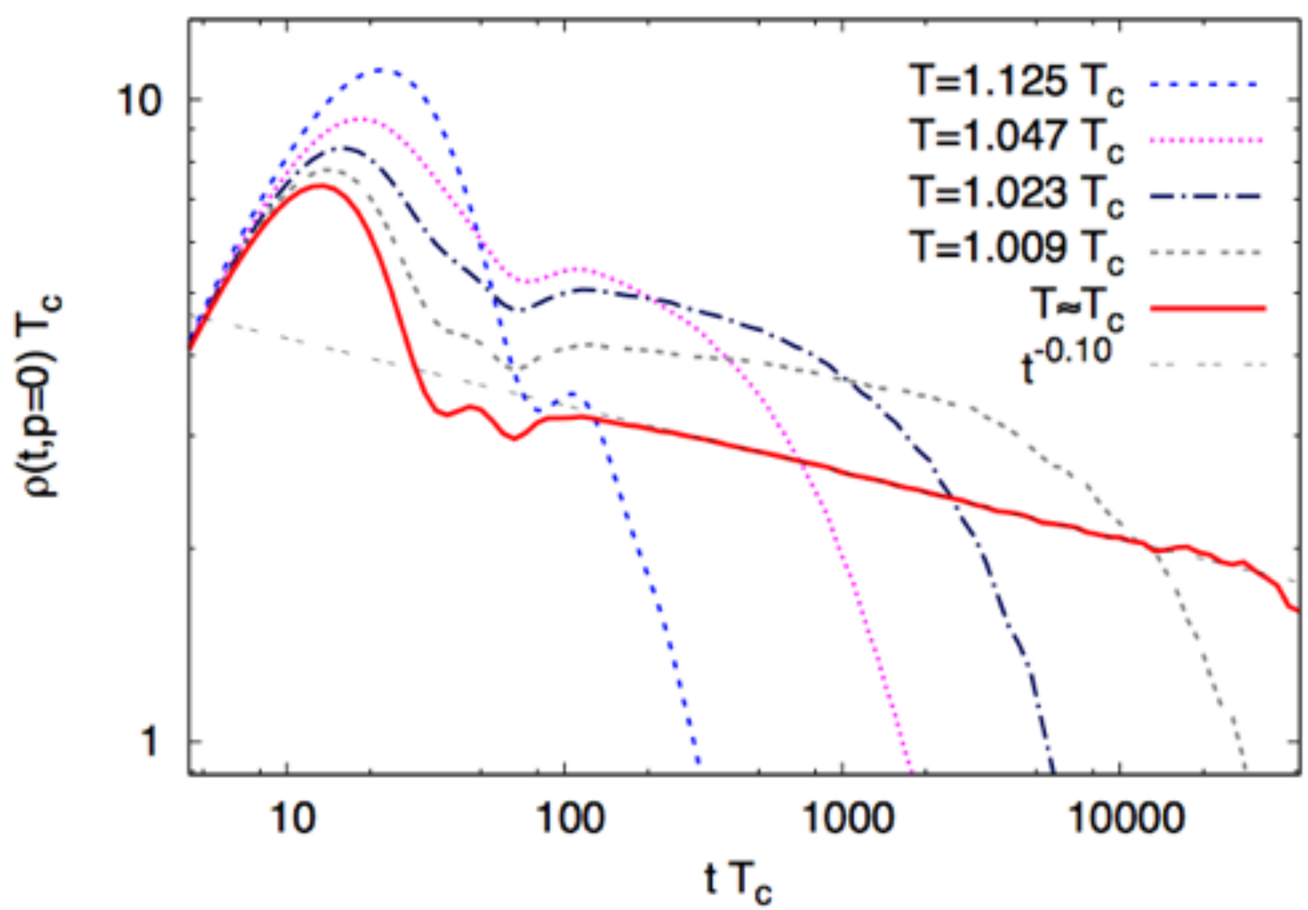
-> Change from relativistic quasi-particle to relaxation dynamics

Critical dynamics of relativistic scalar theory

Spectral function $\rho(t, p=0, T_r)$ at finite temperature
from real-time lattice simulation



High temperature

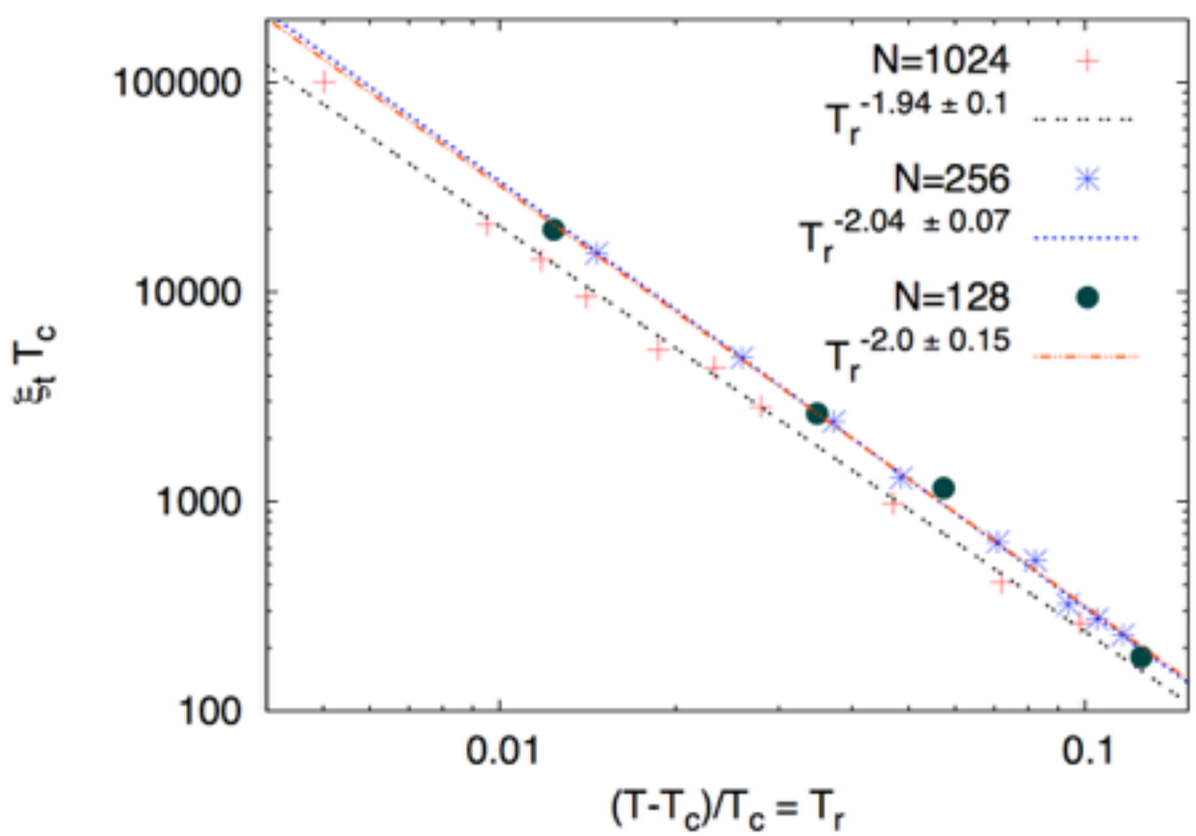


Critical region

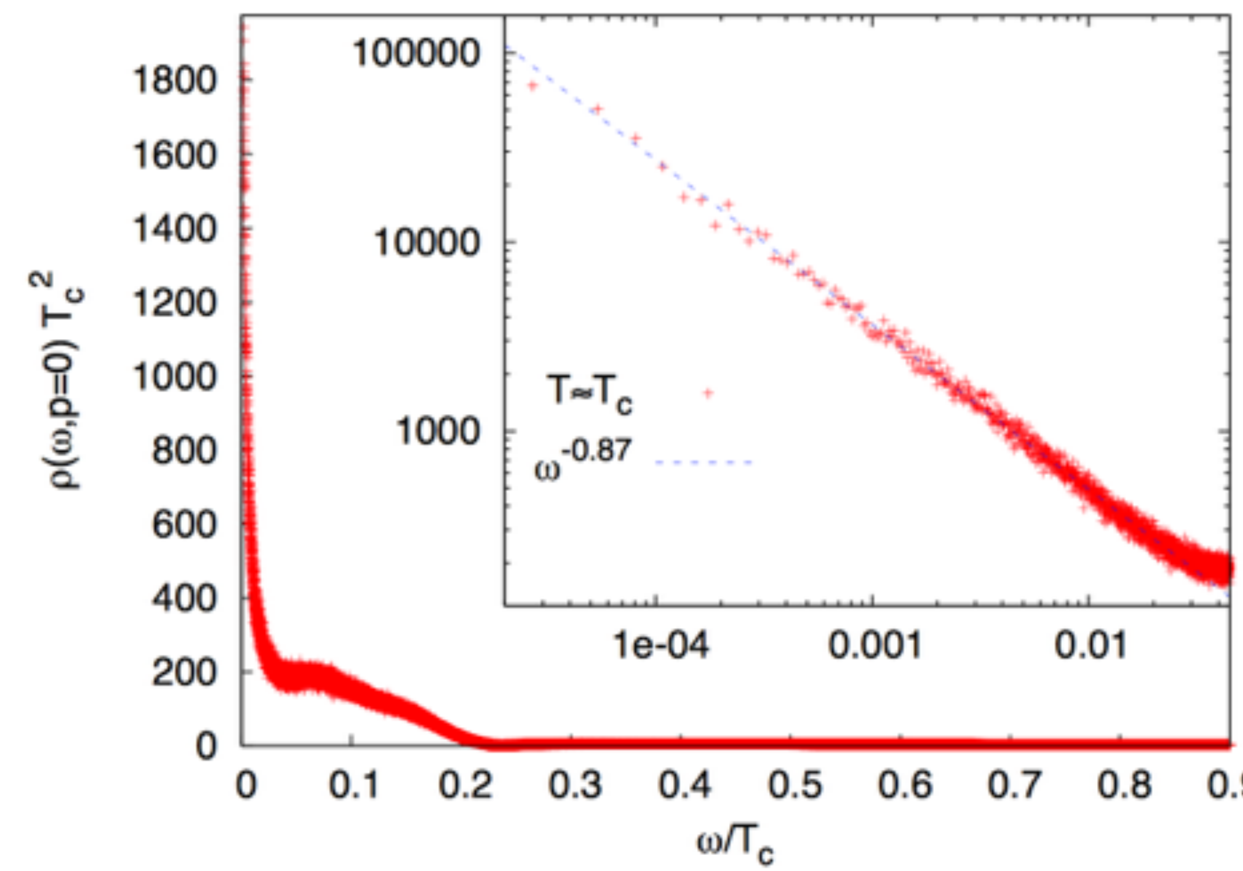
-> Change from relativistic quasi-particle to relaxation dynamics with a divergent (temporal) correlation length

Critical dynamics of relativistic scalar theory

Extraction of dynamic critical exponent *z*



Divergence of temporal correlation length



Critical spectral function

—> Different extractions yield z=2.05±0.15 in agreement with Model C

Critical dynamics of relativistic scalar theory

- Classical-statistical lattice field theory provides an efficient way to study dynamic critical behavior from first principles

- First application to (2+1)D relativistic scalar field theory.

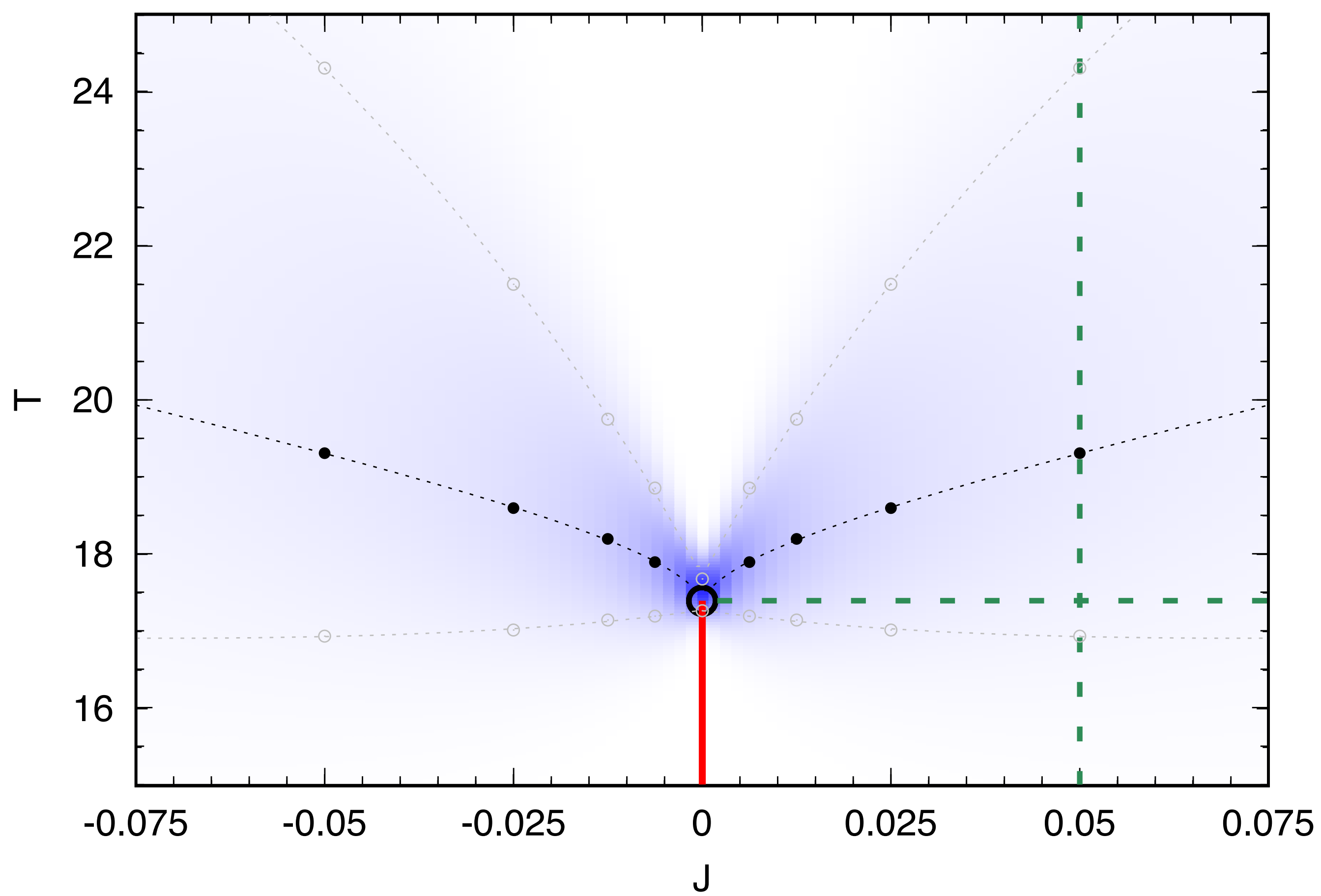
  -> Critical dynamics governed by diffusive degree of freedom which emerges in the vicinity of the critical point.

  -> Dynamic critical exponent of the relativistic theory is consistent with the classification scheme of Halperin & Hohenberg

- Interesting applications e.g. to effective models of low-energy QCD

Spectral functions in 3D O(4) model

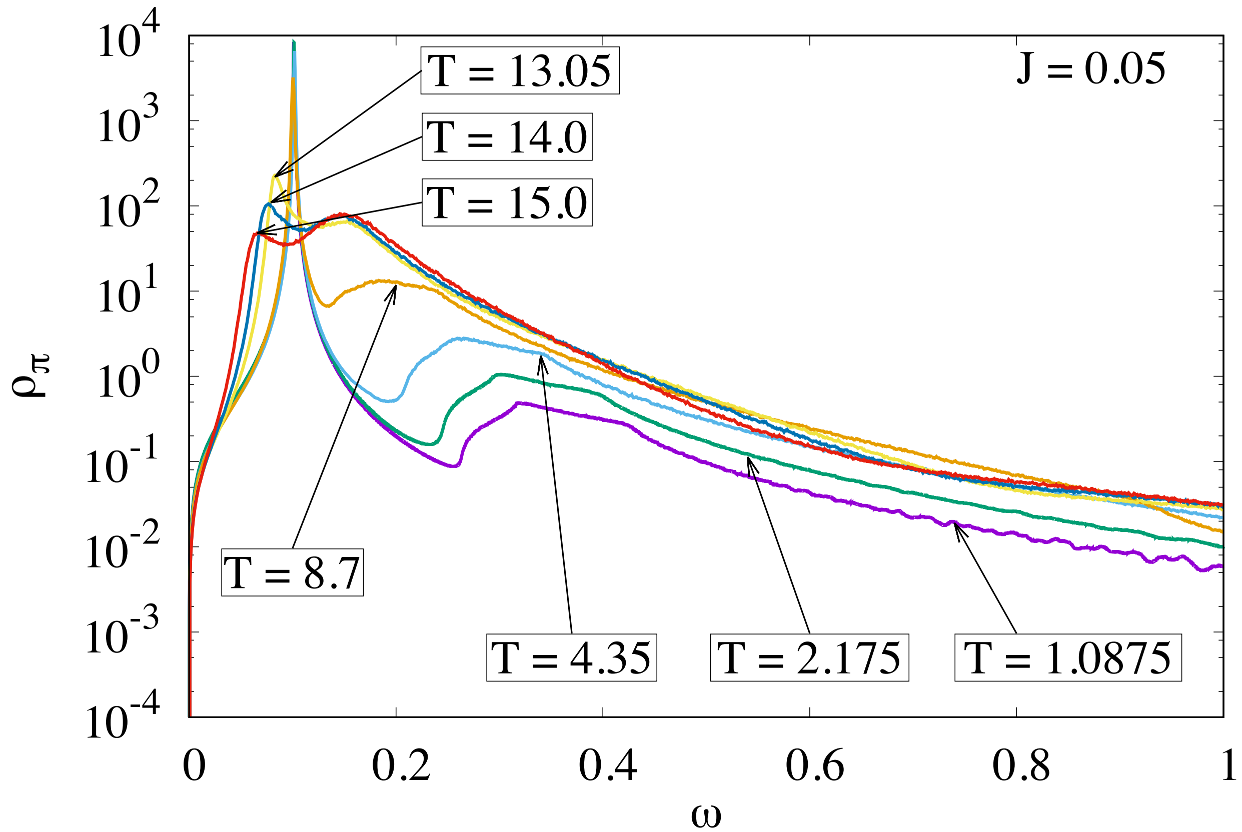
Distinction between Pion and Sigma modes difficult in finite volume
-> only possible with explicit symmetry breaking

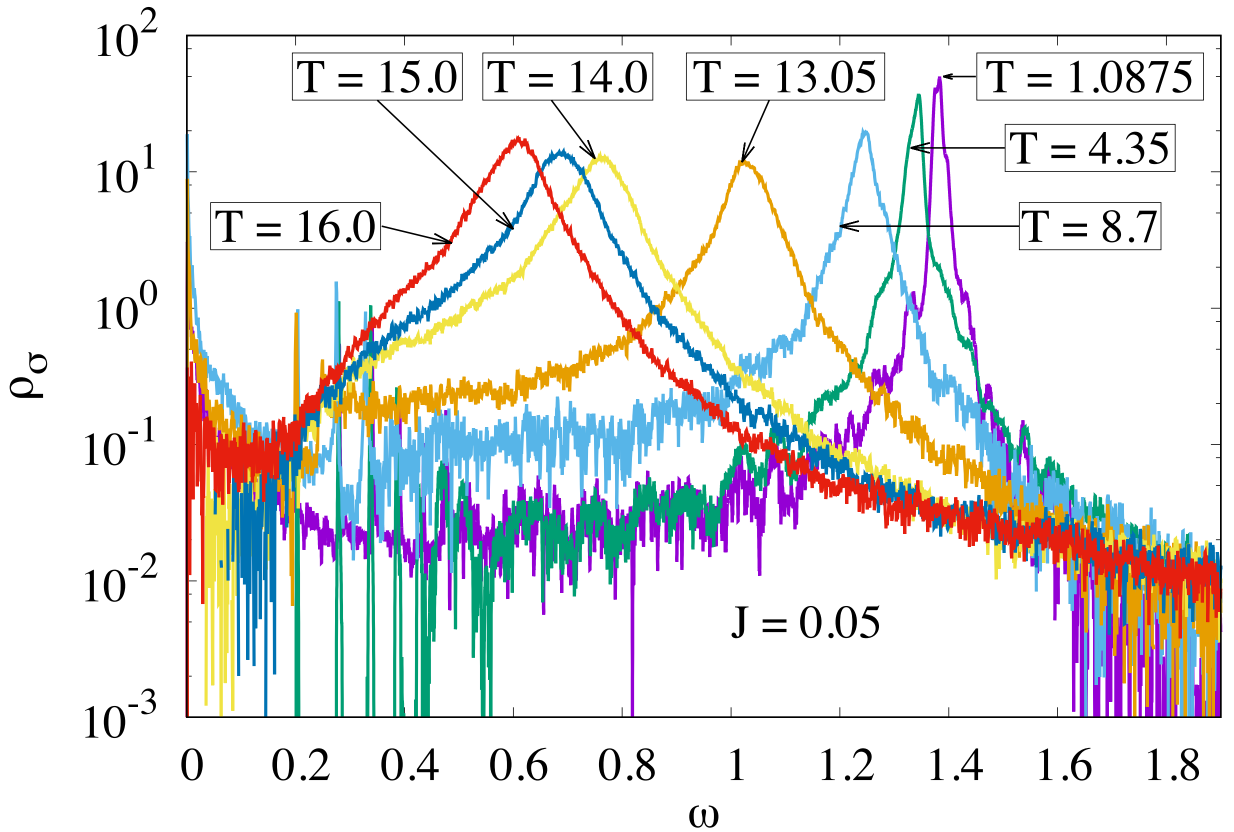


1) Scan temperature axis for pseudo-critical behavior at finite symmetry breaking

2) Scan critical behavior by tuning explicit symmetry breaking to zero at T=Tc

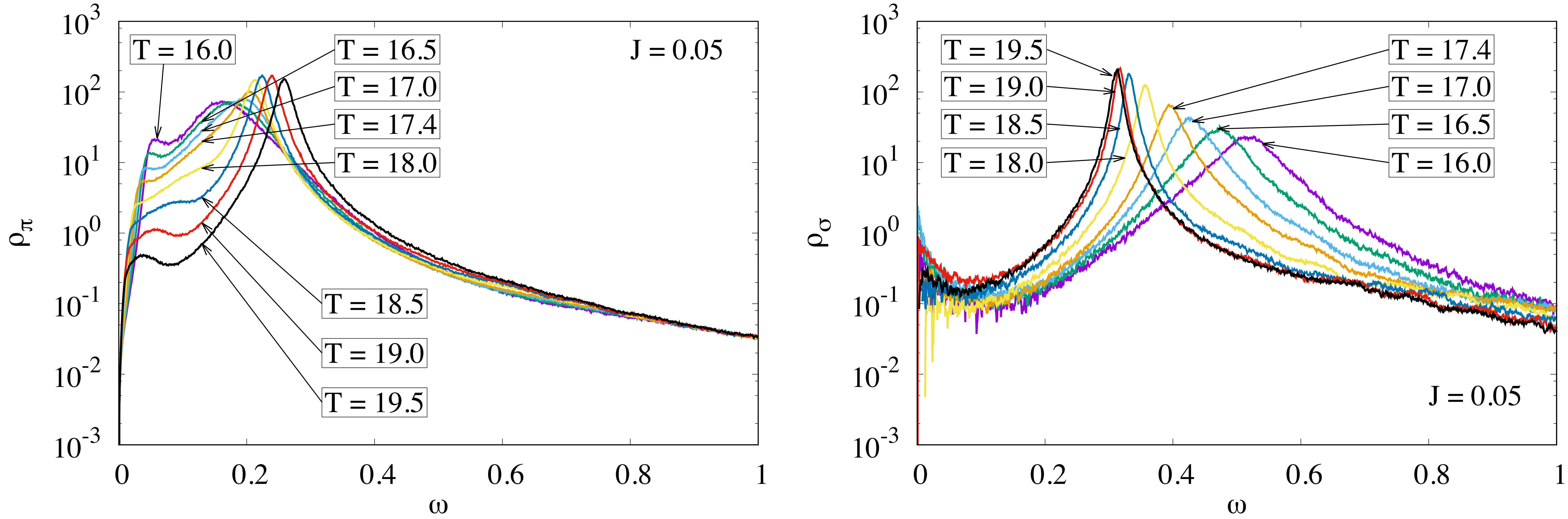
Spectral functions in 3D O(4) model





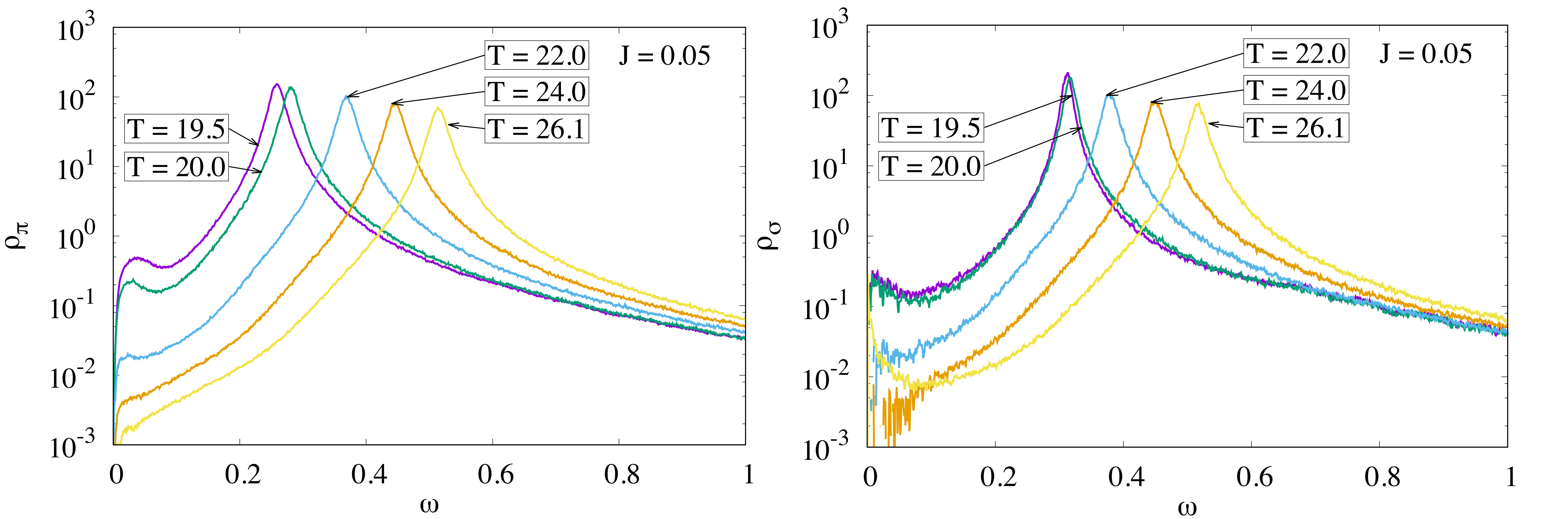
low temperature

Spectral functions in 3D O(4) model



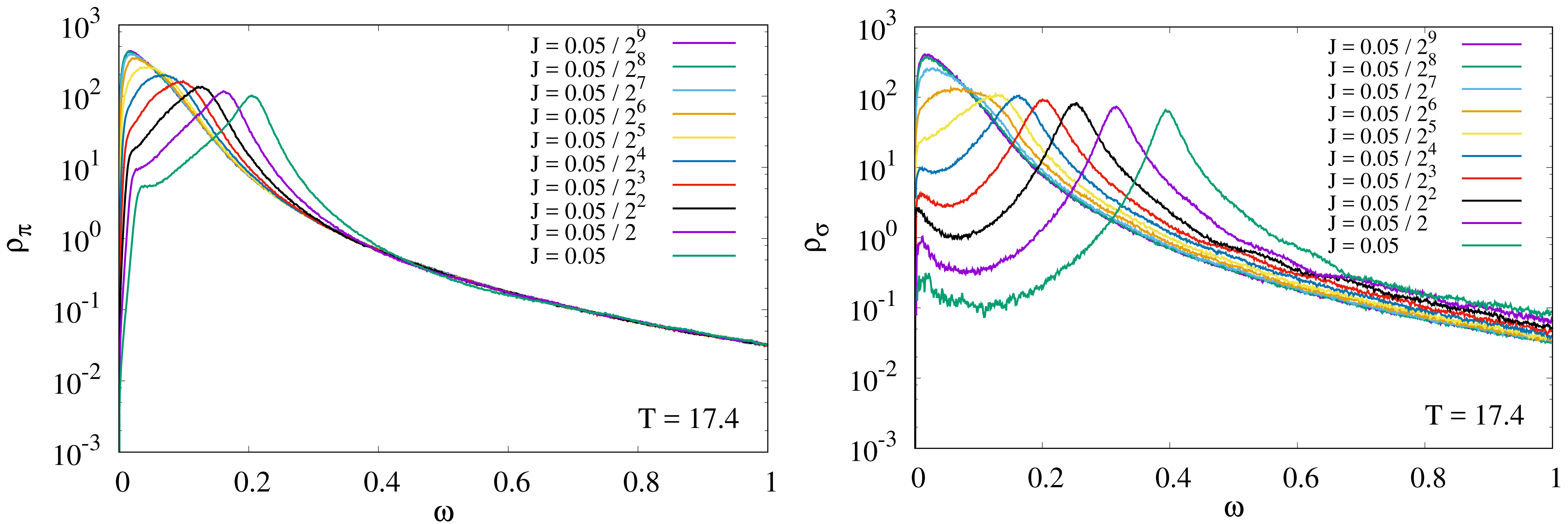
pseudo-critical region

Spectral functions in 3D O(4) model



high temperature

Spectral functions in 3D O(4) model



near critical region

Summary & Conclusion

- Exploited classical nature of low energy excitations to study real-time critical properties of relativistic scalar field theories (Z_2,O(4)) in 2D and 3D

  Several interesting extensions:

- Non-equilibrium phase transitions (work in progress)

- Explicit construction of classical-statistical low energy eff. theory by integrating out high energy modes (work in progress)