# <span id="page-0-0"></span>Many-body dynamics with quantum computers

## Alessandro Roggero



figure credit: IBM



Advances in MC Techniques for Many-Body Quantum Systems 22 Aug, 2018



# Goal: exclusive cross sections for  $\nu$  oscillation experiments





$$
P(\nu_{\alpha} \to \nu_{\alpha}) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E_{\nu}}\right)
$$

• need to use measured reaction products to constrain  $E_{\nu}$  of the event

DUNE, MiniBooNE, T2K, Minerνa, NOνA,. . .





# Inclusive cross section and the response function

• xsection completely determined by response function

$$
R_O(\omega) = \sum_{f} \left| \langle f|\hat{O}|0\rangle \right|^2 \delta(\omega - E_f + E_0)
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• excitation operator  $\hat{O}$  specifies the vertex



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Extremely challenging classically for strongly correlated quantum systems



Many body dynamics with Integral Transforms

$$
R_O(\omega) = \sum_f \left| \langle f|\hat{O}|0\rangle \right|^2 \delta(\omega - E_f + E_0)
$$

PROBLEM: need lots of detailed informations to compute this ab-initio

## A possible way out: integral transform techniques

**•** integrated quantities can be much easier to compute

$$
T(\sigma) = \int d\omega K(\sigma, \omega) R_O(\omega) = \sum_f |\langle f|\hat{O}|0\rangle|^2 K(\sigma, E_f - E_0)
$$
  
=  $\langle 0|\hat{O}^\dagger K(\sigma, \hat{H} - E_0) \hat{O}|0\rangle$ 

 $K(\sigma,\omega) = \omega^{\sigma} \quad \Rightarrow \quad$  energy weighted sum-rules  $K(\sigma, \omega) = e^{-\sigma \omega} \quad \Rightarrow \quad$  Laplace Transform (euclidean time/QMC)  $K(\sigma, \omega; \Gamma) = \frac{\Gamma}{\Gamma^2 + (\sigma - \omega)^2} \;\; \Rightarrow \;\;$  Lorentz Integral Transform (NCSM,CC)

$$
L(\sigma) = \int K(\sigma, \omega) R(\omega) d\omega = \int_0^\infty e^{-\sigma \omega} R(\omega) d\omega
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Bayesian methods are usually used to select the "best" resconstruction

$$
G(\sigma,\beta) = \int K(\sigma,\omega,\beta)R(\omega)d\omega = \int_0^\infty e^{-\frac{(\sigma-\omega)^2}{2\beta}} R(\omega)d\omega
$$

• We have now one more parameter:  $\beta$ .



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The transform  $G(\sigma)$  is a smoothened version of the original signal!

PROBLEM: computational cost scales exponentially with  $1/\beta$  !!!

# Additional challanges: the nuclear many-body problem

$$
H = \sum_{i} \frac{p^2}{2m} + \frac{1}{2} \sum_{i,j} V_{ij} + \frac{1}{6} \sum_{i,j,k} W_{ijk} + \cdots
$$

- **•** much easier to deal with than not the QCD lagragian
- being non-perturbative it is still extremely challenging
	- nuclear states live in huge Hilbert spaces:  $\,dim\left(\mathcal{H}\right)>4^A$

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# What is a Quantum computer?

JQI@Univ. of MD

Intel





Google



Alessandro Roggero (LANL) [Many-body dynamics with QC](#page-0-0) INT - 22 Aug, 2018 7/15

Microsoft?

 $\bullet$ 

## Stages of quantum computations

- prepare the initial state
- **•** perform unitary operations
- **•** measure the final state



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Can access ALL unitary matrices via a small set of universal gates

- integer factorization Schor (1994)
- database search Grover (1996)
- Hamiltonian simulation Lloyd (1996)
- linear equations Harrow et al. (2009)

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Quantum Adiabatic Algorithm Farhi et al. (2000,2001), McClean et al. (2016)

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H(\lambda) = (1 - \lambda)H_A + \lambda H_B
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PROBLEM:  $\bullet$  number of steps scales with gap  $\Delta$ :  $N_s = \frac{\lambda}{\delta \lambda} \approx \Delta^{-2}$ 

**•** gap could scale exponentially with system size

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## Spectral Combing Algorithm Kaplan, Klco, Roggero (2017)

IDEA: couple target to bath



- **•** bath prepared in a cold state
- unitary evolution could entangle the 2 systems such that entropy has maximum at  $|GS\rangle$ <sub>targ</sub>

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## PROBLEM

needs huge density of states

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N_{\rm bath}\gg N_{\rm targ}
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- $\bullet$  comb energies decreasing with  $t$
- energy transferred to the comb through avoided level crossings



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Kitaev (1996), Brassard et al. (2002), Svore et. al (2013), Weibe & Granade (2016),...

QPE is a general algorithm to estimate eigenvalues of a unitary operator

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U|\xi_k\rangle = \lambda_k|\xi_k\rangle, \lambda_k = e^{2\pi i \phi_k} \quad \Leftarrow \quad U = e^{-itH}
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- starting vector  $|\psi\rangle = \sum_k c_k |\xi_k\rangle$
- store time evolution  $|\psi(t)\rangle$  in auxiliary register of  $M$  qubits
- perform (Quantum) Fourier transform on the auxiliary register
- measures will return  $\lambda_n$  with probability  $P(\lambda_n) \approx |c_n|^2$

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BONUS: final state after measurement is  $|\psi_{fin}\rangle \approx \sum_k \delta(\lambda_k - \lambda_n) c_k |\xi_k\rangle$ 

Response functions as a probability distribution

$$
R_O(\omega) = \sum_f \left| \langle f|\hat{O}|0\rangle \right|^2 \delta(\omega - E_f + E_0)
$$

- positive definite quantity with finite integral  $\int_{-\infty}^{\infty} R_O(\omega) < \infty$
- properly normalized version  $\overline{R_O}(\omega)$  defines a probability density

 $\rightarrow$  scattering events with energy transfer  $\omega$  happen with probability  $\overline{R_O}(\omega)$ 

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## Strategy on a Quantum Computer (Roggero & Carlson (2018)

**e** generate final states  $|f\rangle$  with energy transfer  $\omega$  distributed as

$$
P(\omega) \propto \sum_{f} \left| \langle f | \hat{O} | 0 \rangle \right|^2 \delta_{\Delta} (\omega - E_f + E_0)
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• finite width  $\Delta$  can be made small at will with only modest resources

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- direct access to final states with given  $\omega \rightarrow$  exclusive information

## Additional ingredient:

**•** quantum circuit that prepares  $|E\rangle = \hat{O}(q)|0\rangle$  (Roggero & Carlson (2018))

$$
P(\nu) = \sum_{f} |\langle f|E \rangle|^2 \, \delta_W \, (\nu - E_f + E_0)
$$

- **•** finite width approximation of  $R(q, \omega)$
- need only  $W \sim \log_2 \left( 1 / \Delta \omega \right)$  ancillae
- evolution time  $t \sim Poly(\Omega)/\Delta\omega$



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**• quantum circuit that prepares**  $|E\rangle = \hat{O}(q)|0\rangle$  (Roggero & Carlson (2018))

By performing quantum phase estimation with  $W$  ancilla qubits we will measure frequency  $\nu$  with probability:

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## We need around  $\sim 10^4$  samples to get within  $1\%$  error

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$$
|out\rangle_{\nu} \sim \sum_{f} \langle f|\hat{O}(q)|0\rangle|f\rangle \quad \text{ with } E_f - E_0 = \nu \pm \Delta \omega
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we can then measure eg. 1- and 2-particle momentum distributions

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- pionless EFT on a  $10^3$  lattice of size  $20$  fm  $[a=2.0$  fm]
- 10x faster gates and negligible error correction cost (very optimistic)
- want  $R(q,\omega)$  with 20 MeV energy resolution

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we need a quantum device with  $\approx 4000$  qubits (current record is 72)





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classical  $\gtrsim 10^{81}$  PB and an exascale machine for  $\approx \frac{1}{4}$  $\frac{1}{4}$  age of universe

# Summary

- accurate input from nuclear physics is critical to extract reliable informations from current and planned neutrino experiments
- **•** current ab-initio techniques are getting better especially for ground state properties and inclusive scattering cross sections
	- still not enough, need new ideas: quantum computing?
- QC is an emerging technology with the potential of revolutionarize the way many-body theory is done
- we already know how to simulate efficiently the time-evolution of non relativistic systems and progress on field theories is on the way
- more work has to be done to make all this viable in the near term

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### Collaborators:

- N.Klco, D.Kaplan (INT)
- J.Carlson (LANL)



# Application to the 1D Ising model in a transverse field

## The Spectral Combing algorithm

- initialize system in  $|\psi\rangle \otimes |\downarrow\downarrow \cdots\rangle$ 
	- **1** propagate state from  $t = 0$  to  $t = t_f$ using full Hamiltonian  $H = H_{\text{targ}} + H_{\text{comb}} + H_{\text{int}}$
	- <sup>2</sup> if more iteration needed perform a measurement of z-projection of spins in the comb otherwise exit
	- <sup>3</sup> return spins in the comb to their ground-state and repeat



$$
H_{\text{targ}} = -h \sum_{i}^{N_{\text{targ}}} \sigma_i^x - \sum_{i}^{N_{\text{targ}}} \sigma_i^z \sigma_{i+1}^z
$$
  
•  $N_{\text{targ}} = 3$ ,  $N_{\text{comb}} = 3$  and  $h = 2.0$   
 $N_{\text{comb}} = 3$  sufficient for  $N_{\text{targ}} = 3, 4, 5$   
and variety of h across phase transition

# Non-unitary operators on a quantum computer

#### Measurement based non-unitary gates with ancilla

Gingrich & Williams (2004), Terashima & Ueda (2005)



- **•** entangle system with ancilla
- measure ancilla
- if ancilla is  $|0\rangle$  system left in

 $|\psi\rangle \propto \hat{N}|\phi\rangle$ 

• probability of success 
$$
P(|0\rangle) \leq 1
$$

For our purpose we can very easily prepare in this way the wanted state

$$
|\Phi_O\rangle \propto \hat O |\psi_0\rangle + O(\delta)
$$

paying the price that  $P(|0\rangle) = O(\delta)$ .

One can raise  $P(|0\rangle) \approx 1$  deterministically! Roggero & Carlson (2018)