Neutrino Mean Free Path in neutron matter from QMC equation of state

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Advances in Monte Carlo Techniques for Many-Body Quantum Systems

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- For neutron stars physics and, in part for supernova explosions, it is possible to approximate baryonic matter with pure neutron matter. The presence of magnetic fields might suggest that spin polarization could play a role.
- It is possible to use ab initio calculations for ground state properties, while for excited states in matter we still need to use mean field approximation.
- This work follows a previous work focused on asymmetry in the isospin channel.

Some references...

- Tamm-Dancoff response function and ν mean free path
  Cowell and Pandhariphande, *Phys. Rev. C* **70**, 035801;
  A. Lovato, O. Benhar, S. Gandolfi, C. Losa, *Phys. Rev. C 70*, 025804 (2014)
- TDLSDA for many electron systems:
  E. Lipparini and L. Serra, *Phys. rev. B* **57** R6830(R) (1998)
- This work:
  E. Lipparini and F. Pederiva, *Phys. Rev. C* **88**, *024318 (2013);* **94**, *024323 (2016)*
  E. Lipparini and F. Pederiva,

Overview

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Equation of State (EoS)
for Pure Neutron Matter (PNM) and
Spin Polarized Pure Neutron Matter (SPPNM)

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Energy-density functional

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Neutron matter

Neutron matter can be modeled as a periodic system of N neutrons interacting by a Hamiltonian of the form:

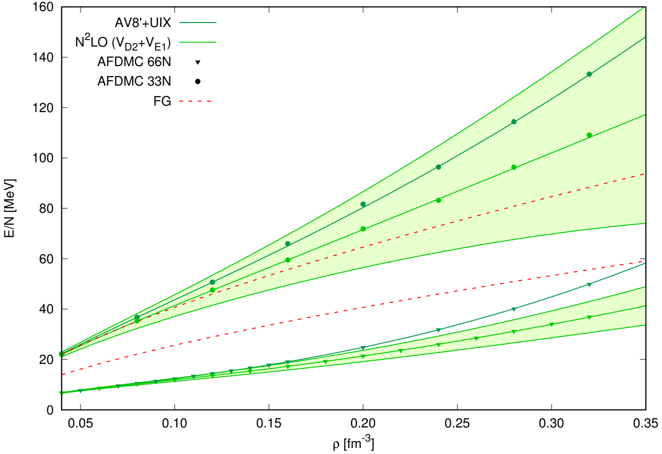
$$H = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \cdots \ .$$

In our calculations we used two kinds of potentials:

- phenomenological AV8'+UIX.
- chiral EFT N2LO local (D2,E1 and with R_0=R3N=1.0 fm) [Lynn et al. PRL 116, 062501 (2016)].

EoS computed by Auxiliary Field Diffusion Monte Carlo (AFDMC).

Neutron matter with AV8'+ UIX and N2LO



Uncertainties on χ-EFT (green bands) have been computed following Epelbaum et al. Eur. Phys. J. A, 51, 53 (2015).

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Energy density functional

The interaction part of the EDF is assumed to be of the form:

$$\epsilon_V(\rho,\xi) = \epsilon_0(\rho) + \xi^2 \left[\epsilon_1(\rho) - \epsilon_0(\rho)\right] \ ,$$

where:

$$\epsilon_q(\rho) = \epsilon_q^0 + a_q(\rho - \rho_0) + b_q(\rho - \rho_0)^2 + c_q(\rho - \rho_0)^3$$

where q=0,1 (spin polarization). The saturation density is *assumed* to be $\rho_0 = 0.16$ fm^{-3}.

This parametrization reproduces very well the AFDMC calculations in a wide range of density ρ (from $\rho_0/2$ to $3\rho_0$) and for both $\xi = 0, 1$.

General density excitations in nuclear matter

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We are interested in studying the density response of the system. For nucleons the response can be splitted in different channels, described by the following operators:

$$
\begin{aligned}
O_F &= \sum_i O_F(i) = \sum_i \tau^{\pm} e^{i\mathbf{q}\cdot\mathbf{r_i}} \quad \text{"Fermi"} \\
\mathbf{O}_{GT} &= g_A \sum_i \mathbf{O}(i) = \sum_i \sigma_i \tau_i^{\pm} e^{i\mathbf{q}\cdot\mathbf{r_i}} \quad \text{"Gamow-Teller"} \\
O_{NV} &= \sum_i O_{NV}(i) \quad \text{"Neutral-vector"} \\
&= \sum_i \left[-\sin^2\theta_W + \tfrac{1}{2}(1-2\sin^2\theta_W)\tau_i^z \right] e^{i\mathbf{q}\cdot\mathbf{r_i}} \\
\mathbf{O}_{NA} &= g_A \sum_i \mathbf{O}_{NA}(i) = g_A \sum_i \tfrac{1}{2}\tau_i^z \sigma_{\mathbf{i}} e^{i\mathbf{q}\cdot\mathbf{r_i}} \quad \text{"Neutral-axial-vector"}
\end{aligned}
$$

Weinberg-Salam model

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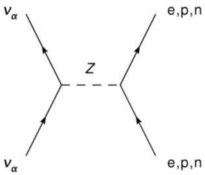
The relation between weak scattering processes and nuclear density response descends from the *Weinberg-Salam Lagrangian* coupling a nucleon of mass m with neutrinos through weak currents.

Weinberg-Salam model

The relation between weak scattering processes and nuclear density response descends from the *Weinberg-Salam Lagrangian* coupling a nucleon of mass m with neutrinos through weak currents. E.g., for a lepton weak neutral current the coupling Lagrangian density would be:

$$\mathcal{L}_W = \frac{G_W}{\sqrt{2}} \bar{\psi}_\nu(x)\gamma_\mu(1-\gamma_5)\psi_\nu(x)\frac{1}{2}\bar{\psi}_n(x)\gamma^\mu(1-C_A\gamma_5)\psi_n$$



ν scattering rate

The WS Lagrangian couples neutrinos to *density* and *spin density* fluctuations of neutrons.

In the **non-relativistic limit** the baryonic current can be approximated by:

$$\bar{\psi}_n(x)\gamma^\mu(1 - C_A\gamma_5)\psi_n \sim \psi_n^\dagger(x)\psi_n(x)\delta_0^\mu - C_A\psi_n^\dagger(x)\sigma_i\psi_n(x)\delta_i^\mu .$$

We have two contributions: density fluctuations, and spin-density fluctuations.
The scattering rate from a system of neutrons of a neutrino with 4-momentum $q^\mu \equiv (q^0, \vec{q})$ can be computed from the Fermi golden rule, averaging on the initial (neutron and/or proton) states and summing over all the final states.

ν scattering rate

The result gives the *neutrino scattering rate*. For a neutrino of incident energy E, the contribution to the scattering rate σ in a given channel can be writtten as:

$$\sigma = \frac{G^2}{2}\frac{1}{E}\int dq \int d\omega (E-\omega) q \left(1 + \frac{E^2 + (E-\omega)^2 - q^2}{2E(E-\omega)}\right) S(q,\omega),$$

where $S(q,\omega)$ is the *dynamical structure factor (DSF)* for the excitation operators describing the process. These in turn can be written as a combination of the DSF relative to density, and spin-density excitations.

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Time Dependent Local Density Approximation

We use the Time Dependent Local Spin Density Approximation (TDLSDA) approach to compute the response function and the DSF.

We have worked out the response function in the transverse and longitudinal spin channels.

Following the *Kohn-Sham* method, we introduce a Local Spin Density Approximation (LSDA) for the homogeneous neutron matter defining the energy density functional as:

$$E(\rho, \xi) = T_0(\rho, \xi) + \int \epsilon_V(\rho, \xi)\ \rho\ d\mathbf{r},$$

and we applied the Hohenberg-Kohn theorem which provides a *variational principle on the energy-density functional*.

Response functions

For the longitudinal channel we can write the expression for $\chi^{n\uparrow}$ and $\chi^{n\downarrow}$, in terms χ_0 (longitudinal response function of the free Fermi gas). Summing and subtracting $\chi^{n\uparrow}$ and $\chi^{n\downarrow}$ we obtain the density-density (χ_s) and vector-density/vector-density (χ_v) response functions for arbitrary spin polarization.

A similar derivation can be done for the transverse channel. The LSDA-KS equations in the transverse channel include an effective vector potential accounting for the equilibrium spin polarization and one gets the response function χ_t in terms of the transverse response of the free Fermi gas $\chi_t^0(q, \omega)$.

Excitation strengths and sum rules

- From the response function it is possible to determine the dynamic structure factor via the relation:

$$S^{s,v}(q,\omega) = -\frac{1}{\pi}\Im m[\chi^{s,v}]$$

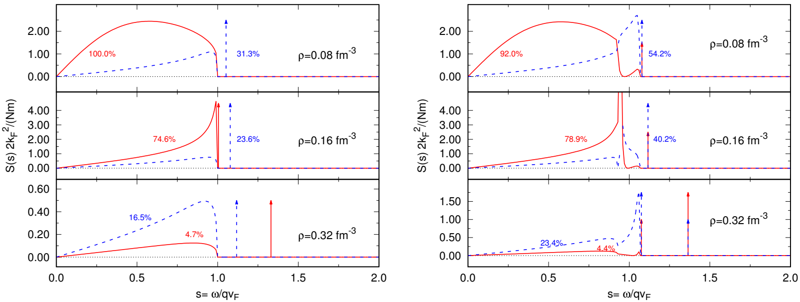
- From the DSF it is also possible to compute the energy weighted sum rules:

$$m_k^{s,v} = \int_0^\infty d\omega\, \omega^k S^{s,v}(q,\omega) = \sum_n \omega_{no}^k |\langle 0|F^{s,v}|n\rangle|^2$$

In particular the ratio m_{-1}/m_0 gives the *compressibility* of the system.

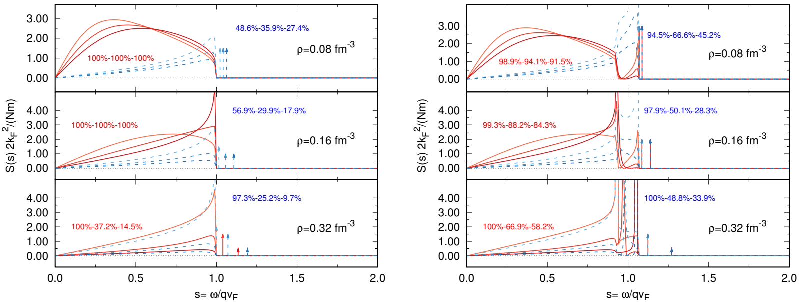
- The poles of $\chi(q,\omega)$ give the spectrum and the dispersion $\omega(q)$ of the collective excitations, for which we can also evaluate the strength.

Longitudinal response (AV8'+UIX)



TDLSDA Dynamical structure factor for **density** (solid lines) and **spin density** (dashed lines) in the longitudinal channel. Left panel: results for PNM. Right panel: results for $\xi = 0.2$. Arrows indicate the location of the collective excitations. The percentages represent the fraction of the total strength pertinent to the particle-hole excitations.

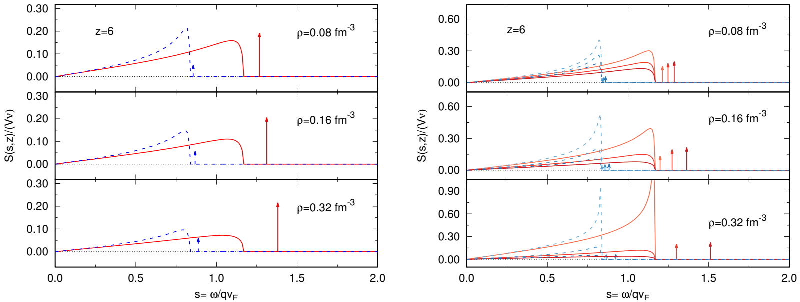
Longitudinal response (χ-EFT)



TDLSDA Dynamical structure factor for **density** (solid lines) and **spin density** (dashed lines) in the longitudinal channel. Left panel: results for PNM. Right panel: results for $\xi = 0.2$. Arrows indicate the location of the collective excitations. The percentages represent the fraction of the total strength pertinent to the particle-hole excitations.

Transverse response

$$\nu = mk_F/(2\pi^2)$$



Excitation strengths for $z = 3q/(2k_F\xi) = 6$. The full and dashed lines indicate the particle/hole and collective strengths in the $\Delta S_z = -1$ ($s > 0$ - red) and $\Delta S_z = +1$ ($s < 0$ - blue) channels.

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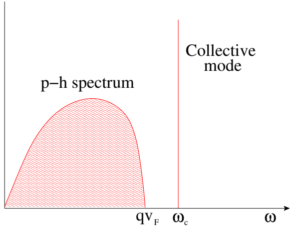
Neutrino mean free path

As previously discussed, the scattering rate of neutrinos can be obtained by computing the integral:

$$\sigma = \frac{G^2}{2}\frac{1}{E}\int dq \int d\omega (E-\omega) q \left(1 + \frac{E^2 + (E-\omega)^2 - q^2}{2E(E-\omega)}\right) S(q,\omega)$$

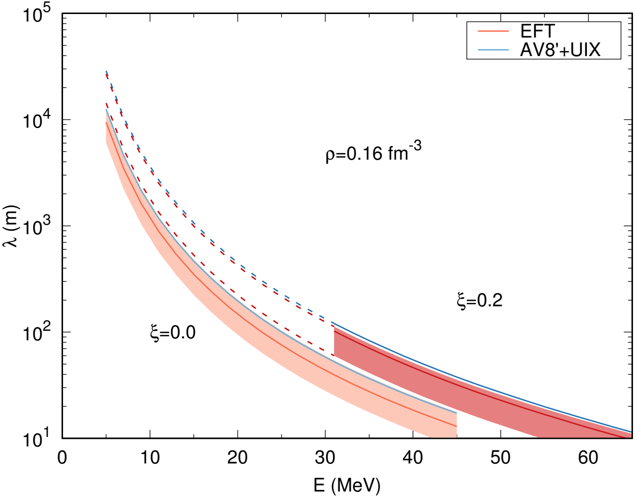
The neutrino mean free path λ is related to σ by the following relation:

$$\lambda = \frac{1}{\sigma\rho}$$



The integration has to be performed on the values of momentum kinematically accessible to neutrinos (kinematic limits).

Neutrino mean free path (total)



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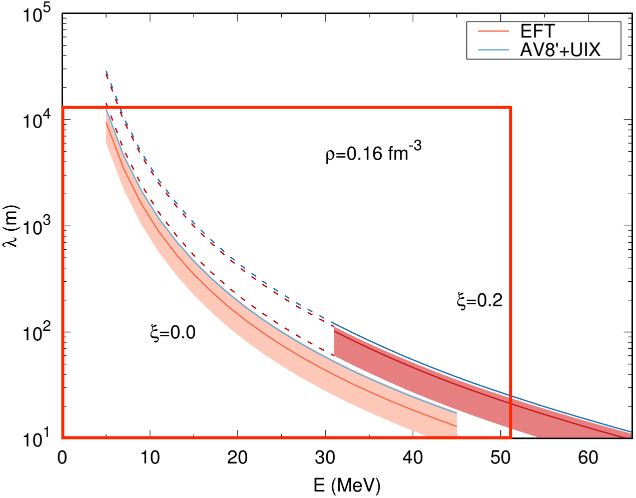
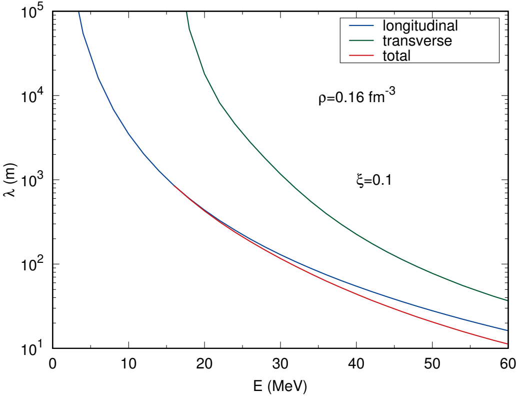
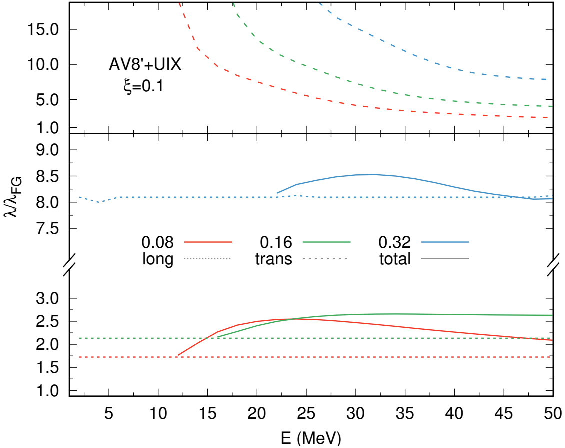


Neutrino mean free path (contribution)



The dominant channel is the longitudinal channel, if the system is slightly spin polarized. The graph shows the results for the potential AV8′+UIX.

Neutrino mean free path (contribution)



Channels contribution at different densities.

Neutrino mean free path (contribution)

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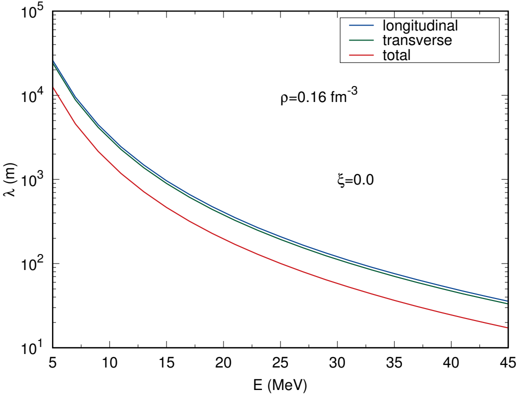

In case of PNM, we have a similar contribution coming from both channels.

Neutrino mean free path (total)

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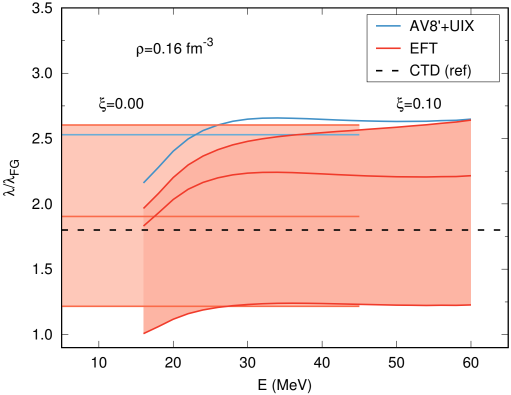

Ratio of the NMFP in an interacting neutron matter and in a free Fermi gas at density $\rho/\rho_0 = 1$.

Conclusions

- We computed the response function in the longitudinal and transverse channel in pure neutron matter, starting from accurate QMC calculations of (spin polarized) neutron matter.
- The time dependent local density approximation was successfully applied to estimate the response function of arbitrary spin polarized neutron matter.
- We computed the contribution of the longitudinal and transverse channels to the suppression of the neutrino mean free path in neutron matter. At the NS core conditions matter is essentially transparent, while relevant effects could be seen in the NS crust.