Entangled Pauli Principles and Parton Wavefunctions in Quantum Hall Fluids

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Jorge Dukelsky: CSIC - Madrid

PRB **88**, 65303 (2013), PRB **91**, 085115 (2015), arXiv:1803.00975, …

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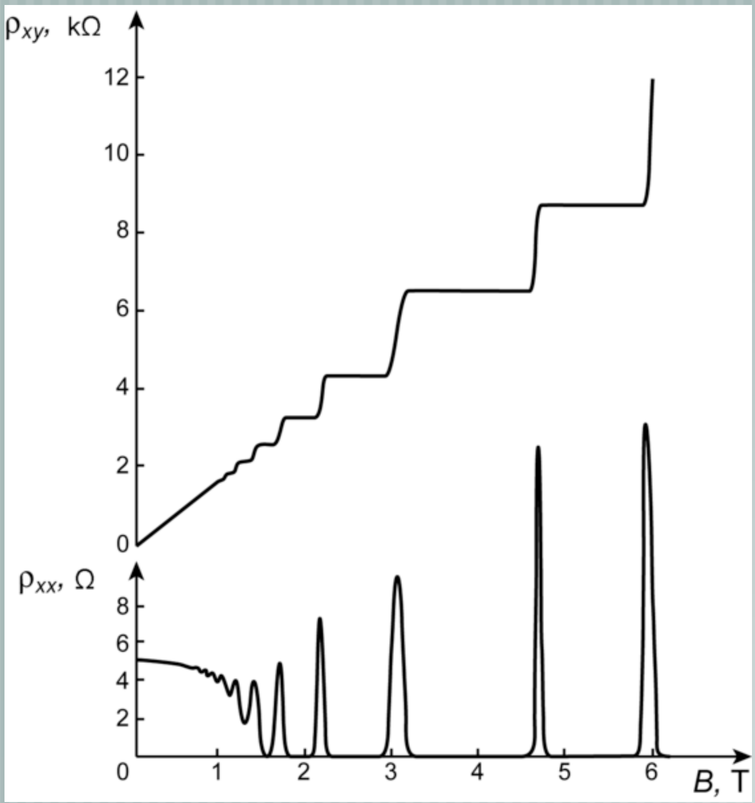
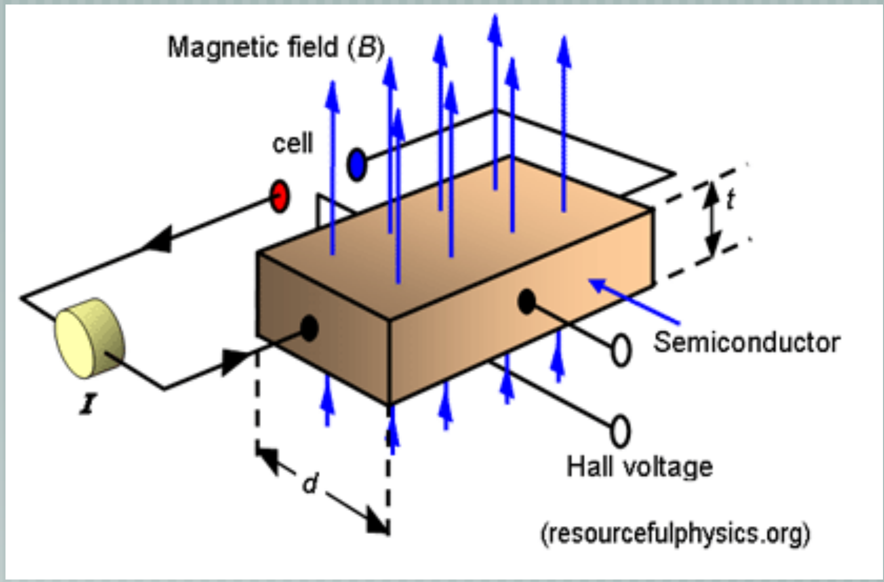
Quantum Hall Fluids 101

- Quantum Hall Liquids



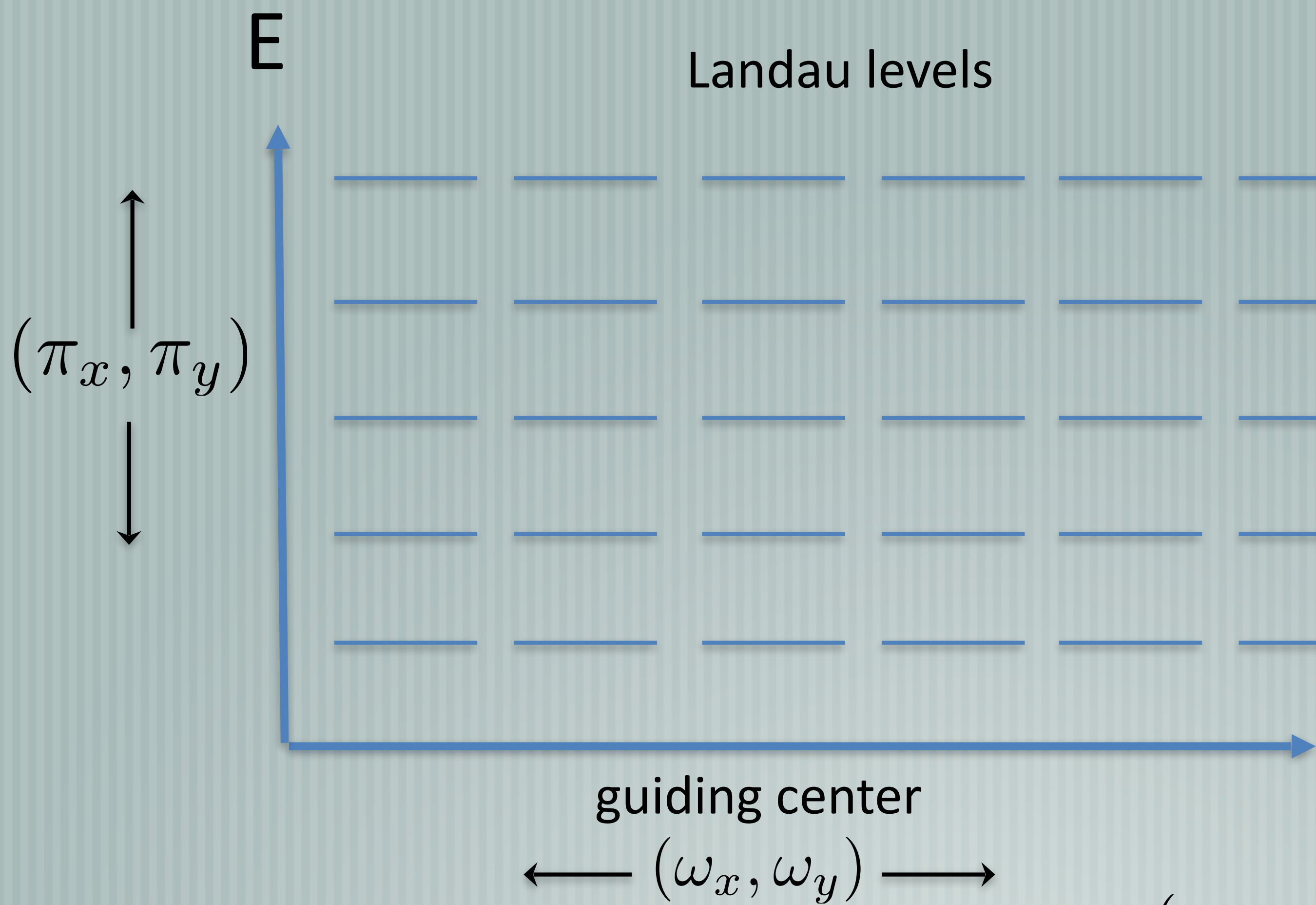


We know the Hamiltonian!

$$H_{\text{QHE}} = \sum_{i=1}^{N} \frac{\vec{\pi}_i^2}{2m} + \sum_{i<j} V(\vec{r}_{ij}) \text{ + dirt}$$

Exhibit Topological Quantum Order (TQO)

Quantum Hall Physics



$$H = \frac{1}{2m}\vec{\pi}^2$$

$$\vec{\pi} = \vec{p} - \vec{A}$$

$$[\pi_x, \pi_y] = i$$

$$\omega_i = x_i + \epsilon_{ij}\pi_j$$

$$[\omega_i, \pi_j] = 0$$

$$[\omega_x, \omega_y] = -i$$

Lowest Landau level:

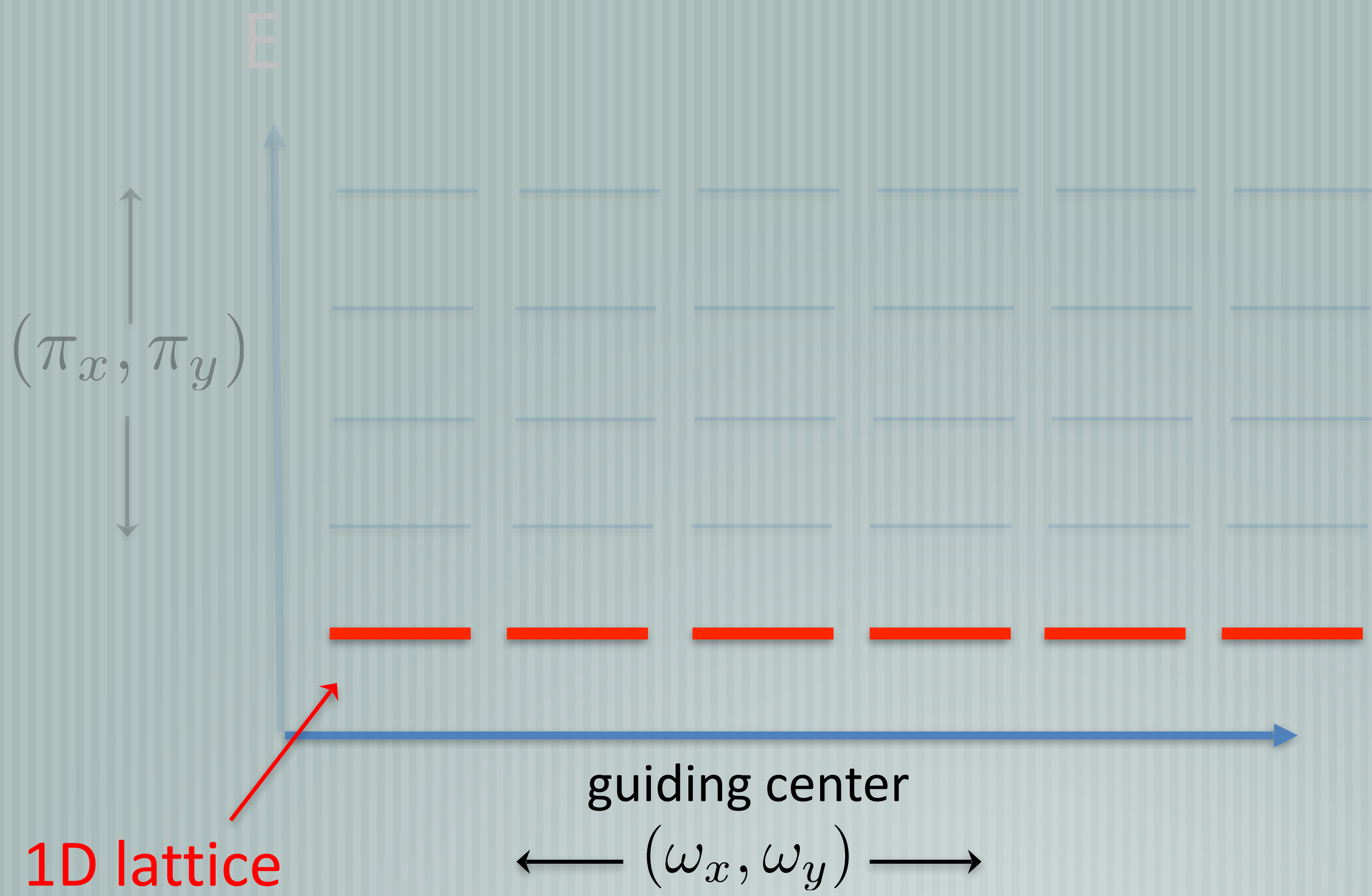
$$\psi_n(z) = \mathcal{N}_n z^n e^{-|z|^2/4}$$

$$(\pi_x - i\pi_y)|\psi_n\rangle = 0$$

$$a^\dagger a|\psi_n\rangle = n|\psi_n\rangle$$

Linear combinations of ω_x, ω_y.

Landau Level Projection



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$\nwarrow\ \nearrow$

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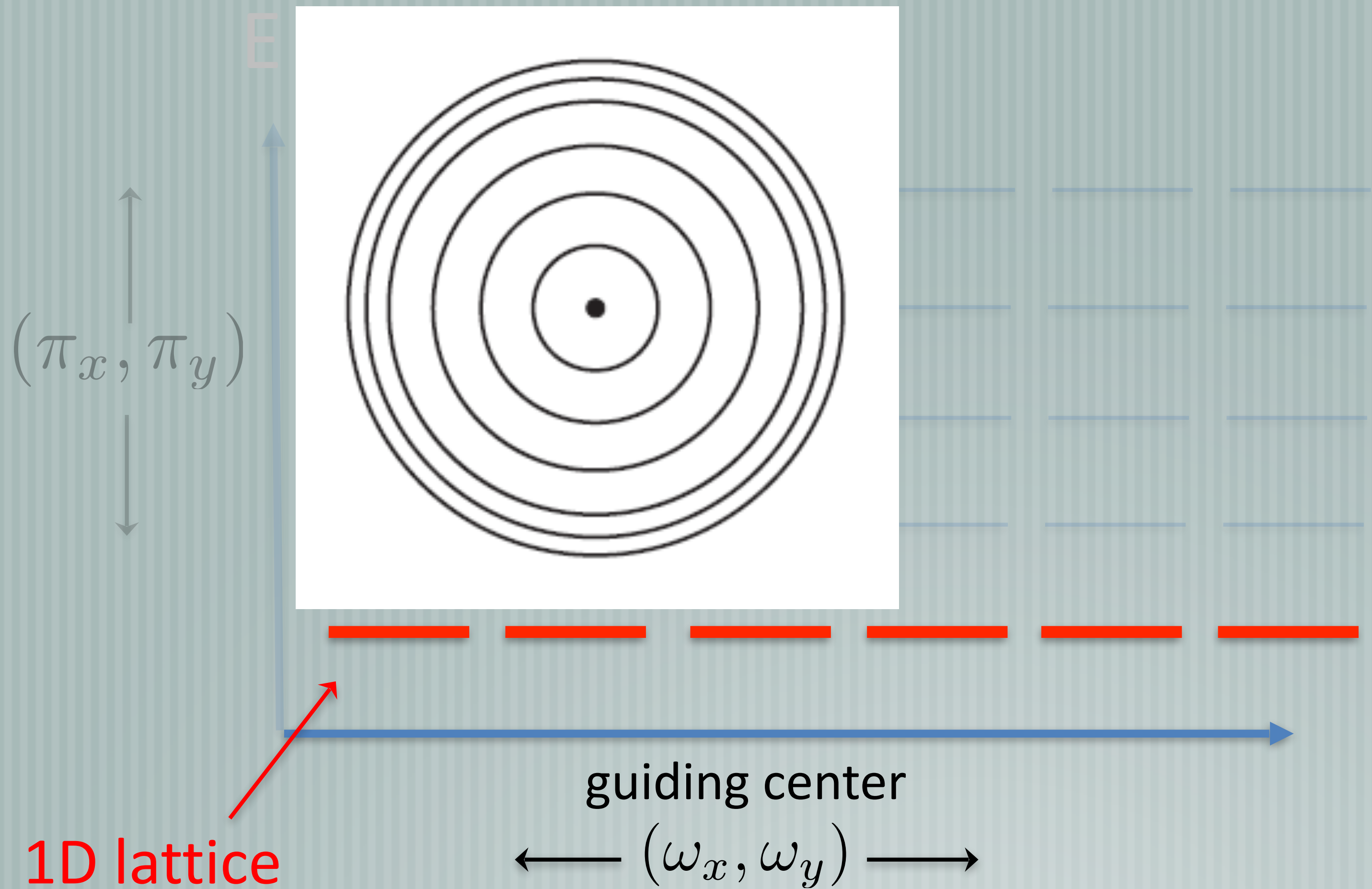

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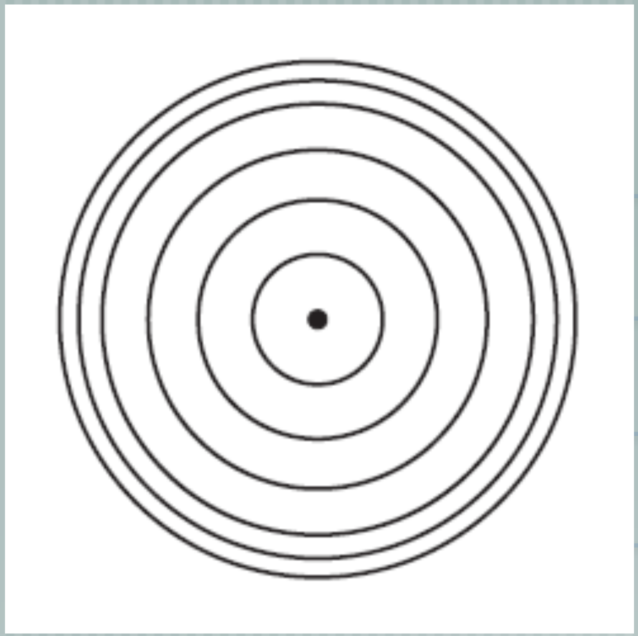
Linear combinations of ω_x, ω_y.

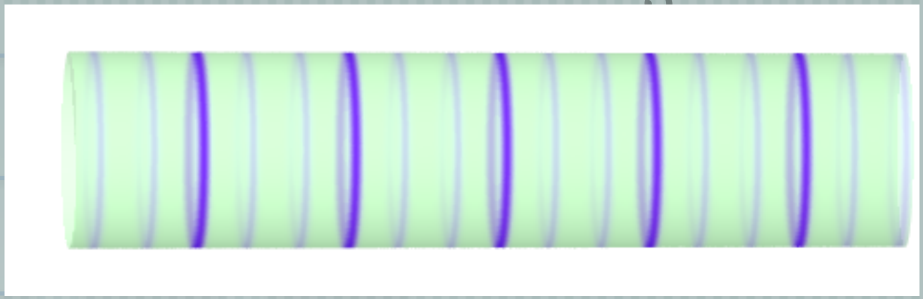
Landau Level Projection

E

$$H = \frac{1}{2}\vec{\pi}^2$$

(π_x, π_y)





$$\omega_i = x_i + \epsilon_{ij}\pi_j$$

$$[\omega_i, \pi_j] = 0$$

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1D lattice

guiding center

$\longleftarrow (\omega_x, \omega_y) \longrightarrow$

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Linear combinations of ω_x, ω_y.

Fractional Quantum Hall States

$$\psi_{\mathsf{Laughlin}} = \prod_{i<j}(z_i - z_j)^M e^{-\sum_i |z_i|^2/4}$$

$$\psi_{\mathsf{Moore-Read}} = \mathrm{Pfaff}\left[\frac{1}{z_i - z_j}\right]\prod_{i<j}(z_i - z_j)^M e^{-\sum_i |z_i|^2/4}$$

$$\psi_{\mathsf{Gaffnian}} = \mathsf{Symm} \prod_{a<b\leq\frac{N}{2}}(z_a - z_b)^{2+q} \prod_{\frac{N}{2}<c<d}(z_c - z_d)^{2+q} \prod_{e\leq\frac{N}{2}<f}(z_e - z_f)^{1+q} \prod_{g\leq\frac{N}{2}}\frac{1}{z_g - z_{g+\frac{N}{2}}} e^{-\sum_i |z_i|^2/4}$$

$$\psi_n(z) = \mathcal{N}_n z^n e^{-|z|^2/4}$$

$$\text{“interesting polynomial”} \times e^{-\sum_i |z_i|^2/4}$$

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"Interesting" usually means "Clustering Conditions"

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Strongly-correlated states of matter: Everything is about Interactions

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$$\text{“interesting polynomial”} \times e^{-\sum_i |z_i|^2/4}$$

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Bottom-Up Approach to QH

QH states

Analytic Prop.

CFT

(Parent)Hamiltonian

Bottom-Up Approach to QH

QH states

Algebraic

QH Hamiltonian

QH states

Analytic Prop.

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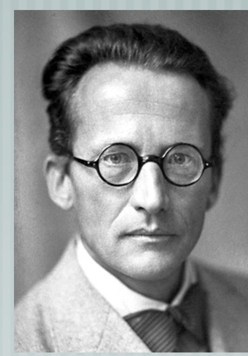

(Parent)Hamiltonian

Bottom-Up Approach to QH

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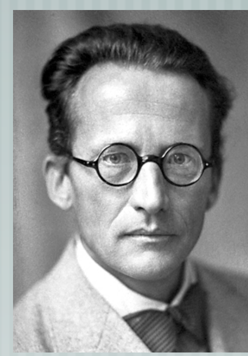


Analytic Prop.

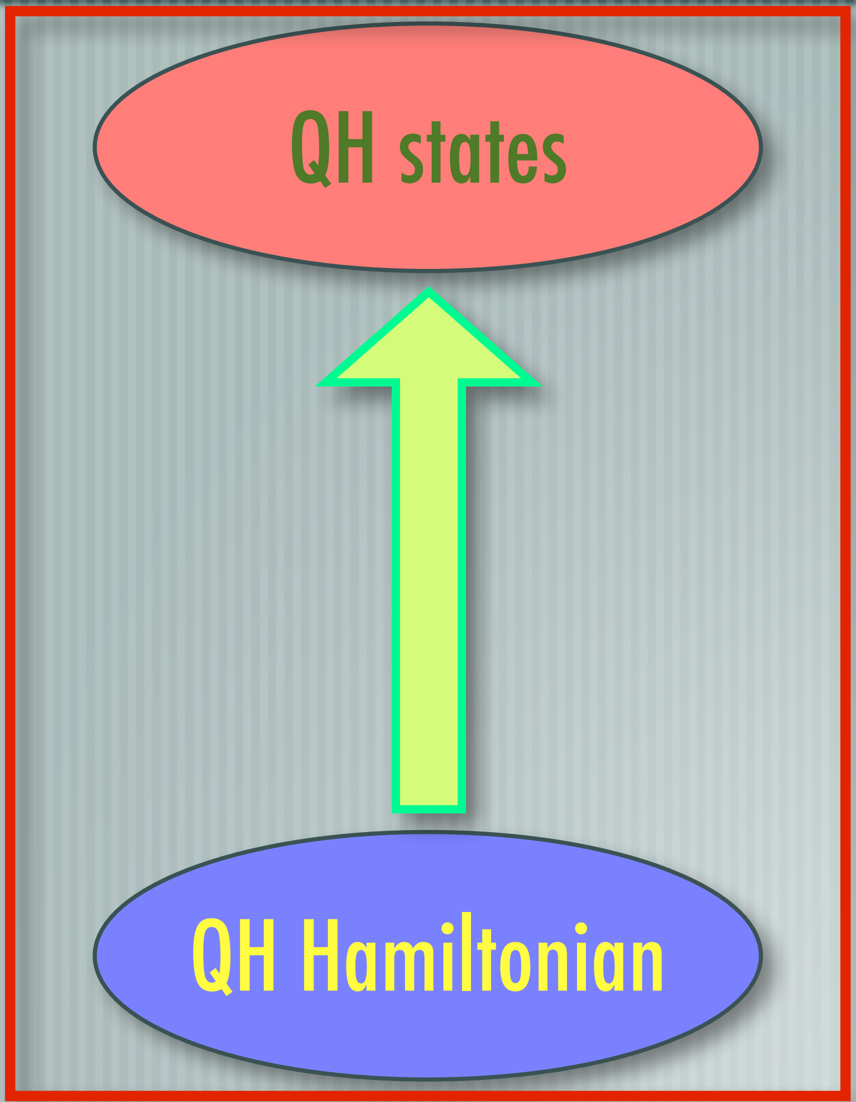
CFT

(Parent)Hamiltonian

Would like to:

- Understand the nature of the TQO defining FQH fluids
- Understand its Abelian and non-Abelian excitations (TQC)
- Derivation of states with filling fractions other than Laughlin's? Need some organizing principle (two-body parent Hamiltonians?) Composite Fermions? Parton states?
- How about quasihole operators and edge modes?
- Nature of correlations in the $\nu = \frac{1}{2}$ state (strange metal)?

Outline



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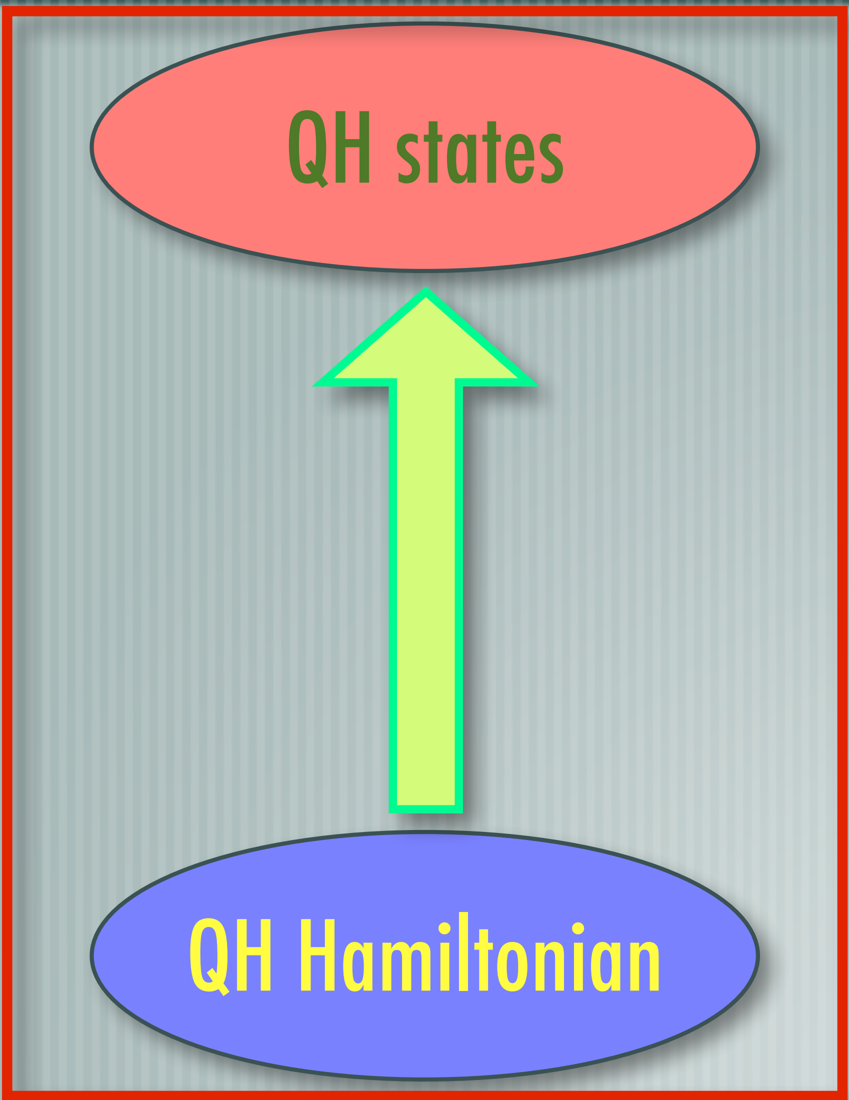
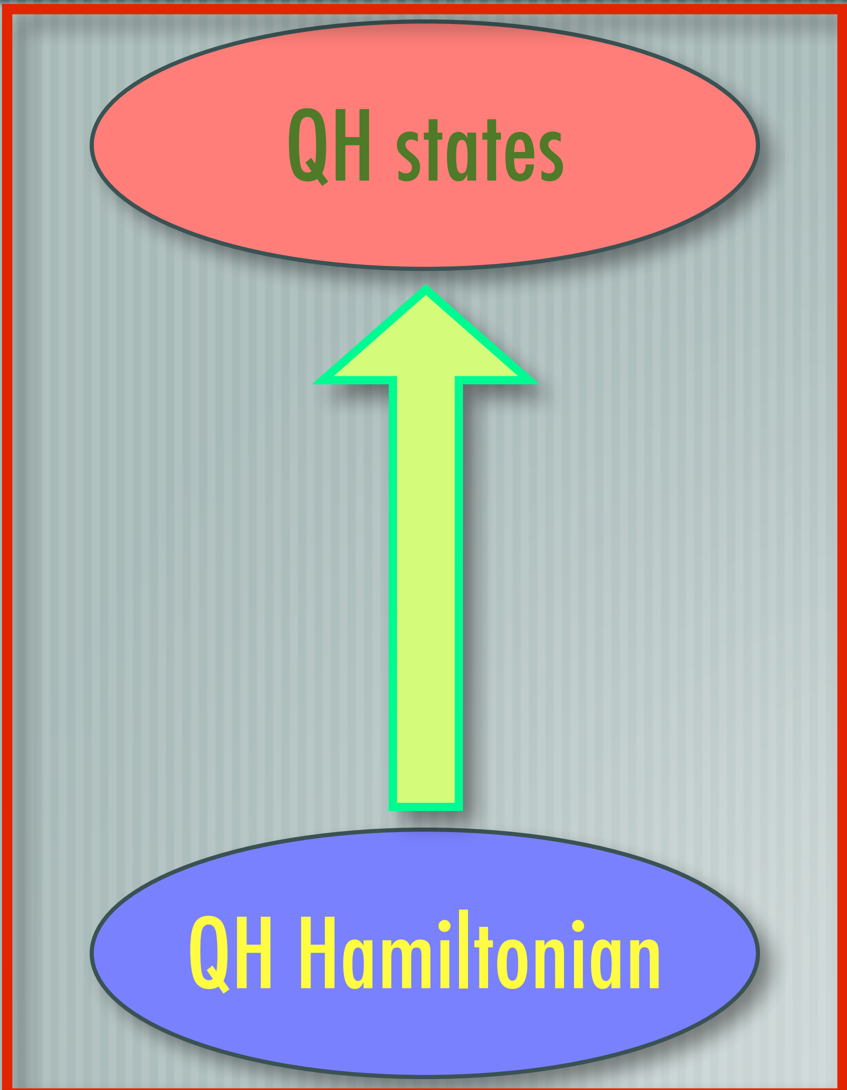


- Setup the QH Hamiltonian in second quantization

Outline



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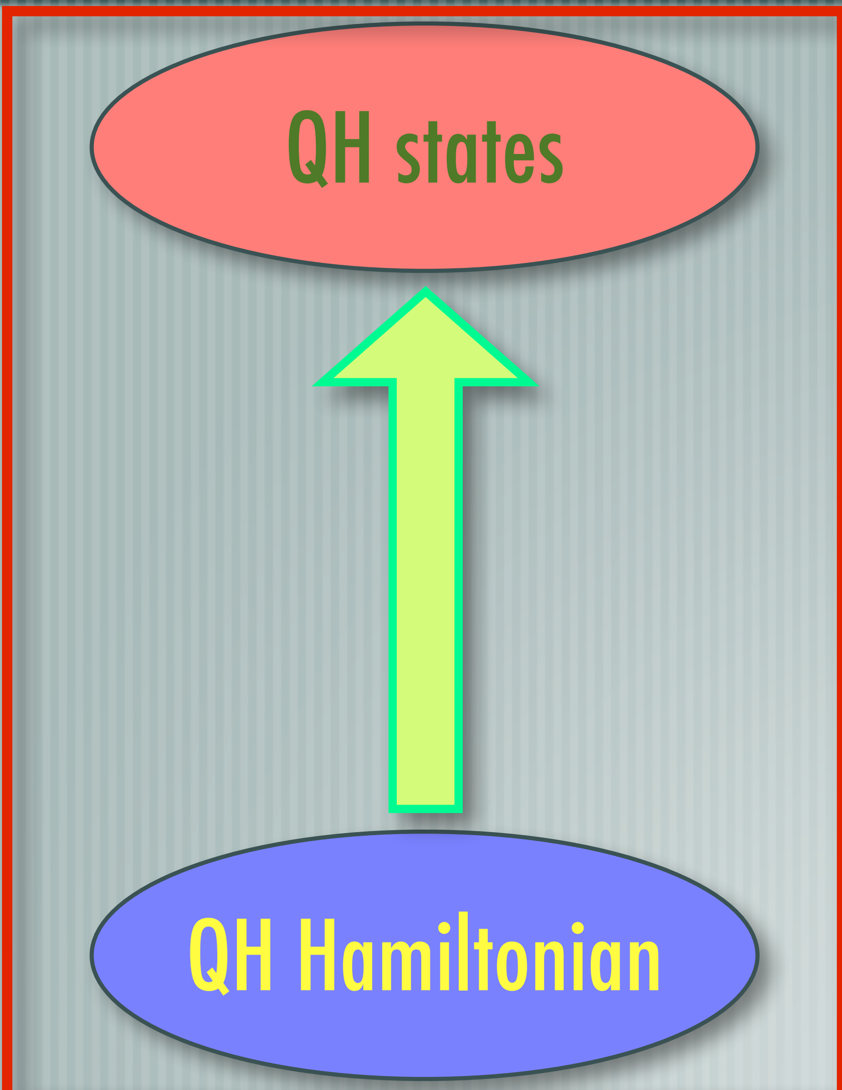

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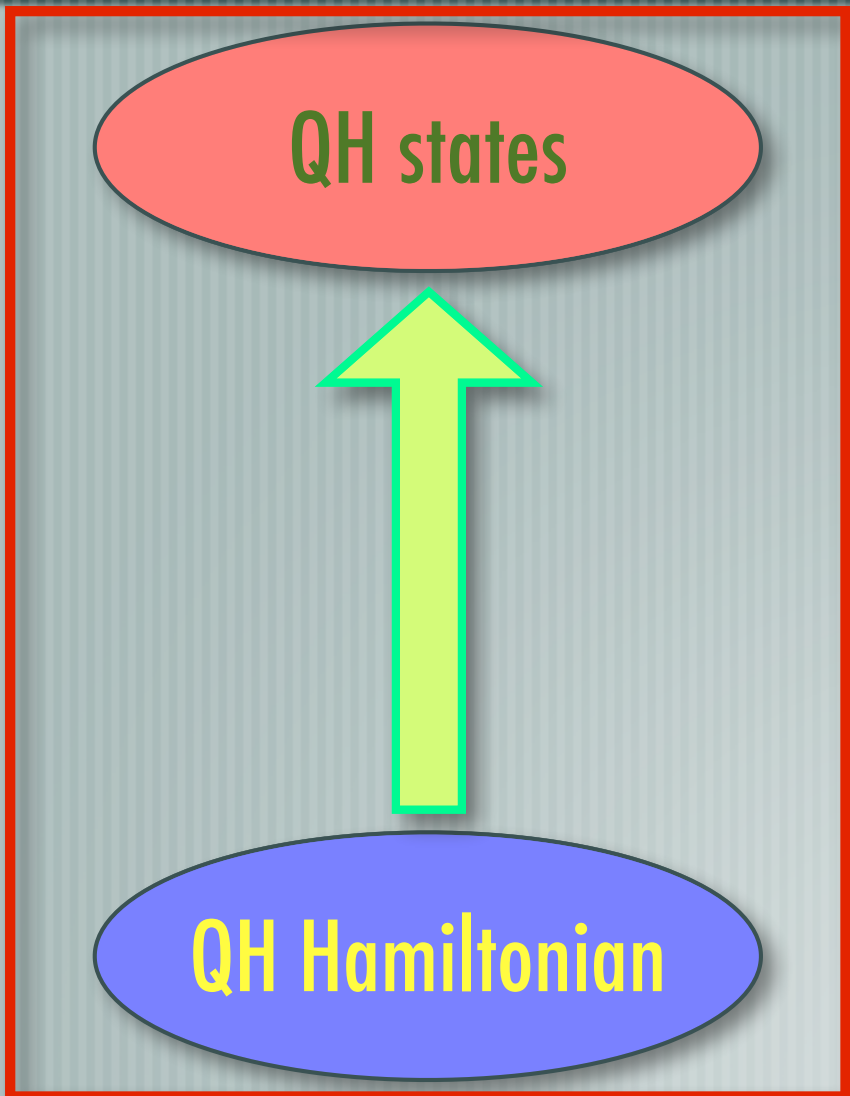
Entangled-Pauli-Pples and Parton States

Outline



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Entangled-Pauli-Pples and Parton States

- Charge, Statistics, Quasi-hole operators and String orders

Quantum Hall Physics

An Exercise in Second Quantization

Quantum Hall Physics

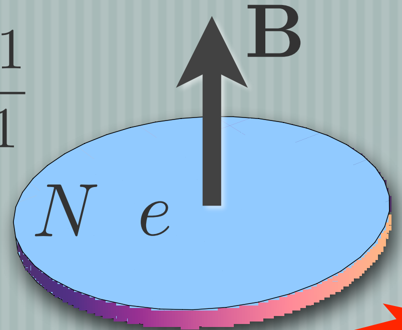
An Exercise in Second Quantization

Let us start with a projection onto the LLL

Dimensional Reduction - QH Physics

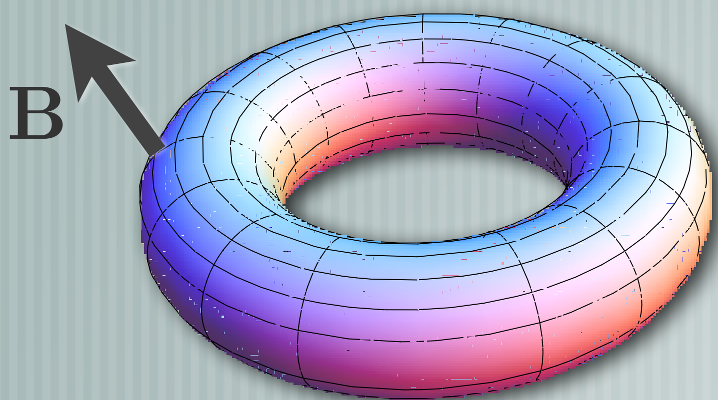
First Quantization

$$\nu = \frac{N-1}{L-1}$$



dynamical momenta

$$H_{\mathsf{QH}} = \sum_{\mathsf{i}=1}^{N} \frac{\mathbf{\Pi}_{\mathsf{i}}^2}{2m} + \sum_{\mathsf{i}<\mathsf{j}} V(\mathbf{x}_{\mathsf{i}} - \mathbf{x}_{\mathsf{j}})$$



2D continuous geometries

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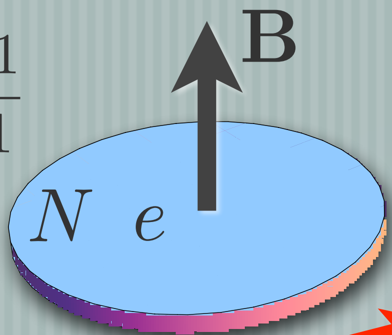

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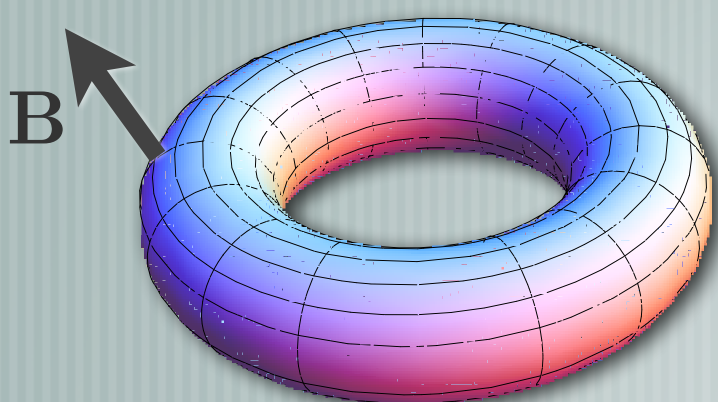
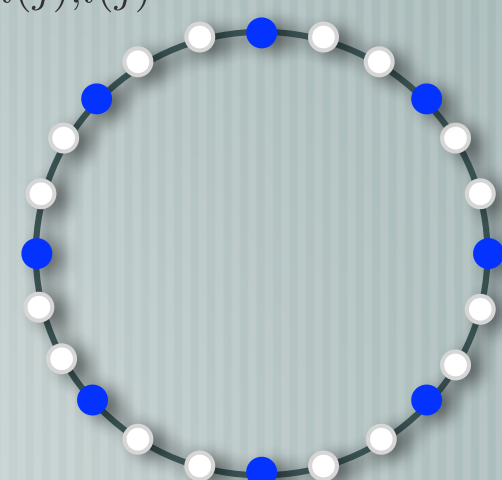


2D continuous geometries

Second Quantization

$$\hat{P}_{\mathsf{LLL}} H_{\mathsf{QH}} \hat{P}_{\mathsf{LLL}}$$

$$\widehat{H}_{\mathsf{QH}} = \sum_{0<j<L-1} \sum_{k(j),l(j)} V_{j;kl} \; c^{\dagger}_{j+k} c^{\dagger}_{j-k} c_{j-l} c_{j+l}$$



1D orbital lattices

Separability of Pseudopotentials

Given an arbitrary spherically symmetric interaction:

$$V(\mathbf{x}_\mathsf{i} - \mathbf{x}_\mathsf{j}) = \sum_{m\geq 0} g_m V_m = \sum_{m\geq 0} g_m \sum_{\mathsf{i}<\mathsf{j}} P_m(\mathsf{ij})$$

with $g_m \geq 0$ and $P_m(\mathsf{ij})$ a projector onto the subspace of relative angular momentum m of the pair (ij)

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We have shown that in second quantization: $\hat{H}_{\mathsf{QH}} = \sum_{m\geq 0} g_m \, \hat{H}_{V_m}$

with $$\hat{H}_{V_m} = \sum_{0<j<L-1} \sum_{k(j),l(j)} \eta_k \eta_l \; c^\dagger_{j+k} c^\dagger_{j-k} c_{j-l} c_{j+l}$$

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For the 1st Haldane pseudopotential or Trugman-Kivelson model:

Geometry	L (Laughlin)	N_Φ	$\eta_k(j,1)$	$\phi_r(z)$
Disk	$\mathsf{q}N-\mathsf{q}+1$	L	$k\,2^{-j+1}\sqrt{\frac{1}{j}\binom{2j}{j+k}}$	$\frac{1}{\sqrt{2\pi 2^r r!}}\,z^r e^{-\frac{1}{4}\|z\|^2}$
Cylinder	$\mathsf{q}N-\mathsf{q}+1$	L	$2(8/\pi)^{1/4}\kappa^{3/2}\,k\,e^{-\kappa^2 k^2}$	$(4\pi^3)^{-1/4}\sqrt{\kappa}\,e^{-\frac{1}{2}(x-r\kappa)^2+ir\kappa y}$
Sphere	$\mathsf{q}N-\mathsf{q}+1$	$L-1$	$k\sqrt{\frac{2N_\Phi-2}{j(N_\Phi-j)}\binom{N_\Phi}{j-k}\binom{N_\Phi}{j+k}/\binom{2N_\Phi}{2j}}$	$\sqrt{\frac{N_\Phi+1}{4\pi}\binom{N_\Phi}{r}}[e^{-i\frac{\varphi}{2}}\sin(\frac{\theta}{2})]^r[e^{i\frac{\varphi}{2}}\cos(\frac{\theta}{2})]^{N_\Phi-r}$
Torus	$\mathsf{q}N$	L	$2(8/\pi)^{1/4}\kappa^{3/2}\sum_{s\in\mathbb{Z}}(k+sL)\,e^{-\kappa^2(k+sL)^2}$	$\sum_{s\in\mathbb{Z}}\phi^{\text{cylinder}}_{r+sL}$

- In the case of the cylinder for arbitrary m

$$\eta_k = \frac{e^{-\kappa^2 k^2}}{2^{\frac{m}{2}}\sqrt{m!}}\,H_m[\sqrt{2}\,\kappa k] \rightarrow \text{Hermite poly}$$

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We have shown that geometries with the same genus number can be related through similarity transformations

Gaudin for Quantum Hall

Exactly-Solvable Model: Strong Coupling

Consider the general class of hyperbolic Gaudin models with:

$$\mathsf{S}^z(x) = -\frac{1}{2} - \sum_{k(j)} Z(x,\eta_k) S^z_{jk}, \quad \mathsf{S}^\pm(x) = \sum_{k(j)} X(x,\eta_k) S^\pm_{jk}$$

In this rep one can define $\mathcal{C}(j)$ constants of motion: (Fix j)

$$R_{jk} = S^z_{jk} - \sum_{l(j),l\neq k} X(\eta_k,\eta_l)\left(S^+_{jk}S^-_{jl} + S^-_{jk}S^+_{jl}\right) - 2\sum_{l(j),l\neq k} Z(\eta_k,\eta_l) S^z_{jk} S^z_{jl}$$

And from their linear combination obtain:

$$H_{\mathsf{G}j} = \sum_{k(j)} \epsilon_k S^z_{jk} - \sum_{k(j),l(j)} (\epsilon_k - \epsilon_l) X(\eta_k,\eta_l) S^+_{jk} S^-_{jl} - \sum_{k(j),l(j)} (\epsilon_k - \epsilon_l) Z(\eta_k,\eta_l) S^z_{jk} S^z_{jl}$$

The following parametrization (satisfying Jacobi's relation):

$$X(x,y) = -\bar{g}\frac{xy}{x^2-y^2}\ , \quad Z(x,y) = -\frac{\bar{g}}{2}\frac{x^2+y^2}{x^2-y^2}$$

and $\epsilon_k = \lambda_j \eta_k^2$ leads to the Hamiltonian:

$$H_{\mathsf{G}j} = \lambda_j \left(1+\bar{g}(S_j^z - 1)\right) \sum_{k(j)} \eta_k^2 S_{jk}^z + \lambda_j \bar{g} \sum_{k(j),l(j)} \eta_k \eta_l S_{jk}^+ S_{jl}^-$$

where

$\mathbf{S}_{jk}^2$ and $\mathbf{S}_j = \sum_{k(j)} \mathbf{S}_{jk}$ are good quantum numbers

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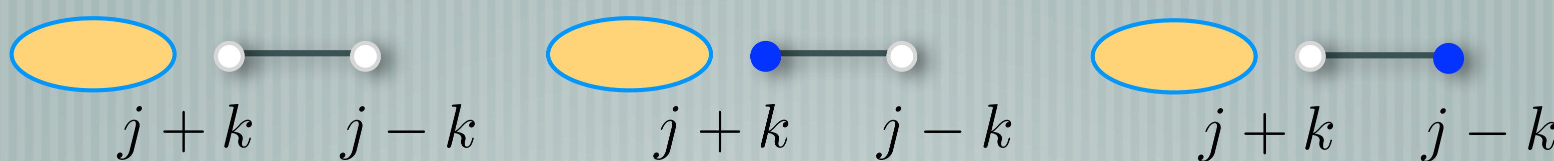
We want to consider the special case where vanishes

One can choose the SU(2) fermionic representation:

$$S^+_{jk} = c^\dagger_{j+k} c^\dagger_{j-k} \;,\; S^-_{jk} = c_{j-k} c_{j+k} \;,\; S^z_{jk} = \frac{1}{2}(n_{j+k} + n_{j-k} - 1)$$

such that acting on the vacuum $|\nu(j)\rangle$ containing only unpaired e-

$$S^-_{jk}|\nu(j)\rangle = 0 \qquad S^z_{jk}|\nu(j)\rangle = \frac{1}{2}(|\nu_{jk}| - 1)|\nu(j)\rangle \equiv -s_{jk}|\nu(j)\rangle$$



$\nu_{jk} = 0$ $\qquad$ $\nu_{jk} = +1$ $\qquad$ $\nu_{jk} = -1$

$$N = 2M + N_{\text{b}} + N_{\text{inactive}}$$

(L orbitals)

Paired $\qquad$ **Unpaired**= $N_{\text{b}} = \sum_{k(j)} |\nu_{jk}|$ $\qquad$ **Inactive levels**

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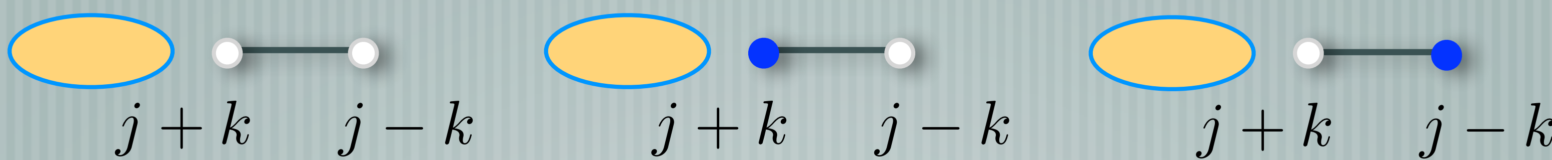
seniority



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By choosing: $\bar{g} = -1/(M - \sum_{k(j)} s_{jk} - 1)$

one obtains: $(g = \lambda_j \bar{g})$

$$H_{\mathsf{G}j} = g \sum_{k(j),l(j)} \eta_k \eta_l \; c^\dagger_{j+k} c^\dagger_{j-k} c_{j-l} c_{j+l} = g \; T^+_{j1} T^-_{j1}$$

Arbitrary Haldane pseudopotential

This model is exactly solvable for any η_k , the QH information is in part in their specific values

By choosing: $\bar{g} = -1/(M - \sum_{k(j)} s_{jk} - 1)$

one obtains: $(g = \lambda_j \bar{g})$

$$H_{\mathsf{G}j} = g \sum_{k(j),l(j)} \eta_k \eta_l \; c^\dagger_{j+k} c^\dagger_{j-k} c_{j-l} c_{j+l} = g \; T^+_{j1} T^-_{j1}$$

Arbitrary Haldane pseudopotential

Geometry	L (Laughlin)	N_Φ	$\eta_k(j,1)$	$\phi_r(z)$
Disk	$\mathsf{q}N - \mathsf{q} + 1$	L	$k\, 2^{-j+1} \sqrt{\frac{1}{j} \binom{2j}{j+k}}$	$\frac{1}{\sqrt{2\pi 2^r r!}} z^r e^{-\frac{1}{4}\|z\|^2}$
Cylinder	$\mathsf{q}N - \mathsf{q} + 1$	L	$2(8/\pi)^{1/4} \kappa^{3/2}\, k\, e^{-\kappa^2 k^2}$	$(4\pi^3)^{-1/4} \sqrt{\kappa}\, e^{-\frac{1}{2}(x - r\kappa)^2 + ir\kappa y}$
Sphere	$\mathsf{q}N - \mathsf{q} + 1$	$L-1$	$k \sqrt{\frac{2N_\Phi - 2}{j(N_\Phi - j)} \binom{N_\Phi}{j-k} \binom{N_\Phi}{j+k} / \binom{2N_\Phi}{2j}}$	$\sqrt{\frac{N_\Phi + 1}{4\pi} \binom{N_\Phi}{r}} [e^{-i\frac{\varphi}{2}} \sin(\frac{\theta}{2})]^r [e^{i\frac{\varphi}{2}} \cos(\frac{\theta}{2})]^{N_\Phi - r}$
Torus	$\mathsf{q}N$	L	$2(8/\pi)^{1/4} \kappa^{3/2} \sum_{s \in \mathbb{Z}} (k + sL)\, e^{-\kappa^2 (k+sL)^2}$	$\sum_{s \in \mathbb{Z}} \phi^{\text{cylinder}}_{r+sL}$

Eigenvectors:

$$|\Phi_{M\nu(j)}\rangle = \prod_{\alpha=1}^{M} \mathsf{S}_j^+(E_\alpha)|\nu(j)\rangle \ , \ \mathsf{S}_j^+(E_\alpha) = \sum_{k(j)} \frac{\eta_k}{\eta_k^2 - E_\alpha} \ c_{j+k}^\dagger c_{j-k}^\dagger$$

There exists two classes of solutions:

All finite pairons: $\mathcal{E}_{M\nu(j)} = 0$

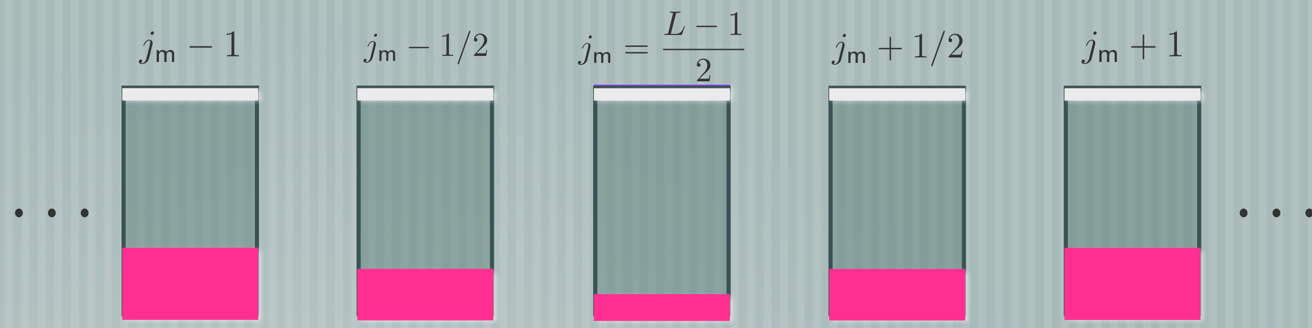
One infinite pairon: $\mathcal{E}_{M\nu(j)} = 2g \left(\sum_{k(j)} s_{jk}\, \eta_k^2 - \sum_{\alpha=1}^{M-1} E_\alpha \right)$

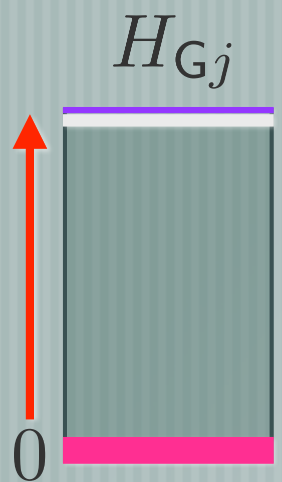
The Gaudin (Bethe) equation is:

$$\sum_{\beta(\neq\alpha)=1}^{M} \frac{E_\beta}{E_\beta - E_\alpha} - \sum_{k(j)} s_{jk} \ \frac{\eta_k^2}{\eta_k^2 - E_\alpha} = 0, \ \ \forall\alpha$$

Spectrum of Gaudin-Quantum Hall

Repulsive case $(g > 0)$:

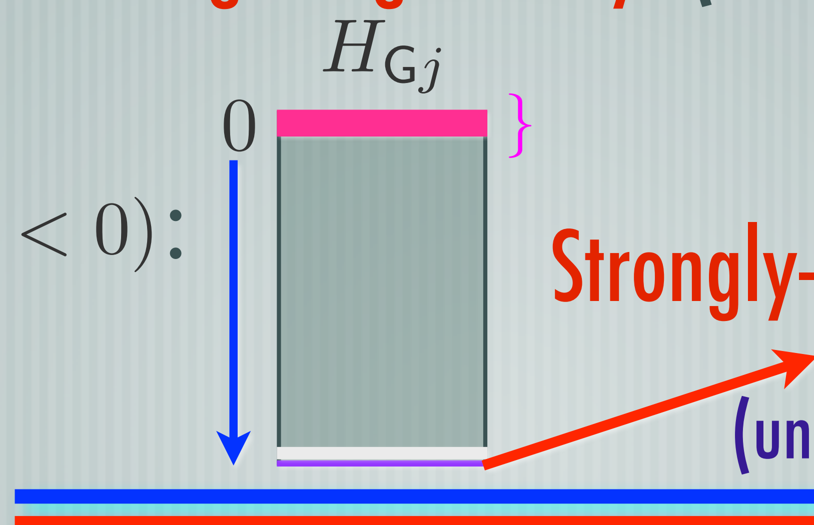




$\dim \mathcal{H}_L(N, J) - \dim \mathcal{H}_L(N-2, J-2j)$ (independent of η_k)

} # zeros: Large degeneracy (Null space)

Attractive case $(g < 0)$:

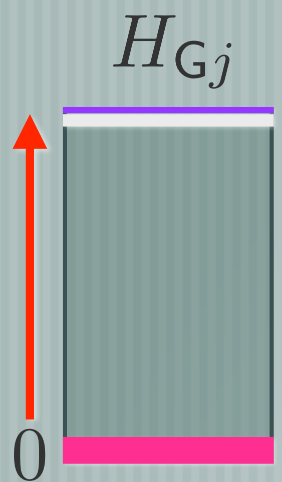


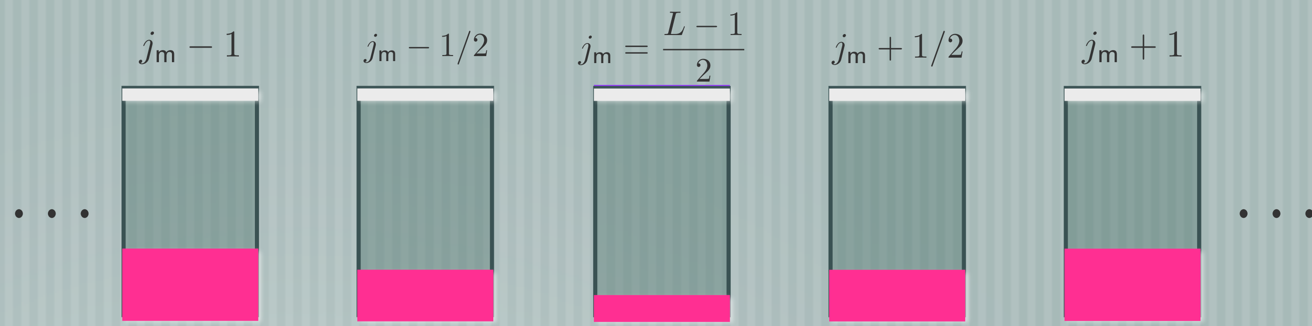
Strongly-coupled Superconductor

(unique ground state)

Spectrum of Gaudin-Quantum Hall

Repulsive case $(g > 0)$:

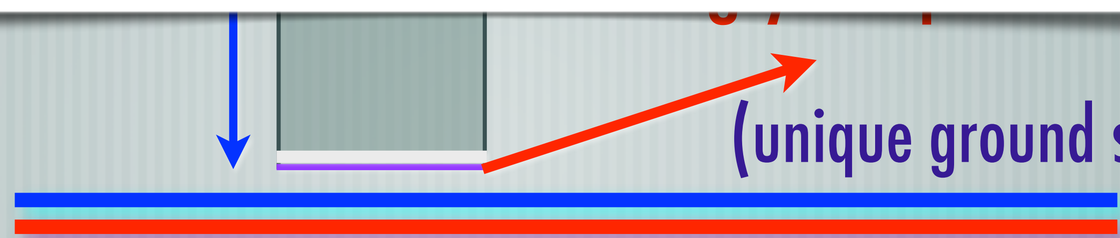




$\dim \mathcal{H}_L(N, J) - \dim \mathcal{H}_L(N-2, J-2j)$ (independent of η_k)

} # zeros: Large degeneracy (Null space)

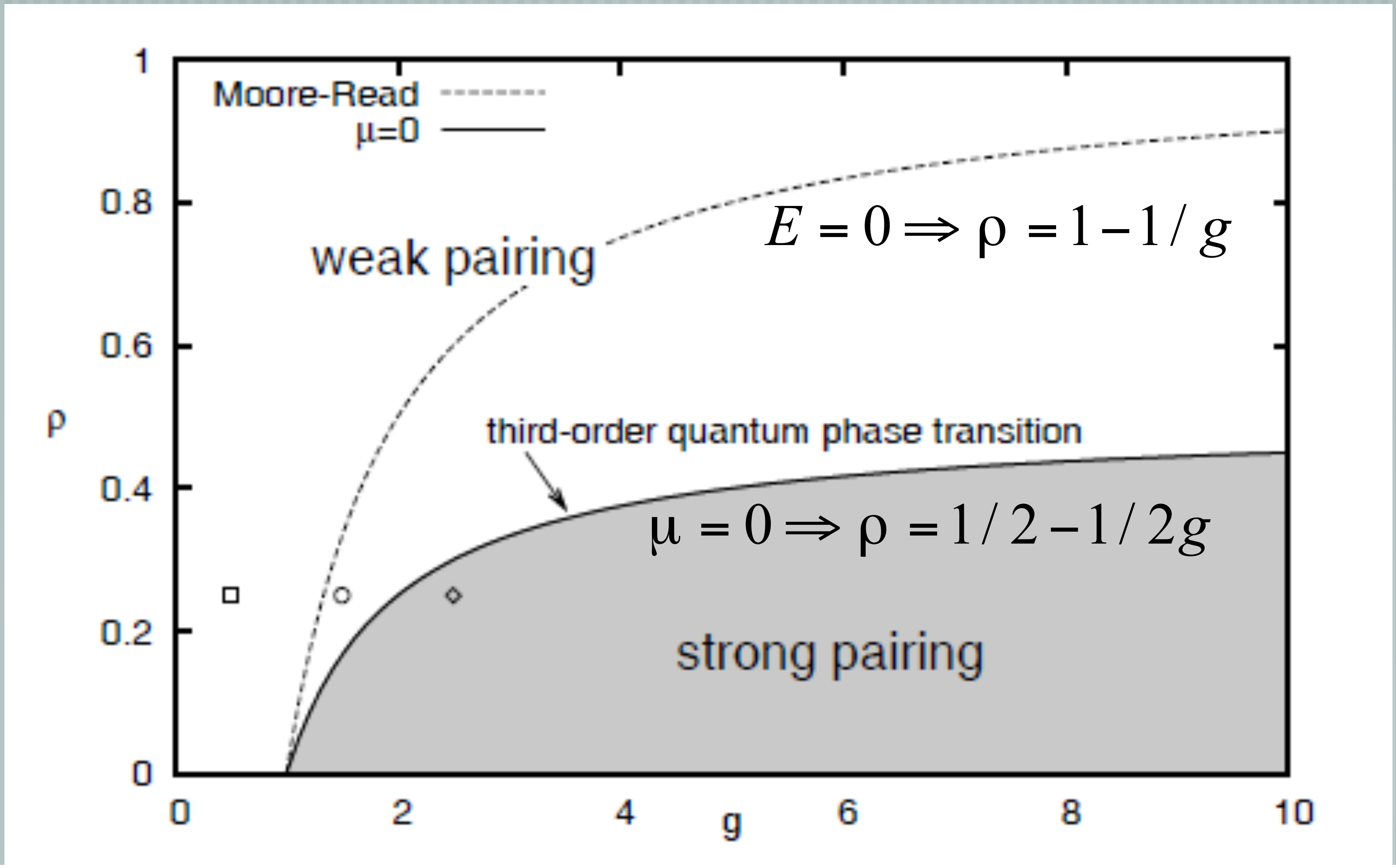
The Gaudin (Bethe) equation has a symmetry that relates two states with different filling fractions, and makes $\nu = \frac{1}{2}$ "special"



(unique ground state)

Quantum Phase Diagram

The phase diagram can be parametrized in terms of the density $\rho = M/L$ and the rescaled coupling $g = GL$



Quantum Phase Diagram

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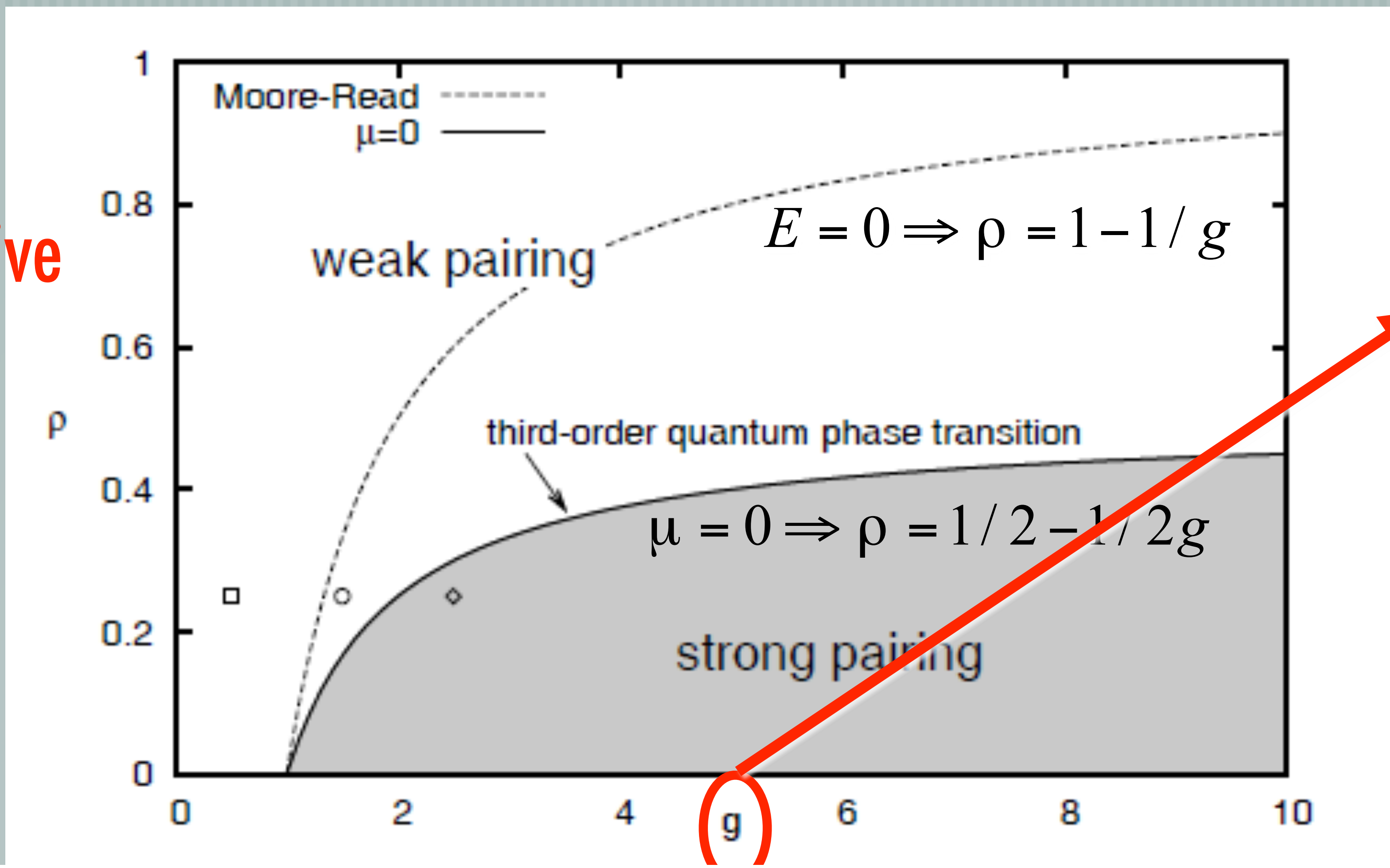

attractive

Ground States of the Full Pseudopotential Problem

Ground States of the Full Pseudopotential Problem

No gauge symmetry: Seniority no longer a good quantum number

Frustration-Free Properties

We have shown that in second quantization:

$$\hat{H}_{\mathsf{QH}} = \sum_{0<j<L-1} \sum_{m\geq 0} H^m_{\mathsf{G}j} = \sum_{m\geq 0} g_m \, \hat{H}_{V_m}$$

$\mathsf{Ker}(\hat{H}_{\mathsf{QH}})$ is the common null space of all the null spaces $\mathsf{Ker}(H^m_{\mathsf{G}j})$

Given N, L , the Hamiltonian $\hat{H}_{V_1}$ displays zero energy ground states $|\Psi^J_\nu\rangle$, whenever $\nu = \frac{p}{q} \leq \frac{1}{3}$. The zero energy state is unique when $\nu = \frac{1}{3}$, it is in the sector $J = J_{\mathsf{m}}$, and it is the Laughlin state

$\hat{H}_{V_1}$ is a frustration-free Hamiltonian for $\nu = \frac{p}{q} \leq \frac{1}{3}$

$$H_{\mathsf{G}j}|\Psi_\nu^J\rangle = 0, \quad \text{for all } j,\ j_{\mathsf{min}} \leq j \leq j_{\mathsf{max}} \Rightarrow T_{j1}^{-}|\Psi_\nu^J\rangle = 0$$

positive semi-definite

Corollary: All zero energy states have zero coefficients, in a Slater determinant expansion, for the basis states with:

$$(n_0 = 1, n_1 = 1)\ ,\ (n_0 = 1, n_2 = 1)\ ,\ (n_{L-3} = 1, n_{L-1} = 1),\ (n_{L-2} = 1, n_{L-1} = 1)$$

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What is the Organizing Principle?

Zero Modes and Root Patterns

Zero Modes and Root Patterns

Frustration-free Quantum Hall Systems

Fractional Quantum Hall effect and Laughlin State

$$\prod_{i>j}^{N}(z_i - z_j)^3 = \begin{vmatrix} 1 & 1 & \cdots & 1 & 1 \\ z_1 & z_2 & \cdots & z_{N-1} & z_N \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ z_1^N & z_2^N & \cdots & z_{N-1}^N & z_N^N \end{vmatrix}^3 ; \qquad z_i = x_i + iy_i$$

For three particles,

$$\begin{vmatrix} 1 & 1 & 1 \\ z_1^3 & z_2^3 & z_3^3 \\ z_1^6 & z_2^6 & z_3^6 \end{vmatrix} + 3\begin{vmatrix} z_1 & z_2 & z_3 \\ z_1^2 & z_2^2 & z_3^2 \\ z_1^6 & z_2^6 & z_3^6 \end{vmatrix} + 3\begin{vmatrix} 1 & 1 & 1 \\ z_1^4 & z_2^4 & z_3^4 \\ z_1^6 & z_2^5 & z_3^5 \end{vmatrix} - 6\begin{vmatrix} z_1 & z_2 & z_3 \\ z_1^3 & z_2^3 & z_3^3 \\ z_1^5 & z_2^5 & z_3^5 \end{vmatrix} + 15\begin{vmatrix} z_1 & z_2 & z_3 \\ z_1^2 & z_2^2 & z_3^2 \\ z_1^6 & z_2^6 & z_3^6 \end{vmatrix}$$

$$1001001 \quad + \quad 3 * 0110001 \quad + \quad 3 * 1000110 \quad - \quad 6 * 0101010 \quad + \quad 15 * 0011100$$

Root pattern: Generalized Pauli Principle , no two particles are allowed in three consecutive sites.

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Non-Expandable

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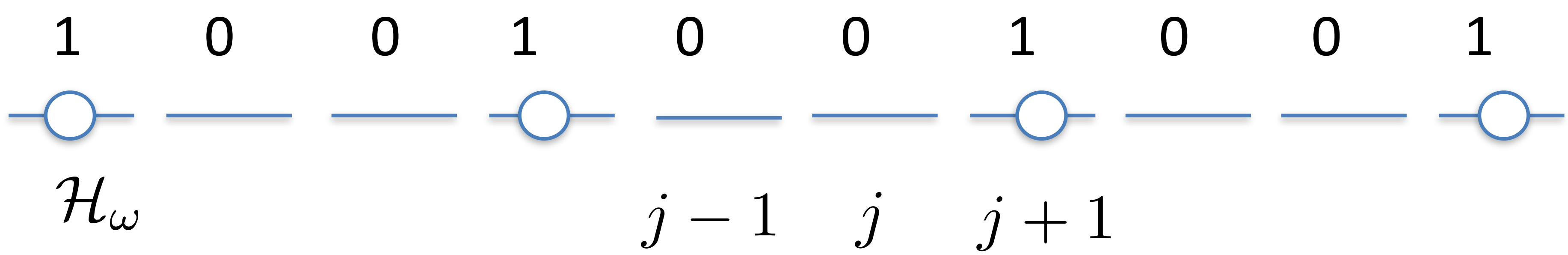
Expandables

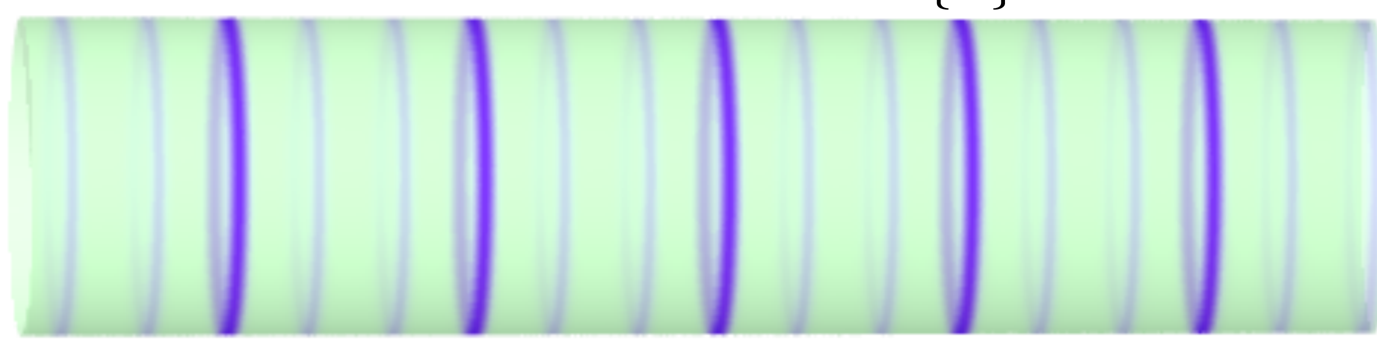
$$1001001 \quad + \quad 3 * 0110001 \quad + \quad 3 * 1000110 \quad - \quad 6 * 0101010 \quad + \quad 15 * 0011100$$

Root pattern: Generalized Pauli Principle , no two particles are allowed in three consecutive sites.

Guiding center/second quantized presentation of quantum Hall Hamiltonians



Use cylinder Hamiltonian geometry: $$|\psi\rangle = \sum_{\{n\}} C_{\{n\}} |\{n\}\rangle$$



Focus on V_1 pseudo potential:

$$H_{V_1} = \sum_j T_{j1}^{+} T_{j1}^{-} \qquad T_j^{-} = \sum_{k(j)} \eta_k(j,1) c_{j-k} c_{j+k}$$

$$H_{V_1} |\psi_{\frac{1}{3},\mathrm{QH}}\rangle = 0 \qquad T_j^{-} |\psi_{\frac{1}{3},\mathrm{QH}}\rangle = 0 \quad \forall j$$

Laughlin 1/3 state with any number of quasi-holes added

A frustration-free Hamiltonian

(Ground state is a ground state of each individual term at fixed j)

Famous frustration free lattice models:

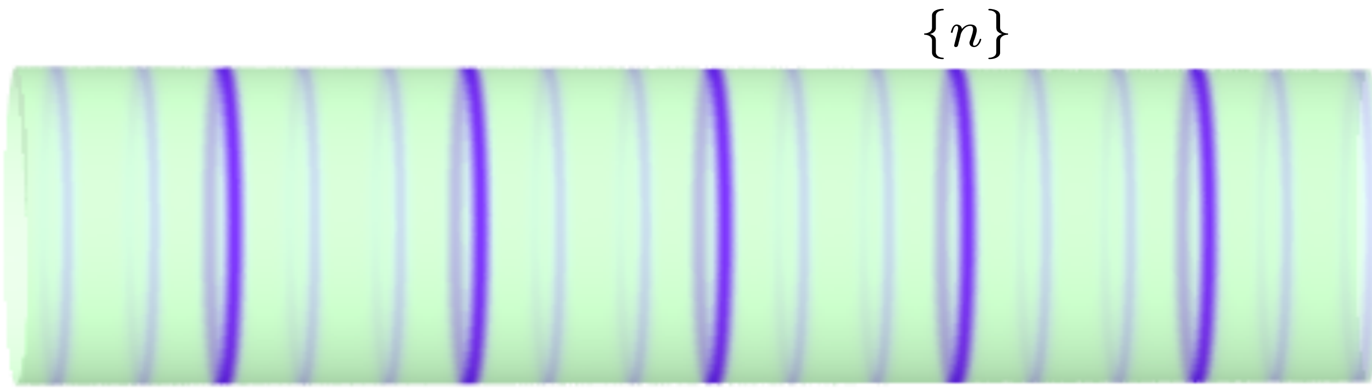
(linked to tensor product ground states)

all short ranged!

AKLT (1D)
Majumdar-Gosh(1D)

quantum dimer (2D)
Kitaev (2D)

...



Focus on V_1 pseudo potential:

$$H_{V_1} = \sum_j T^+_{j1} T^-_{j1} \qquad T^-_j = \sum_{k(j)} \eta_k(j,1) c_{j-k} c_{j+k}$$

$$H_{V_1} |\psi_{\frac{1}{3},\text{QH}}\rangle = 0 \qquad T^-_j |\psi_{\frac{1}{3},\text{QH}}\rangle = 0 \quad \forall j$$

Laughlin 1/3 state with any number of quasi-holes added

Squeezing as a result of the general zero mode property

Assume: $H|\psi\rangle = 0$ where $|\psi\rangle = \sum_{\{n\}} C_{\{n\}} |\{n\}\rangle \quad (*)$

Can show: Every $|\{n\}\rangle$ that appears in $(*)$ can be "inward-squeezed"

G. Ortiz, et al
PRB 13

from a $|\{n'\}\rangle$ that also appears in $(*)$ and that satisfies the

generalized Pauli principle (GPP) of "no more than 1 particle

in any M adjacent orbitals"

$M = 3:$

$|\{n'\}\rangle$ 100100001000100010010001001000100001

inward squeezing

satisfies $M = 3$ GPP

$|\{n\}\rangle$ 100001001010000010000011001100000001

does not satisfy $M = 3$ GPP

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from a $|\{n'\}\rangle$ that also appears in $(*)$ and that satisfies the generalized Pauli principle (GPP) of "no more than 1 particle in any M adjacent orbitals"

densest pattern satisfying $M = 3$ GPP:

$|\{n'\}\rangle$ 100100100100100100100100100100100101001 $\nu = 1/3$

inward squeezing

$|\{n\}\rangle$ 100001100110000100001100100110000 1001

does not satisfy $M = 3$ GPP

Squeezing as a result of the general zero mode property

Assume: $H|\psi\rangle = 0$ where $|\psi\rangle = \sum_{\{n\}} C_{\{n\}} |\{n\}\rangle \quad (*)$

- The M=3 subclass of Hamiltonians can have zero modes only up to filling factor $\nu = 1/3$
- The zero mode at $\nu = 1/3$ must be unique, if it exists
- Analogous statements for general M

in any M adjacent orbitals”

densest pattern satisfying $M = 3$ GPP:

$|\{n'\}\rangle$ 1001001001001001001001001001001001001 $\quad \nu = 1/3$

inward squeezing

$|\{n\}\rangle$ 1000011001100001000011001001100001001

does not satisfy $M = 3$ GPP

These statements apply to the entire class of Hamiltonians, where zero modes may not at all be related to first quantized wavefunctions with nice analytic clustering properties

- The M=3 subclass of Hamiltonians can have zero modes only up to filling factor $\nu = 1/3$
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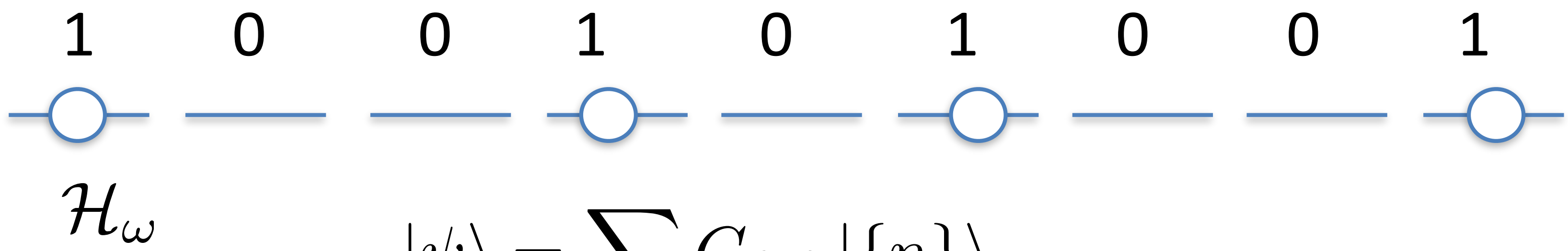
does not satisfy $M = 3$ GPP

Guiding center/second quantized presentation of quantum Hall states

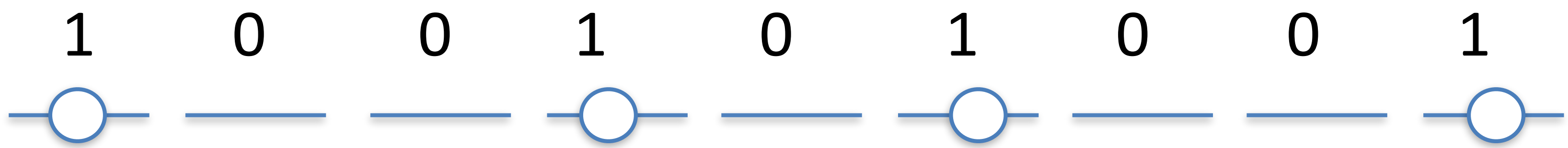
$|\{n\}\rangle =$



$$|\psi\rangle = \sum_{\{n\}} C_{\{n\}} |\{n\}\rangle$$

Guiding center/second quantized presentation of quantum Hall states

$$|\{n\}\rangle =$$



$\mathcal{H}_\omega$

$$|\psi\rangle = \sum_{\{n\}} C_{\{n\}} |\{n\}\rangle$$

Squeezing Principle for Zero Modes

"inward squeezing"

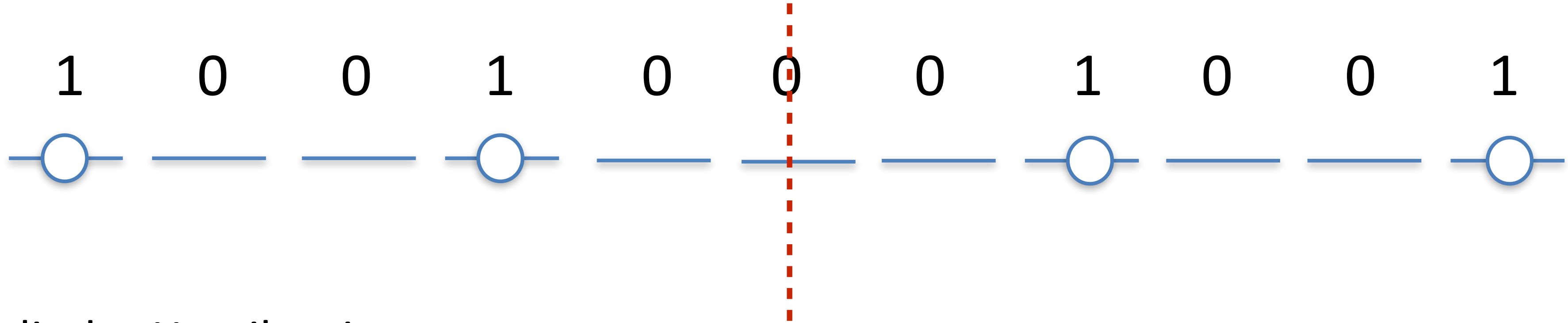
$$|\psi_{\text{Laughlin},1/3}\rangle = |1001001001001001001\ldots\rangle + \text{rest}$$

root partition

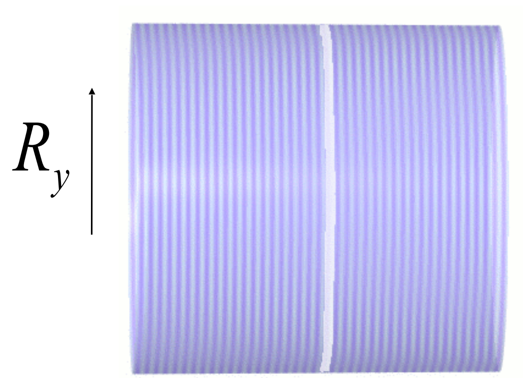
(Unentangled)

All states appearing in "rest" can be obtained from the root partition via "inward squeezing" processes

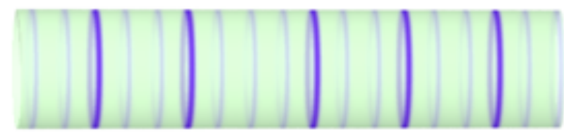
Guiding center/second quantized presentation of quantum Hall Hamiltonians



Cylinder Hamiltonian geometry:



$1/R_y$



"Tao-Thouless" state on thin cylinder

ν=1/3 Laughlin state on cylinder

charge 1/3 quasi-hole

(Necessary for multi-Landau Levels)

Entangled Pauli Principles

(Entangled Root Partition)

3 Landau levels

$$H_{\mathsf{TK}} = P_n \, \nabla^2 \delta^2(z_1 - z_2) P_n$$

P_n : projection onto first n Landau levels

3 Landau levels

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P_n : projection onto first n Landau levels

Recall: n=1 $\longleftrightarrow$ 1/3 Laughlin state

n=2 $\longleftrightarrow$ 2/5 Jain state

3 Landau levels

$$H_{\mathsf{TK}} = P_n \nabla^2 \delta^2(z_1 - z_2) P_n$$

P_n: projection onto first n Landau levels

Recall: p=1 n=1 $\longleftrightarrow$ 1/3 Laughlin state

p=1 n=2 $\longleftrightarrow$ 2/5 Jain state

$$\nu = \frac{n}{2np+1}$$

3 Landau levels

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$$\nu = \frac{n}{2np+1}$$

Seems like: The n=3 Hamiltonian should stabilize the n=3, p=1 (3/7) Jain state

3 Landau levels

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p=1 n=2 $\longleftrightarrow$ 2/5 Jain state

$$\nu = \frac{n}{2np+1}$$

Seems like: The n=3 Hamiltonian should stabilize the n=3, p=1 (3/7) Jain state

not quite true!!

Derivation of entangled Pauli principles

$$\hat{H}_{\mathsf{TK}} = \sum_J \sum_{\lambda=1}^{8} E_\lambda \mathcal{T}_J^{(\lambda)\dagger} \mathcal{T}_J^{(\lambda)}$$

zero mode condition:

$$\hat{H}_{\mathsf{TK}} |\psi\rangle = 0 \Leftrightarrow \mathcal{T}_J^{(\lambda)} |\psi\rangle = 0 \quad \forall \quad J, \lambda$$

$$\mathcal{T}_J^{(\lambda)} = \sum_{x, n_1, n_2} \eta^{\lambda}_{J,x,n_1,n_2} c_{n_1, J-x} c_{n_2, J+x}$$

Derivation of entangled Pauli principles

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$$\hat{H}_{\mathsf{TK}} |\psi\rangle = 0 \Leftrightarrow \mathcal{T}_J^{(\lambda)} |\psi\rangle = 0 \quad \forall \quad J, \lambda$$

Slater-determinant expansion:

$$|\psi\rangle = \sum_{\{(n_1,J_1)\dots(n_N,J_N)\}} \mathcal{C}_{(n_1,J_1)\dots(n_N,J_N)} c^{\dagger}_{(n_1,J_1)} \dots c^{\dagger}_{(n_N,J_N)} |0\rangle$$

$$\mathcal{T}_J^{(\lambda)} = \sum_{x,n_1,n_2} \eta^{\lambda}_{J,x,n_1,n_2} c_{n_1,J-x} c_{n_2,J+x}$$

Derivation of entangled Pauli principles

Slater-determinant expansion:

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Derivation of entangled Pauli principles

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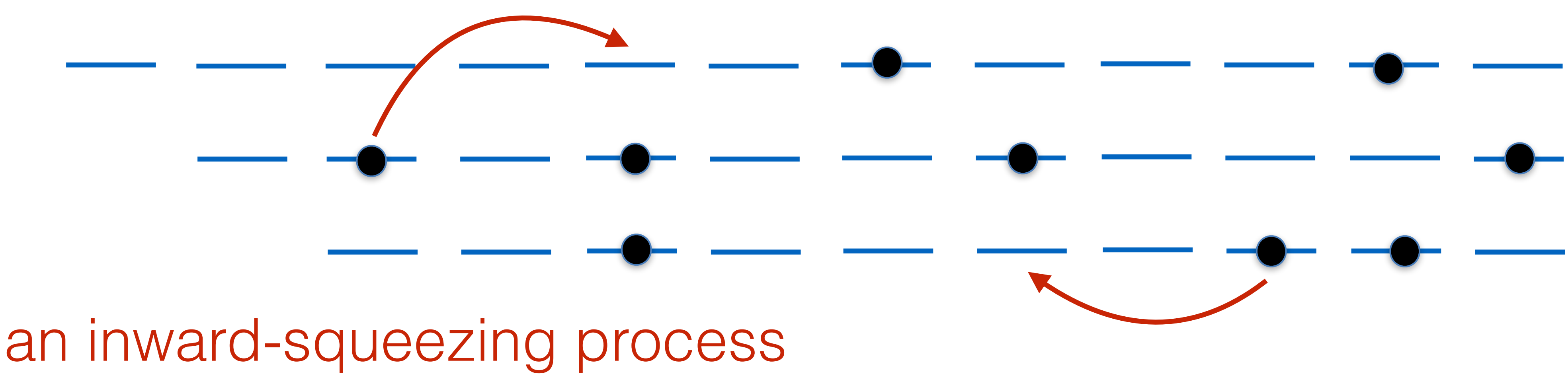
"maximal" or "non-expandable" Slater determinants:

Those that cannot be obtained from others in the expansion through "inward squeezing processes"



Derivation of entangled Pauli principles

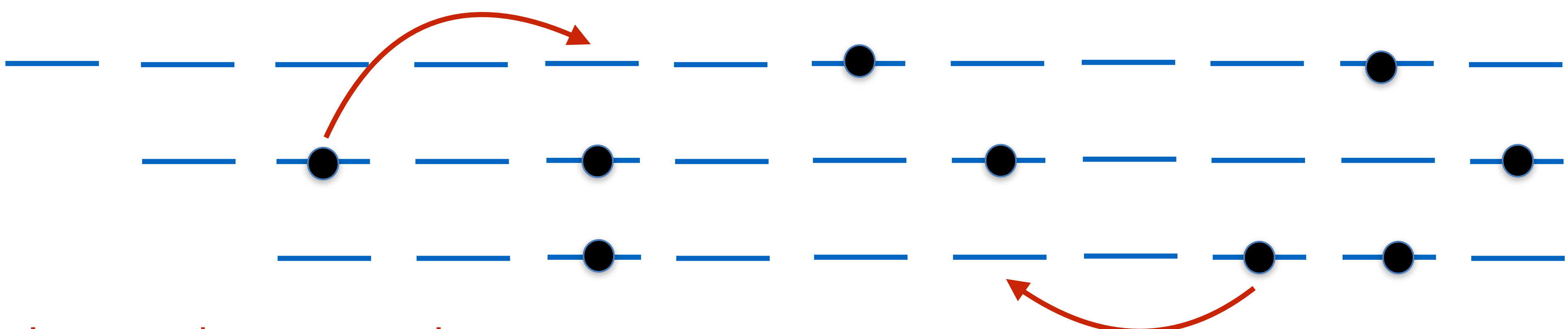
Slater-determinant expansion:

$$|\psi\rangle = |\text{root}\rangle + |\text{rest}\rangle$$

orthogonal to “root”

“maximal” or “non-expandable” Slater determinants:

Those that cannot be obtained from others in the expansion through “inward squeezing processes”



an inward-squeezing process

Physical properties from EPP: degeneracies

two "densest" patterns at $\nu = 1/2$:

…1100110011001100110011001100…

singlet

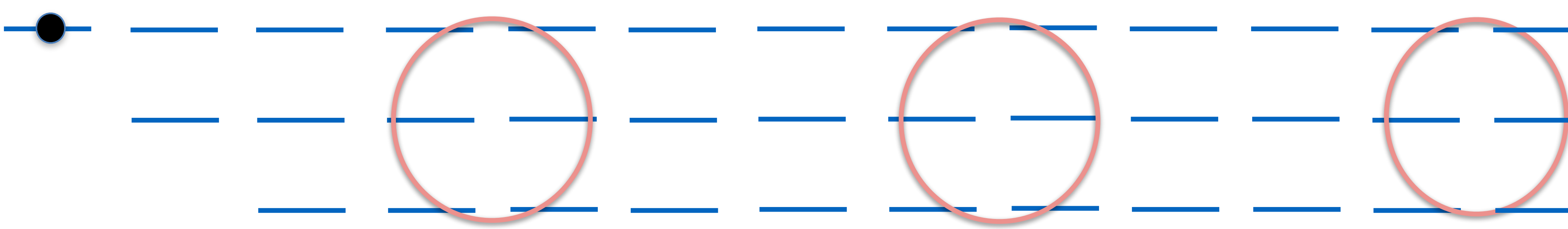
…1010101010101010101010101010…

AKLT MPS-type ground state!!

boundary condition on **disk**: leading orbital cannot participate in entanglement!

unique "densest" pattern for disk=root state of Jain 211 wave function

1 0 0 1 1 0 0 1 1 0 0 1 1



Physical properties from EPP: braiding statistics

…1100110011001100101010101010101…

$\uparrow$

charge-1/4 domain-wall *with* spin

if we ignore spin, the situation is very much
like for the $\nu = 1/2$ Moore-Read state

Turns out, the patterns "know" something about the statistics:

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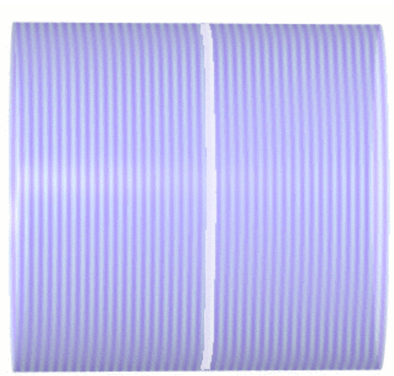

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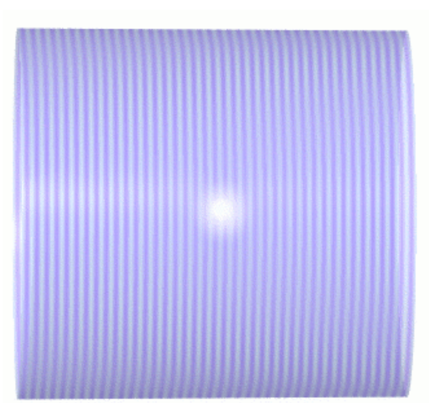

→ …1100110011001100101010101010101…

delocalize
(over magnetic length)

Parton Wavefunctions

$$Z = \{z_1, z_2, \cdots, z_N\}$$

$$\Psi_\nu(Z,\bar{Z}) = \Phi_{\nu_1}(Z,\bar{Z})\Phi_{\nu_2}(Z,\bar{Z})\cdots\Phi_{\nu_M}(Z,\bar{Z}) \equiv [\nu_1, \nu_2, \cdots, \nu_M],$$

$$\Phi_{\nu_i}(Z,\bar{Z}) = \frac{1}{\sqrt{N!}}\begin{vmatrix} \varphi_{\alpha_1}(z_1,\bar{z}_1) & \varphi_{\alpha_1}(z_2,\bar{z}_2) & \cdots & \varphi_{\alpha_1}(z_N,\bar{z}_N) \\ \varphi_{\alpha_2}(z_1,\bar{z}_1) & \varphi_{\alpha_2}(z_2,\bar{z}_2) & \cdots & \varphi_{\alpha_2}(z_N,\bar{z}_N) \\ \vdots & \vdots & \cdots & \vdots \\ \varphi_{\alpha_N}(z_1,\bar{z}_1) & \varphi_{\alpha_N}(z_2,\bar{z}_2) & \cdots & \varphi_{\alpha_N}(z_N,\bar{z}_N) \end{vmatrix}$$

$$\nu = (\sum_{i=1}^{M} \nu_i^{-1})^{-1}$$

Nice symmetry properties

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The densest zero-energy (unique) ground states of Frustration-free Quantum Hall Hamiltonians are parton states.

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Conclusions

- QH Systems can be viewed as a soup of Pairing Systems
- We determined the exact spectrum of the QH-Gaudin problem
- Frustration-free property (zero modes) of QH Hamiltonians and a Squeezing Principle for zero modes
- Systematic construction of Frustration-free QH Hamiltonians for several interesting filling fractions: Charge-Statistics
- Quasi-hole generators and String Order Parameter (sym-poly)