

# Quantum Monte Carlo and local chiral interactions: light and not-so-light nuclei

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MICHIGAN STATE  
UNIVERSITY



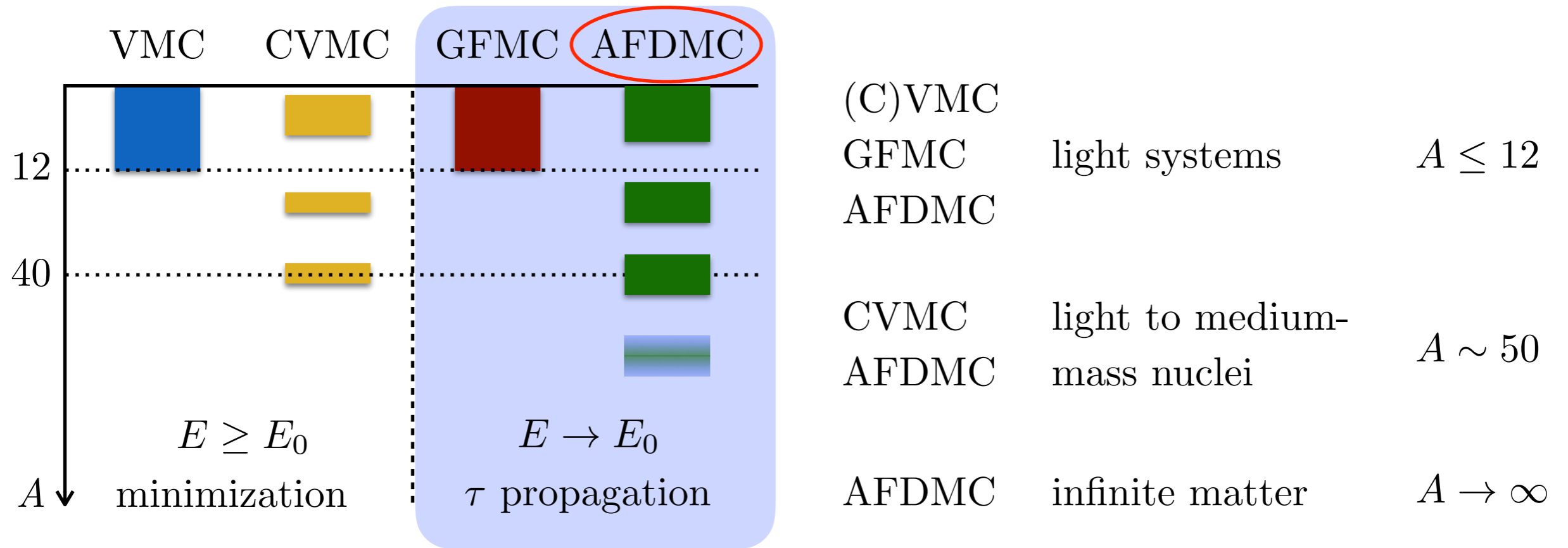
NUCLEI  
Nuclear Computational Low-Energy Initiative



INT Program INT-18-2b, Seattle, August 28, 2018

- ✓ Quantum Monte Carlo
  - ▶ AFDMC
  
- ✓ Nuclear Hamiltonians
  - ▶ Phenomenological potentials
  - ▶ Local chiral potentials
  
- ✓ AFDMC & local chiral potentials
  - ▶ Results
  
- ✓ Summary & Perspectives

**Goal:** solve the many-body problem for correlated systems in a non perturbative fashion



**Pros:**

- ▶ Truly *ab-initio*: work with bare interactions.
- ▶ Good for strongly correlated systems.
- ▶ Stochastic method: errors quantifiable and systematically improvable.  $\sigma \sim 1/\sqrt{\mathcal{N}}$

**Cons:**

- ▶ Limitations in the systems and/or in the interaction to be used.
- ▶ Can be computationally expensive.

## Auxiliary Field Diffusion Monte Carlo (AFDMC)

$$|\Psi_V\rangle = \left[ 1 + \sum_{i<j<k} U_{ijk} \right] \left[ 1 + \sum_{i<j} U_{ij} \right] \left[ \prod_{i<j} f_c(r_{ij}) \right] \mathcal{A}|\Phi\rangle$$

3-body corr
2-body corr (linear)
mean field

$\mathcal{N}_{\text{det}}$  determinants of single-particle orbitals + spinors: proper  $(J^\pi, T)$

1. minimization: variational search of optimal parameters for  $|\Psi_V\rangle \longrightarrow$  SR, LM
2. propagation in imaginary time:  $e^{-H\tau}|\Psi_V\rangle \xrightarrow{\tau \rightarrow \infty} |\Psi_0\rangle$ 

$\tau = \mathcal{M}d\tau$ 
 $\mathcal{M} \gg 1$ 
 $d\tau \ll 1$

- ▶ spatial degrees of freedom: diffusion of positions in coordinate space
- ▶ spin-isospin degrees of freedom: Hubbard-Stratonovich transformation

$$e^{-\frac{1}{2}\lambda d\tau \mathcal{O}^2} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx e^{-\frac{1}{2}x^2} e^{\sqrt{-\lambda d\tau} x \mathcal{O}}$$

*Note:* 2-body operators only

auxiliary fields
spin-isospin rotations

- ▶ sign problem: constrained path approximation + unconstrained evolution

ground-state energies within 1-2% with respect to GFMC

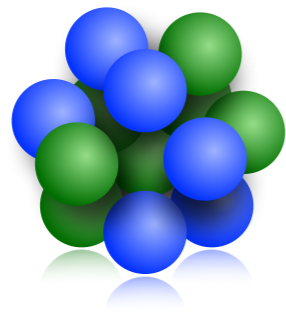
**Model:** non-relativistic nucleons interacting with an effective nucleon-nucleon (NN) force and three-nucleon interaction (NNN)

$$H = -\frac{\hbar^2}{2m_N} \sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk} + \dots$$

$v_{ij}$  fit to NN scattering data & deuteron

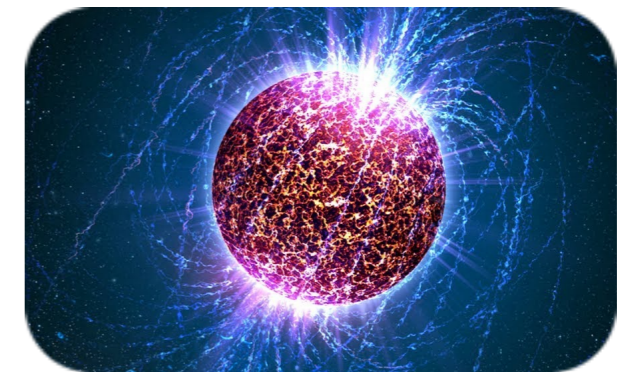
$v_{ijk}$  fit to properties of nuclei

**Question:** is it possible to describe nuclei and their global properties from microscopic nuclear Hamiltonians constructed to reproduce only few-body observables, while simultaneously predicting properties of matter?



$R \sim \text{fm} \sim 10^{-15} \text{ m}$   
 $M \sim 10^{-27} \text{ kg}$

← nuclear potentials ? →

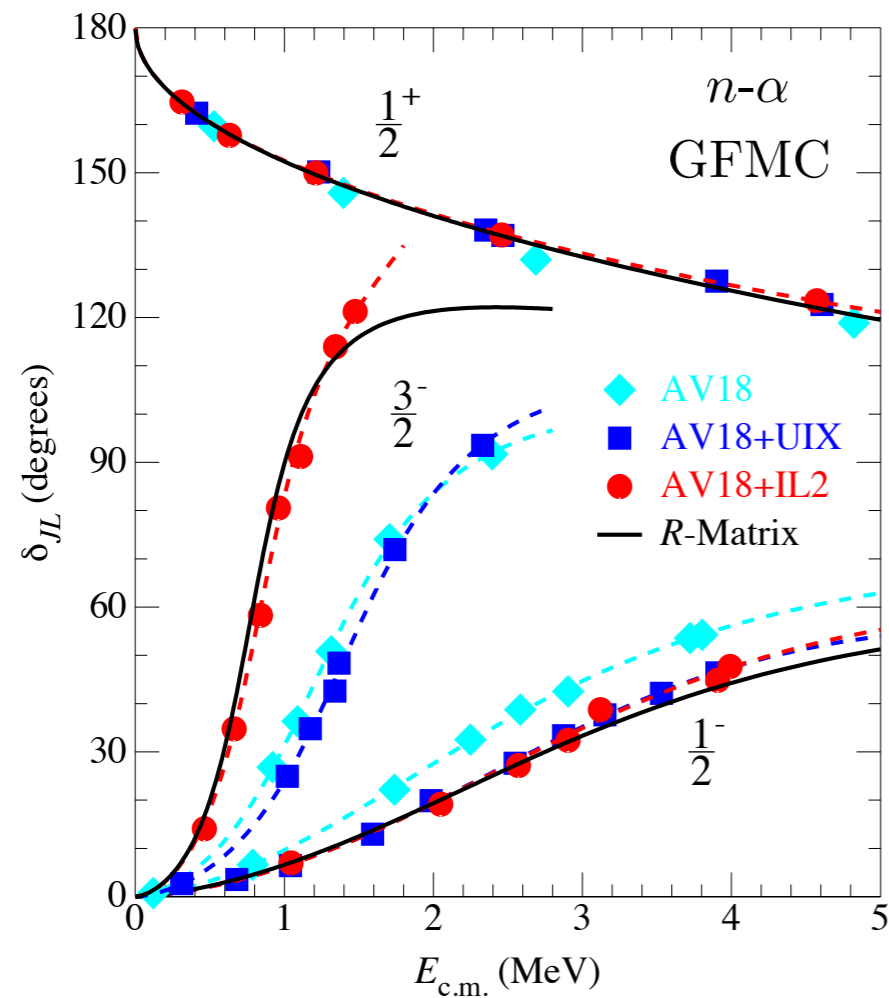


$R \sim 10 \text{ km} \sim 10^4 \text{ m}$   
 $M \sim 1.4 M_{\odot} \sim 10^{30} \text{ kg}$

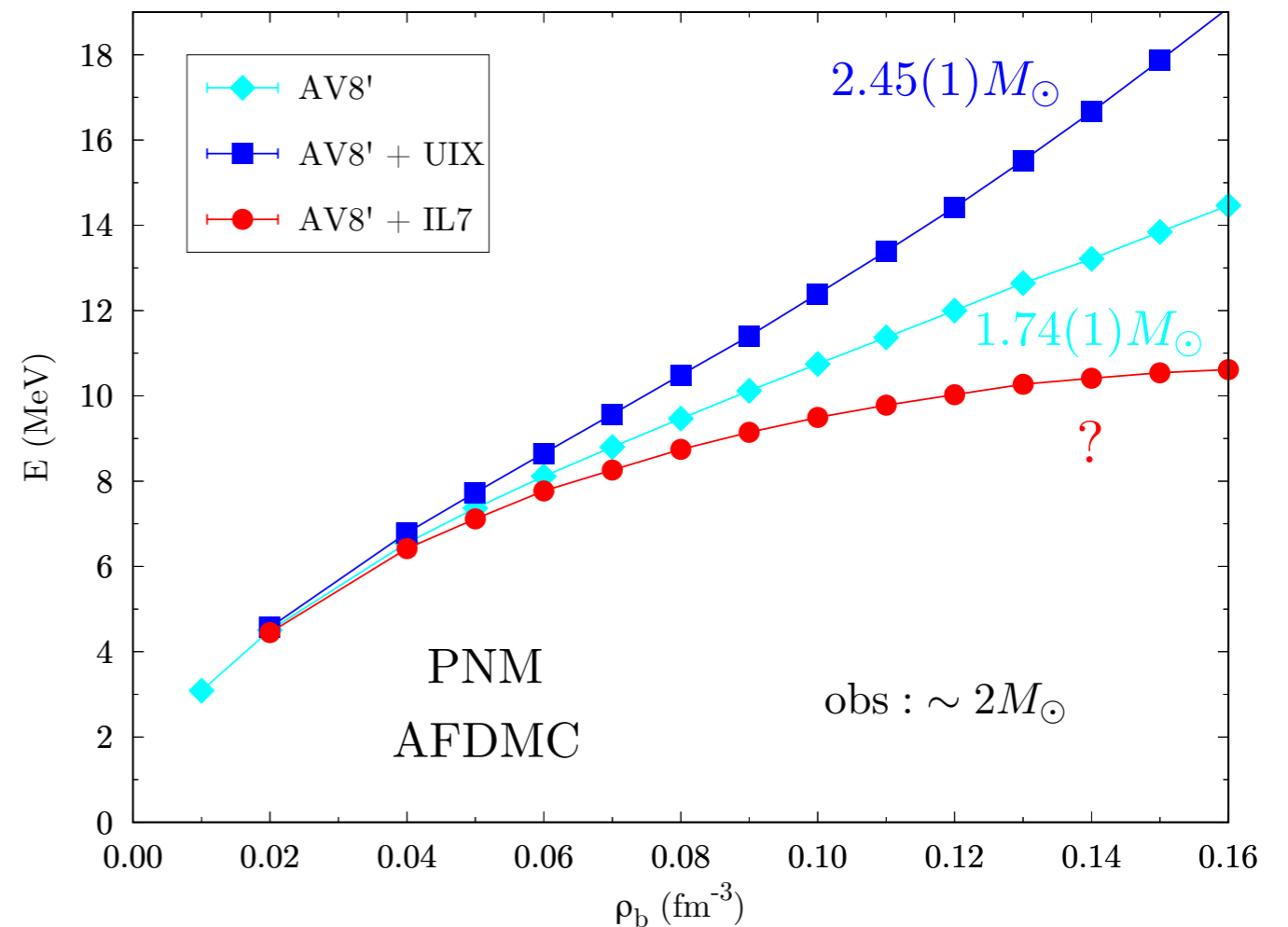
- ✓ remarkable description of the physics of light nuclei up to  $^{12}\text{C}$

J. A. Carlson *et al.*, Rev. Mod. Phys. **87**, 1067 (2015)

- ✦ unrealistic description of pure neutron matter & neutron star structure



K. M. Nollett *et al.*, Phys. Rev. Lett. **99**, 022502 (2007)



P. Maris *et al.*, Phys. Rev. C **87**, 054318 (2013)

	$NN$	$NNN$
LO $\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^0$		—
NLO $\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^2$		—
N <sup>2</sup> LO $\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^3$		

local in  
coordinate  
space



good for  
QMC

AV7

operator  
structure



ok in  
propagator

mostly

2-body  
operators



doable in  
propagator

- ▶  $\chi$ EFT: expansion in power of  $Q/\Lambda_b$ 
  - $Q \sim m_\pi \sim 140$  MeV soft scale
  - $\Lambda_b \sim m_\rho \sim 800$  MeV hard scale
- ▶ Long-range physics: given explicitly (no parameters to fit) by pion-exchanges
- ▶ Short-range physics: parametrized through contact interactions with low-energy constants (LECs) fit to low-energy data
- ▶ Many-body forces enter systematically and are related via the same LECs
- ▶ Possibility for error quantification

	$NN$	$NNN$
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<b>N<sup>2</sup>LO <math>\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^3</math></b>		

local in  
coordinate  
space



good for  
QMC

$c_D$  &  $c_E$  fit to:

- ✓  $^4\text{He}$  binding energy
- ✓  $n$ - $\alpha$  scattering phase shifts

- ▶  $\chi$ EFT: expansion in power of  $Q/\Lambda_b$ 
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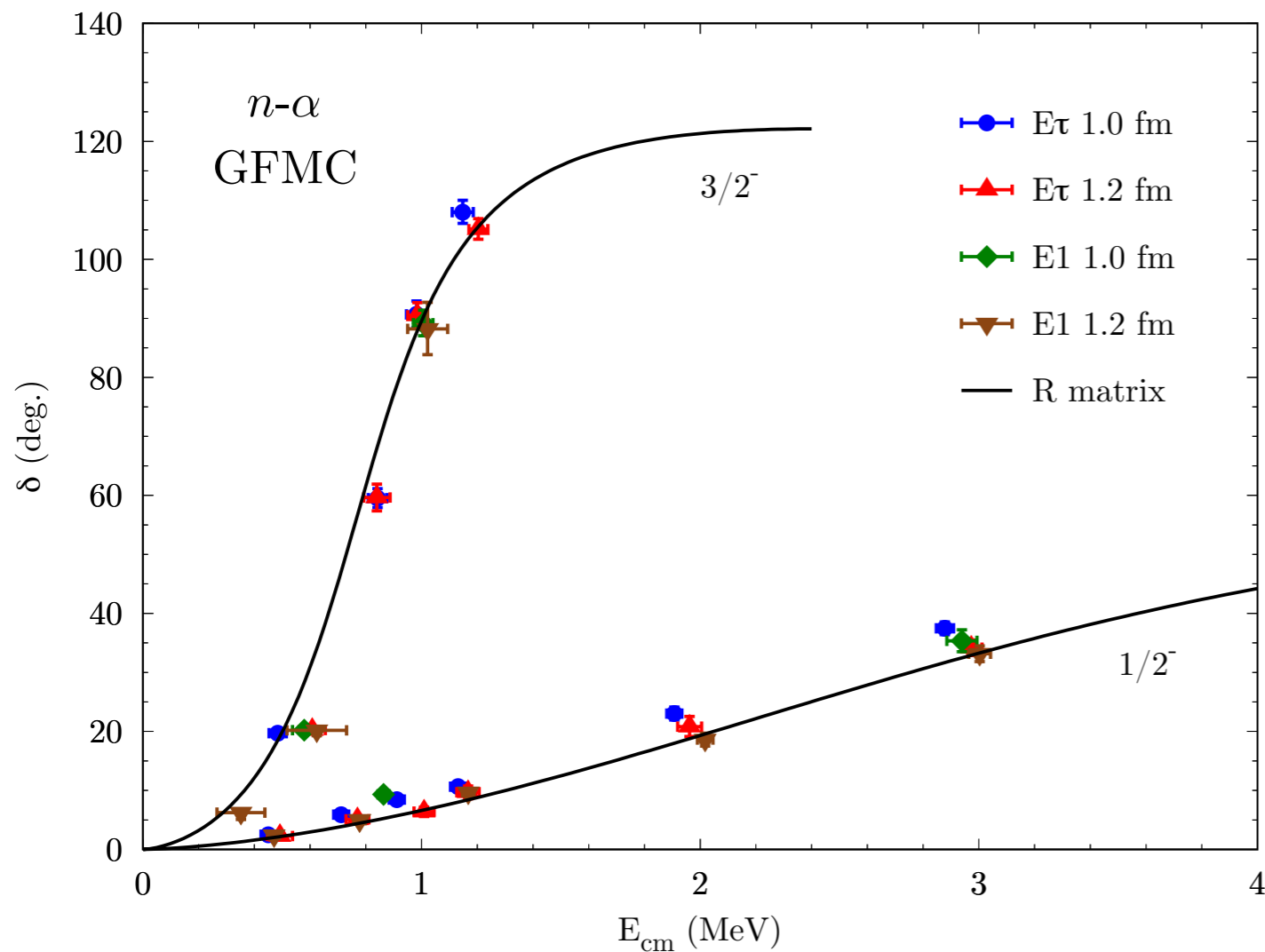
info on  $T = \frac{3}{2}$  physics



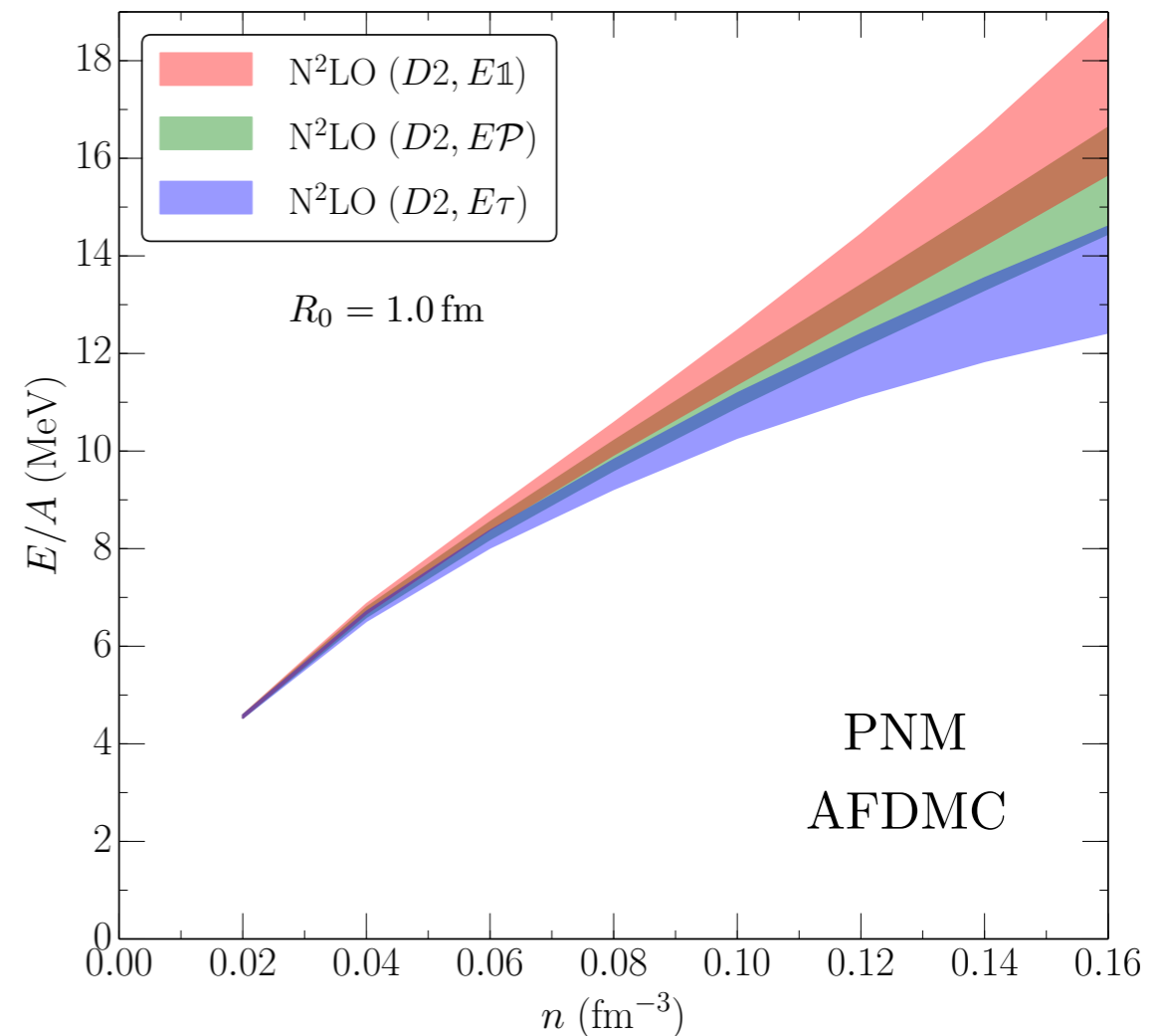
Some details:

✓ coordinate-space cutoff:  $R_0 = 1.0$  fm (harder)       $R_0 = 1.2$  fm (softer)

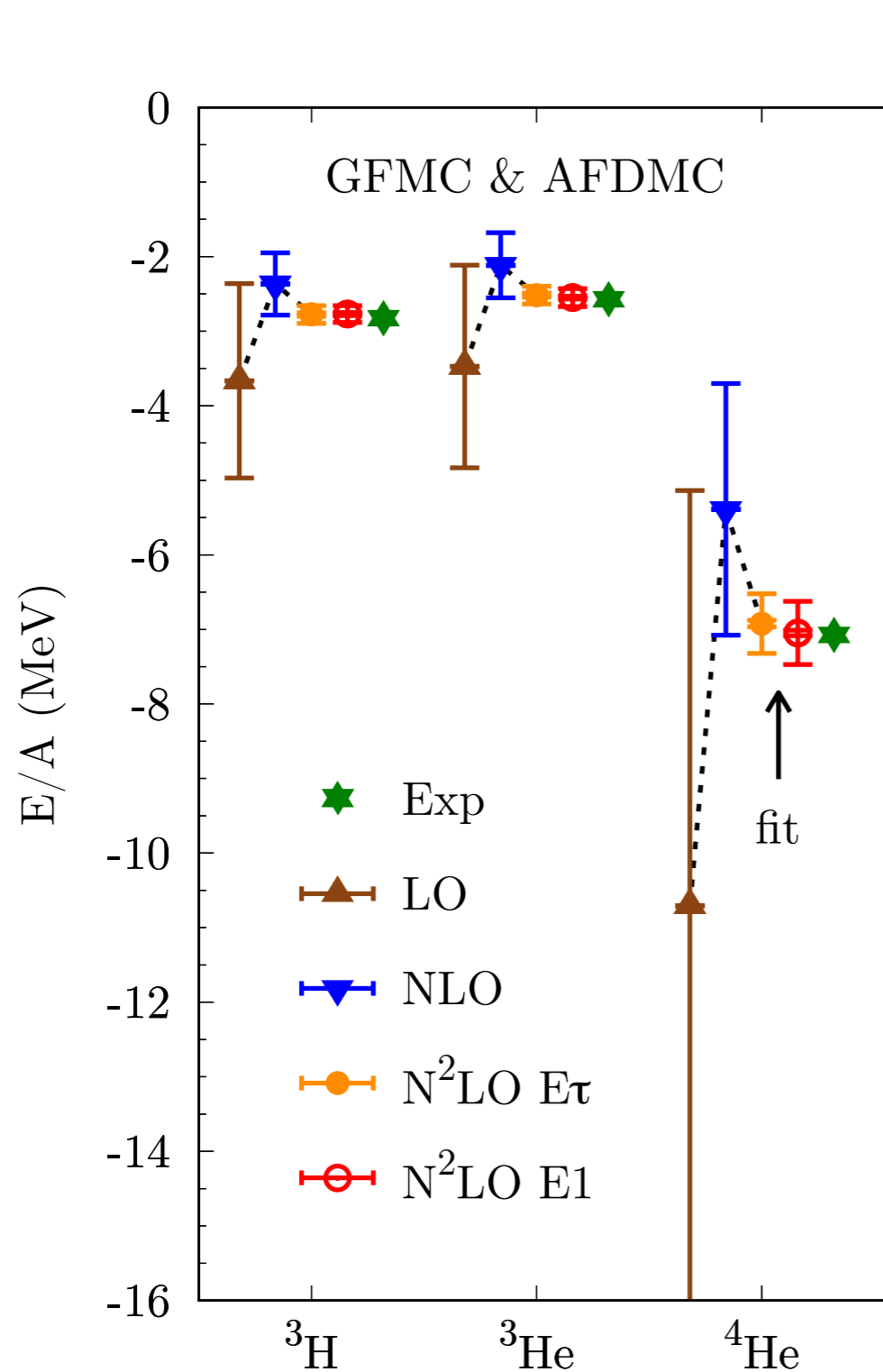
✓ different possible operator structures:  $V_D \longrightarrow D1, D2$        $V_E \longrightarrow E\tau, E\mathbb{1}$



D.L. *et al.*, Phys. Rev. C **97**, 044318 (2018)

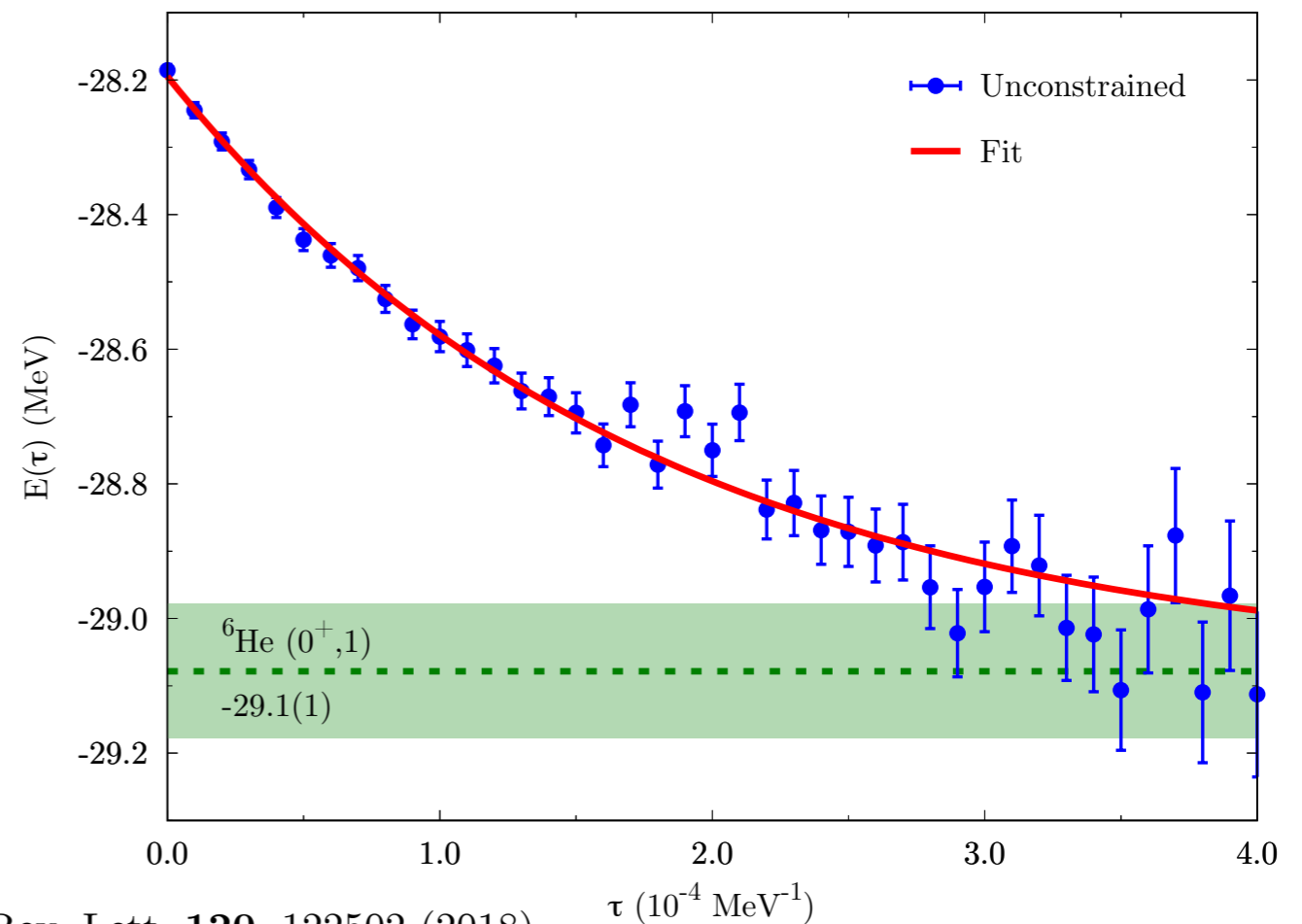


J. E. Lynn *et al.*, Phys. Rev. Lett. **116**, 062501 (2016)



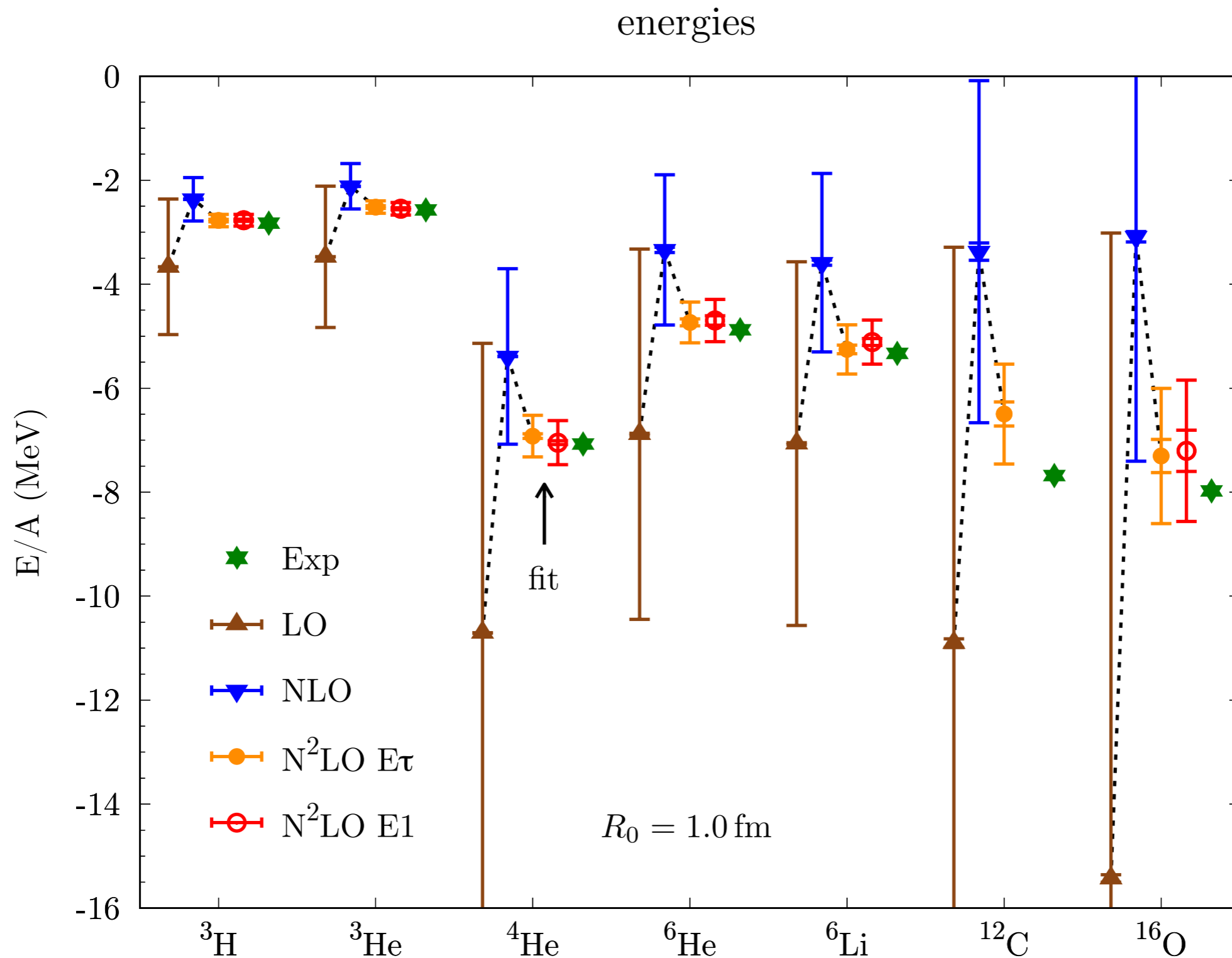
first AFDMC calculations:

- ✓ full 2+3-body potentials & correlations
- ✓ closed- and open-shell systems
- ✓ unconstrained evolution
- ✓ statistical and theoretical error estimate



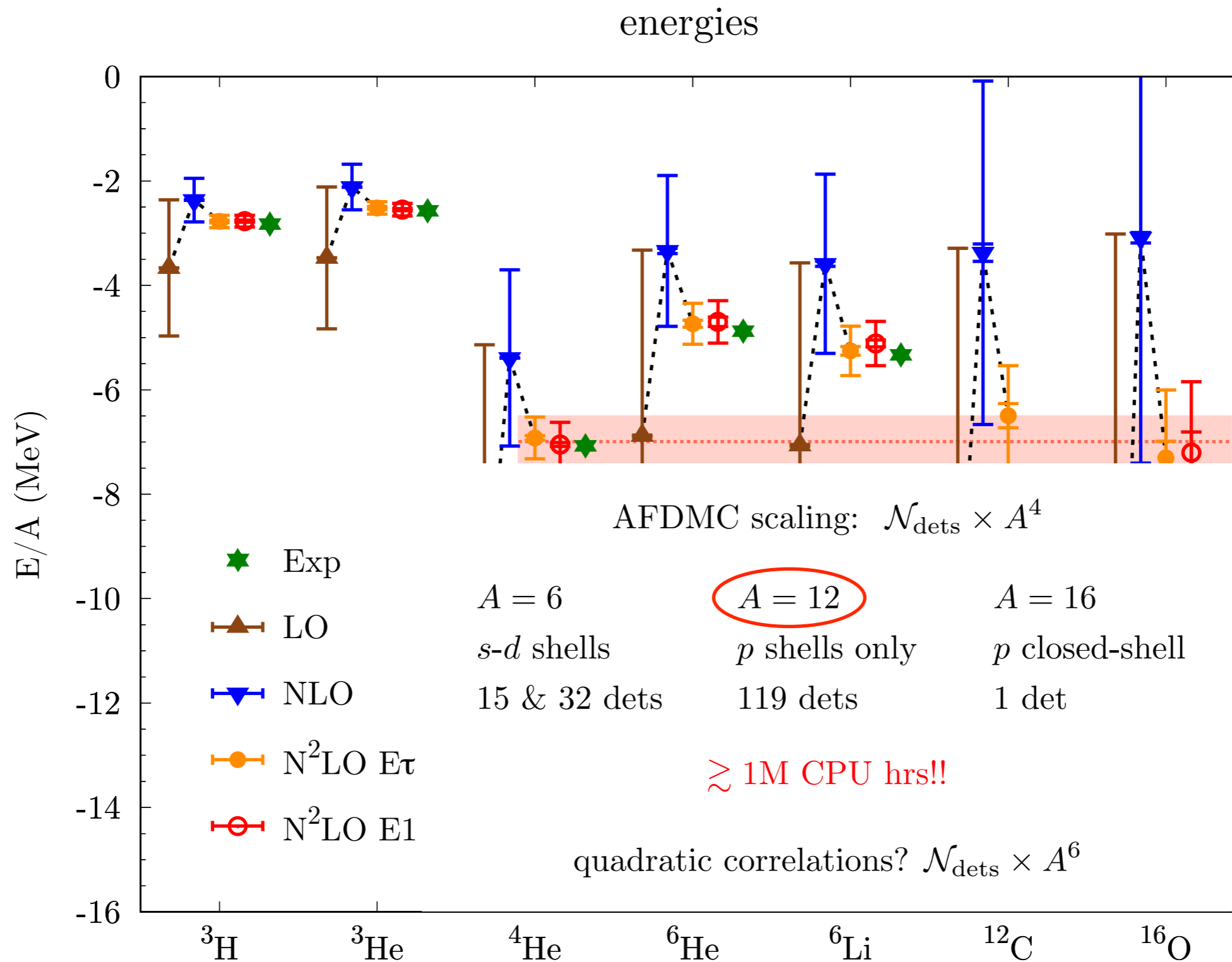
D.L. *et al.*, Phys. Rev. Lett. **120**, 122502 (2018)

D.L. *et al.*, Phys. Rev. C **97**, 044318 (2018)



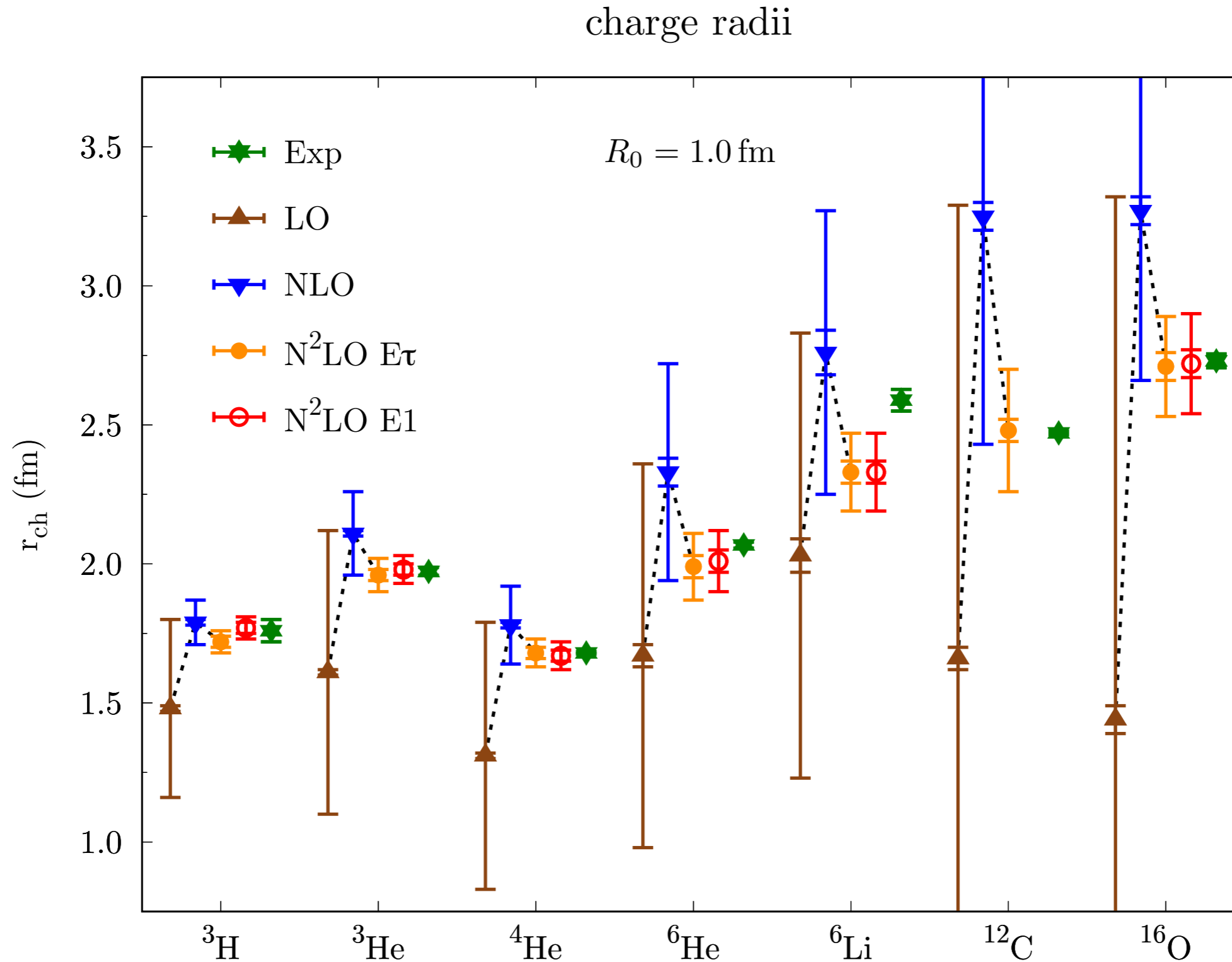
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D.L. *et al.*, Phys. Rev. C **97**, 044318 (2018)



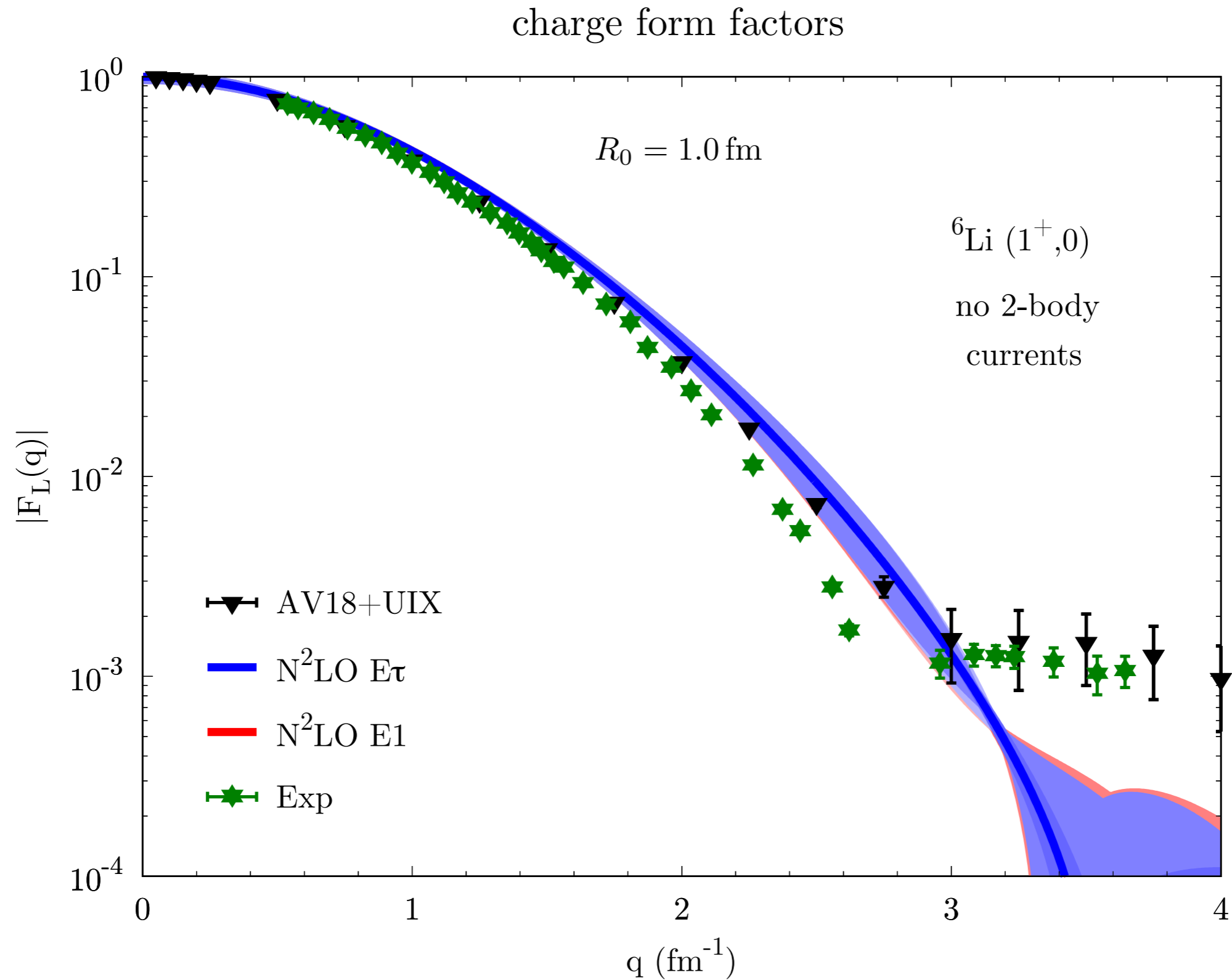
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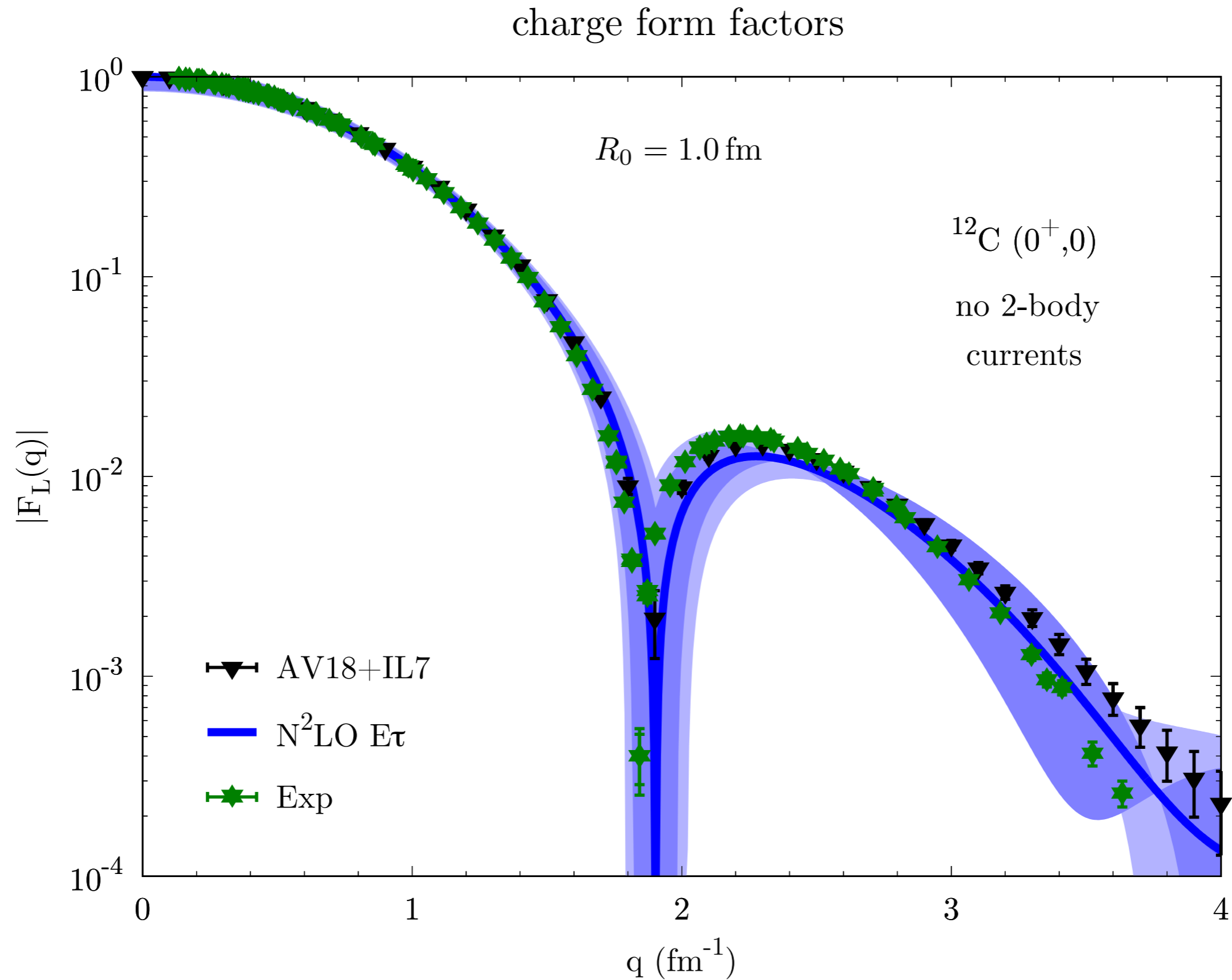
D.L. *et al.*, Phys. Rev. C **97**, 044318 (2018)

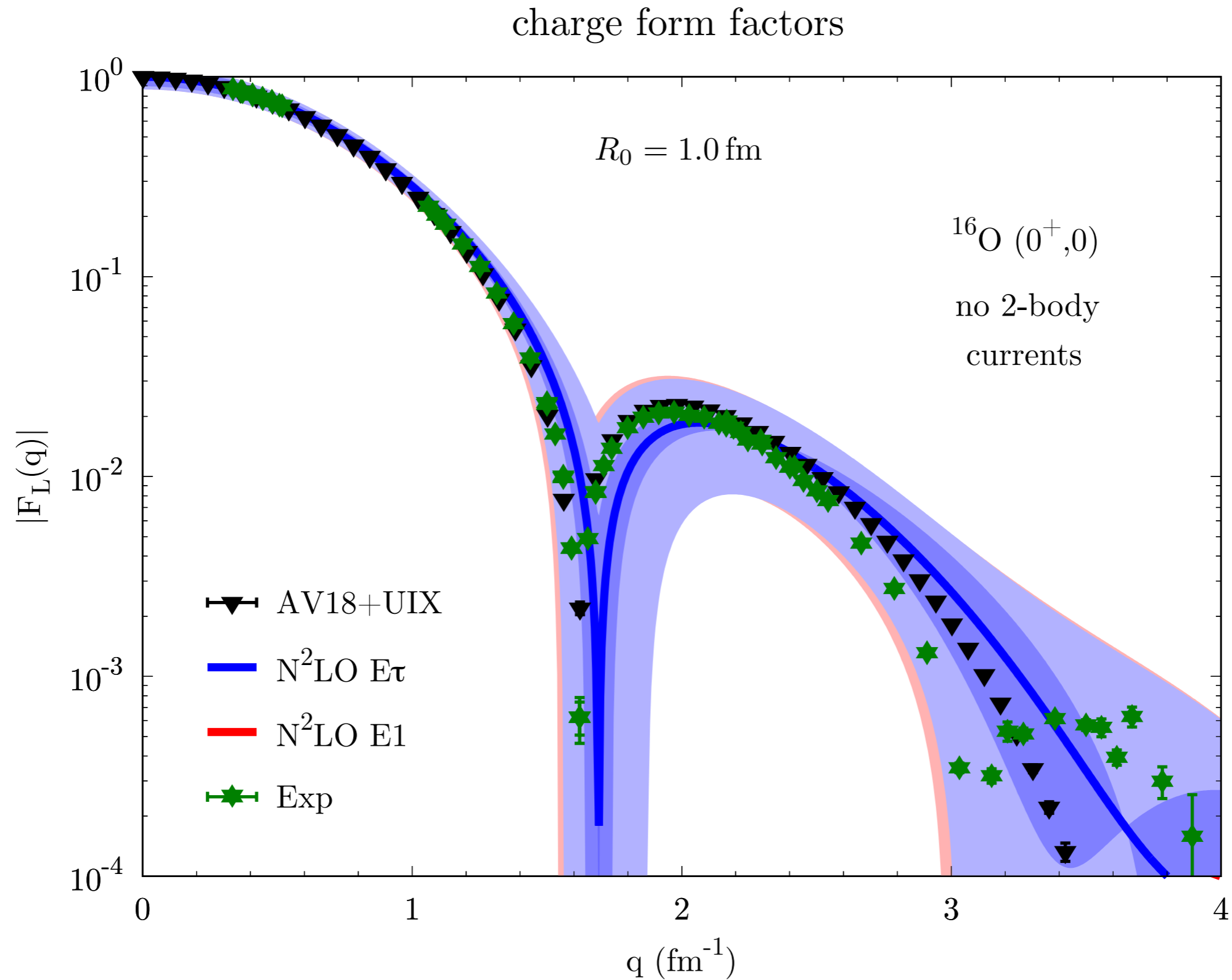


D.L. *et al.*, Phys. Rev. Lett. **120**, 122502 (2018)

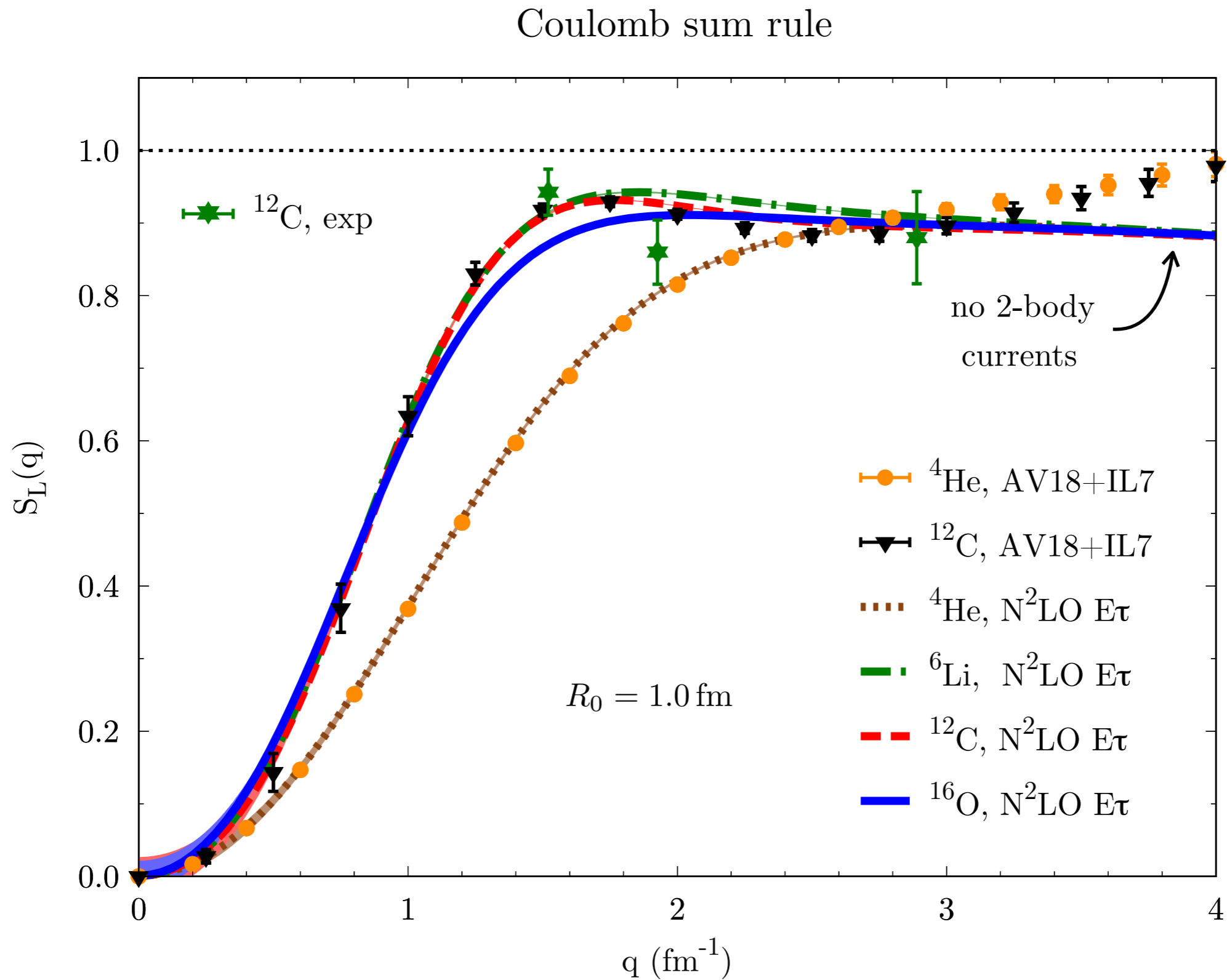
D.L. *et al.*, Phys. Rev. C **97**, 044318 (2018)



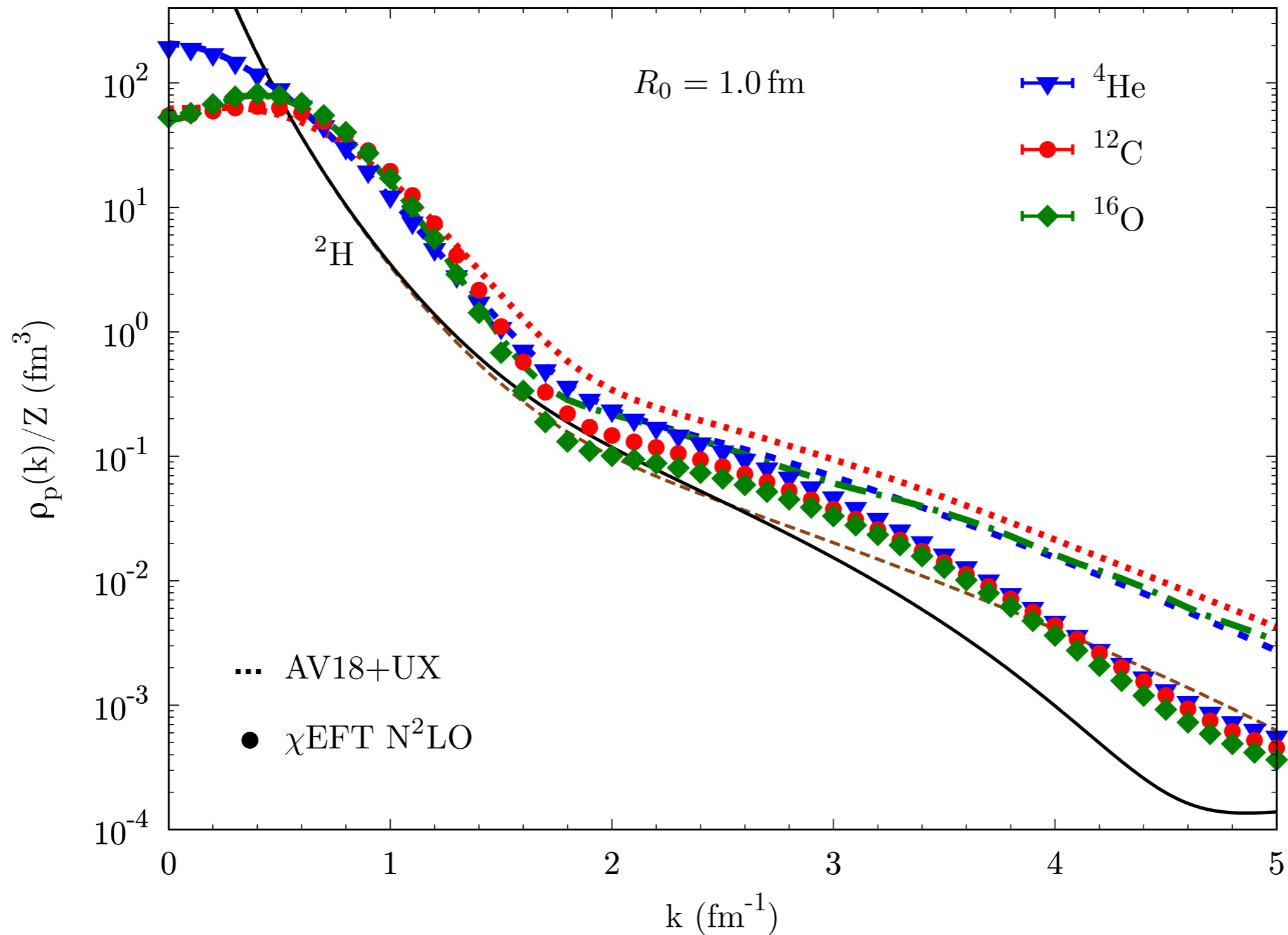




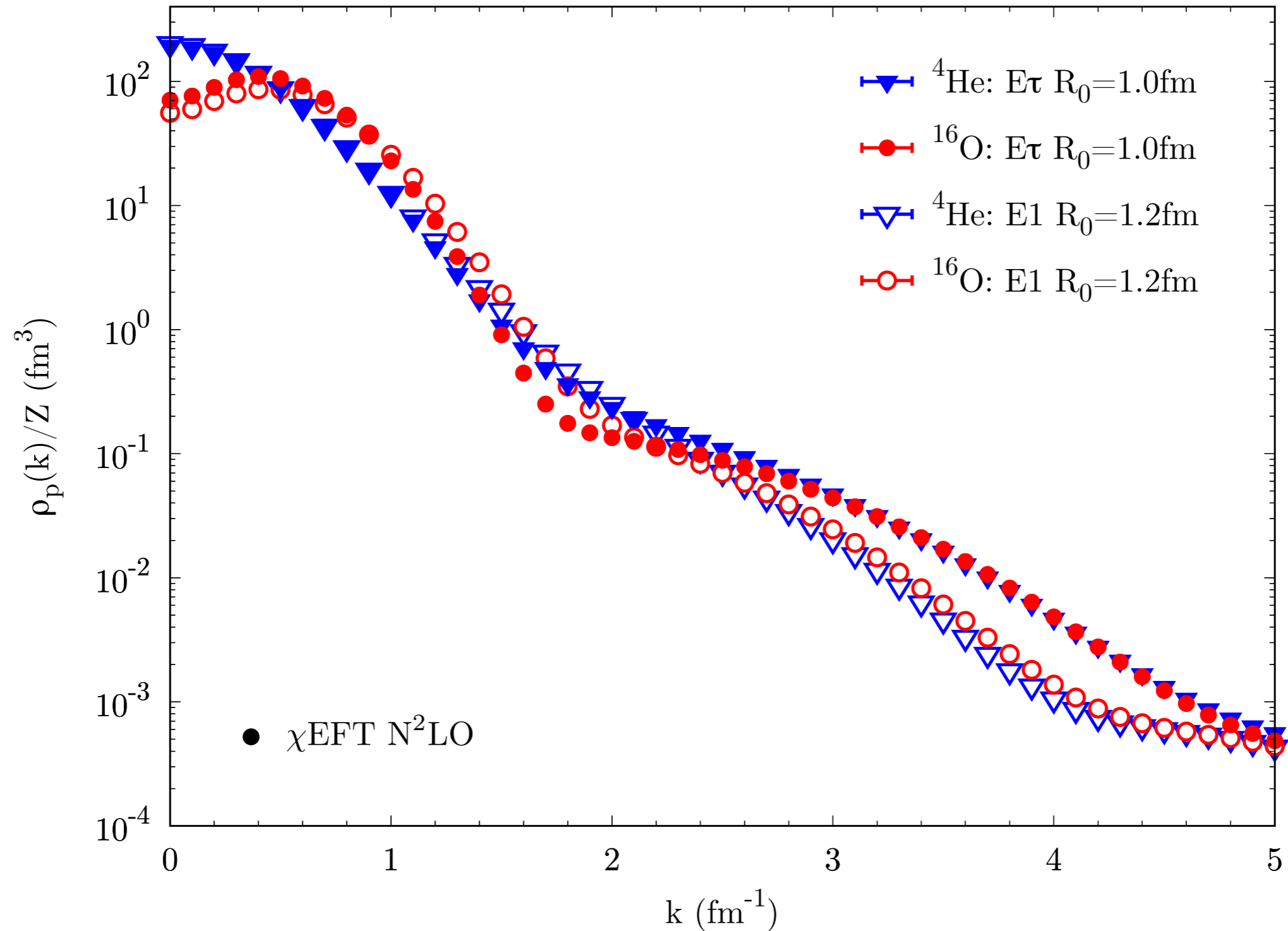




## single-nucleon momentum distributions

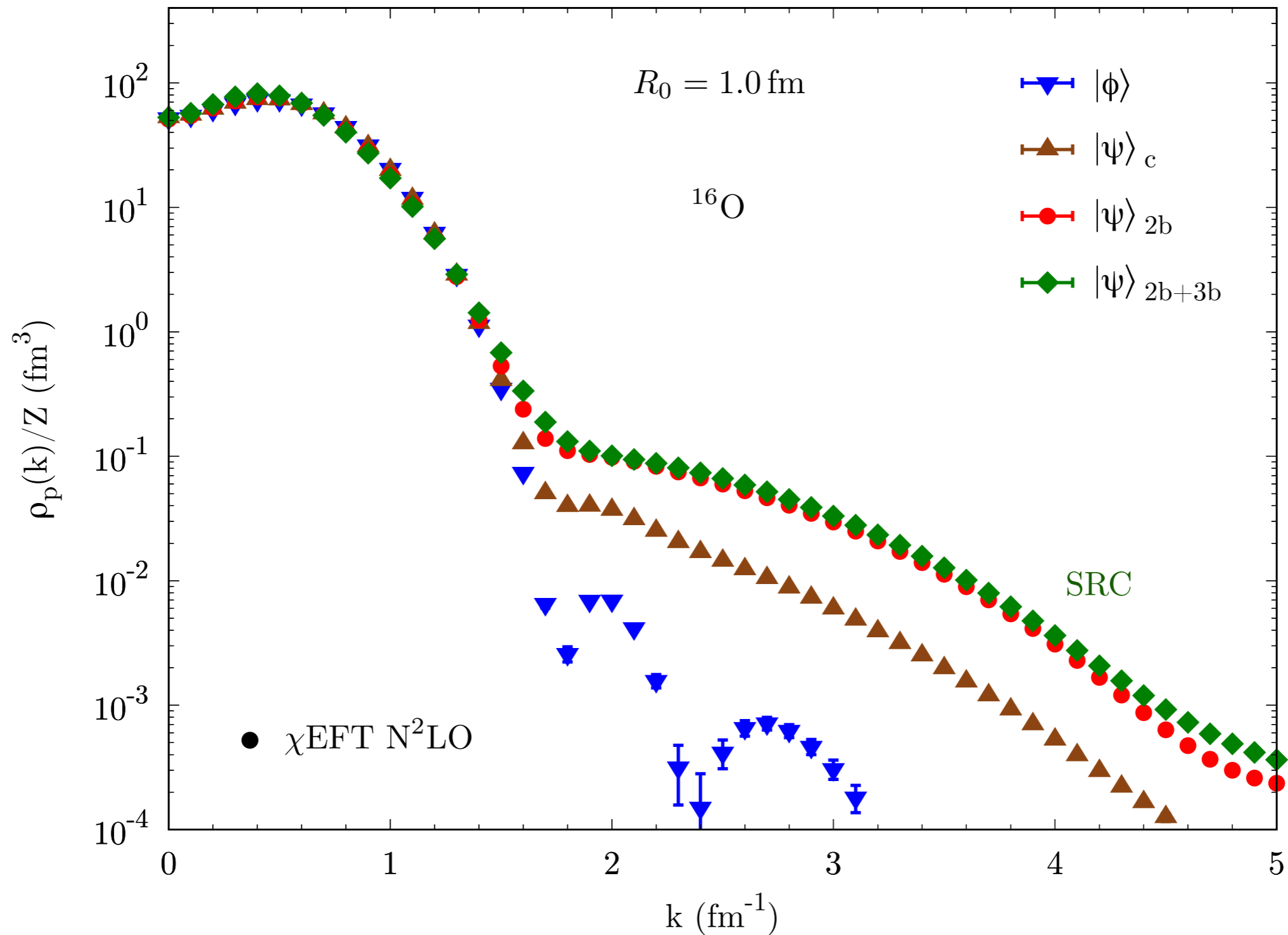


single-nucleon momentum distributions

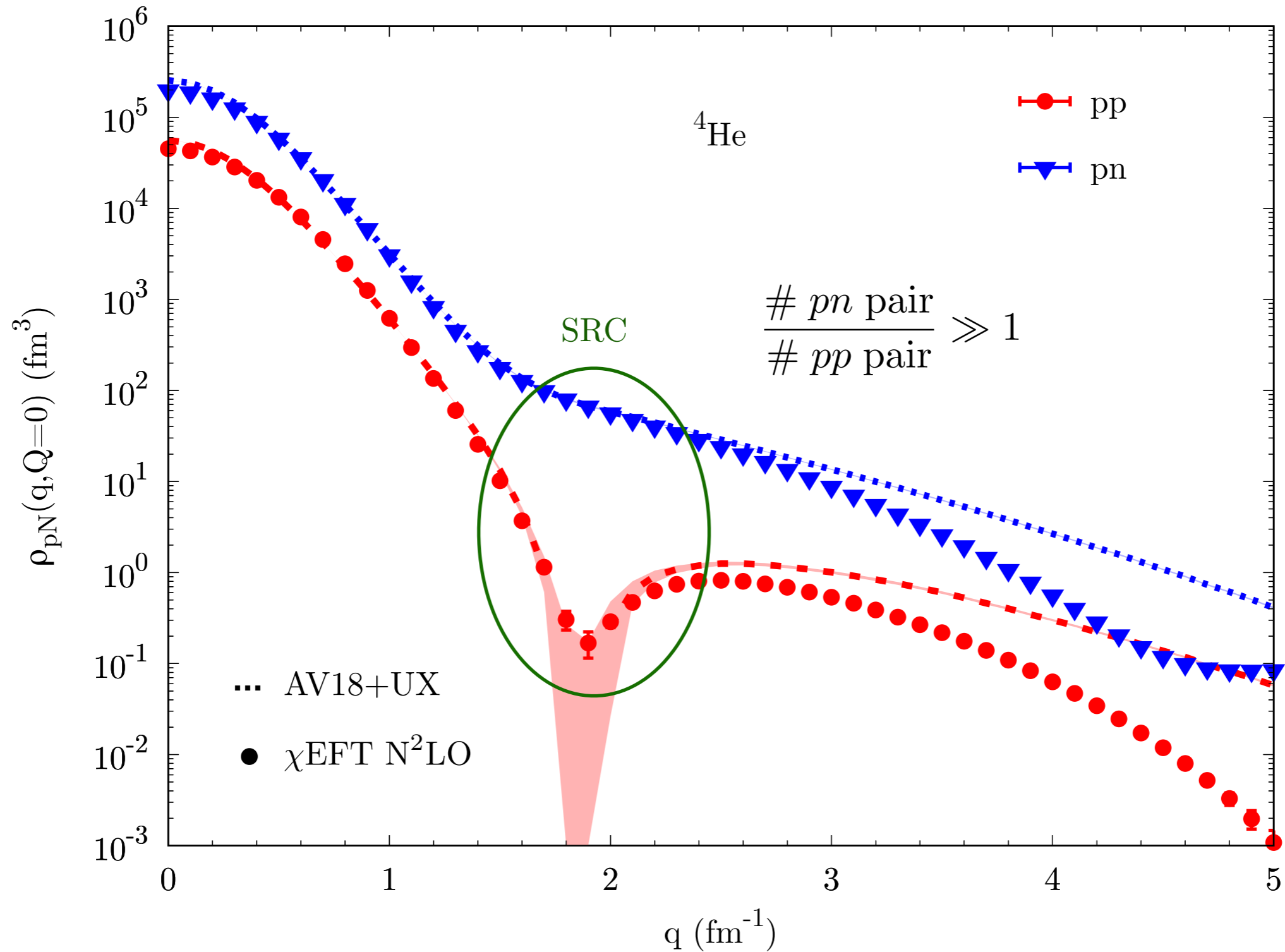


D.L. *et al.*, Phys. Rev. C **98**, 014322 (2018)  
(tables available as Supplemental Material)

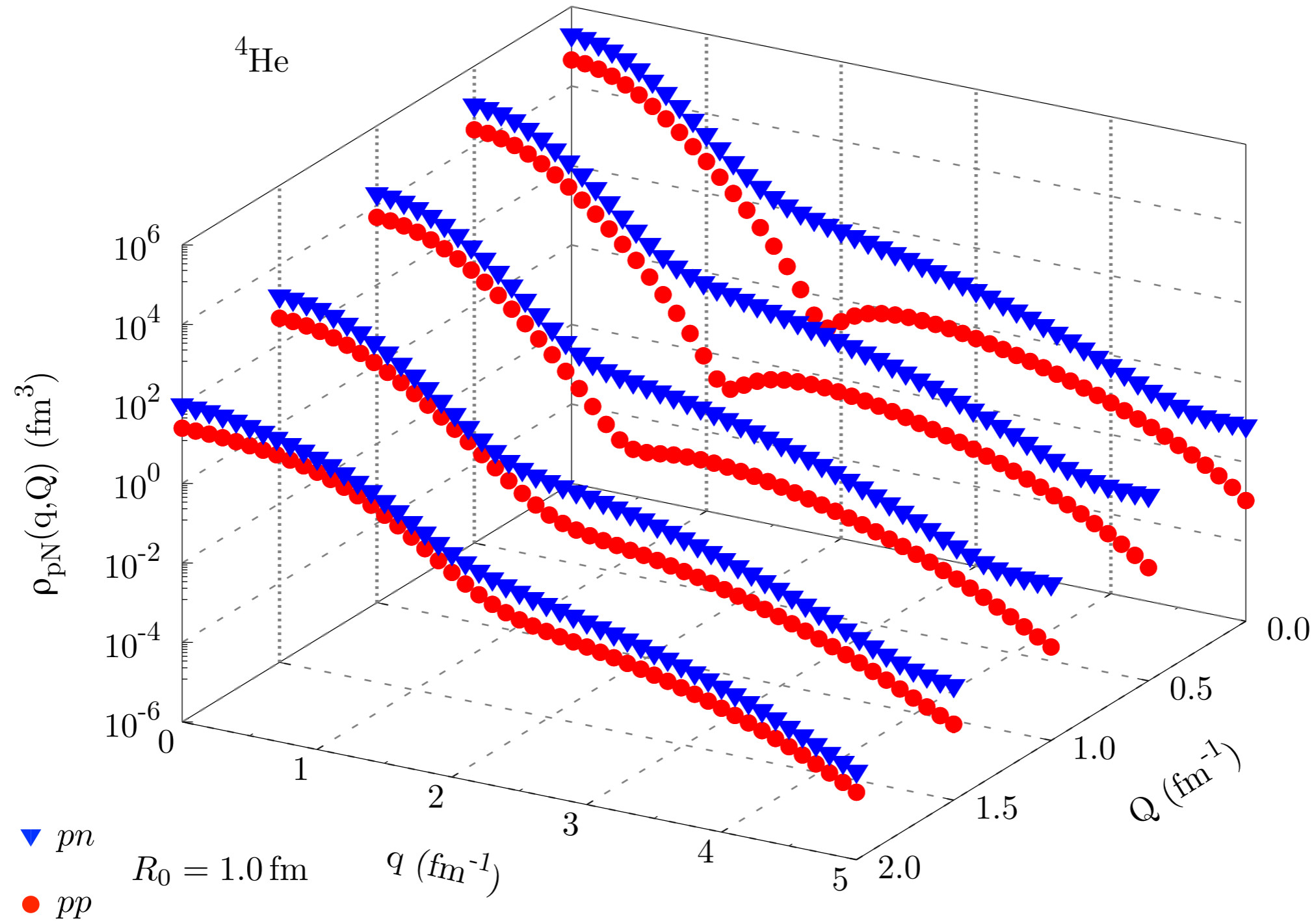
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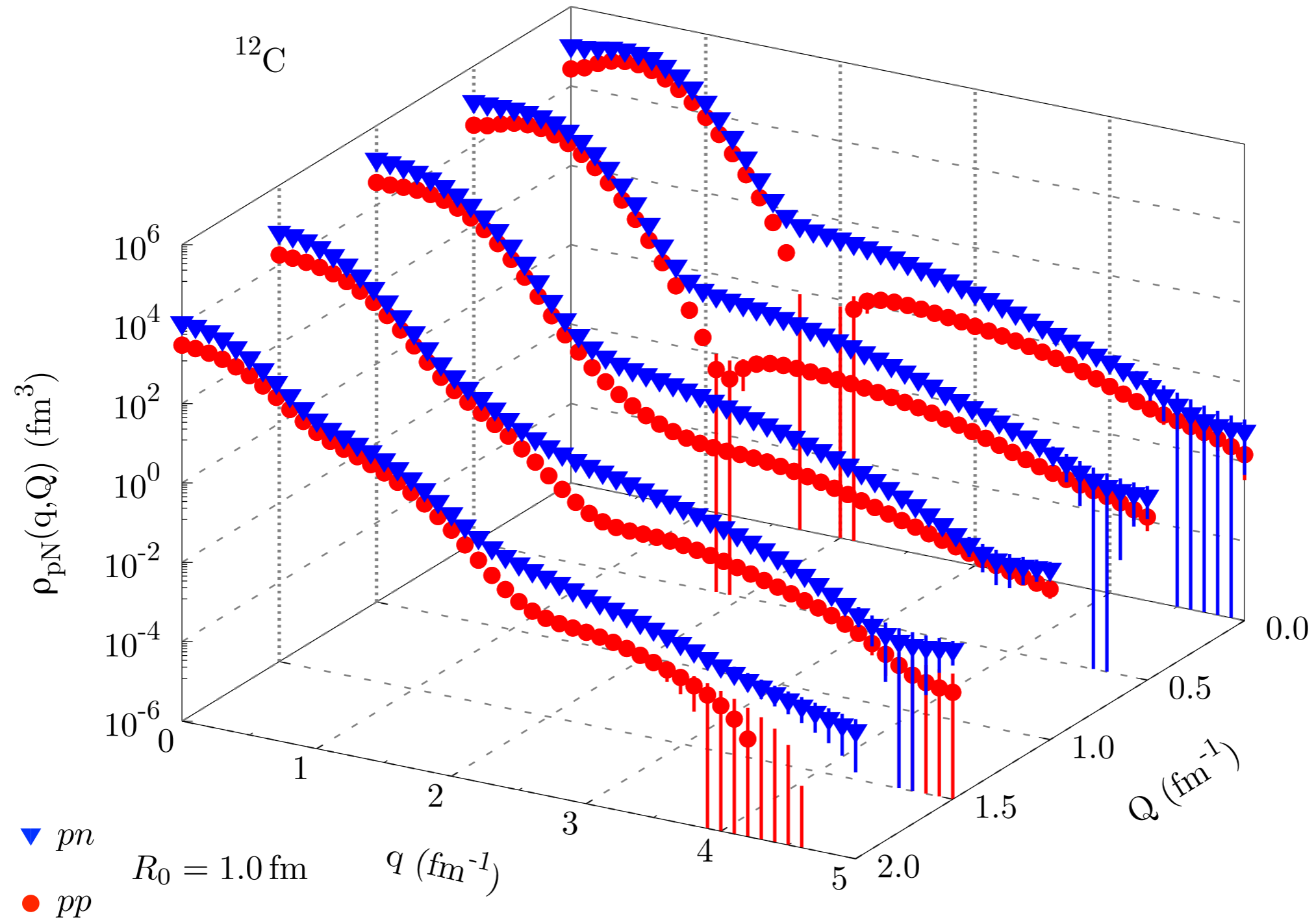
## two-nucleon momentum distributions



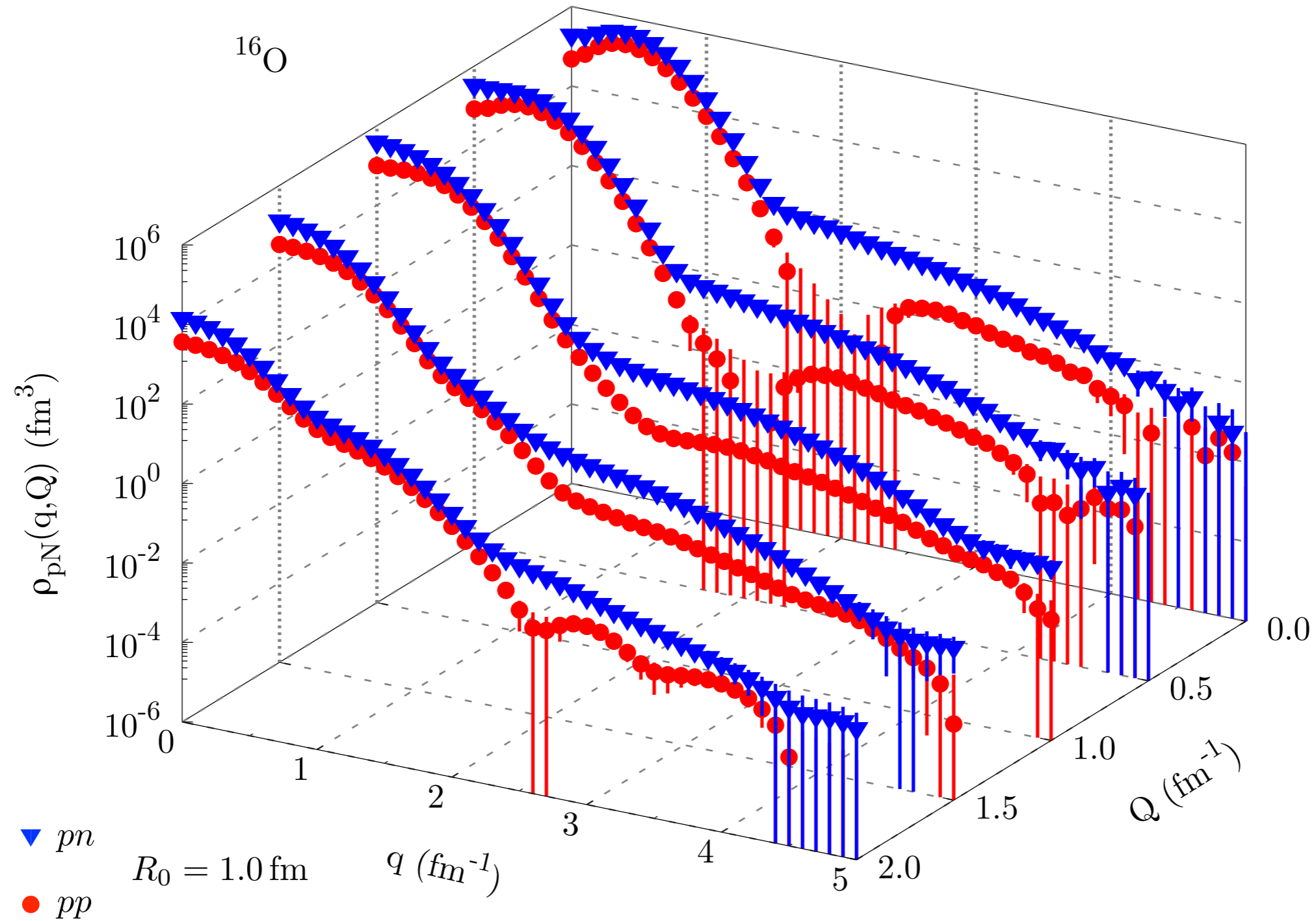
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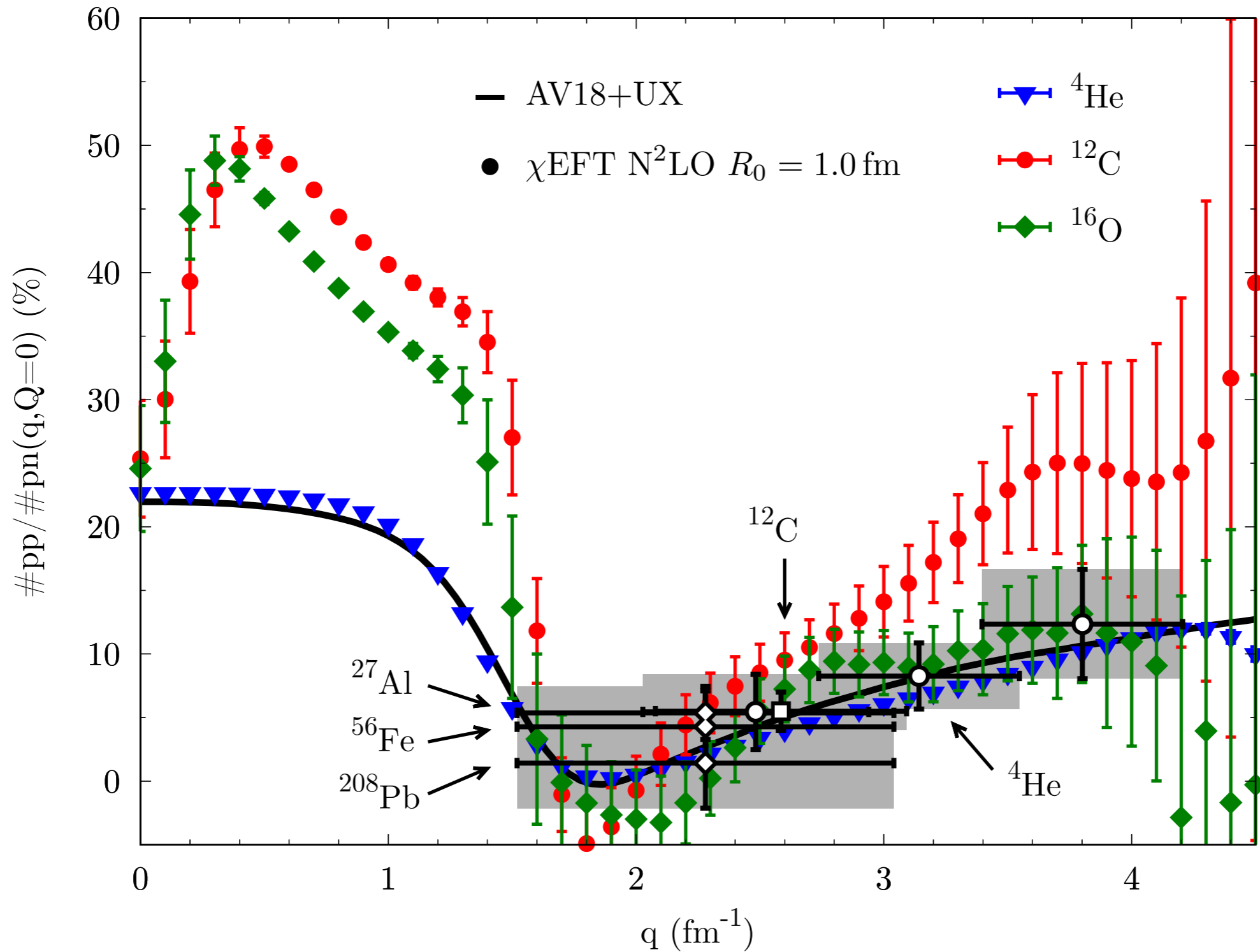


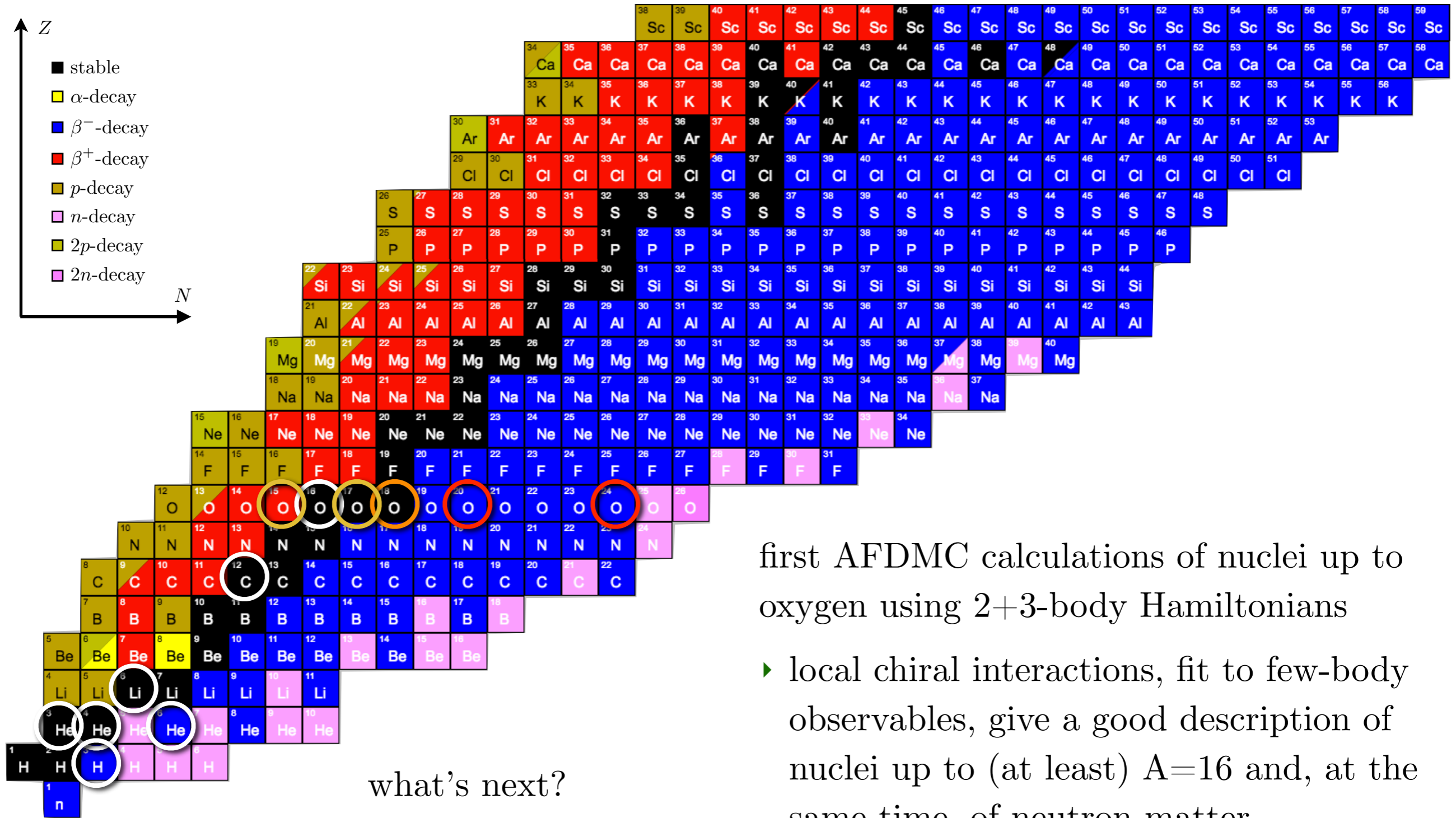
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## two-nucleon momentum distributions

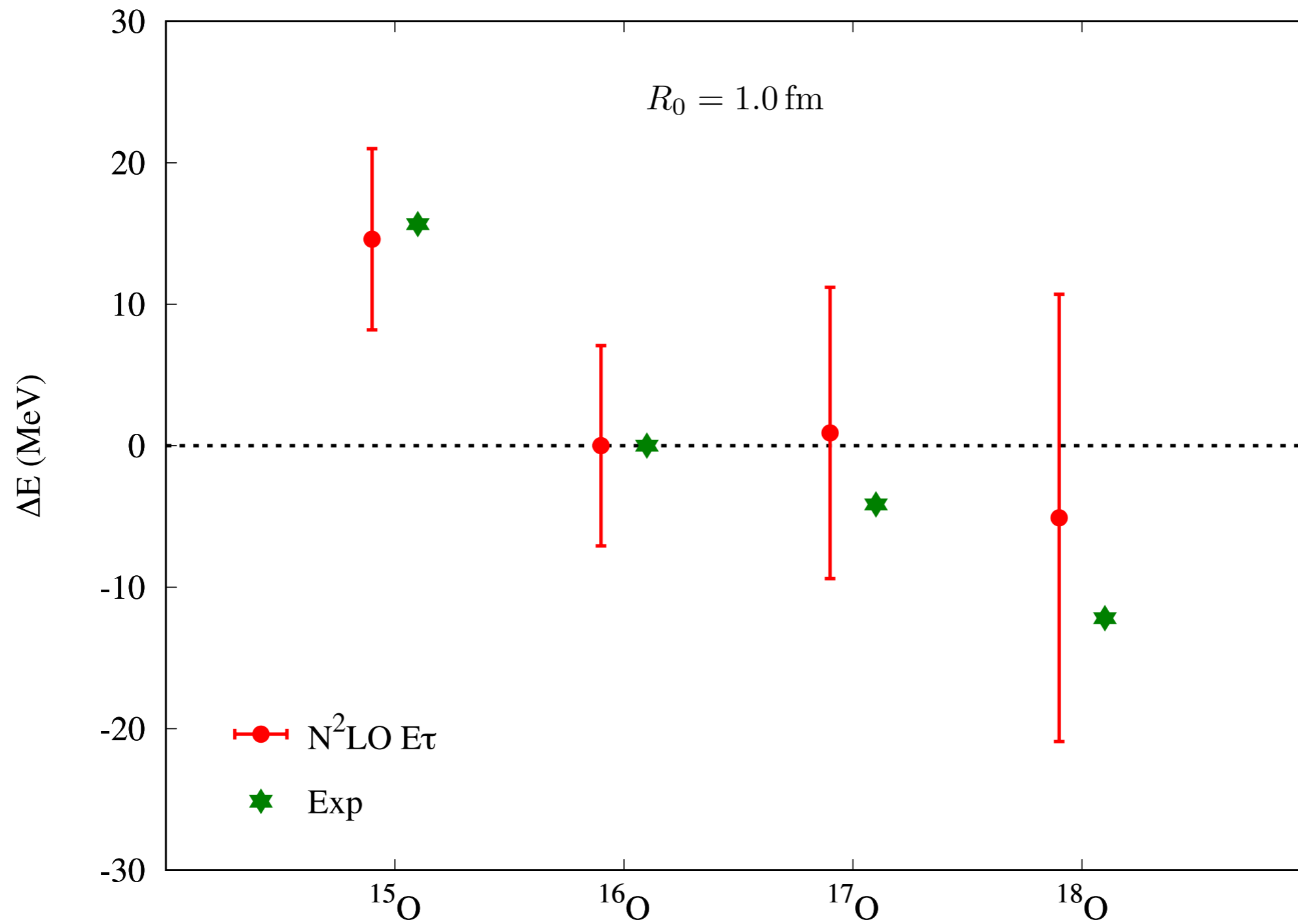




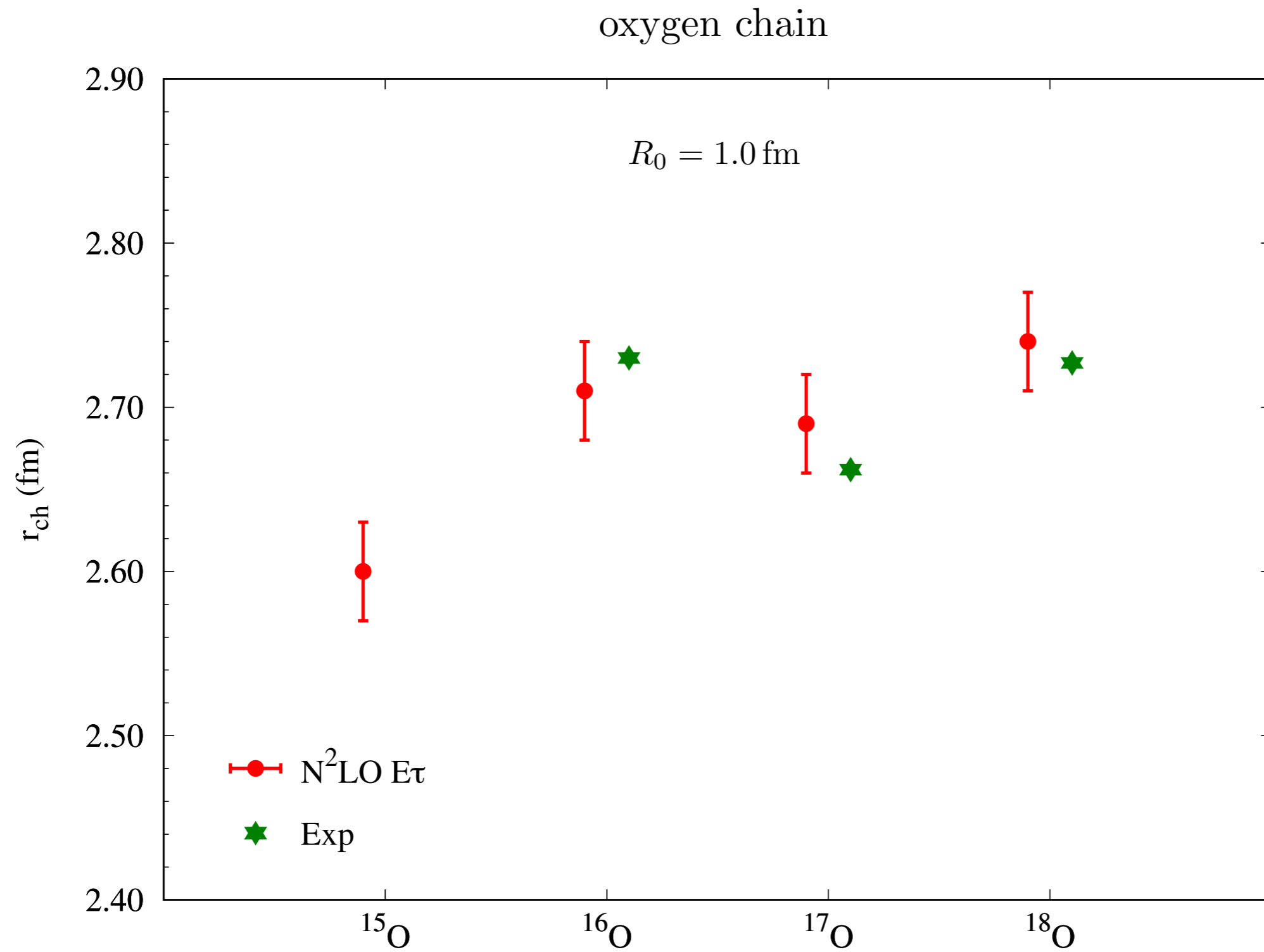
what's next?  
 oxygen/other isotopic chains?  
 exotic nuclei?  
 symmetric nuclear matter?

- first AFDMC calculations of nuclei up to oxygen using 2+3-body Hamiltonians
- ▶ local chiral interactions, fit to few-body observables, give a good description of nuclei up to (at least) A=16 and, at the same time, of neutron matter
- ▶ test of different parametrizations and cutoffs on several observables

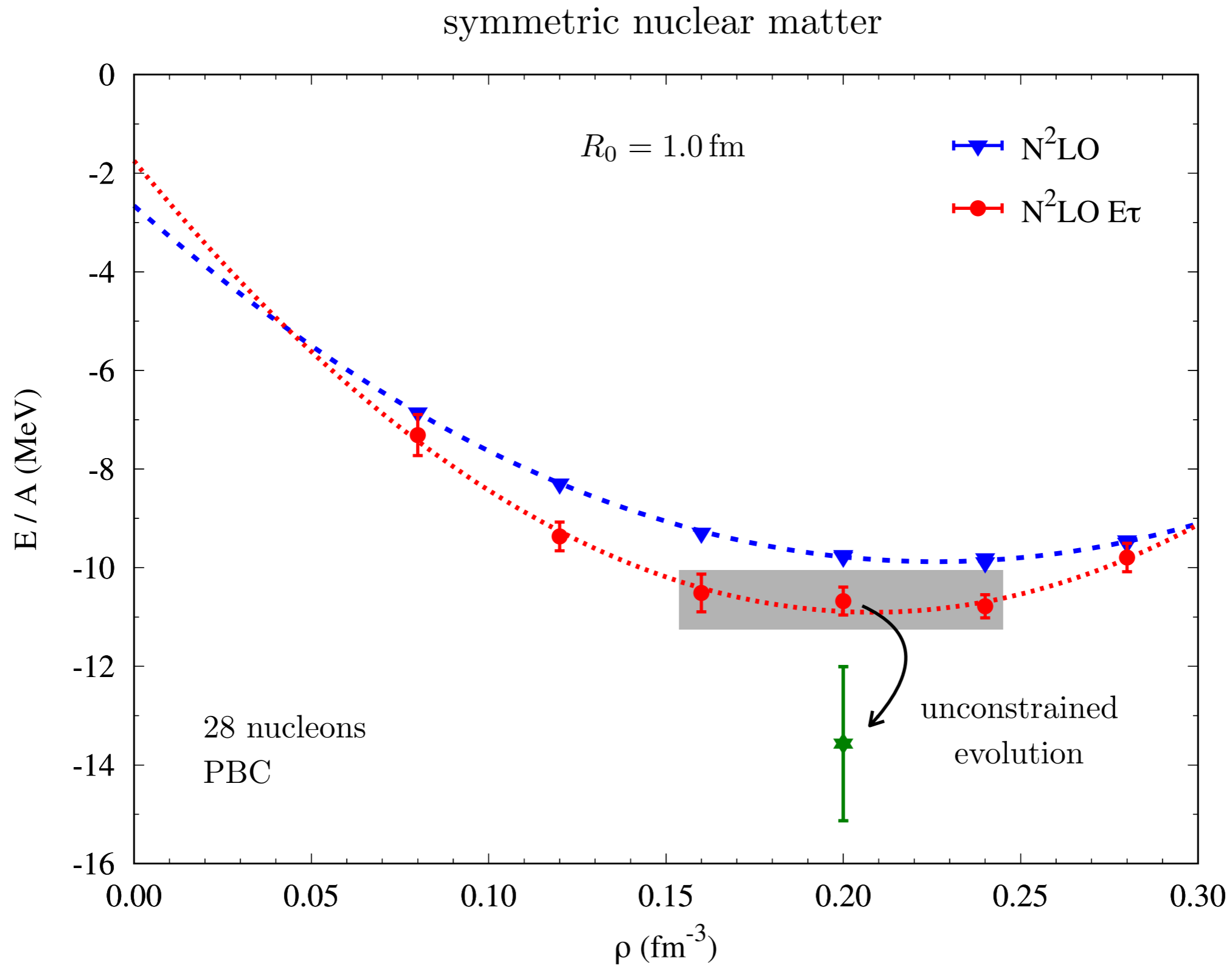
## oxygen chain



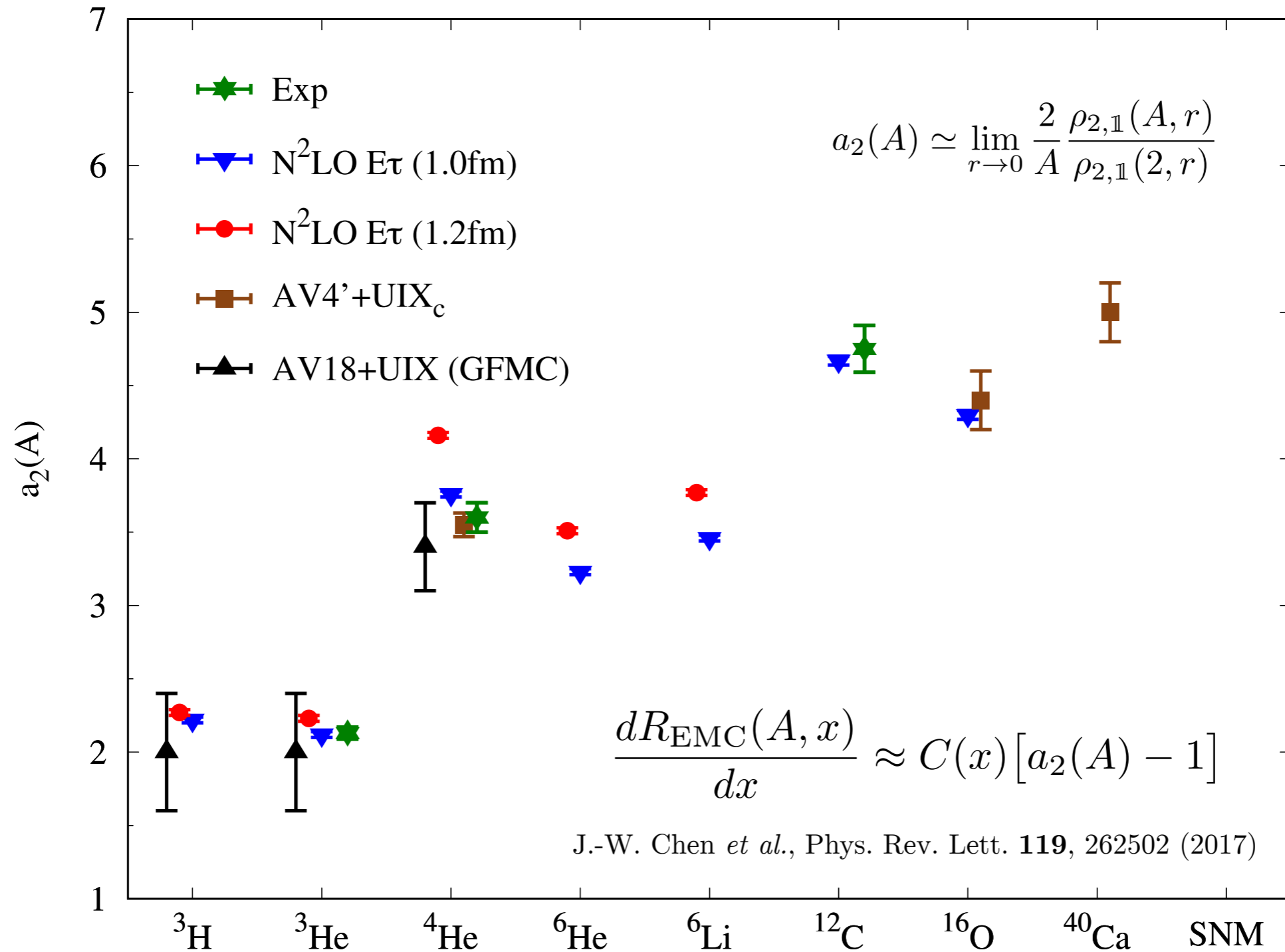
preliminary!!



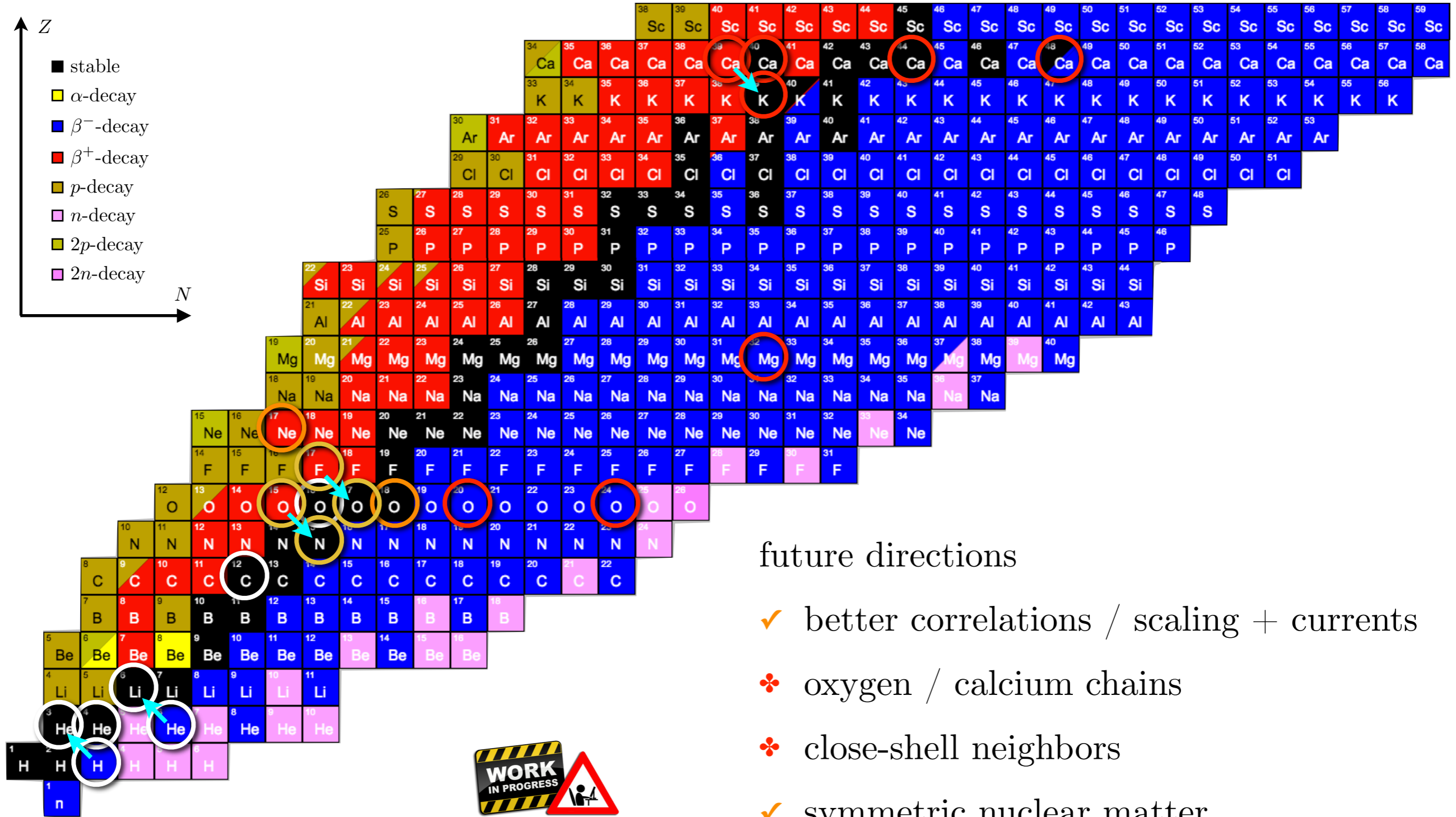
preliminary!!



short-range correlation scaling factor



preliminary!!



future directions

- ✓ better correlations / scaling + currents
- ✿ oxygen / calcium chains
- ✿ close-shell neighbors
- ✓ symmetric nuclear matter
- ✿ asymmetric nuclear matter
- ✿ beta-decay

*Thank you!!*