Thermodynamics of non-relativistic matter from complex Langevin in one and two dimensions

Andrew C. Loheac

University of North Carolina at Chapel Hill – North Carolina, USA

August 1, 2018 | INT Program 18-2B

For further details…

Polarized fermions in one dimension: density and polarization from complex Langevin calculations, perturbation theory, and the virial expansion

Andrew C. Loheac, Jens Braun, and Joaquín E. Drut

arXiv:1804.10257

Third-order perturbative lattice and complex Langevin analyses of the finite-temperature equation of state of nonrelativistic fermions in one dimension

Andrew C. Loheac and Joaquín E. Drut

Phys. Rev. D **95,** 094502 (2017)

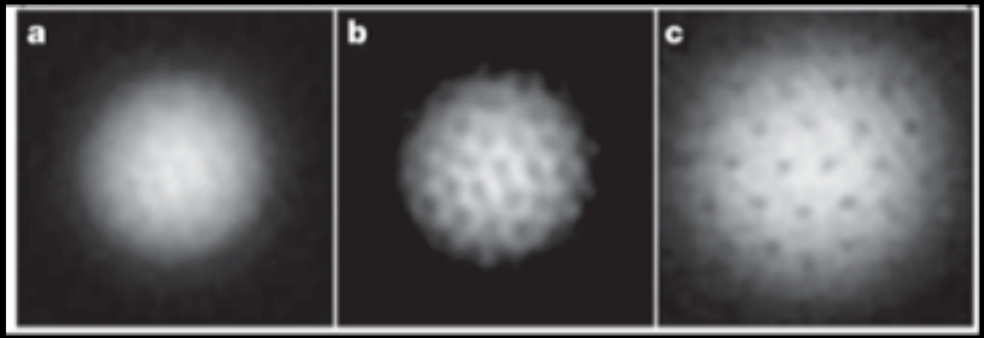
Also see for the polarized unitary gas: *Finite-temperature equation of state of polarized fermions at unitarity,* arXiv:1807.04664

Ultracold atoms and many-body theory

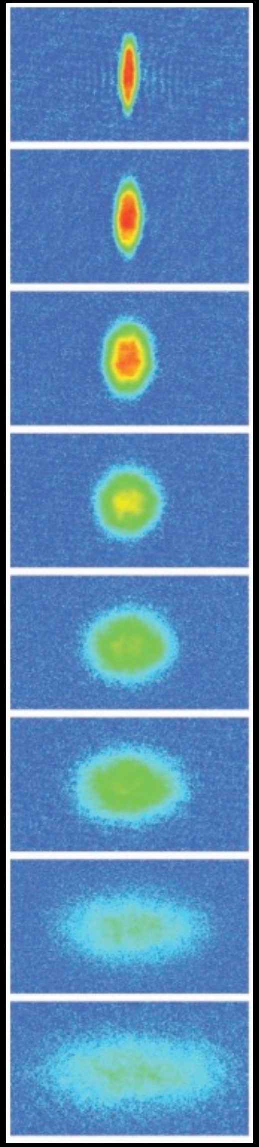
Ultracold atoms provide **clean** and **malleable** systems to develop and test new methods for many-body theory outside of lattice QCD.

Experiments can

- **control interactions** (harmonic traps, optical lattices, coupling strength), introduce spin-polarization and mass-imbalance,
- **study phenomena** including superfluidity, quantized vortices, phases of matter (e.g. FFLO, BEC, BCS states).



MIT



K. M. O'Hara et al. Nature (2002)

Hamiltonian and particle content

- We are studying the many-body system of fermions with contact interactions.
- The non-relativistic Hamiltonian we study is (applies in any dimension):

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 - g\sum_{i<j}\delta(x_i - x_j)$$

short-range interaction (studied in dilute limit)

bare coupling: renormalized to 2-body binding energy (1D, 2D)

- We compute the finite-temperature equation of state for unpolarized and polarized fermions in one and two dimensions.

- Use a continuous and compact H. S. transformation: $\mathcal{Z} = \int \mathcal{D}\sigma \det{}^2 M[\sigma]$

Complex Langevin formalism

Hybrid Monte Carlo requires a *positive-definite probability measure* in order to propose new field configurations -- to circumvent the sign problem we applied the **complex Langevin** method.

We make the auxiliary field complex: $\sigma \to \sigma_R + i\sigma_I$

Dynamical equations of motion are simply:

$$\delta\sigma_R = -\mathrm{Re}\left[\frac{\delta S[\sigma]}{\delta\sigma}\right]\delta t + \eta\sqrt{dt}$$

where the action is $S = -\ln(\det^2 M)$

$$\delta\sigma_I = -\mathrm{Im}\left[\frac{\delta S[\sigma]}{\delta\sigma}\right]\delta t$$

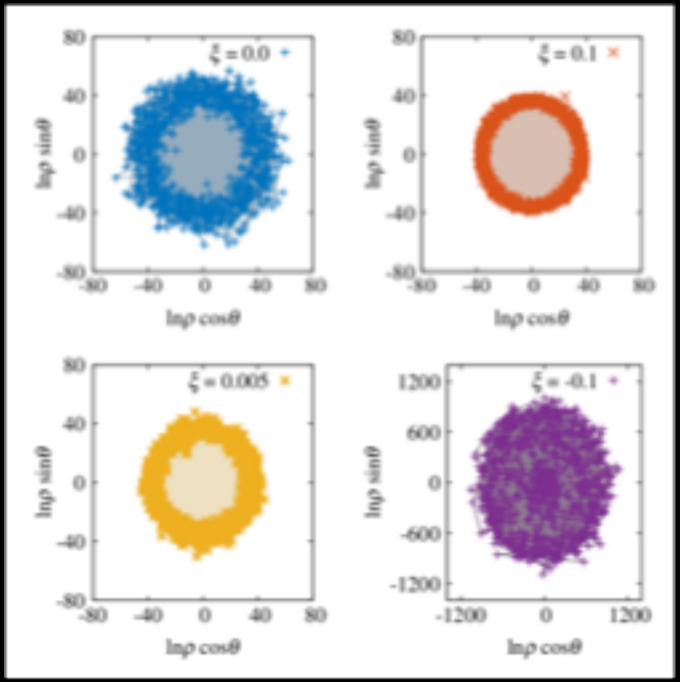
Complex Langevin formalism

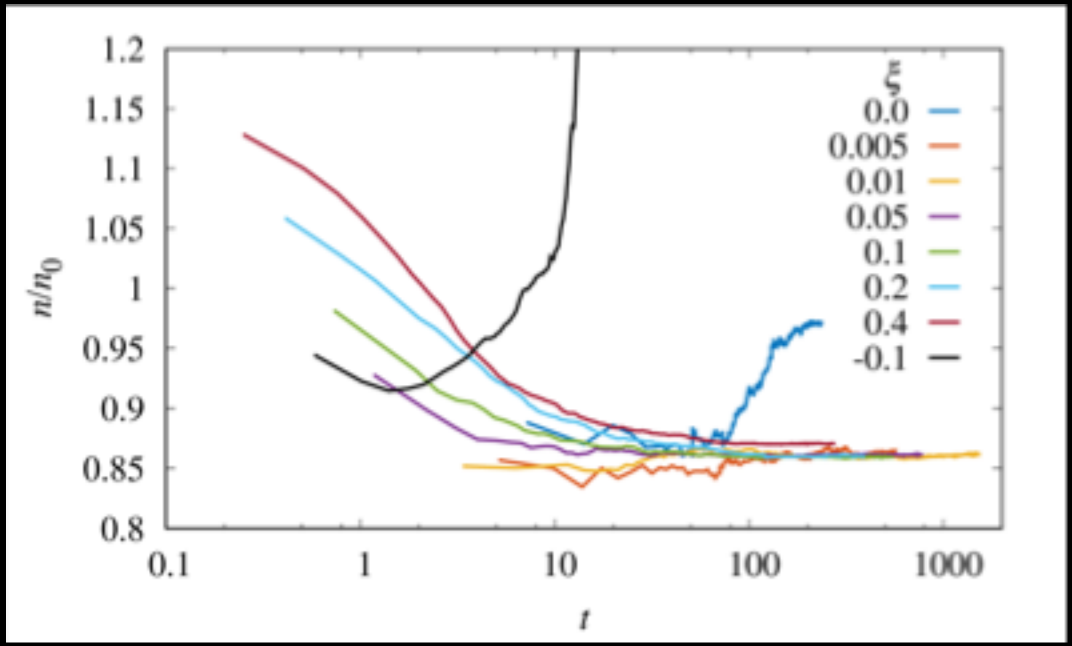
Dynamical equations of motion are simply:

$$\delta\sigma_R = -\mathrm{Re}\left[\frac{\delta S[\sigma]}{\delta\sigma}\right]\delta t + \eta\sqrt{dt} - 2\xi\sigma_R\delta t$$

$$\delta\sigma_I = -\mathrm{Im}\left[\frac{\delta S[\sigma]}{\delta\sigma}\right]\delta t - 2\xi\sigma_I\delta t$$

To avoid uncontrolled excursions in the complex plane, we modify the action with a regulating term.





Lattice perturbation theory to N3LO

To check our CL results, we developed software which analytically computes the perturbative expansion of the pressure – we currently have results to N3LO.

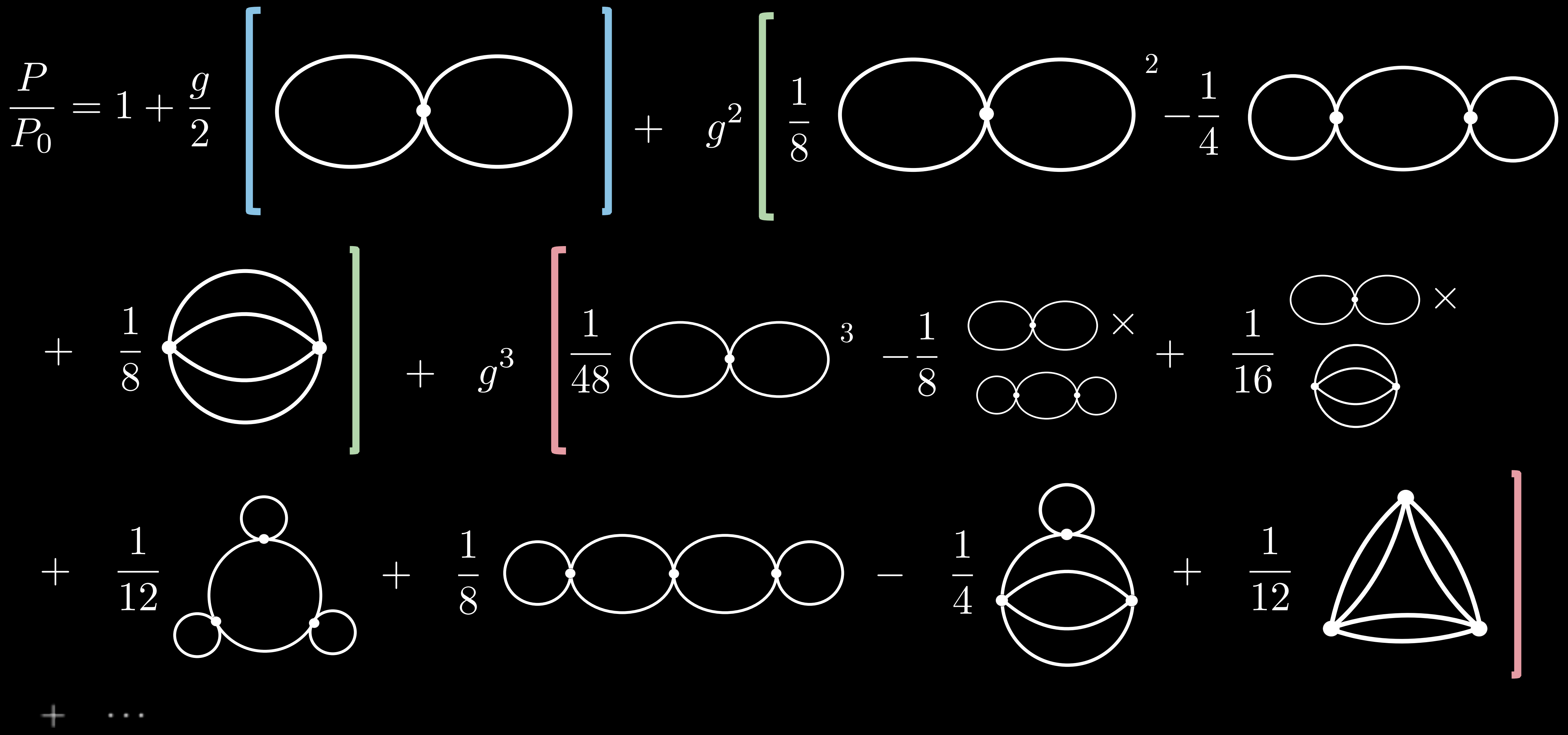
- Expansion is performed on the lattice using a Hubbard-Stratonovitch transformation (can compare methods *lattice to lattice*),

- the fermion determinant is expanded in powers of the coupling about the non-interacting limit,

$$\mathcal{Z} = \int \mathcal{D}\sigma \, \det{}^2 M[\sigma] = \int \mathcal{D}\sigma \, \det{}^2 M_0 \left[1 + g f_1(\sigma) + g^2 f_2(\sigma) + \cdots\right]^2$$

- path integral is computed exactly and resulting expressions are computed numerically to obtain EoS.

Perturbative expansion of the pressure



Fermions in 1D: attractive & repulsive interactions

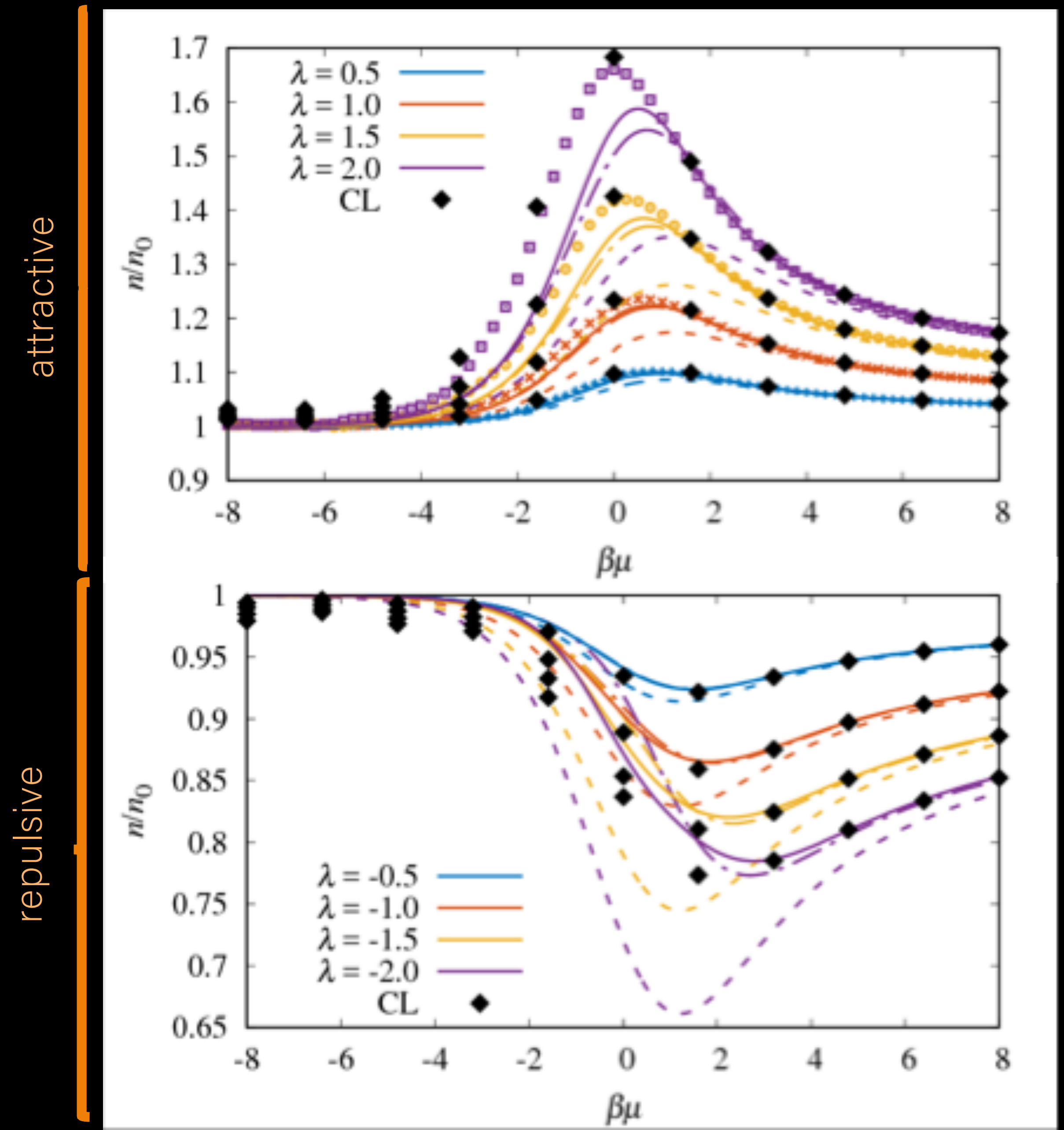
We have computed the **density** equation of state for **unpolarized** fermions in 1D for:

- attractive interactions using PT, complex Langevin (CL), and HMC.
- repulsive interactions using PT and CL.

where PT is up to N3LO.

Spatial lattice size: $N_x = 100$ points
Temporal lattice size : $N_t = 160$ points

Dimensionless coupling: $\lambda = \sqrt{\beta} g$



Equation of state for polarized fermions in 1D

We have also computed the **density** equation of state for **polarized** fermions in 1D for:

- attractive interactions using PT, complex Langevin (CL), and HMC under imaginary polarization.
- repulsive interactions using PT and CL.

where PT is shown at N3LO.

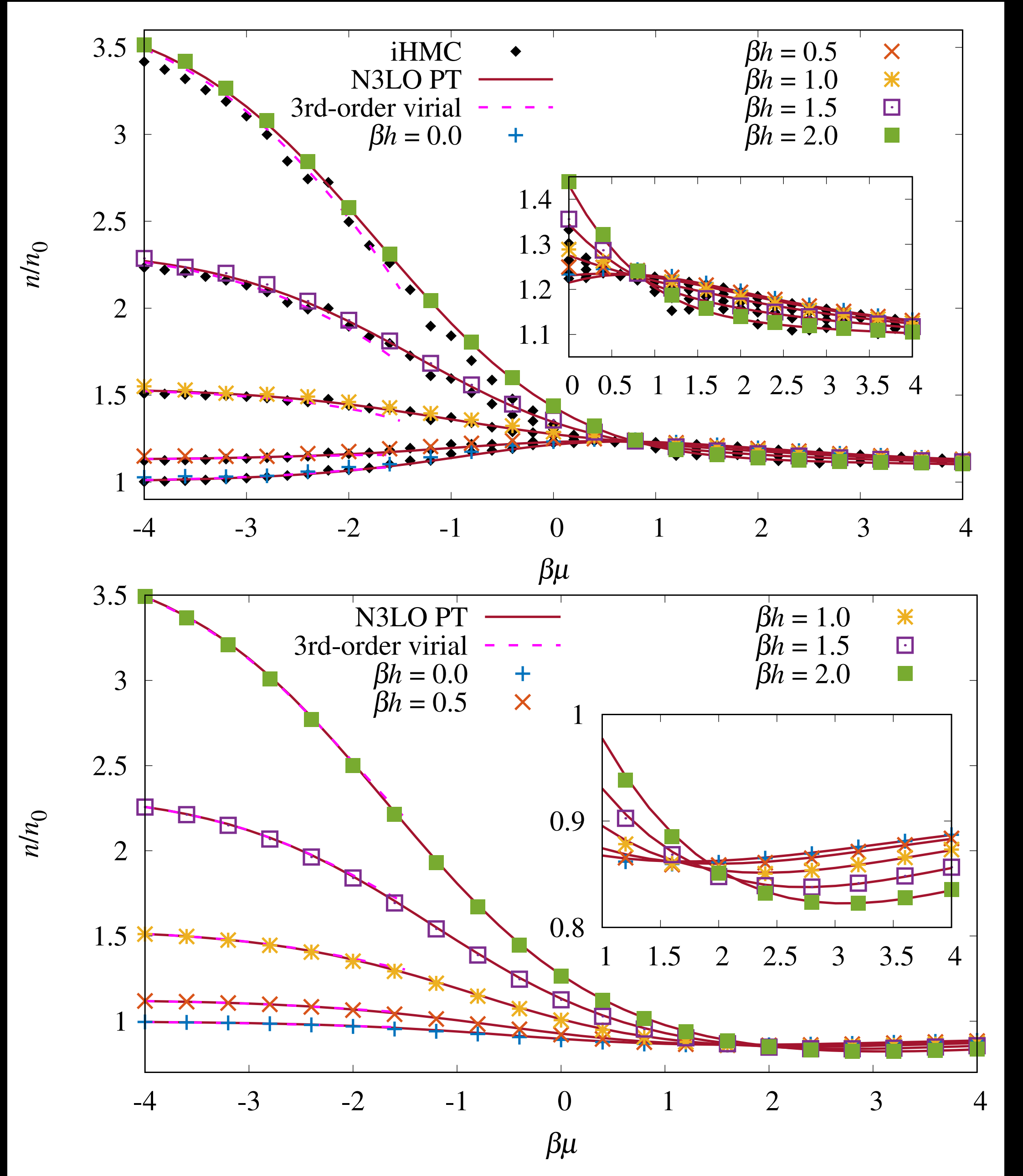
Chemical potential polarization:

$$\mu = \frac{\mu_\uparrow + \mu_\downarrow}{2} \quad , \quad h = \frac{\mu_\uparrow - \mu_\downarrow}{2}$$

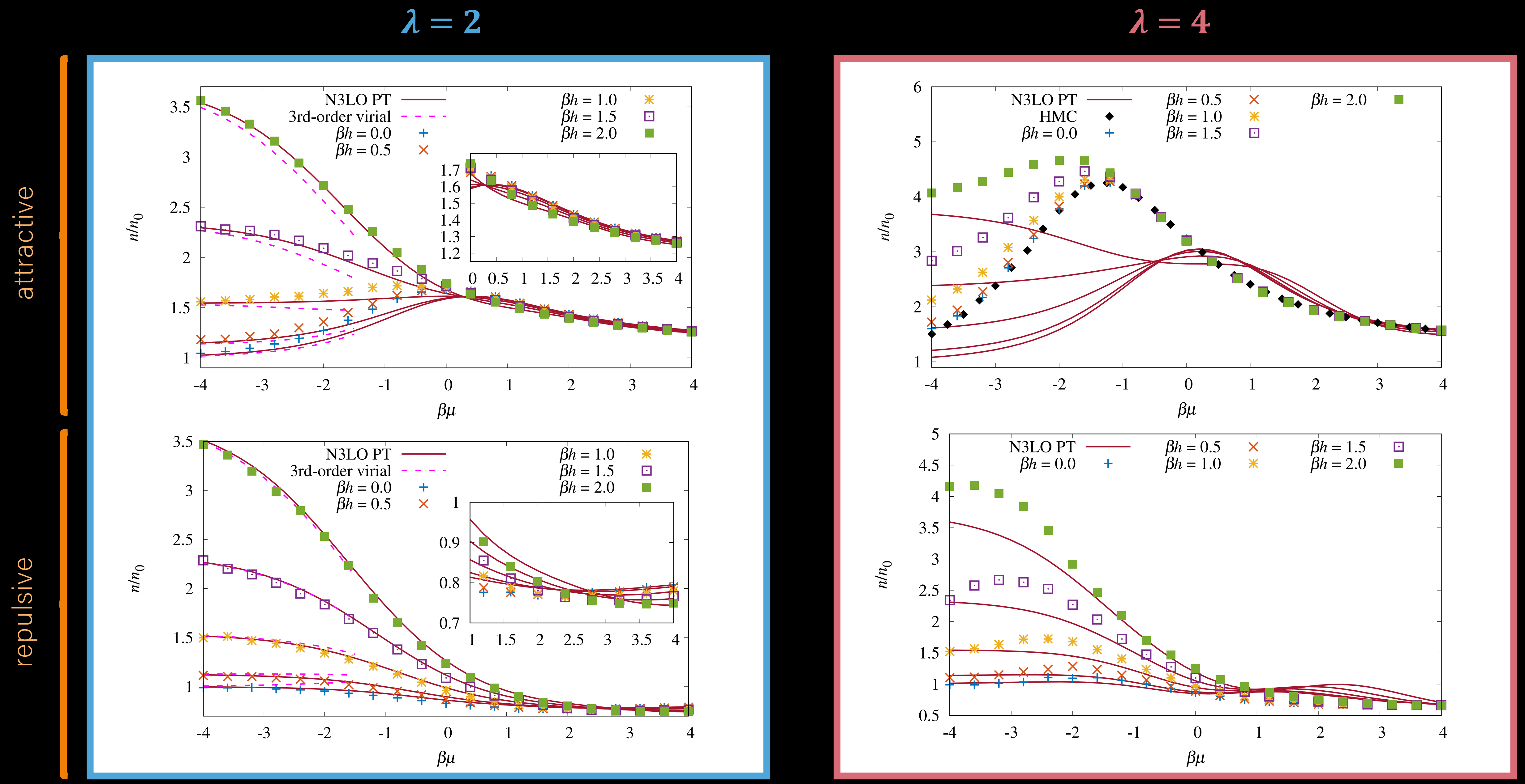
Constant coupling strength: $\lambda = 1$
Lattice size: $N_x = 60$ points, $N_t = 160$ points.

attractive

repulsive



Moving beyond perturbative regimes



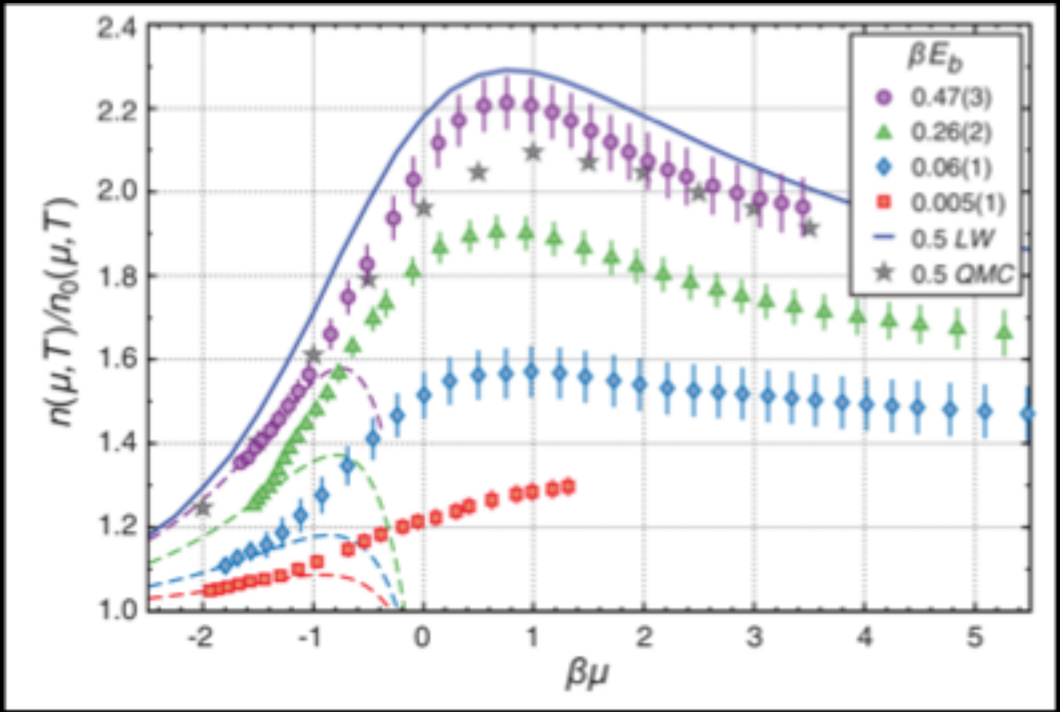
Polarized gas in two dimensions

We are studying the same non-relativistic Hamiltonian with contact interactions in two spatial dimensions – however, the coupling constant is dimensionless, and the system is classically scale invariant.

Unlike in 3D, a bound state is immediately formed for non-zero attractive interactions in 1D and 2D.

2D unpolarized systems have been realized experimentally:

Fenech et al. PRL ***116****, 045302 (2016)*



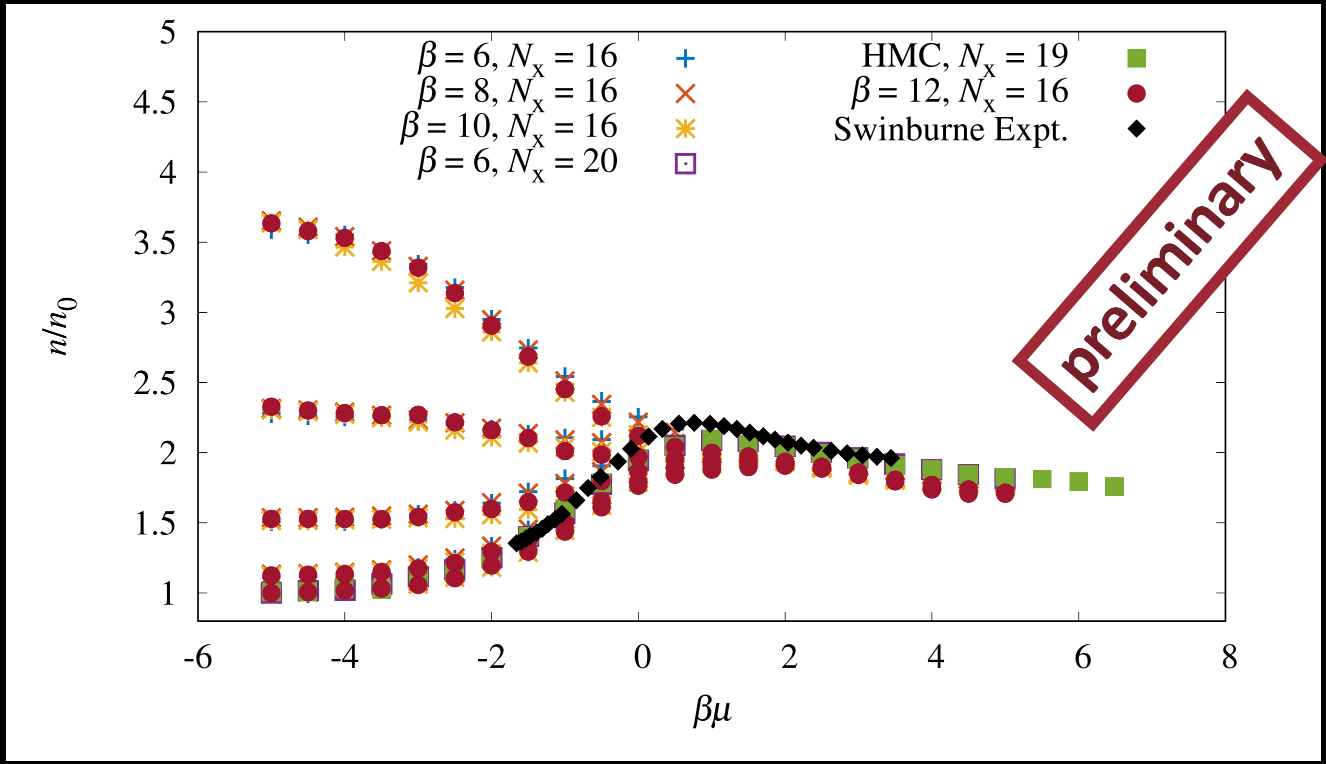
Polarized gas in two dimensions

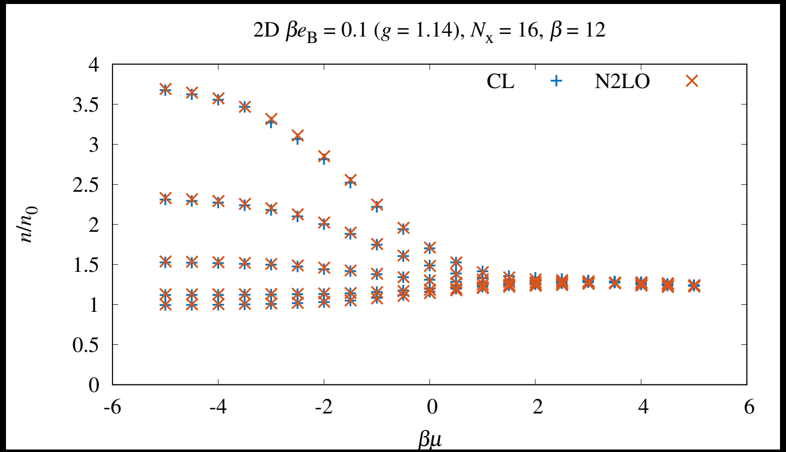
Data collection is still underway.

Perform extrapolation in N_x and β:

- Spatial lattice: 12, 16, 20, 24
- Temporal lattice: 120, 160, 200, 240
- Couplings: $\beta\varepsilon_B$ of 0.1 to 2
- Asymmetry: βh of 0 to 2

$$1 = \ell \ll \lambda_F, \lambda_T \ll L = N_x$$





Summary and conclusions

- We are able to compute equations of state for systems with a sign problem,
  - perturbatively up to N3LO
  - non-perturbatively using complex Langevin.
- We have studied polarized and unpolarized Fermi gases with in one, two and three dimensions.
- We are able to look at both repulsive and attractive interactions.

Thank you for your time and attention.

This work was funded by the U. S. National Science Foundation Graduate Research Fellowship Program, Nuclear Theory Program, and Computational Physics Program.

For further information, see Phys. Rev. D **95,** 094502 (2017) and arXiv:1804.10257.