

Structure and Electroweak Transitions of Light Nuclei

Robert B. Wiringa, Physics Division, Argonne National Laboratory

Alessandro Baroni, South Carolina

Joe Carlson, Los Alamos

Stefano Gandolfi, Los Alamos

Alessandro Lovato, Argonne

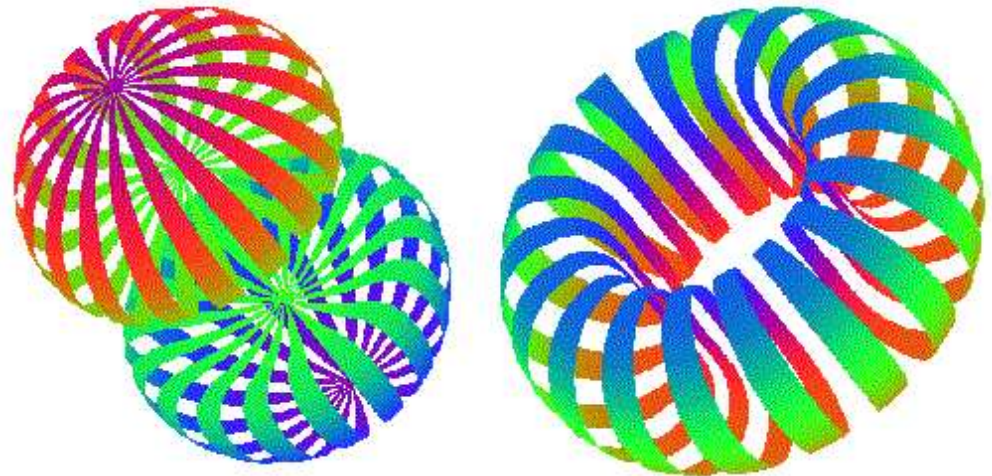
Ken Nollett, San Diego State

Saori Pastore, Los Alamos

Maria Piarulli, Argonne

Steve Pieper, Argonne

Rocco Schiavilla, JLab & ODU



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Ab Initio CALCULATIONS OF LIGHT NUCLEI

GOALS

Understand nuclei at the level of elementary interactions between individual nucleons, including

- Binding energies, excitation spectra, relative stability, matter properties
- Densities, electromagnetic moments, transition amplitudes, spectroscopic overlaps
- Low-energy NA & AA' scattering, asymptotic normalizations, astrophysical reactions

REQUIREMENTS

- Two-nucleon potentials that accurately describe elastic NN scattering data
- Consistent three-nucleon potentials and electroweak current operators
- Accurate methods for solving the many-nucleon Schrödinger equation

RESULTS

- Quantum Monte Carlo methods can evaluate realistic Hamiltonians accurate to $\sim 1-2\%$
- About 100 states calculated for $A \leq 12$ nuclei in good agreement with experiment
- Applications to elastic & inelastic e, ν, π scattering, $(e, e'p)$, (d, p) reactions, etc.
- Electromagnetic moments, $M1$, $E2$, F, GT transitions, electroweak response
- ${}^5\text{He} = n\alpha$ scattering and $3 \leq A \leq 9$ ANCs and widths

NUCLEAR HAMILTONIAN

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

K_i : Non-relativistic kinetic energy, m_n - m_p effects included

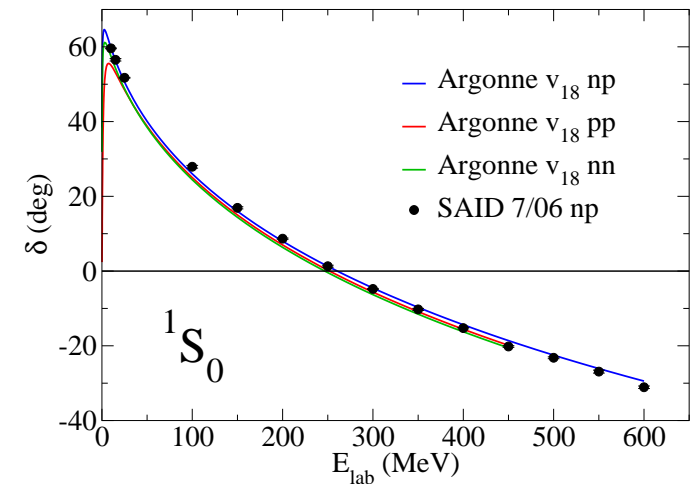
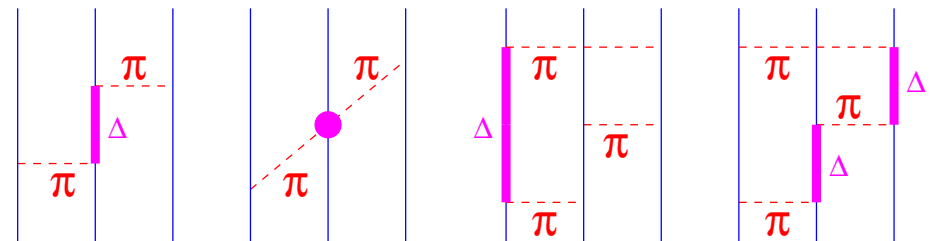
Argonne v₁₈: $v_{ij} = v_{ij}^\gamma + v_{ij}^\pi + v_{ij}^I + v_{ij}^S = \sum v_p(r_{ij}) O_{ij}^p$

- 18 spin, tensor, spin-orbit, isospin, etc., operators
- full EM and strong CD and CSB terms included
- predominantly local operator structure
- fits Nijmegen PWA93 data with $\chi^2/\text{d.o.f.}=1.1$

Wiringa, Stoks, & Schiavilla, PRC **51**, (1995)

Urbana & Illinois: $V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi} + V_{ijk}^R$

- Urbana has standard 2π P -wave + short-range repulsion for matter saturation
- Illinois adds 2π S -wave + 3π rings to provide extra $T=3/2$ interaction
- Illinois-7 has four parameters fit to 23 levels in $A \leq 10$ nuclei



Pieper, Pandharipande, Wiringa, & Carlson, PRC **64**, 014001 (2001)

Pieper, AIP CP **1011**, 143 (2008)

Norfolk NV2: $v_{ij} = v_{ij}^{\gamma} + v_{ij}^{\pi} + v_{ij}^{2\pi} + v_{ij}^{CT} = \sum v_p(r_{ij})O_{ij}^p$

- derived in chiral effective field theory with Δ -intermediate states
- 17 spin, tensor, spin-orbit, isospin, etc., operators
- full EM and strong CD and CSB terms included
- predominantly local operator structure suitable for quantum Monte Carlo
- multiple models with different regularization fit to Granada PWA2013 data
- Ia,b,c fit to $E_{\text{lab}} = 125$ MeV with $\chi^2/\text{d.o.f.} \sim 1.1$
- IIa,b,c fit to $E_{\text{lab}} = 200$ MeV with $\chi^2/\text{d.o.f.} \sim 1.4$

Piarulli, Girlanda, Schiavilla, Kievsky, Lovato, Marcucci, Pieper, Viviani, & Wiringa, PRC **94**, 054007 (2016)

Norfolk NV3: $V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{CT}$

- standard 2π S -wave and 2π P -wave terms consistent with chiral NN potential
- contact terms of c_D (π -short range) and c_E (short-short range $\tau_i \cdot \tau_k$) type
- two parameters fit to ${}^3\text{H}$ binding and nd scattering length

Piarulli, Baroni, Girlanda, Kievsky, Lovato, Marcucci, Pieper, Schiavilla, Viviani, & Wiringa, PRL **120**, 052503 (2018)

QUANTUM MONTE CARLO

Variational Monte Carlo (VMC): construct Ψ_V that

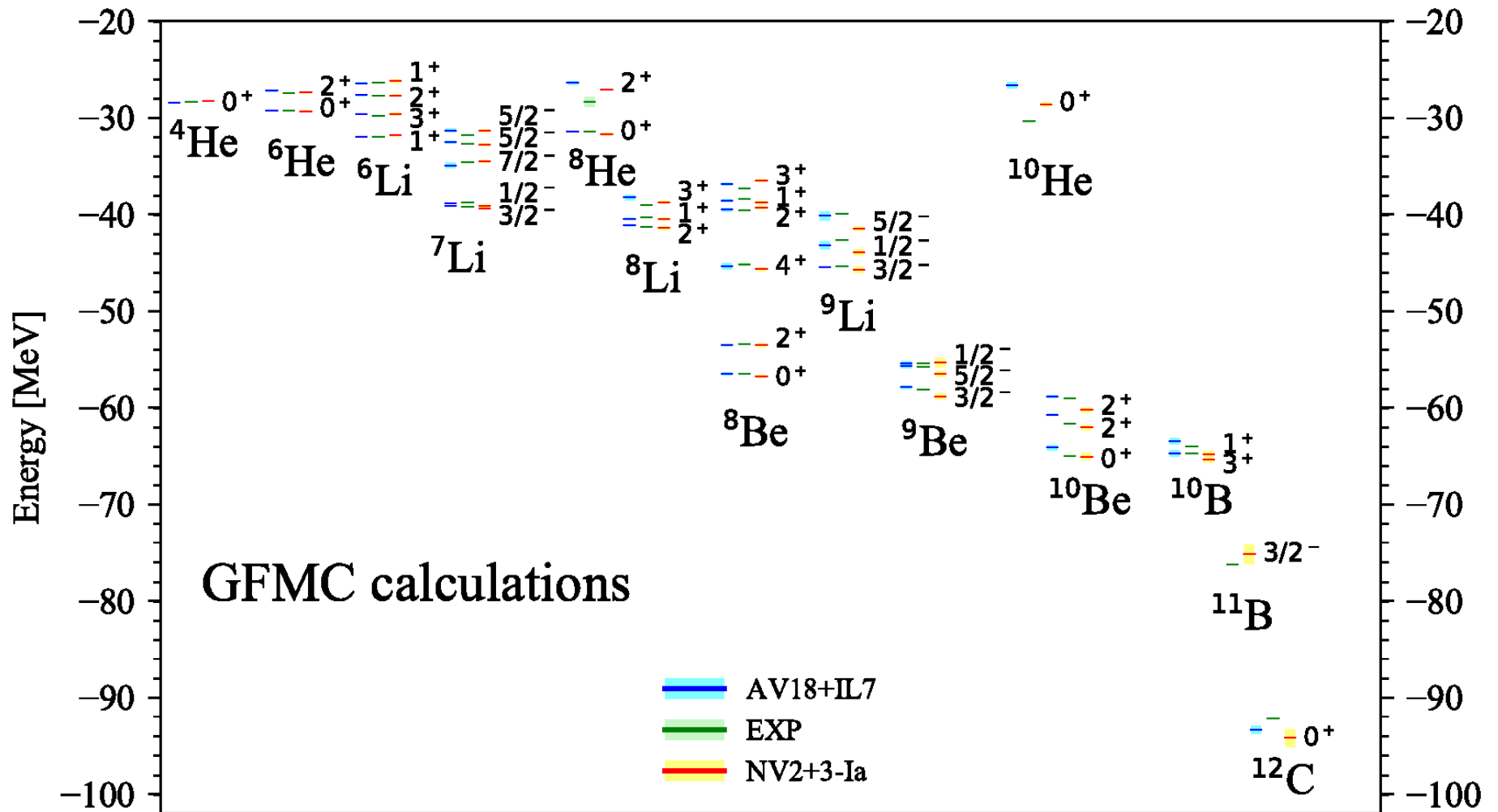
- Are fully antisymmetric and translationally invariant
- Have cluster structure and correct asymptotic form
- Contain non-commuting 2- & 3-body operator correlations from v_{ij} & V_{ijk}
- Are orthogonal for multiple J^π states
- Minimize $E_V = \langle \Psi_V | H | \Psi_V \rangle \geq E$

These are $\sim 2^A \left(\frac{A}{Z}\right)$ component (540,672 for ^{12}C) spin-isospin vectors in $3A$ dimensions

Green's function Monte Carlo (GFMC): project out the exact eigenfunction

- $\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau]a_n \Psi_n \Rightarrow \Psi_0$ at large τ
- Propagation done stochastically in small time slices $\Delta\tau$
- Exact $\langle H \rangle$ for local potentials; mixed estimates for other $\langle O \rangle$
- Constrained-path propagation controls fermion sign problem for $A \geq 8$
- Multiple excited states for same J^π stay orthogonal

Many tests demonstrate 1–2% accuracy for realistic $\langle H \rangle$



RMS ΔE for 36 states: AV18+IL7 = 0.80 MeV ; NV2+3-Ia = 0.72 MeV
 with signed average deviation: -0.23 MeV and +0.15 MeV

NOLEN-SCHIFFER ANOMALY

Nuclear forces are mostly charge-independent [$\text{CI} \propto 1, \tau_i \cdot \tau_j$], but have small charge-dependent [$\text{CD} \propto T_{ij}$] and charge-symmetry-breaking [$\text{CSB} \propto (\tau_i + \tau_j)_z$] components, while electromagnetic forces are a mix of CI , CD , & CSB terms. Evidence for strong charge-independence-breaking (CIB) comes from the energy differences of isobaric multiplets:

$$E_{A,T}(T_z) = \sum_{n \leq 2T} a_n(A, T) Q_n(T, T_z)$$

$$Q_0 = 1 ; Q_1 = T_z ; Q_2 = \frac{1}{2}(3T_z^2 - T^2)$$

For example,

$$a_1(3, \frac{1}{2}) = E(^3\text{He}) - E(^3\text{H}) \quad a_2(6, 1) = \frac{1}{3}[E(^6\text{Be}) - 2E(^6\text{Li}^*) + E(^6\text{He})]$$

The **Nolen-Schiffer anomaly** is the difference not explained by Coulomb force; strong CIB and other electromagnetic terms in Argonne v_{18} explain much of the remainder (shown in keV):

$a_n(A, T)$	K^{CSB}	$v_{C1}(pp)$	$v^{\gamma,R}$	$v^{CSB} + v^{CD}$	δH^{CI}	Total	Expt.
$a_1(3, \frac{1}{2})$	14	642(1)	26	65(0)	8(1)	755(1)	764
$a_1(7, \frac{1}{2})$	23	1442(2)	36	83(1)	27(10)	1611(10)	1645
$a_1(8, 1)$	25	1652(3)	18	77(1)	33(11)	1813(11)	1770
$a_2(6, 1)$		140(1)	18	100(2)	17(2)	273(3)	223
$a_2(8, 1)$		133(1)	3	-3(2)	10(5)	139(5)	127

Isospin-mixing in ${}^8\text{Be}$

Experimental energies of 2^+ states

$$E_a = 16.626(3) \text{ MeV} \quad \Gamma_a^\alpha = 108.1(5) \text{ keV}$$

$$E_b = 16.922(3) \text{ MeV} \quad \Gamma_b^\alpha = 74.0(4) \text{ keV}$$

Isospin mixing of $2^+;1$ and $2^+;0^*$

states due to isovector interaction H_{01} :

$$\Psi_a = \beta\Psi_0 + \gamma\Psi_1; \quad \Psi_b = \gamma\Psi_0 - \beta\Psi_1$$

decay through $T = 0$ component only

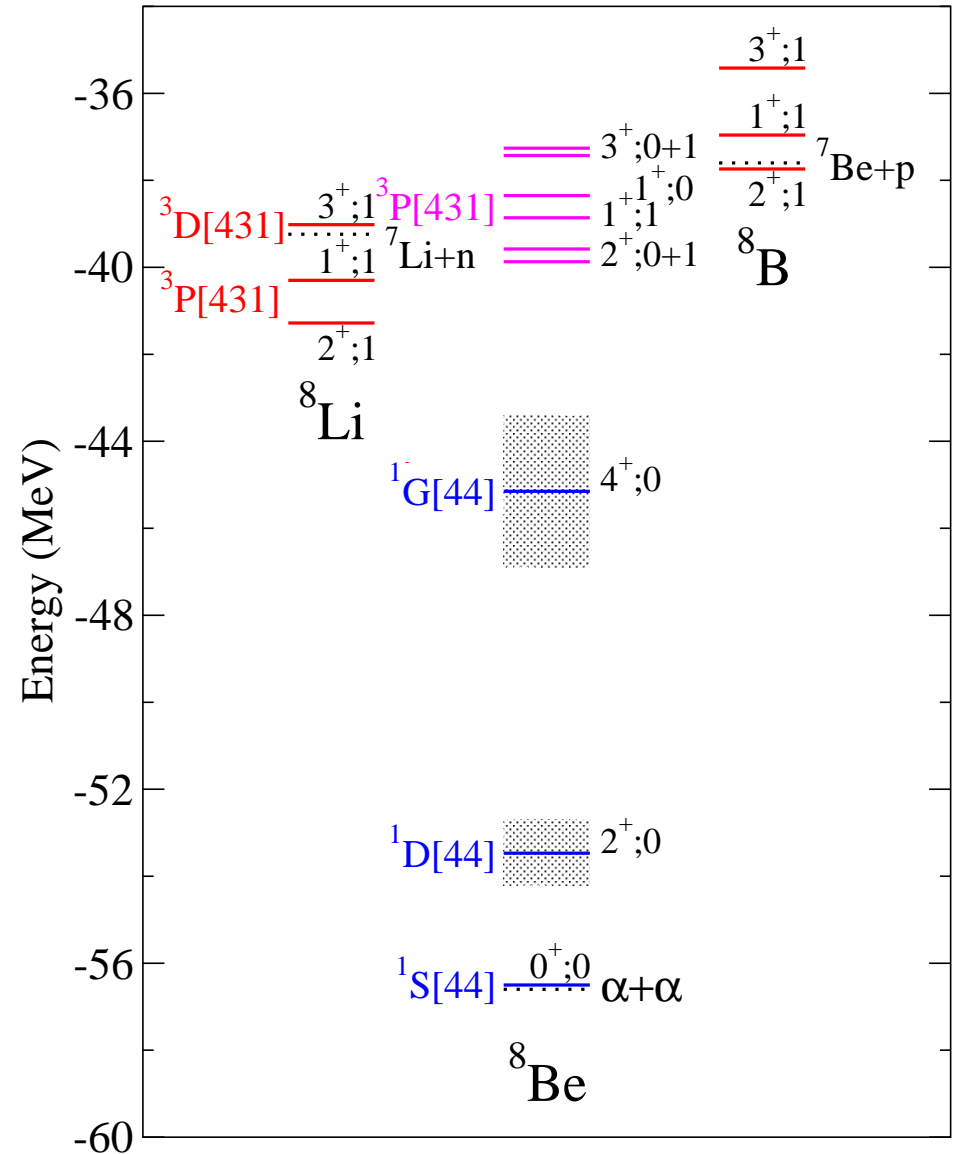
$$\Gamma_a^\alpha / \Gamma_b^\alpha = \beta^2 / \gamma^2 \Rightarrow \beta = 0.77; \quad \gamma = 0.64$$

$$E_{a,b} = \frac{H_{00} + H_{11}}{2} \pm \sqrt{\left(\frac{H_{00} - H_{11}}{2}\right)^2 + (H_{01})^2}$$

$$H_{00} = 16.746(2) \text{ MeV}$$

$$H_{11} = 16.802(2) \text{ MeV}$$

$$H_{01} = -145(3) \text{ keV}$$



Isospin-mixing matrix elements in keV

	K^{CSB}	$v_{C1}(pp)$	$v^{\gamma,R}$	$v^{CSB}+v^{CD}$	H_{01}	Expt.
$2^+;1 \Leftrightarrow 2_2^+;0$	−3.6(1)	−89.3(11)	−11.0(2)	−23.4(4)	−127(2)	−145(3)
$1^+;1 \Leftrightarrow 1^+;0$	−2.8(1)	−73.4(11)	1.0(1)	−18.5(4)	−94(1)	−103(14)
$3^+;1 \Leftrightarrow 3^+;0$	−3.0(1)	−74.6(12)	−16.8(2)	−16.6(4)	−111(2)	−59(12)
$2^+;1 \Leftrightarrow 2_1^+;0$					−7(2)	
$0^+;2 \Leftrightarrow 0_3^+;0$		−32.2(2)	−8.9(1)	−83.8(22)	−125(2)	

Coulomb terms are 70% of H_{01} , but magnetic moment and strong **Type III CSB** are relatively more important than in Nolen-Schiffer anomaly; still missing $\approx 10\%$ of strength.

Strong **Type IV CSB** also contribute (probably best nuclear structure place to look):

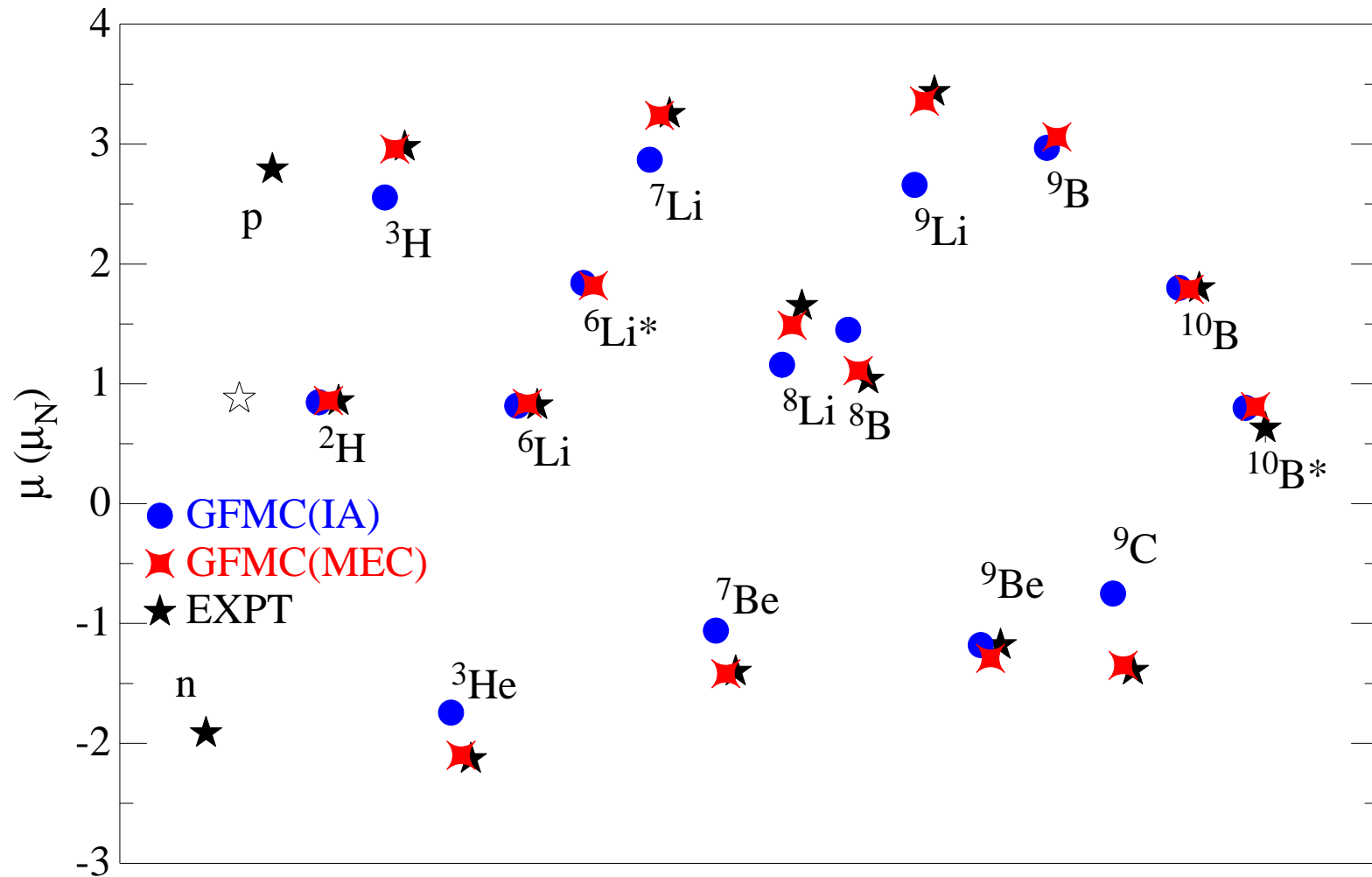
$$\begin{aligned}
 V_{IV}^{CSB} &= (\tau_1 - \tau_2)_z (\sigma_1 - \sigma_2) \cdot \mathbf{L} v(r) \\
 &+ (\tau_1 \times \tau_2)_z (\sigma_1 \times \sigma_2) \cdot \mathbf{L} w(r)
 \end{aligned}$$

These contributions are model-dependent with $V_{IV}^{CSB} \sim \pm$ few keV.

$A \leq 10$ MAGNETIC MOMENTS W/ χ EFT EXCHANGE CURRENTS

Hybrid calculations using AV18+IL7 wave functions and χ EFT exchange currents developed in:

Pastore, Schiavilla, & Goity, PRC **78**, 064002 (2008) ; Pastore, *et al.*, PRC **80**, 034004 (2009)

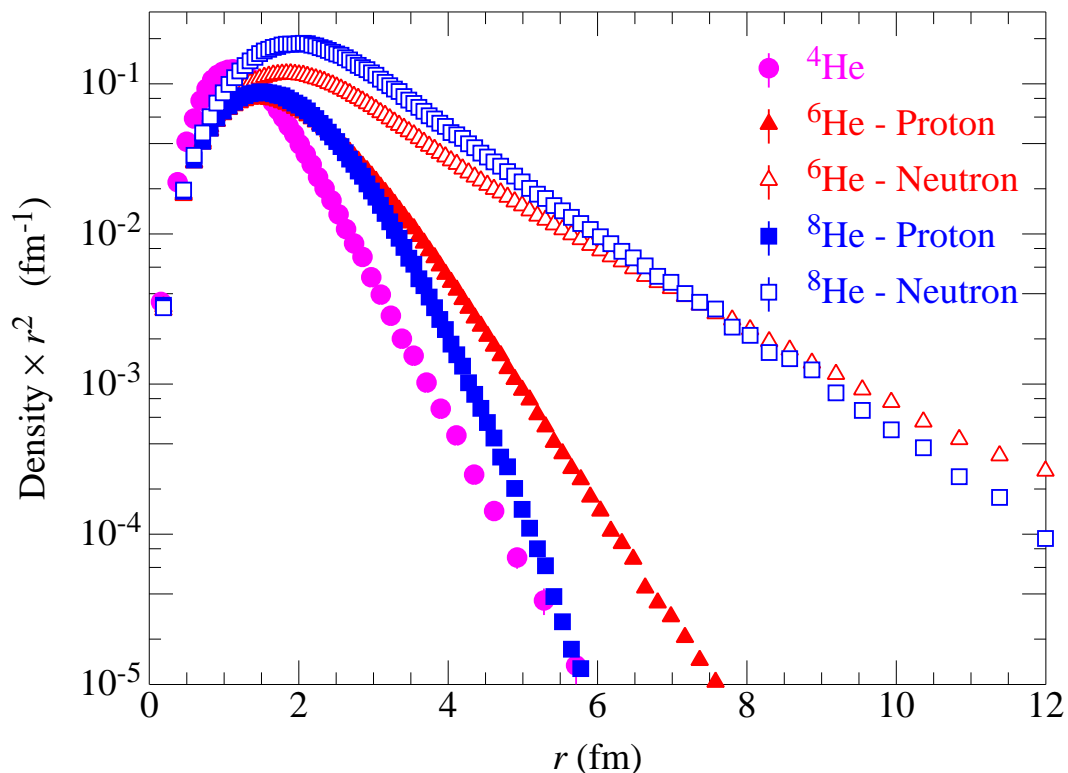
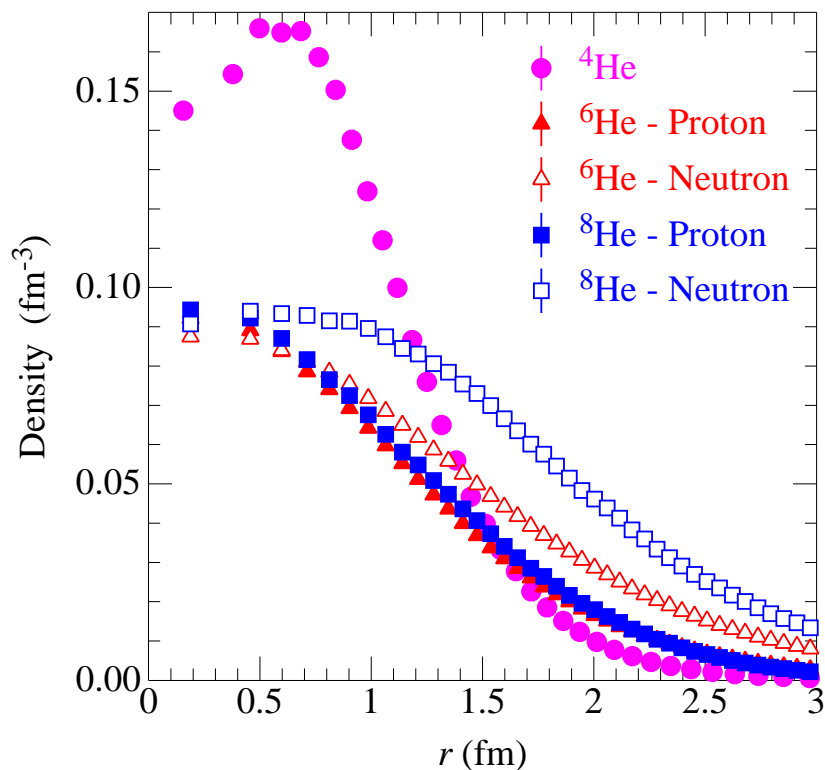


Pastore, Pieper, Schiavilla & Wiringa, PRC **87**, 035503 (2013)



SINGLE-NUCLEON DENSITIES

$$\rho_{p,n}(r) = \sum_i \langle \Psi | \delta(r - r_i) \frac{1 \pm \tau_i}{2} | \Psi \rangle$$



RMS radii

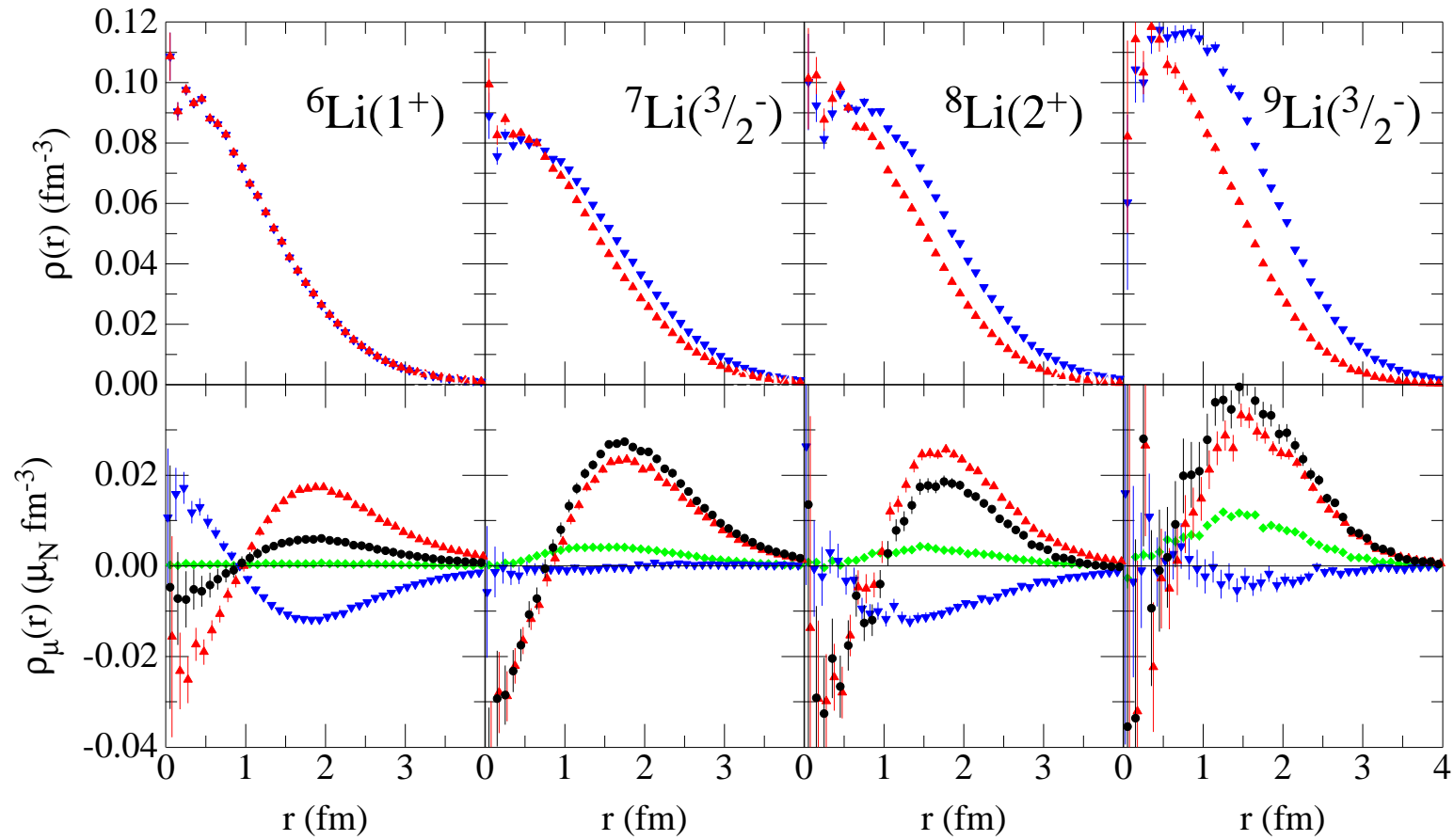
	r_n	r_p	r_c	Expt
${}^4\text{He}$	1.45(1)	1.45(1)	1.67(1)	1.681(4)*
${}^6\text{He}$	2.86(6)	1.92(4)	2.06(4)	2.060(8)†
${}^8\text{He}$	2.79(3)	1.82(2)	1.94(2)	1.959(16)‡

*Sick, PRC **77**, 041302(R) (2008)

†Wang, *et al.*, PRL **93**, 142501 (2004)

‡Mueller, *et al.*, PRL **99**, 252501 (2007)

Brodeur, *et al.*, PRL **108**, 052504 (2012)



RMS radii

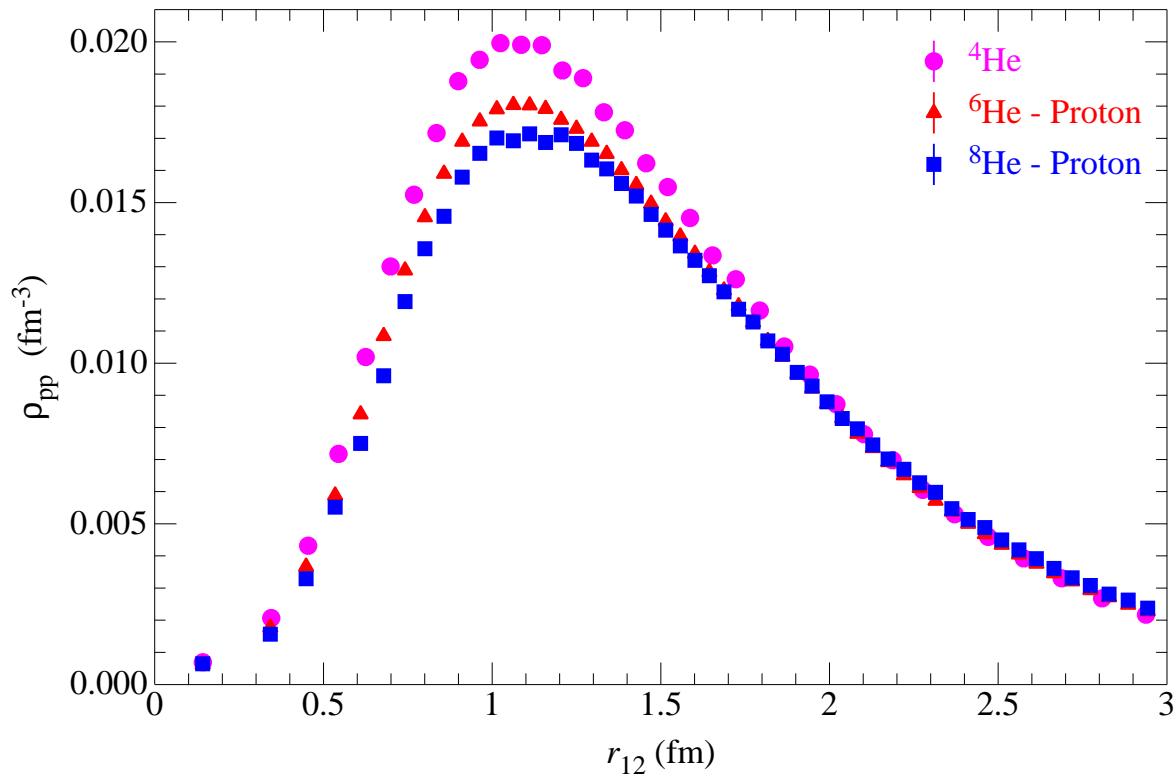
	r_c	Expt	r_m	Expt
${}^6\text{Li}$	2.53(1)	2.589(39)*	3.30(2)	
${}^7\text{Li}$	2.38(1)	2.444(43)*	2.86(2)	2.98(5) †
${}^8\text{Li}$	2.24(1)	2.339(45)*	1.85(2)	
${}^9\text{Li}$	2.10(1)	2.245(47)*	2.38(2)	

* Nörtershauser, *et al.*, PRC **84**, 024307(R) (2011)

† Van Niftrik, *et al.*, NPA **174**, 173 (1971)

TWO-NUCLEON DENSITIES

$$\rho_{pp}(r) = \sum_{i < j} \langle \Psi | \delta(r - |\mathbf{r}_i - \mathbf{r}_j|) \frac{1 + \tau_i}{2} \frac{1 + \tau_j}{2} | \Psi \rangle$$



RMS radii

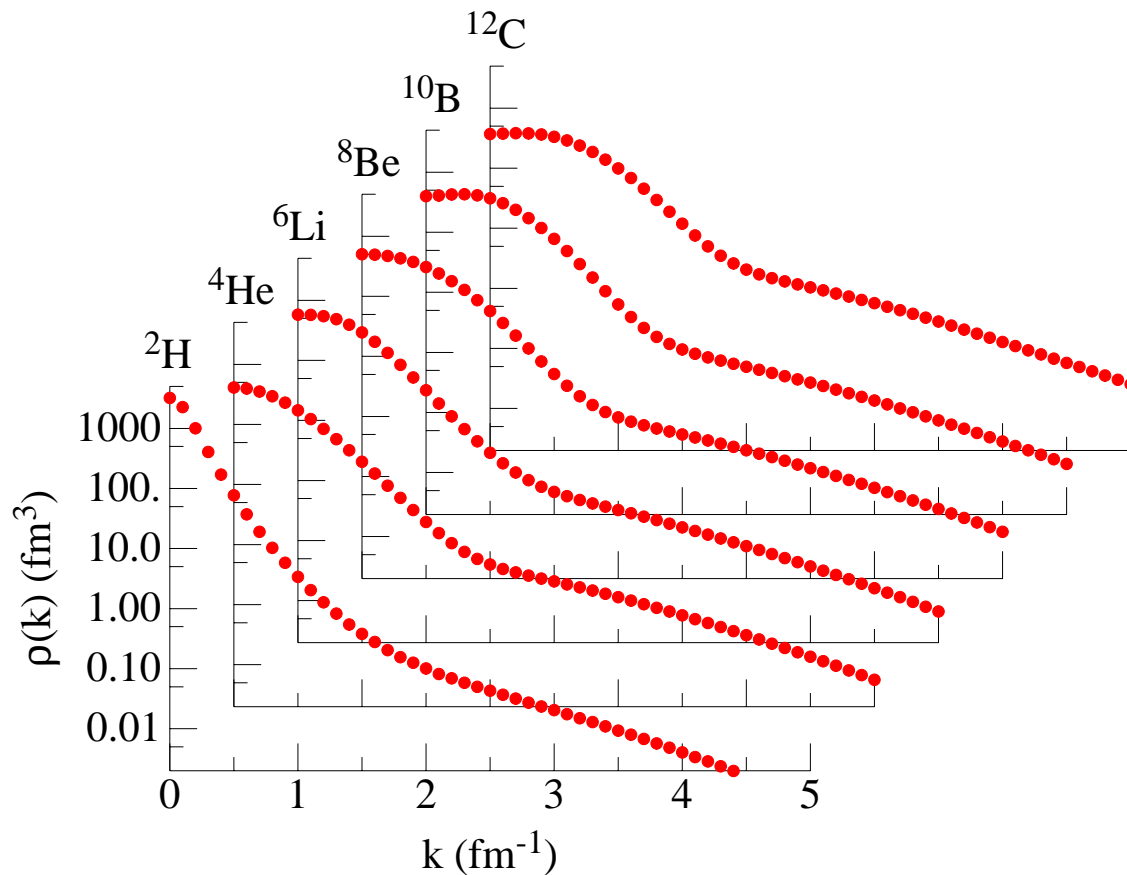
	r_{pp}	r_{np}	r_{nn}
${}^4\text{He}$	2.41	2.35	2.41
${}^6\text{He}$	2.51	3.69	4.40
${}^8\text{He}$	2.52	3.58	4.37

SINGLE-NUCLEON MOMENTUM DISTRIBUTIONS

Probability of finding a nucleon in a nucleus with momentum k in a given spin-isospin state:

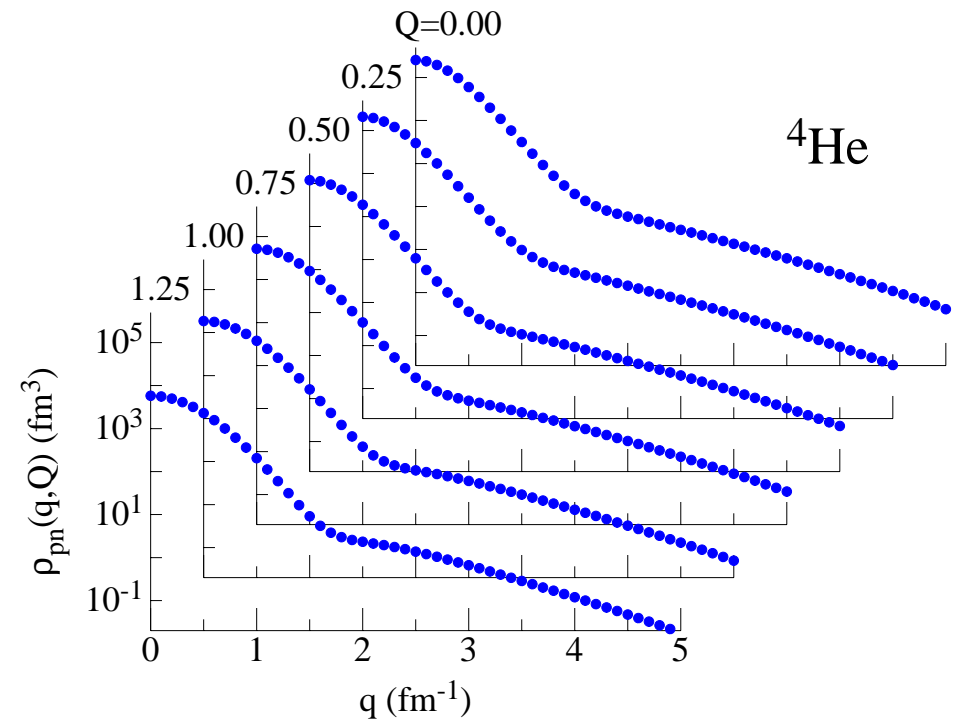
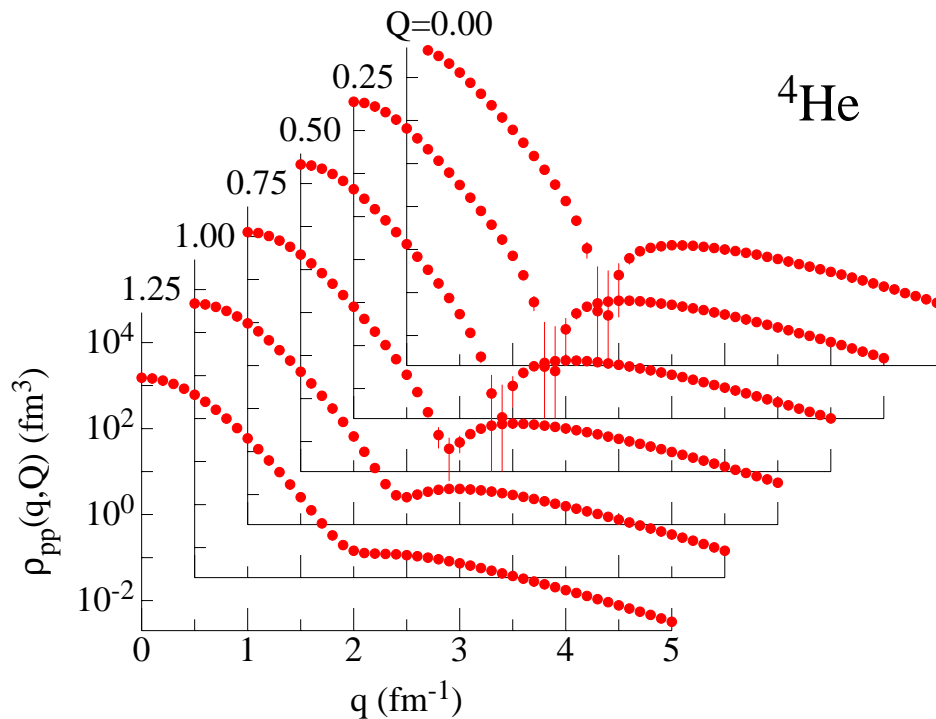
$$\rho_{\sigma\tau}(k) = \int d\mathbf{r}'_1 d\mathbf{r}_1 d\mathbf{r}_2 \cdots d\mathbf{r}_A \psi_A^\dagger(\mathbf{r}'_1, \mathbf{r}_2, \dots, \mathbf{r}_A) e^{-i\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}'_1)} P_{\sigma\tau} \psi_A(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$$

- Useful input for electron scattering studies
- Universal character of high-momentum tails from np tensor interaction



TWO-NUCLEON MOMENTUM DISTRIBUTIONS

Probability $\rho_{NN}(q, Q)$ of finding a pair of nucleons with relative momentum q and total momentum Q can be defined in a similar fashion:



- Large ratio $\rho_{pn}(q, Q = 0) / \rho_{pp}(q, Q = 0)$ has been observed in ¹²C($e, e'pN$) scattering
- Results in good agreement with recent ⁴He($e, e'pN$) experiment

M1, E2, F, GT transitions

NO EFFECTIVE CHARGES!

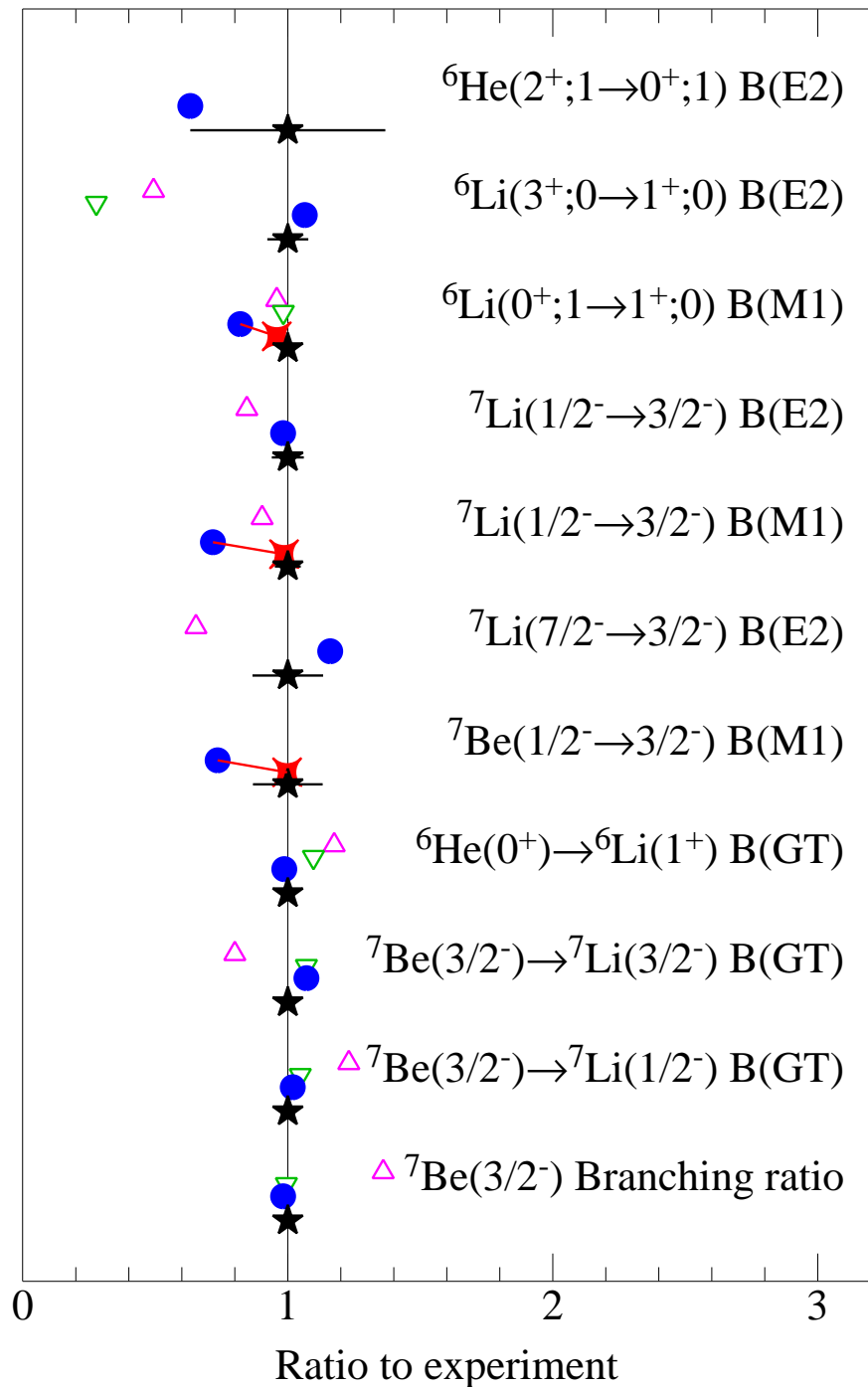
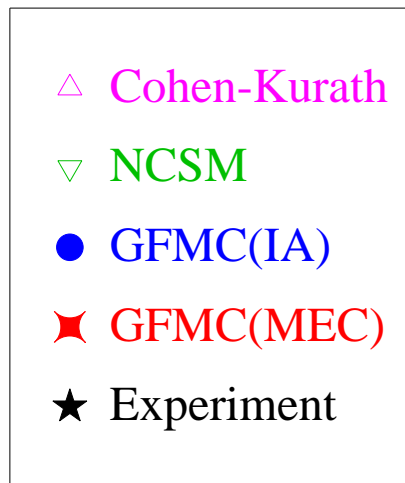
$$E2 = e \sum_k \frac{1}{2} [r_k^2 Y_2(\hat{r}_k)] (1 + \tau_{kz})$$

$$M1 = \mu_N \sum_k [(L_k + g_p S_k)(1 + \tau_{kz})/2 + g_n S_k (1 - \tau_{kz})/2]$$

$$F = \sum_k \tau_{k\pm} ; \text{ GT} = \sum_k \sigma_k \tau_{k\pm}$$

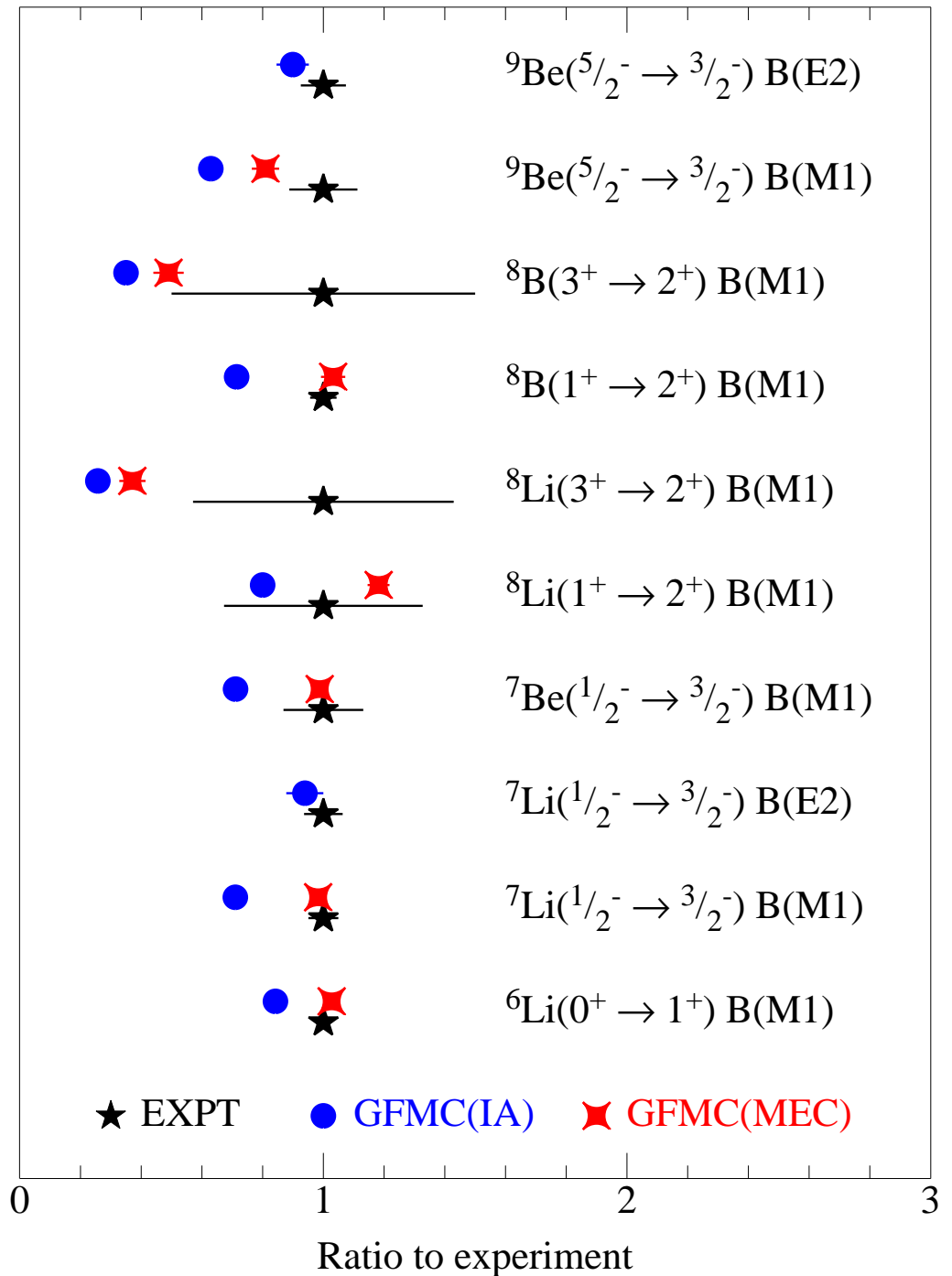
Pervin, Pieper, & Wiringa, PRC **76**, 064319 (2007)

Marcucci, Pervin, *et al.*, PRC **78**, 065501 (2008)



M1 TRANSITIONS W/ χ EFT

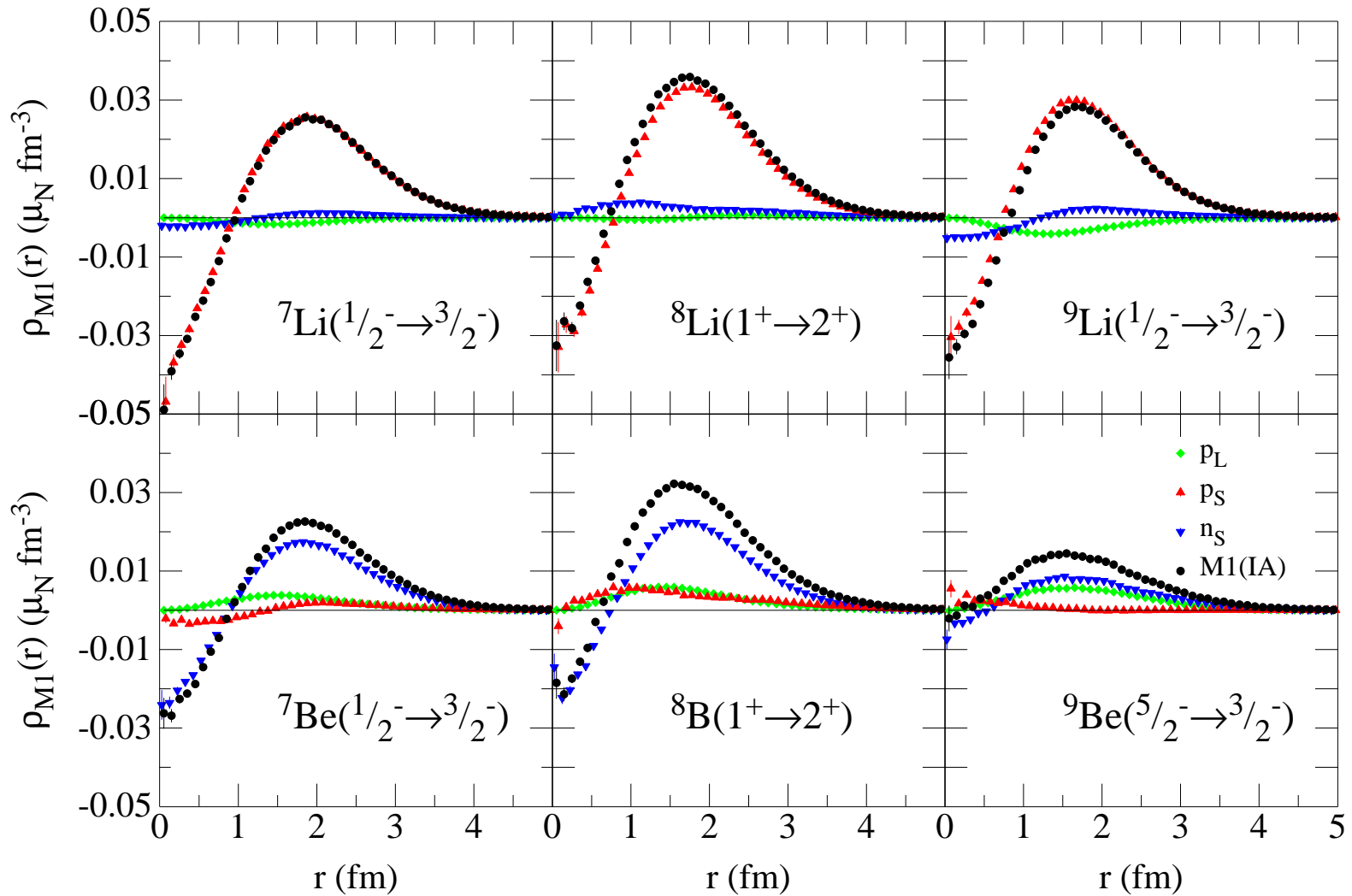
- dominant contribution is from OPE
- five LECs at N3LO
- d_2^V and d_1^V are fixed assuming Δ resonance saturation
- d^S and c^S are fit to experimental μ_d and $\mu_S(^3\text{H}/^3\text{He})$
- c^V is fit to experimental $\mu_V(^3\text{H}/^3\text{He})$
- $\Lambda = 600$ MeV



Pastore, Pieper, Schiavilla, & Wiringa

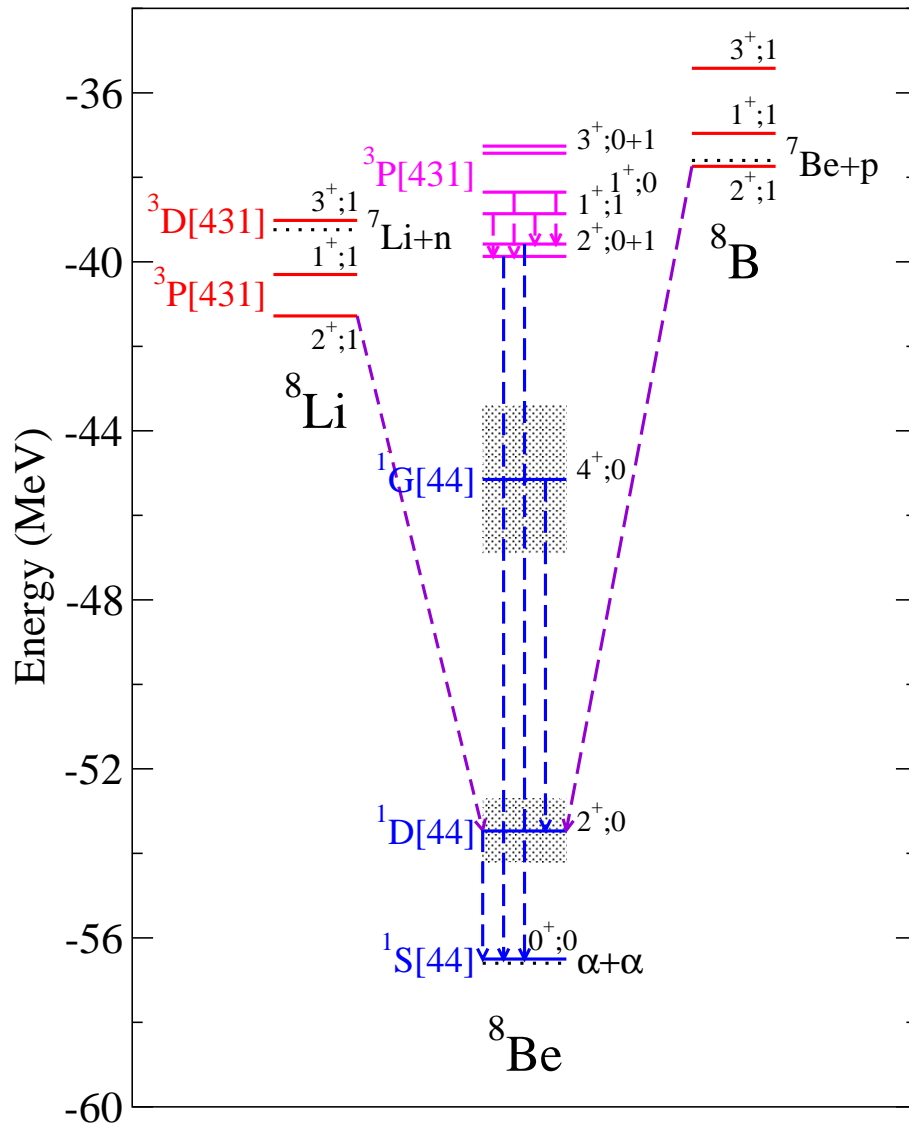
PRC **87**, 035503 (2013)

M1 TRANSITION DENSITIES



$$\mu_p[\rho_{p\uparrow}(r) - \rho_{p\downarrow}(r)] \quad \mu_n[\rho_{n\uparrow}(r) - \rho_{n\downarrow}(r)] \quad \mu_p \rho_{pL}(r)$$

TRANSITIONS IN/TO ^8Be

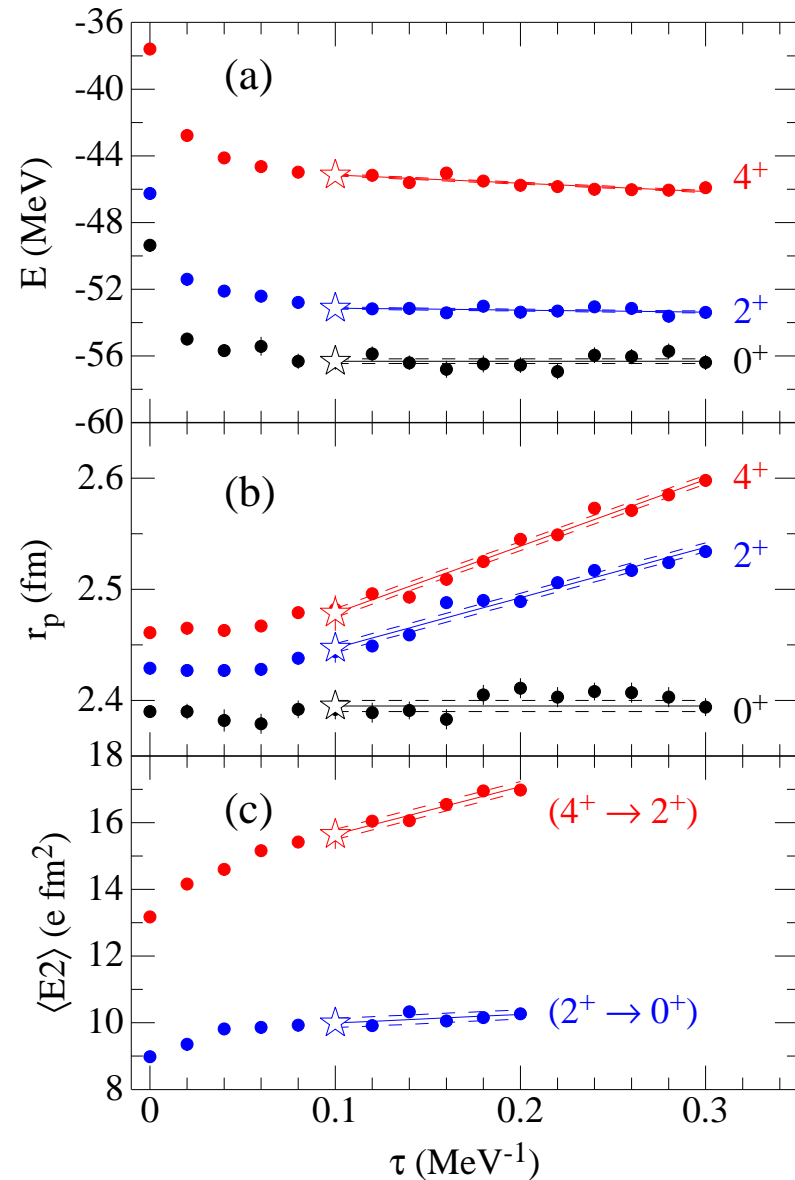


- ^8Be presents new challenges in transition calculations
- $E2$ transitions between rotational band states which have large widths
- $M1$ transitions involving isospin-mixed states
- GT transitions that are not super-allowed and go to a broad final state

$J^\pi; T$	GFMC	Expt
0^+	-56.3(2)	-56.50
2^+	+ 3.2(2)	+ 3.03(1)
4^+	+11.2(2)	+11.35(15)
$2^+; 0$	+16.8(3)	+16.746(3) \rightarrow 16.626
$2^+; 1$	+16.8(3)	+16.802(3) \rightarrow 16.922
$1^+; 1$	+17.4(2)	+17.66(1) \rightarrow 17.64
$1^+; 0$	+18.0(3)	+18.13(1) \rightarrow 18.15

$E2$ TRANSITIONS IN ${}^8\text{Be}$

- New experiment at Tata Institute, Mumbai for $4^+ \rightarrow 2^+$ transition
- Experimental AND theoretical challenge: 4^+ and 2^+ states are wide and breakup into two α s
- GFMC calculation is extrapolated back to $\tau = 0.1 \text{ MeV}^{-1}$; predicts $B(E2) = 27.2(15)$
- Experiment detects $\alpha+\alpha+\gamma$ in coincidence for range of beam energies
- Assuming Breit-Wigner shape, simple analysis gives $B(E2) = 21.3(23)$



M1 TRANSITIONS IN ^8Be BETWEEN ISOSPIN-MIXED STATES

We calculate between states of pure isospin:

matrix element	IA	MEC	TOT
$\langle 1^+; 1 M1 2^+; 0 \rangle$	2.29(1)	0.62(1)	2.91(1)
$\langle 1^+; 1 M1 2^+; 1 \rangle$	0.14(0)	0.04(1)	0.18(1)
$\langle 1^+; 0 M1 2^+; 0 \rangle$	0.17(0)	0.02(0)	0.19(0)
$\langle 1^+; 0 M1 2^+; 1 \rangle$	2.60(1)	0.29(1)	2.89(1)

Then have to combine them using the physical states:

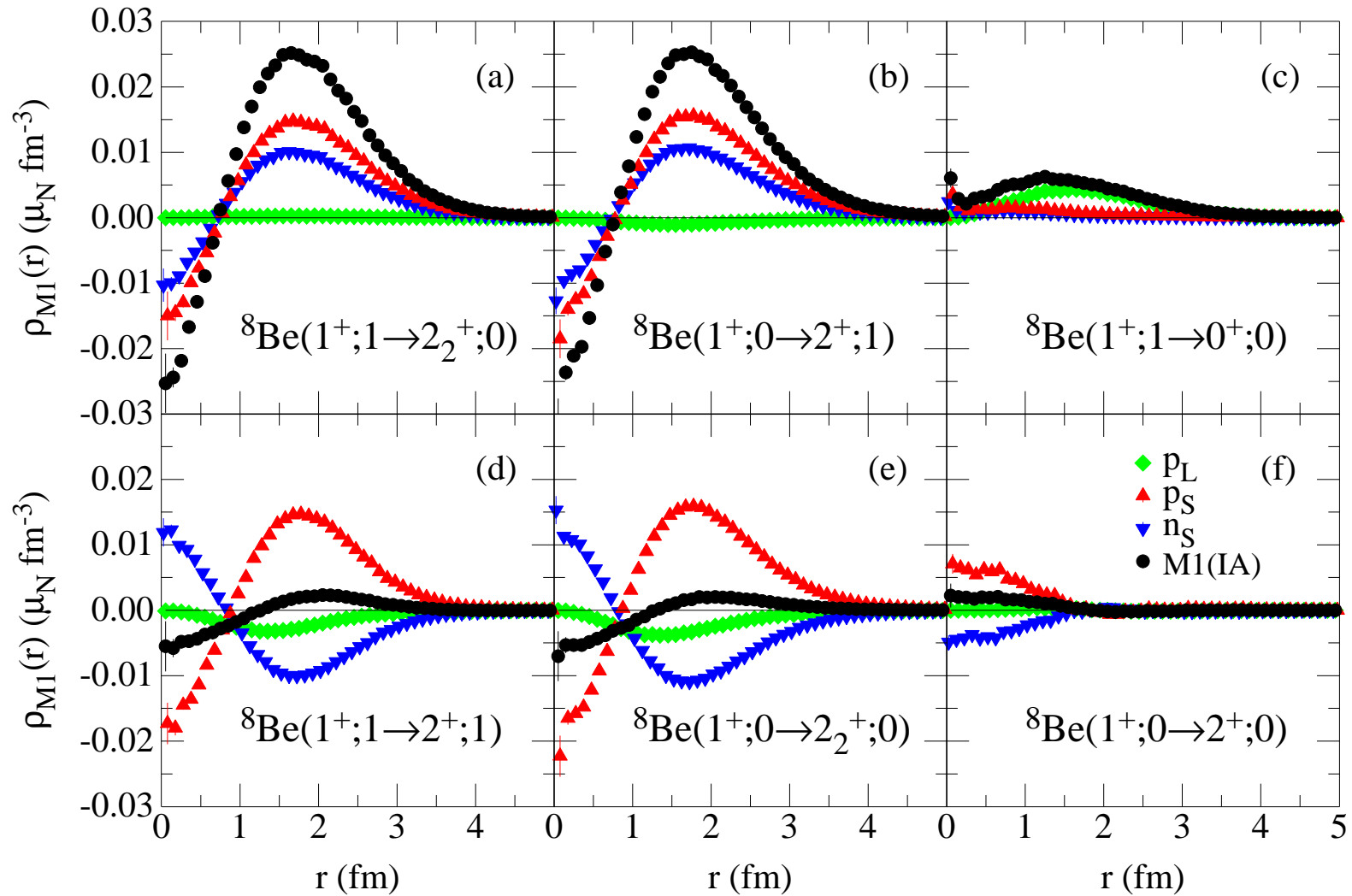
$$\begin{aligned}
 |16.626\rangle &= 0.77|2^+; 0\rangle + 0.64|2^+; 1\rangle & |17.64\rangle &= 0.24|1^+; 0\rangle + 0.97|1^+; 1\rangle \\
 |16.922\rangle &= 0.64|2^+; 0\rangle - 0.77|2^+; 1\rangle & |18.15\rangle &= 0.97|1^+; 0\rangle - 0.24|1^+; 1\rangle
 \end{aligned}$$

to get the final results:

$B(M1)$	IA	TOT	Expt
17.64 \rightarrow 16.626	1.65(2)	2.54(3)	2.65(25)
17.64 \rightarrow 16.922	0.25(1)	0.46(1)	0.30(7)
18.15 \rightarrow 16.626	0.56(1)	0.62(1)	1.88(46)
18.15 \rightarrow 16.922	1.56(2)	2.01(2)	2.89(33)

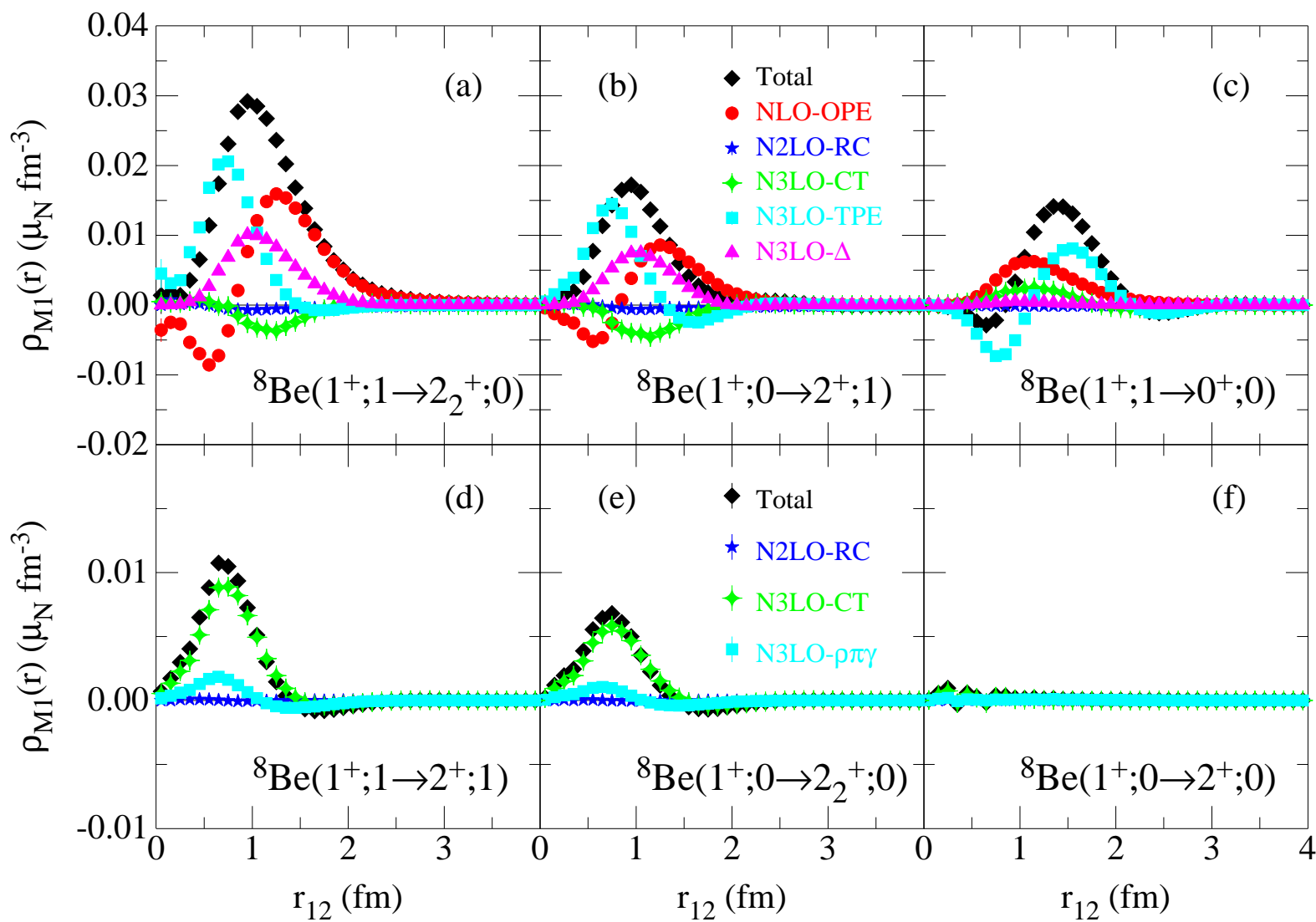
We evaluate the isospin-mixing matrix elements $\langle H_{01} \rangle$ to make sure we have the correct relative signs of our wave functions.

M1 TRANSITION DENSITIES - IMPULSE APPROXIMATION

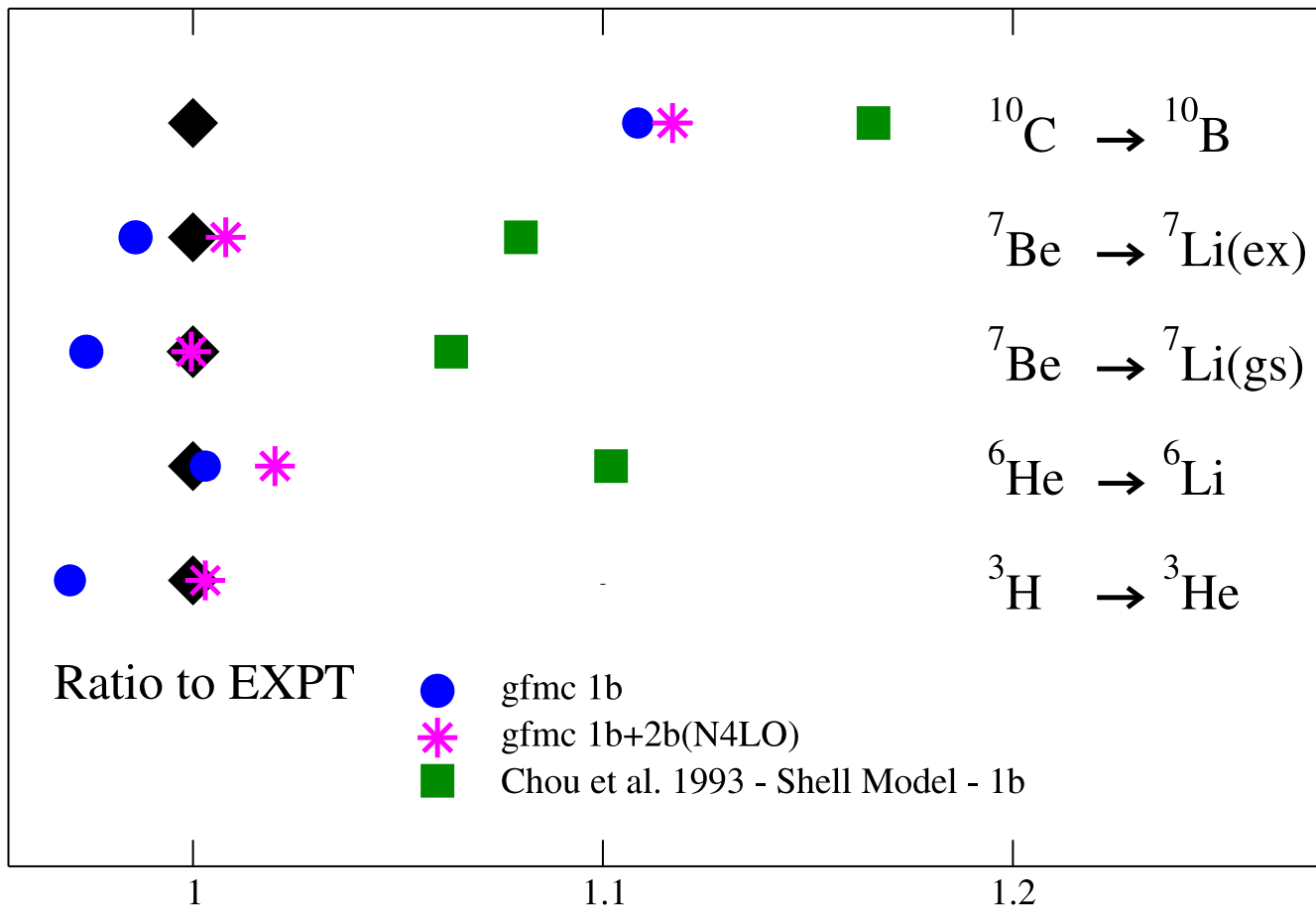


$$\mu_p [\rho_{p\uparrow}(r) - \rho_{p\downarrow}(r)] \quad \mu_n [\rho_{n\uparrow}(r) - \rho_{n\downarrow}(r)] \quad \mu_p \rho_{pL}(r)$$

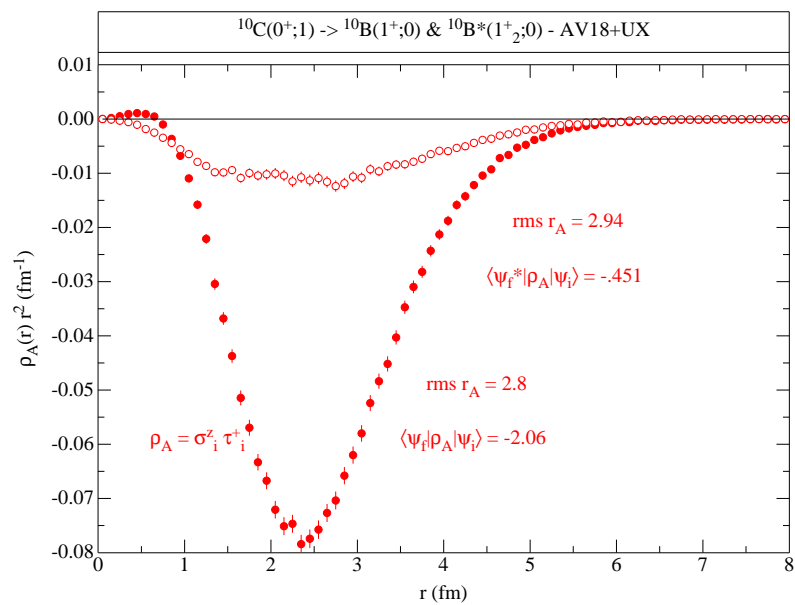
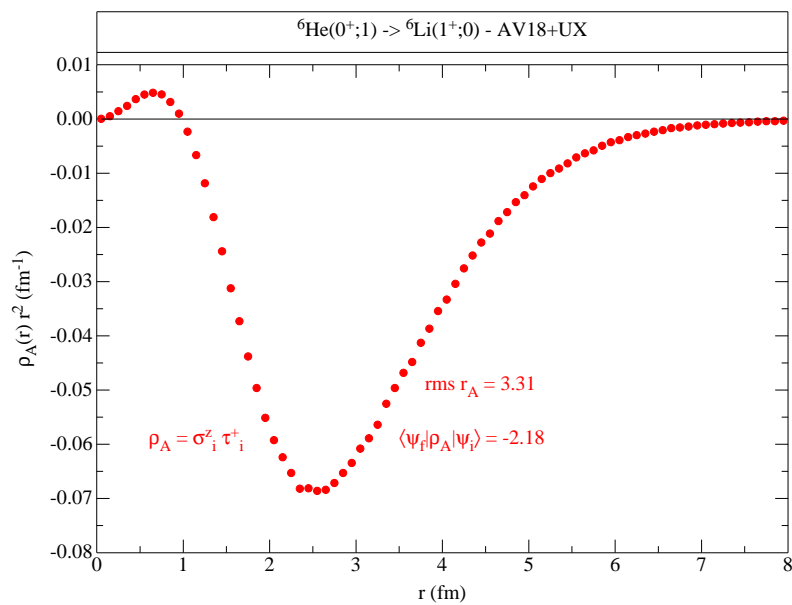
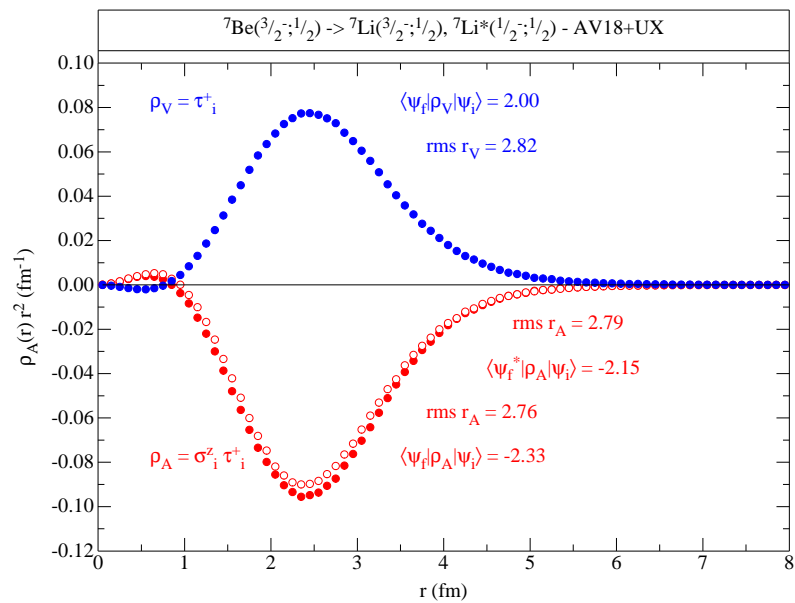
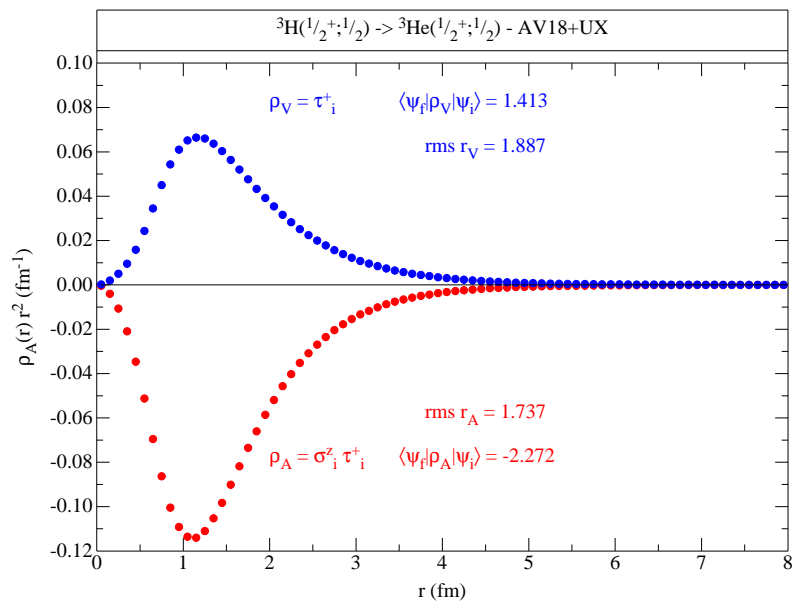
M1 TRANSITION DENSITIES - TWO-BODY CURRENTS



WEAK DECAYS



WEAK TRANSITION DENSITIES



OBSERVATIONS AND ONGOING WORK

- The $T_i + v_{ij}$ for AV18 and all NV2 underbind light nuclei so need net attraction from V_{ijk}
- AV18 and all NV2 saturate symmetric nuclear matter (SNM) at $\approx 2\rho_0$ so SNM needs net repulsion from V_{ijk}
- The $V_{ijk}^{2\pi}$ is attractive in nuclei and SNM, but weakly repulsive in pure neutron matter (PNM), so saturation in SNM and robust PNM need significant repulsion from shorter-range c_D and c_E
- The sign of NV3 c_D term is not well determined by binding energy alone; new fits to GT matrix element give all NV3 $c_D^* < 0$ which provides repulsion in nuclei and SNM; in PNM its effect is much smaller due to weaker tensor correlations
- The $\langle \tau_i \cdot \tau_k \rangle$ of NV3 c_E term is negative in light nuclei, but will change sign in PNM; new GT fits have small $c_E < 0$ (> 0) for soft -Ia, -IIa (hard -Ib, -IIb) giving weak repulsion (attraction) in nuclei
- Calculations of light nuclear spectra, SNM, PNM for new NV2+3* models are in progress, but more general 3NF may be required
- Set of 10 subleading chiral 3NF terms are being evaluated; also more general Urbana with c_D -like and multiple operator c_E -like terms
- Surveys of magnetic moments, $M1$, $E2$, and GT matrix elements in impulse approximation to check for possible problems are ongoing:

VMC IA MAGNETIC MOMENTS FOR NV2+3 POTENTIALS

Nucleus	AV18+UX	NV2+3-Ia	NV2+3-Ib	NV2+3-IIa	NV2+3-IIb	Expt
2H(1+)	0.8470	0.8499	0.8485	0.8501	0.8501	0.8574
3H(1/2+)	2.5901	2.5854	2.5886	2.5874	2.5895	2.9790
3He(1/2+)	-1.7753	-1.7642	-1.7723	-1.7687	-1.7687	-2.1270
6Li(1+)	0.8211	0.8245	0.8173	0.8233	0.8228	0.8220
7Li(3/2-)	2.8999	2.9149		2.8982		3.2564
7Be(3/2-)	-1.0773	-1.0863		-1.0701		-1.3980
8Li(2+)	1.2576	1.3296	1.1743	1.2545	1.3186	1.6536
8B(2+)	1.3319	1.2890		1.3478	1.2967	1.0355
9Li(3/2-)	2.7522	2.3664		2.9109		3.4391
9C(3/2-)	-0.7556	-0.1988		-1.0979		-1.3914
9Be(3/2-)	-1.0957	-0.9622		-1.0211		-1.1778
9B(3/2-)	2.9415	2.6904		2.7803		
10B(3+)	1.8047	1.7869		1.7933		1.8006
10B(1+)	0.7893	0.7863		0.8100		0.6300
11B(3/2-)	2.9125	1.9414		2.1873		2.6886
11C(3/2-)		-0.3094		-0.5018		-0.9640

VMC IA GAMOW-TELLER MES FOR NV2+3 POTENTIALS

Nucleus	AV18+UX	NV2+3-Ia	NV2+3-Ib	NV2+3-IIa	NV2+3-IIb	Expt	
						Suzuki	Chou
6He -> 6Li 0+;1 -> 1+;0	2.193	2.200	2.254	2.206	2.213	2.182	2.174
7Be -> 7Li 3/2- -> 3/2-	2.335	2.318	2.294	2.294	2.309	2.290	2.280
7Be -> 7Li* 3/2- -> 1/2-	2.150	2.158	2.120	2.127	2.143	2.128	2.119
8He -> 8Li* 0+;1 -> 1+;1	0.342	0.386		0.467		0.514	0.512
8Li -> 8Be* 2+;1 -> 2+;0	0.082	0.147		0.144			0.288
8B -> 8Be* 2+;1 -> 2+;0	0.082	0.146		0.145			0.269
9Li -> 9Be 3/2- -> 3/2-	0.044	0.244		0.129		0.276	0.275
9Li -> 9Be* 3/2- -> 5/2-	0.119	0.232		0.284		0.338	0.336
9C -> 9B 3/2- -> 3/2-	0.028	0.259		0.124		0.287	0.286
10C -> 10B 0+;1 -> 1+;0	2.062	1.942		2.158		1.862	1.854
11C -> 11B 3/2- -> 3/2-	1.92	1.02				1.18	1.17