

# Electroweak Probes of Three-Nucleon Systems

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- ▶ Form factors of three-nucleon systems
- ▶ Hadronic parity-violation in three-nucleon systems

# Pionless Effective Field Theory

- ▶ Ideally suited for momenta  $p < m_\pi$  since pions can be integrated out as degrees of freedom and only nucleons and external currents are left.
- ▶ Power counting organizes terms by their relative importance and allows for error estimation of calculation.
  - ▶ Organized by counting powers of  $Q^n = \left(\frac{p}{m_\pi}\right)^n$ .
  - ▶ Ensure order-by-order results are renormalization group invariant (converge to finite values for  $\Lambda \rightarrow \infty$ ).
- ▶ Power counting gives relative size of two- and three-body currents.

Dibaryon fields make three-body calculations easier

$$\begin{aligned} \mathcal{L} = & \hat{N}^\dagger \left( i\partial_0 + \frac{\vec{\nabla}^2}{2M_N} \right) \hat{N} - \hat{t}_i^\dagger \left( i\partial_0 + \frac{\vec{\nabla}^2}{4M_N} - \Delta_{(-1)}^{(3S_1)} - \Delta_{(0)}^{(3S_1)} \right) \hat{t}_i \\ & - \hat{s}_a^\dagger \left( i\partial_0 + \frac{\vec{\nabla}^2}{4M_N} - \Delta_{(-1)}^{(1S_0)} - \Delta_{(0)}^{(1S_0)} \right) \hat{s}_a + y \left[ \hat{t}_i^\dagger \hat{N}^T P_i \hat{N} + \text{H.c.} \right] \\ & + y \left[ \hat{s}_a^\dagger \hat{N}^T \bar{P}_a \hat{N} + \text{H.c.} \right]. \end{aligned}$$

- ▶  $\hat{N}$  nucleon fields
- ▶  $\hat{t}_i$  (dibaryon field) two nucleons in  $^3S_1$  channel
- ▶  $\hat{s}_a$  (dibaryon field) two nucleons in  $^1S_0$  channel
- ▶ Can be matched to theory of only nucleons by integrating out dibaryon fields

# Two- and Three-Body Inputs of $EFT_{\pi}$

Two-body inputs for  $EFT_{\pi}$ :

- ▶ LO scattering lengths  $a_1$  ( $^3S_1$ ) and  $a_0$  ( $^1S_0$ ) **non-perturbative**
- ▶ NLO range corrections  $r_1$  and  $r_0$  **perturbative**

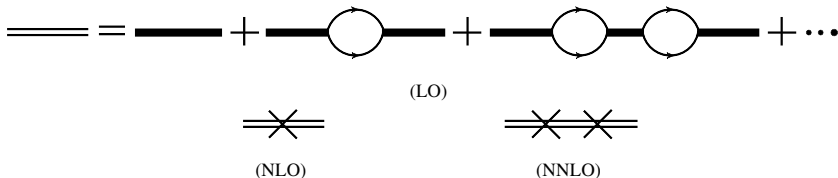
Three-body inputs for  $EFT_{\pi}$ :

- ▶ LO three-body force  $H_0$  fit to doublet  $S$ -wave  $nd$  scattering length **non-perturbative** (Bedaque et al.) nucl-th/9906032
- ▶ NNLO three-body energy dependent three-body force  $H_2$  fit to triton binding energy **perturbative**

Total of **6** NNLO parameters predicts observables to  $\sim 3\%$

- ▶ two-body LECs fit using  $Z$ -parametrization: LO fits to  $^3S_1$  and  $^1S_0$  poles and NLO to their residues.

The LO dressed deuteron propagator is given by a sum of bubble diagrams

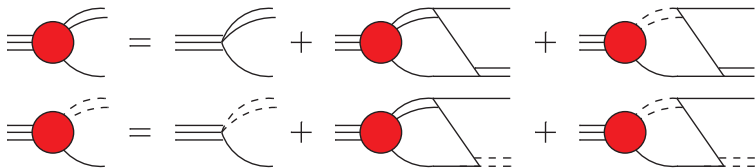


yielding the LO dibaryon propagator

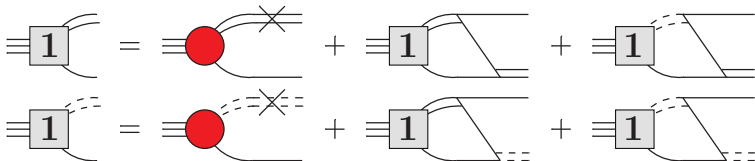
$$iD_{t,s}^{LO}(p_0, \vec{p}) = \frac{4\pi i}{M_N y^2} \frac{1}{\gamma_{t,s} - \sqrt{\frac{\vec{p}^2}{4} - M_N p_0} - i\epsilon}$$

# LO and NLO vertex function

LO three-nucleon vertex function

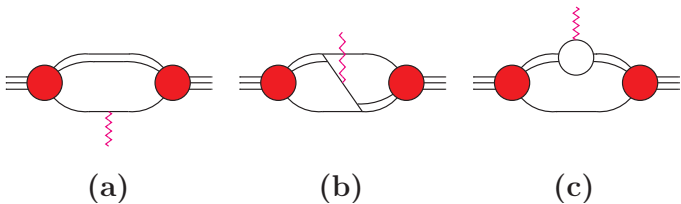


NLO correction to three-nucleon vertex function

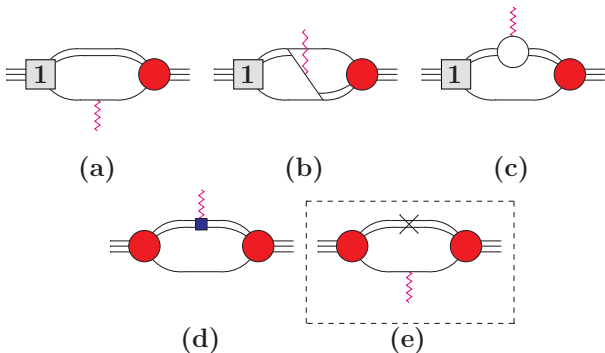


Resulting integral equations are solved numerically

## LO form factor (arbitrary probe)



## NLO form factor





# Form Factor Couplings

Generic form factor can be expanded as

$$F(Q^2) = a \left( 1 - \frac{1}{6} \langle r^2 \rangle Q^2 + \dots \right)$$

The couplings for the form factors of interest are given by

	Charge	Magnetic	Axial
1B LO	$-e \hat{N}^\dagger \hat{N} \hat{A}_0$	$\hat{N}^\dagger (\kappa_0 + \tau_3 \kappa_1) \boldsymbol{\sigma} \cdot \mathbf{B} \hat{N}$	$\frac{g_A}{\sqrt{2}} \hat{N}^\dagger \sigma_i \tau_+ \hat{N} \hat{A}_i^+$
2B NLO	$e c_{0t} \hat{t}_i^\dagger \hat{t}_i \hat{A}_0$	$e \frac{L_1}{2} \hat{t}_i^\dagger \hat{s}_3 \mathbf{B}_j - e \frac{L_2}{2} i \epsilon^{ijk} \hat{t}_i^\dagger \hat{t}_j \mathbf{B}_k$	$l_{1,A} \hat{s}_-^\dagger \hat{t}_i \hat{A}_i^-$
$a$	$Z$	$\mu$	$\langle \mathbf{GT} \rangle$

**Table:** List of couplings for form factors of interest and their physical values at  $Q^2 = 0$ .

The iso-scalar and iso-vector combination of magnetic moments are

$$\mu_s = \frac{1}{2} (\mu_{^3\text{He}} + \mu_{^3\text{H}}) \quad , \quad \mu_v = \frac{1}{2} (\mu_{^3\text{He}} - \mu_{^3\text{H}}) ,$$

$\mu_s$  only depends on  $\kappa_0$  and  $L_2$ , while  $\mu_v$  only depends on  $\kappa_1$  and  $L_1$  at NLO.

	$\mu_s$	$\mu_v$	$L_1$ fit
LO	0.440(152)	-2.31(78)	N/A
NLO	0.421(50)	-2.20(26)	$\sigma_{np}$
NLO	0.421(50)	-2.56(31)	$\mu_{^3\text{H}}$
NLO	0.421(50)	-2.50(30)	$\sigma_{np}$ and $\mu_{^3\text{H}}$
Exp	0.426	-2.55	N/A

**Table:** Table of three-nucleon iso-scalar and iso-vector magnetic moments compared to experiment. The different NLO rows are different fits for  $L_1$ .

Magnetic moments and polarizabilities also calculated to NLO by (Kirscher et al. (2017)) arXiv:1702.07268

# Bound State Observables for $3N$ Systems

([Vanasse \(2016\)+\(2017\)](#)). arXiv:1512.03805 + arXiv:1706.02665

Observable	LO	NLO	NNLO	Exp.
${}^3\text{H}: r_C$ [fm]	1.14(19)	1.59(8)	1.62(3)	1.5978(40)
${}^3\text{He}: r_C$ [fm]	1.26(21)	1.72(8)	1.74(3)	1.7753(54)
${}^3\text{H}: r_m$ [fm]	1.40(24)	1.78(11)	–	1.840(181)
${}^3\text{He}: r_m$ [fm]	1.49(26)	1.85(11)	–	1.965(153)
${}^3\text{H}: \mu_m$ [ $\mu_N$ ]	2.75(92)	2.92(35)	–	2.98
${}^3\text{He}: \mu_m$ [ $\mu_N$ ]	-1.87(73)	-2.08(25)	–	-2.13

Calculation of LO triton charge radius in unitary limit gives

$$mE_{3B} \langle r_C^2 \rangle = 0.224\dots$$

Using analytical techniques in

([Braaten and Hammer \(2006\)](#)) cond-mat/0410417 it can be shown that  $mE_{3B} \langle r_C^2 \rangle = (1 + s_0^2)/9 = 0.224\dots$  in the unitary limit.

# Tritium $\beta$ -decay

Half life  $t_{1/2}$  of tritium given by

$$\frac{(1 + \delta_R)f_V}{K/G_V^2} t_{1/2} = \frac{1}{\langle \mathbf{F} \rangle^2 + f_A/f_V g_A^2 \langle \mathbf{GT} \rangle^2}$$

The Gamow-Teller matrix element is

$$\frac{\langle \mathbf{GT} \rangle_{\text{Exp}}}{\sqrt{3}} = 0.9551 \quad , \quad \frac{\langle \mathbf{GT} \rangle_0}{\sqrt{3}} = 0.9807 \quad , \quad \frac{\langle \mathbf{GT} \rangle_{0+1}}{\sqrt{3}} = 0.9935$$

Fitting  $L_{1A}$  to the GT-matrix element gives

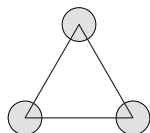
$$L_{1A} = 3.46 \pm 1.19 \text{ fm}^3$$

Compares well to lattice prediction

$$L_{1A} = 3.9(0.1)(1.0)(0.3)(0.9) \text{ fm}^3$$

# Wigner-symmetry

**Wigner-limit:**  $a_0 = a_1$  and  $r_0 = r_1$   
**Wigner-breaking  $\mathcal{O}(\delta)$ :**  $\delta = \frac{1/a_1 - 1/a_0}{1/a_1 + 1/a_0}$



The GT-matrix element is given by

$$\langle \mathbf{GT} \rangle \simeq \sqrt{3}(P_S + P_D/3 - P_{S'}/3)$$

In Wigner-SU(4) limit  $P_{S'} = 0$  and  $P_S = 1$  hence  $\langle \mathbf{GT} \rangle = \sqrt{3}$

	Wigner	$\mathcal{O}(\delta)$
LO EFT $_{\not{r}}$	1.22	1.08/1.19
$\mathcal{O}(r)$	1.66	1.58/1.70
Experiment		1.5978(40)/1.775(5)

Table:  ${}^3\text{H}/{}^3\text{He}$  charge radius in Wigner-limit  
(**Vanasse and Phillips (2016)**) arXiv:1607.08585

In Wigner-limit it can be shown both analytically and numerically

$$\mu({}^3\text{H}) = \mu_p$$

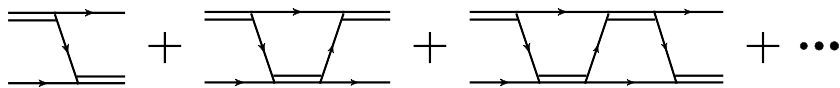
$$\mu({}^3\text{He}) = \mu_n$$

## Conclusions and Future directions

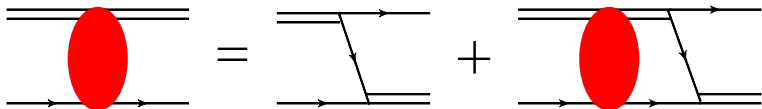
- ▶ Charge radii of  $^3\text{H}$  and  $^3\text{He}$  reproduced well at NNLO in  $\text{EFT}_{\pi}$ .
- ▶ Magnetic moments and radii reproduced within errors at NLO in  $\text{EFT}_{\pi}$ .
- ▶ Better prediction for  $L_{1A}$  will further constrain  $\text{EFT}_{\pi}$  prediction for  $pp$  fusion. Slight inconsistencies with how LECs are fit in two and three-body, differences likely higher order.
- ▶ Wigner-symmetry gives good expansion for charge radii and is interesting limit for three-nucleon magnetic moments and GT-matrix element. Results should be used as benchmark.
- ▶ Reproduce analytical results in unitary limit for charge radii. Should be used as benchmark for all such calculations.
- ▶ An  $\text{N}^3\text{LO}$  calculation will result in observables with roughly  $\sim 1\%$  uncertainty.

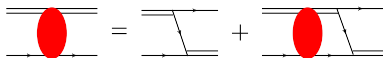
## Quartet Channel ( $nd$ Scattering)

At LO in the quartet channel,  $nd$  scattering is given by an infinite sum of diagrams.



This infinite sum of diagrams can be represented by an integral equation.





Projecting spin and isospin in the quartet channel and projecting out in angular momentum gives

$$\begin{aligned}
 t_{0,q}^{\ell}(k, p) = & -\frac{y_t^2 M_N}{pk} Q_{\ell} \left( \frac{p^2 + k^2 - M_N E - i\epsilon}{pk} \right) - \\
 & + \frac{2}{\pi} \int_0^{\Lambda} dq q^2 t_{0,q}^{\ell}(k, q) \frac{1}{\gamma_t - \sqrt{\frac{3q^2}{4} - M_N E - i\epsilon}} \frac{1}{qp} \times \\
 & Q_{\ell} \left( \frac{p^2 + q^2 - M_N E - i\epsilon}{pq} \right),
 \end{aligned}$$

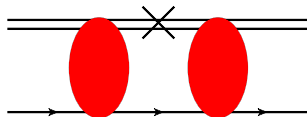
where

$$Q_{\ell}(a) = \frac{1}{2} \int_{-1}^1 dx \frac{P_{\ell}(x)}{x + a}.$$

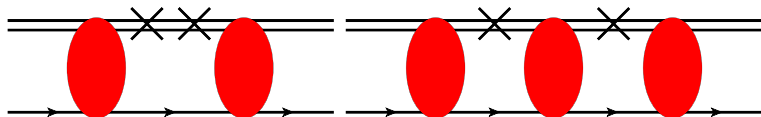


# Higher Orders

NLO correction is



NNLO corrections are



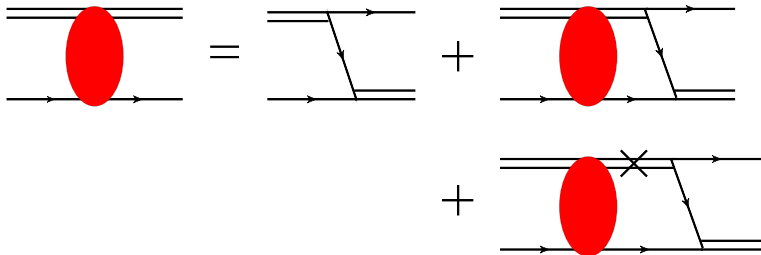
Note the second diagram contains full off-shell scattering amplitude.

# Partial Resummation Technique

Denoting  $t_{NLO}^\ell = t_{0,q}^\ell + t_{1,q}^\ell$ , for the partial resummation technique one finds (Bedaque, Rupak, Grißhammer, and Hammer (2003))

$$t_{NLO}^\ell(k, p) = B_0^\ell(k, p) + B_1^\ell(k, p) + (K_0^\ell(q, p, E) + K_1^\ell(q, p, E)) \otimes t_{NLO}^\ell(k, q),$$

with the diagrammatic representation

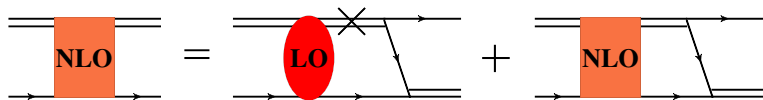


# New Full Perturbative technique

Picking out only NLO pieces gives (Vanasse (2013))

$$t_{1,q}^{\ell}(k, p) = B_1^{\ell}(k, p) + K_1^{\ell}(q, p, E) \otimes t_{0,q}^{\ell}(k, q) + K_0^{\ell}(q, p, E) \otimes t_{1,q}^{\ell}(k, q).$$

Terms are reshuffled to inhomogeneous term. Kernel at each order is the same. Diagrammatically NLO correction is now given by



Note all corrections are half off-shell.

# Doublet Channel $nd$ scattering

At LO in the doublet channel,  $nd$  scattering is given by a coupled set of integral equations

The diagram illustrates the integral equations for  $nd$  scattering in the doublet channel at LO. It consists of two rows of equations, each with three terms. The first row shows the LO scattering amplitude as a sum of two diagrams: a single exchange and a contact term. The second row shows the next-order corrections, which are products of the LO diagrams with themselves, representing two-loop diagrams.

$$\begin{aligned} & \text{Diagram 1} = \text{Diagram 2} + \text{Diagram 3} \\ & + \text{Diagram 1} \times (\text{Diagram 2} + \text{Diagram 3}) \\ & + \text{Diagram 1} \times (\text{Diagram 2} + \text{Diagram 3}) \\ & \text{Diagram 1} = \text{Diagram 2} + \text{Diagram 3} \\ & + \text{Diagram 1} \times (\text{Diagram 2} + \text{Diagram 3}) \\ & + \text{Diagram 1} \times (\text{Diagram 2} + \text{Diagram 3}) \end{aligned}$$

# Two-Body Parity Violation

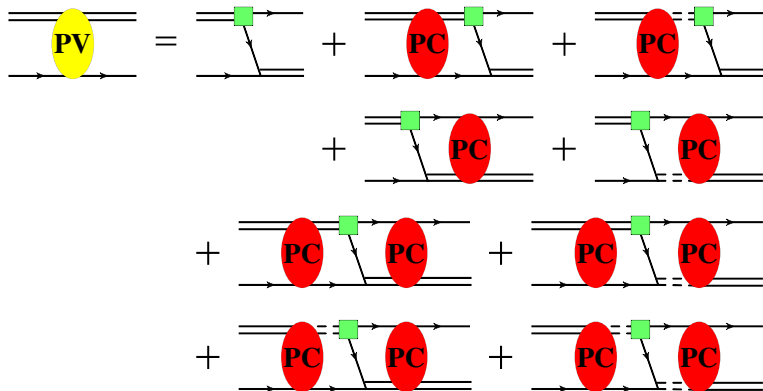
The LO PV Lagrangian in EFT <sub>$\pi$</sub>  has five LECs (**Girlanda (2008)**)

$$\begin{aligned}\mathcal{L}_{PV} = & - \left[ g^{(3S_1-1P_1)} t_i^\dagger \left( N^t \sigma_2 \tau_2 i \overleftrightarrow{\nabla}_i N \right) \right. \\ & + g^{(1S_0-3P_0)} s_a^\dagger \left( N^T \sigma_2 \vec{\sigma} \cdot \tau_2 \tau_a i \overleftrightarrow{\nabla} N \right) \\ & + g^{(1S_0-3P_0)} \epsilon^{3ab} (s^a)^\dagger \left( N^T \sigma_2 \vec{\sigma} \cdot \tau_2 \tau^b \overleftrightarrow{\nabla} N \right) \\ & + g^{(1S_0-3P_0)} \mathcal{I}^{ab} (s^a)^\dagger \left( N^T \sigma_2 \vec{\sigma} \cdot \tau_2 \tau^b i \overleftrightarrow{\nabla} N \right) \\ & \left. + g^{(3S_1-3P_1)} \epsilon^{ijk} (t_i)^\dagger \left( N^T \sigma_2 \sigma^k \tau_2 \tau_3 \overleftrightarrow{\nabla}^j N \right) \right] + h.c.,\end{aligned}$$

where  $\mathcal{I}^{ab} = \text{diag}(1, 1, -2)$  and  $a \overleftrightarrow{\nabla} b = a(\overrightarrow{\nabla} b) - (\overrightarrow{\nabla} a)b$ . Contains all possible  $S \rightarrow P$  transition operators and isospin structures

# LO PV $Nd$ Scattering

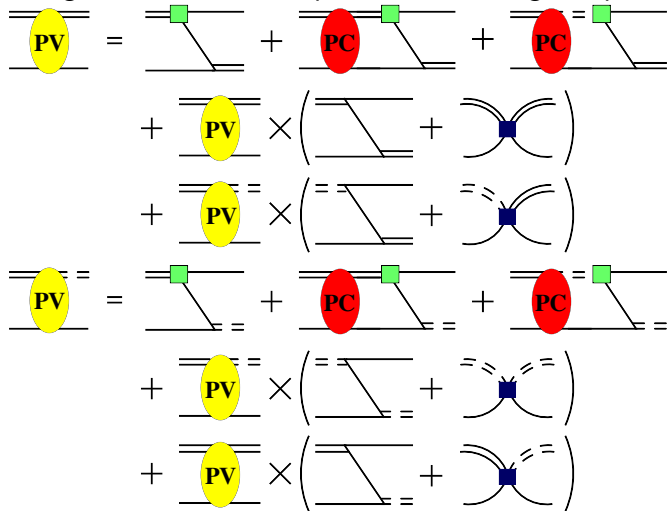
LO PV  $Nd$  scattering given by sum of diagrams



Diagrams with lower two-body PV vertex not shown

# LO PV Nd Scattering (Alternative)

Sum of diagrams can also be represented via integral equation

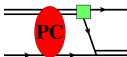


Makes asymptotic analysis easier

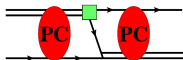
The amplitude can be projected in partial waves of  $\vec{J} = \vec{L} + \vec{S}$



$$t_{PV L'S', LS}^{JM}(k, p) \sim \mathcal{K}(k, p)_{L'S', LS}^{JM}$$



$$t_{PV L'S', LS}^{JM}(k, p) \sim \int_0^\infty dq q^2 \mathcal{K}(q, p)_{L'S', LS}^{JM} \mathbf{D} \left( E - \frac{q^2}{2M_N}, \vec{\ell} \right) \left( t_{PCL S, LS}^{JM}(k, q) \right)$$



$$t_{PV L'S', LS}^{JM}(k, p) \sim \int_0^\infty dq q^2 \int_0^\infty d\ell \ell^2 \left( t_{PCL'S', L'S'}^{JM}(p, \ell) \right)^T \mathbf{D} \left( E - \frac{\ell^2}{2M_N}, \vec{\ell} \right) \mathcal{K}(q, \ell)_{L'S', LS}^{JM} \mathbf{D} \left( E - \frac{q^2}{2M_N}, \vec{q} \right) \left( t_{PCL S, LS}^{JM}(k, q) \right)$$



One term of projected  $\mathcal{K}(k, p)_{L'S',LS}^{JM}$



is given by

$$\left[ \mathcal{K}(k, p)_{L'S',LS}^J \right]_{22} = -y_t \left( 3g_{(\Delta I=0)}^{1S_0-3P_0} - 2g_{(\Delta I=1)}^{1S_0-3P_0} \right) 4\pi\sqrt{6}(-1)^{1/2-L-J} \delta_{S1/2} \delta_{S'1/2} \sqrt{\bar{L}'} \\ \times C_{L',1,L}^{0,0,0} \left\{ \begin{matrix} L' & 1 & L \\ S & J & S' \end{matrix} \right\} \frac{1}{kp} (kQ_{L'}(a) + pQ_L(a))$$

where

$$\bar{x} = 2x + 1$$

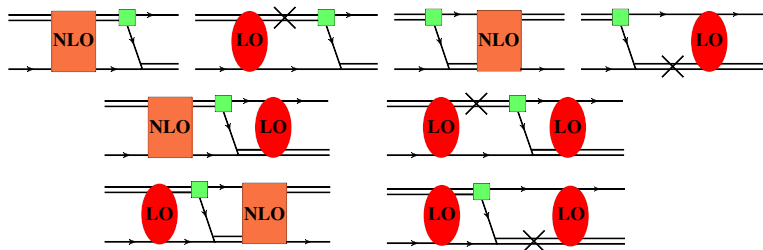
and

$$a = \frac{k^2 + p^2 - M_N E - i\epsilon}{kp}$$

All Projections given in (Vanasse (2012)). Agree with  $S$ -wave to  $P$ -wave projections in (Grießhammer, Schindler, and Springer (2012)).

# NLO 3-Body PV

NLO PV amplitude is given by type of diagrams below. NLO box is the half off shell NLO amplitude.



(Note not all diagrams given here)

As shown by (Schindler and Grißhammer (2010)) no NLO PV three-body force for *Nd* scattering should exist.

Using Fierz rearrangements only single derivative PV 3B forces in  ${}^2S_{1/2} - {}^2P_{1/2}$  are (**Grießhammer and Schindler (2010)**)

$$i\mathcal{M} \left[ {}^2S_{\frac{1}{2}} \rightarrow {}^2P_{\frac{1}{2}}, p, q \right]_{3\text{NI}}^{\text{Wigner}} = A_{3\text{NI}} \left( H_{\text{PV}}^{(\Delta I=0)} + \tau^3 H_{\text{PV}}^{(\Delta I=1)} \right) \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$

tree-level PV diagrams are given by

$$i\mathcal{M} \left[ {}^2S_{\frac{1}{2}} \rightarrow {}^2P_{\frac{1}{2}}, p, q \right]_{2\text{NI}}^{\text{Wigner}} = A_{2\text{NI}}^{(a)} \begin{pmatrix} 0 & 0 \\ \mathcal{S}_1 + \mathcal{T} & \mathcal{S}_1 - \mathcal{T} \end{pmatrix} + A_{2\text{NI}}^{(b)} \begin{pmatrix} 0 & \mathcal{S}_1 + \mathcal{T} \\ 0 & \mathcal{S}_1 - \mathcal{T} \end{pmatrix}$$

where

$$\mathcal{S}_1 = 3g^{({}^3S_1 - {}^1P_1)} + 2\tau_3 g^{({}^3S_1 - {}^3P_1)}, \mathcal{T} = 3g_{(\Delta I=0)}^{({}^1S_0 - {}^3P_0)} + 2\tau_3 g_{(\Delta I=1)}^{({}^1S_0 - {}^3P_0)}$$

PC scattering amplitudes are diagonal in Wigner basis, therefore NLO PV diagrams do not contain element in upper left of Wigner basis matrix. Hence, **no NLO 3B PV force**

# Asymptotic Behavior (Bedaque Numbers)

Going to Wigner basis asymptotic form of  $nd$  scattering integral equation is

$$t_{\lambda}^{(\ell)}(p) = \frac{8\lambda}{\sqrt{3\pi}} (-1)^{\ell} \int_0^{\infty} \frac{dq}{q} Q_{\ell} \left( \frac{p}{q} + \frac{q}{p} \right) t_{\lambda}^{(\ell)}(q)$$

( $\lambda = 1$ ): Wigner-symmetric combination

( $\lambda = -1/2$ ): Wigner antisymmetric combination

Equation is scaleless and must have solution of form

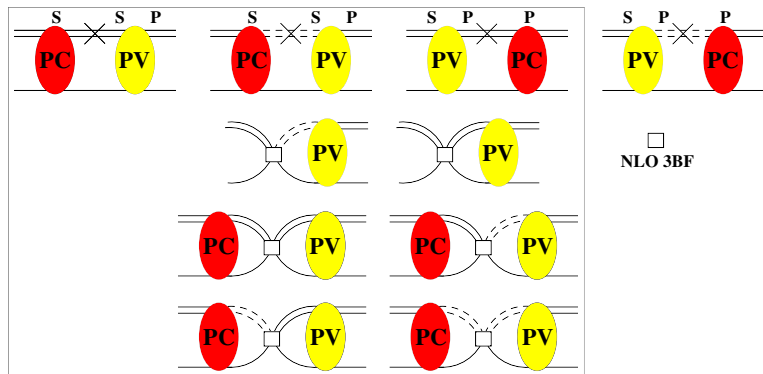
$$t_{\lambda}^{(\ell)}(p) = p^{-s_{\ell}^{\lambda} - 1}$$

partial wave $\ell$	$s_{\ell}(\lambda = 1)$	$s_{\ell}(\lambda = -\frac{1}{2})$
0	1.00624... $i$	2.16622...
1	2.86380...	1.77272...
2	2.82334...	3.10498...

(Grißhammer 2005)

# NLO PV *Nd* Scattering

The NLO PV amplitude can be calculated with



Note, three-body force is PC

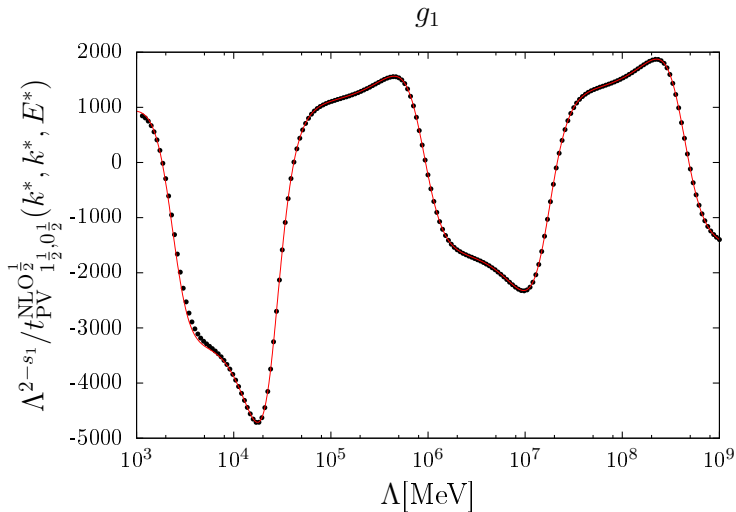
## Asymptotic behavior of NLO PV ${}^2S_{1/2} - {}^2P_{1/2}$ scattering

$$\begin{aligned} & \frac{1}{16\pi} \frac{\Lambda^{2-s_1}}{\sqrt{s_0^2 + (s_1 - 2)^2}} \left[ (\rho_t + \rho_s) \left\{ C D^{2P_{1/2}} \sin \left( s_0 \ln \left( \frac{\Lambda}{\Lambda^*} \right) + \arctan \left( \frac{s_0}{s_1 - 2} \right) \right) \right. \right. \\ & \quad \left. \left. + B^{2P_{1/2}} |H^{2P_{1/2}}| \sin \left( s_0 \ln \left( \frac{\Lambda}{\Lambda^*} \right) + \arctan \left( \frac{s_0}{s_1 - 2} \right) + \text{Arg}(H^{2P_{1/2}}) \right) \right\} \right. \\ & \quad \left. + (\rho_t - \rho_s) C E^{2P_{1/2}} \sin \left( s_0 \ln \left( \frac{\Lambda}{\Lambda^*} \right) + \arctan \left( \frac{s_0}{s_1 - 2} \right) \right) \right] \\ & \quad + \frac{4H_{\text{NLO}}(\Lambda)}{3\pi^2\Lambda^2} C D^{2P_{1/2}} \frac{1}{\sqrt{1 + s_0^2(2 - s_1)}} \Lambda^{3-s_1} \sin \left( s_0 \ln \left( \frac{\Lambda}{\Lambda^*} \right) - \arctan(s_0) \right) + b, \end{aligned}$$

where  $H_{\text{NLO}}(\Lambda)$  is NLO PC 3B force

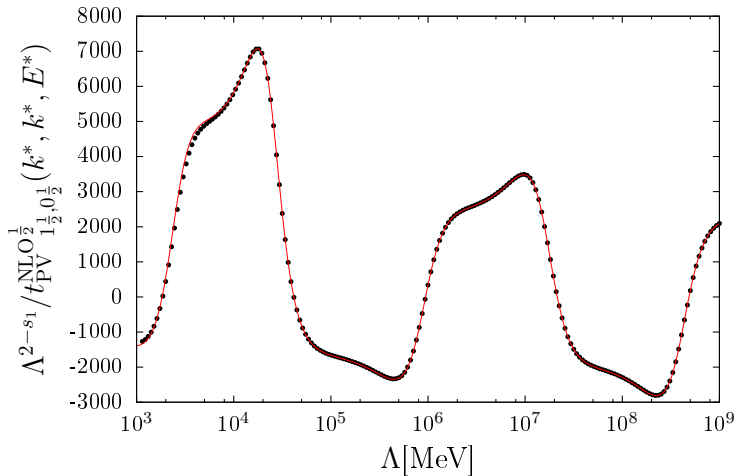
$$H_{\text{NLO}}(\Lambda) = -\Lambda \frac{3\pi(1 + s_0^2)}{128} (\rho_t + \rho_s) \frac{\left( 1 - \frac{1}{\sqrt{1+4s_0^2}} \sin \left( 2s_0 \ln \left( \frac{\Lambda}{\Lambda^*} \right) + \arctan \left( \frac{1}{2s_0} \right) \right) \right)}{\sin^2 \left( s_0 \ln \left( \frac{\Lambda}{\Lambda^*} \right) - \arctan(s_0) \right)} + \dots$$

$$g_1 \rightarrow g^{3S_1-1P_1}$$



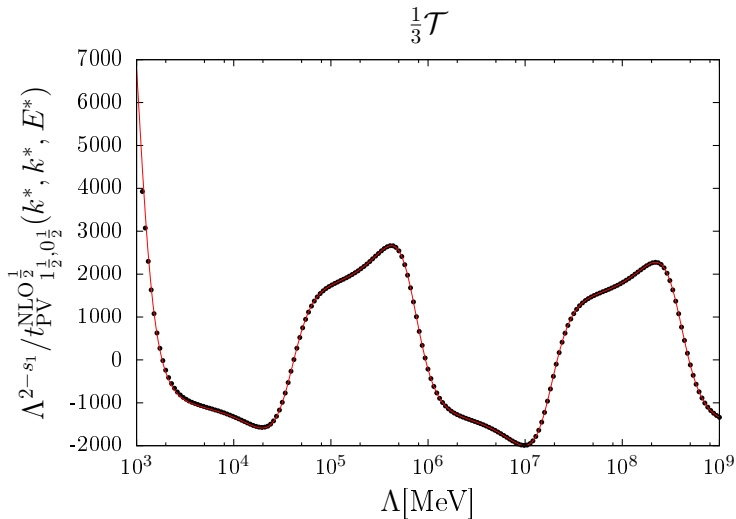
$$g_2 \rightarrow g^{3S_1-3P_1}$$

$g_2$





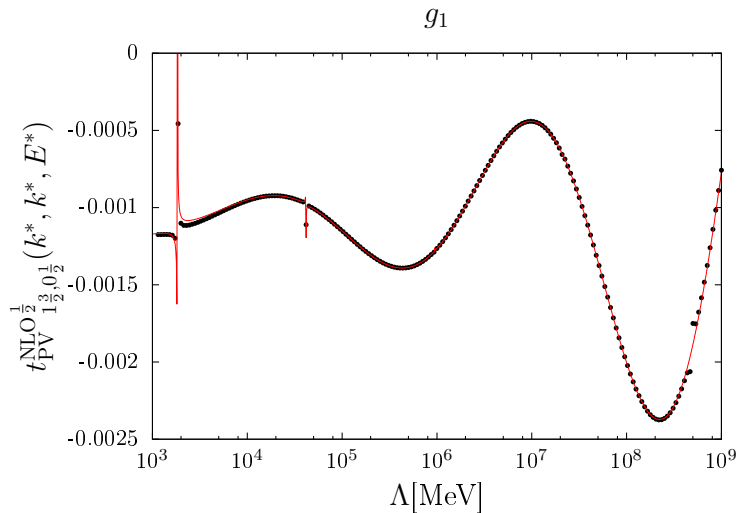
$$\frac{1}{3}\mathcal{T} \rightarrow g_{(\Delta I=0)}^{1S_0-3P_0} + \frac{2}{3}\tau_3 g_{(\Delta I=1)}^{1S_0-3P_0}$$



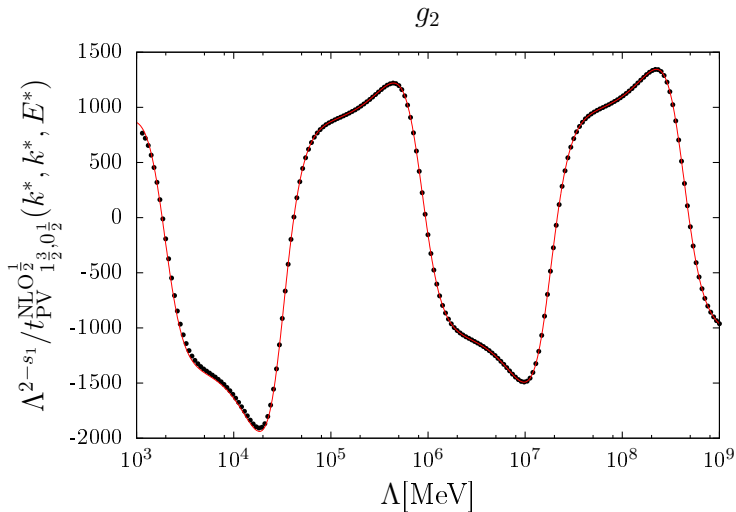
## Asymptotic behavior of NLO PV ${}^2S_{1/2} - {}^4P_{1/2}$ scattering

$$\begin{aligned}
 & \frac{1}{16\pi} \frac{\Lambda^{2-s_1}}{\sqrt{s_0^2 + (s_1 - 2)^2}} \left[ (\rho_t + \rho_s) C D^{4P_{1/2}} \sin \left( s_0 \ln \left( \frac{\Lambda}{\Lambda^*} \right) + \arctan \left( \frac{s_0}{s_1 - 2} \right) \right) \right. \\
 & \quad + (\rho_t - \rho_s) C E^{4P_{1/2}} \sin \left( s_0 \ln \left( \frac{\Lambda}{\Lambda^*} \right) + \arctan \left( \frac{s_0}{s_1 - 2} \right) \right) \\
 & \quad \left. + 4\rho_t B^{4P_{1/2}} |H^{4P_{1/2}}| \sin \left( s_0 \ln \left( \frac{\Lambda}{\Lambda^*} \right) + \arctan \left( \frac{s_0}{s_1 - 2} \right) + \text{Arg}(H^{4P_{1/2}}) \right) \right] \\
 & + \frac{4H_{\text{NLO}}(\Lambda)}{3\pi^2 \Lambda^2} C D^{4P_{1/2}} \frac{1}{\sqrt{1 + s_0^2(2 - s_1)}} \Lambda^{3-s_1} \sin \left( s_0 \ln \left( \frac{\Lambda}{\Lambda^*} \right) - \arctan(s_0) \right) + b
 \end{aligned}$$

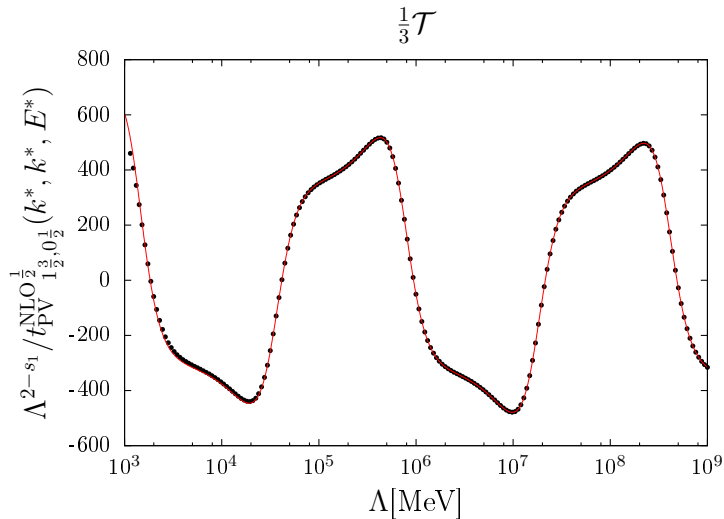
$$g_1 \rightarrow g^3S_1 - ^1P_1$$



$$g_2 \rightarrow g^{3S_1-3P_1}$$



$$\frac{1}{3}\mathcal{T} \rightarrow g_{(\Delta I=0)}^{1S_0-3P_0} + \frac{2}{3}\tau_3 g_{(\Delta I=1)}^{1S_0-3P_0}$$



Convenient to rescale PV LECs by dibaryon nucleon-nucleon coupling  $y$

$$g_1 = \frac{g^{3S_1-1P_1}}{y}, g_2 = \frac{g^{3S_1-3P_1}}{y}, g_3 = \frac{g^{1S_0-3P_0}(\Delta I=0)}{y}, g_4 = \frac{g^{1S_0-3P_0}(\Delta I=1)}{y}, g_5 = \frac{g^{1S_0-3P_0}(\Delta I=2)}{y}$$

Large- $N_C$  basis of LECs ([Schindler, Springer, and Vanasse \(2015\)](#))  
([Gardner, Haxton, and Holstein \(2017\)](#))

$$g_1^{(N_C)} = \frac{1}{4}g_1 + \frac{3}{4}g_3, \quad g_2^{(N_C)} = g_5 \quad \text{LO}$$

$$g_3^{(N_C)} = \frac{1}{4}g_1 - \frac{3}{4}g_3, \quad g_4^{(N_C)} = g_2, \quad g_5^{(N_C)} = g_4 \quad \text{N}^2\text{LO,}$$

Parity violating asymmetry in  $pd$  scattering in large- $N_C$  basis

$$A_L^{\vec{p}d}(E_{\text{lab}} = 15 \text{ MeV}) = [-943g_1^{(N_C)} - 636g_3^{(N_C)} + 568g_4^{(N_C)} + 345g_5^{(N_C)}] \text{ MeV}$$

Dominant contribution from only LO large- $N_C$  LEC. Experiment (Nagle et. al (1979)) gives bound

$$A_L^{\vec{p}d}(E_{\text{lab}} = 15 \text{ MeV}) = (-3.5 \pm 8.5) \times 10^{-8}$$

Gives bound

$$g_1^{(N_C)} = (3.7 \pm 9.0) \times 10^{-11} \text{ MeV}^{-1}$$

## $\vec{p}\vec{p}$ scattering

Asymmetry in  $\vec{p}\vec{p}$  scattering in large- $N_C$  basis (**Phillips, Schindler, and Springer (2009)**)

$$A_L^{\vec{p}\vec{p}}(E_{\text{lab}} = 13.6 \text{ MeV}) = [603g_1^{(N_C)} + 904g_2^{(N_C)} - 603g_3^{(N_C)} + 904g_5^{(N_C)}] \text{ MeV}$$

Experiment (**Eversheim et. al (1991)**) gives

$$A_L^{\vec{p}\vec{p}}(E_{\text{lab}} = 13.6 \text{ MeV}) = (-0.93 \pm 0.21) \times 10^{-7}$$

Dropping subleading LECs in large- $N_C$  basis and using bound on  $g_1^{(N_C)}$  gives

$$g_2^{(N_C)} = (-1.3 \pm 0.83) \times 10^{-10} \text{ MeV}^{-1}$$

Consistent with experimental bound (**Knyaz'kov (1984)**) and theoretical EFT $_{\pi}$  prediction (**Schindler and Springer (2010)**) for  $P_{\gamma}(np \rightarrow d\vec{\gamma})$



## $np$ and $nd$ spin rotation

$np$  spin rotation to NLO in the large- $N_C$  basis gives (Grießhammer, Schindler, and Springer (2012))

$$\frac{d\phi_{np}}{dz} = \left[ -6.0g_1^{(N_C)} + 67g_2^{(N_C)} + 38g_3^{(N_C)} + 16g_4^{(N_C)} \right] \text{rad cm}^{-1}$$

Using prediction for  $g_2^{(N_C)}$  gives

$$\frac{d\phi_{np}}{dz} \sim -8.7 \times 10^{-9} \text{ rad cm}^{-1}$$

LO  $nd$  spin rotation in large- $N_C$  basis is

$$\frac{d\phi_{nd}}{dz} = \left[ -50g_1^{(N_C)} - 27g_3^{(N_C)} - 38g_4^{(N_C)} + 11g_5^{(N_C)} \right] \text{rad cm}^{-1}$$

# Conclusions and Future Directions

- ▶ Any PV observable of interest can be calculated for  $nd$  scattering at LO in  $EFT_{\not{p}}$ .
- ▶ PV three-body force needed at NLO
- ▶ New perturbative technique can be used with external currents making calculations of PV in  $nd \rightarrow {}^3H + \gamma$  and  $pd \rightarrow {}^3He + \gamma$  feasible.
- ▶ Need to add Coulomb to investigate  $pd$  at lower energies