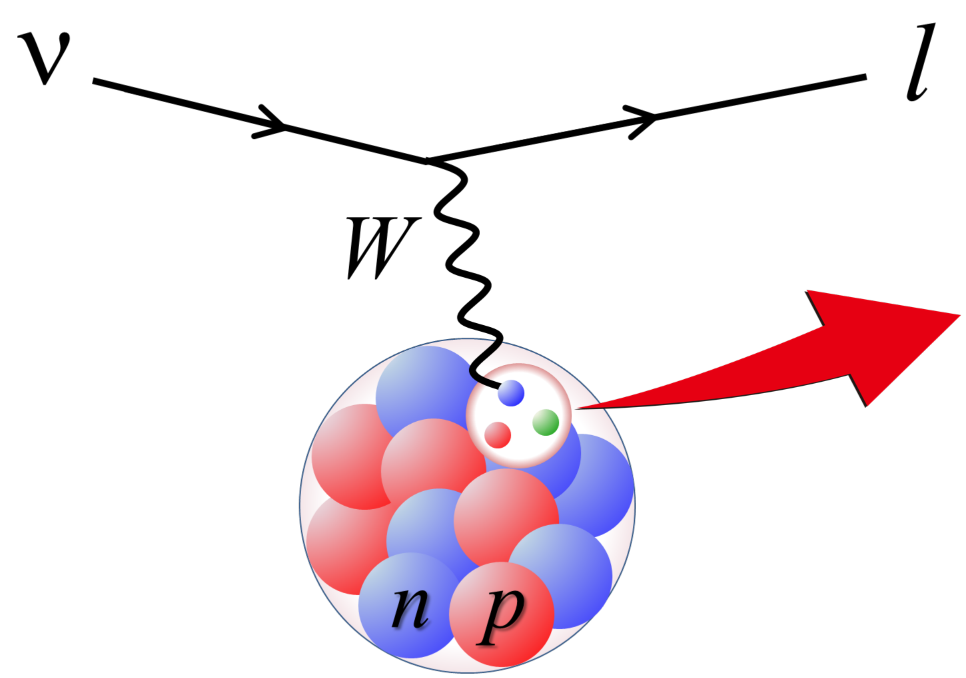
Lattice QCD input for neutrino physics

and also
new results for scalar matrix elements in light nuclei



Phiala Shanahan

William & Mary
CHARTERED 1693
Jefferson Lab
Massachusetts Institute of Technology

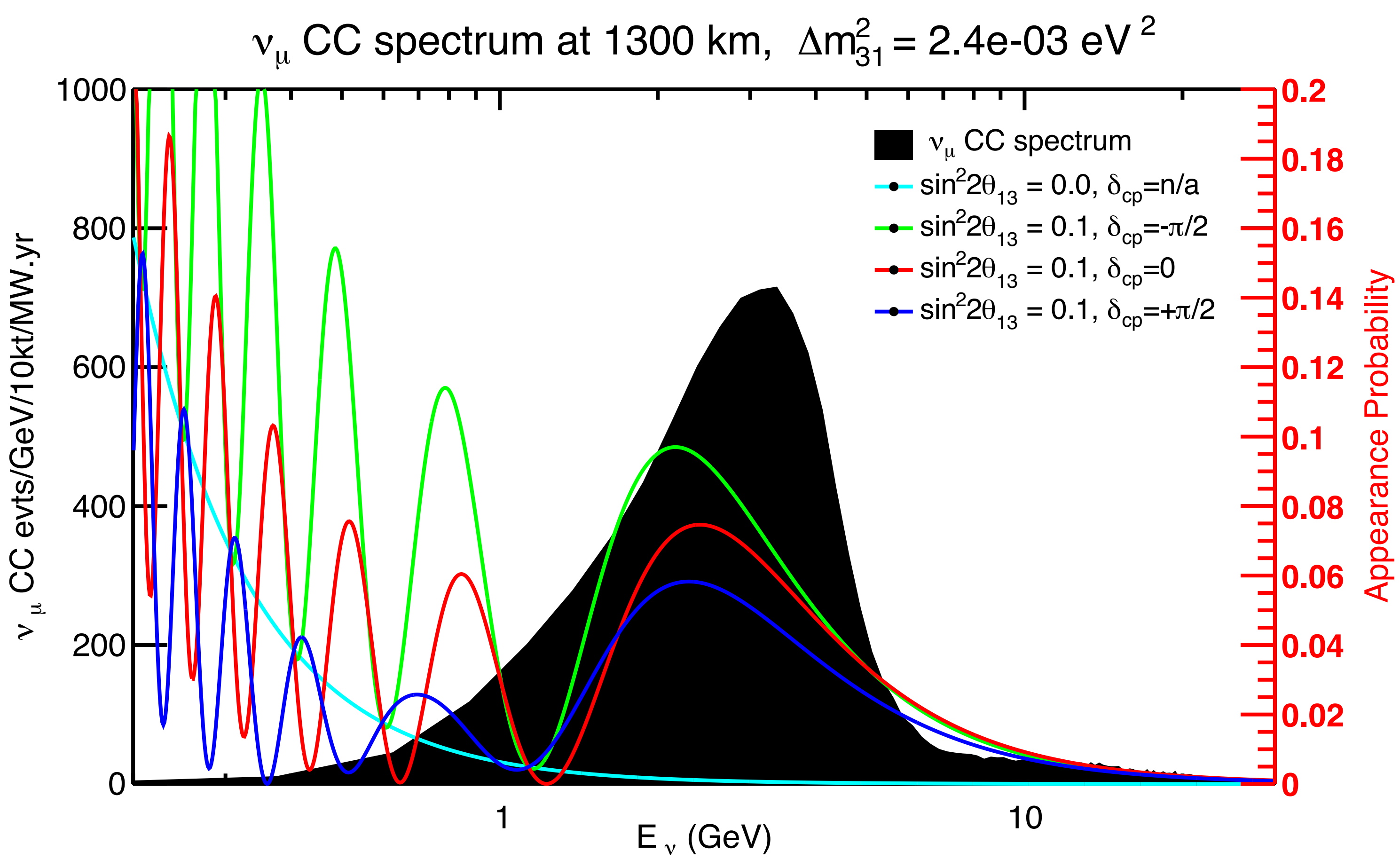
Long-baseline neutrino experiments

Neutrinos produced as secondary decay products of hadrons from primary reactions of protons with nuclei

→ neutrino energy must be reconstructed event-by-event from the final state of the reaction

DUNE
Need energy reconstruction to better than 100 MeV

Robust understanding of both nucleon and nuclear level amplitudes essential



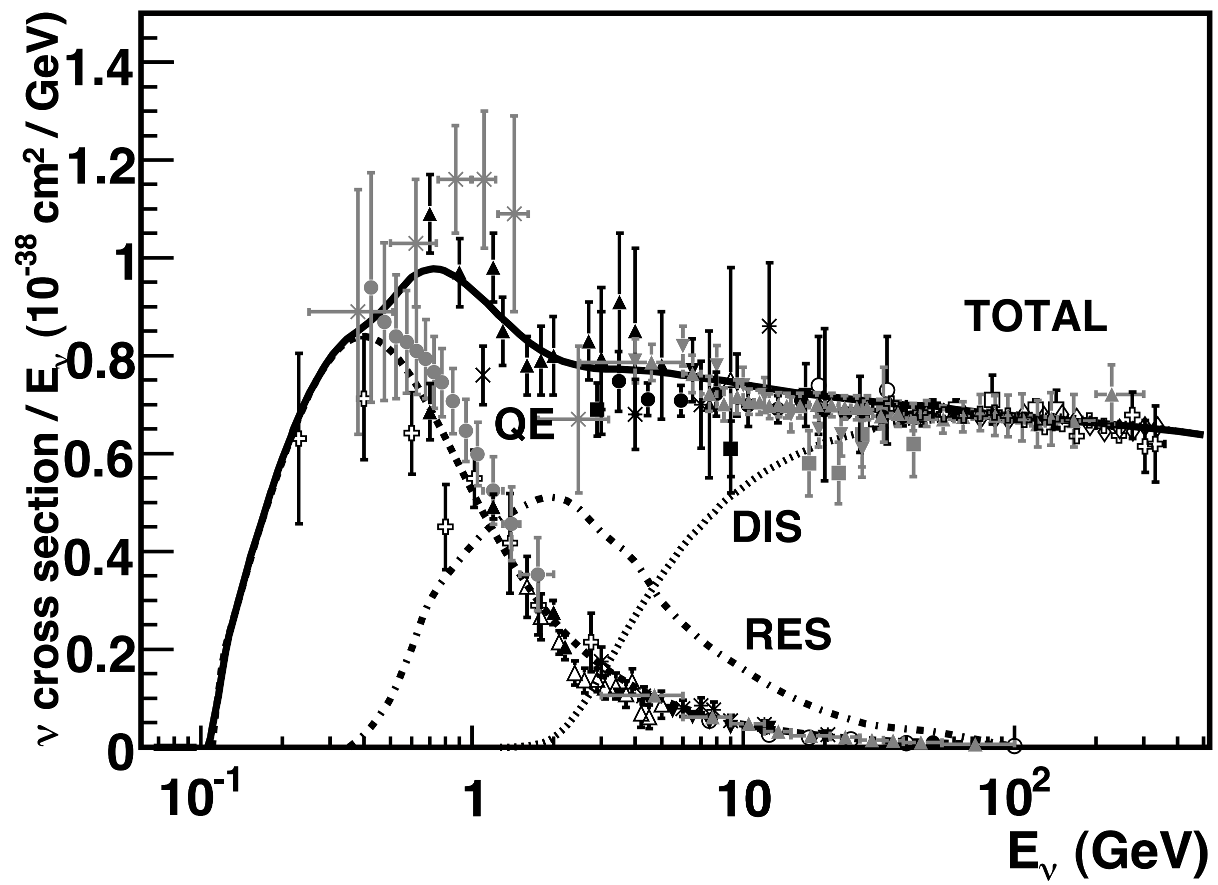
Adams C, et al. arXiv:1307.7335

Constraining ν-nucleus interactions

- For LBNEs neutrino energy distributions peak at 1-10 GeV
- Challenging region: several processes contribute
  - Quasielastic lepton scattering
  - Inelastic continuum / shallow-inelastic region
  - Resonances
- Lattice QCD can provide direct non-perturbative QCD predictions of nucleon and nuclear matrix elements

Neutrino charged-current cross-section

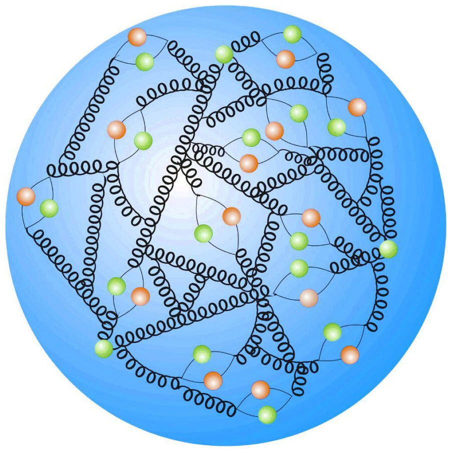
J.A. Formaggio, G.P. Zeller, Rev. Mod. Phys. 84 (2012) 1307



LQCD input for ν-nucleus interactions

1. **Directly access QCD single-nucleon form factors without nuclear corrections**

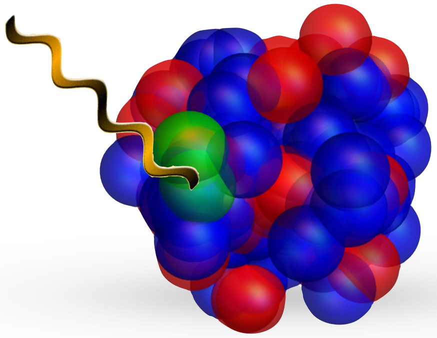
   Reliable calculations with fully-controlled uncertainties



2. **Calculate matrix elements in light nuclei from first principles**

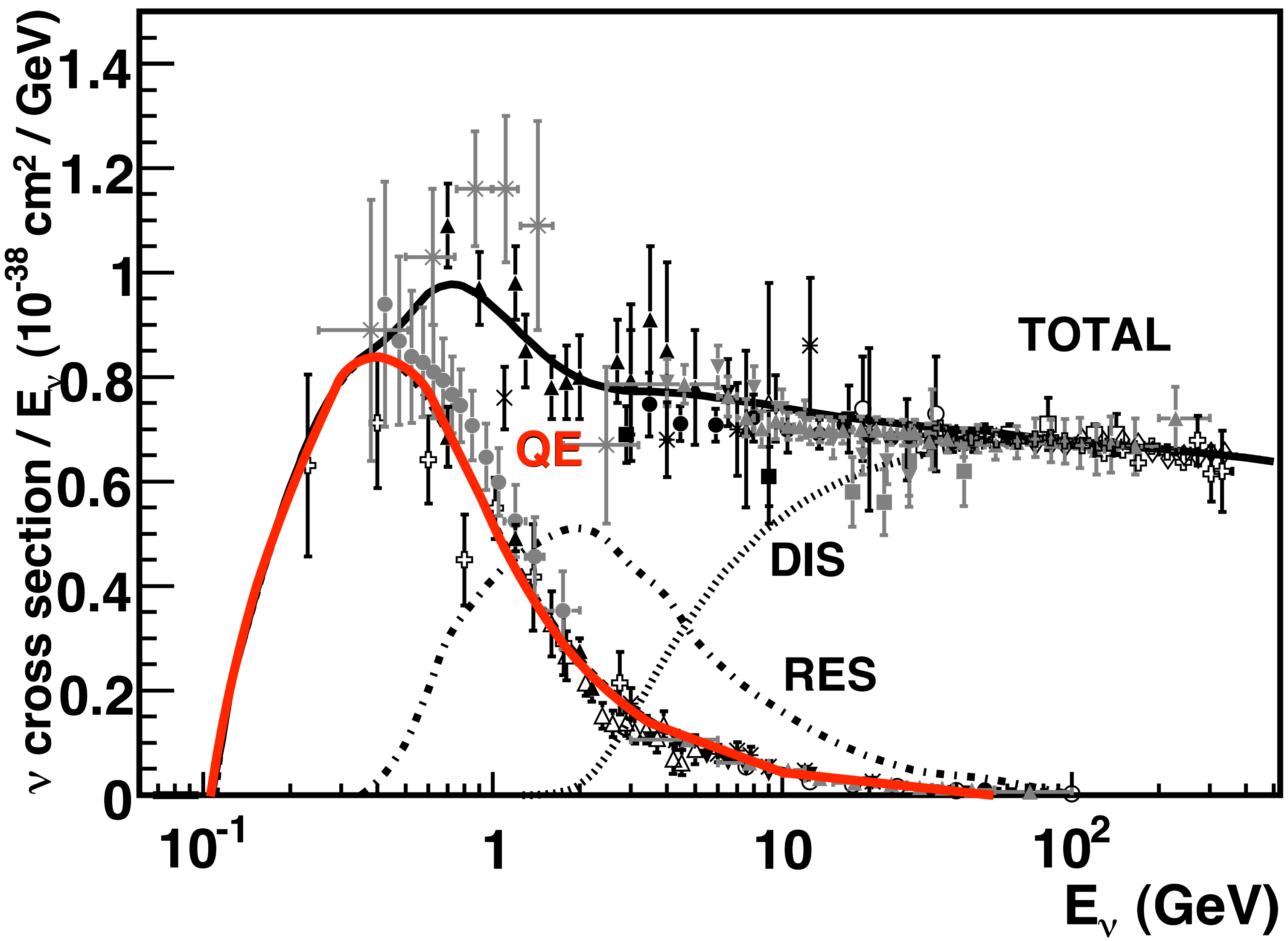
   ➡ EFT to reach heavy nuclear targets relevant to experiment

   First calculations of axial charge of light nuclei



Constraining ν-nucleus interactions

Neutrino charged-current cross-section

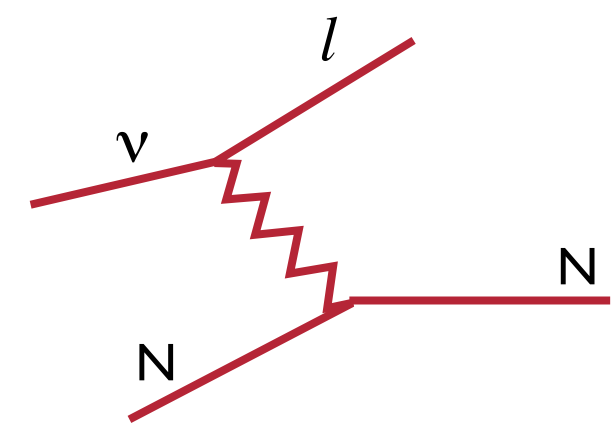


J.A. Formaggio, G.P. Zeller, Rev. Mod. Phys. 84 (2012) 1307

Quasi-elastic scattering

- Cross-section for quasi-elastic neutrino-nucleon scattering

$$\frac{d\sigma}{dQ^2} = \frac{G_f^2 M^2 \cos^2\theta_C}{8\pi E_\nu^2}\left[A \mp \frac{(s-u)}{M^2}B + \frac{(s-u)^2}{M^4}C\right]$$



$$A = \frac{(m^2+Q^2)}{M^2}\Big[(1+\tau)G_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1F_2 - \frac{m^2}{4M^2}\Big((F_1+F_2)^2 + (G_A+2G_P)^2 - \Big(\frac{Q^2}{M^2}+4\Big)G_P^2\Big)\Big]$$

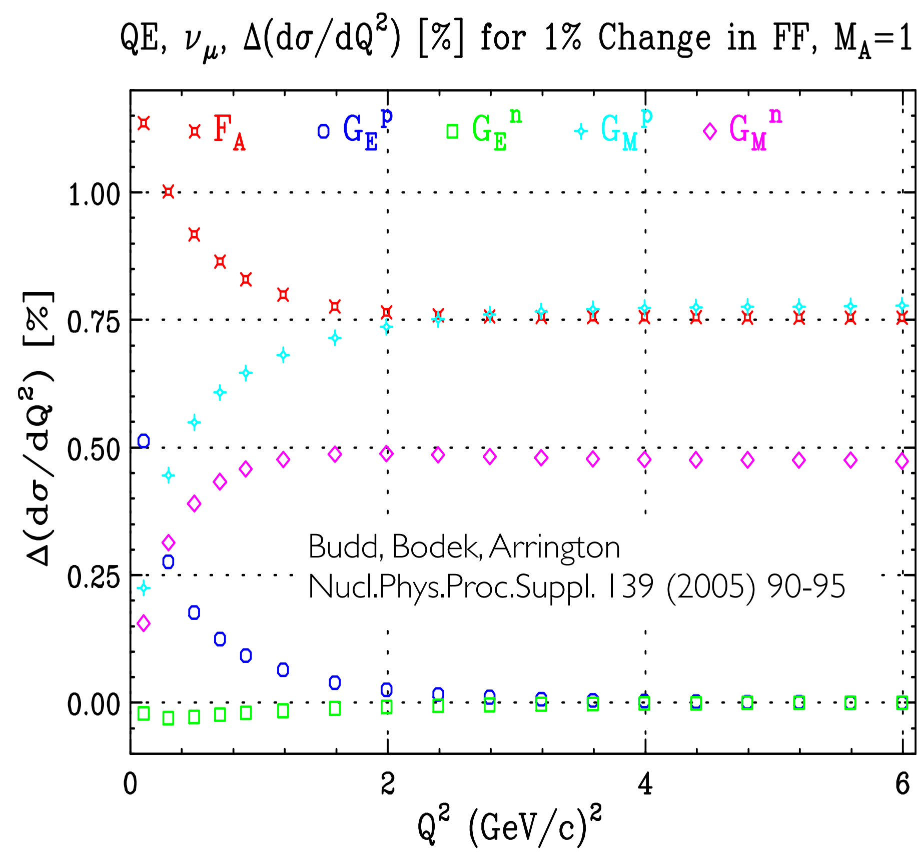
$$B = \frac{Q^2}{M^2}G_A(F_1+F_2)$$

$$C = \frac{1}{4}(G_A^2 + F_1^2 + \tau F_2^2)$$

G_A
- dominant contribution
- largest uncertainty

$F_{1,2}$ Well-determined from electron scattering expts

G_P can be related to G_A by pion pole dominance



Axial form factor

- Traditionally assumed to have dipole form

$$G_A(Q^2) = \frac{g_A}{\left(1+Q^2/M_A^2\right)^2}$$

- $g_A = 1.2671$ determined with high precision from nuclear beta decay
- axial mass M_A must be determined experimentally

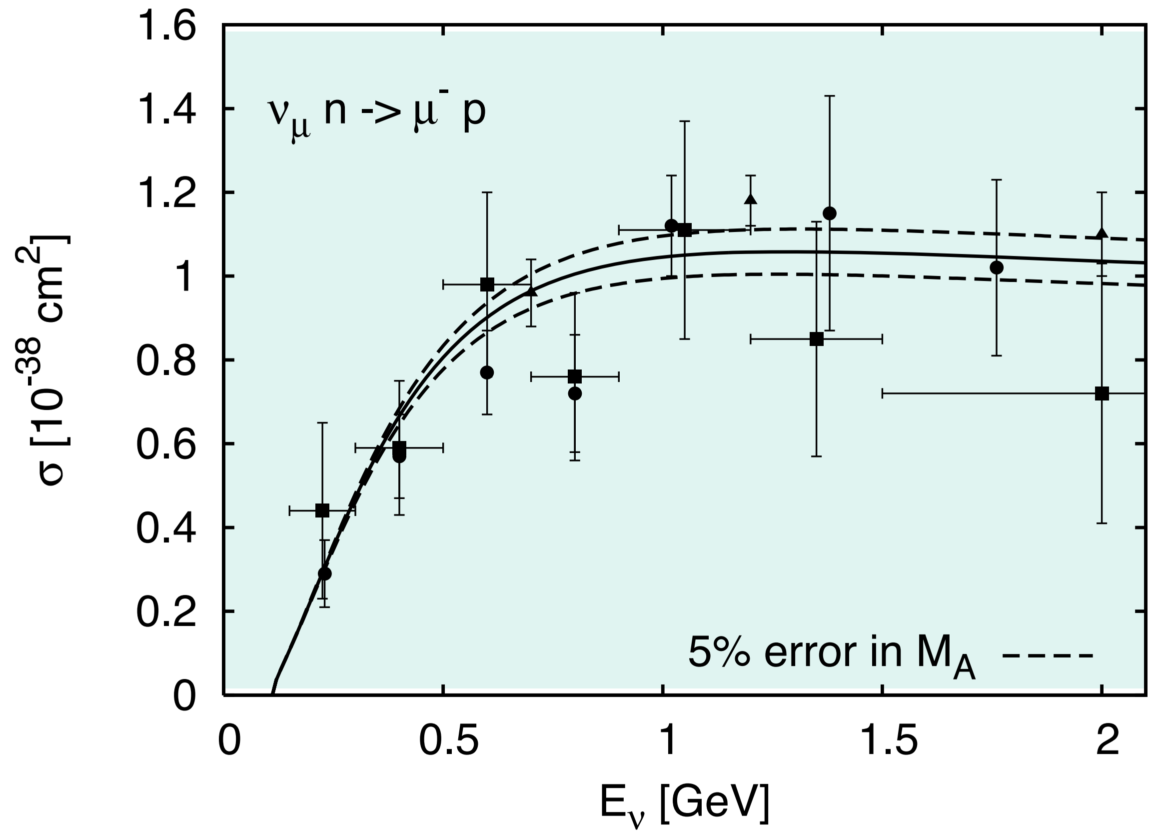
BUT

- Electromagnetic FFs show significant deviation from dipole parametrisation form

More general alternatives

- Model-indep z-expansion
  Hill & Paz (2010), Bhattacharya (2011)
- Direct LQCD results

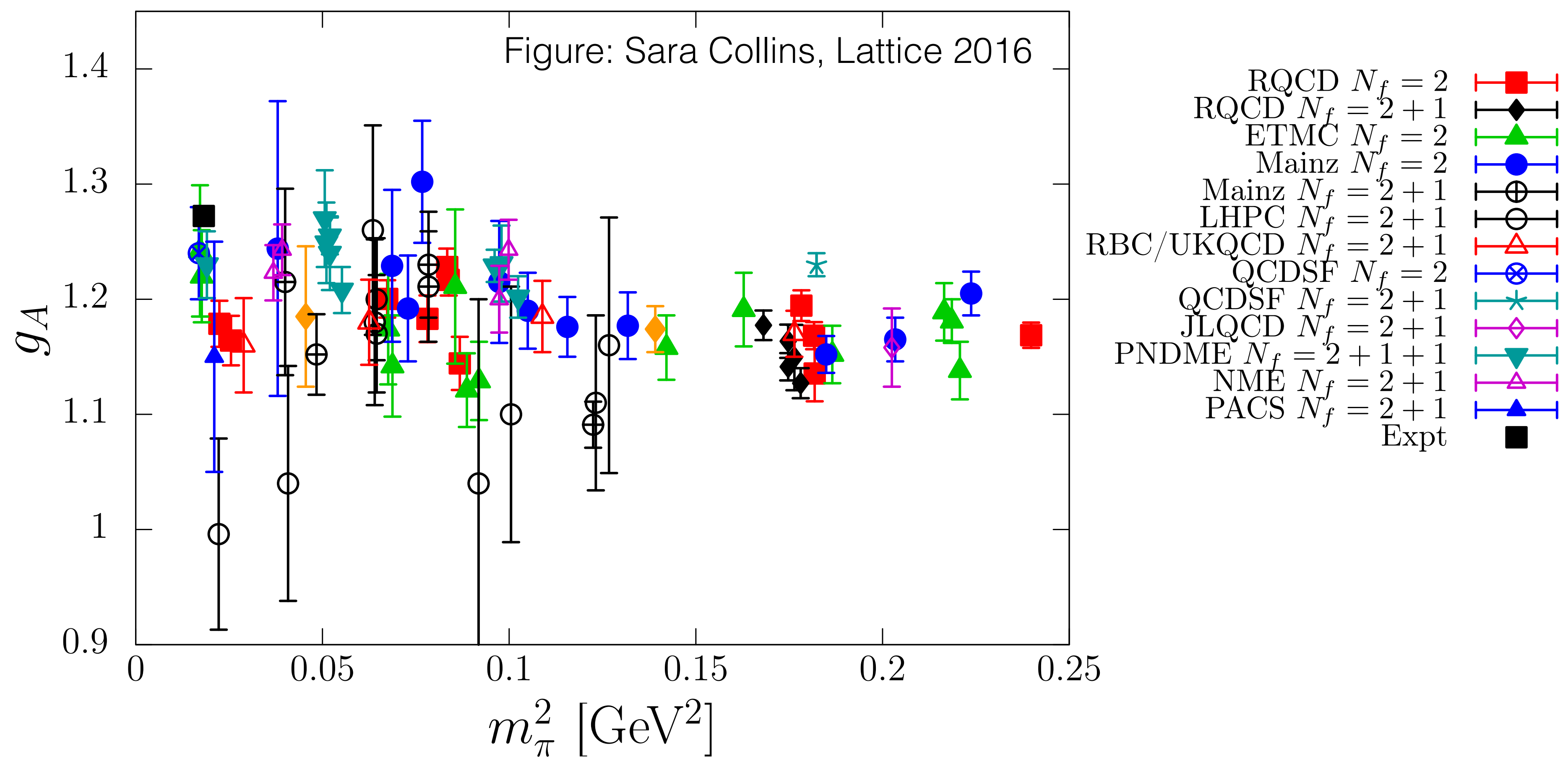
Total QE cross-section sensitive to the axial mass:



Mosel, Ann. Rev. Nucl. Part. Sci. 66, 171 (2016)

Nucleon Axial FFs from LQCD

- $g_A = G_A(Q^2 = 0)$ is a historically difficult calculation
- Recent calculations in agreement with experiment with fully-controlled uncertainties
- Q^2 -dependence well-determined in LQCD — competitive with experiment
- z-parameterisations remove assumption of dipole form



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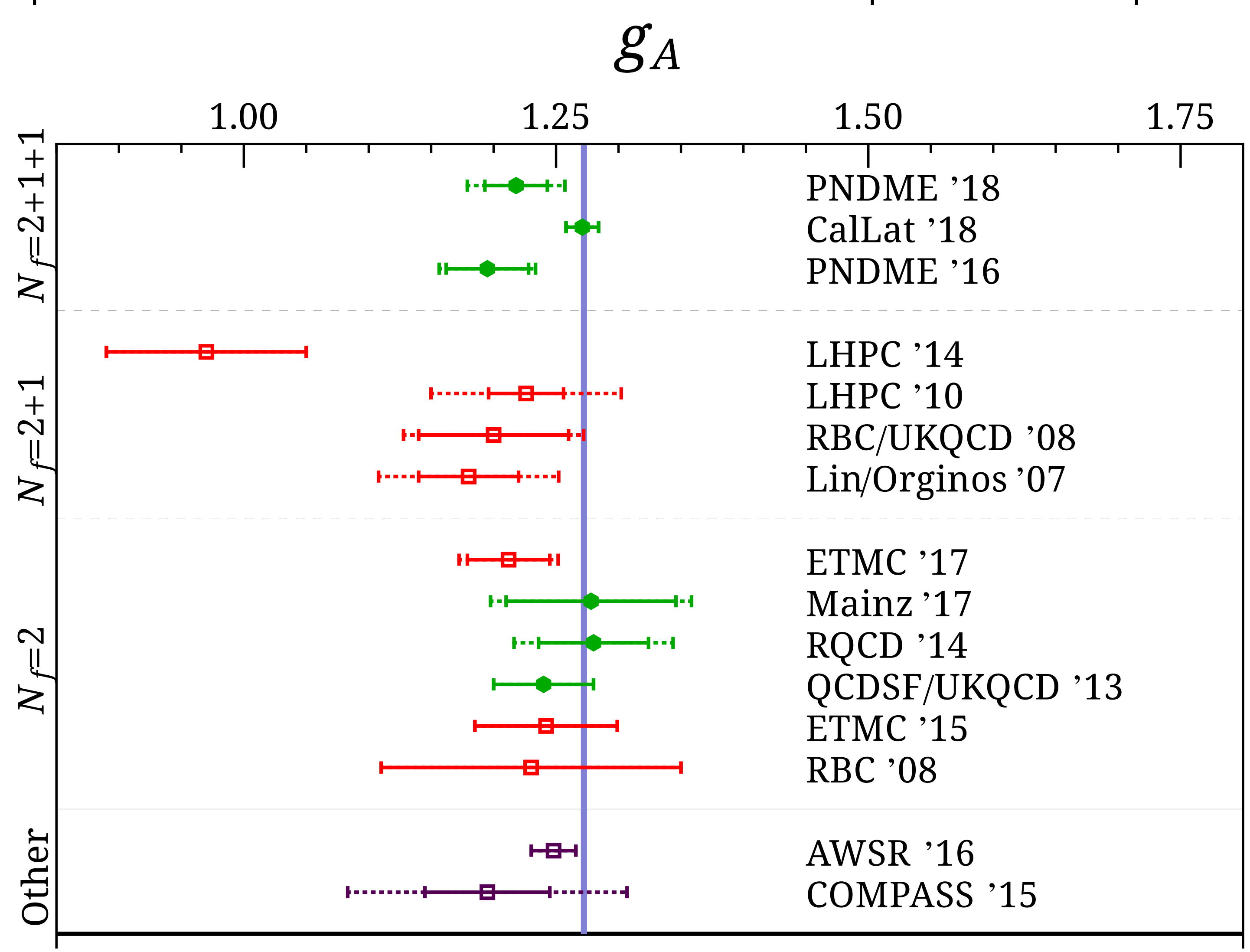

Summary figure from paper Huey-Wen discussed yesterday
Gupta et al, arXiv:1806.09006

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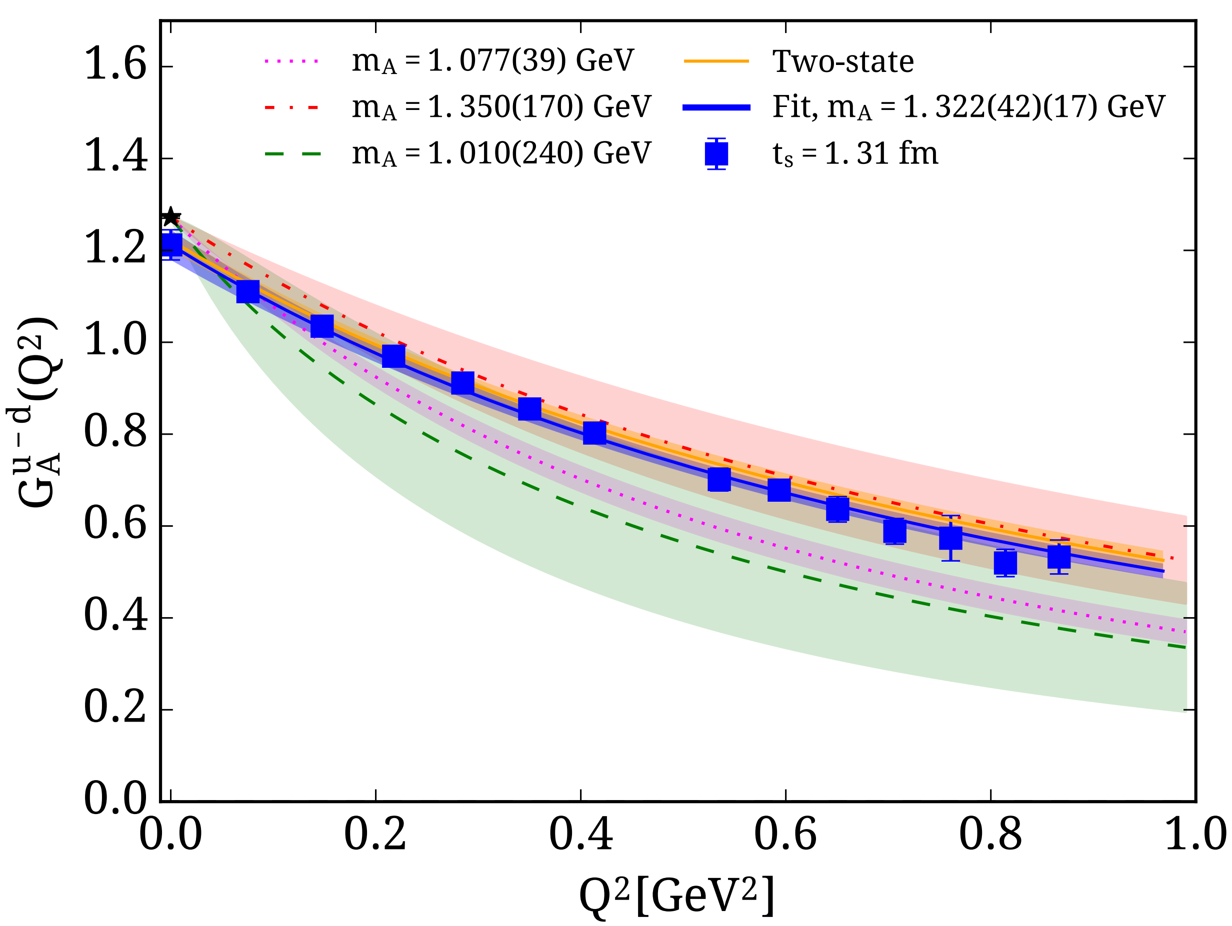
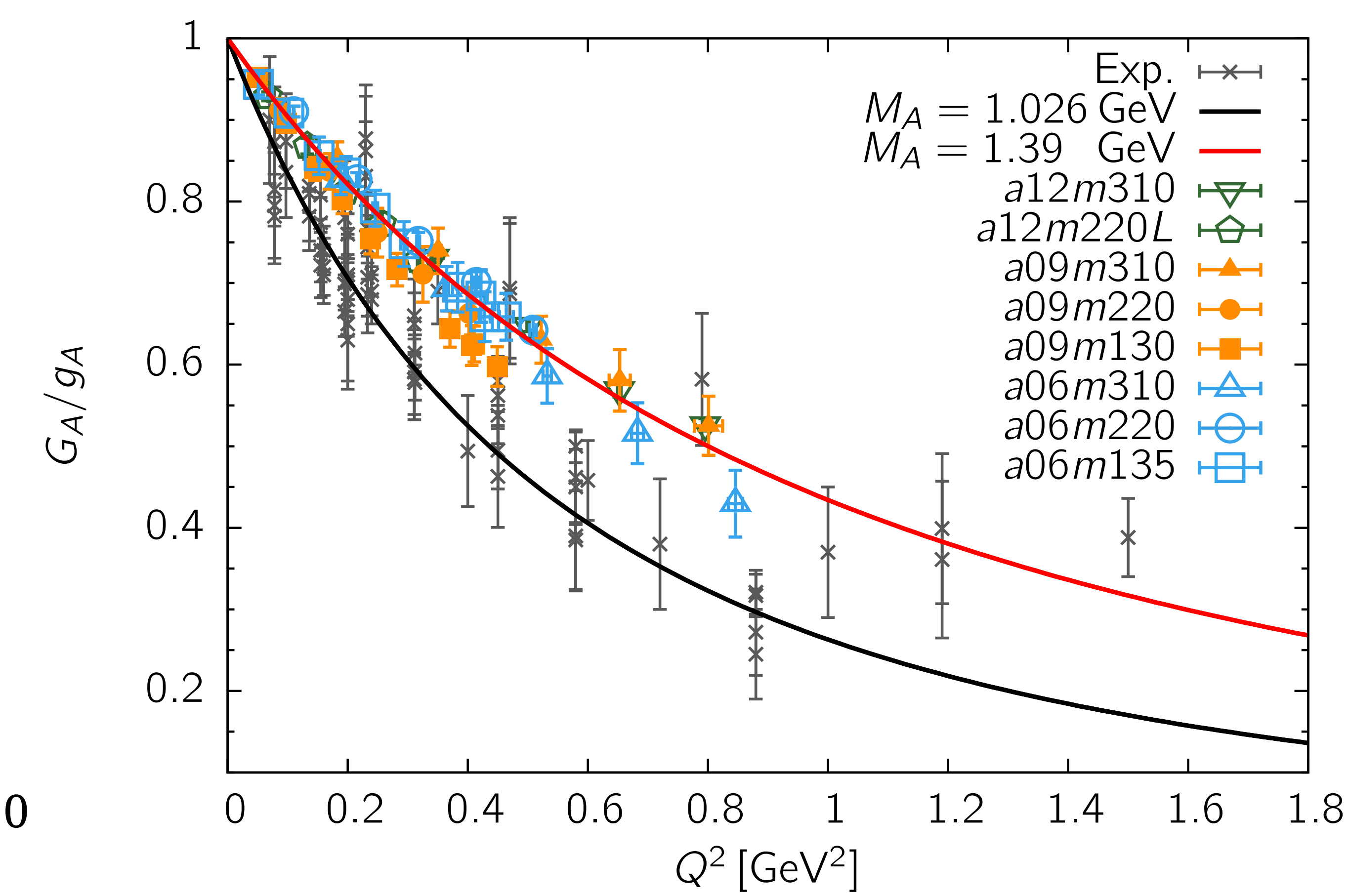


Alexandrou et al., arXiv:1705.03399, Phys.Rev. D96 (2017), 054507



Gupta et al., arXiv:1705.06834, Phys.Rev. D96 (2017), 114503

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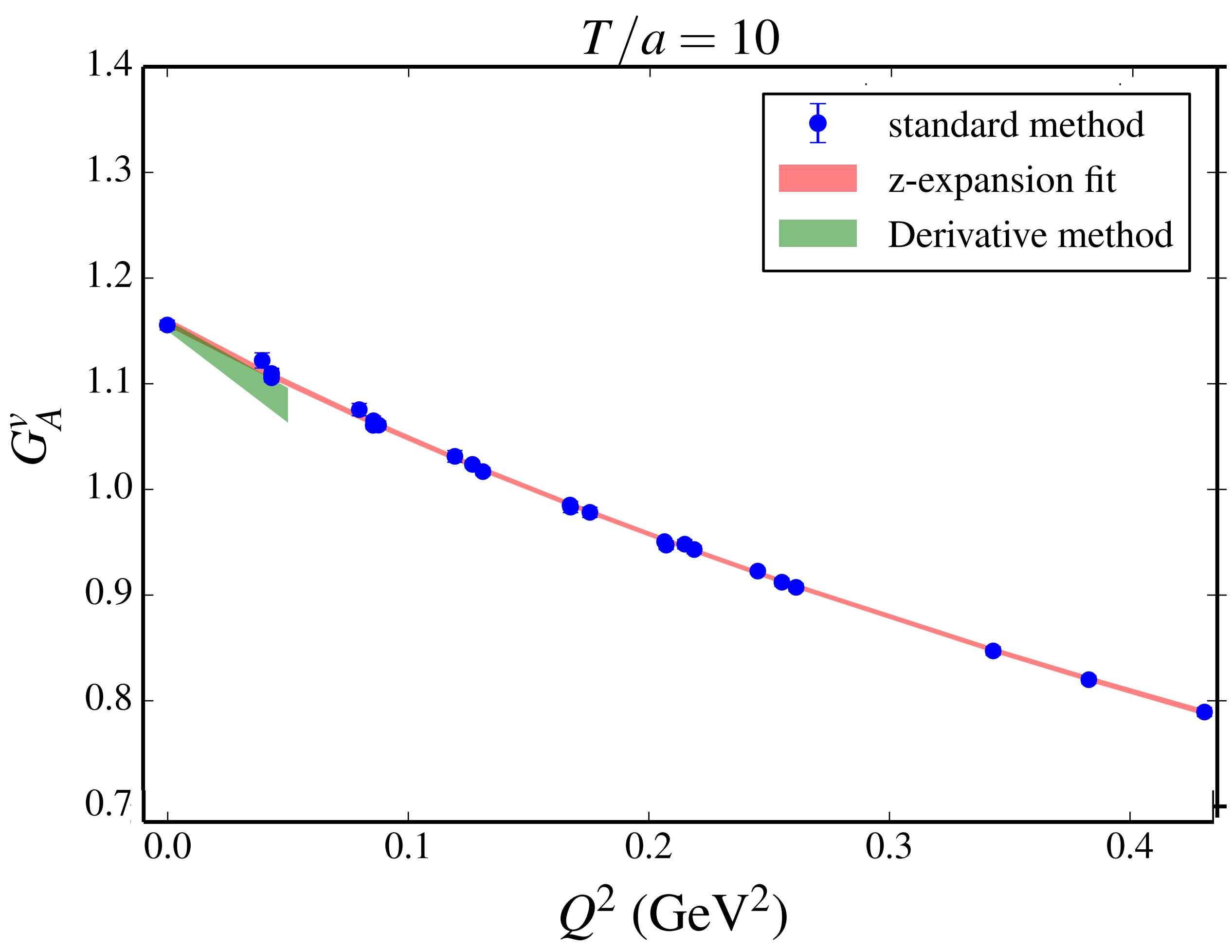
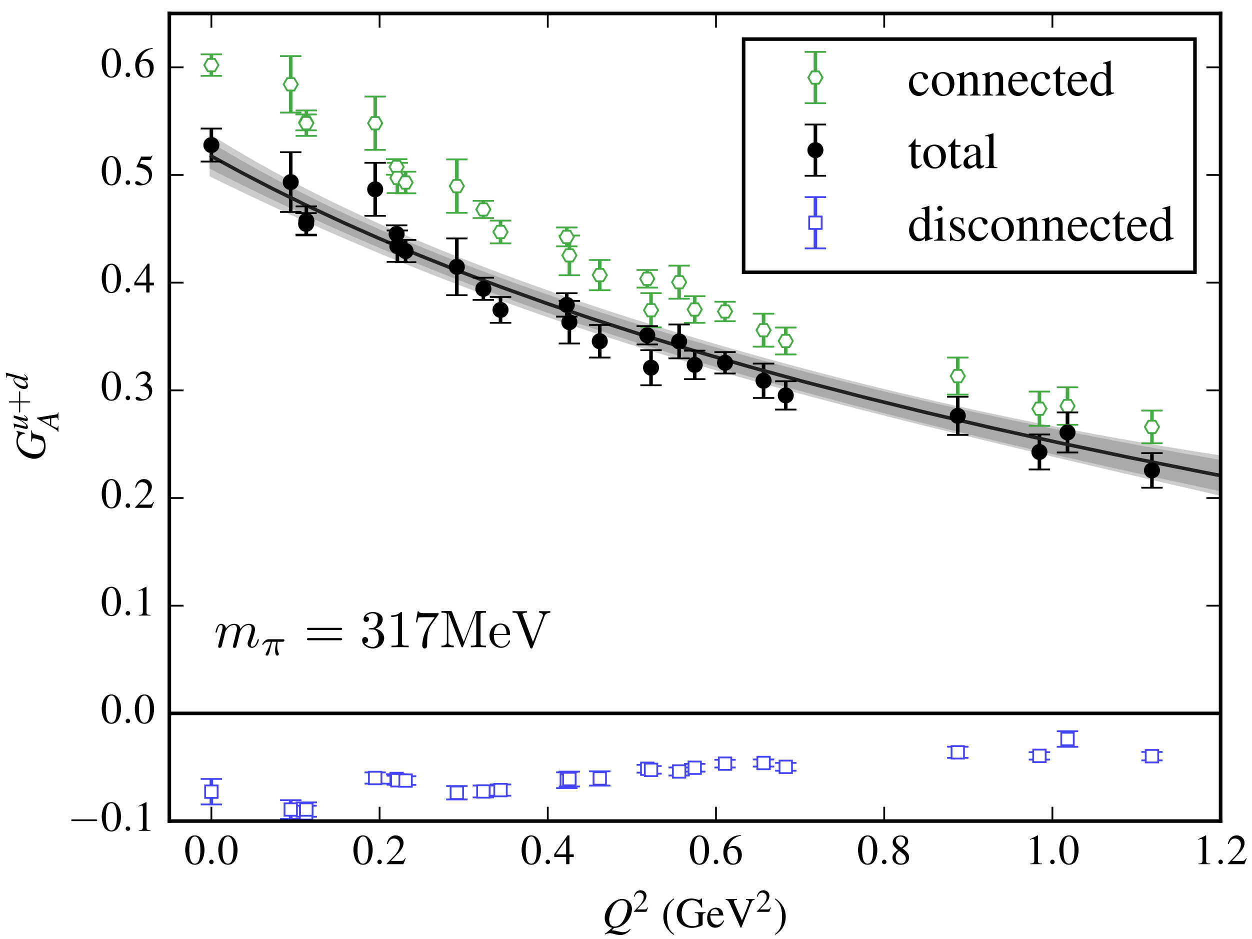


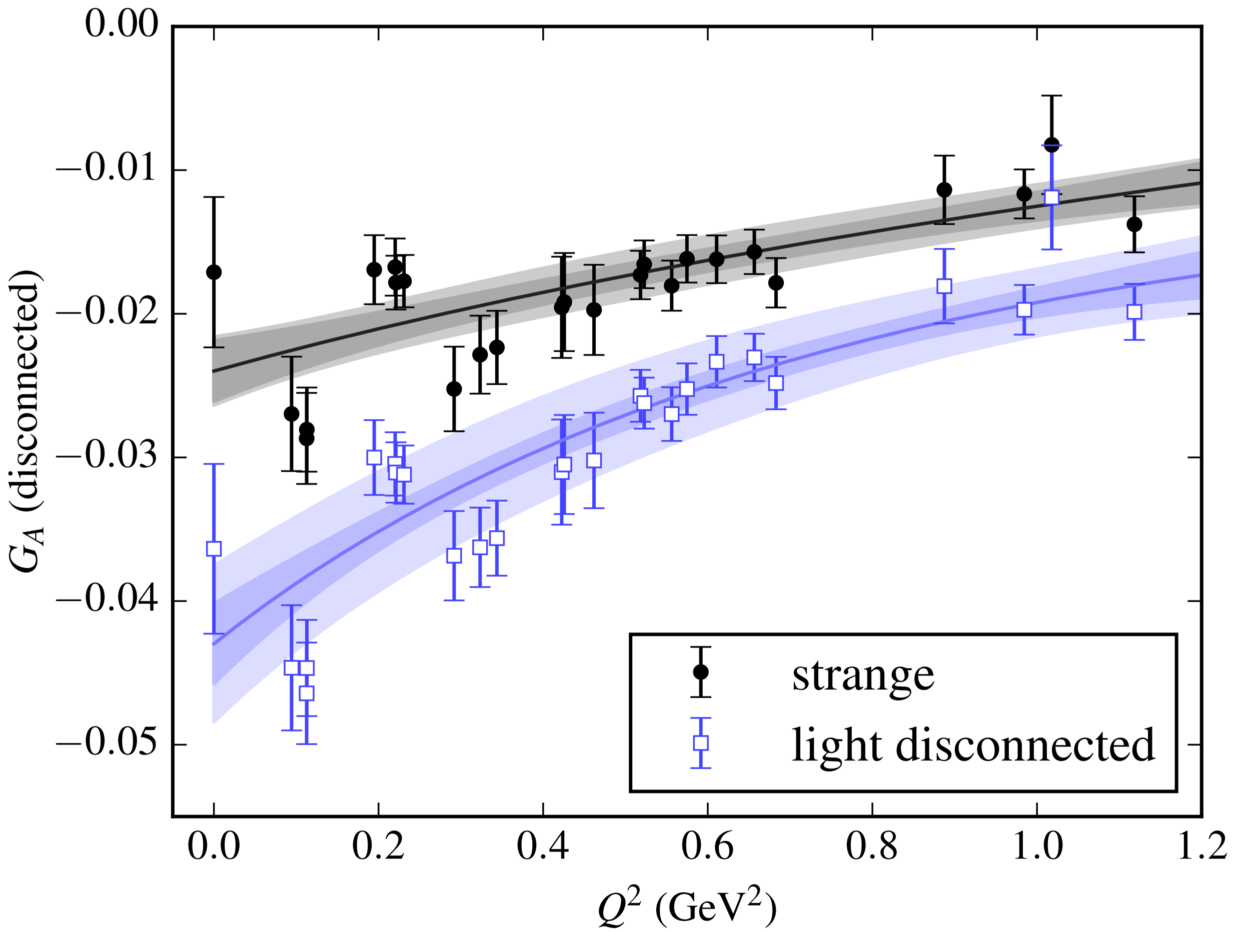
Direct calculation of slope (green band)
Uncertainties larger, but model-independent

Hasan et al., arXiv:1711.11385, Phys.Rev. D97 (2018), 034504

Nucleon Axial FFs from LQCD

- Strange quark contributions determined separately and can be isolated



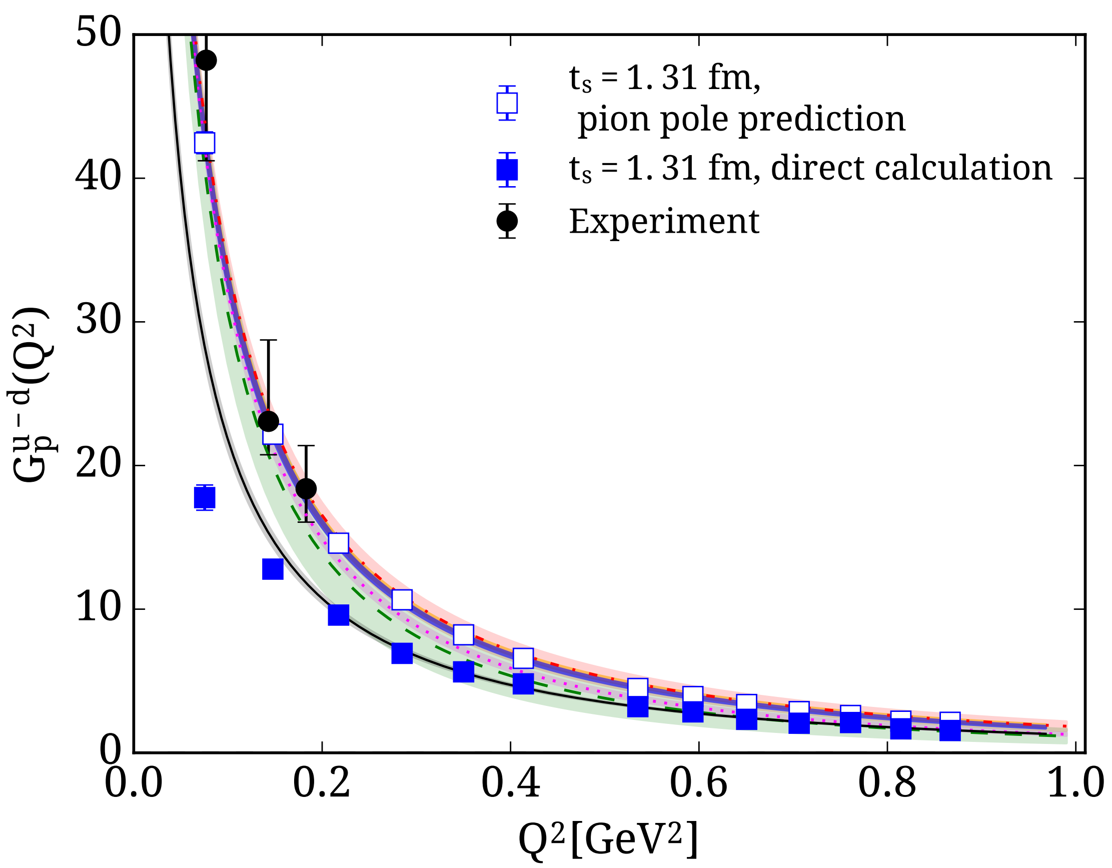


Green et al., Phys. Rev. D 95, 114502 (2017)

Also physical-point strange quark axial charge: Gupta et al., EPJ Web Conf. 175 (2018) 06029, Form factors Alexandrou et al., arXiv:1705.03399, Phys.Rev. D96 (2017), 054507

Nucleon pseudoscalar FF

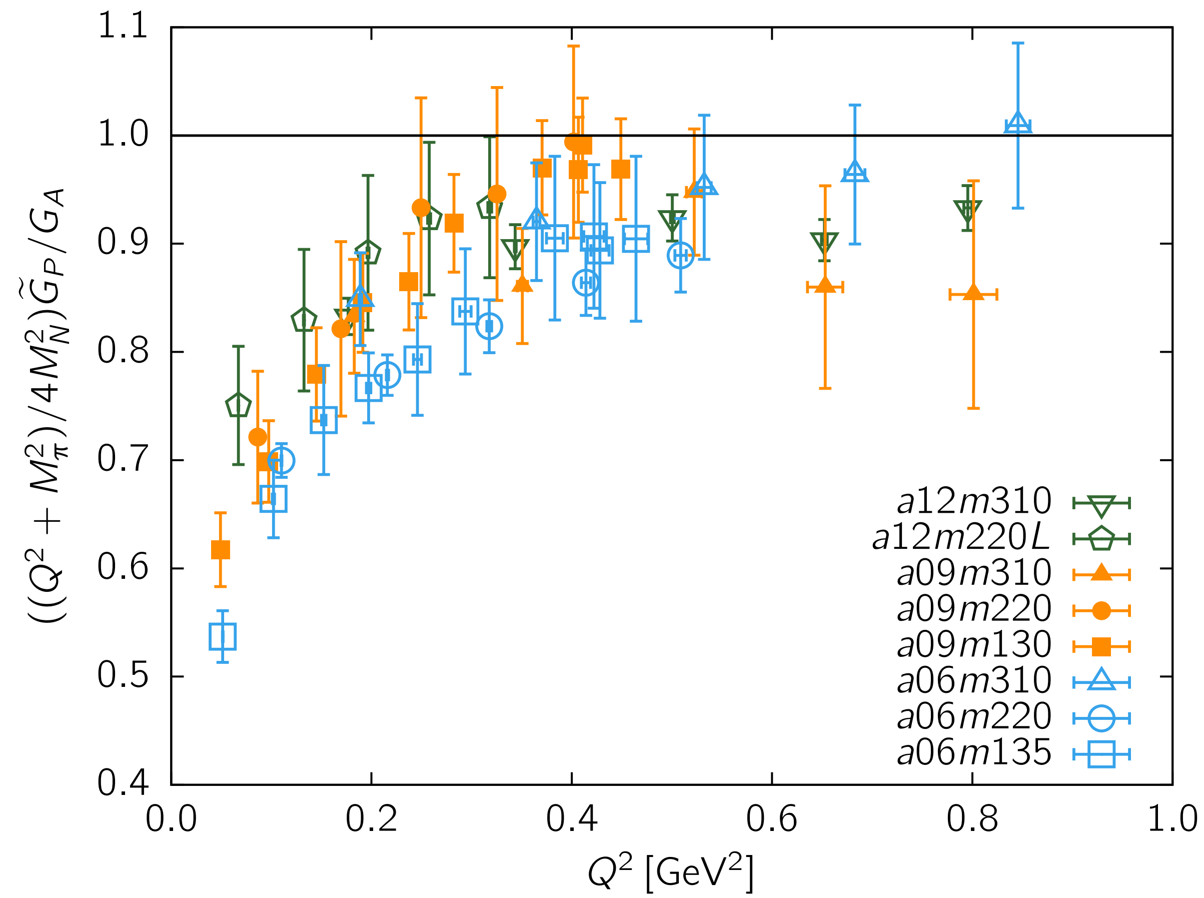
First calculations with controlled uncertainties



Alexandrou et al., arXiv:1705.03399, Phys.Rev. D96 (2017), 054507

Deviations from pion-pole dominance ansatz at low-Q^2

$$\tilde{G}_P(Q^2) = G_A(Q^2)\left[\frac{4M_N^2}{Q^2 + M_\pi^2}\right]$$



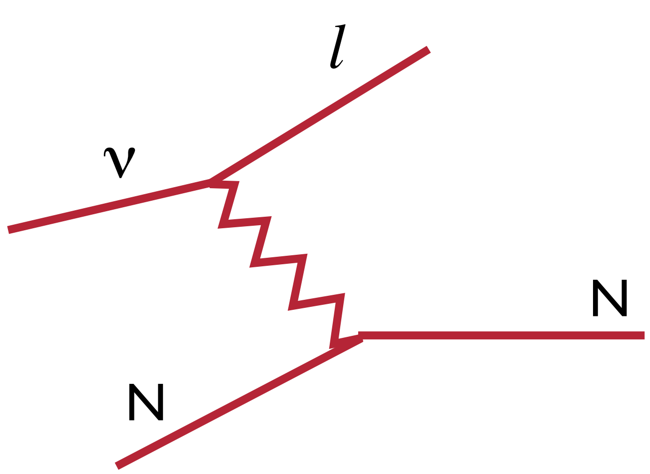
Gupta et al., arXiv:1705.06834, Phys.Rev. D96 (2017), 114503

Quasi-elastic scattering

LQCD input for the quasi-elastic scattering region:

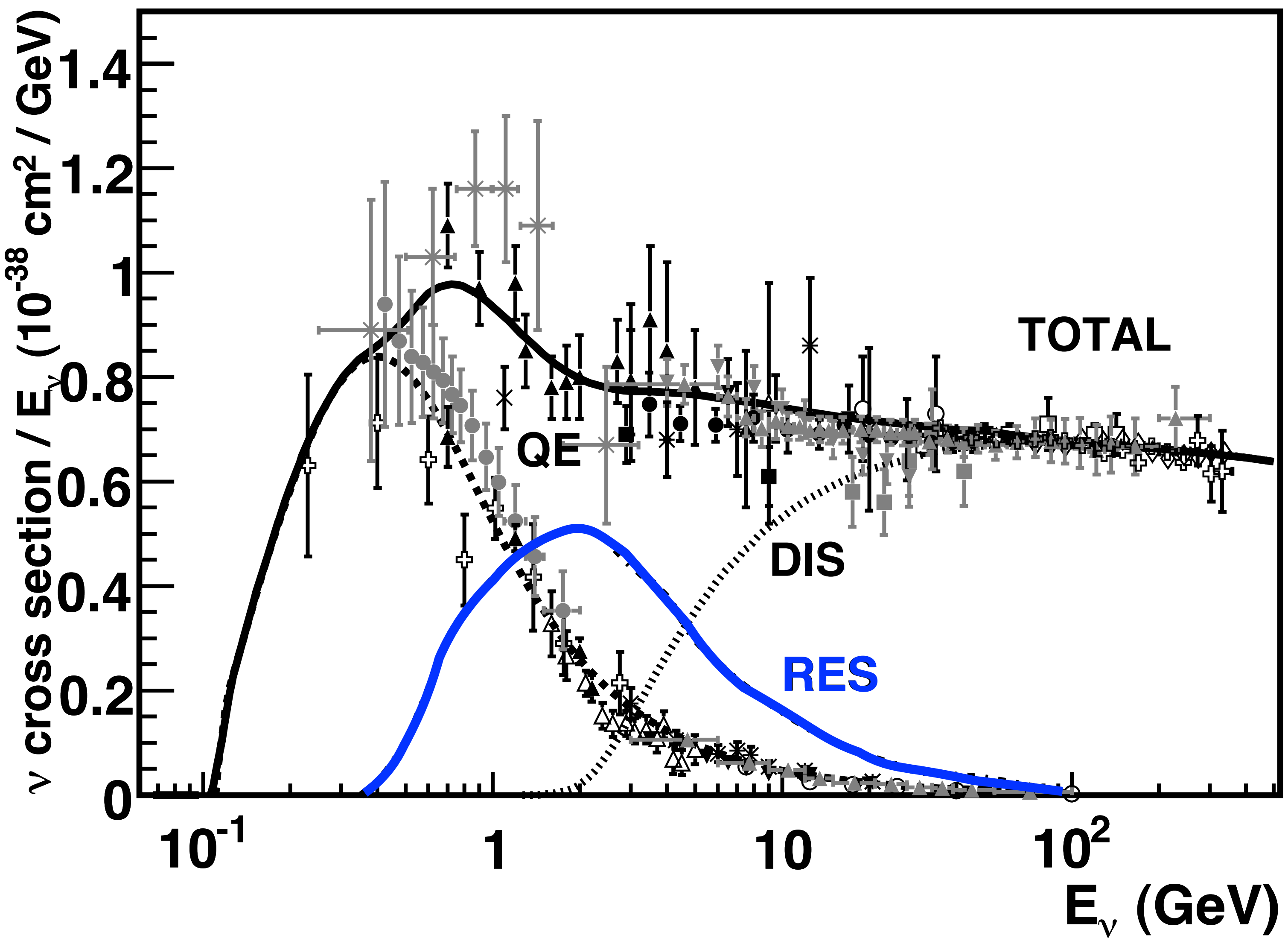
- Q^2 dependence of nucleon axial form factor
  - fully-controlled uncertainties
  - competitive with experiment
  - z parameterisation removes assumption of dipole form

- Nucleon pseudo scalar form factor
  - fully-controlled uncertainties
  - competitive with experiment
  - deviations from pion-pole ansatz observed



Constraining ν-nucleus interactions

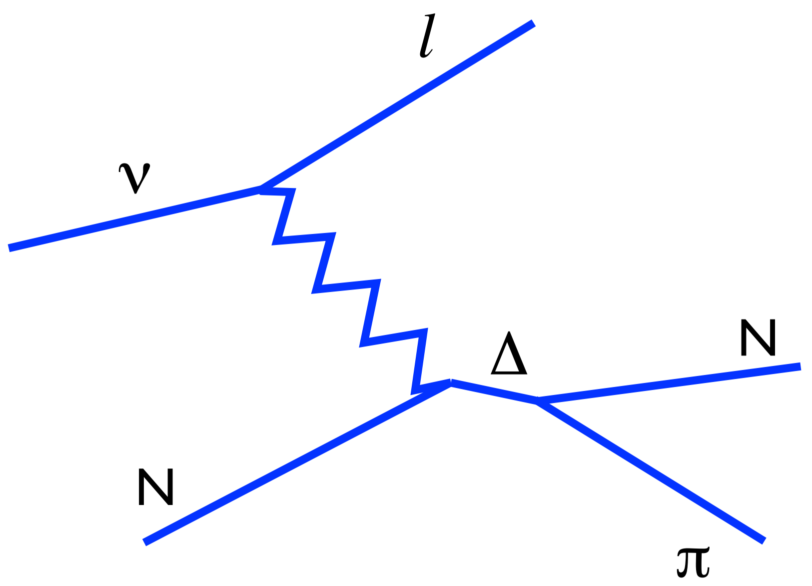
Neutrino charged-current cross-section



J.A. Formaggio, G.P. Zeller, Rev. Mod. Phys. 84 (2012) 1307

Resonance region

- Energies above ~200 MeV, inelastic excitations from pion production
- Dominant contribution from Δ resonance
- N*'s also important at high E_ν
- Very difficult to access experimentally
  Constrained only from PCAC
- QCD calculations possible
- Need to account for unstable nature of resonance: extract $N \to N\pi$ transition FFs

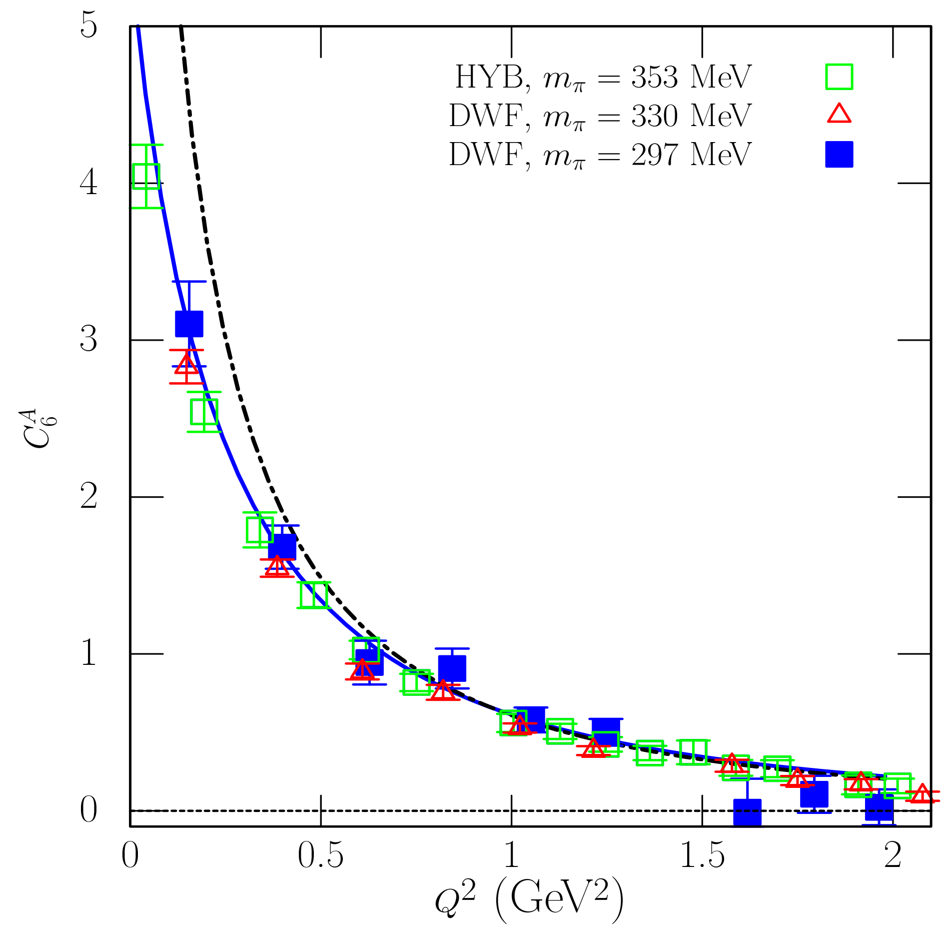


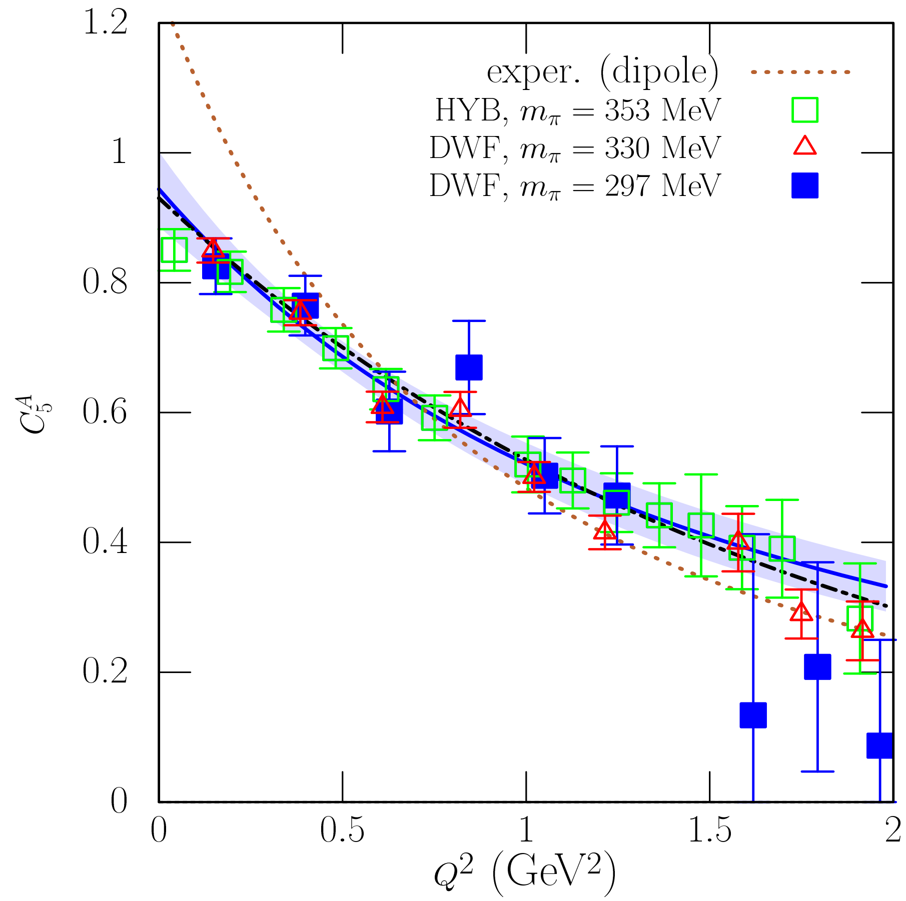
Resonance region

Lattice QCD calculation of axial N Δ transition form factor:

$$\langle\Delta(p',s')|A^3_\mu|N(p,s)\rangle = i\sqrt{\frac{2}{3}}\left(\frac{m_\Delta m_N}{E_\Delta(\mathbf{p}')E_N(\mathbf{p})}\right)^{1/2}\bar{u}^\lambda_{\Delta^+}(p',s')\left[\left(\frac{C^A_3(q^2)}{m_N}\gamma^\nu+\frac{C^A_4(q^2)}{m_N^2}p'^\nu\right)(g_{\lambda\mu}g_{\rho\nu}-g_{\lambda\rho}g_{\mu\nu})\,q^\rho + C^A_5(q^2)g_{\lambda\mu}+\frac{C^A_6(q^2)}{m_N^2}q_\lambda q_\mu\right]u_P(p,s)$$

C Alexandrou et al., Phys.Rev. D83 (2011) 014501

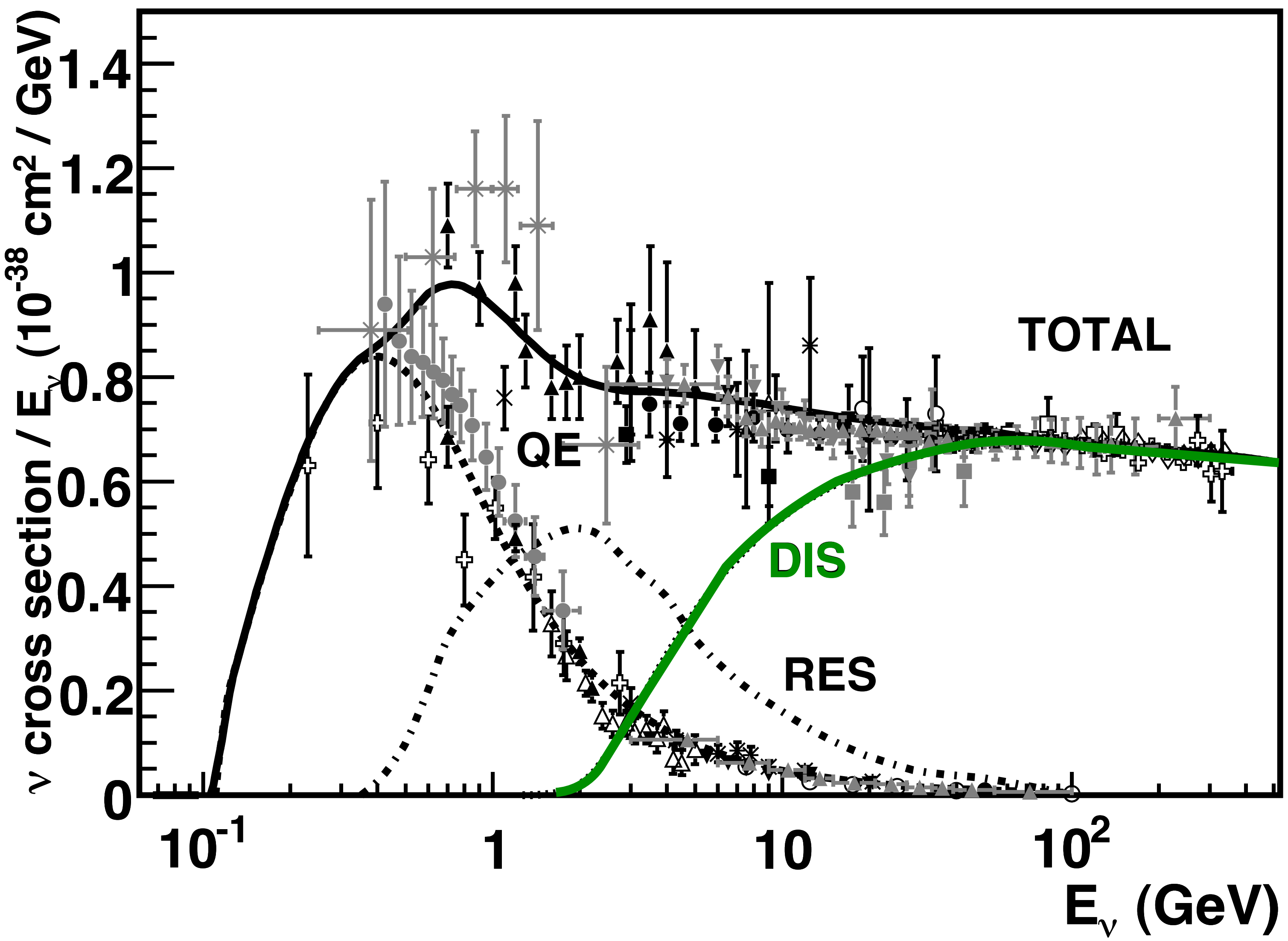




CAVEAT: Complexities at physical point with unstable resonances, but formalism exists: Lellouch-Lüscher hep-lat/0003023

Constraining ν-nucleus interactions

Neutrino charged-current
cross-section



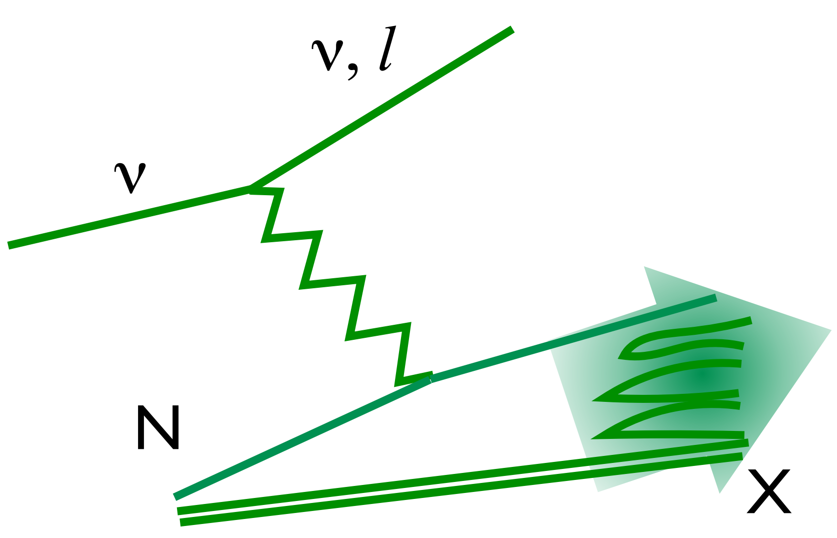
J.A. Formaggio, G.P. Zeller, Rev. Mod. Phys. 84 (2012) 1307

Shallow inelastic region

- In inelastic regime, quark PDFs of the nucleon control scattering cross-section
- In shallow inelastic region, both resonances and DIS are important
- Multi-meson channels may become important
- Nuclear effects are different in **ν**A vs. eA (MINER**ν**A)
- DIS structure functions accessible in lattice QCD
  - low moments of structure functions controlled

$$M_n = \int_{-1}^{1} x^n f(x) dx, \quad n \lessapprox 4$$

  - x-dependence difficult but promising

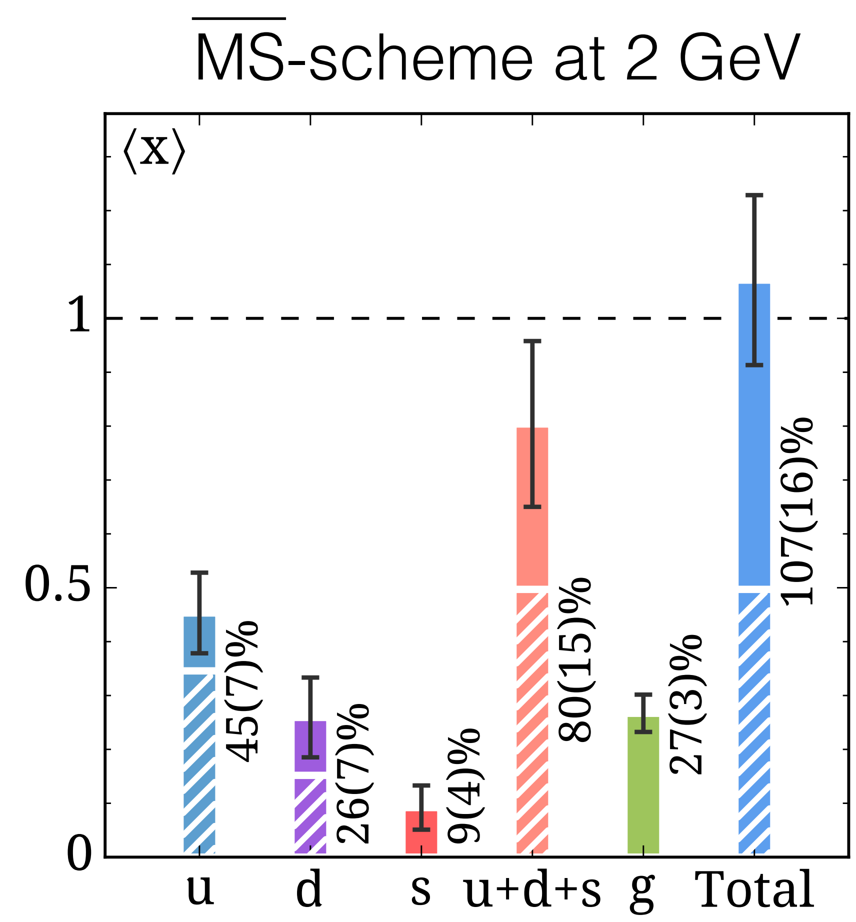


Nucleon PDFs

- Lattice QCD typically calculates low moments of PDFS
- Can separate and isolate contributions from
  - Strangeness
  - Charge symmetry violation
  - Gluons

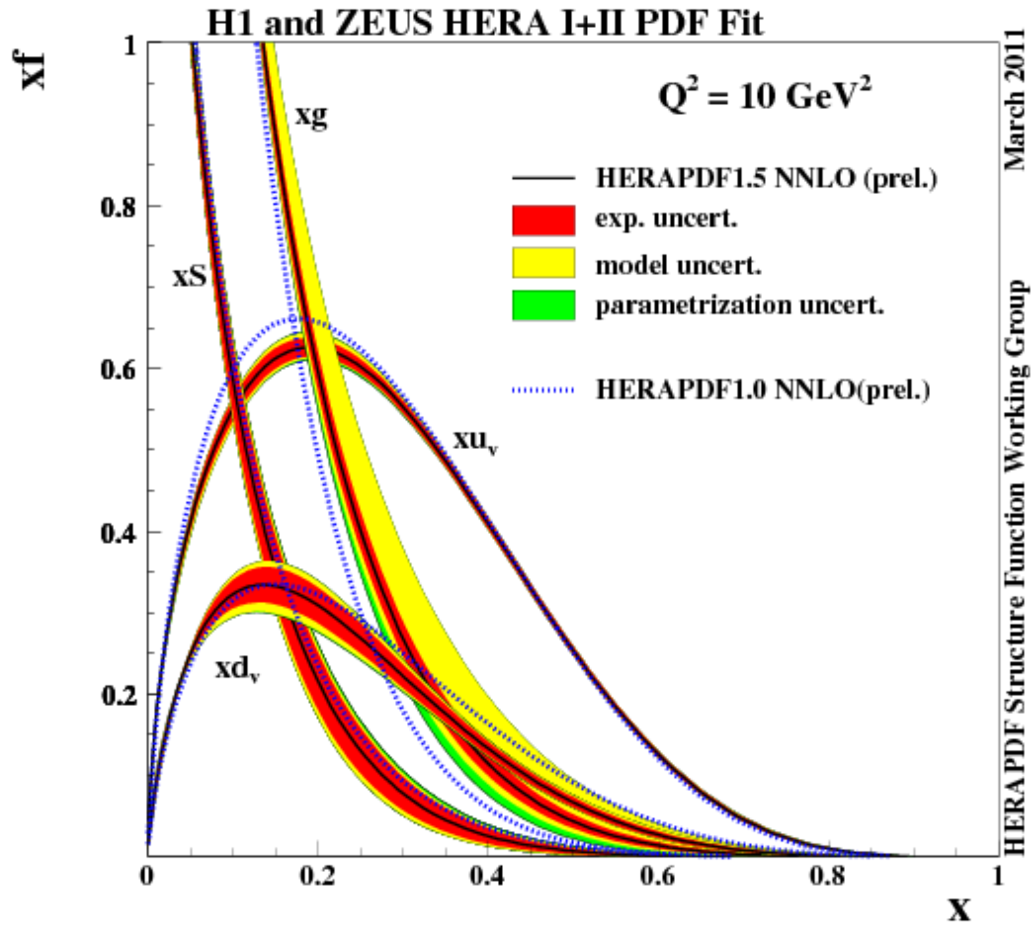
e.g., all terms of nucleon momentum decomposition calculated with controlled systematics

C. Alexandrou et al., arXiv:1706.02973

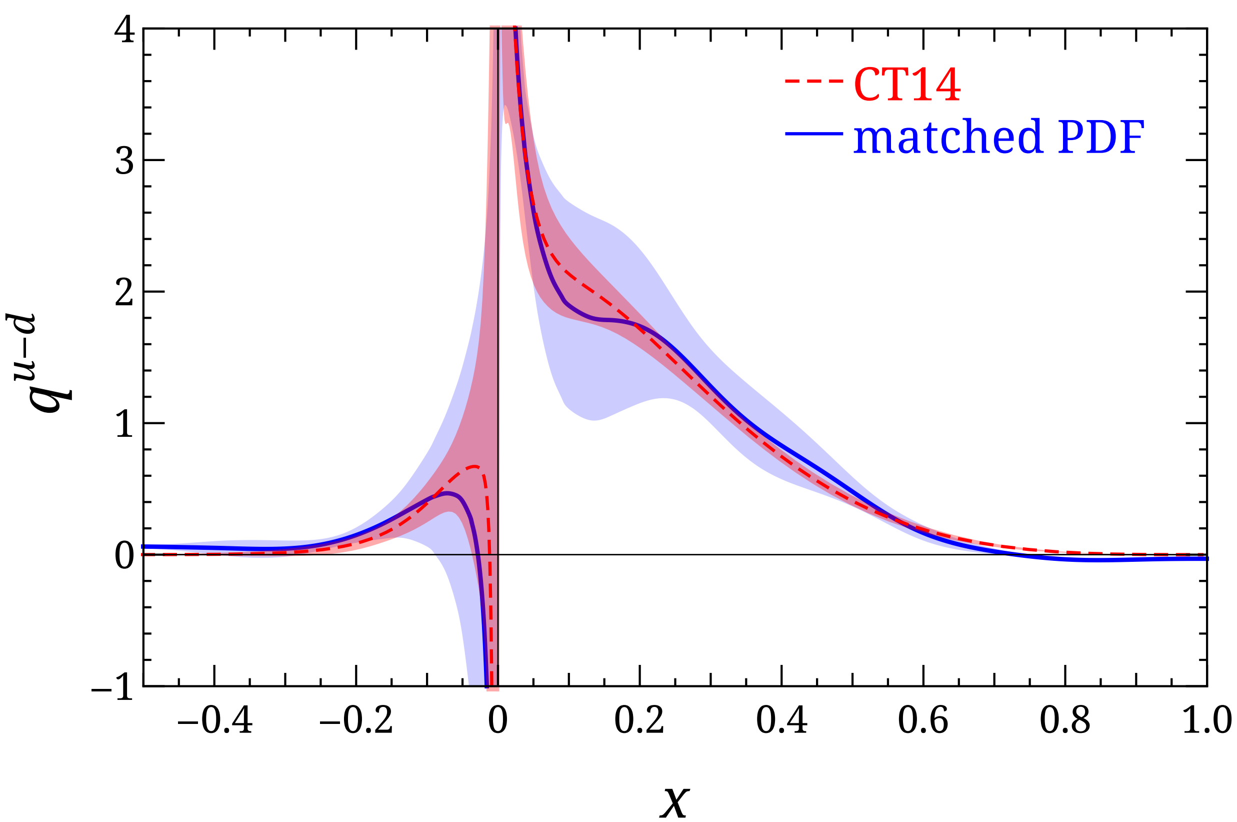


Nucleon PDFs

- First calculations of x-dependence of nucleon PDFs
- Rapid progress, but many systematics to be controlled
- Will not improve on experimental constraints in near future



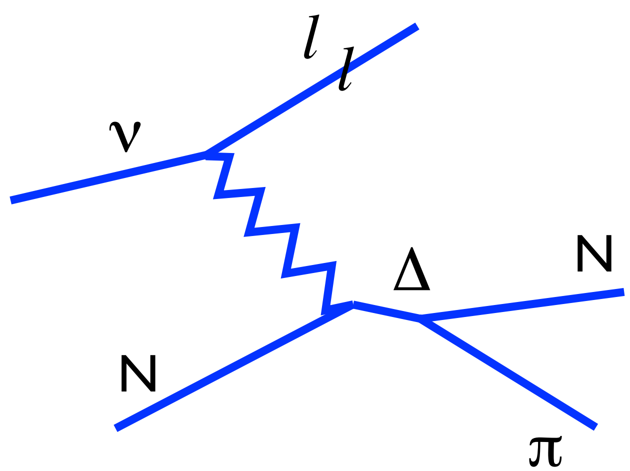
J-W Chen et al., arXiv:1803.04393, physical pion mass



Resonance region

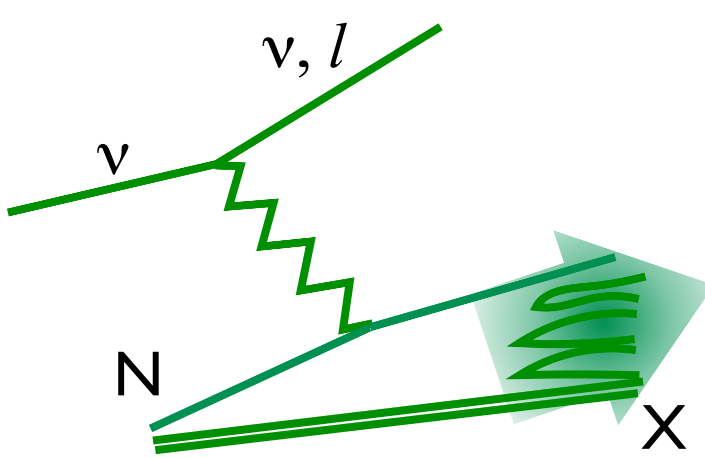
LQCD input for the resonance region:

- First calculations of axial transition form factors
  - resonances difficult for lattice QCD
  - currently: uncontrolled systematic uncertainties, unphysical values of quark masses
  - formalism in place to move to physical case



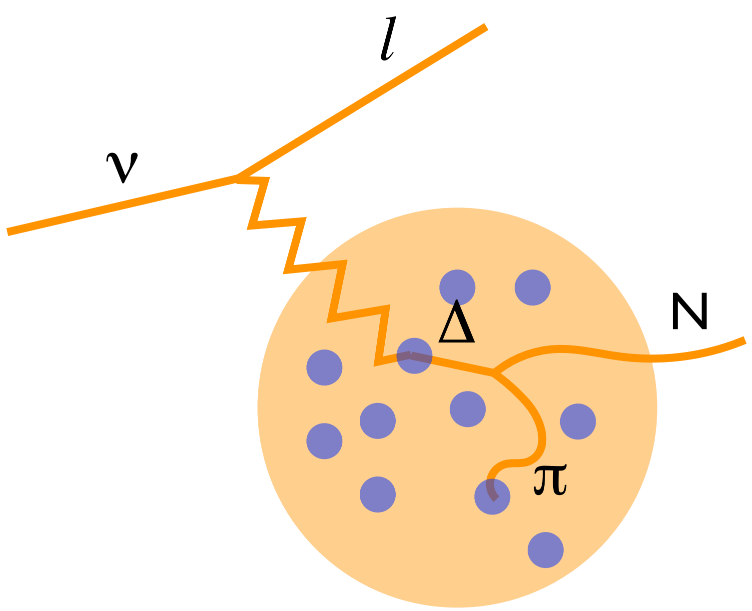
LQCD input for the inelastic scattering region:

- Much recent progress, but challenging region for direct input to neutrino program



Nuclear effects

- Targets are nuclei (C, Fe, Ar, Pb, H_2O) so how relevant are nucleon FFs, PDFs?
  - EMC effect
  - Quenching of g_A in GT transitions
- Experimental investigations: MINERνA



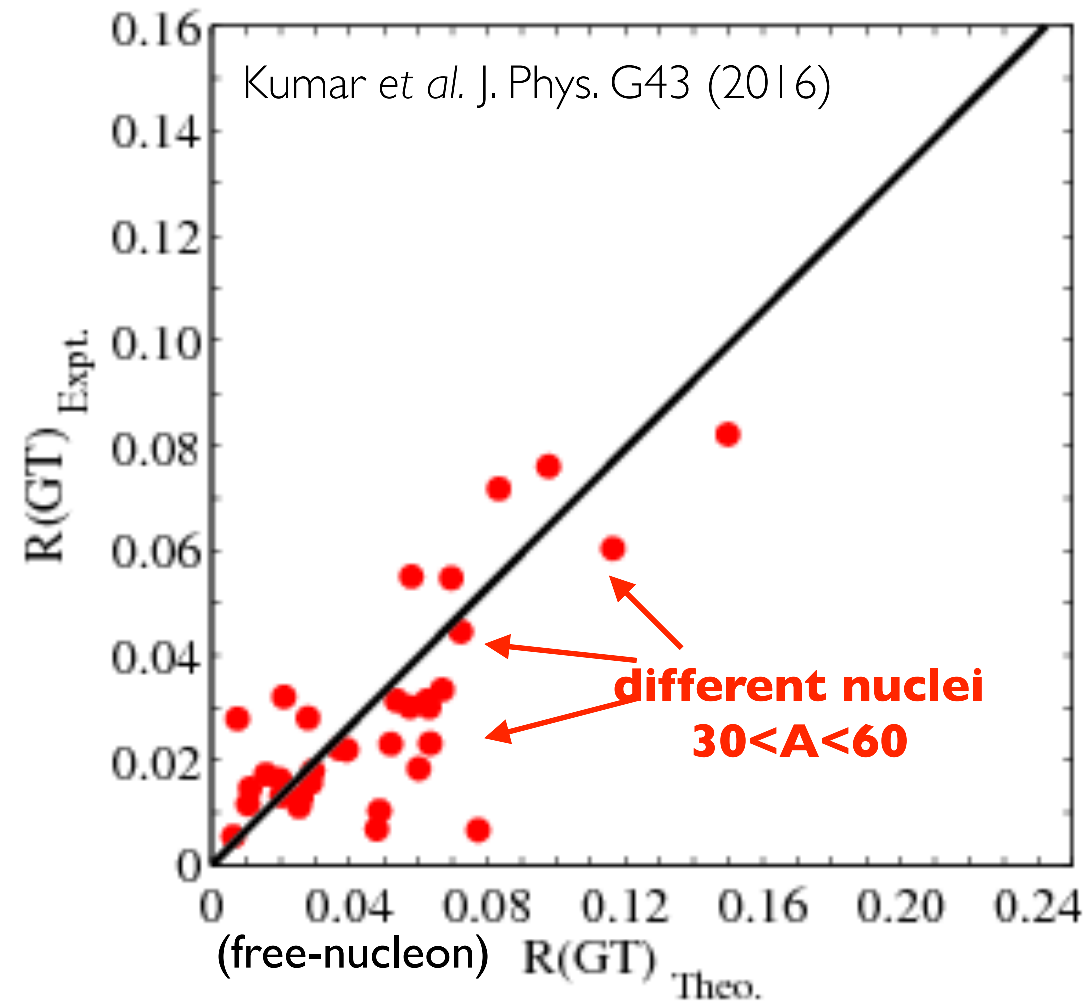
Calculate matrix elements in light nuclei from first principles

➡ EFT to reach heavy nuclear targets relevant to experiment

First calculations of axial charge of light nuclei

Nuclear effects

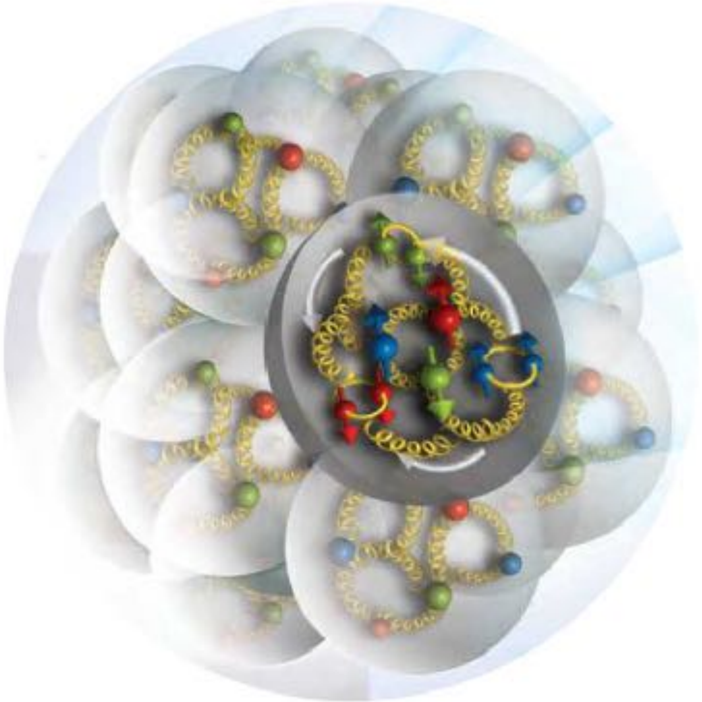
- Gamow-Teller transitions in nuclei are a stark example of problems
- Well-measured
- Best nuclear structure calculations are systematically off by 20–30%
  - Large range of nuclei (30<A<60) where spectrum is well described
  - QRPA, shell-model,...
  - Correct for it by "quenching" axial charge in nuclei ...



Nuclear physics from LQCD

Nuclei on the lattice

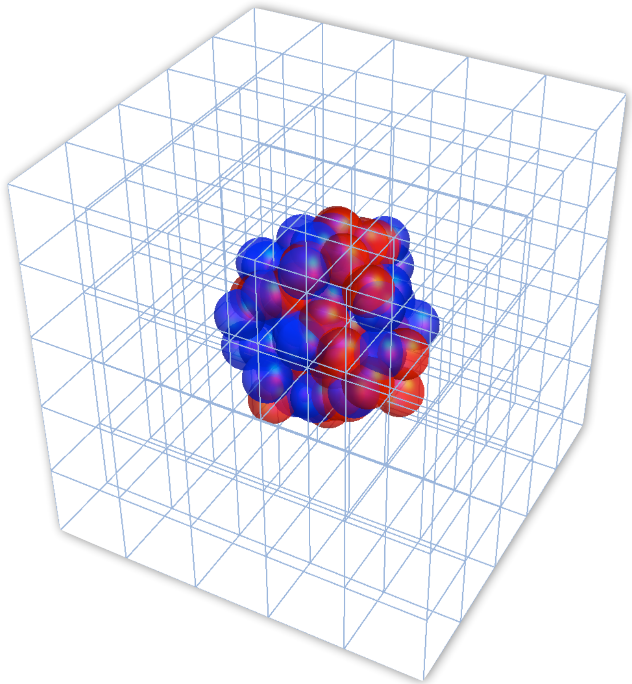
- Calculations of matrix elements of currents in light nuclei just beginning
- Deeply bound nuclei:
  use the same techniques as for single hadron matrix elements
- Near threshold states:
  need to be careful with volume effects

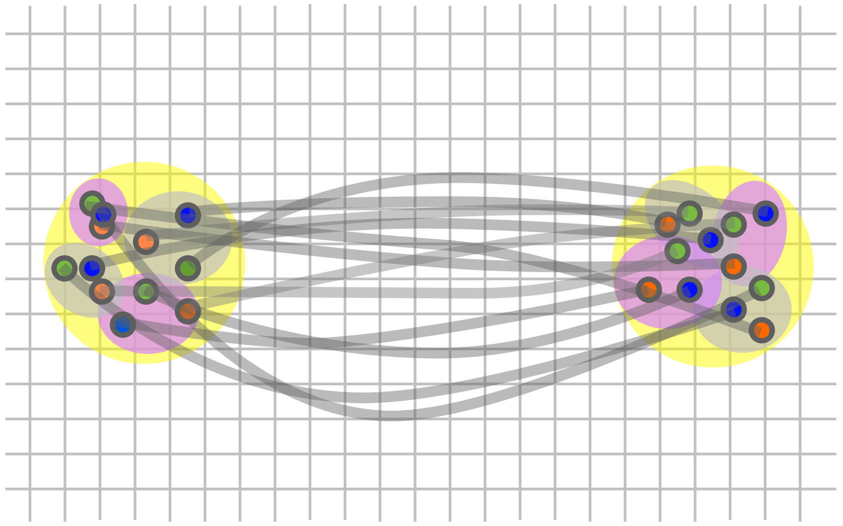


Nuclear physics from LQCD

Nuclei on the lattice

- Hard problem
  - Noise:
    Statistical uncertainty grows exponentially with number of nucleons
  - Complexity:
    Number of contractions grows factorially





Unphysical nuclei

NPLQCD collaboration

- Nuclei with A<5
- QCD with unphysical quark masses

m_π~800 MeV, m_N~1,600 MeV

m_π~450 MeV, m_N~1,200 MeV

- Nuclear structure: magnetic moments, polarisabilities
  [PRL **113**, 252001 (2014), PRD 92, 114502 (2015)]
- First nuclear reaction: np→dγ
  [PRL **115**, 132001 (2015)]
- Proton-proton fusion and tritium β-decay
  [PRL **119**, 062002 (2017)]
- Double β-decay
  [PRL **119**, 062003 (2017), PRD **96**, 054505 (2017)]
- Gluon structure of light nuclei
  [PRD **96** 094512 (2017)]
- Scalar, axial and tensor MEs
  [PRL **120**, 152002 (2018)]

Unphysical nuclei

NPLQCD collaboration

- Nuclei with $A<5$
- QCD with unphysical quark masses

  $m_\pi \sim 800$ MeV, $m_N \sim 1{,}600$ MeV

  $m_\pi \sim 450$ MeV, $m_N \sim 1{,}200$ MeV

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  [PRD **96** 094512 (2017)]
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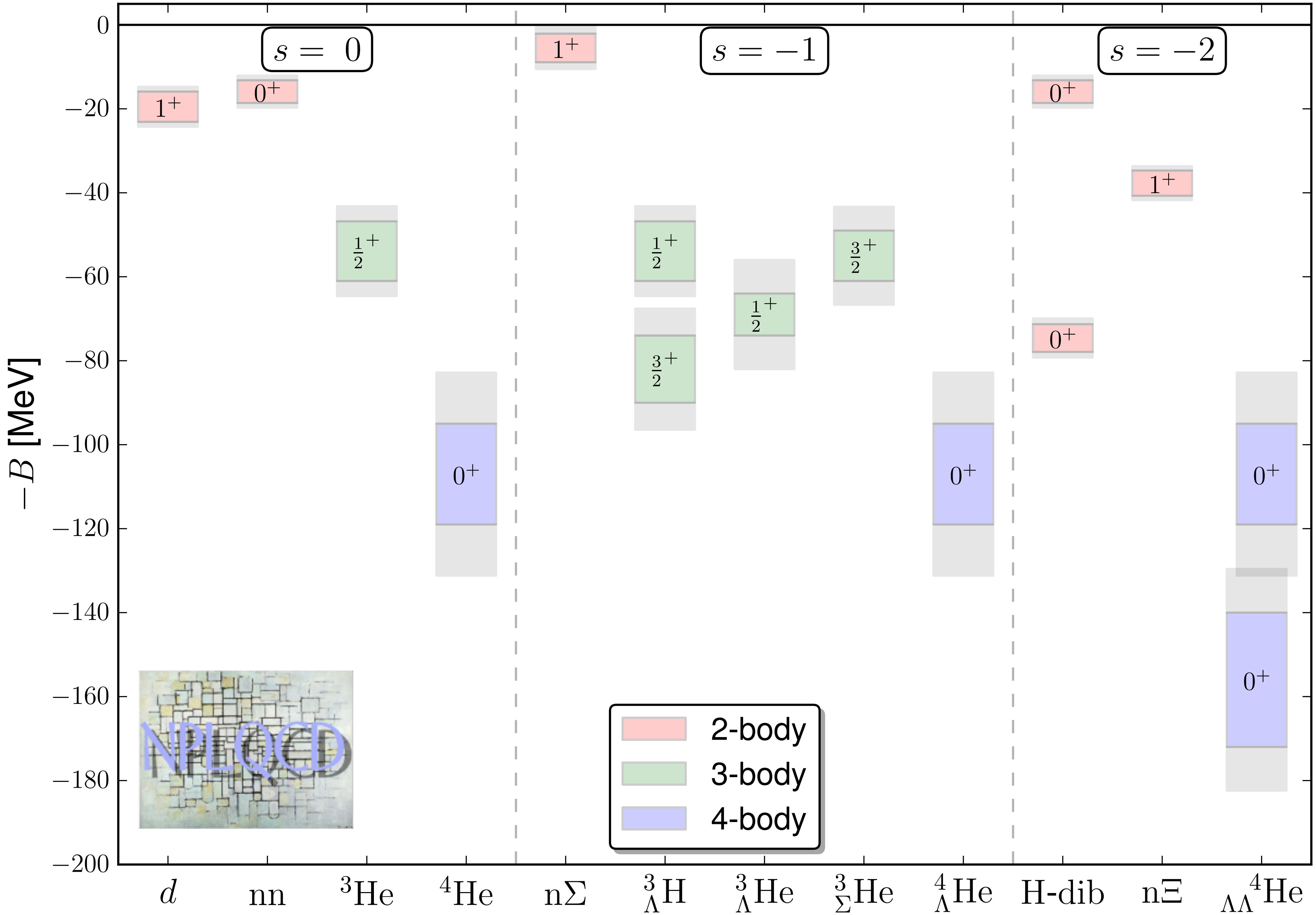
  [PRL **120**, 152002 (2018)]

Spectrum of light nuclei

@ m_π = 800 MeV



NPLQCD Phys.Rev. D87 (2013), 034506

Background field method

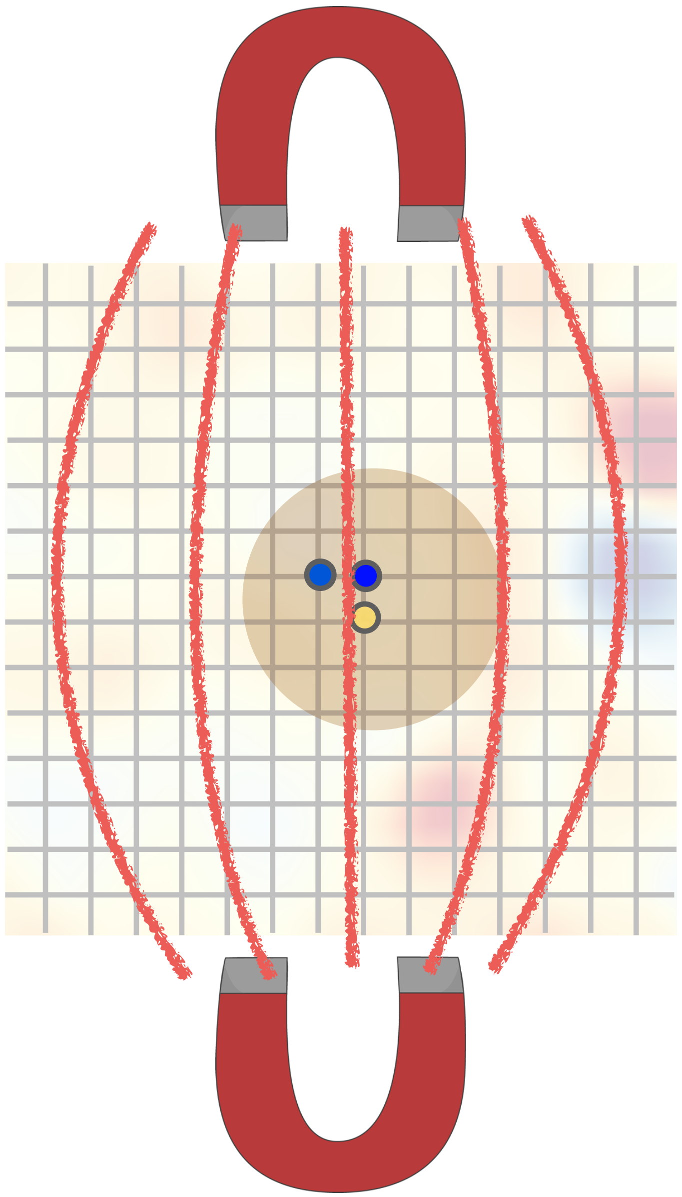
Hadron/nuclear energies are modified by presence of fixed/constant external fields

Example: fixed magnetic field

landau level — mag. mmt

$$E(\vec{B}) = \sqrt{M^2 + (2n+1)|Qe\vec{B}|} - \vec{\mu}\cdot\vec{B} - 2\pi\beta_{M0}|\vec{B}|^2$$

mag. polarisability



- Calculations with multiple fields
  ➔ extract coefficients of response
  e.g., magnetic moments, polarisabilities, …
- Not restricted to simple EM fields
  Axial MEs: uniform axial background field

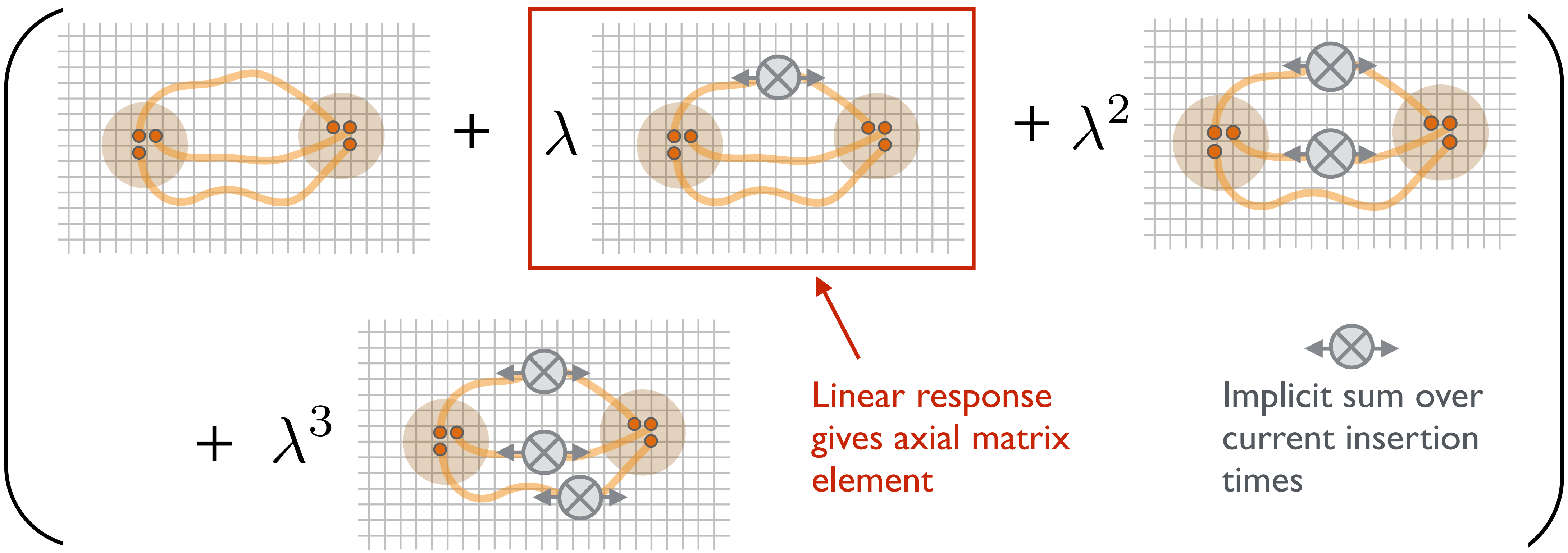
[Detmold, Savage PRL **113**, 252001 (2014)]

Axial background field

Example: fixed magnetic field ➡ moments, polarisabilities

Axial MEs: fixed axial background field ➡ axial charges, other matrix elts.

$C_{\lambda_u;\lambda_d}(t) =$

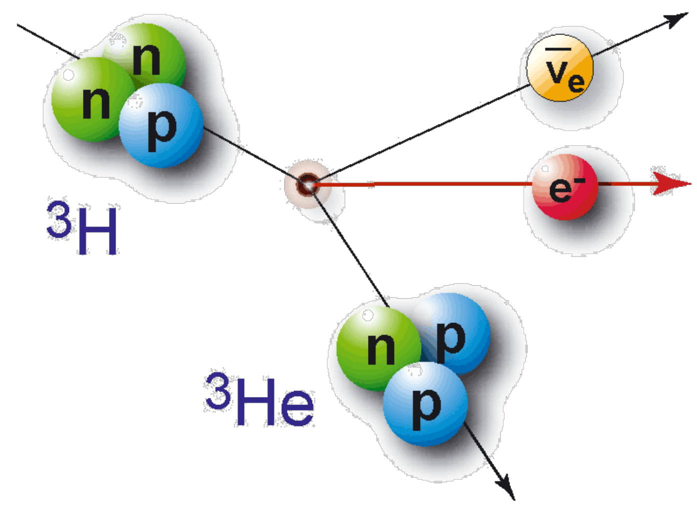


Second order piece: being used for calculations of double-beta decay

[NPLQCD Nucl. Phys. **A743**, 170 (2004)]

Tritium β-decay

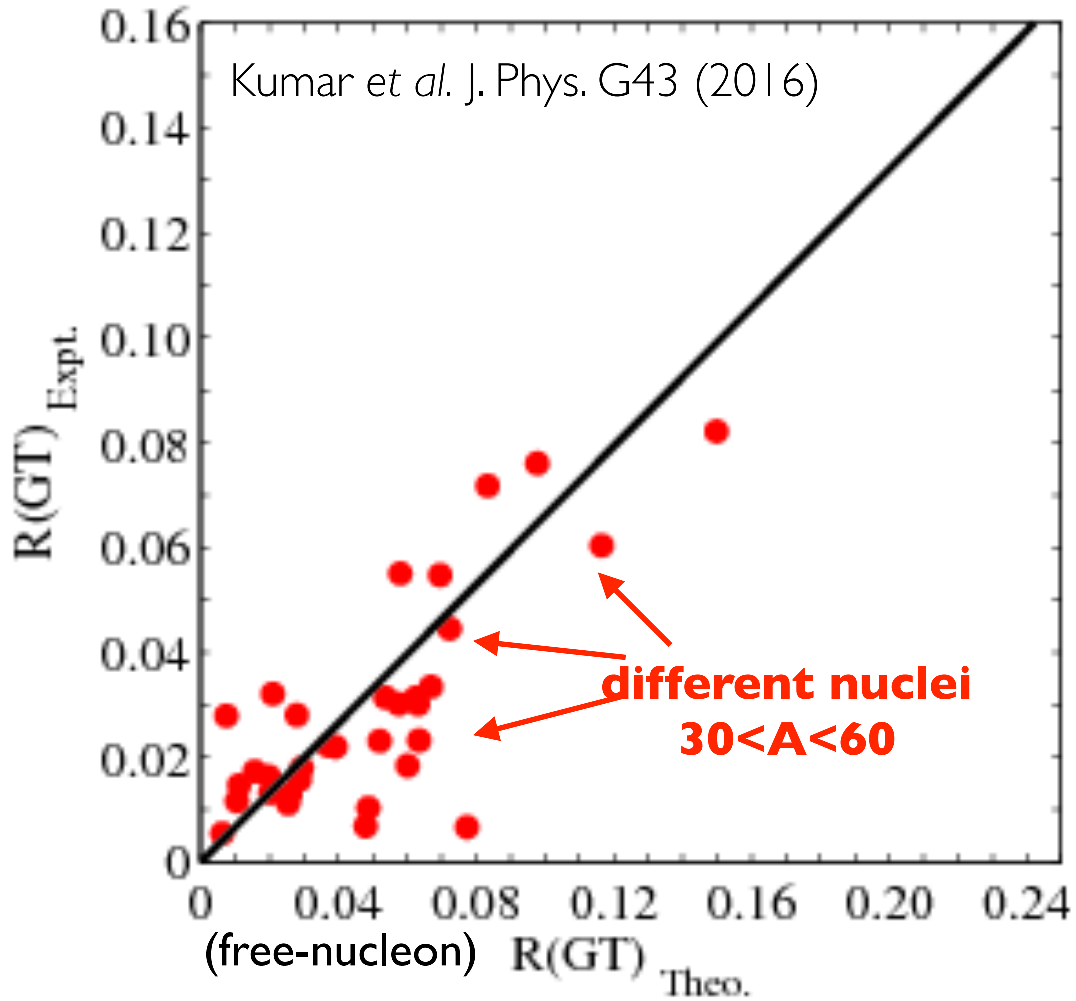
- Simplest semileptonic weak decay of a nuclear system



- Gamow-Teller (axial current) contribution to decays of nuclei not well-known from theory
- Understand multi-body contributions to $\langle \mathbf{GT} \rangle$ ➡ better predictions for decay rates of larger nuclei

We calculate

$$g_A \langle \mathbf{GT} \rangle = \langle {}^{\mathbf{3}}\mathrm{He} | \overline{\mathbf{q}} \gamma_{\mathbf{k}} \gamma_{\mathbf{5}} \tau^- \mathbf{q} | {}^{\mathbf{3}}\mathrm{H} \rangle$$



Tritium β-decay

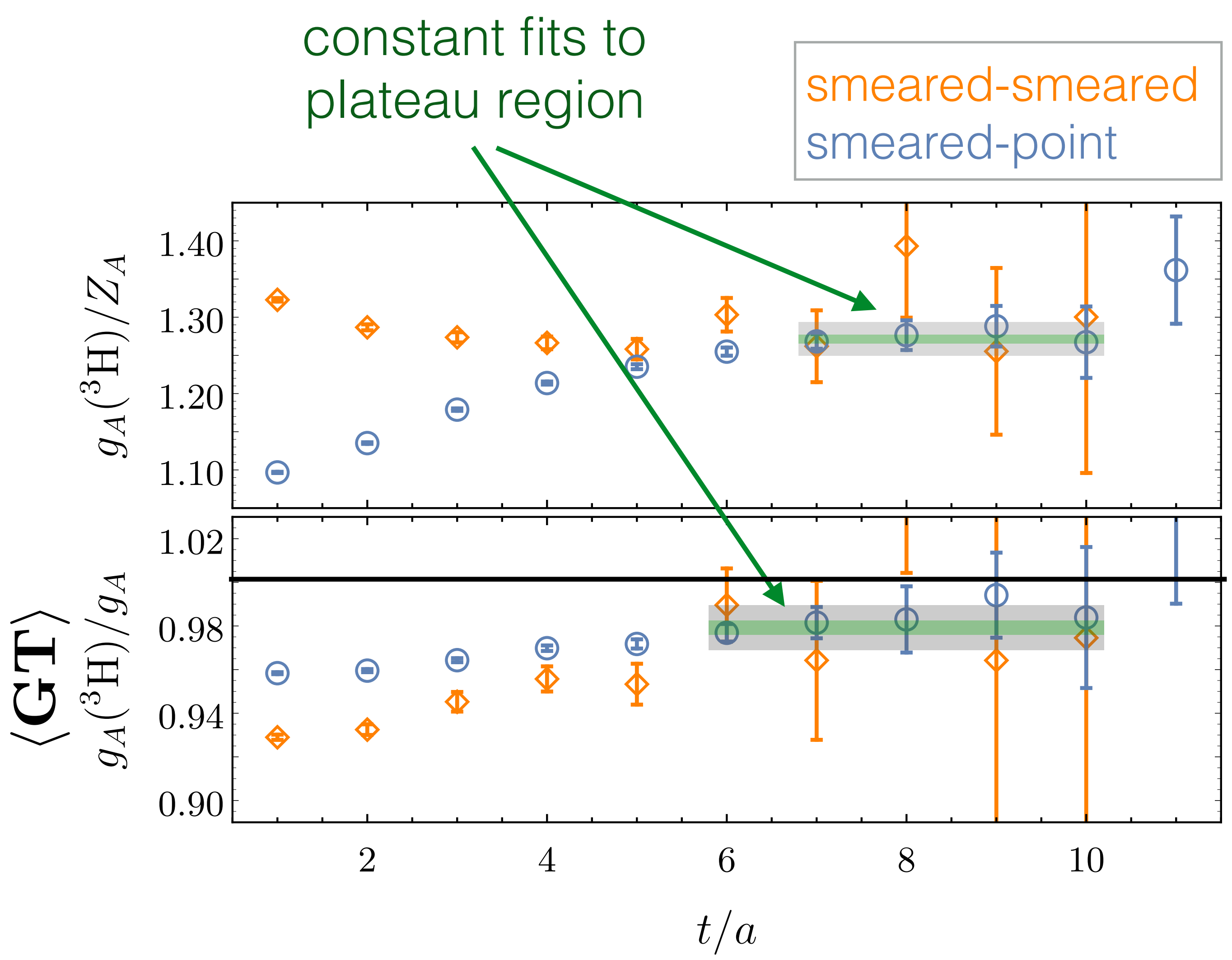
$$\frac{(1+\delta_R)f_V}{K/G_V^2}\, t_{1/2} = \frac{1}{\langle \mathbf{F} \rangle^2 + f_A/f_V\, g_A^2 \langle \mathbf{GT} \rangle^2}$$

known from theory or expt. · half-life · vector ME · axial ME

- Form ratios of compound correlators to cancel leading time-dependence:

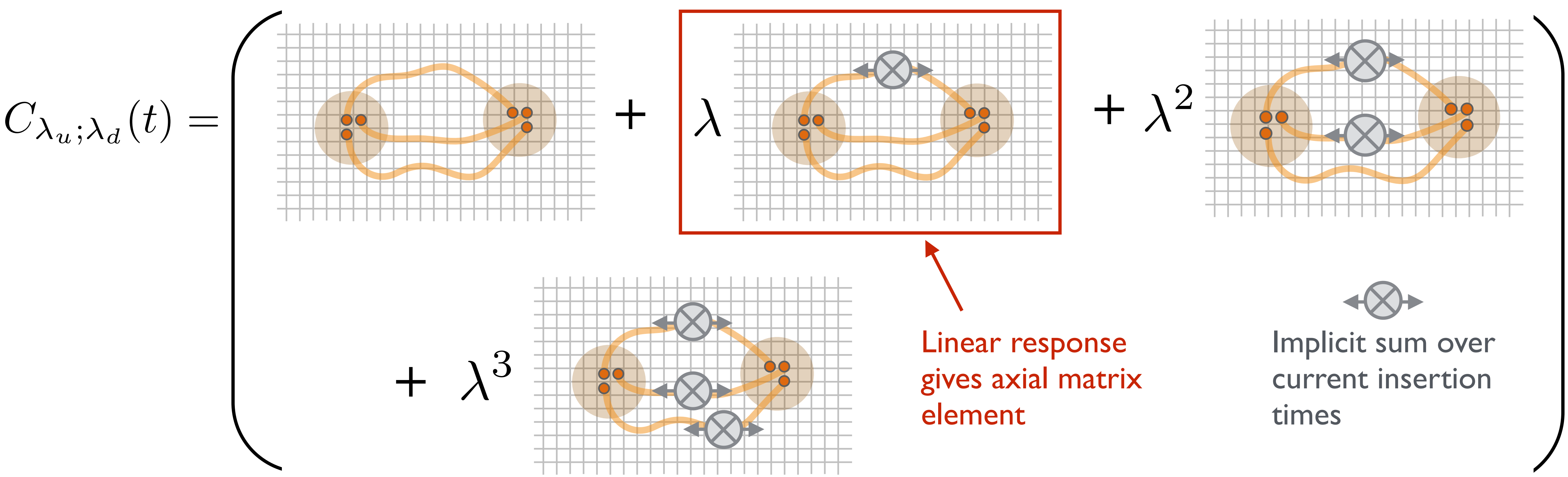
$$\frac{\overline{R}_{^3\mathrm{H}}(t)}{\overline{R}_p(t)} \xrightarrow{t\to\infty} \frac{g_A(^3\mathrm{H})}{g_A} = \langle \mathbf{GT} \rangle$$

- "Quenching" of the axial charge emerges from LQCD calculation



Higher-order insertions

- Can access terms with more current insertions from same calculations
- Recall: background field correlation function

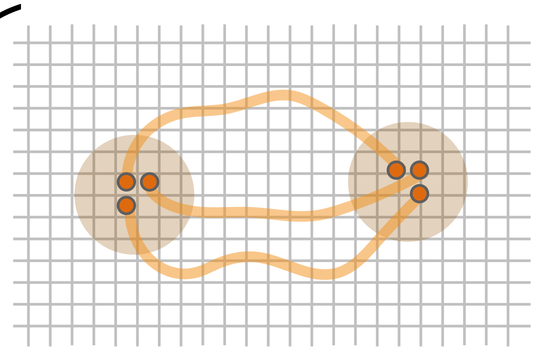


Higher-order insertions

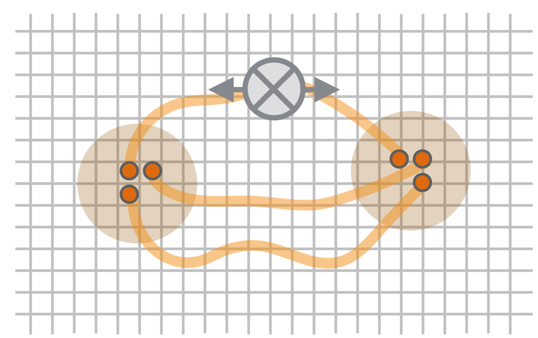
- Can access terms with more current insertions from same calculations
- Recall: background field correlation function

Quadratic response from two insertions on different quark lines

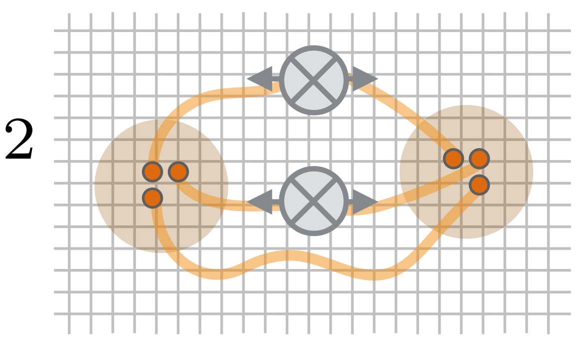
$$C_{\lambda_u;\lambda_d}(t) = \Bigg($$



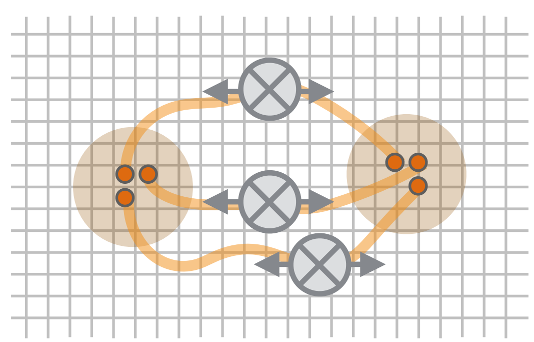
$+ \; \lambda$



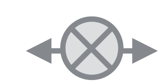
$+ \; \lambda^2$



$+ \; \lambda^3$



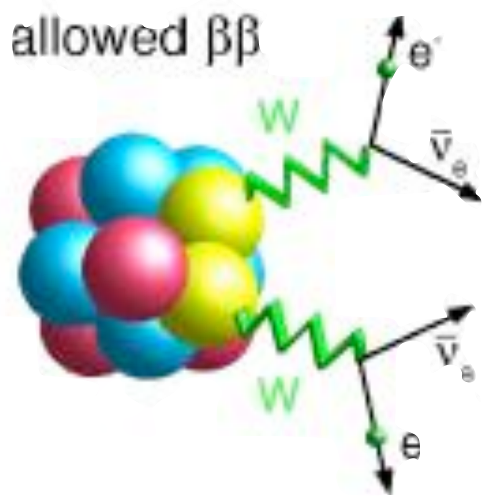
$$\Bigg)$$



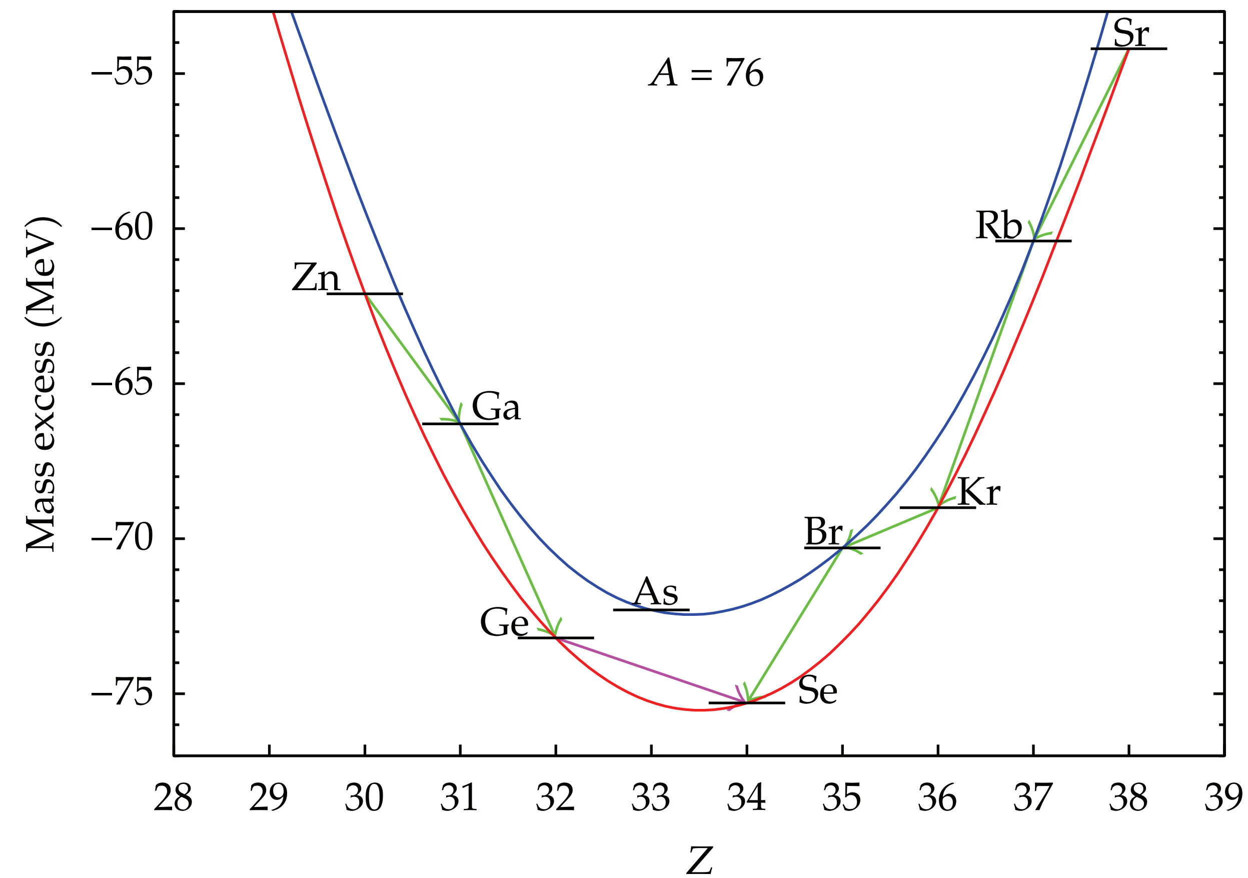
Implicit sum over current insertion times

Double β-decay

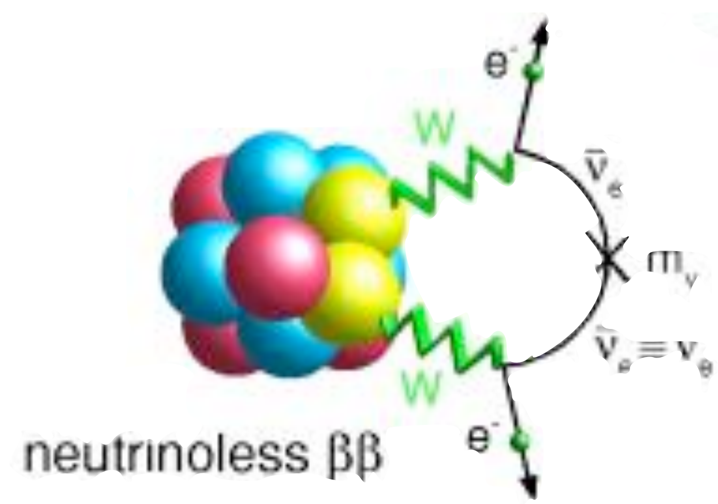
- Certain nuclei allow observable ββ decay



$T_{1/2}^{2\nu\beta\beta} \gtrsim 10^{19}$ y



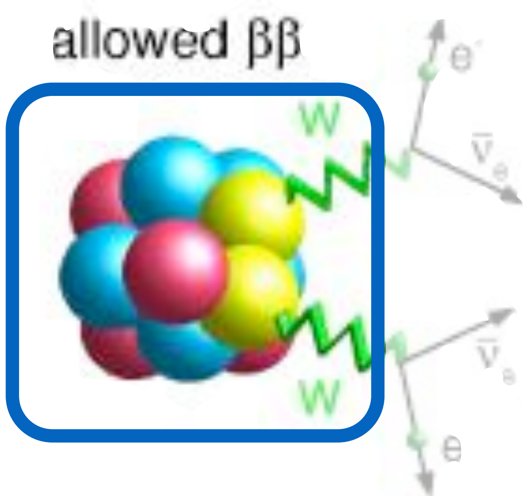
- If neutrinos are massive Majorana fermions 0νββ decay is possible



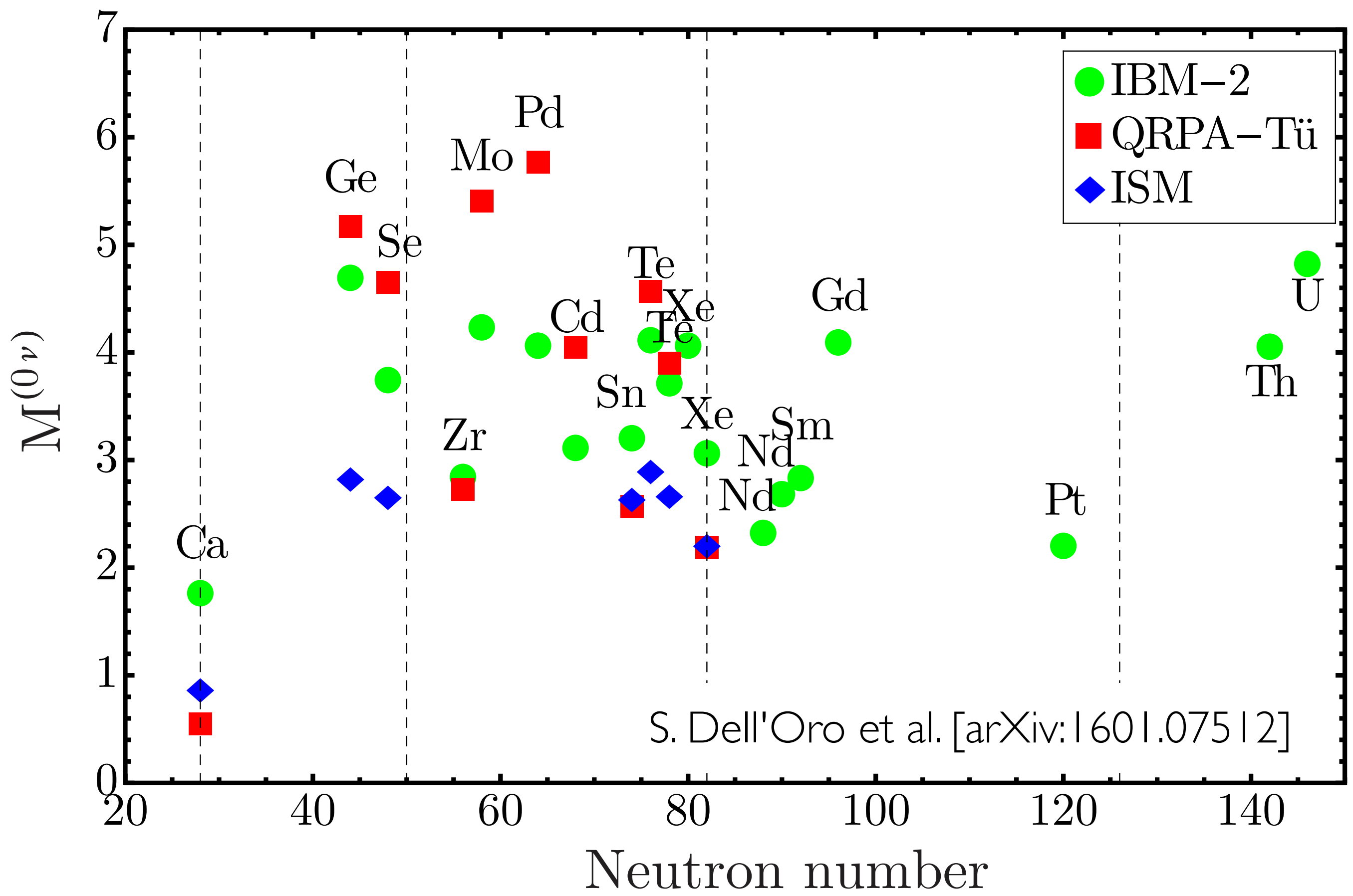
$T_{1/2}^{0\nu\beta\beta} > 10^{25}$ y

Double β-decay

Want to understand **2νββ** and **0νββ** decay from theory



Model calculations have large uncertainties



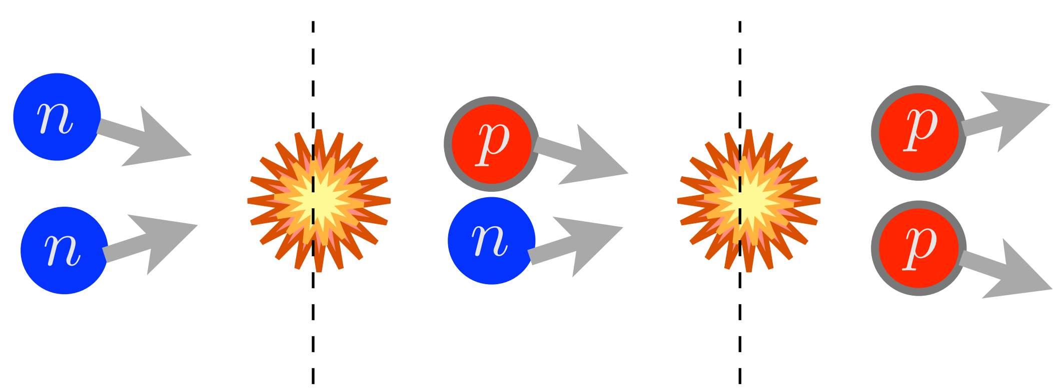
Calculate two-current nuclear matrix elements

→ dictate half-life

Second order weak interactions

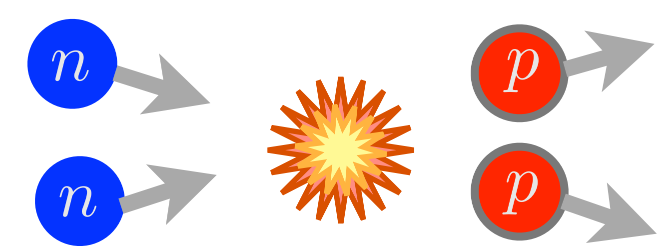
NPLQCD PRL **119**, 062003 (2017), PRD **96**, 054505 (2017)

Lattice QCD: Calculate nn→pp transition matrix element



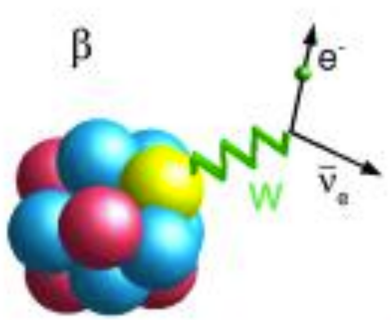
LONG-DISTANCE PIECE

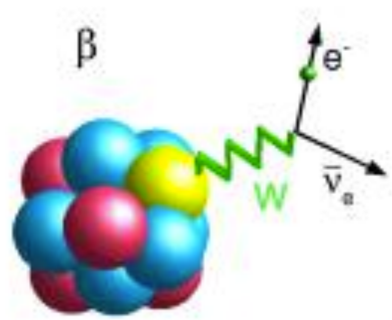
Two single-beta decays

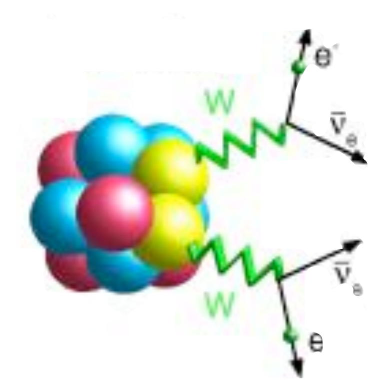


SHORT-DISTANCE PIECE

Two-body effect







Second order weak interactions

NPLQCD PRL **119**, 062003 (2017), PRD **96**, 054505 (2017)

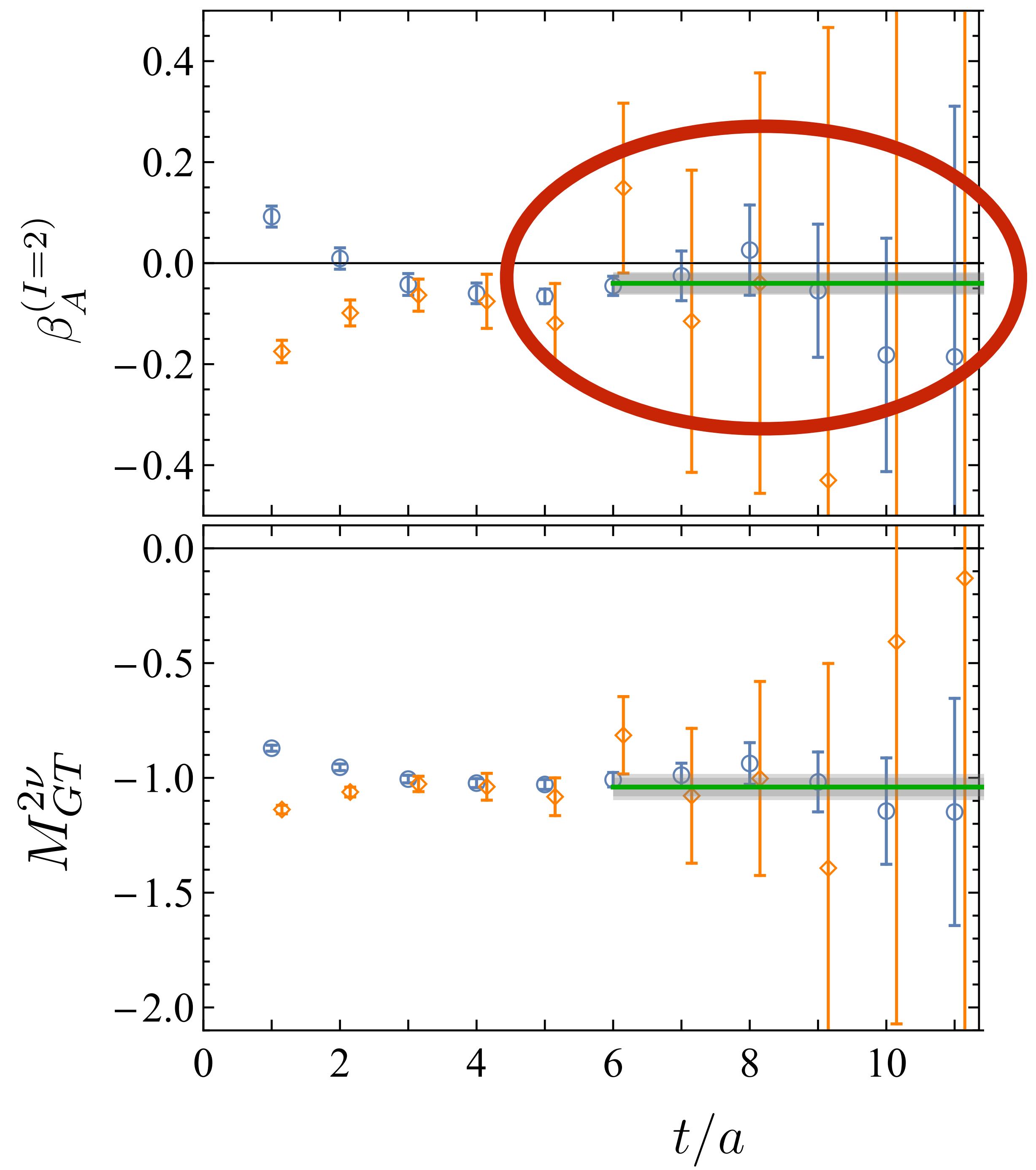
- Non-negligible deviation from long distance deuteron intermediate state contribution

Isotensor axial polarisability

$$M_{GT}^{2\nu} = -\frac{|M_{pp\to d}|^2}{E_{pp} - E_d} + \beta_A^{(I=2)}$$

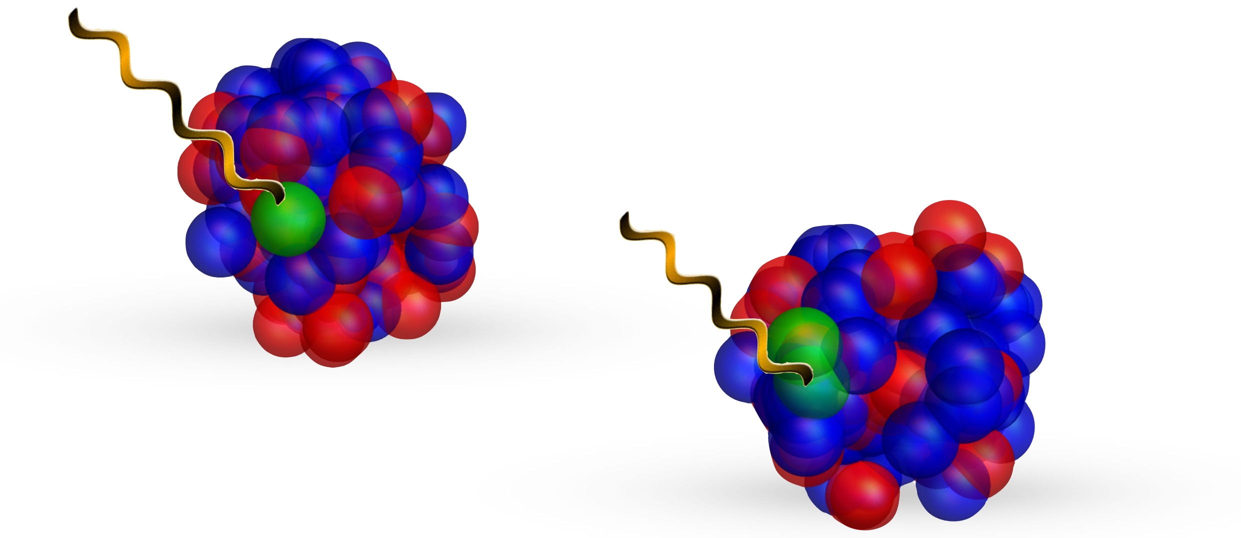
➡ Multi-body effects can't be neglected!

- TBD: connect to models / effective field theory for larger systems



Larger nuclei

- What about larger (phenomenologically-relevant) nuclei?
- Nuclear effective field theory:
  - 1-body currents are dominant
  - 2-body currents are sub-leading
    but non-negligible



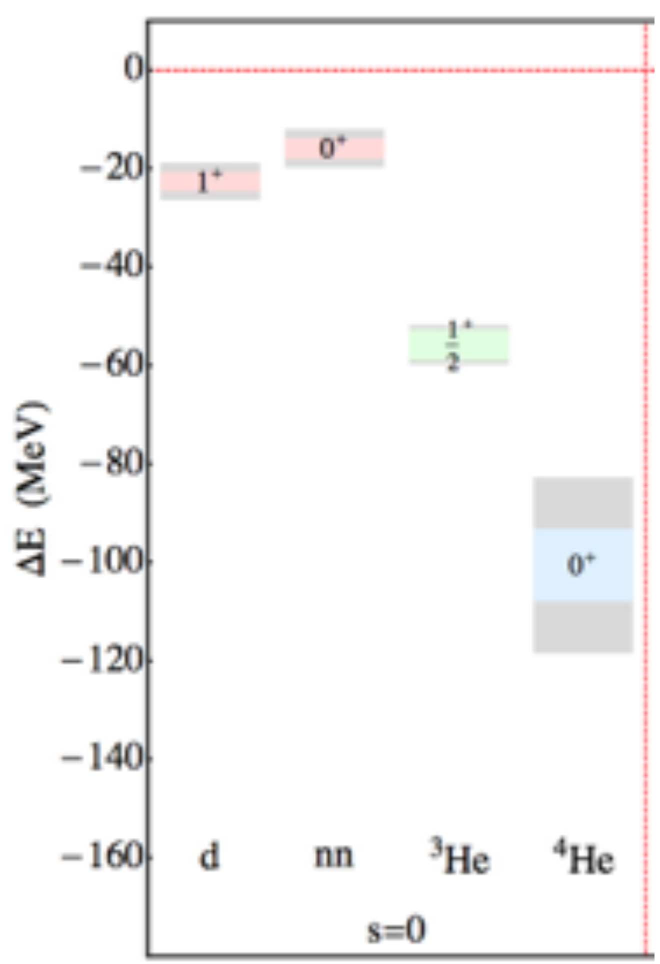
- Determine one body contributions from single nucleon
- Determine few-body contributions from A=2,3,4...
- Match EFT and many body methods to LQCD to make predictions for larger nuclei

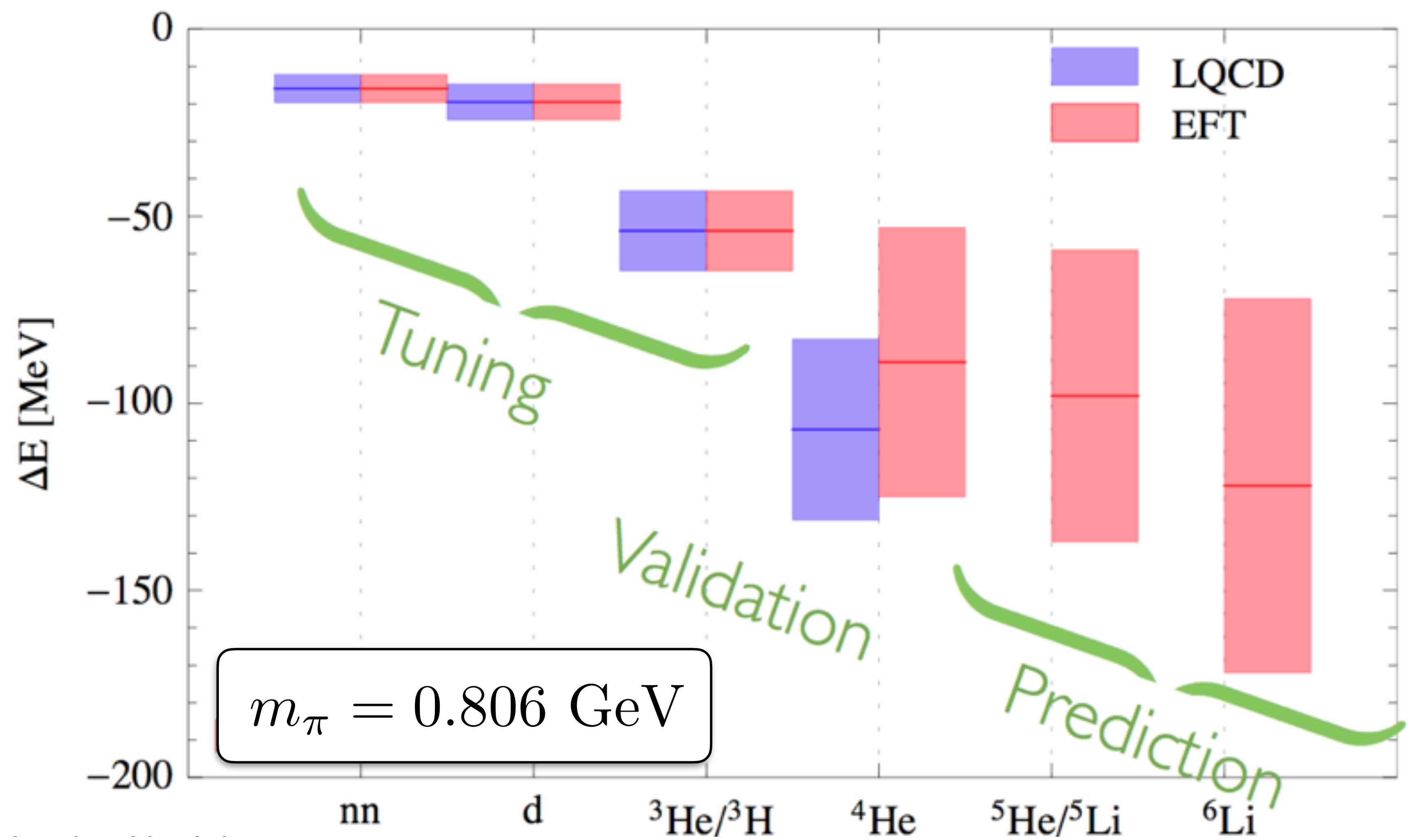
Larger nuclei

e.g., Barnea et al., arXiv:1311.4966
Effective field theory for lattice nuclei

Contessi et al., arXiv:1701.06516
Ground-State Properties of ^{4}He and ^{16}O Extrapolated from Lattice QCD with Pionless EFT

Bansal et al., arXiv:1712.10246
Pion-less effective field theory for atomic nuclei and lattice nuclei

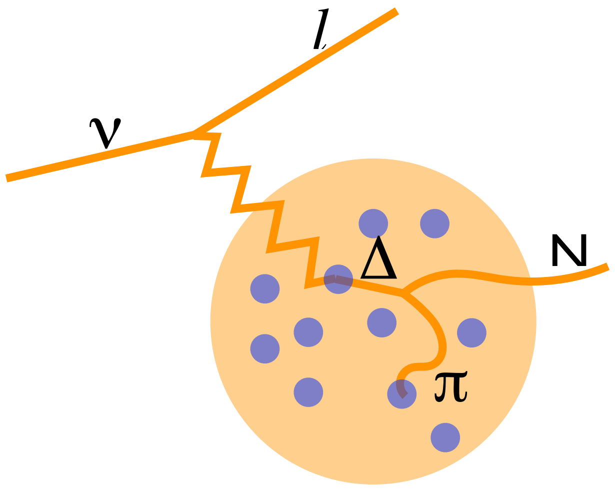




+Lorenzo Contessi's PhD thesis

Summary - Part I

- Lattice efforts have potential to impact ν energy determinations



- Precise determinations with controlled percent-level uncertainties within ~5 years
  - Axial and pseudoscalar FFs determined with momenta less than a few GeV
  - BUT: large momentum FFs (≳3 GeV) more difficult. Novel ideas exist, need testing
- Early results with promising applications
  - Transition FFs
    Formalism exists but developments still necessary for higher states above $N\pi\pi$ inelastic threshold
  - Application of EFT using 2-, 3- body matrix elements to constrain nuclear effects

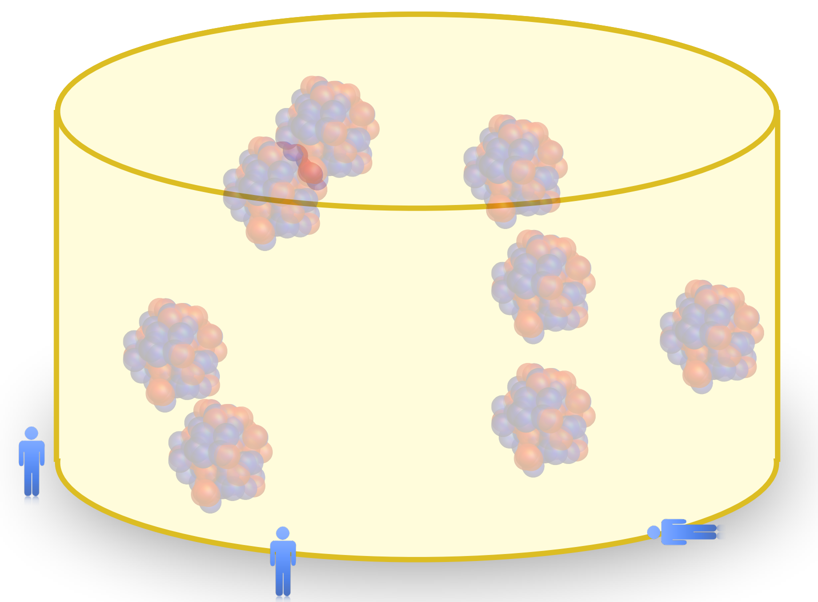
Dark matter

How do we find dark matter?

- Dark (does not interact with light)
- Interacts through gravity

WIMP
Weakly-interacting massive particles

Direct detection
Wait for DM to hit us



Detection rate depends on

- Dark matter properties
- Probability for interaction with nucleus

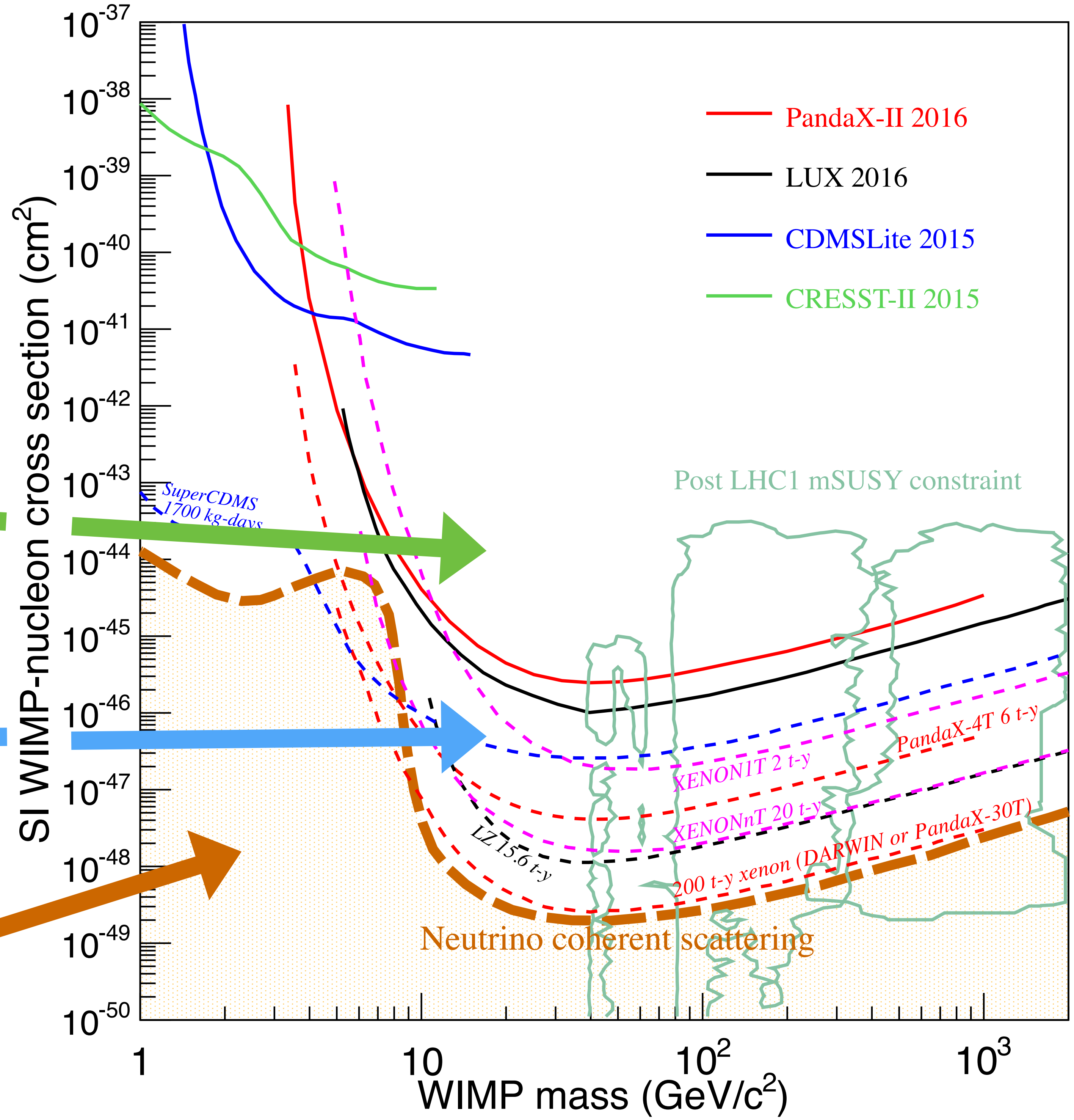
Dark matter direct detection

Limits on WIMP-nucleon interaction from direct detection experiments

Ruled out above the solid lines

Projected limits from future experiments

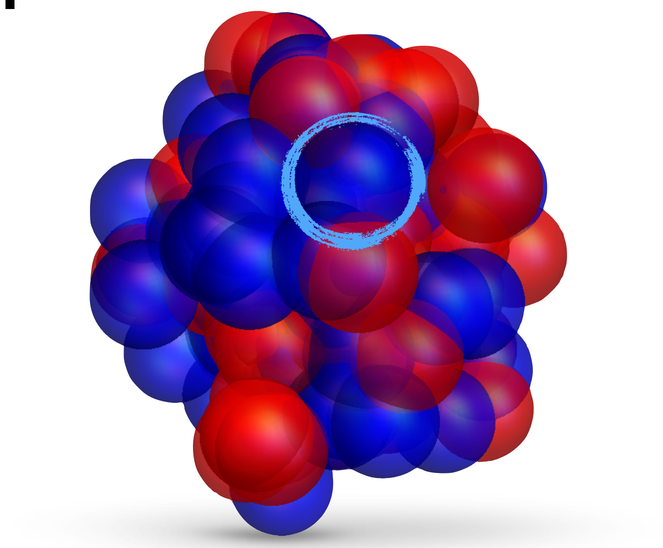
Background



Dark matter

Determine interaction cross-section (with nucleus) for a given dark matter model

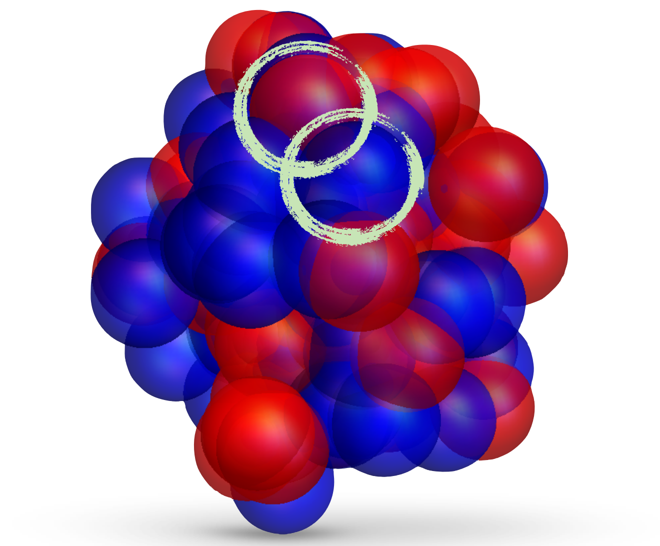
- Born approximation – interacts with a single nucleon



$$\sigma \sim |A \langle N|DM|N\rangle|^2$$

known from LQCD

- Interacts non-trivially with multiple nucleons

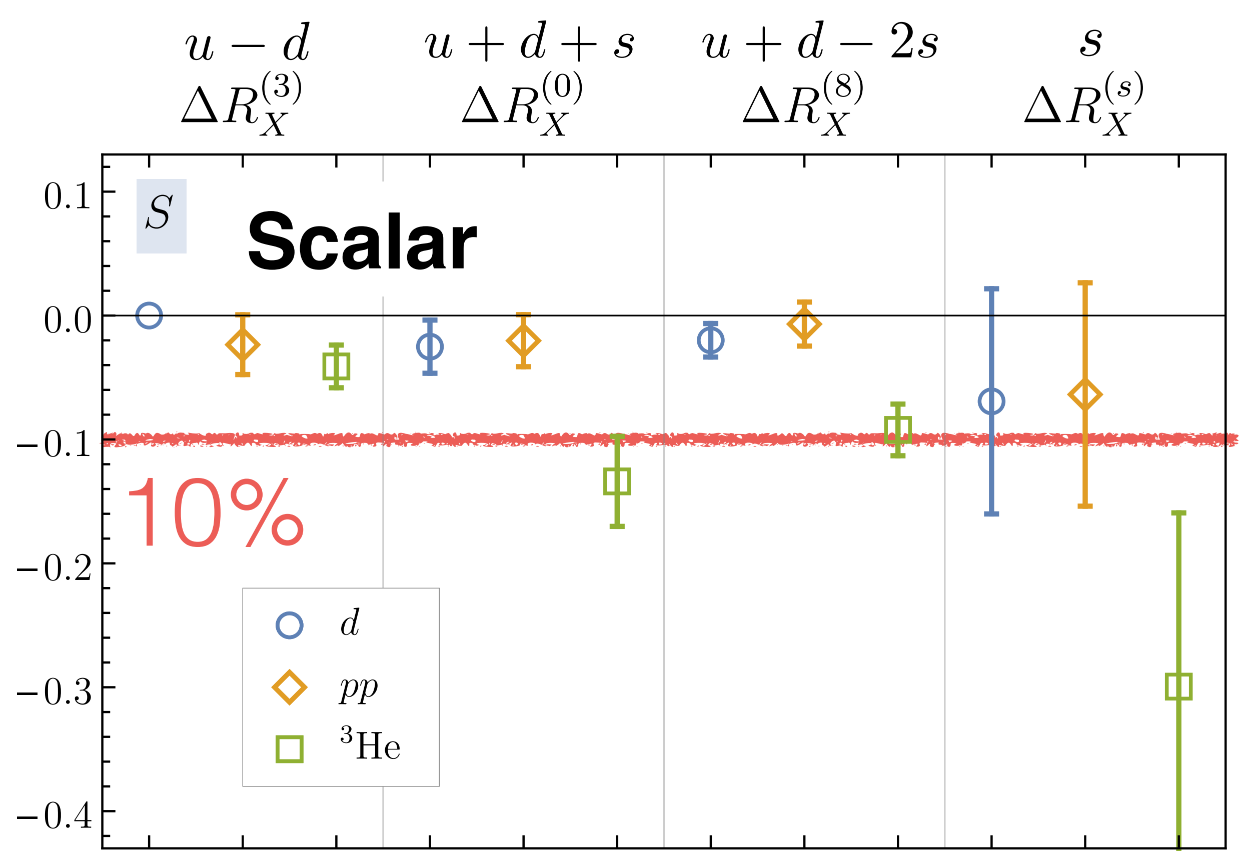


$$\sigma \sim |A \langle N|DM|N\rangle + \alpha \langle NN|DM|NN\rangle + \ldots|^2$$

poorly known!

Scalar matrix elements

- Spin-independent scattering of many WIMP candidates governed by scalar matrix elements
- Lattice QCD calculation with m_π~800 MeV shows 10% nuclear effects! (Naive expectation determined by baryon#, isospin, spin)
- Same calculation gives axial and tensor nuclear effects around ~1%



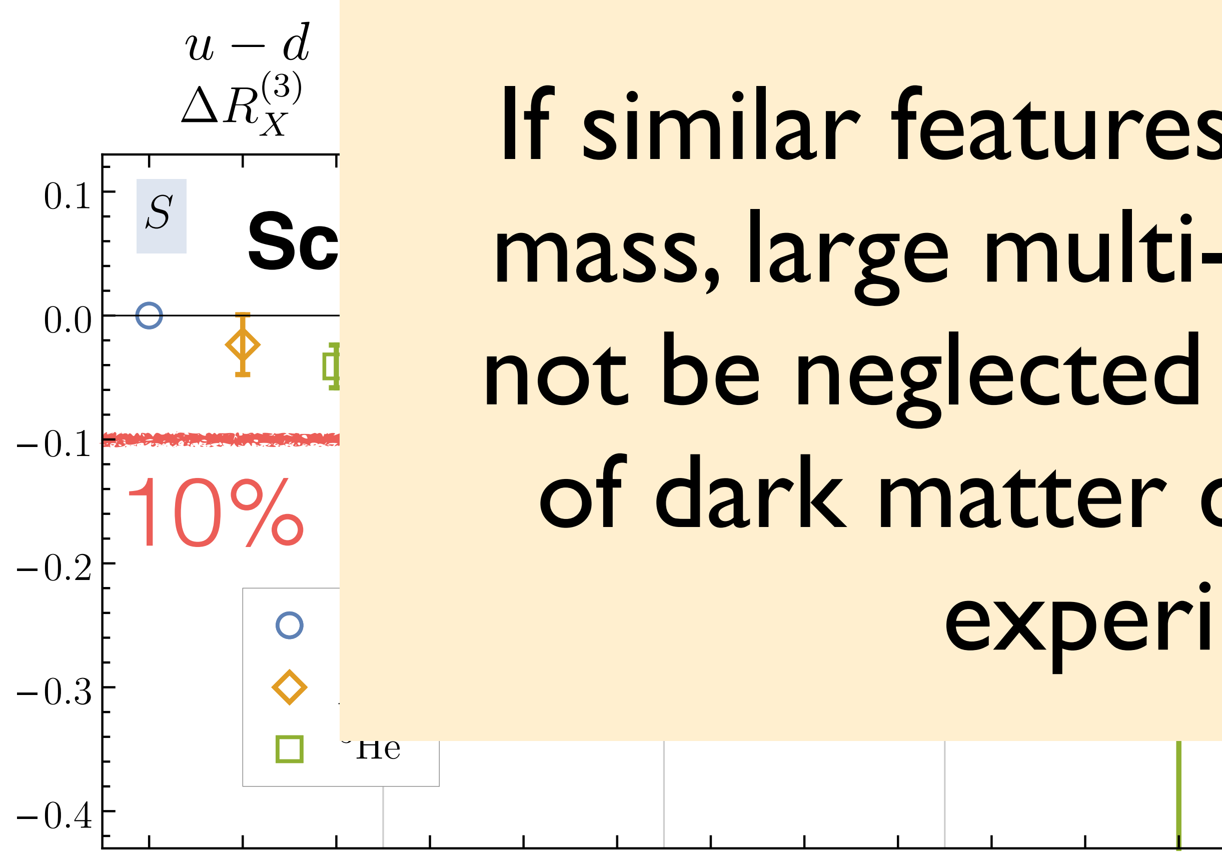
NPLQCD Phys. Rev. Lett. 120 (2018), 152002

$$\frac{\textbf{ME}}{\textbf{Nucleon ME}} - \textbf{naive expectation}$$

"Multiply up" nucleon expectation

Scalar matrix elements

- Spin-independent scattering of many WIMP candidates governed by scalar matrix elements
- Lattice QCD calculation with m_π~800 MeV shows 10% nuclear effects! (Naive expectation determined by baryon#, isospin, spin)
- Same calculation gives axial and tensor nuclear effects around ~1%



If similar features at physical pion mass, large multi-body effects can not be neglected in interpretation of dark matter direct detection experiments

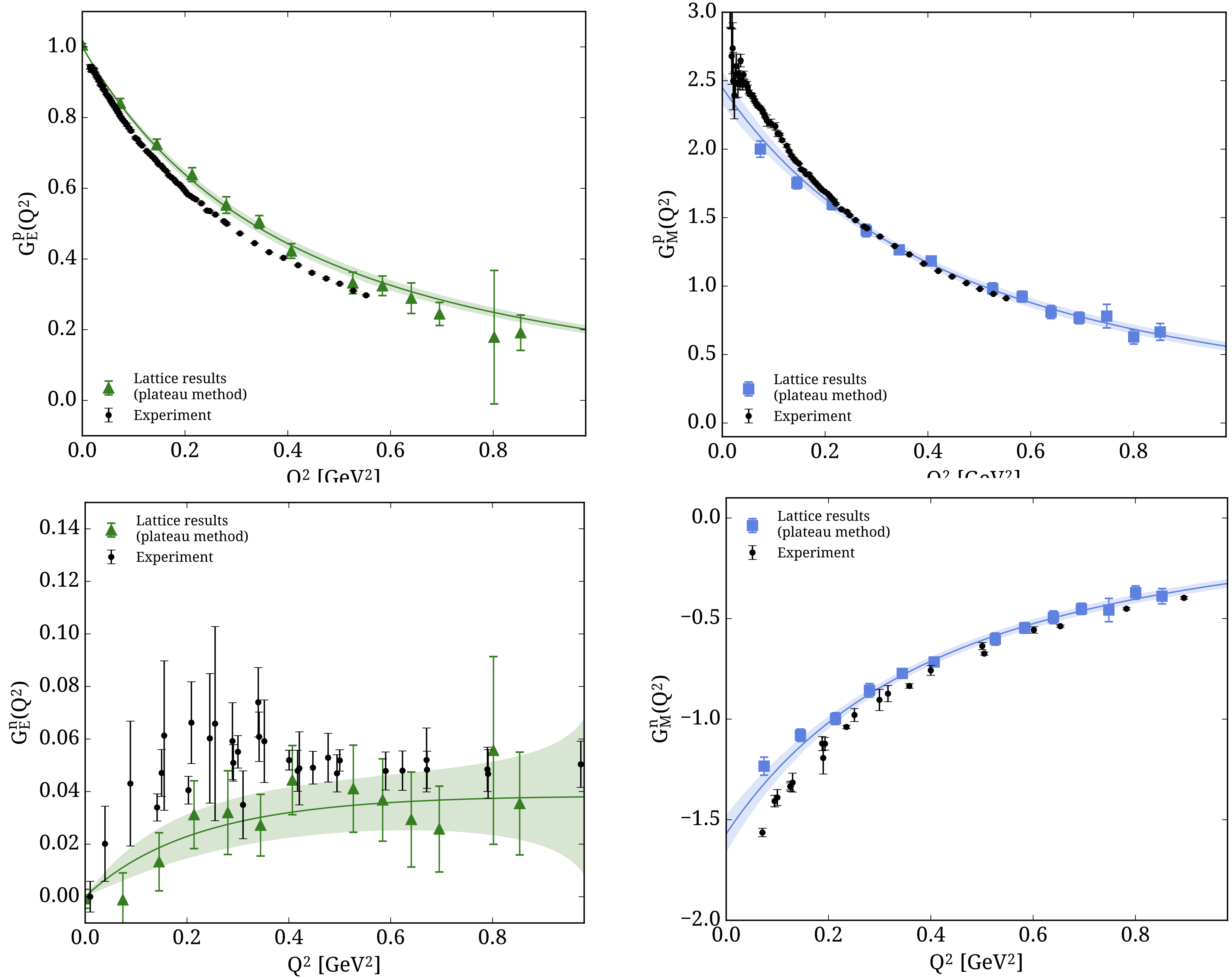
naive
ectation

p''
nucleon expectation

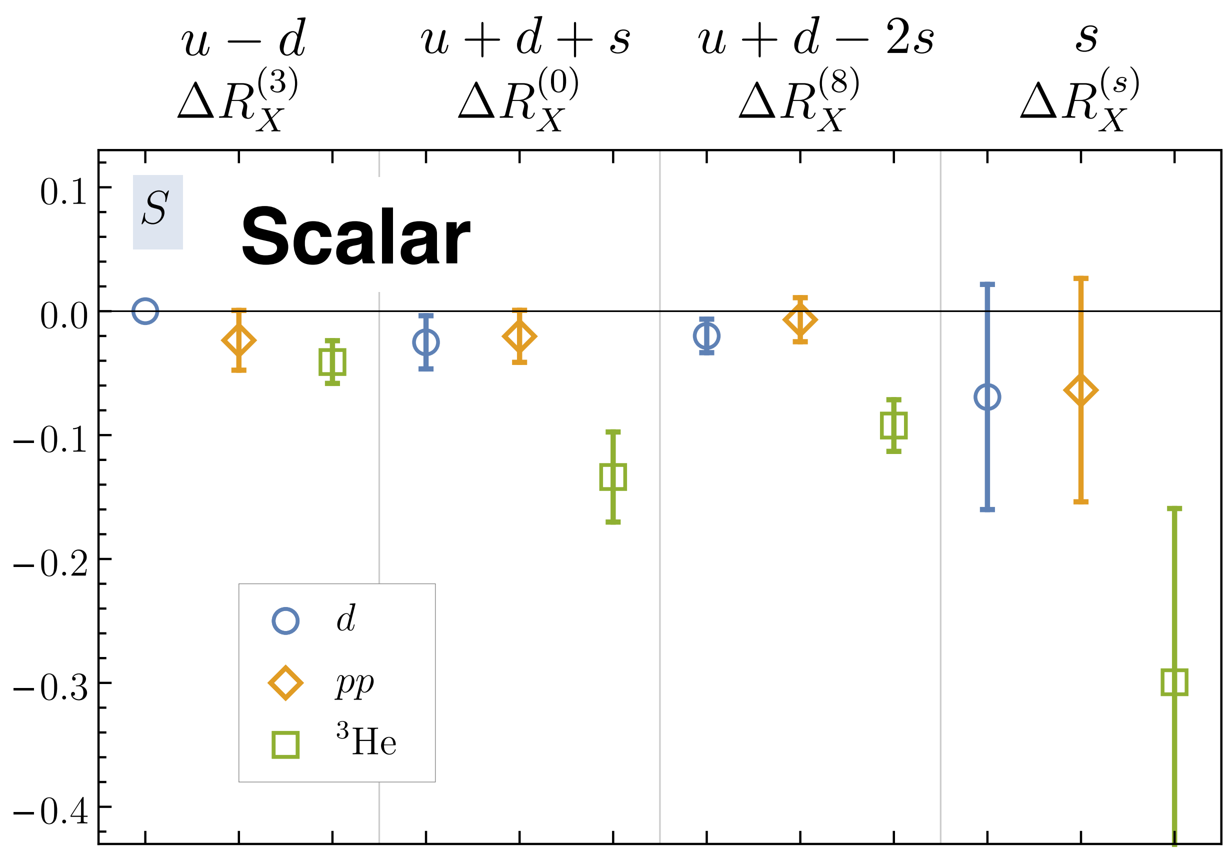
NPLQCD Phys. Rev. Lett. 120 (2018), 152002

Nucleon EMFFs

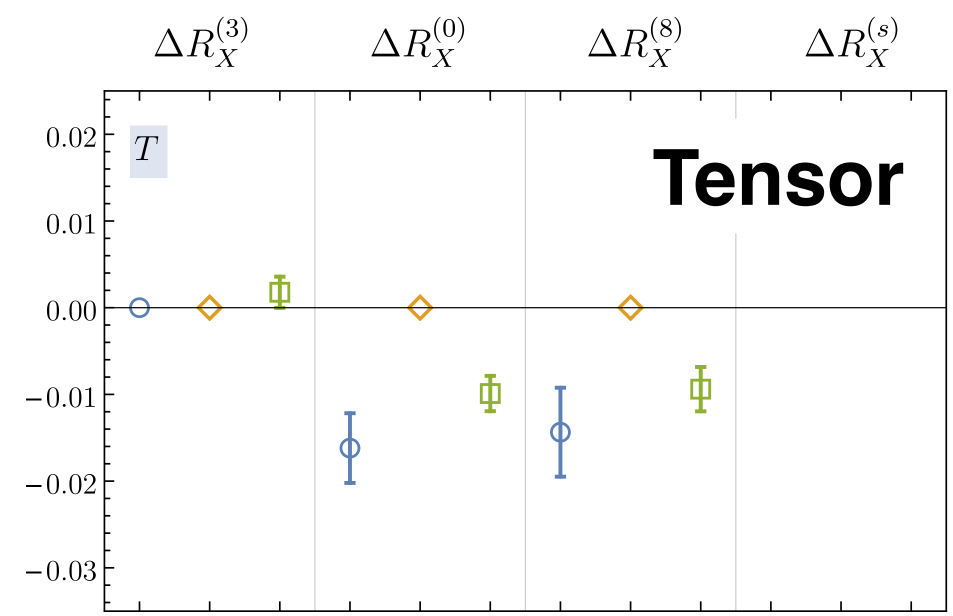
Alexandrou et al., arXiv:1706.00469

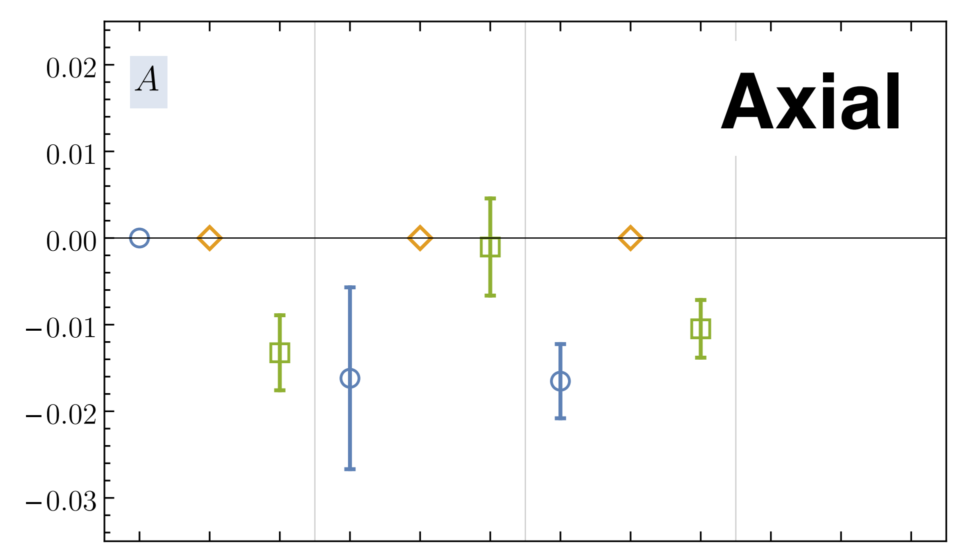


Scalar & tensor nuclear MEs



$$\frac{\textbf{ME}}{\textbf{Nucleon ME}} - \textbf{naive expectation}$$





- Naive expectation determined by baryon#, isospin, spin
- O(10%) nuclear effects in the scalar charges
- Nuclear modifications scale with magnitude of corresponding charge (i.e., baryon# for scalar, spin for tensor, axial)

NPLQCD Phys. Rev. Lett. 120 (2018), 152002

Second order weak interactions

Differentiate terms by their distinct time-dependences in correlation function

Expand contributions to the correlation function

$$
\begin{aligned}
a^2 C_{nn\to pp}(t) = 2 Z_{pp} Z^\dagger_{nn} e^{-E_{nn}t} \Bigg\{ & \left[\frac{e^{\Delta t}-1}{\Delta^2} - \frac{t}{\Delta}\right] \langle pp|\tilde{J}_3^+|d\rangle\langle d|\tilde{J}_3^+|nn\rangle \\
& + \sum_{\mathfrak{l}'\neq d} \left[\frac{t}{\delta_{\mathfrak{l}'}} - \frac{1}{\delta_{\mathfrak{l}'}^2}\right] \langle pp|\tilde{J}_3^+|\mathfrak{l}'\rangle\langle \mathfrak{l}'|\tilde{J}_3^+|nn\rangle \\
& + \sum_{\mathfrak{n}\neq nn,pp} \left[\frac{e^{\Delta t}}{\Delta(\Delta+\delta_\mathfrak{n})} - \frac{1}{\Delta\delta_\mathfrak{n}}\right] \left(\frac{Z_\mathfrak{n}}{Z_{pp}} \langle \mathfrak{n}|\tilde{J}_3^+|d\rangle\langle d|\tilde{J}_3^+|nn\rangle + \frac{Z^\dagger_\mathfrak{n}}{Z^\dagger_{nn}} \langle pp|\tilde{J}_3^+|d\rangle\langle d|\tilde{J}_3^+|\mathfrak{n}\rangle\right) \\
& + \sum_{\mathfrak{n}\neq nn,pp} \sum_{\mathfrak{l}'\neq d} \frac{1}{\delta_{\mathfrak{l}'}\delta_\mathfrak{n}} \left(\frac{Z_\mathfrak{n}}{Z_{pp}} \langle \mathfrak{n}|\tilde{J}_3^+|\mathfrak{l}'\rangle\langle \mathfrak{l}'|\tilde{J}_3^+|nn\rangle + \frac{Z^\dagger_\mathfrak{n}}{Z^\dagger_{nn}} \langle pp|\tilde{J}_3^+|\mathfrak{l}'\rangle\langle \mathfrak{l}'|\tilde{J}_3^+|\mathfrak{n}\rangle\right) \\
& + \sum_{\mathfrak{n},\mathfrak{m}\neq nn,pp} \frac{e^{\Delta t}}{(\Delta+\delta_\mathfrak{n})(\Delta+\delta_\mathfrak{m})} \frac{Z_\mathfrak{n}}{Z_{pp}} \frac{Z^\dagger_\mathfrak{m}}{Z^\dagger_{nn}} \langle \mathfrak{n}|\tilde{J}_3^+|d\rangle\langle d|\tilde{J}_3^+|\mathfrak{m}\rangle + \mathcal{O}(e^{-\delta t}, e^{-\delta' t}) \Bigg\}.
\end{aligned}
$$

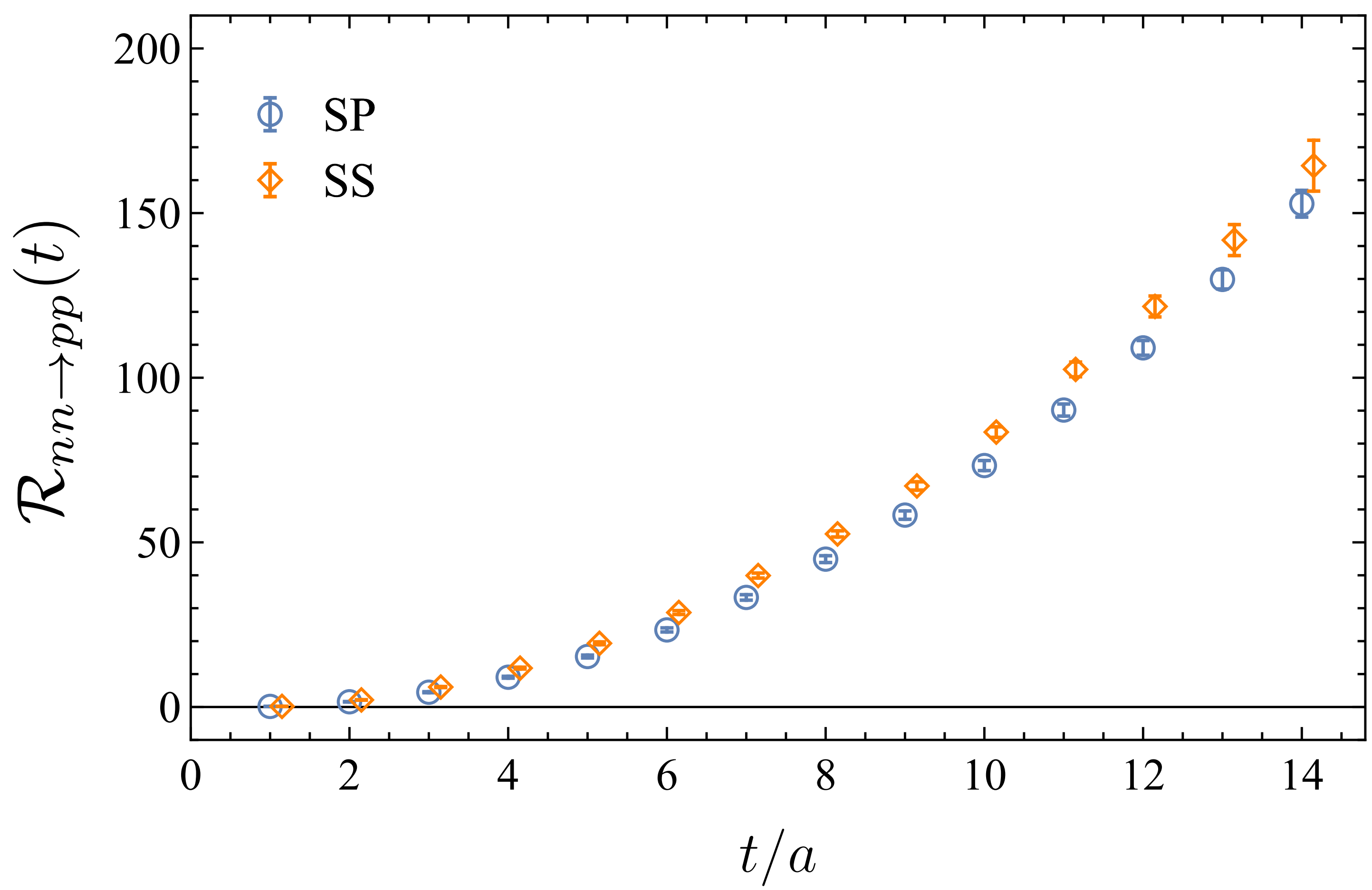
$$
\begin{aligned}
\mathcal{R}_{nn\to pp}(t) = \frac{C_{nn\to pp}(t)}{2C^{(nn)}_{0;0}(t)} = \left[-t + \frac{e^{\Delta t}-1}{\Delta}\right] \frac{\langle pp|\tilde{J}_3^+|d\rangle\langle d|\tilde{J}_3^+|nn\rangle}{\Delta} \; + \; t \sum_{\mathfrak{l}'\neq d} \frac{\langle pp|\tilde{J}_3^+|\mathfrak{l}'\rangle\langle \mathfrak{l}'|\tilde{J}_3^+|nn\rangle}{\delta_{\mathfrak{l}'}} \\
+ \; C \; + \; D\, e^{\Delta t} + \mathcal{O}(e^{-\delta t}, e^{-\delta' t})
\end{aligned}
$$

C,D irrelevant constants

Second order weak interactions

Challenging!

- Correlation function ratio clearly dominated by exponential
- BUT: Deuteron contribution well-determined by calculations with single axial current insertions



Second order weak interactions

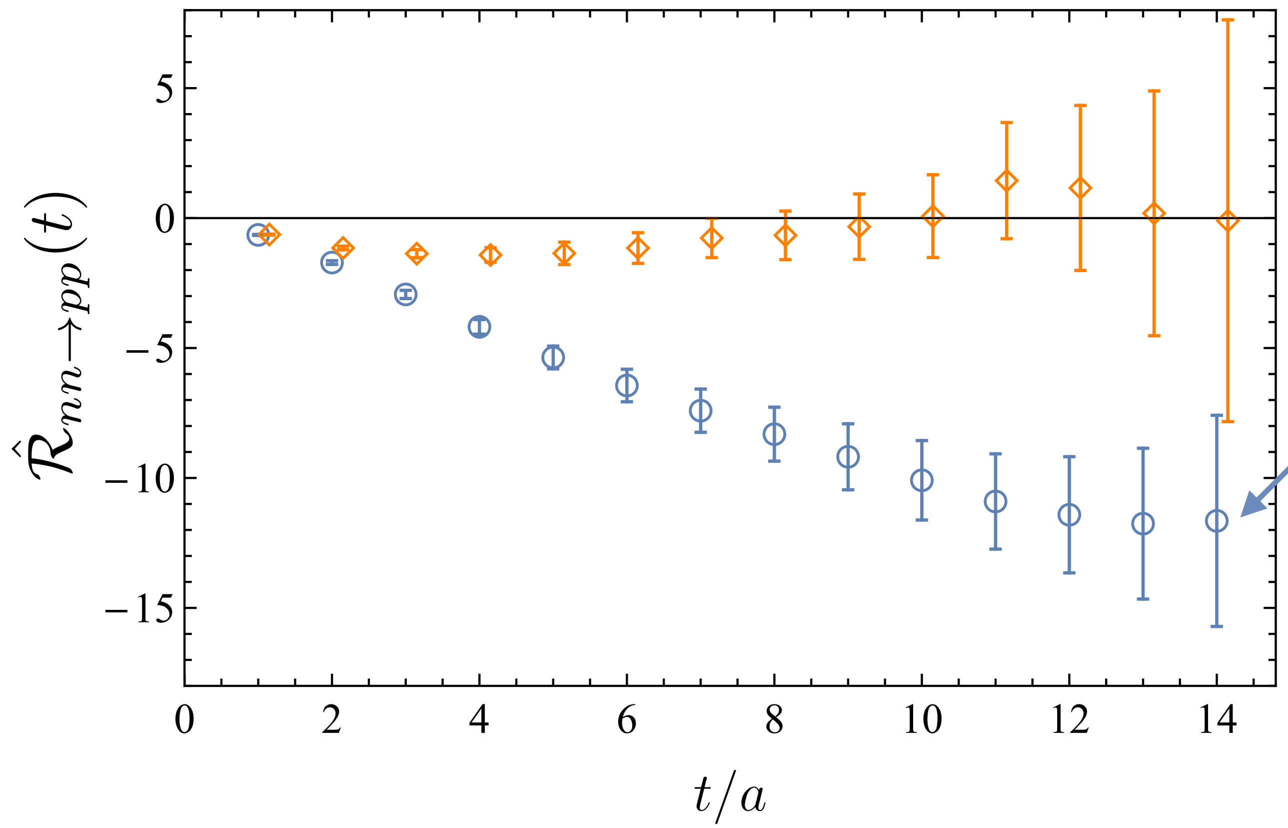
Subtract deuteron pole term determined from (correlated) single-insertion calculations i.e., bootstrap-level subtraction

$$\hat{\mathcal{R}}_{nn\to pp}(t) = \mathcal{R}_{nn\to pp}(t) - \frac{|\langle pp|\tilde{J}_3^+|d\rangle|^2}{a\Delta}\left[-\frac{t}{a} + \frac{e^{\Delta t}-1}{a\Delta}\right]$$

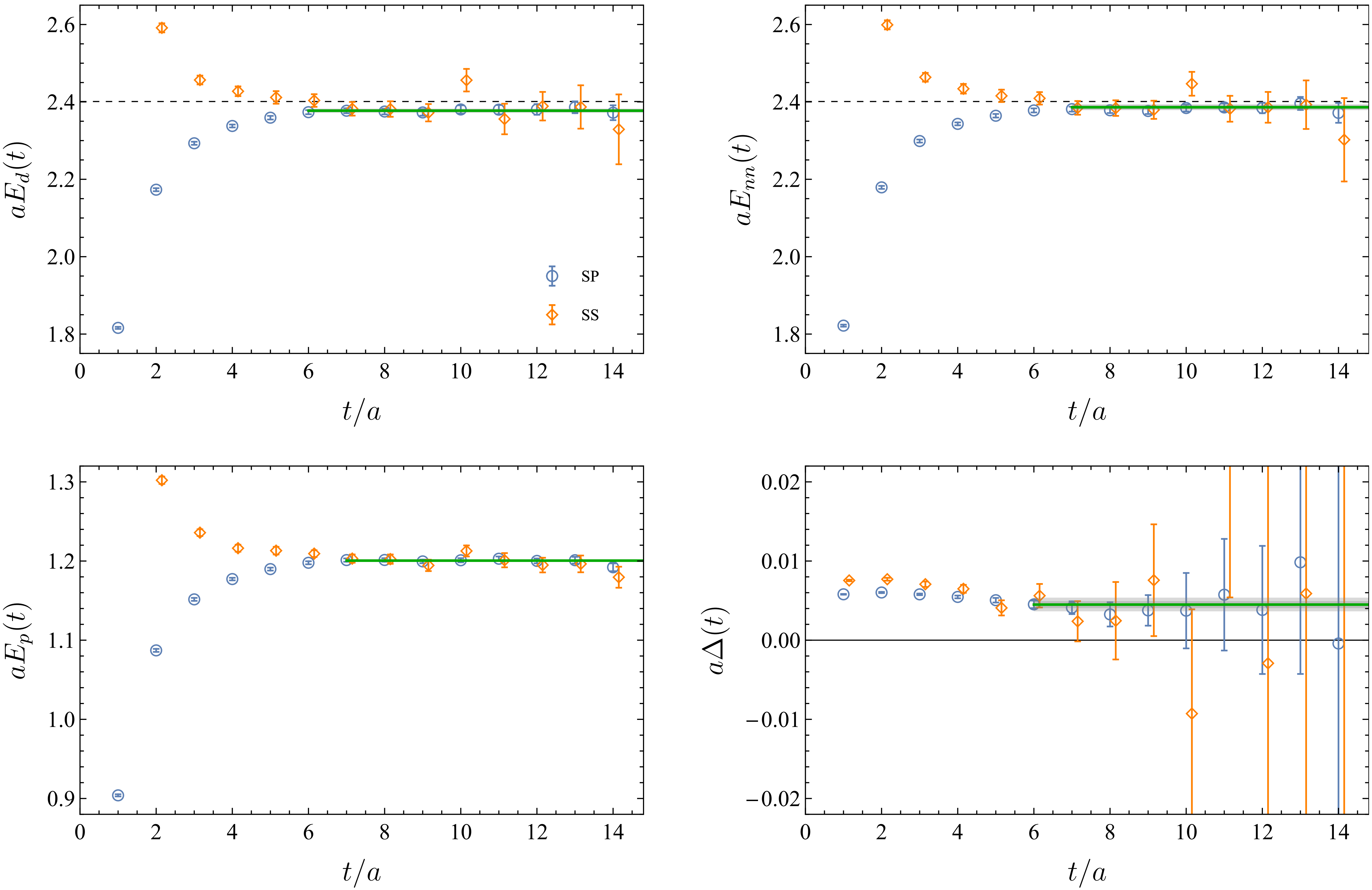
$$= \frac{t}{a}\sum_{\mathfrak{l}'\neq d}\frac{\langle pp|\tilde{J}_3^+|\mathfrak{l}'\rangle\langle\mathfrak{l}'|\tilde{J}_3^+|nn\rangle}{a\delta_{\mathfrak{l}'}} + c + d\ e^{\Delta t}.$$

c,d irrelevant constants

$\Delta = E_{nn} - E_d$ determined from ratios of two-point functions



Linear in t:
SP sink has a highly suppressed overlap onto the nn scattering states

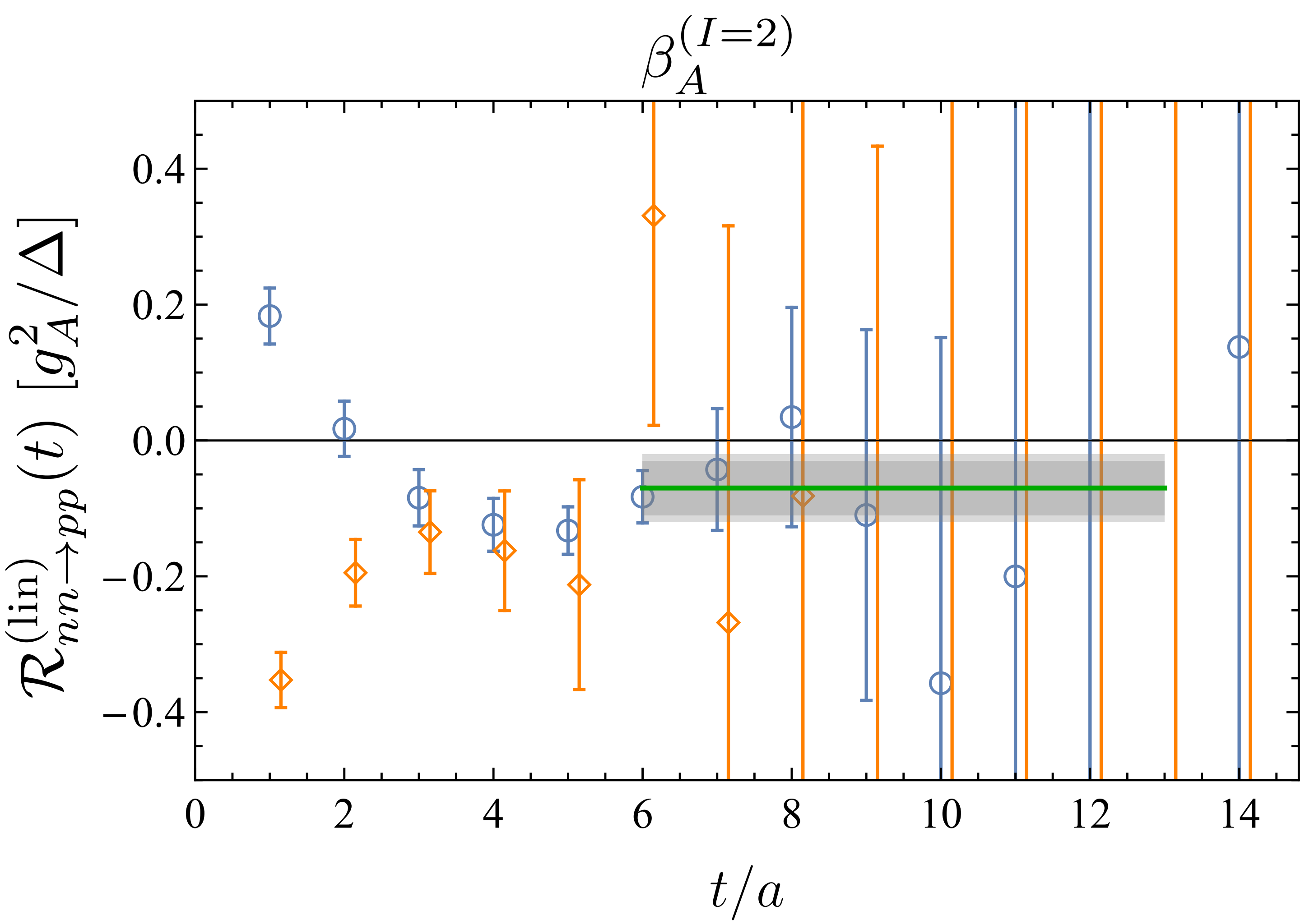


$aE_d(t)$
t/a
SP
SS
$aE_{nn}(t)$
t/a
$aE_p(t)$
t/a
$a\Delta(t)$
t/a

Second order weak interactions

Form ratios to extract **isotensor axial polarisability**

$$\mathcal{R}^{(\text{lin})}_{nn\to pp}(t) = \frac{(e^{a\Delta}+1)\hat{\mathcal{R}}_{nn\to pp}(t+a) - \hat{\mathcal{R}}_{nn\to pp}(t+2a) - e^{a\Delta}\hat{\mathcal{R}}_{nn\to pp}(t)}{e^{a\Delta}-1} \xrightarrow{t\to\infty} \frac{1}{aZ_A^2}\frac{\beta_A^{(2)}}{6}$$



Bootstrap-level combination with

$\Delta = E_{nn} - E_d$

determined from ratios of two-point functions

Matrix element fitting

Treatment of uncertainties: MEs at $m_\pi \sim$ 800MeV

- Statistical

  bootstrap/jackknife over configs.

  correlated ratios of correlation functions

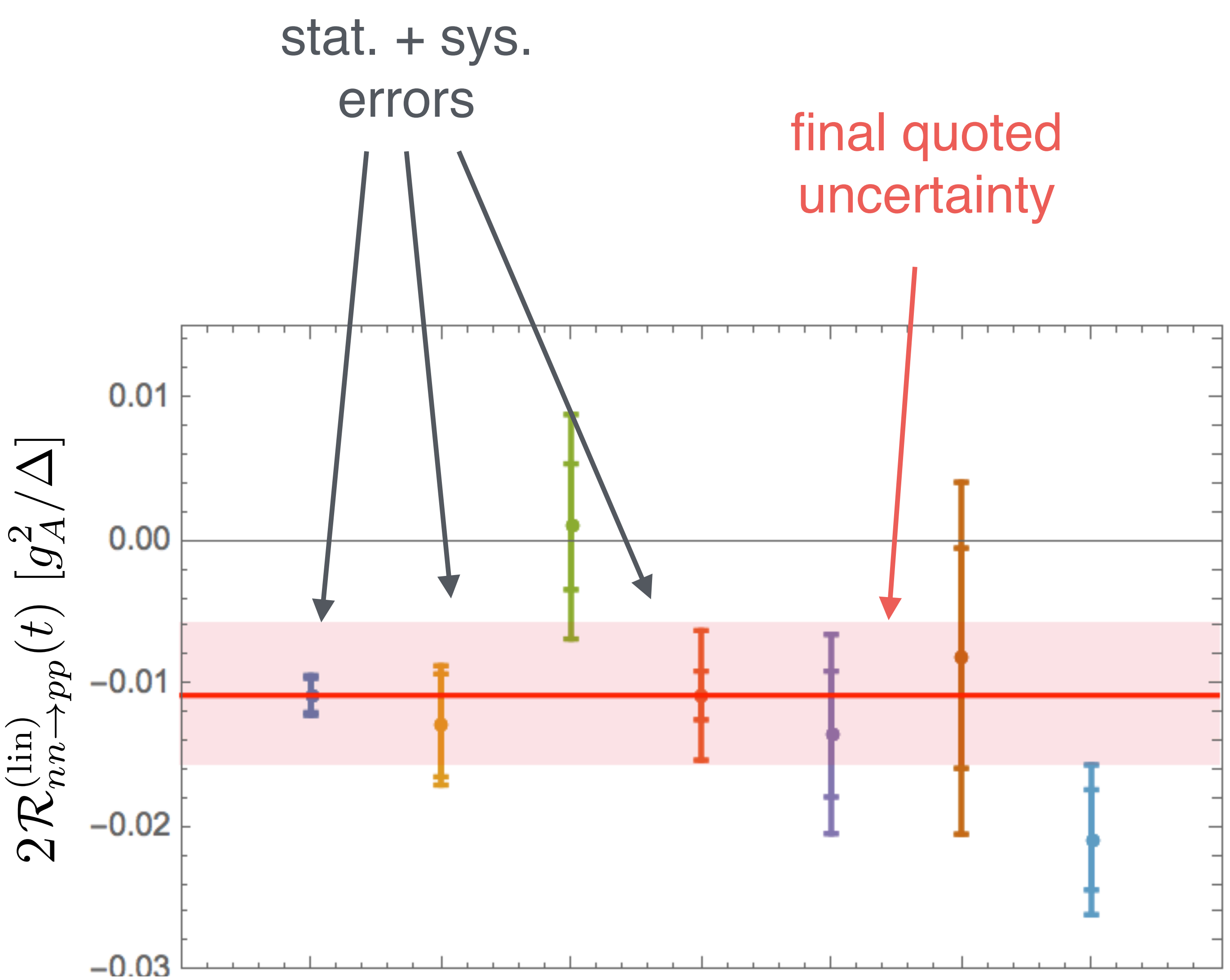
- Systematics in fit

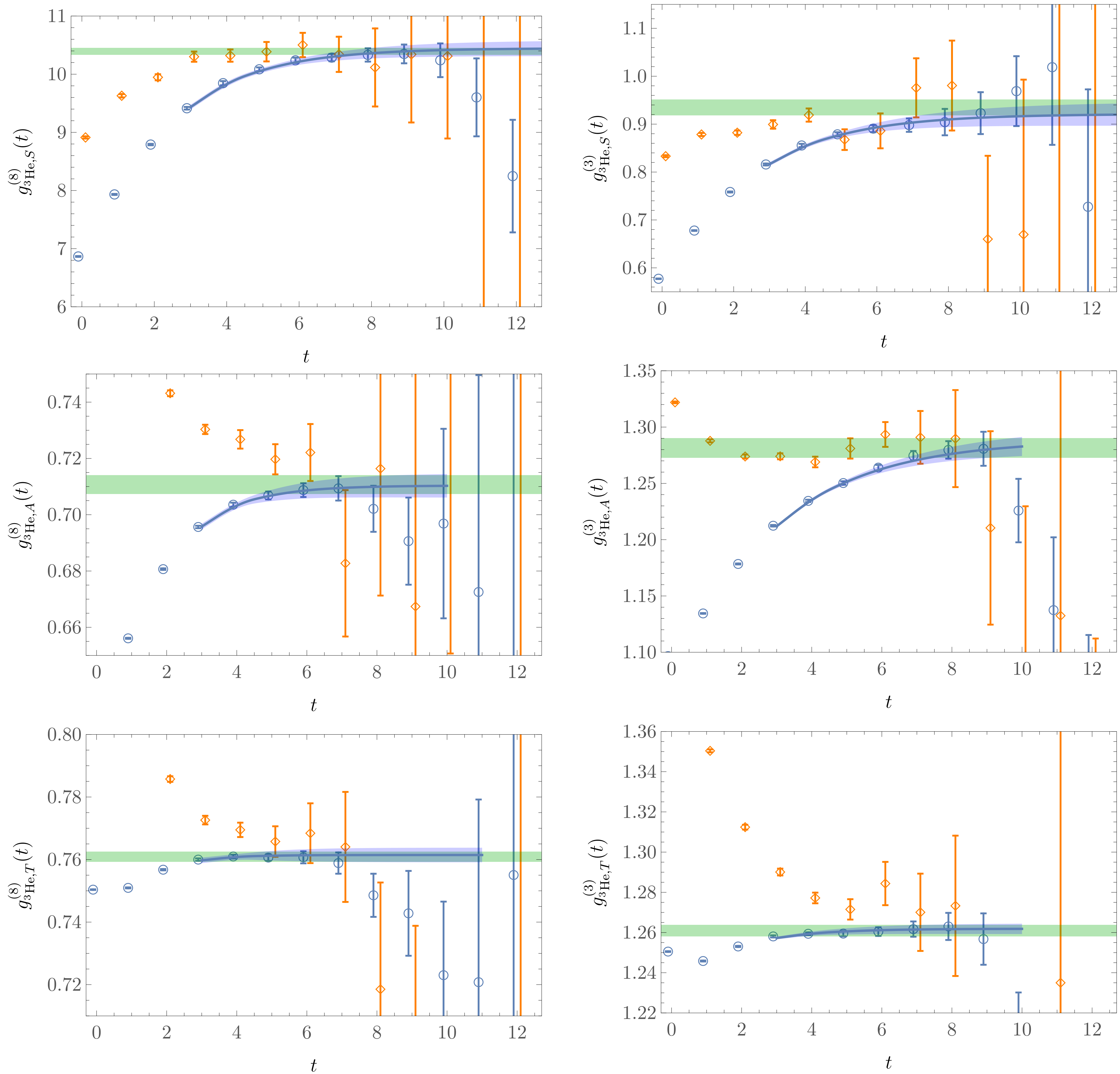
  Take every 'reasonable' fit with $\chi^2/d.o.f \leq 1$

  - Fit time begins after 2pt function is consistent with appropriate (1 or 2 state) form
  - Minimum 5 timeslices
  - Extending fits to later times gives consistent results
  - Variation over all central values taken as systematic uncertainty on result

- Systematic in analysis method

  Variation over range of analysis procedures, performed independently by different collaboration members, taken as additional systematic uncertainty

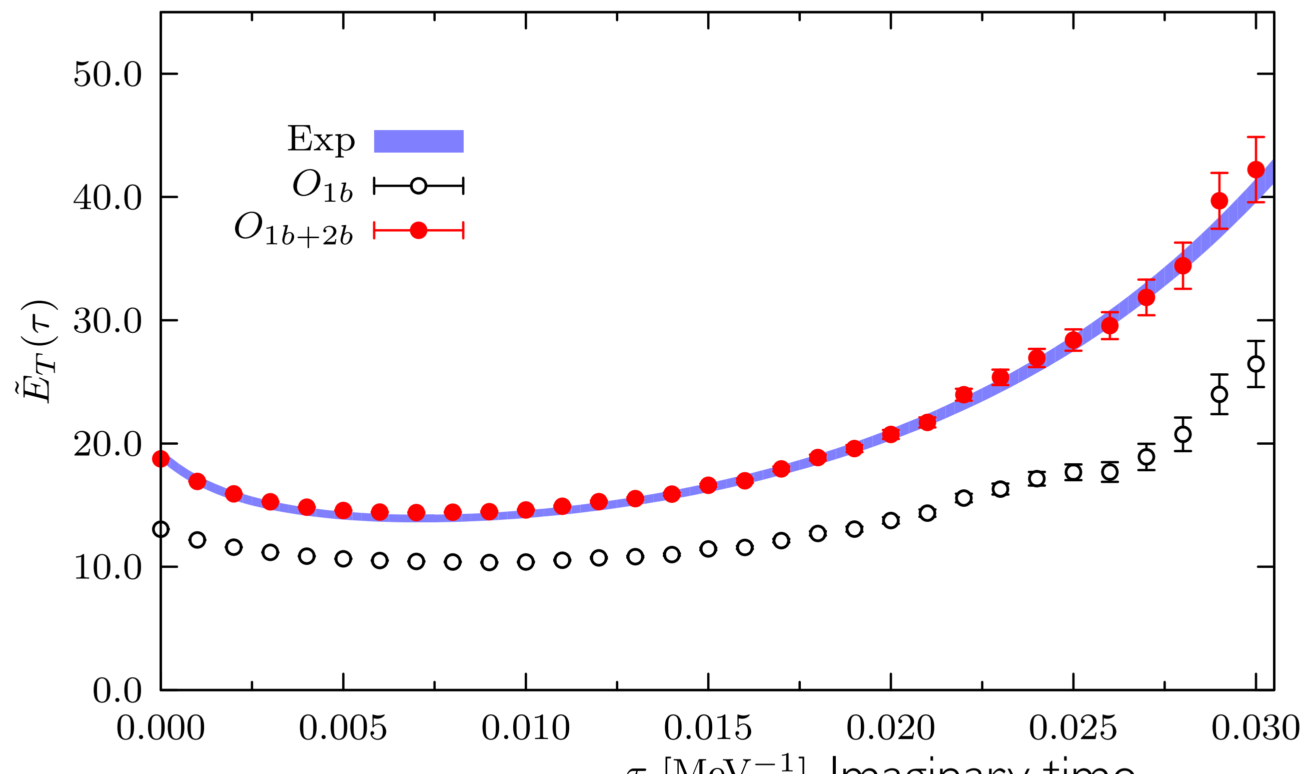




$g^{(8)}_{^3\mathrm{He},S}(t)$
$g^{(3)}_{^3\mathrm{He},S}(t)$
$g^{(8)}_{^3\mathrm{He},A}(t)$
$g^{(3)}_{^3\mathrm{He},A}(t)$
$g^{(8)}_{^3\mathrm{He},T}(t)$
$g^{(3)}_{^3\mathrm{He},T}(t)$
t

Two-body effects

- EM transverse response function shows important two-body effects: ^{12}C at q = 570 MeV



Imaginary time

Lovato et al., Phys. Rev. C 91, 062501 (2015)

Ab-initio calculation
Two- and three-body forces and external electroweak probes via one- and two-body currents

- Expect to be similarly important for axial