Lattice QCD input for neutrino physics

and also

new results for scalar matrix elements in light nuclei

Massachusetts lechnology

Phiala Shanahan

Long-baseline neutrino experiments Because all modern experiments use nuclear targets, such as H2O, CH*ⁿ* and ⁴⁰Ar, the

Neutrinos produced as secondary decay products of hadrons from primary reactions of protons with nuclei description of the final state of the neutrino-nucleus interaction. To get a sense for the accuracy needed for the energy reconstruction in our the energy reconstruction in oscillation experience in our the energy reconstruction in our the energy reconstruction in our the energy reconstruc

DENE as a function energy must be reconstructed event-by-event from the final state of the reaction iments, it is helpful to look at Fig. 1. The figure shows the expected oscillation signal for expected oscillatio

DUNE Need energy reconstruction to better than 100 MeV

Robust understanding of both nucleon and nuclear level amplitudes essential

Adams C, et al. arXiv:1307.7335

Constraining v -nucleus interactions

- **For LBNEs neutrino energy** distributions peak at 1-10 GeV
- Challenging region: several processes contribute
	- Quasielastic lepton scattering
	- Inelastic continuum / shallowinelastic region
	- Resonances
- Lattice QCD can provide direct non-perturbative QCD predictions of nucleon and nuclear matrix elements

Neutrino charged-current cross-section

$LQCD$ input for ν -nucleus interactions

- 1. Directly access QCD single-nucleon form factors without nuclear corrections σ il i σ ilil **ONS**
	- Reliable calculations with fully-controlled uncertainties ILI OIICU

-
- Calculate matrix elements in light nuclei **2.** from first principles
	- EFT to reach heavy nuclear targets relevant to experiment
	- First calculations of axial charge of light nuclei

Constraining v -nucleus interactions

Neutrino charged-current cross-section

Quasi-elastic scattering

Cross-section for quasi-elastic neutrino-nucleon scattering where $(x_i)_{i\in\mathbb{N}}$ is the fermion scattering, (see u) α , and m is the lepton is the lept

$$
\frac{d\sigma}{dQ^2} = \frac{G_f^2 M^2 \cos^2 \theta_c}{8\pi E_v^2} \left[A \mp \frac{(s-u)}{M^2} B + \frac{(s-u)^2}{M^4} C \right]
$$

$$
A = \frac{(m^{2} + Q^{2})}{M^{2}}[(1 + \tau)G_{A}^{2} - (1 - \tau)F_{1}^{2} + \tau(1 - \tau)F_{2}^{2} + 4\tau F_{1}F_{2}
$$

\n
$$
- \frac{m^{2}}{4M^{2}}\left((F_{1} + F_{2})^{2} + (G_{A} + 2G_{P})^{2} - \left(\frac{Q^{2}}{M^{2}} + 4\right)G_{P}^{2}\right)
$$

\n
$$
B = \frac{Q^{2}}{M^{2}}G_{A}(F_{1} + F_{2})
$$

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$$
C = \frac{1}{4}(G_{A}^{2} + F_{1}^{2} + \tau F_{2}^{2})
$$

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$$
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$$

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Well-determined from electron scattering expts can be related to $\,G_A$ by pion pole dominance *F*1*,*² *G^P* $\begin{array}{cc} \texttt{t}, \texttt{t} \texttt{t} \texttt{t} & \texttt{t} \texttt{t} & \texttt{t} \texttt{t} \texttt{t} & \texttt{t} \texttt{t} & \texttt{t} \texttt{t} \texttt{t} & \texttt{t} \texttt{t} \texttt{t} & \texttt{t} \texttt{$ σ_I can be related to σ_A by pion pole dominance

QE, ν_{μ} , $\Delta(d\sigma/dQ^2)$ [%] for 1% Change in FF, $M_A=1$

Axial form factor \blacksquare could be determined from the neutrino scattering, thus leaving the neutrino experiments to neutrino experiments

Traditionally assumed to have dipole form: Traditionally assumed to have dipole form:

measure the axial٬vector form factor form factor form factor of the nucleon. Traditionally, the axial sector for

 p_A^{\prime} $G_{A}(Q^{2}) =$ *gA* $\left(1+Q^2/M_A^2\right)^2$

- $g_A = 1.2671$ determined with high to the axial to the axial extendent of the axial precision from nuclear beta decay
- axial mass M_A must be determined experimentally α which mass M_A must be determined $\begin{bmatrix} 1 & 2 \end{bmatrix}$

BUT

• Electromagnetic FFs show significant and $\frac{0.4}{0.2}$ deviation from dipole parametrisation form $\frac{1}{\sqrt{2}}$

More general alternatives \Box for \Box and and matter \Box

- Model-indep z-expansion Fig. 2. Charged-current quasi-elastic cross section for *quasi-elastic cross section* for μ neutrons. The experimental outrons. The experimental outrons. The experimental outrons. The experimental outrons. The experimen conclusions were larger on deutering measurements on deutering measurements on deutering measurements on deuter
Hill & Paz (2010), Bhattacharya (2011)
- Direct LQCD results and other heavier targets also contributed (see Table 1). This is not recently and the contribution of the contribution error bars are clearly much larger than the uncertainties due to using di↵erent values for *M*A; the

 $\lceil \mathbf{w}_A \rceil$ Total QE cross-section sensitive to the axial mass:

(*p, D*) were done approximately 35 years ago with relatively weak neutrino currents. The

Mosel, Ann. Rev. Nucl. Part. Sci. 66, 171 (2016)

Nucleon Axial FFs from LQCD

- $g_A = G_A(Q^2 = 0)$ is a historically difficult calculation
- Recent calculations in agreement with experiment with fully-controlled uncertainties ncertainties **b**¹
- *Q*²-dependence well-determined in LQCD competitive with experiment
- z-parameterisations remove assumption of dipole form parameterisations remove assumption c

Nucleon Axial FFs from LQCD

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0.8

- $\bullet \; Q^2$ -dependence well-determined in LQCD competitive with experiment FIG. 9. The 8-point fit using Eq. (23) without the finite volume correction (*c*⁴ = 0) to the data for the axial radius h*rA*i. The
- z-parameterisations remove assumption of dipole form paon or app

Alexandrou et al., arXiv:1705.03399, Phys.Rev. D96 (2017), 054507

. D96 (2017), 054507 Gupta et al., arXiv:1705.06834, Phys.Rev. D96 (2017), 114503

0.8

Nucleon Axial FFs from LQCD *T/a* = 10 Summation *JULICUII FIXIAI I I S II (*

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Nucleon Axial FFs from LQCD

 T inner error band for the inner error band for the fit show statistical uncertainties, whereas the fit show statistical uncertainties, whereas the fit show statistical uncertainties, whereas the fit show statistical un Strange quark contributions determined separately and can corresponding form factors from factors from the renormalized and disconnected and disconnected and disconnect
The renormalized and disconnected and disconnected diagrams are also shown. The renormalized and disconnected be isolated

Green et al., Phys. Rev. D 95, 114502 (2017) Shapes and the set of the set of the set of the set of the set of

the mixical point strange augriculations of and disconnected light-guarant and form factors, including *according* Also physical politic strain Also physical-point strange quark axial charge: Gupta et al., EPJ Web Conf. 175 (2018) 06029, show put of the statistical and shows the systematic uncertainties. In addition, for the light is the light is
In a factor, the light is set of the light is set o corresponding from the renormalized connected and disconnected and disconnected and disconnected and disconnect Form factors Alexandrou et al., arXiv:1705.03399, Phys.Rev. D96 (2017), 054507

Nucleon pseudoscalar FF \blacksquare M $\overline{}$ Ideesale FE a06m310 a06m220 a06m135 current interacts with the nucleon with strength *gA*. The middle panel shows one of the lowest order Feynman diagrams that contributes to *GA*(*Q*²), and provides the basis for the dipole ansatz. The diagram on the right is the leading contribution to the induced pseudoscalar form factor *G*˜*^P* (*Q*²) that is mediated by a pion intermediate state. Its coupling to the nucleon at the

µ

First calculations with controlled *z z zmax zmax convertainties* value of the constant *t*⁰ is typically chosen to be in First calculations with controlled the *z*-expansion. The *z*-expansion of α -expansion. The *z*-expansion of α

Alexandrou et al., arXiv:1705.03399, Phys.Rev. D96 (2017), 054507 k is the two parallel but the two physical distribution k (k). k , k is the two physical distribution k

dominance ansatz at low- Q^{-} given in Eq. (6), but consistent with 0*.*51(6) fm obtained Deviations from pion-pole dominance ansatz at low- Q^2 cally analyzed assuming the pion-pole dominance ansatz:

$$
\tilde{G}_P(Q^2)=G_A(Q^2)\left[\frac{4M_N^2}{Q^2+M_\pi^2}\right]
$$

Gupta et al., arXIV:1705.06654, Phys.Rev. D96 (2017), 114505 *P* $(2017, 111, 200$ Gupta et al., arXiv:1705.06834, Phys.Rev. D96 (2017), 114503

Quasi-elastic scattering

LQCD input for the quasi-elastic scattering region:

- dependence of nucleon axial form factor *Q*²
	- fully-controlled uncertainties
	- competitive with experiment
	- z parameterisation removes assumption of dipole form
-
- Nucleon pseudo scalar form factor
- fully-controlled uncertainties
- competitive with experiment
- deviations from pion-pole ansatz observed

Constraining v -nucleus interactions

Neutrino charged-current cross-section

Resonance region

- Energies above ~200 MeV, inelastic excitations from pion production
- Dominant contribution from Δ resonance
- N^* 's also important at high E_v
- Very difficult to access experimentally Constrained only from PCAC
- QCD calculations possible
- Need to account for unstable nature of resonance: extract N→Nπ transition FFs

Resonance region A. THE ELECTRO-WEAK AND PSEUDO-SCALAR TREIMAN RELATION COMMUNICATION COMMUNICATION COMMUNICATION COMMUNICATION The nucleon to matrix element of the axial vector current is parameterized in terms of σ A. The Electro-weak and Pseudo-scalar transition matrix element IV. AXIAL N TO TRANSITION FOR THE TRANSITION FOR THE TRANSITION FOR THE TRANSITION \mathcal{A} **GRAMMAN RELATIONS**

 \blacksquare Lattice QCD calculation of axial N △ transition form factor: form factor: The nucleon to matrix element of the axial vector current is parameterized in terms of α d iduon of axidi in Δ transition form facto T nucleon to T axial vector current is parameterized in terms of α axial vector current in terms of four A dilic $\epsilon \gtrsim$ CD calculation of axial in Δ tha

 \lceil but formalism exists: Lellouch-Lüscher hep-lat/0003023 \subset AN/ \subset AT, \subset are shown as function of \subset **CAVEAT:** COMPLEXILIE **CAVEAT:** Complexities at physical point with unstable resonances,

Constraining v -nucleus interactions

Neutrino charged-current cross-section

Shallow inelastic region

- In inelastic regime, quark PDFs of the nucleon control scattering cross-section
- In shallow inelastic region, both resonances and DIS are important
- Multi-meson channels may become important
- Nuclear effects are different in νA vs. eA (MINERνA)
- DIS structure functions accessible in lattice QCD
	- low moments of structure functions controlled

$$
M_n = \int_{-1}^1 x^n f(x) dx, \quad n \lessapprox 4
$$

x-dependence difficult but promising

Nucleon PDFs due to be to exclusive to exclusive the state state in the state state state state state state state state sta 1 ²⌃ *J L* h*x*i

 $u \in \mathbb{R}^3$, $u \in \mathbb{R}^3$

Lattice QCD typically calculates low moments of PDFS \mathcal{L} , typically calculates low moments of PDFS $\,$

- Can separate and isolate contributions from $t \rightarrow t$
	- **Strangeness**
	- Charge symmetry violation show schematically the various contributions to the spin symmetry violation.
	- Gluons

e.g., all terms of nucleon momentum decomposition calculated with controlled systematics

C. Alexandrou et al., arXiv:1706.02973

Nucleon PDFs

- First calculations of x-dependence of nucleon PDFs to plan in plan in proved calculations with the second calculation in the second with the seco JEHUEHCE OF HUCIEVITT LIS
- Rapid progress, but many systematics to be controlled can proceed with similar analyses for the less known porespected as to be controlled as a systematics to be controlled
- Will not improve on experimental constraints in near future \mathcal{G} given by polarized PDFs), where the transversely polarized PDFs (\mathcal{G} and \mathcal ierimental constraints in near iuture

Resonance region

LQCD input for the resonance region:

- First calculations of axial transition form factors
	- resonances difficult for lattice QCD
	- currently: uncontrolled systematic uncertainties, unphysical values of quark masses
	- formalism in place to move to physical case

LQCD input for the inelastic scattering region:

Much recent progress, but challenging region for direct input to neutrino program

Nuclear effects

- Targets are nuclei $(C, Fe, Ar, Pb, H₂O)$ so how relevant are nucleon FFs, PDFs?
	- EMC effect
	- Quenching of g_A in GT transitions
- Experimental investigations: MINERνA

Calculate matrix elements in light nuclei from first principles

 EFT to reach heavy nuclear targets relevant to experiment First calculations of axial charge of light nuclei

Nuclear effects

- Gamow-Teller transitions in nuclei are a stark example of problems
- Well-measured
- Best nuclear structure calculations $\frac{1}{2}$ ^{0.10'}
are systematically off by 20–30% $\frac{1}{2}$ ⁰ are systematically off by 20–30%
	- Large range of nuclei (30<A<60) where spectrum is well described
	- QRPA, shell-model,...
	- Correct for it by "quenching" axial charge in nuclei ...

Nuclear physics from LQCD

Nuclei on the lattice

- Calculations of matrix elements of currents in light nuclei just beginning
- Deeply bound nuclei: use the same techniques as for single hadron matrix elements
- Near threshold states: need to be careful with volume effects

Nuclear physics from LQCD

Nuclei on the lattice

Hard problem

● Noise: Statistical uncertainty grows exponentially with number of nucleons

Complexity: Number of contractions grows factorially

Unphysical nuclei

NPLQCD collaboration

- Nuclei with A<5
- QCD with unphysical quark masses m_{π} ~800 MeV, $m_{\rm N}$ ~1,600 MeV m_{π} ~450 MeV, m_N~1,200 MeV
- Nuclear structure: magnetic moments, polarisabilities [PRL **113**, 252001 (2014), PRD 92, 114502 (2015)]
- First nuclear reaction: np→dγ [PRL **115**, 132001 (2015)]
- Proton-proton fusion and tritium **β**-decay [PRL **119**, 062002 (2017)] Double β-decay [PRL **119**, 062003 (2017), PRD **96**, 054505 (2017)] Gluon structure of light nuclei [PRD **96** 094512 (2017)] Scalar, axial and tensor MEs [PRL**120**, 152002 (2018)]

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Spectrum of light nuclei

NPLQCD Phys.Rev. D87 (2013), 034506

Background field method

Hadron/nuclear energies are modified by presence of fixed/constant external fields

Example: fixed magnetic field

 $E(\vec{B}) = \sqrt{M^2 + (2n + 1)|Qe\vec{B}|} - |\vec{\mu} \cdot \vec{B}|$ landau level **mag. mmt mag. polarisability** $-2\pi \beta_{M0}|\vec{B}|^2$ ² ²⇡*M*2*TijBiB^j* ⁺ *...*

- Calculations with multiple fields extract coefficients of response e.g., magnetic moments, polarisabilities, …
- Not restricted to simple EM fields Axial MEs: uniform axial background field

Axial background field

Example: fixed magnetic field **notice** moments, polarisabilities

Axial MEs: fixed axial background field \rightarrow axial charges, other matrix elts.

Second order piece: being used for calculations of double-beta decay

Tritium β-decay

Simplest semileptonic weak decay of a nuclear system

- Gamow-Teller (axial current) contribution to decays of nuclei not well-known from theory
- **Understand multi-body contributions** to $\langle \mathbf{GT} \rangle$ \longrightarrow better predictions for decay rates of larger nuclei

We calculate $g_A \langle \mathbf{GT} \rangle = \langle ^3\mathrm{He} | \overline{\mathbf{q}} \gamma_\mathbf{k} \gamma_5 \tau^- \mathbf{q} | ^3\mathrm{H} \rangle$

Tritium β-decay *The GT Matrix Element for Tritium -decay:* The half-life of tritium, *t*1*/*2, is related to the F and GT matrix

Form ratios of compound correlators to cancel leading time-dependence:

$$
\frac{\overline{R}_{^3\mathrm{H}}(t)}{\overline{R}_p(t)} \stackrel{t\rightarrow\infty}{\longrightarrow} \frac{g_A(^3\mathrm{H})}{g_A} = \langle \mathbf{GT} \rangle
$$

"Quenching" of the axial charge emerges from LQCD calculation

Higher-order insertions

- Can access terms with more current insertions from same calculations
- Recall: background field correlation function

Higher-order insertions

- Can access terms with more current insertions from same calculations
- Recall: background field correlation function

Quadratic response from two insertions on different quark lines

Double β-decay

Certain nuclei allow observable ββ decay If neutrinos are massive −55 −60 −65 −70 −75 28 29 30 31 32 33 34 35 36 37 38 39 *Z* Mass excess (MeV) $A = 76$ Zn Ga As Ge Br Kr Se <u>Rb</u> Sr **¹/_{***R***}** between 76Ge and 76Se is energetically forbidden, hence leaving double beta "*ββ*#—pink arrow—as the $massive$ channel. The two mass parabolas exists because of the pairing interaction that \bar{v} of even *Z*—even *N* nuclei with respect to odd *Z*—odd *N* nuclei. For odd *A* nuclei there is a single mass C ertain purclai allows -55 $\sum_{\substack{p=1 \ p \text{ odd}}}^{\infty}$ and $\sum_{\substack{p=1 \ p \text{ odd}}}^{\infty}$ is energy is energy in $\sum_{p=1}^{\infty}$ is energy in $\sum_{p=1}^{\infty}$ allowed ββ October 24, 2012 $\frac{1}{\sqrt{2}}$ $\frac{12000}{1/2} \approx 10^{13} y$
 -75 \mathcal{L}_{max} forbidden or hindered by large *J* difference.

 $neutrinoless $\beta\beta$$

beta decay "0*νββ*#, proposed by Furry \$4%after the Majorana theory of the neutrino \$5%. The

 $V = V$

45

 $> 10^{25}$ y

 $T^{0\nu\beta\beta}_{1/2}$

Majorana fermions 0νββ decay is possible parabola, and all single-beta transitions \mathbb{R}^n are \mathbb{R}^n to \mathbb{R}^n from \mathbb{R}^n

Double β-decay

Want to understand $2\nu\beta\beta$ and $0\nu\beta\beta$ decay from theory

where \mathcal{L}_{max} background rate from the single beta decay. | Calculate two-current even-even-event nuclear pairing force, the nuclear pairing force, the nuclear pairing for are lighter than the original terms of the original terms of the *nuclear matrix elements* and dictate half-life worth noting that, since the 000 candidates are even-best are even-best are even-

0⌫ will be too dicult to be observed due to the over-

even nuclei, it follows immediately that their spin is al-

.
15 Model calculations have large uncertainties

Second order weak interactions

NPLQCD PRL **119**, 062003 (2017), PRD **96**, 054505 (2017)

Lattice QCD: Calculate nn→pp transition matrix element

Second order weak interactions -5 condender is straightforward in the contract of the contract o -5 \bullet

NPLQCD PRL **119**, 062003 (2017), PRD **96**, 054505 (2017) *^R*ˆ(*t*) = *^R*(*t*) *[|]*h*pp|J*⁺ ³ *|d*i*|* $\overline{\mathcal{L}}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ *n*PLQCD PRL 119,062003 (2017), PRD 96,05450 \blacktriangledown, \vee *^e^t* ¹ *t*, PRD **96**, 05 $\left(13\right)$

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Non-negligible deviation from long distance deuteron intermediate state contribution ³ *[|]*lihl*|J*⁺ ³ *|nn*i *E*^l *Enn* ⁺ *^c* ⁺ *d e^t* ⁺ *^O*(*e* ˆ*t*)*,* where *c* and *d* involve complicated combinations of exintermediate state contribution ^h*pp|J*⁺ ³ *[|]*lihl*|J*⁺ ³ *|nn*i *E*^l *Enn* $\frac{1}{2}$ *deviation*)*,* where *c* and *d* involve complicated combinations of exintermediate state contributi-

Isotensor axial polarisability the ground- and first excited- state in either channels and first excited- state in either channels and, and, a \mathfrak{p} the ground- and first excited- state in either channels in either channels in either channels in either channel
State in either channels in either

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$$
M_{GT}^{2\nu} = -\frac{|M_{pp \to d}|^2}{E_{pp} - E_d} + \beta_A^{(I=2)}
$$

TBD: connect to models / effective field theory for larger systems *^R*(full)(*t*) = *^R*(lin)(*t*) *[|]*h*pp|J*⁺ ³ *|d*i*|* 2 *^R*(full)(*t*) = *^R*(lin)(*t*) *[|]*h*pp|J*⁺ $t = \frac{1}{\sqrt{2}}$ \Box i b \Box : connect to mod τ deuteron-pole contribution to give a quantity that τ \Box i b \Box : connect to models /

Larger nuclei

What about larger (phenomenologically-relevant) nuclei?

- Nuclear effective field theory:
	- I-body currents are dominant
	- 2-body currents are sub-leading *but non-negligible*

- Determine one body contributions from single nucleon
- Determine few-body contributions from A=2,3,4...
- Match EFT and many body methods to LQCD to make predictions for larger nuclei

 d round-State Romerties of 4 He and 160 Extrapolated from Lattice MCD with Pleakess EFT :

Summary - Part I

- Lattice efforts have potential to impact ν energy determinations
- Precise determinations with controlled percent-level uncertainties within \sim 5 years

- Axial and pseudoscalar FFs determined with momenta less \bigcirc than a few GeV
- BUT: large momentum FFs (≳3 GeV) more difficult. Novel ideas exist, need testing \bigcirc
- Early results with promising applications
	- Transition FFs \bigodot Formalism exists but developments still necessary for higher states above $N\pi\pi$ inelastic threshold
	- Application of EFT using 2-, 3- body matrix elements to constrain nuclear effects

Dark matter

How do we find dark matter?

- Dark (does not interact with light)
- Interacts through gravity

WIMP Weakly-interacting massive particles

Direct detection Wait for DM to hit us

Detection rate depends on

- Dark matter properties
- Probability for interaction \bigcirc with nucleus

Dark matter direct detection

Dark matter

Determine interaction cross-section (with nucleus) for a given dark matter model

Born approximation – interacts with a single nucleon

$$
\sigma \sim |A| \langle N | DM | N \rangle |^2
$$

known from LQCD

Interacts non-trivially with multiple nucleons

 $\sigma \sim |A\ \langle N|DM|N\rangle + \alpha\ \langle NN|DM|NN\rangle + \ldots |^2$

poorly known!

Scalar matrix elements

- Spin-independent scattering of many WIMP candidates governed by scalar matrix elements
- Lattice QCD calculation with m_{π} ~800 MeV shows 10% nuclear effects! (Naive expectation determined by baryon#, isospin, spin)
- Same calculation gives axial and tensor nuclear effects around \sim 1%

0.02 NPLQCD Phys. Rev. Lett. 120 (2018), 152002

Scalar matrix elements

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0.02 NPLQCD Phys. Rev. Lett. 120 (2018), 152002

Nucleon EMFFs

Alexandrou et al., arXiv:1706.00469

Scalar & tensor nuclear MEs -0.3

- Naive expectation determined by baryon#, isospin, spin 0.22
- O(10%) nuclear effects in the scalar charges 0.00
- Nuclear modifications scale with magnitude of corresponding charge (i.e., baryon# for scalar, spin for tensor, axial) -0.01

0.02 NPLQCD Phys. Rev. Lett. 120 (2018), 152002

 \Box ifferentiates weak correlation function calculated in the ratin system in Ref. (1200 S) \Box Second order weak interactions eigenstates with the quantum numbers of the *pp*, *nn* and deuteron systems, respectively. With *C*_{μ} E_{μ} $-$ *p* $\left(\frac{E_{V} - E_{\mathfrak{n}}}{E_{V} - E_{\mathfrak{n}}} \right)$ $\frac{1}{2}$ *a*2 *x* because the contract of $\left\{\begin{matrix}x_1 \\ y_2\end{matrix}\right\}$ in the contract of $\left\{\begin{matrix}x_1$ eigenstates with the quantum numbers of the pp , $|nn|$ and deuteron systems, respectively. With the example of isospin symmetry and in the absence of electromorpotism, which is the esse for *E*

₀ 1 1 4 \mathcal{F}^{t} each \mathcal{F}^{t} **E**l 0-**1** \mathbb{C} ¹ \mathbb{C} ⁰ **E** \blacktriangleright *,* (31) the assumption of isospin symmetry and in the absence of electromagnetism, which is the case for the calculations presented in this work, the nn and pp states are degenerate. Eq. (31) resembles a is "Dy their distinct ume-dependences" in the absolution of the absence of the case for the case for the magnituditions presented in this work. The *n*₁ and *p* states are degree and *p* states are degenerate. Equal to the state and *p* states are degree and *p* in the state and *p* in the state and *p* in the state and Second tighter weak interactions n*,*m*,*l 0 $E_{\mathfrak{l}'}-E_{\mathfrak{m}}$ $E_{\mathfrak{l}'}-E_{\mathfrak{n}}$ $E_{\mathfrak{n}}-E_{\mathfrak{m}}$ where the summations over r he have been performed as integrals (the analysis is \pm at altered significantly if the discrete summation is used). Here, $\langle n \rangle$ and *l'* energy and content design second-order weak correlation function calculated in the kaon system in Ref. [22]. In order to make the matrix element between ground-state dinucleons explicit, the sums over

י
⁄m *t*

 $Z_n Z_m e \longrightarrow F_n - F$

t

correlates in this correlation fi states in this correlation function are partially expanded, giving

 $\frac{1}{2}$

*a*2

 \mathbf{v}

$$
a^{2}C_{nn \to pp}(t) = 2Z_{pp}Z_{nn}^{\dagger}e^{-E_{nn}t} \left\{ \left[\frac{e^{\Delta t} - 1}{\Delta^{2}} - \frac{t}{\Delta} \right] \langle pp|\tilde{J}_{3}^{+}|d\rangle \langle d|\tilde{J}_{3}^{+}|nn\rangle \right\}
$$
¹²
gompared, with the inverse of the *time*-separation, between the space and the sink used to ex-

^e(*E*l0*E*n)*^t* ¹

 $\begin{array}{ccc} \uparrow & \begin{array}{c} \hline \text{ } & \text{ } & \text{ } \text{ } & \text{ } \text{ } \\ \hline \end{array} \end{array}$

, (31)

12

response in Eq. 2099 swound-state overlap vactors and the overancexpenental unterdependence chocassion and
can be removed by 40rming the catoEq. (34) ch heldeut resugaine ut see, quergy repliq times, is assumed show empoul a^2 *C*_{nn} \Rightarrow pp(*t*)=2*Z*_{pp}*Z*_{nn}e
traditionounces $\left\{\begin{array}{l} \text{the }\text{finite-sا}\ \text{the } \text{a} \text{+} \text{the }$ ^h*pp|J*˜⁺ ³ *[|]d*ih*d|J*˜⁺ ³ *|nn*i $+$ $\frac{1}{2}$ **140** *t* E
delweg \int ^h*pp|J*˜⁺ ³ *|*l 0 ihl 0 *|J*˜+ ³ *|nn*i $\frac{d}{dt}$ inu $\sum_{i=1}^{N}$ n6=*nn,pp z z*
 *z*_{*z*} *z*^{*z*} *z*_{*z*} *z*_{*z*} *z*^{*z*} *z*² *z*² *z*² *z*² *z*² *z* $\frac{1}{2}$ $\frac{1}{2}$ *Z† nn* ^h*pp|J*˜⁺ ³ *[|]d*ih*d|J*˜⁺ ³ *|*ni \mathcal{V}^{\dagger} HITCH n6=*nn,pp* \sum_{1} l ⁰6=*d* \overline{d} tuls
Ol¹On **De R**
Zna *Z***ⁿ¹</u> ***Z*¹^{*i*} *Z*^{*n*}^{*i*} *Z*^{*l*}^{*i*} *<i>Z*^{*l*}^{*i*} *<i>Z*^{*l*}^{*i*} *<i>Z* *Z†* n *Z† nn* ^h*pp|J*˜⁺ ³ *|*l 0 ihl 0 *|J*˜+ ³ *|*ni δ $\frac{\Phi}{\Phi}$ n*,*m6=*nn,pp e^t* $(\mathbb{X}\boxtimes\mathbb{Y})$ $(\mathbb{X}\subseteq\mathbb{Y})$ *Z*n *Zpp Z†* m *Z† nn* $\lim_{\delta \to 0} \frac{\partial f}{\partial x} = \lim_{\delta \to 0} \lim_{\delta \to 0} \lim_{\delta \to 0} \lim_{\delta \to 0} \frac{\partial f}{\partial y}$ ($e^{-\delta t}$, $e^{-\delta' t}$) \mathcal{L} $\frac{02}{1}$ response in Eq.1e(260)er cirsumd-state la priast@astors and the overall the opprevious time investige discussion after $\frac{1}{4}$ 2 compared with χ to arrange of the time separation algority the source and the source of the sink used to extract the matrix elements, while the energy splittings between ground and the initial states in both channel channels are assumed to be set is the distribution of the solution of the set of the set \mathbb{R}^n of \mathbb{R}^n and \mathbb{Z}^n is not \mathbb{Z}^n is set uation, the correlation functions with background-field insertions on all timeslices cannot be used to un- α ambiguously extract the terms relevant for this analysis. The numerical calculations discussed below , the requisite hierarchy is found δ_{O} be satisfied. As the deuteron in energy than the dinucleon external states, and hence gives bise to a growing expression stated contribution (after the $\rm{overall~exponential~} e^{-E_{nn}t}$ is $\rm{factored~out~}$ of \rm{Edd} \rm{de} (32)), \rm{Edd} of \rm{Edd} in the singled out in the \sum_{s} summation over states in Eq. (32). (The deuteron contribution is close to quadratic in t (it would be exactly quadratic if $\Delta = 0$, and the coefficient of this term is known from the pulse-order axial $t_{\rm H}$ ant $\bar{t}_{\rm H}$ the matrix elements, while the Δ hergy splittings between ground and exited states in both channels are assumed to be large, so that $e^{-\delta t} \to 0$ and $e^{-\delta_0 t} \to 0$. If this is not the situation, the correlation functions with background-field insertions on all timeslices cannot be used to un- \widetilde{A} is the \widetilde{A} the terms relevant for this \widetilde{A} palysis.⁷ In the numerical calculations discussed \widetilde{b} the region is \widetilde{b} and \widetilde{b} is \widetilde{b} and \widetilde{b} is \widetilde{b} and \wid the dinucleon external states, and hence gives rise to a growing exponential contribution (after the overall exponential *eEnn^t* is factored out of Eq. (32)), this contribution has been singled out in the summation of the deuteron of the deuteron of the deuteron contribution is close to quadratic in *t* (it would be exactly quadratic if $\Delta \neq 0$), and the coefficient of the constraint of known from the first-order axial response type grovy re Ground-state coeftap factors and the e_qverall exponential time dependence can be removed by containing the participal can be reated as $\begin{pmatrix} \mathcal{L}_{nn}^{in} & C_{nn \rightarrow pp}^{in} (t) \\ \mathcal{L}_{nn}^{in} & C_{nn \rightarrow pp}^{in} (t) \end{pmatrix}$ (33) response in Eche(200)er giroundstater laughter that the coefficient of this term is known hour marginust-order
response in Eche(200)er giroundstater laughter facture and under chain the perspectively that interested discus Expande Contributions, which relation functions with back
The correlation fuisite thierar
Nexternal stat $+\sum$ $l' \neq d$ $\lceil t \rceil$ $\frac{t}{\delta_{\mathfrak{l}^\prime}}-\frac{1}{\delta_{\mathfrak{l}^\prime}^2}$ $\overline{}$ *h* radt[Hilder]

(pp| J³₃ | lett[|] J₃ | *nn*₎

channels are assumed $\frac{\pi}{2}$ $\frac{\pi}{2}$ $\widetilde{\mathfrak{n}}$ \neq $\widetilde{\mathfrak{nn}}, \widetilde{\mathfrak{pp}}$ $\left\{\frac{\text{he energy}}{\text{A0}\text{A0}}\right\}$ *Z*^{*Q*}*n I*² *Z*^{*n*} *I*² *Z*^{*n*} *Z* **Zuweeze**
Jextrekin *Zpp* ^hn*|J*˜⁺ ³ *[|]d*ih*d|J*˜⁺ ³ *|nn*i + **Z**^{*I*}nalysis₁⁷ In the numer
satistic *J*3 *dH(d|J*+|n)
satistic is fighed the deuter $\sum_{n=1}^{\infty}$ + X $\widetilde{\mathfrak{h}} \widetilde{\neq} \widetilde{n}^{\prime} n, \widetilde{p} \widetilde{p}$ $\overline{\textbf{R}^{\text{V}\xi}}$ $\int_{0}^{\infty} \vec{f} \cdot d\vec{g}$ 1 rdi/ d_{ih} $\vec{\gamma}$ *Leg*l
Zzgr *Zpp* **2000 | John Hotel Burne**
0 **1000 | Syder states (10000** 0 - 11
1 **D** G al C al C + 13 b a d Burset O£¢
≩¢a *Z† nn |Q***a ||ations (iscus)
| pp deuterut (2011)
| University en effetted** $\sqrt{\frac{1}{2}}$ $\stackrel{nn^t}{\leftarrow}$ is $\stackrel{\leftarrow}{\mathbf{\mathcal{R}}^{\mathbf{C}}}$ $\overrightarrow{n,m}$ $\overrightarrow{m,n}$ read
fall $({\Delta\hbox{h}\ensuremath{\mathbf{t}}})$ on $\delta\hspace{-1.4mm}\delta\hspace{-1.4mm}\alpha$ *Z*n *Zpp Z†* m *Z† nn* overall exponential $e^{-E_{nn}t}$ is vactored out of hold be $\frac{\partial}{\partial p}$ with $\frac{\partial}{\partial p}$ (h) $\frac{\partial}{\partial p}$ in $\frac{\partial}{\partial p}$ is $\frac{\partial}{\partial p}$ is $\frac{\partial}{\partial p}$ on $\frac{\partial}{\partial p}$ is $\frac{\partial}{\partial p}$ of $\frac{\partial}{\partial p}$ is $\frac{\partial}{\partial p}$ of $\frac{\partial}{\partial p}$ p $\frac{1}{\delta u} + \sum_{\alpha} \frac{1}{\delta u} - \frac{1}{\delta^2} \frac{1$ t_{normal} traction $\sum_{k=0}^{\lfloor t \end{matrix}$ the time setter that between $\frac{1}{2}$ the property of the setter of t channels are assumed to be large to be large, so that the terms relevant for the tract the matrix elements, while $\left\{\frac{\Delta t}{\text{heo}}\right\}$ is not the series relevant for the situation of the situation of the situation, $\frac{\Delta t$ $\text{chann@}\text{Order}$ is what Top $\text{Exp}_\mathcal{A}$ and $\text{Rep}_{\mathcal{A}}$ is $\text{Rep}_{\mathcal{A}}$ in $\text{Rep}_{\mathcal{A}}$. The uni-field $\text{Rep}_{\mathcal{A}}$ the correlation functions with background-field insertions on all times, and negle gives
The correlation functions with background-field insertions on all times and cannot $\begin{equation} \begin{minipage}[c]{0.9\textwidth} \includegraphics[width=0.9\textwidth]{Figs/0.9\textwidth} \includegraphics[width=0.9\textwidth]{Figs/0.9\textwidth} \includegraphics[width=0.9\textwidth]{Figs/0.9\textwidth} \includegraphics[width=0.9\textwidth]{Figs/0.9\textwidth} \includegraphics[width=0.9\textwidth]{Figs/0.9\textwidth} \includegraphics[width=0.9\textwidth]{Figs/0.9\textwidth} \includegraphics[width=0.9\textwidth]{Figs/0.9\textwidth} \includegraphics[width=0.9\textwidth]{Figs/0.9\textwidth} \includegraphics[width=0.9\textwidth]{F$ $t_{\rm H}$ dinucleon external states of $\sum_{i=1}^{n} \sum_{i=1}^{n} \frac{1}{i}$ and $\sum_{i=1}^{n} \sum$ overall exponential *eEnn^t* is factored out of Eq. (32)), this contribution has been singled out in the be exactly quadratic if $\Delta = 0$, and the coefficient of this term is known from the θ compared with the inverse of the effice-separation, between, the source and the sink under $\left\{ \begin{array}{l} 1 \quad 0 \quad \text{where} \quad \mathcal{L}_{n} \neq 0 \quad$ $t\rightarrow \left(\frac{\varepsilon}{\varepsilon}-\frac{1}{\varepsilon^2} \right)$ (page $J_{3,1}^{\text{eff}}$) (figure $J_{3,1}^{\text{eff}}$) $J_{3,1}^{\text{eff}}$) $J_{3,1}^{\text{eff}}$ in all times $J_{3,1}^{\text{eff}}$ in all times $J_{3,1}^{\text{eff}}$ is Ω in $J_{3,1}^{\text{eff}}$ insertions on $J_{3,1}^{\text{eff$ $\alpha = \sqrt{\frac{V}{\tau}} d_c$ or $\alpha = \frac{V}{\tau}$ is channels are assumed to be large, so that $e^{-\tau} \to 0$ and $e^{-\tau} \to 0$. It this is not the θ below, the rate of the separation is found to be satisfied. The same set θ can be seen to be seen to be satisfied. $\lim_{\Delta t \to \infty} \frac{\ln t}{\Delta t}$ $\lim_{\Delta t \to \infty} \frac{\ln t}{\ln t}$ from $\lim_{\Delta t \to \infty} \frac{\ln t}{\ln t}$ for $\lim_{\Delta t \to \infty} \frac{\ln t}{\ln t}$ and $\lim_{\Delta t \to \infty} \frac{\ln t}{\ln t}$ is lower in exponential $\lim_{\Delta t \to \infty} \frac{\ln t}{\ln t}$ is lower in exponential $\lim_{\Delta t \to \$ $\limsup_{n\to\infty} \frac{1}{n}$ is $\limsup_{n\to\infty} \frac{1}{n}$ for $\limsup_{n\to\infty} \frac{1}{n}$ or $\limsup_{n\to\infty} \limsup_{n\to\infty} \frac{1}{n}$ is the $\liminf_{n\to\infty} \frac{1}{n}$ on $\limsup_{n\to\infty} \frac{1}{n}$ is $\liminf_{n\to\infty} \frac{1}{n}$ on $\liminf_{n\to\infty} \frac{1}{n}$ on $\liminf_{n\to\infty} \$ Γ at $\tilde{\Omega}$ $\tilde{\Omega}$ at $\tilde{\Omega}$ overall exponential $e^{-\mu_{n}t}$ is factored out of Eq. (32)), this contribution has been singled $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial$ $\frac{1}{2}$ response in Eq. (26). The exactly duality of $\frac{1}{2}$ of $\frac{1}{2}$ over and the overlap state of $\frac{1}{2}$ and the dependence of $\frac{1}{2}$ and the overall time dependence of $\frac{1}{2}$ and the overall time depen e^{-Emt} is $\frac{1}{2}$ factored out of $\frac{1}{2}$ of $\frac{1}{2}$ $\frac{1}{2}$ *R*_{*nn*} $\frac{1}{2}$ *(s*₂^{*C*}*l*(*c*) = *C_{<i>pp*</sup>} $\frac{1}{2}$ _{*nn*}</sup> + *C_{nn}*</sup> + *C*_{*nn*} + *C*_{*nn*} + *C_{<i>nn*} + *C_{nn}* + *C_* trom*m*
2022ءan $\lim_{n\to pp^{(l)}}$ \int $\lim_{n\to pp^{(l)}}$

$$
\begin{array}{ll}\n\overbrace{\mathcal{R}_{nn}^{\text{Sospin}} \mathcal{N}_{tp}^{\text{th}} \mathbb{H}^{\text{th}} \mathbb{E}(\mathcal{L}_{\lambda_{u};\mathcal{H}_{d}}^{(h)}(t) \text{ and } \mathcal{L}_{\lambda_{u};\mathcal{H}_{d}}^{(h)} \mathbb{E}(\mathcal{L}_{\lambda_{u};\mathcal{H}_{d}}^{(h)}) \text{ with } \mathcal{L}_{\lambda_{u};\mathcal{H}_{d}}^{(h)} \mathbb{E}(\mathcal{L}_{\lambda_{u};\mathcal{H}_{d}}^{(h)} \mathbb{E}(\mathcal{L}_{\lambda_{u};\mathcal{H}_{d}}^{(h)}) \text{ with } \mathcal{L}_{\lambda_{u};\mathcal{H}_{d}}^{(h)} \mathbb{E}(\mathcal{L}_{\lambda_{u};\mathcal{H}_{d}}^{(h)}) \text{ with } \mathcal{L}_{\lambda_{u};\mathcal
$$

 Δ Δ Intermediate states coupling to the axial durrent, i.e., the isotensor axial polarizability as defined in ⁰6=*d* $\frac{1}{2}$ $\frac{1}{2}$ Eq. (4). The coefficients C and D_t are complicated terms involving ground-state and excited-state overlap factors and matrix dependence $S_{\alpha}(\mathcal{E}_{2n})$ be read from $\mathcal{E}_{\alpha}(\mathcal{E}_{3n})$, but have no time dependence. ³ *|nn*i $\frac{1}{2}$ inte
Eq. \int_{0}^{∞} fi Eq. (4). The coefficients C and Q_t are complicated terms involving ground-state and ex

Second order weak interactions this quantity returns a very small value in the *N*^c scaling from α

 $\mathcal{L} = \{ \mathcal{L} \mid \mathcal{L} \in \mathcal{L} \}$ with the SU(4) Wigner-symmetry limit. With this term

Challenging! supporting evidence, it is reasonable to conclude that the contaminating term *c* in Eq. (20) is

Sec. III B 2 that this quantity vanishes as 1*/N*⁴

Correlation function ratio clearly dominated by exponential BUT: Deuteron contribution well-determined by calculations with single axial current insertions Second order weak interactions Fits to both the mass di↵erence, , and to the bare *pp* ! *d* matrix element on each booton function ratio clearly dominated by exponential t *R*
R
R R^{*n*+b cinale axial current incortions.}

required linear and quadratic field-strength dependences of the correlation functions determined, transition matrix element and in the right panel of The right panel of Fig. 4. The late-(lower-left panel), and the quantity = *Enn E^d* (lower-right panel). Blue circles and orange diamonds Γ and spectrum set and SS correlations, respectively. The dashed lines in the upper-particular plots correspond to the mass of the mass of the nucleon. In all figures, the horizontal bands Second order weak interactions \bigcap nce and short-distance and short-distance components. Since the long-distance components. Since the long-distance components. Since the long-distance components. Since the long-distance components. Since the long-dis \bullet and \bullet

Subtract deuteron pole term determined from (correlated) single-insertion calculations i.e., bootstrap-level subtraction moe deuterond subtraction political challenge is to isolate both its long-distance and short-distance components. Since the long-distance single-insertion calculations i.e., bootstrap-level subtractioi Subtract deuteron pole term determined from *^c*) ⇠ *O*(1%) of the dominant term. the combinations of correlation functions derived in Sec. III B. As the first-order responses have been presented in Ref. [18], the primary focus of this work is the second-order axial matrix element challenge is to isolate both its long-distance and short-distance and short-distance components. Since the long-Sec. III B 2 that this quantity vanishes as 1*/N*⁴ Subtract deuteron pole term determ single-insertion calculations i.e. hoots

 $S_{\rm eff}$ interactions are conditions of \sim 10 μ m and μ m and μ m and μ m and μ

Sec. III B 2 that this quantity vanishes as 1*/N*⁴

transition matrix element and is shown in the right panel of \mathcal{A} and \mathcal{A}

elated SP-SS fits to the late-time behavior of the FIG. 5. The ratio *^Rnn*!*pp*(*t*) (left panel) and the subtracted ratio *^R*ˆ*nn*!*pp*(*t*) (right panel) that are con-Fits to both the mass di↵erence, , and to the bare *pp* ! *d* matrix element on each boottum constant correlated SP-SS fits to the late-time behavior of the quantities.

complished straightforwardly by forming the following combination of $\hat{\mathcal{D}}$ at three complished straightforwardly by forming the following combination of *^R*ˆ*nn*!*pp* at three neighboring tim constant correlated SP-SS fits to the late-time behavior of the quantities. denote results determined using SP and SS correlations, respectively. The horizontal bands show \mathbf{v} Fig. 3. E $\frac{1}{2}$ ective-mass plots for the deuteron (upper-right panel), nucleon (upper-right panel), nu

complished straightforwardly by forming the following combination of $\hat{\mathcal{R}}_{nn\to pp}$ at timeslices: $\lim_{\epsilon \to 0} \text{untimes}$ κ_n compussied straightforwardly by forming the following combination of $\kappa_{nn} \rightarrow$ $\sum_{n=1}^{\infty}$ functions, as $\sum_{n=1}^{\infty}$ structed straightforwardly by forming the following combination of $\mathcal{R}_{nn\rightarrow}$ $\frac{\kappa_{nn}}{\kappa_{\text{com}}}$ timeslices: Com \overline{v} $\overline{$ complished straightforwardly by forming the following combination of $\mathcal{R}_{nn\rightarrow nn}$ at the $\frac{R}{m}$ timeslices:
 $\frac{R}{m}$ com $\mathop{\mathrm{Com}}$ complished straightforwardly by forming the following combination of $\hat{\mathcal{R}}_{nn\to pp}$ at three timeslices: *ea* 1 *A t*!1 !*pp* complished straightforwardly by forming the following !*pp* g combination of $\mathcal{R}_{nn\to pn}$ at three $\frac{d}{dt}$ denote respect to $\frac{d}{dt}$ and $\frac{d}{dt}$

Matrix element fitting Take combinations to pull out isotensor axial polarisability (two complished straightforwardly by forming the following combination of *^R*ˆ*nn*!*pp* at three neighboring FIG. 5. The (a) ratio *^Rnn*!*pp*(*t*) and (b) subtracted ratio *^R*(sub) $\mathbf{S} \cap \mathbf{S}$ denote results determined using SP and SS correlation functions, respectively. The horizontal bands show constant correlated SP-SS fits to the late-time behavior of the quantities. orange diamonds denote results denote results determined using $\mathcal{L}_{\mathcal{A}}$ correlations, respectively. The SS correlations, respectively. The SS correlations, respectively. The SS correlations, respectively. The SS cor points are slightly o↵set in *t* for clarity.

Treatment of uncertainties: MEs at $m_\pi \sim 800$ MeV body contribution) *^R*(lin) *nn*!*pp*(*t*) = (*ea* + 1)*R*ˆ*nn*!*pp*(*^t* ⁺ *^a*) *^R*ˆ*nn*!*pp*(*^t* + 2*a*) *^eaR*ˆ*nn*!*pp*(*t*) M ρ \mathbf{t} $\mathsf{B.}$ iils denoting the outliness !*pp*

Statistical

bootstrap/jackknife over configs. correlated ratios of correlation functions

Systematics in fit

Take every 'reasonable' fit with $\,\chi^2/d.o.f \leq 1\,$

- Fit time begins after 2pt function is consistent with appropriate (1 or 2 state) form
- Minimum 5 timeslices
- Extending fits to later times gives consistent results
- Variation over all central values taken as systematic uncertainty on result
- Systematic in analysis method

Variation over range of analysis procedures, performed independently by different collaboration members, taken as additional systematic uncertainty

complished straightforwardly by forming the following combination of *^R*ˆ*nn*!*pp* at three neighboring

Two-body effects 1tT dinal and transverse Euclidean response functions are in excellent agreement with data. $\overline{}$

EM transverse response function shows important two-body effects: $12C$ at q = 570 MeV \mathcal{L} \bigvee Insverse response function shows important two-body τ (3) needs to be removed in order to account for the theoretical to account for the theoretical to account for the theoretical term of the theoretical term of the theoretical term of the theoretical term of the theore

ato et al.. Phys. Rev. C 91.062501 (2015 Lovato et al., Phys. Rev. C 91, 062501 (2015)

 $A_{\rm eff}$ the statistical errors associated by the statistical errors associated by the statistical errors associated by Ab-initio calculation
T in the longitudinal response for which the elastic contriand external electroweak proces via orie- and two-
beducuments body. Carlotts Two- and three-body forces and external electroweak probes via one- and twobody currents: the transverse functions: the transverse (top)

Expect to be similarly important for axial

*O*R*b*Yk*^b*