Parity Violation in Nuclei with Large N_c

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Fundamental Physics with Electroweak Probes of Light Nuclei

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Beyond the Standard Model physics at low energies

- New physics at scale Λ_{new}
- Below Λ_{new}: effective interactions with SM degrees of freedom
- In particular: local 4-quark operators

$$\bar{q}\mathcal{O}_1 q \; \bar{q}\mathcal{O}_2 q$$

- In nuclei: isolate through symmetry violations
- QCD nonperturbative

Manifestation of 4-quark operators at hadronic level?

Prototype: Weak Interactions

- Mediated by W, Z exchange
- Short-ranged $\sim$ 0.002 fm
- Parity violating
- Well-tested in leptonic and semileptonic sectors
- Strangeness-conserving hadronic sector at low energies

$$\mathcal{L}_{weak}^{\Delta S=0} = \frac{G}{\sqrt{2}} \left[\underbrace{\cos^2\theta_C J_W^{0,\dagger} J_W^0}_{\Delta I=0,2} + \underbrace{\sin^2\theta_C J_W^{1,\dagger} J_W^1}_{\Delta I=1} + J_Z^\dagger J_Z \right]$$

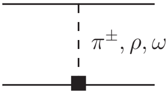
- $\Delta I = 1$ dominated by neutral current J_Z ($\sin^2\theta_C \sim 0.05$)
- Neutral currents highly suppressed in flavor-changing hadronic decays

Hadronic parity violation

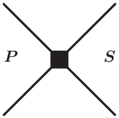
- Parity-violating component in NN interactions
- Manifestation of PV quark interactions at hadronic level
- Interplay of weak and nonperturbative strong interactions
- Short range of weak interactions $\Rightarrow$
  - Sensitive to quark-quark correlations inside nucleon
  - No need to go to high energy
  - "Inside-out probe"
- Relative strength for NN case: $\sim G_F m_\pi^2 \approx 10^{-7}$
- Isolate through pseudoscalar observables ($\vec{p} \cdot \vec{\sigma}$)

PV NN interactions

- Approaches
  - Meson-exchange models



  - EFT($\not\pi$): Pionless effective field theory



  - Chiral effective field theory

$\rightarrow$ PV meson-nucleon and/or nucleon-nucleon couplings

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Danilov (1965,'71); Desplanques, Donoghue, Holstein (1980); Kaplan, Savage (1993); Savage, Springer (1998); Zhu et al. (2005)

Parity violation in EFT($\not\pi$)

Structure of interaction

- At very low energies: pion exchange not resolved
- Only nucleons as explicit degrees of freedom
- Contact terms with increasing number of derivatives
- Parity determined by orbital angular momentum $L : (-1)^L$
- Simplest parity-violating interaction: $L \to L \pm 1$
- Leading order: $S - P$ wave transitions
- Spin, isospin: 5 independent structures

Danilov (1965, '71, '72); Girlanda (2008)

Lowest-order parity-violating Lagrangian

Partial wave basis

$$
\begin{aligned}
\mathcal{L}_{PV} = -\Big[& \mathcal{C}^{(^3S_1-^1P_1)} \left(N^T\sigma_2\, \vec{\sigma}\tau_2 N\right)^\dagger \cdot \left(N^T\sigma_2\tau_2 i\overleftrightarrow{\nabla} N\right) \\
& + \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=0)} \left(N^T\sigma_2\tau_2\vec{\tau}N\right)^\dagger \left(N^T\sigma_2\, \vec{\sigma}\cdot i\overleftrightarrow{\nabla}\tau_2\vec{\tau}N\right) \\
& + \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=1)}\, \epsilon^{3ab} \left(N^T\sigma_2\tau_2\tau^a N\right)^\dagger \left(N^T\sigma_2\, \vec{\sigma}\cdot \overleftrightarrow{\nabla}\tau_2\tau^b N\right) \\
& + \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)}\, \mathcal{I}^{ab} \left(N^T\sigma_2\tau_2\tau^a N\right)^\dagger \left(N^T\sigma_2\, \vec{\sigma}\cdot i\overleftrightarrow{\nabla}\tau_2\tau^b N\right) \\
& + \mathcal{C}^{(^3S_1-^3P_1)}\, \epsilon^{ijk} \left(N^T\sigma_2\sigma^i\tau_2 N\right)^\dagger \left(N^T\sigma_2\sigma^k\tau_2\tau_3 \overleftrightarrow{\nabla}^j N\right)\Big] + h.c.
\end{aligned}
$$

Phillips, MRS, Springer (2009)

PV low-energy couplings

- 5 independent LECs at leading order
- Parameterize short-distance details
- Determine from
  - Underlying theory $\rightarrow$ Nonperturbative QCD calculation
  - Experimental results
    - Suite of observables in unified framework
    - High-precision measurements
    - Few-nucleon systems
    - Low energies
- Currently only weakly constrained
- Additional theoretical constraints?

Large-N_c QCD

QCD in limit $N_c \to \infty$

- Taken with $g^2 N_c$ fixed
- Simplifications
  - Color-singlet physical states
  - Mesons, glueballs: Weakly interacting $\sim 1/\sqrt{N_c}$
- Systematic expansion in $1/N_c$
- Seems to work well phenomenologically
- Baryons
  - Baryon mass $M \sim N_c$
  - SU(4) spin-flavor symmetry: $u\uparrow$, $u\downarrow$, $d\uparrow$, $d\downarrow$

't Hooft (1974); Witten (1979); Dashen, Jenkins, Manohar (1994)

NN potential in large-N_c expansion

$$V(\vec{p}_-, \vec{p}_+) = \langle (\vec{p}_1', C), (\vec{p}_2', D) | H | (\vec{p}_1, A), (\vec{p}_2, B) \rangle$$

with $\vec{p}_\pm = \vec{p}\,' \pm \vec{p}$

- Effective Hamiltonian

$$H = N_c \sum_{s,t,u} v_{stu} \left(\frac{S}{N_c}\right)^s \left(\frac{I}{N_c}\right)^t \left(\frac{G}{N_c}\right)^u$$

- Building blocks

$$S^i = q^\dagger \frac{\sigma^i}{2} q\,, \quad I^a = q^\dagger \frac{\tau^a}{2} q\,, \quad G^{ia} = q^\dagger \frac{\sigma^i \tau^a}{4} q$$

- Coefficients v_{stu}
  - Momentum dependent
  - Constrained by symmetries

Witten (1979); Kaplan, Savage (1996); Kaplan, Manohar (1997); Cohen et al. (2002)

Large-N_c scaling

- Nucleon matrix elements

$$\langle N'|S^i|N\rangle \sim \langle N'|I^a|N\rangle \sim 1,$$
$$\langle N'|G^{ia}|N\rangle \sim \langle N'|\mathbb{1}|N\rangle \sim N_c$$

- Momenta (in t-channel)

$$\vec{p}_- \sim 1$$
$$\vec{p}_+ \sim 1/M_N \sim 1/N_c$$

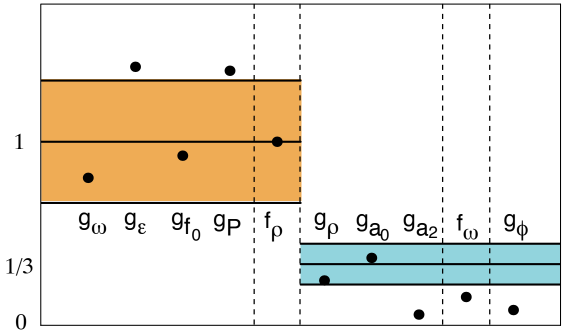
- Coefficients excluding momenta

$$\tilde{v}_{stu} \sim 1$$

Dashen, Jenkins, Manohar (1994,95); Kaplan, Savage (1996); Kaplan, Manohar (1997)

$1/N_c$ expansion of NN potential

Comparison large-N_c scaling vs Nijmegen potential



Kaplan, Manohar (1997)

Large-N_c expansion and pionless EFT

- Leading-order parity-conserving EFT($\not\pi$) interactions

$$\mathcal{L} = -\frac{1}{2} C_S (N^\dagger N)(N^\dagger N) - \frac{1}{2} C_T (N^\dagger \sigma^i N)(N^\dagger \sigma^i N)$$

- Large-N_c scaling

$$C_S \sim N_c, \quad C_T \sim 1/N_c$$

- In partial-wave basis

$$C_0^{(^1S_0)} = (C_S - 3C_T), \quad C_0^{(^3S_1)} = (C_S + C_T)$$

In large-N_c limit

$$C_0^{(^1S_0)} = C_0^{(^3S_1)}$$

Kaplan, Savage (1996)

Parity-conserving S-wave couplings

- In field theory LECs renormalization-scale dependent
- In PDS renormalization

$$\frac{C_0^{({}^1S_0)}}{C_0^{({}^3S_1)}} = \frac{\frac{1}{a^{({}^3S_1)}} - \mu}{\frac{1}{a^{({}^1S_0)}} - \mu}$$

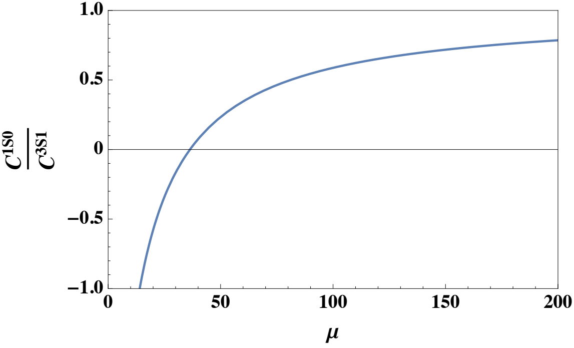
$$\xrightarrow{\mu\to 0} \frac{a^{({}^1S_0)}}{a^{({}^3S_1)}} \approx -4.4$$

- Magnitude $\neq 1$
- Wrong sign

Kaplan, Savage (1996); Kaplan, Savage, Wise (1998)

Parity-conserving S-wave couplings

- Large-N_c + EFT($\not\pi$) requires suitable renormalization scale
- Agreement with large-N_c predicted errors for $\mu \gtrsim m_\pi$



Two-derivative interactions

- Next order in EFT($\not\pi$): two-derivative interactions
- Apply large-N_c constraints

$$\begin{aligned}\mathcal{L}_{\text{LO-in-}N_c} = {} & C_{1\cdot 1}\nabla_i(N^\dagger N)\nabla_i(N^\dagger N) \\ & + C_{G\cdot G}\nabla_i(N^\dagger\sigma_j\tau_a N)\nabla_i(N^\dagger\sigma_j\tau_a N) \\ & + C'_{G\cdot G}\nabla_i(N^\dagger\sigma_i\tau_a N)\nabla_j(N^\dagger\sigma_j\tau_a N)\end{aligned}$$

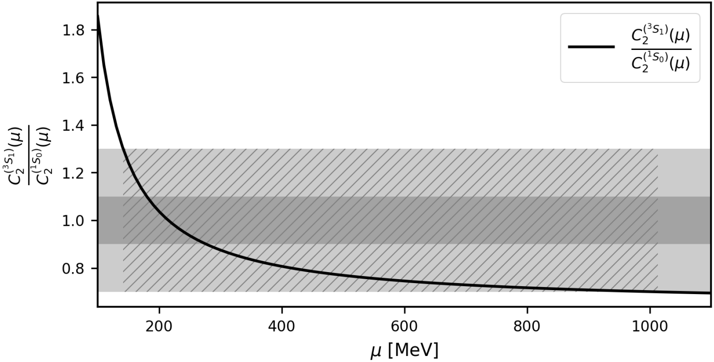
MRS, Singh, Springer (2018)

$$
\begin{aligned}
\mathcal{L}_{\mathrm{N^2LO\text{-}in\text{-}}N_c} &= C_{\tau\cdot\tau}\nabla_i(N^\dagger\tau_a N)\nabla_i(N^\dagger\tau_a N) \\
&+ \overleftrightarrow{C}_{1\cdot 1}(N^\dagger\overleftrightarrow{\nabla}_i N)(N^\dagger\overleftrightarrow{\nabla}_i N) \\
&+ C_{\sigma\cdot\sigma}\nabla_i(N^\dagger\sigma_j N)\nabla_i(N^\dagger\sigma_j N) \\
&+ \overleftrightarrow{C}_{G\cdot G}(N^\dagger\overleftrightarrow{\nabla}_i\sigma_j\tau_a N)(N^\dagger\overleftrightarrow{\nabla}_i\sigma_j\tau_a N) \\
&- \frac{i}{2}\overleftrightarrow{C}_{1\cdot\sigma}\ \epsilon_{ijk}\left[\nabla_j(N^\dagger\sigma_i N)(N^\dagger\overleftrightarrow{\nabla}_k N) + \nabla_j(N^\dagger N)(N^\dagger\overleftrightarrow{\nabla}_k\sigma_i N)\right] \\
&- \frac{i}{2}\overleftrightarrow{C}_{G\cdot\tau}\ \epsilon_{ijk}\left[\nabla_j(N^\dagger\sigma_i\tau_a N)(N^\dagger\overleftrightarrow{\nabla}_k\tau_a N) + \nabla_j(N^\dagger\tau_a N)(N^\dagger\overleftrightarrow{\nabla}_k\sigma_i\tau_a N)\right] \\
&+ C'_{\sigma\cdot\sigma}\nabla_i(N^\dagger\sigma_i N)\nabla_j(N^\dagger\sigma_j N) \\
&+ \overleftrightarrow{C}'_{G\cdot G}(N^\dagger\overleftrightarrow{\nabla}_i\sigma_i\tau_a N)(N^\dagger\overleftrightarrow{\nabla}_j\sigma_j\tau_a N)
\end{aligned}
$$

S-waves

At LO-in-N_c:

$$\left.\frac{C_2^{(^3S_1)}}{C_2^{(^1S_0)}}\right|_{\text{LO-in-}N_c} = 1$$



P-waves

- No renormalization scale dependence
- At LO-in-N_c:

$$\left.\frac{C^{(^3P_0)} - \frac{4}{3}C^{(^3P_2)}}{-C^{(^3P_1)} + 2C^{(^3P_2)}}\right|_{\text{LO-in-}N_c} = 1$$

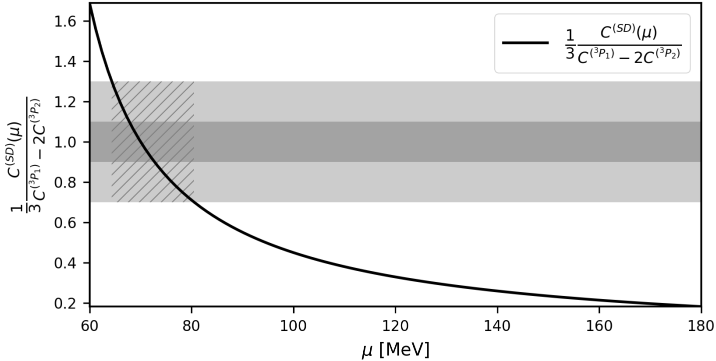
- Using values extracted from NN scattering

$$\frac{C^{(^3P_0)} - \frac{4}{3}C^{(^3P_2)}}{-C^{(^3P_1)} + 2C^{(^3P_2)}} \approx 0.82$$

S-D mixing

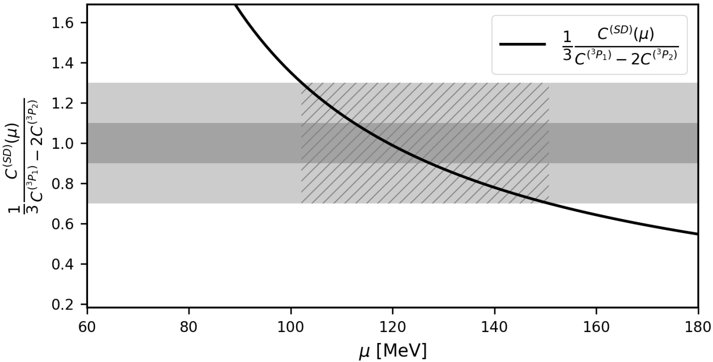
- At LO-in-N_c

$$\frac{1}{3}\frac{C^{(SD)}}{C^{(^3P_1)} - 2C^{(^3P_2)}}\bigg|_{\text{LO-in-}N_c} = 1$$



S-D mixing

- S-D term unnaturally small?
- Increase S-D term by factor 3



- Additional physics can impact large-N_c analysis

Rupak (2000)

PV operators in $1/N_c$ expansion

- Leading order $[\mathcal{O}(N_c)]$

$$\vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\, \vec{\tau}_1 \cdot \vec{\tau}_2$$

- Leading order′ $[\mathcal{O}(N_c) \sin^2\theta_W]$

$$\vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\, [\tau_1\, \tau_2]_2^{zz}$$

- Next-to-leading order $[\mathcal{O}(N_c^0) \sin^2\theta_W]$

$$\vec{p}_+ \cdot (\vec{\sigma}_1 \tau_1^3 - \vec{\sigma}_2 \tau_2^3)$$
$$\vec{p}_- \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)\, (\vec{\tau}_1 \times \vec{\tau}_2)^3$$
$$\vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\, (\vec{\tau}_1 + \vec{\tau}_2)^3$$
$$\left[(\vec{p}_+ \times \vec{p}_-) \cdot \vec{\sigma}_1\, \vec{p}_- \cdot \vec{\sigma}_2 + (\vec{p}_+ \times \vec{p}_-) \cdot \vec{\sigma}_2\, \vec{p}_- \cdot \vec{\sigma}_1 \right]\, (\vec{\tau}_1 \times \vec{\tau}_2)^3$$

- Multiplied by independent functions $U_i(\vec{p}_-^2) \sim \mathcal{O}(1)$

Phillips, Samart, Schat (2015), MRS, Springer, Vanasse (2016)

Parity violation in pionless EFT

- In 'Girlanda basis'

$$
\begin{aligned}
\mathcal{L}_{PV}^{\text{min}} = & \mathcal{G}_1 (N^\dagger \vec{\sigma} N \cdot N^\dagger i \overset{\leftrightarrow}{\nabla} N - N^\dagger N N^\dagger i \overset{\leftrightarrow}{\nabla} \cdot \ \vec{\sigma} N) \\
& - \tilde{\mathcal{G}}_1 \epsilon_{ijk} N^\dagger \sigma_i N \nabla_j (N^\dagger \sigma_k N) \\
& - \mathcal{G}_2 \epsilon_{ijk} \left[N^\dagger \tau_3 \sigma_i N \nabla_j (N^\dagger \sigma_k N) + N^\dagger \sigma_i N \nabla_j (N^\dagger \tau_3 \sigma_k N) \right] \\
& - \tilde{\mathcal{G}}_5 \mathcal{I}_{ab} \epsilon_{ijk} N^\dagger \tau_a \sigma_i N \nabla_j (N^\dagger \tau_b \sigma_k N) \\
& + \mathcal{G}_6 \epsilon_{ab3} \vec{\nabla} (N^\dagger \tau_a N) \cdot N^\dagger \tau_b \vec{\sigma} N
\end{aligned}
$$

Girlanda (2008)

Large-N_c scaling of LECs?

$$
\begin{aligned}
V^{\text{min}} = & - \mathcal{G}_1 \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) - i\tilde{\mathcal{G}}_1 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \\
& - i\mathcal{G}_2 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)(\tau_1 + \tau_2)^3 \\
& - i\tilde{\mathcal{G}}_5 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\mathcal{I}_{ab}\tau_1^a \tau_2^b \\
& + \frac{i}{2}\mathcal{G}_6 \vec{p}_- \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)(\tau_1 \times \tau_2)^3
\end{aligned}
$$

Extracted N_c scaling?

$$
\begin{aligned}
&\tilde{\mathcal{G}}_5 \sim N_c \sin^2\theta_W, \\
&\mathcal{G}_2 \sim \mathcal{G}_6 \sim N_c^0 \sin^2\theta_W, \\
&\mathcal{G}_1 \sim \tilde{\mathcal{G}}_1 \sim N_c^{-1}
\end{aligned}
$$

- Only one term at LO in N_c?
- Isoscalar coupling suppressed?

Vanasse, MRS, Springer (2016)

Fierz identities and large-N_c scaling

- Minimal form of Lagrangian derived using Fierz identities
- Fierz identities
  - Relate different (iso-)spin and momentum structures
  - Do not change EFT power counting
  - Change large-N_c counting
- Identify large-N_c scaling from non-minimal form of $\mathcal{L}$
- Example:

$$\mathcal{A}_1^+ \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)$$
$$\mathcal{A}_3^+ \ \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)\vec{\tau}_1 \cdot \vec{\tau}_2$$
$$\mathcal{A}_3^- \ \vec{p}_- \cdot i(\vec{\sigma}_1 \times \vec{\sigma}_2)\vec{\tau}_1 \cdot \vec{\tau}_2$$

- After Fierz transformation contribute to

$$-\mathcal{G}_1 \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) - i\tilde{\mathcal{G}}_1 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$$

Non-minimal potential

- Extract N_c-scaling

$$\mathcal{A}_1^+ \sim N_c^{-1}, \quad \mathcal{A}_3^+ \sim N_c^{-1}, \quad \mathcal{A}_3^- \sim N_c$$

- Relations

$$\mathcal{G}_1 = -\mathcal{A}_1^+ + \mathcal{A}_3^+ - 2\mathcal{A}_3^- \ ,$$
$$\tilde{\mathcal{G}}_1 = -\mathcal{A}_1^- - 2\mathcal{A}_3^+ + \mathcal{A}_3^- \ ,$$

- Maintain most dominant scaling

$$\mathcal{G}_1 \sim N_c \ , \qquad \tilde{\mathcal{G}}_1 \sim N_c \ ,$$
$$\mathcal{G}_2 \sim N_c^0 \sin^2\theta_W \ , \qquad \tilde{\mathcal{G}}_5 \sim N_c \sin^2\theta_W \ ,$$
$$\mathcal{G}_6 \sim N_c^0 \sin^2\theta_W$$

Large-N_c relation

$$\mathcal{G}_1 = -2\tilde{\mathcal{G}}_1[1 + \mathcal{O}(1/N_c^2)]$$

Large-N_c scaling of partial-wave LECs

$$\mathcal{C}^{({}^3S_1-{}^1P_1)} \sim N_c$$

$$\mathcal{C}^{({}^1S_0-{}^3P_0)}_{(\Delta I=0)} \sim N_c$$

$$\mathcal{C}^{({}^1S_0-{}^3P_0)}_{(\Delta I=2)} \sim N_c \ \sin^2\theta_W$$

$$\mathcal{C}^{({}^1S_0-{}^3P_0)}_{(\Delta I=1)} \sim N_c^0 \sin^2\theta_W$$

$$\mathcal{C}^{({}^3S_1-{}^3P_1)} \sim N_c^0 \sin^2\theta_W$$

$$\mathcal{C}^{({}^3S_1-{}^1P_1)} = 3\,\mathcal{C}^{({}^1S_0-{}^3P_0)}_{(\Delta I=0)}[1+\mathcal{O}(1/N_c^2)]$$

PV LECs

- Renormalization-scale dependent
- Scale dependence driven by S-wave interactions
- "Wrong" choice of scale can hide large-N_c scaling

Application to measurements: LO-in-N_c

Longitudinal asymmetry in $\vec{p}p$ scattering

- Experimental result

$$A_L^{\vec{p}p}(E = 13.6\ \text{MeV}) = (-0.93 \pm 0.21) \times 10^{-7}$$

- Constraint on LECs in large-N_c limit

$$(-1.5 \pm 0.3) \times 10^{-10}\ \text{MeV}^{-1} = \frac{4\left[\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=0)} + \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=1)} + \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)}\right]}{C_0^{(^1S_0)}}$$

$$\to \frac{4\left[\mathcal{C}^{(^3S_1-^1P_1)}/3 + \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)}\right]}{\mathcal{C}}$$

Eversheim et al (1991); Phillips, MRS, Springer (2009)

- Induced circular polarization in $np \to d\vec{\gamma}$

$$
\begin{aligned}
P_\gamma &= (1.8 \pm 1.8) \times 10^{-7} \\
&\to -\frac{16M_N}{\mathcal{C}} \frac{1}{\kappa_1(1-\gamma a^{^1S_0})} \left(\mathcal{C}^{(^3S_1-^1P_1)}(1-\frac{5}{9}\gamma a^{^1S_0}) \right. \\
&\qquad \left. -\frac{2}{3}\gamma a^{^1S_0} \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)} \right)
\end{aligned}
$$

- In large-N_c limit $\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)} \sim \mathcal{C}^{(^3S_1-^1P_1)}$
- If $\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)} \ll \mathcal{C}^{(^3S_1-^1P_1)}$ predict P_γ larger than current bound
- $\sin^2\theta_W$ suppression for $\Delta I = 2$ coupling not significant?

Knyaz'kov et al (1983/84); Vanasse, MRS, Springer (2016)

Conclusion & Outlook

Large-N_c analysis

- Effects of embedding PV quark interactions in hadrons
- Establishes hierarchy of couplings
- Important constraints in absence of experimental data
- Gives trends, not exact predictions

Parity violation

- Two LECs at LO in combined EFT/large-N_c expansion [$\sin^2\theta_W$ for $\Delta I = 2$]
- Relation between two isoscalar LECs
- Isovector coupling suppressed by $\sin^2\theta_W / N_c$