Understanding astrophysical direct capture reactions through halo EFT and vice versa

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INT Program INT-18-2a
Fundamental Physics with Electroweak Probes of Light Nuclei
June 18, 2018

Radiative direct capture

Direct radiative capture on light nuclei is important in astrophysics
($\sim$ 20 keV Sun, 50–500 keV big bang):

$$d(p,\gamma)^3\mathrm{He} \text{ (Sun \& big bang)}$$
$$^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li} \text{ (big bang)}$$
$$^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be} \text{ (Sun \& big bang)}$$
$$^7\mathrm{Be}(p,\gamma)^8\mathrm{B} \text{ (Sun)}$$

Direct: Not usefully modeled in terms of resonant states
Capture: All particles end up in one nucleus

Two questions:

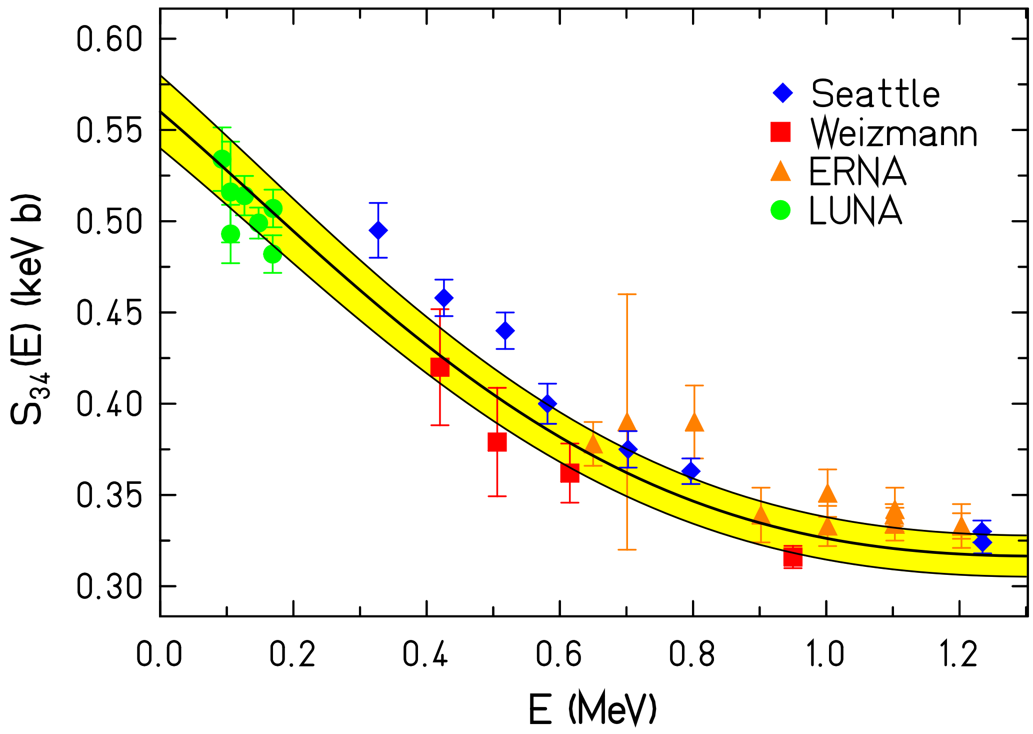
How can we understand the reaction processes?

What rates should be used in astrophysical modeling?

Determining rates

For the big bang, you can measure cross sections right at the energies where you need them

You do still need theory to stitch data together and compute $\langle \sigma v \rangle$



For solar neutrinos, you need a well-validated theory to predict/extrapolate cross sections to $E \lesssim 30$ keV

Let's talk about S

In the 1930s, the quantity needed for stellar astrophysics was identified as the "low-energy cross section"

Energies are well-below the Coulomb barrier, σ dominated by tunneling

Beginning(?) with Salpeter, $\sigma(E) = \frac{S(E)}{E} e^{-2\pi\alpha Z_1 Z_2 / v}$

For s-wave entrance channels, the S-factor is a slowly-varying function

The quantity used in astrophysics is often called $S(0)$, but it's more like $S(20\ \mathrm{keV})$

Parameters of $S(E) \approx s_0 + s_1 E + s_2 E^2$ are sometimes useful ways to disseminate results but sometimes not; used to be important for rate integral

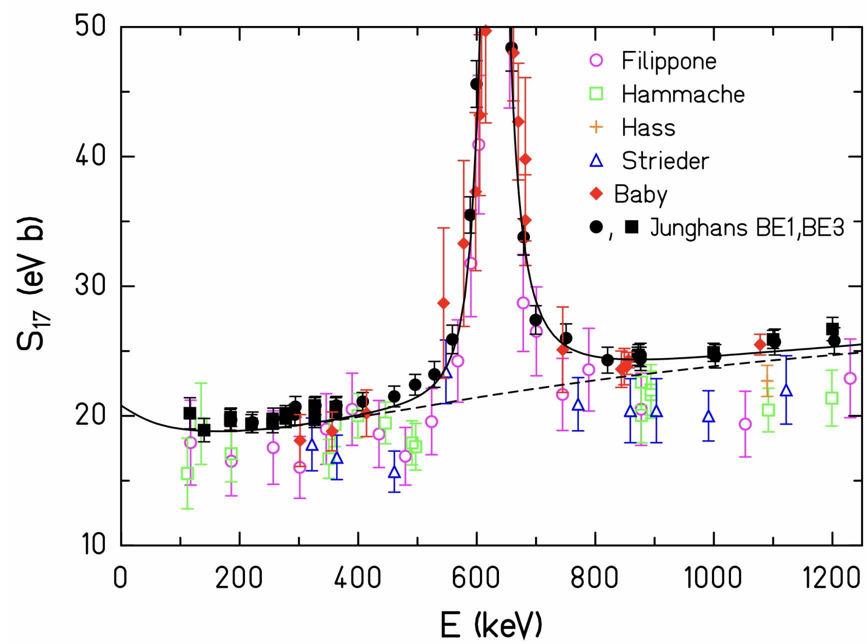
It's important to figure out what's useful & not get hung up on "how things have always been done"

^{7}Be$(p,\gamma)^8$B: The basic task

From threshold to $\sim$ 400 keV, the cross section is purely nonresonant

Reaction process is E1, starting from s- & d-waves

M1 p-wave resonances are negligible below 400 keV



There's a gap between data at $\gtrsim$ 125 keV & solar Gamow peak at 20 keV

Upturn at low E is Coulomb physics & turns up in all models

We want the most accurate extrapolation or *ab initio* model possible

Models of radiative capture

There is a long history of theoretical models for $^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$

Workhorses for fitting have been very restricted RGM-type models & potential models

There was some careful R-matrix fitting by Barker

There is now a fully *ab initio* model from Navrátil et al.

There's also an extensive history of CI or Hartree-Fock models, not pitched or viewed as tools for data extrapolation

I won't be talking about indirect constraints like Coulomb dissociation

Potential models

The simplest quantum mechanical model you could come up with for nonresonant captures is qualitatively pretty good:

The initial nuclei are simple particles interacting through a potential

The final state is a bound state of that potential

The cross section comes from multipole matrix elements (E1, M1, E2, M2,. . .)

This is a “potential model” – dates back to at least Christy & Duck (1961)

Salient features of potential models

Matrix element density at 20 keV peaks well beyond range of nuclear interaction

You can get to 10% accuracy assuming purely external capture

Then the $S(E)$ shape is known & only an ANC is needed to normalize it

Empirical constraints are sparse – sometimes just a binding energy & some resonance levels

There are few constraints on short-distance amplitudes, which get baked in through model assumptions

Where do potential models come from?

Start with standard optical-potential lore for Woods-Saxon geometry as a function of A & for spin-orbit term – full parameter space not explored

Sort-of match scattering data if they exist (often placing a Pauli node matches observed hard-sphere phase shifts)

In $^7\mathrm{Be}+p$ there were no scattering data before 2003, so well depths came from p-wave binding & resonance energies

Nodal structure implies very different potentials for different parities

Aurdal (1970) noticed that $^7\mathrm{Be}+p$ s-waves need a node & made a deep s-wave potential

Projection into the 2-body space implies spectroscopic factors, which came mostly from shell model, with a final rescaling to match data (where available)

Potential model matrix element (200 keV)

A potential model is dominated by asymptotic region ($r \gtrsim 6$ fm)

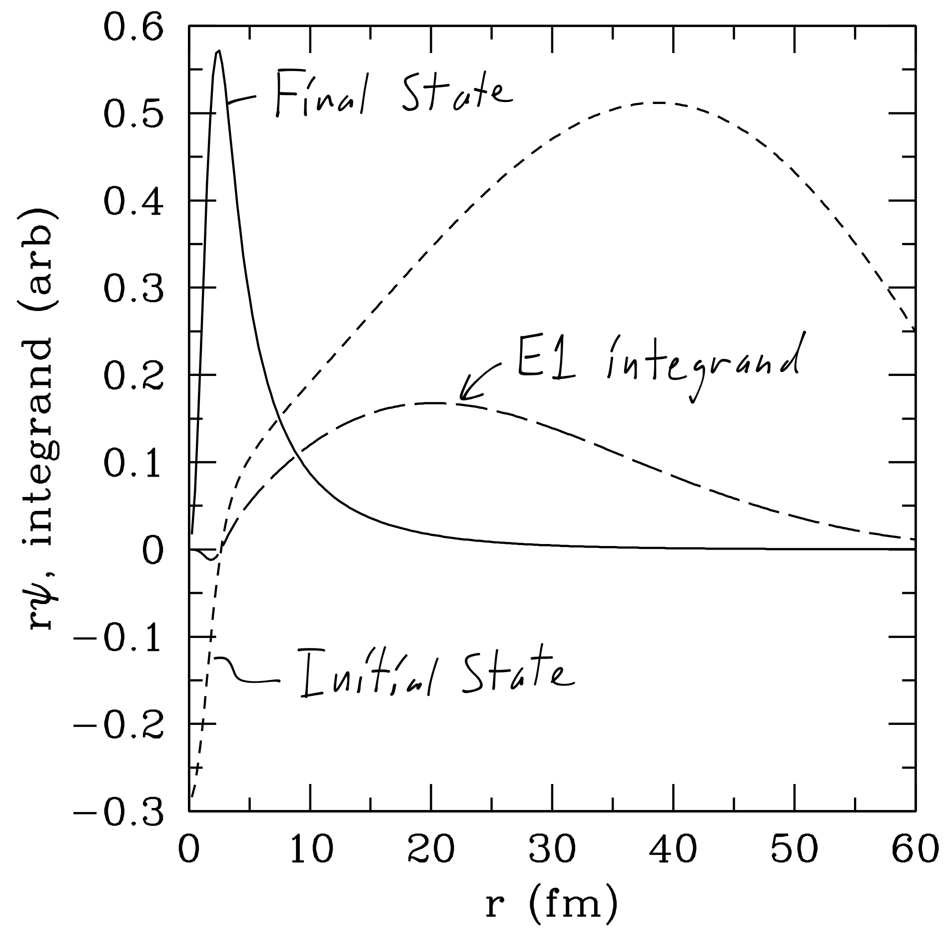
There is a contribution inside the potential well

Nodal structure in s-waves suppresses it

But it's still a few percent of total at $E = 0$

If you care about 5% precision, it matters

Potential model fitting isn't designed to get it right



Traditional cluster models

RGM/GCM cluster models of $A = 6, 7, 8$ captures have been around since the 1980s

For $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$, basis is built from antisymmetrized products of ^{4}He & ^{3}He as $0s$ clusters, using a 2-term nuclear interaction

The ^{7}Be-p correlation solves a Schrödinger-like equation – more rigorous version of the potential-model wave function

Nodal structure in projection onto clusters arises naturally from antisymmetry & size of clusters – $S(E)$ shape should be more reliable

But amplitude of $S(E)$ is often too large by 25% or more

That's usually viewed as overestimated spectroscopic factors for included channels, & $S(E)$ is renormalized to extrapolate data

A related alternative to potential models: Halo EFT

Small binding energy & small reaction energy make these systems amenable to methods of effective field theory

You treat all nuclei as particles in quantum field theory & develop a Lagrangian

There is a high momentum scale Λ where the model breaks down from neglected inelasticities

Lagrangian is expanded & truncated in terms of $(k/\Lambda)^n$

It's "halo" EFT because it's only accurate for small binding energy – cartoon version would involve a halo nucleus

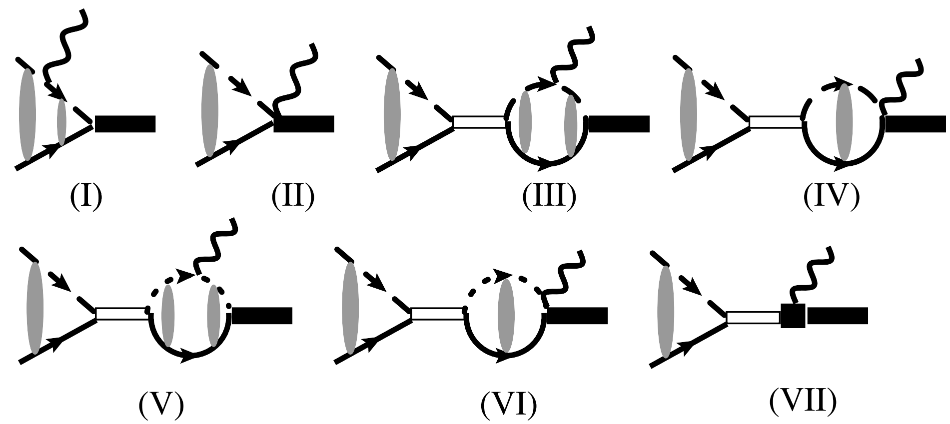
Halo EFT of $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$

Over a few papers, Xilin Zhang developed an EFT of $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ at next-to-leading order (NLO)

Key ingredients: Sum Coulomb at all orders & organize field theory renormalization in terms of physical parameters

The renormalized theory is in terms of ANCs, scattering lengths, effective ranges

The S-factor calculation:



$$S(E) = f(E) \sum_s C_s^2 \Big[\; \Big| \mathcal{S}_{\mathrm{EC}}\left(E;\delta_s(E)\right) + \overline{L}_s \mathcal{S}_{\mathrm{SD}}\left(E;\delta_s(E)\right) + \epsilon_s \mathcal{S}_{\mathrm{CX}}\left(E;\delta_s(E)\right) \Big|^2 + |\mathcal{D}_{\mathrm{EC}}(E)|^2 \; \Big]$$

Halo EFT at next-to-leading order (NLO)

At NLO there are 9 parameters for $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$

2 ANCs: C_s $(s = 1, 2)$

2 short-distance couplings to the photon (like R-matrix radiative widths): $\bar{L}_s$

1 coupling to excited ^{7}Be (essentially an ANC): ϵ_s

2-term effective-range expansion in each s-wave channel, modeled as an unbound "dimer" similar to bound state pole: (a_s & r_s – yields phase shifts δ_S)

$$S(E) = f(E) \sum_s C_s^2 \Big[\; \Big| \mathcal{S}_{\mathrm{EC}}(E; \delta_s(E)) + \bar{L}_s \mathcal{S}_{\mathrm{SD}}(E; \delta_s(E)) + \epsilon_s \mathcal{S}_{\mathrm{CX}}(E; \delta_s(E)) \Big|^2 + |\mathcal{D}_{\mathrm{EC}}(E)|^2 \; \Big]$$

The $\mathcal{S}$ & $\mathcal{D}$ matrix elements are very close to parts of Barker & Kajino R-matrix

Mapping potential models onto EFTs

If the EFT works as claimed, any non-EFT model should be a point in the 9-dimensional EFT parameter space (near threshold)

Every term of the EFT $S(E)$ maps easily into a part of a potential-model calculation, & involves nearly the same integrals

$\mathcal{S}_{\mathrm{EC}}\,(E;\delta_s(E))$ contains phase-shifted Coulomb waves, close to dominant external capture part of potential model

$\overline{L}_s\mathcal{S}_{\mathrm{SD}}\,(E;\delta_s(E))$ for deviation from $\mathcal{S}_{\mathrm{EC}}$ calculation at short distance (dominated by Pauli nodes)

We computed several potential models with our own radiative capture code & located them explicitly in the EFT space

Potential model matrix element (200 keV)

A potential model is dominated by asymptotic region ($r \gtrsim 6$ fm)

There is a contribution inside the potential well

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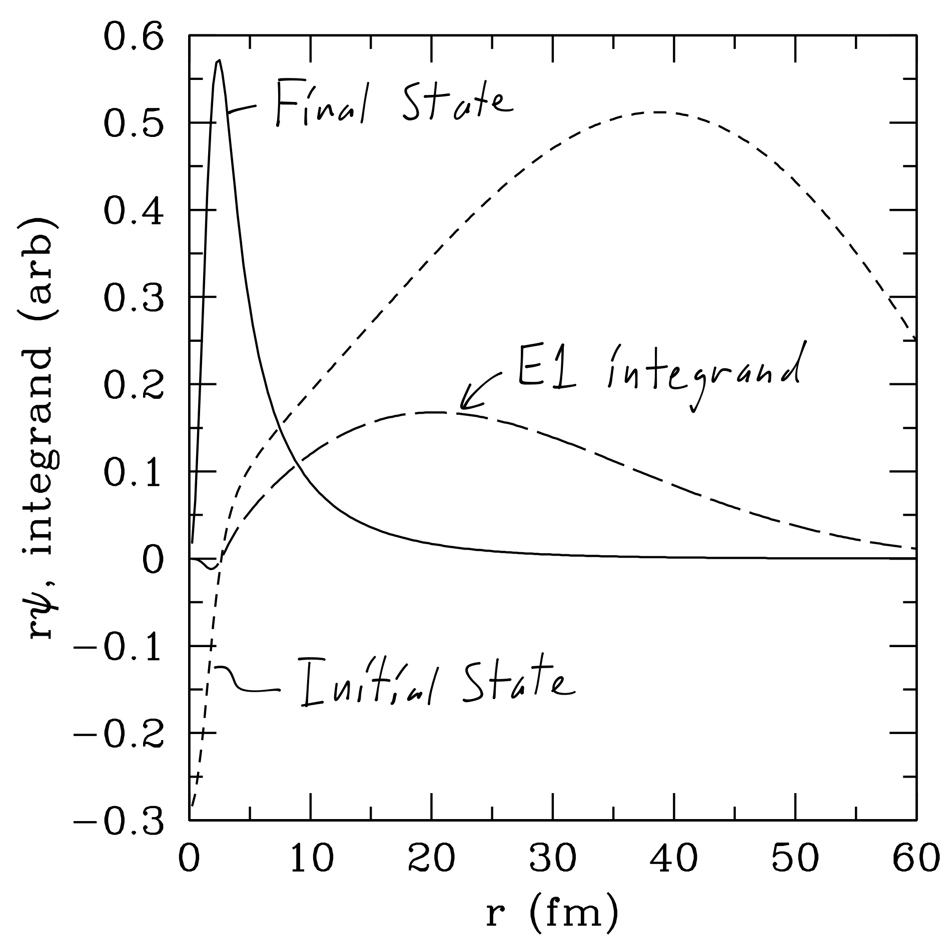


Potential & cluster models of $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ for comparison

Central, lower limit, & upper limit potential models from Davids & Typel (2003) that propagate scattering length error into well depths (minus a bug)

Navrátil, Bertulani & Caurier (2006) potential model with p-waves tuned to match *ab initio* final-state overlap functions

We also looked at the Descouvemont (2004) 8-body GCM model

(Esbensen "traditional" Woods-Saxon model with spin-orbit term & tuning to resonance energies)

Descouvemont, Davids-Typel "upper," & Navrátil defined Adelberger et al. (2011) recommendation for solar-neutrino work

Mapping potential models onto EFTs

Models are easier to fit than data: there's full information & only numerical error

We used both scattering & capture outputs of potential models to fit EFT parameters

Note: adjustment of short-distance $\bar{L}_{1,2}$ deviation from Coulomb waves doesn't affect long-range ANCs & effective-range expansion parameters

You can't do that with a potential model

Tuning a potential to phase shifts changes short-range amplitudes & vice versa

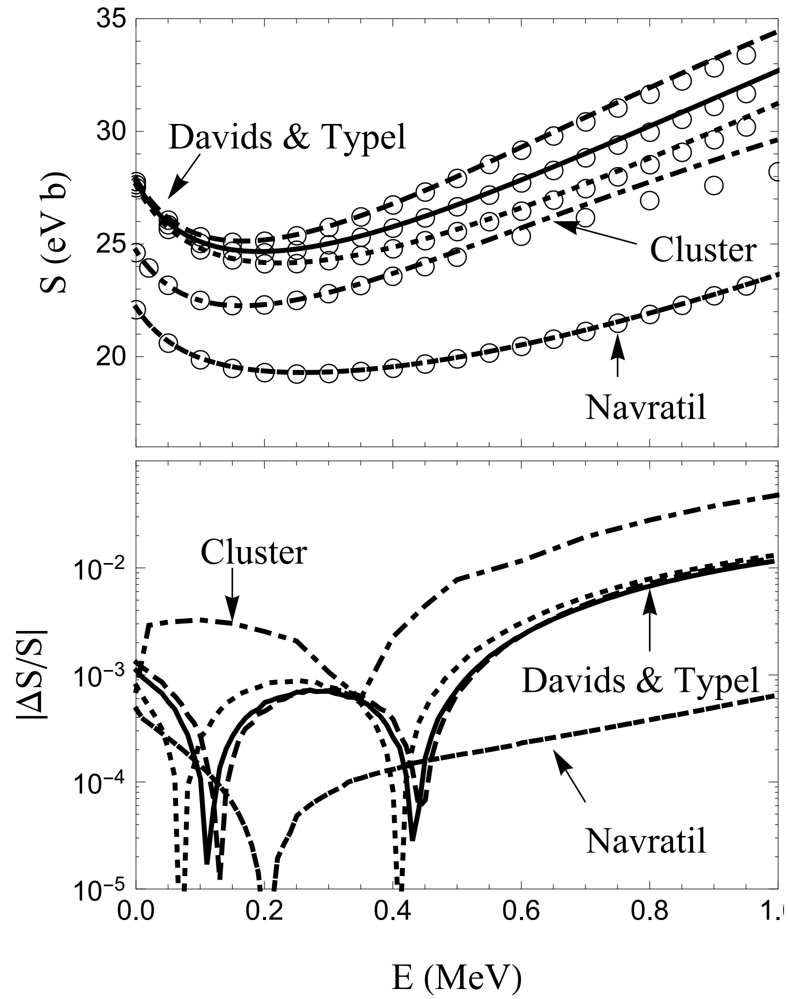
Quod erat demonstrandum

We can match the models to $\lesssim 0.5\%$ at $E \leq 500$ keV

Fitted parameters mainly have “natural” values

So then the EFT power counting looks consistent

Circles: Fitted   Curves: EFT



Lower panel: $\dfrac{|\text{EFT minus original}|}{\text{original}}$

Quod erat demonstrandum

Extrapolation of each fitted EFT to 1 MeV still matches its model at percent level

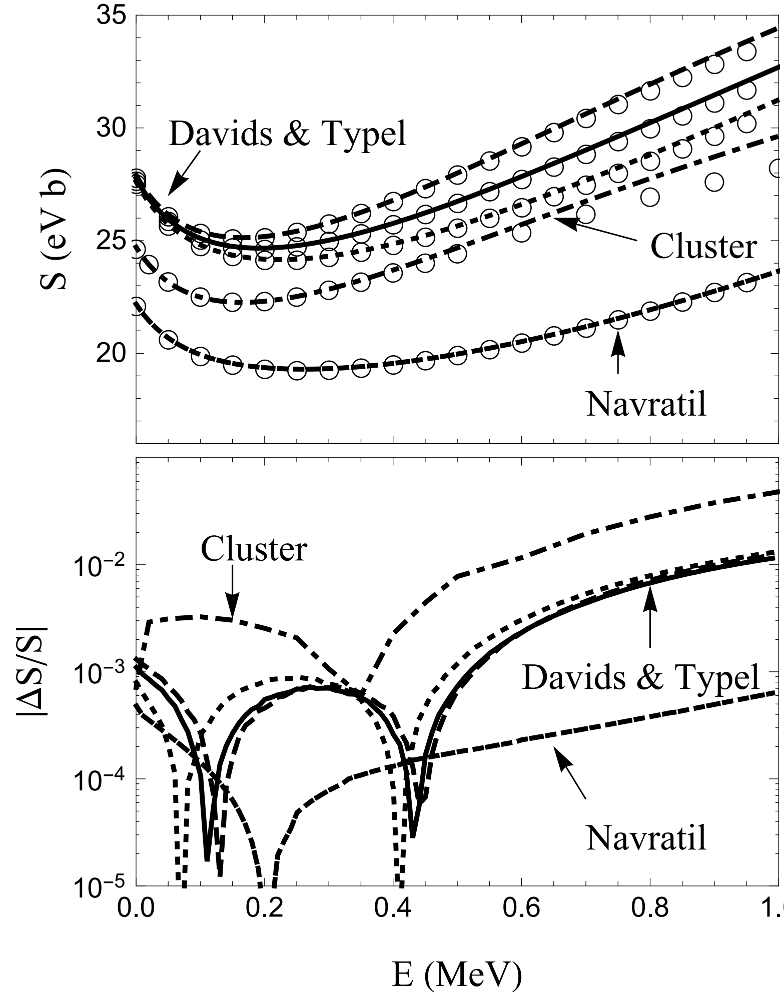
(1% for potential models, 5% for cluster model)

That suggests N^3LO as next EFT term

Consistent with k^2 expansion: overall structure of the EFT looks good

Potential models also helped estimate M1 & E2 strengths of EFT

Circles: Fitted Curves: EFT



Lower panel: $\frac{|\text{EFT minus original}|}{\text{original}}$

Implications for model selection

Adelberger et al. (2011) didn't select models based on data: available models for sure didn't span the space of possible models

With only a discrete subset, it didn't make sense to use data for model selection

Adelberger et al. tried to identify a fairly complete model, recommend based on it, & estimate errors by identifying plausible outlier models

Halo EFT defines a family of models that spans the space of low-energy theories

Error in EFT model construction appears to be $< 1\%$ at $E < 500$ keV

Simultaneous limits on the 9 parameters from capture data are feasible

(EFT results are easy to recompute when you adjust parameters)

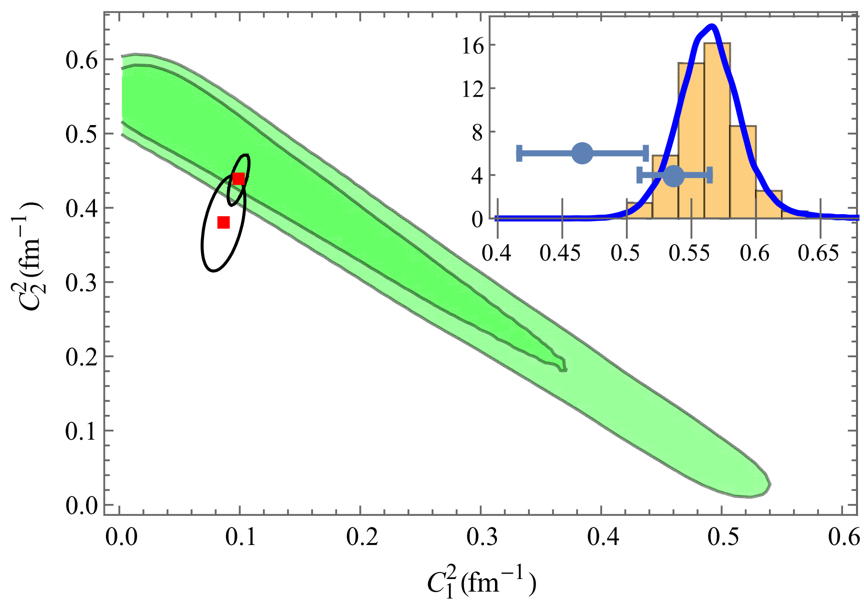
Bayesian treatment of parameters

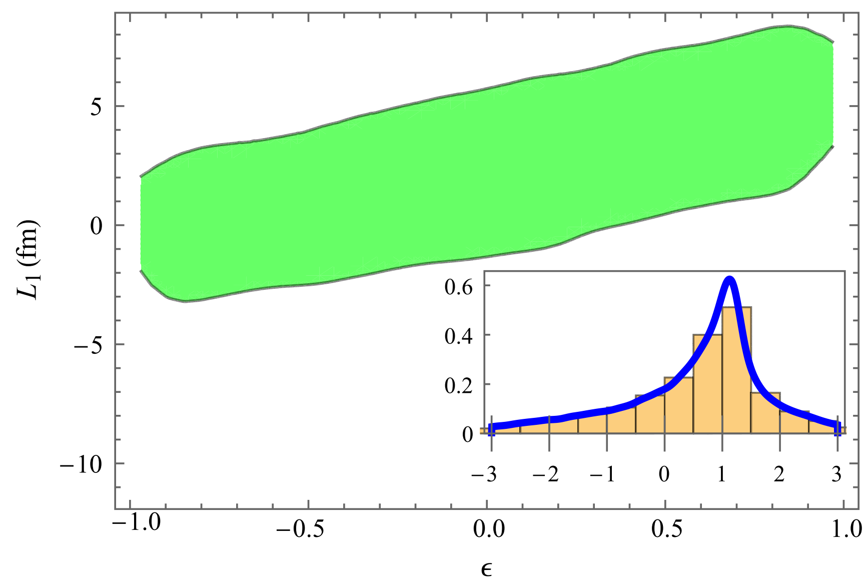
None of the 9 parameters are well-determined by data, but $S(E < 500 \text{ keV})$ is

We computed Bayesian posterior probability of $S(E)$ from capture data, with scattering lengths & floating norms as Gaussian-distributed priors

We fitted at $E < 500$ keV to avoid resonances $\longrightarrow (k/\Lambda)^2 \lesssim 4\%$ estimates truncation error conservatively (marginalizes out to 0.2% on $S(0)$)

We also tried experiment & *ab initio* ANC priors, but eventually left them out





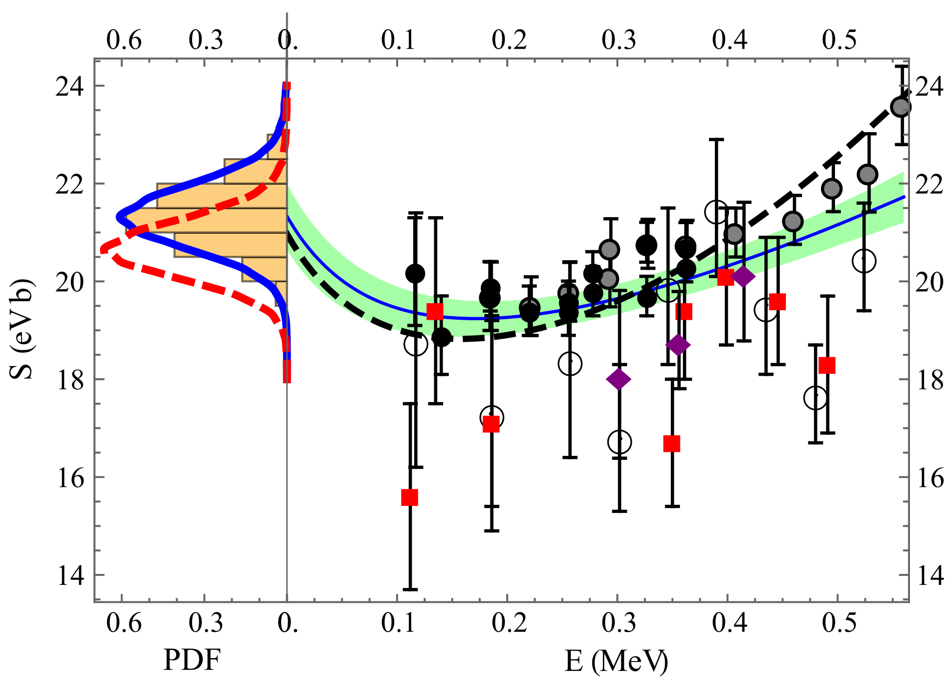
Zhang, Nollett, Phillips, PLB **751**, 535 (2015)

What we really want for $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ is $S(0)$ or $S(20\text{ keV})$

Marginalizing over all parameters, we find $S(0) = 21.3 \pm 0.7$ eV b

Solar Fusion II recommends $S(0) = 20.8 \pm 0.7\,(\mathrm{ex}) \pm 1.4\,(\mathrm{th})$ eV b

Navrátil et al. compute $S(0) = 19.4 \pm 0.7$ eV b *ab initio*, error from truncation



Zhang, Nollett, Phillips, PLB **751**, 535 (2015)

Full histogram: $S(0)$

Dashed histogram: $S(20\text{ keV})$

Green band: Marginalized $S(E)$

Solid curve: Parameters matching band median

Dashed curve: Keeping only LO parameters from solid curve

Coming attractions

We are nearing completion of a similar analysis of $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$

Final state is well-modeled as p-wave bound state of reactants

So then it's completely analogous to $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ but with:

1. Different corner of the parameter space of charges & masses
2. Two bound ^{7}Be final states
3. No resonances up to ~ 3 MeV
4. Only one spin channel (^{4}He has $J = 0$)

Otherwise, it's again E1 capture from s- & d-waves & gives similar shape

Coming attractions: matching to models

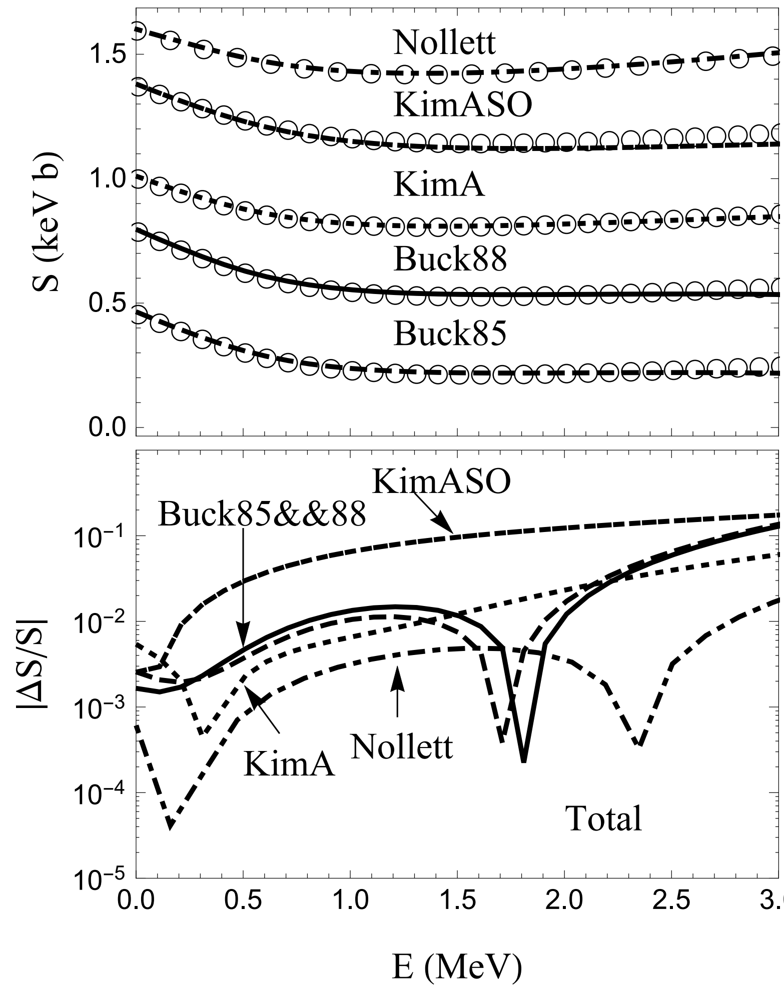
We've repeated the exercise of matching existing models

This also works well & mostly seems to confirm the power counting

But NLO isn't always enough to fit d-waves

This is either a mistake or a weird fine-tuning

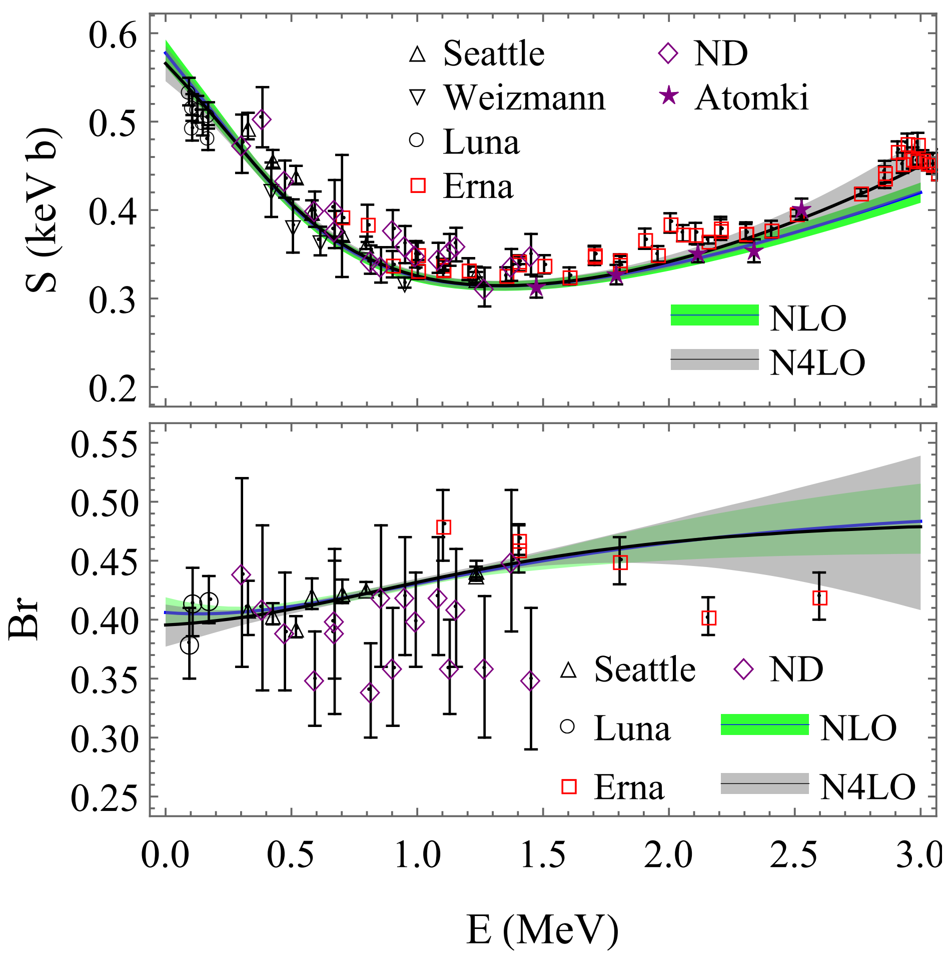
Currently digging through the content of the potential models

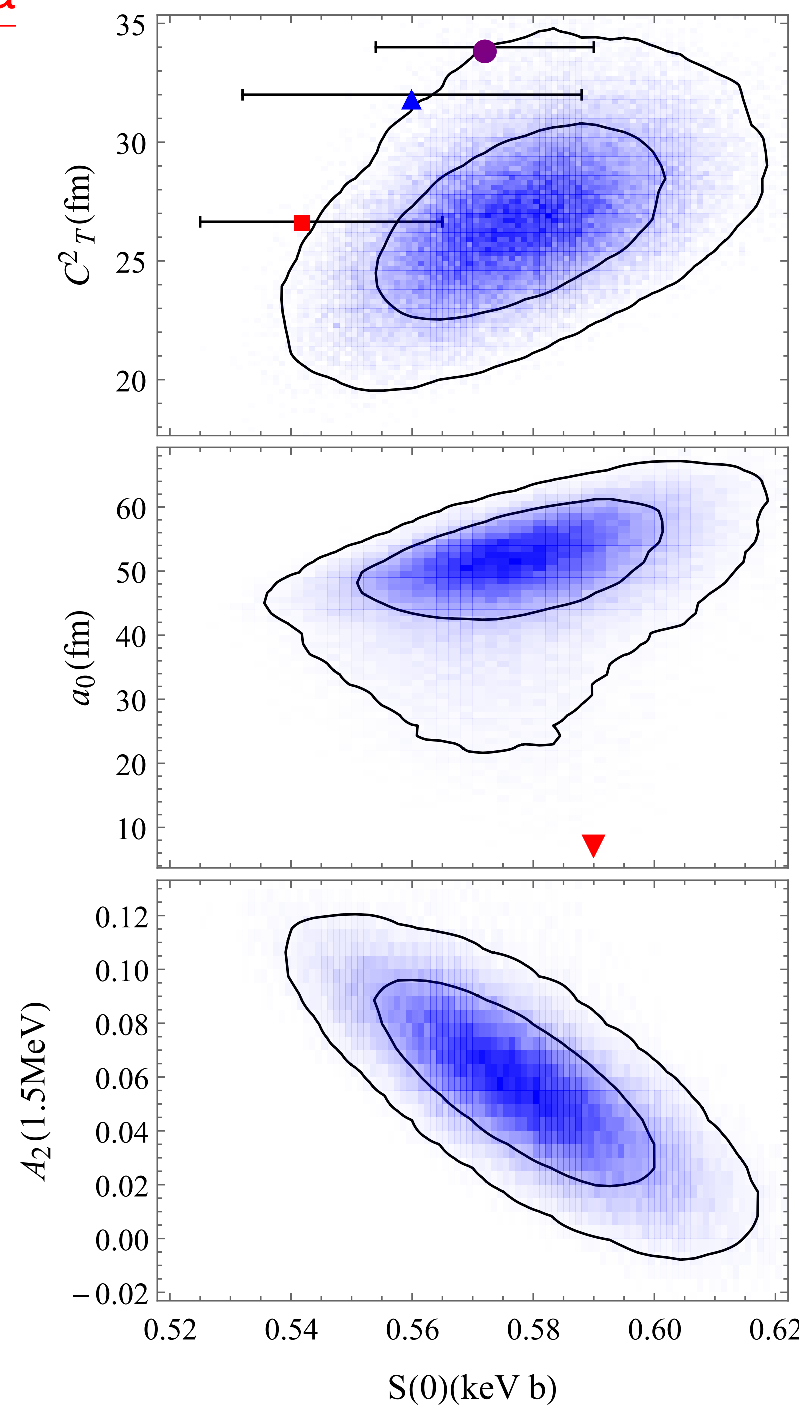


Coming attractions: fitting ${}^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ data

Unlike ${}^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$, we see strong sensitivity to scattering length

You can go “backwards” from $S(E)$ to ANC & scattering length





Concluding thoughts (I)

Ab initio calculations are unlikely to replace data fitting, even where they're feasbible (fine tuning, sociology)

So the question is: how do you use *ab initio* to constrain data fitting?

And there are (at least) two answers:

EFT can be renormalization can be set up (sometimes, anyway) in terms of calculable numbers

The *ab initio* information *complementary to experiment* can be brought in as Bayesian priors, not rigid $S(E)$ shapes

Should be able to avoid fine-tuning things like threshold energies in the *ab initio* calculation

Concluding thoughts (II)

Halo EFT as we've done it is nearly the same calculations as potential models & R-matrix

Halo EFT avoids spurious coupling of short-range capture amplitude to phase shift fitting

Parameter fitting of full space is easy in EFT because integrals can all be reused as parameters vary

We've now found two reactions where Halo EFT expansion converges faster than R-matrix pole expansion

The math done in halo EFT & R-matrix radiative capture is very nearly the same

But effective-range expansion built into EFT is at least simpler algebraically

BONUS MATERIAL

Details of the EFT Lagrangian

The Lagrangian looks like this:

$$\mathcal{L}_0 \;=\; \psi^\dagger \left[i\partial_0 - e\,\hat{Q}\,A_0 + \frac{\left(\vec{\nabla} - ie\vec{A}\hat{Q}\right)^2}{2\hat{M}} + \hat{\Delta} \right] \psi$$

The field ψ includes [7]Be core & its excited state, the proton, & s- & p-wave dimers

The nuclear part of the interaction:

$$\mathcal{L}_S \;=\; h_{(^3S_1)} \phi^{\dagger i}_{(1)} T_i^{\sigma a} n_\sigma c_a + h_{(^3S_1^*)} \phi^{\dagger i}_{(1)} T_i^{\sigma \delta} n_\sigma d_\delta + h_{(^5S_2)} \phi^{\dagger \alpha}_{(2)} T_\alpha^{\sigma a} n_\sigma c_a + \text{c.c.}$$

$$\mathcal{L}_P \;=\; \pi^{\dagger\alpha} \left[h_{(^3P_2)} T_\alpha^{ij} T_i^{\sigma a} + h_{(^5P_2)} T_\alpha^{\beta j} T_\beta^{\sigma a} \right] n_\sigma \tilde{\vec{V}}_{Rj} c_a + h_{(^3P_2^*)} \pi^{\dagger\alpha} T_\alpha^{jk} T_k^{\delta\sigma} n_\sigma \tilde{\vec{V}}_{Rj} d_\delta + \text{c.c.}$$