TRIUMF

Canada's national laboratory
for particle and nuclear physics
and accelerator-based science

Nuclear structure corrections in muonic atoms (To appear in J. Phys. G)

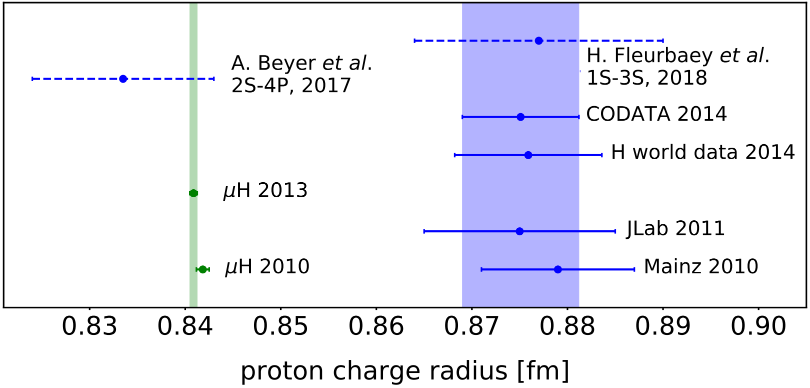
Nir Nevo Dinur[1]

Chen Ji[2], Javier Hernandez[1,3,5], Sonia Bacca[1,4,5], Nir Barnea[6]

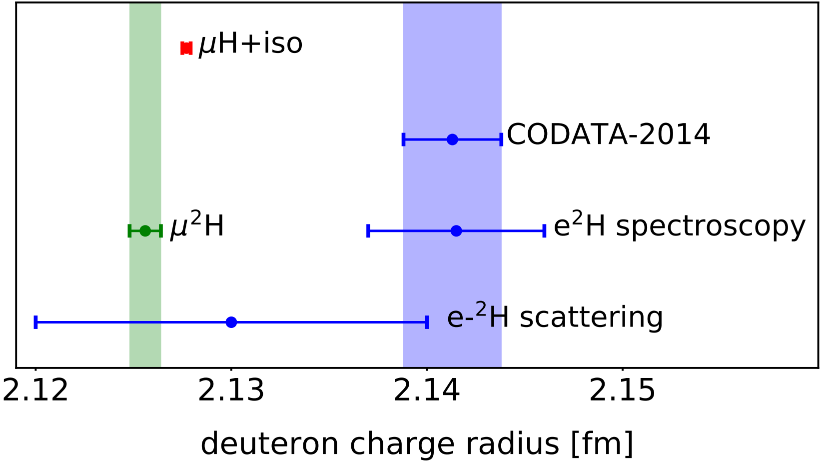
[1]TRIUMF, [2]Central China Normal University,
[3]University of British Columbia, [4]University of Manitoba,
[5]Gutenberg-Universität Mainz, [6]The Hebrew University of Jerusalem

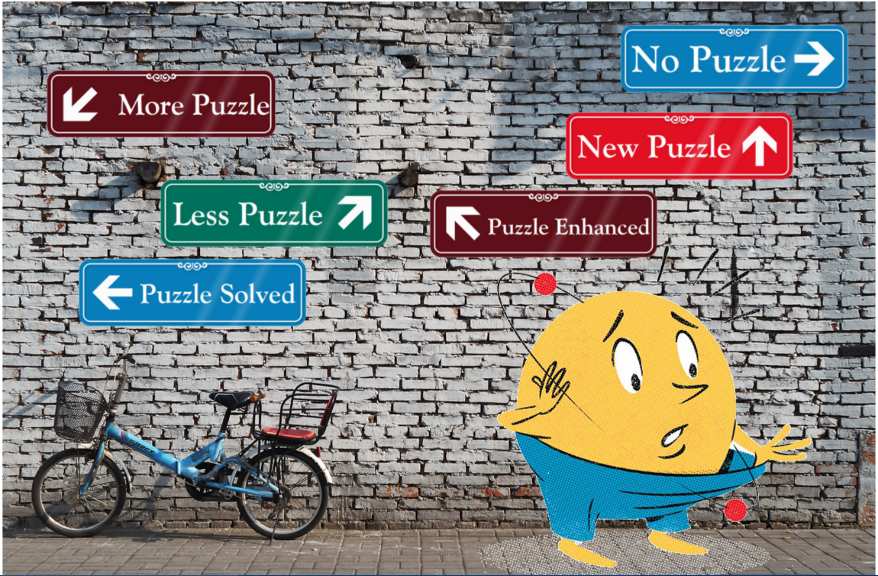
INT-18-2a — Fundamental Physics with Electroweak Probes of Light Nuclei

How big is the proton?



How big is the deuteron?




No Puzzle
More Puzzle
New Puzzle
Less Puzzle
Puzzle Enhanced
Puzzle Solved

CREMA @ PSI

Extract precise **charge radii** $\boldsymbol{R_c}$ from Lamb shift (LS) in:

- μH (published 2010,2013: **proton radius puzzle)**
- μD (published 2016: **deuteron radius puzzle)**
- $\mu\,^4$He$^+$ (measured 2014, finalizing: **agreement with** $e-^4$**He ?!**)
- $\mu\,^3$He$^+$ (measured 2014, analyzing: **???)**

  $\Longrightarrow$ radius puzzle(s), QED tests, He isotope shift, nuclear *ab initio*, ...
- $\mu\,^3$H, $\mu\,^6$He$^+$, $\mu\,^{6,7}$Li^{+2} ... (possible?)

CREMA @ PSI

Extract precise **charge radii** R_c from Lamb shift (LS) in:

- μH (published 2010,2013: **proton radius puzzle)**
- μD (published 2016: **deuteron radius puzzle)**
- $\mu\,^4$He$^+$ (measured 2014, finalizing: **agreement with** $e-^4$**He ?!**)
- $\mu\,^3$He$^+$ (measured 2014, analyzing: **???)**

  $\Longrightarrow$ radius puzzle(s), QED tests, He isotope shift, nuclear *ab initio*, ...
- $\mu\,^3$H, $\mu\,^6$He$^+$, $\mu\,^{6,7}$Li^{+2} ... (possible?)

Extract **magnetic radii** R_m from Hyper-fine splitting (HFS) in:

- μH & $\mu\,^3$He$^+$ (approved)

CREMA @ PSI

Extract precise **charge radii** $\boldsymbol{R_c}$ from Lamb shift (LS) in:

- μH (published 2010,2013: **proton radius puzzle)**
- μD (published 2016: **deuteron radius puzzle)**
- $\mu\,^4$He$^+$ (measured 2014, finalizing: **agreement with $e{-}^4$He ?!**)
- $\mu\,^3$He$^+$ (measured 2014, analyzing: **???)**

  $\Longrightarrow$ radius puzzle(s), QED tests, He isotope shift, nuclear *ab initio*, ...
- $\mu\,^3$H, $\mu\,^6$He$^+$, $\mu\,^{6,7}$Li^{+2} ... (possible?)

Extract **magnetic radii** $\boldsymbol{R_m}$ from Hyper-fine splitting (HFS) in:

- μH & $\mu\,^3$He$^+$ (approved)

FAMU @ RIKEN-RAL / J-PARC

- HFS in μH in two new methods (planned)

Precise R_c (R_m) from μA LS (HFS)

Require accurate theoretical inputs from QED, hadron and nuclear physics

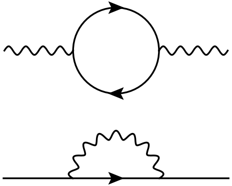
Extract $R_c \equiv \sqrt{\langle r^2 \rangle}$ from Lamb shift measurement

$$\Delta E_{2S-2P} = \delta_{QED} + \mathcal{A}_{OPE} \times R_c^2 + \delta_{\mathrm{Zem}} + \delta_{\mathrm{pol}}$$

Extract $R_c \equiv \sqrt{\langle r^2 \rangle}$ from Lamb shift measurement

$$\Delta E_{2S-2P} = \delta_{QED} + \mathcal{A}_{OPE} \times R_c^2 + \delta_{\text{Zem}} + \delta_{\text{pol}}$$

- QED corrections:
  - vacuum polarization
  - lepton self energy
  - relativistic recoil effects



- Theory of μ-p, D, ^{3,4}He$^+$ reexamined

  Martynenko *et al.* **'07**, Borie **'12**, Krutov *et al.* **'15**

  Karshenboim *et al.* **'15**, Krauth *et al.* **'15** ...

Extract $R_c \equiv \sqrt{\langle r^2 \rangle}$ from Lamb shift measurement

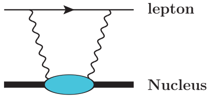
$$\Delta E_{2S-2P} = \delta_{QED} + \mathcal{A}_{OPE} \times R_c^2 + \delta_{\rm Zem} + \delta_{\rm pol}$$

- Nuclear finite-size corrections (elastic):
  - leading term (OPE): $\delta_{size} = \frac{m_r^3}{12} (Z\alpha)^4 \times R_c^2$
  - Zemach/Friar term (TPE): $\delta_{\rm Zem} = -\frac{m_r^4}{24} (Z\alpha)^5 \times \langle r^3 \rangle_{(2)} \quad \propto R_c^3$
    - can be calculated from g.s. charge distribution, Friar **'79**, Borie **'12('14)**, Krutov *et al.* **'15**
    - extracted from experimental form factors, Sick **'14**
    - or avoided due to cancellations with $\delta_{\rm pol}$ Pachucki '11 & Friar '13 (μD)

Extract $R_c \equiv \sqrt{\langle r^2 \rangle}$ from Lamb shift measurement

$$\Delta E_{2S-2P} = \delta_{QED} + \mathcal{A}_{OPE} \times R_c^2 + \delta_{\mathrm{Zem}} + \delta_{\mathrm{pol}}$$

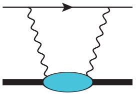
- Nuclear polarization corrections (inelastic TPE):
  - least well-known
  - related to nuclear response functions:
    $S_O(\omega) = \sum\!\!\!\!\!\!\!\int_f |\langle \psi_f | \hat{O} | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$
  - can be calculated (continuum few-body problem)
  - or extracted from data (very imprecise)



Extract $R_c \equiv \sqrt{\langle r^2 \rangle}$ from Lamb shift measurement

$$\Delta E_{2S-2P} = \delta_{QED} + \mathcal{A}_{OPE} \times R_c^2 + \delta_{\text{Zem}} + \delta_{\text{pol}}$$

- Nuclear polarization corrections (inelastic TPE):
  - least well-known
  - related to nuclear response functions:
    $S_O(\omega) = \sum\!\!\!\!\!\!\int_f |\langle \psi_f | \hat{O} | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$
  - can be calculated (continuum few-body problem)
  - or extracted from data (very imprecise)
  - sometimes rewritten as:
    $\boldsymbol{\delta_{\text{TPE}} \equiv \delta_{\text{Zem}} + \delta_{\text{pol}}}$



The accuracy of R_c is limited by δ_{TPE}

Example — μD:

$$\begin{aligned}
\Delta E^{\mathrm{LS}}_{\mathrm{QED}} &= 228.77356(75)\,\mathrm{meV} \\
\Delta E^{\mathrm{LS}}_{\mathrm{rad.-dep.}} &= -6.11025(28)\, r_{\mathrm{d}}^2\,\mathrm{meV/fm^2} + 0.00300(60)\,\mathrm{meV} \\
\Delta E^{\mathrm{LS}}_{\mathrm{TPE}} &= 1.70910(2000)\,\mathrm{meV}
\end{aligned}$$

J. Krauth *et al.* (CREMA), **Ann. Phys. (2016)**; R. Pohl *et al.* (CREMA), **Science 2016**

Status — prior to $\mu^{3,4}$He$^+$ measurements:

- Uncertainty in δ_{pol}: $\sim 20\%$
- Required: $\sim 5\%$
  (to determine R_c with $\sim 10^{-4}$ accuracy)

We have performed the first *ab-initio* calculation of $\boldsymbol{\delta_{\mathrm{Zem}}}$ and $\boldsymbol{\delta_{\mathrm{pol}}}$ for $\boldsymbol{A = 3, 4}$

we used state-of-the-art nuclear forces

- **AV18+UIX**
- **χEFT: N3LO (Entem & Machleidt) + N2LO (Navrátil)**

$\Longrightarrow$ estimate nuclear physics uncertainty

we employ established few-body methods

- **EIHH**: Effective interaction Hyperspherical Harmonics **(bound method)**
- **LIT**: Lorentz Integral Transform **(continuum method)**
- **LSR**: **A new method** based on the Lanczos algorithm
  NND *et al.*, **Phys. Rev. C (2014)**

We have performed the first *ab-initio* calculation of δ_{Zem} and δ_{pol} for $A = 3, 4$

we used state-of-the-art nuclear forces (at the time...)

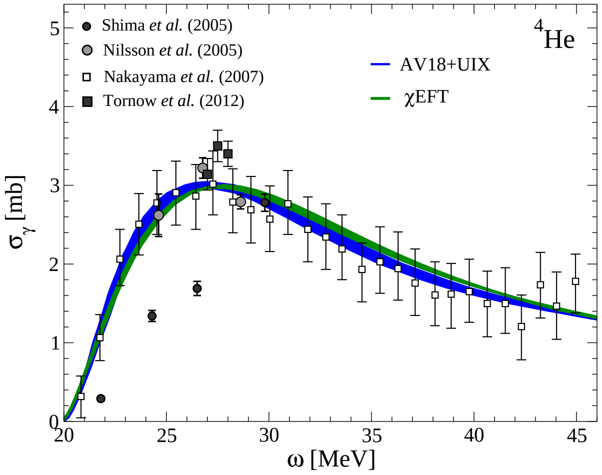
- **AV18+UIX**
- **χEFT: N3LO (Entem & Machleidt) + N2LO (Navrátil)**

$\Longrightarrow$ estimate nuclear physics uncertainty

we employ established few-body methods

- **EIHH**: Effective interaction Hyperspherical Harmonics **(bound method)**
- **LIT**: Lorentz Integral Transform **(continuum method)**
- **LSR**: **A new method** based on the Lanczos algorithm
  NND *et al.*, **Phys. Rev. C (2014)**

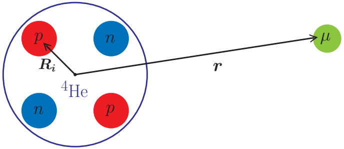
electric dipole photoabsorption cross section $\sigma_\gamma(\omega) = 4\pi^2\alpha\omega S_{D1}(\omega)$ — from LIT



- Hamiltonian for muonic atoms

$$H = H_{nucl} + H_\mu + \Delta H$$

$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



- Corrections to the point Coulomb from protons

$$\Delta H = \alpha \sum_i^Z \left(\frac{1}{r} - \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|} \right)$$

- Evaluate inelastic effects of ΔH on muonic spectrum in 2^{nd}-order perturbation theory

$$\delta_{\rm pol} = \sum_{N \neq N_0, \mu} \langle N_0 \mu_0 | \Delta H | N \mu \rangle \frac{1}{E_{N_0} - E_N + \epsilon_{\mu_0} - \epsilon_\mu} \langle N \mu | \Delta H | \mu_0 N_0 \rangle$$

$|\mu_0\rangle$: muon wave function for $2S/2P$ state

Systematic contributions to nuclear polarization

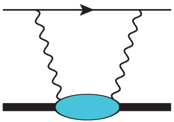
δ_{NR} **Non-Relativistic** limit

δ_{Rel} **Relativistic corrections**

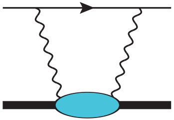
δ_{C} **Coulomb** distortions

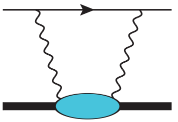
δ_{NS} Corrections from **finite Nucleon Size**

- **Neglect Coulomb interactions in the intermediate state**



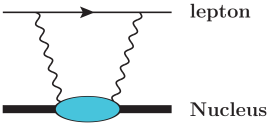
- **Neglect Coulomb interactions in the intermediate state**
- **Expand muon matrix element in powers of** $\eta \equiv \sqrt{2m_r\omega}|\boldsymbol{R} - \boldsymbol{R}'|$





- **Neglect Coulomb interactions in the intermediate state**
- **Expand muon matrix element in powers of** $\eta \equiv \sqrt{2m_r\omega}|\boldsymbol{R} - \boldsymbol{R}'|$
  - $|\boldsymbol{R} - \boldsymbol{R}'| \implies$ "virtual" distance the proton travels in 2γ exchange
  - uncertainty principal $|\boldsymbol{R} - \boldsymbol{R}'| \sim 1/\sqrt{2m_N\omega}$
  - $\eta \sim \sqrt{\frac{m_r}{m_N}} \approx 0.3$

- **Neglect Coulomb interactions in the intermediate state**
- **Expand muon matrix element in powers of** $\eta \equiv \sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$



  - $|\boldsymbol{R}-\boldsymbol{R}'| \Longrightarrow$ “virtual” distance the proton travels in 2γ exchange
  - uncertainty principal $|\boldsymbol{R}-\boldsymbol{R}'| \sim 1/\sqrt{2m_N\omega}$
  - $\eta \sim \sqrt{\frac{m_r}{m_N}} \approx 0.3$

$$P_{NR}(\omega,\boldsymbol{R},\boldsymbol{R}') \simeq \frac{m_r^3(Z\alpha)^5}{12}\sqrt{\frac{2m_r}{\omega}}\left[|\boldsymbol{R}-\boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega}}{4}|\boldsymbol{R}-\boldsymbol{R}'|^3 + \frac{m_r\omega}{10}|\boldsymbol{R}-\boldsymbol{R}'|^4\right]$$

$$\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)} \sim \eta^2 + \eta^3 + \eta^4$$

Dynamical (polarization) contributions to δ_{TPE}

$$\delta_{pol} = \sum_a I_a = \sum_a \int d\omega \, S_{\hat{O}a}(\omega) \, g_a(\omega)$$

Dynamical (polarization) contributions to δ_{TPE}

$$\delta_{pol} = \sum_a I_a = \sum_a \int d\omega \, S_{\hat{O}_a}(\omega) \, g_a(\omega)$$

$$S_{\hat{O}}(\omega) = \sum\!\!\!\!\!\!\!\int |\langle f|\hat{O}|i\rangle|^2 \delta(\omega_f - \omega) \qquad \Longrightarrow \qquad I = \langle i|\hat{O}^\dagger \, g(\hat{H}) \, \hat{O}|i\rangle$$

Dynamical (polarization) contributions to δ_{TPE}

$$\delta_{pol} = \sum_a I_a = \sum_a \int d\omega\, S_{\hat{O}_a}(\omega)\, g_a(\omega)$$

$$S_{\hat{O}}(\omega) = \sum\!\!\!\!\!\!\int |\langle f|\hat{O}|i\rangle|^2 \delta(\omega_f - \omega) \qquad \Longrightarrow \qquad I = \langle i|\hat{O}^\dagger\, g(\hat{H})\, \hat{O}|i\rangle$$

- E.g., the leading polarization contribution relates to the dipole response

$$\delta^{(0)}_{D1} \propto \int_{\omega_{\rm th}}^{\infty} d\omega\, S_{D_1}(\omega)\, \omega^{-1/2}$$

Dynamical (polarization) contributions to δ_{TPE}

$$\delta_{pol} = \sum_a I_a = \sum_a \int d\omega \, S_{\hat{O}_a}(\omega) \, g_a(\omega)$$

$$S_{\hat{O}}(\omega) = \sum\!\!\!\!\!\!\int |\langle f|\hat{O}|i\rangle|^2 \delta(\omega_f - \omega) \qquad \Longrightarrow \qquad I = \langle i|\hat{O}^\dagger \, g(\hat{H}) \, \hat{O}|i\rangle$$

- E.g., the leading polarization contribution relates to the dipole response

$$\delta^{(0)}_{D1} \propto \int_{\omega_{\rm th}}^{\infty} d\omega \, S_{D_1}(\omega) \, \omega^{-1/2}$$

$\Rightarrow$ can calculate I_a using LIT: Calculate $\mathcal{L}$, invert, integrate, extrapolate to ∞

Dynamical (polarization) contributions to δ_{TPE}

$$\delta_{pol} = \sum_a I_a = \sum_a \int d\omega\, S_{\hat{O}a}(\omega)\, g_a(\omega)$$

$$S_{\hat{O}}(\omega) = \sum\!\!\!\!\!\!\int |\langle f|\hat{O}|i\rangle|^2 \delta(\omega_f - \omega) \qquad \Longrightarrow \qquad I = \langle i|\hat{O}^\dagger\, g(\hat{H})\, \hat{O}|i\rangle$$

- E.g., the leading polarization contribution relates to the dipole response

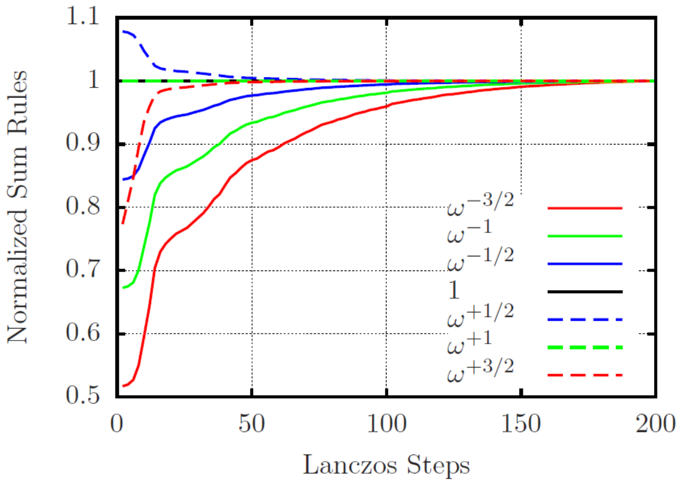
$$\delta^{(0)}_{D1} \propto \int_{\omega_{\rm th}}^{\infty} d\omega\, S_{D_1}(\omega)\, \omega^{-1/2}$$

$\Rightarrow$ can calculate I_a using LIT: Calculate $\mathcal{L}$, invert, integrate, extrapolate to ∞

$\Rightarrow$ or calculate I_a directly from $\hat{O}|i\rangle$ using LSR:
fast, precise, efficient $\rightarrow$ automatized

LSR: Lanczos sum rule method

- For example, from $S_{D_1}(\omega)$ calculated with $M \sim 10^5$, we get

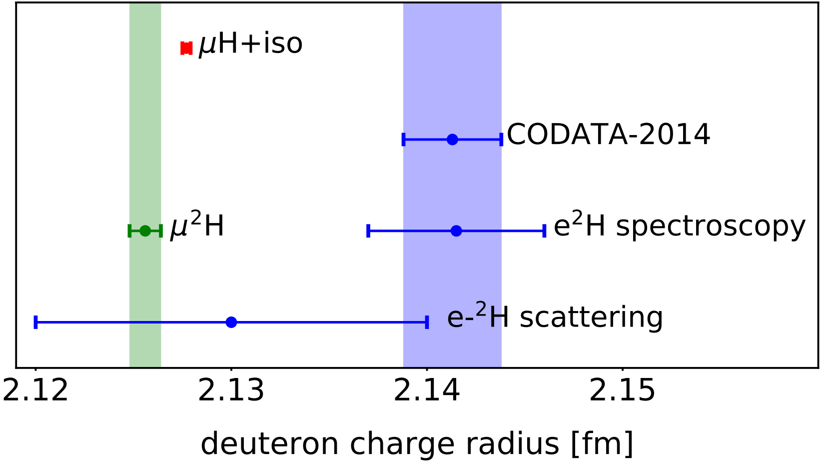


NND, Barnea, Ji, and Bacca, PRC (2014)

Results: μD benchmark

	Pachucki [36] (2011)	Hernandez et al. [41] (2014)	Pachucki and Wienczek [39] (2015)	Friar [28] (2013)
$\delta^{(0)}_{D1}$	-1.910	-1.907	-1.910	-1.925
$\delta^{(0)}_{L}$	0.035	0.029	0.026	0.037
$\delta^{(0)}_{T}$	–	-0.012	–	–
δ_{HO}	–	–	-0.004	–
$\delta^{(0)}_{C}$	0.261	0.262	0.261	–
$\delta^{(0)}_{M}$	0.016	0.008	0.008	0.011
$\delta^{(1)}_{Z3}$	–	0.357	–	–
$\delta^{(2)}_{R2}$	0.045	0.042	0.042	0.042
$\delta^{(2)}_{Q}$	0.066	0.061	0.061	0.061
$\delta^{(2)}_{D1D3}$	-0.151	-0.139	-0.139	-0.137
$\delta^{(1)}_{Z1}$	–	0.064	–	–
$\delta^{(1)}_{np}$	–	0.017	0.018	0.023
$\delta^{(2)}_{NS}$	–	-0.020	-0.020	-0.021
δ^{A}_{pol}	–	-1.240	–	–
δ^{A}_{Zem}	–	-0.421	–	–
δ^{A}_{TPE}	-1.638	-1.661	-1.657	-1.909

How big is the deuteron?



How big is the small puzzle?

382 O.J. Hernandez et al. / Physics Letters B 778 (2018) 377–383

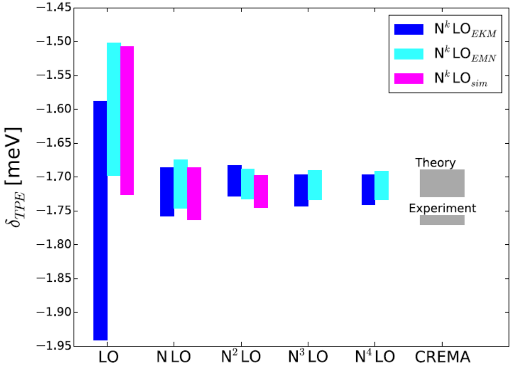
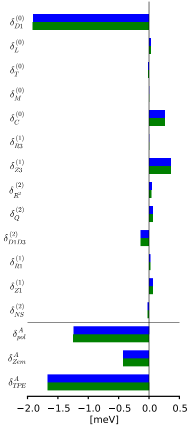


Table 2
Uncertainty breakdown of the final δ_{TPE} value. For the single-nucleon contribution we quote two values, one where we adopted the strategy of Ref. [19] and one where we use the larger uncertainties from Ref. [33] for δ^N_{sub}.

Contribution	Uncertainty in meV	
Nuclear physics (syst)	+0.008 −0.011	
Nuclear physics (stat)	±0.001	
η-expansion	±0.005	
Single-nucleon	±0.0102 [19]	±0.0198 [33]
Atomic physics	±0.0172	
Total	+0.022 −0.024	+0.028 −0.029

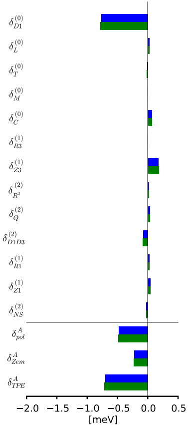
Results: $Z = 1$

Deuteron



	AV18	χEFT
$\delta_{D1}^{(0)}$	-1.907	-1.912
$\delta_{L}^{(0)}$	0.029	0.029
$\delta_{T}^{(0)}$	-0.012	-0.012
$\delta_{M}^{(0)}$	0.003	0.003
$\delta_{C}^{(0)}$	0.262	0.262
$\delta_{R3}^{(1)}$	-	-
$\delta_{Z3}^{(1)}$	0.357	0.359
$\delta_{R^2}^{(2)}$	0.042	0.041
$\delta_{Q}^{(2)}$	0.061	0.061
$\delta_{D1D3}^{(2)}$	-0.139	-0.139
$\delta_{R1}^{(1)}$	0.017	0.017
$\delta_{Z1}^{(1)}$	0.064	0.064
$\delta_{NS}^{(2)}$	-0.020	-0.021
δ_{pol}^{A}	-1.243	-1.248
δ_{Zem}^{A}	-0.421	-0.423
δ_{TPE}^{A}	-1.664	-1.671

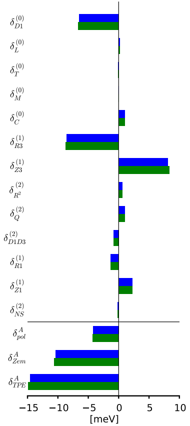
Triton



	AV18/UIX	χEFT
$\delta_{D1}^{(0)}$	-0.7669	-0.7848
$\delta_{L}^{(0)}$	0.0285	0.0296
$\delta_{T}^{(0)}$	-0.0128	-0.0132
$\delta_{M}^{(0)}$	0.0007	0.0007
$\delta_{C}^{(0)}$	0.0718	0.0732
$\delta_{R3}^{(1)}$	-	-
$\delta_{Z3}^{(1)}$	0.1778	0.1844
$\delta_{R^2}^{(2)}$	0.0199	0.0206
$\delta_{Q}^{(2)}$	0.0344	0.0358
$\delta_{D1D3}^{(2)}$	-0.0783	-0.0811
$\delta_{R1}^{(1)}$	0.0280	0.0287
$\delta_{Z1}^{(1)}$	0.0453	0.0463
$\delta_{NS}^{(2)}$	-0.0239	-0.0247
δ_{pol}^{A}	-0.4755	-0.4845
δ_{Zem}^{A}	-0.2231	-0.2307
δ_{TPE}^{A}	-0.6986	-0.7152

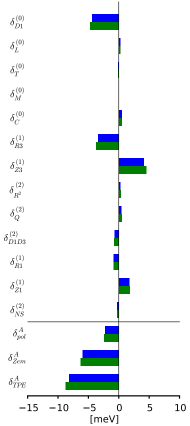
Results: $Z = 2$

^{3}He

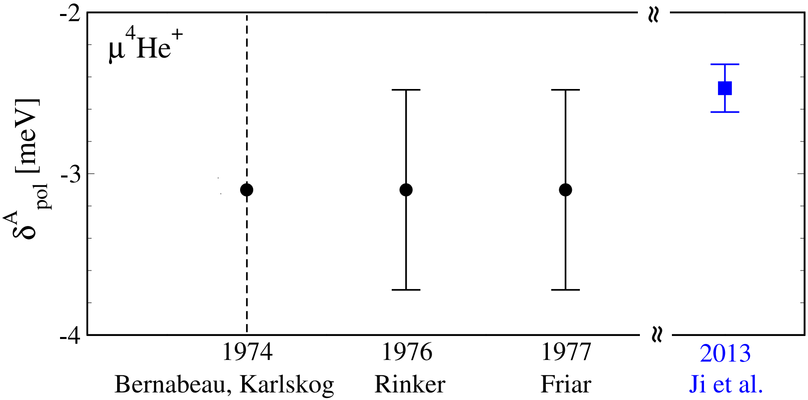


	AV18/UIX	χEFT
$\delta_{D1}^{(0)}$	-6.479	-6.633
$\delta_{L}^{(0)}$	0.232	0.240
$\delta_{T}^{(0)}$	-0.103	-0.107
$\delta_{M}^{(0)}$	0.006	0.006
$\delta_{C}^{(0)}$	1.000	1.020
$\delta_{R3}^{(1)}$	-8.539	-8.711
$\delta_{Z3}^{(1)}$	8.100	8.327
$\delta_{R^2}^{(2)}$	0.632	0.654
$\delta_{Q}^{(2)}$	1.015	1.038
$\delta_{D1D3}^{(2)}$	-0.841	-0.862
$\delta_{R1}^{(1)}$	-1.294	-1.314
$\delta_{Z1}^{(1)}$	2.256	2.291
$\delta_{NS}^{(2)}$	-0.193	-0.199
δ_{pol}^{A}	-4.208	-4.250
δ_{Zem}^{A}	-10.356	-10.618
δ_{TPE}^{A}	-14.564	-14.868

^{4}He

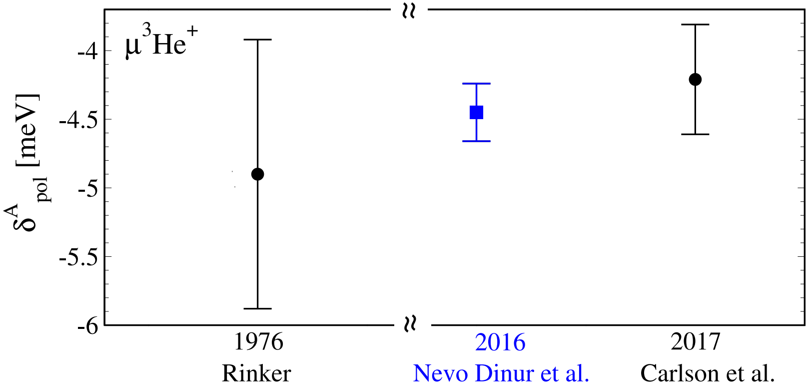


	AV18/UIX	χEFT
$\delta_{D1}^{(0)}$	-4.418	-4.701
$\delta_{L}^{(0)}$	0.289	0.308
$\delta_{T}^{(0)}$	-0.126	-0.134
$\delta_{M}^{(0)}$	0.008	0.006
$\delta_{C}^{(0)}$	0.519	0.553
$\delta_{R3}^{(1)}$	-3.442	-3.717
$\delta_{Z3}^{(1)}$	4.183	4.526
$\delta_{R^2}^{(2)}$	0.259	0.324
$\delta_{Q}^{(2)}$	0.484	0.561
$\delta_{D1D3}^{(2)}$	-0.666	-0.784
$\delta_{R1}^{(1)}$	-0.848	-0.874
$\delta_{Z1}^{(1)}$	1.753	1.811
$\delta_{NS}^{(2)}$	-0.279	-0.293
δ_{pol}^{A}	-2.284	-2.414
δ_{Zem}^{A}	-5.936	-6.337
δ_{TPE}^{A}	-8.220	-8.751



δ^A_{pol} [meV]
-2
-3
-4
μ^4He^+
1974
Bernabeau, Karlskog
1976
Rinker
1977
Friar
2013
Ji et al.

Results: $Z = 2$


μ³He⁺
-4
-4.5
-5
-5.5
-6
δ^A_pol [meV]
1976
Rinker
2016
Nevo Dinur et al.
2017
Carlson et al.

PHYSICAL REVIEW A **95**, 012506 (2017)

Two-photon exchange correction to $2S$-$2P$ splitting in muonic ^{3}He ions

Carl E. Carlson*
Physics Department, College of William and Mary, Williamsburg, Virginia 23187, USA

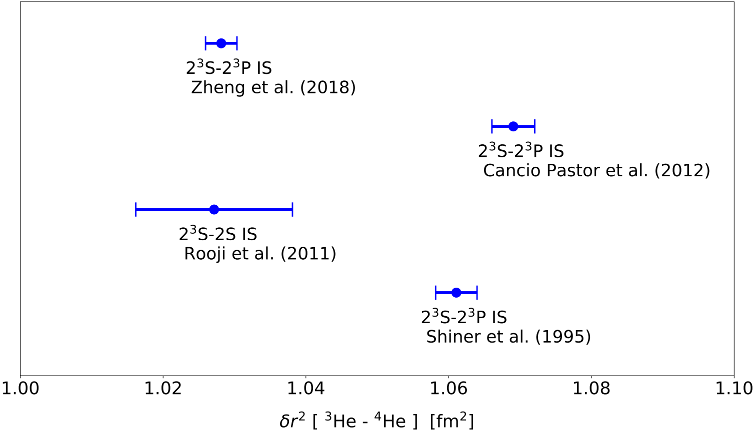
Mikhail Gorchtein† and Marc Vanderhaeghen
Institut für Kernphysik and PRISMA Cluster of Excellence, Johannes Gutenberg-Universität, Mainz, Germany
(Received 2 December 2016; published 27 January 2017)

We calculate the two-photon exchange correction to the Lamb shift in muonic ^{3}He ions within the dispersion relations framework. Part of the effort entailed making analytic fits to the electron-^{3}He quasielastic scattering data set, for purposes of doing the dispersion integrals. Our result is that the energy of the $2S$ state is shifted downwards by two-photon exchange effects by 15.14(49) meV, in good accord with the result obtained from a potential model and effective field theory calculation.

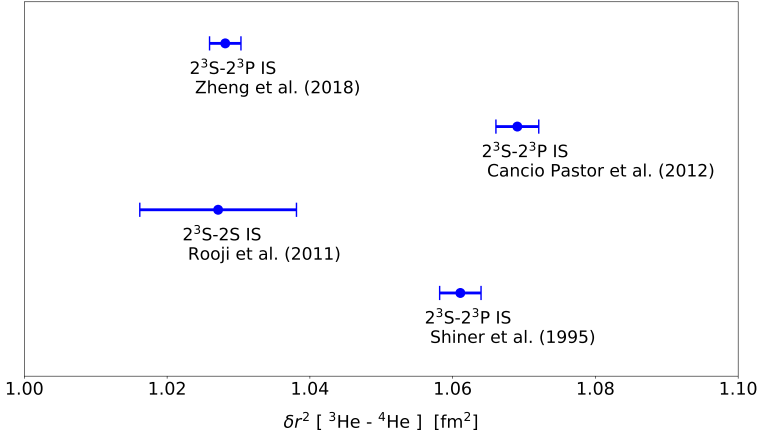
TABLE I. Individual contributions to ΔE_{2S} from two-photon exchange in μ-^{3}He, in units of meV.

Contribution	This work	Refs. [21,22]
Elastic	−10.93(27)	−10.49(24)
$\delta^N_{\rm Zem}$		−0.52(3)
Inelastic	−5.81(40)	−4.45(21)
Nuclear	−5.50(40)	−4.17(17)
Nucleon	−0.31(2)	−0.28(12)
Subtraction	1.60(12)	
Nuclear	1.39(12)	
Nucleon	0.21(3)	
Total TPE	−15.14(49)	−15.46(39)

He isotope shift puzzle



He isotope shift puzzle



μHe precision: ~ 0.03 fm^2 — thanks to correlations between ^{3,4}He calculations

Uncertainty estimates

	μ^2H			μ^3H			μ^3He$^+$			μ^4He$^+$		
	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$
Numerical	0.0	0.0	0.0	0.1	0.0	0.1	0.4	0.1	0.1	0.4	0.3	0.4
Nuclear model	0.3	0.5	0.4	1.3	2.4	1.7	0.7	1.8	1.5	3.9	4.6	4.4
ISB	0.2	0.2	0.2	0.7	0.2	0.5	1.8	0.2	0.5	2.2	0.5	0.5
Nucleon size	0.3	0.8	0.0	0.6	0.9	0.2	1.2	1.3	0.9	2.7	2.0	1.2
Relativistic	0.0	-	0.0	0.1	-	0.1	0.4	-	0.1	0.1	-	0.0
Coulomb	0.4	-	0.3	0.5	-	0.3	3.0	-	0.9	0.4	-	0.1
η-expansion	0.4	-	0.3	1.3	-	0.9	1.1	-	0.3	0.8	-	0.2
Higher $Z\alpha$	0.7	-	0.5	0.7	-	0.5	1.5	-	0.4	1.5	-	0.4
Total	1.0	0.9	0.8	2.3	2.2	2.0	4.2	2.2	2.1	5.5	5.1	4.6

Summary of our results

System	Our Ref.	Unc.	Experimental Status
$\mu\,^2$H	Phys. Lett. B '14, '18	1%	published *Science* '16
$\mu\,^4$He$^+$	Phys. Rev. Lett. '13	20% $\to$ 6%	measured, unpublished
$\mu\,^3$He$^+$	Phys. Lett. B '16	20% $\to$ 4%	measured, unpublished
$\mu\,^3$H	Phys. Lett. B '16	2%	measurable?

- Our results agree with other values and are more accurate

$\Rightarrow$ Unc. comparable with $\sim \mathbf{5\%}$ experimental needs

$\Rightarrow$ Will improve precision of $\boldsymbol{R_c}$ from Lamb shifts

$\Rightarrow$ May help shed light on the "proton (deuteron) radius puzzle"

$\Rightarrow$ ... and on the ^{3,4}He "isotope-shift puzzle"

The work is not completed yet ...

PHYSICAL REVIEW A **94**, 062505 (2016)

Lamb shift in muonic ions of lithium, beryllium, and boron

A. A. Krutov, A. P. Martynenko, F. A. Martynenko, and O. S. Sukhorukova

Samara University, 443086, Moskovskoe shosse 34, Samara, Russia

(Received 1 October 2016; published 15 December 2016)

We present a precise calculation of the Lamb shift $(2P_{1/2} - 2S_{1/2})$ in muonic ions $(\mu^6_3\mathrm{Li})^{2+}$, $(\mu^7_3\mathrm{Li})^{2+}$, $(\mu^9_4\mathrm{Be})^{3+}$, $(\mu^{10}_4\mathrm{Be})^{3+}$, $(\mu^{10}_5\mathrm{B})^{4+}$, $(\mu^{11}_5\mathrm{B})^{4+}$. The contributions of orders $\alpha^3 \div \alpha^6$ to the vacuum polarization, nuclear structure and recoil, and relativistic effects are taken into account. Our numerical results are consistent with previous calculations and improved by additional corrections. The obtained results can be used for the comparison with future experimental data, and extraction more accurate values of nuclear charge radii.

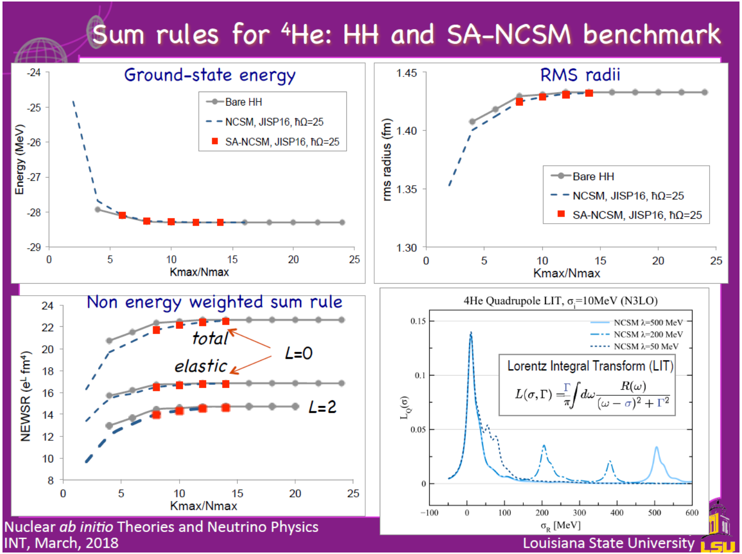
TABLE I. Lamb shift $(2P_{1/2} - 2S_{1/2})$ in muonic ions $(\mu^7_3\mathrm{Li})^{2+}$ and $(\mu^6_3\mathrm{Li})^{2+}$. In parentheses are given the results obtained by other authors, with some references to their works, which discuss the calculation of corrections of this type.

	Contribution to the splitting	$(\mu^7_3\mathrm{Li})^{2+}$ (meV)	$(\mu^6_3\mathrm{Li})^{2+}$ (meV)
1	VP contribution of order $\alpha(Z\alpha)^2$ in 1γ interaction	4682.38 (4682.4 [7])	4664.95 (4665.0 [7])
2	Two-loop VP contribution of order $\alpha^2(Z\alpha)^2$ in 1γ interaction	32.54 (32.44 [7])	32.41(32.27 [7])
3	VP and MVP contribution in one-photon interaction	0.01	0.01
	⋮		
30	HVP contribution	1.17 [7,58–60]	1.16 [7,58–60]
31	Nuclear polarizability	21 ± 4 [7]	15 ± 4 [7]
32	Total contribution	1531.78	1161.85

SA-NCSM-HH-LSR Benchmark

- ^{4}He: $J^{\pi} = 0^{+}$
- N3LO, JISP16
- Quadrupole (E2): IS+IV
  $\rightarrow$ Dipole (E1) & Isoscalar Monopole (E0)

Preliminary Results: ^{4}He



The proton radius puzzle is unsolved despite new data.
More experiments are underway.

Nuclear corrections are the bottleneck in μA spectroscopy.
Extracting them from data was imprecise.
Ab-initio calculations with the LSR method provide precise and efficient results.
We obtained the best nuclear corrections for $A \leqslant 4$.

Our hyperspherical harmonics ^{4}He sum rules are benchmarked with SA-NCSM.
This will allow calculations for $\mu-$A, with $6 \leqslant A$,
as well as other sum rules in heavy open-shell nuclei.

We continue to develop new methods to study and improve these results and related problems (e.g., HFS)

TRIUMF

Canada's national laboratory
for particle and nuclear physics
and accelerator-based science

Thank you!
Merci!