Pion Electroproductions

-- Analysis of electron data and extraction of elementary amplitudes --

Satoshi Nakamura

Universidade Cruzeiro do Sul (Brazil)

Introduction

How electron scattering is relevant to neutrino scattering

Relation between neutrino and electron (photon) interactions

Charged-current (CC) interaction (e.g. $v_{\mu} + n \rightarrow \mu^{-} + p$)

$$L^{cc} = \frac{G_F V_{ud}}{\sqrt{2}} [J_{\lambda}^{cc} \ell_{cc}^{\lambda} + h.c.] \qquad J_{\lambda}^{cc} = V_{\lambda} - A_{\lambda} \qquad \ell_{cc}^{\lambda} = \overline{\psi}_{\mu} \gamma^{\lambda} (1 - \gamma_5) \psi_{\nu}$$

Electromagnetic interaction (e.g. $\gamma^{(*)} + p \rightarrow p$)

$$L^{em} = e J_{\lambda}^{em} A_{em}^{\lambda} \qquad \qquad J_{\lambda}^{em} = V_{\lambda} + V_{\lambda}^{IS}$$

V and *V*^{*IS*} in *J*^{*em*} can be separately determined by analyzing photon for $Q^2=0$ and electron reaction (*e*,*e*' π), (*e*,*e*' X) data for $Q^2 \neq 0$ on both proton and neutron targets, because:

$$= - < n | V_{\lambda} | n > \qquad = < n | V_{\lambda}^{IS} | n >$$

Matrix element for the weak vector current is obtained from analyzing electromagnetic processes

$$= \sqrt{2}$$

Strategy to develop neutrino-nucleon model

• Vector current (form factor) is fixed by analyzing electron scattering data

 \leftarrow abundant precise data are available

- Axial current amplitude can be determined by analyzing:
 - -- Neutrino-nucleon, nucleus data
 - -- Parity-violating inclusive electron scattering data

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(neutrino-deuteron pion data \leftarrow T. Sato's talk)

-- Parity-violating inclusive electron scattering data in the nucleon resonance region (later in this talk)

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• Our analysis is done with dynamical coupled-channels (DCC) model

In the following :

* DCC model * DCC analysis of pion electroproduction data * Comparison with other models * PV electron scattering

Dynamical Coupled-Channels model

Unified description of pion-, electron-, and neutrino-induced meson productions

Dynamical coupled-channels model for meson productions in resonance region



Data for
$$\gamma p 🏓 X$$

Need develop a model to describe these reactions

- Several nucleon resonances form characteristic peaks
- 2π production is comparable to 1π
- η , K productions (multi-channel couplings are important physics)

Dynamical coupled-channels model for resonance region

Resonance excitation + non-resonant meson-exchange mechanisms



Theoretically sound model should also account for:

- Channel-couplings required by unitarity (πN , ηN , $K\Lambda$, $K\Sigma$ stable channels)
- 2 π production mechanisms ($\rho N, \sigma N, \pi \Delta \leftrightarrow \pi \pi N$ channels)

Dynamical Coupled-Channels (DCC) model accounts for these features developed through analyzing data for γN , $\pi N \rightarrow \pi N$, ηN , $K\Lambda$, $K\Sigma \sim 26,000$ data points

Matsuyama et al., Phys. Rep. **439**, 193 (2007) Kamano et al., PRC 88, 035209 (2013)

Coupled-channel Lippmann-Schwinger equation for meson-baryon scattering

$$T_{ab} = V_{ab} + \sum_{c} V_{ac} G_{c} T_{cb}$$

$$\{a, b, c\} = \pi N, \ \eta N, \ \pi \pi N, \ \pi \Delta, \sigma N, \rho N, \ K\Lambda, \ K\Sigma$$

Coupled-channels and hadron rescattering required by unitarity is fully taken into account In addition, γN channel is included perturbatively

Matsuyama et al., Phys. Rep. **439**, 193 (2007) Kamano et al., PRC 88, 035209 (2013)

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for stable channels



for unstable channels

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In addition, γN channel is included perturbatively



γр → π⁰р

$d\sigma/d\Omega$ for W < 2.1 GeV

Comparison of DCC model with data

Kamano, Nakamura, Lee, Sato, PRC 88 (2013)

Reasonable fit to data in the whole resonance region

Vector current (Q²=0) for 1π Production is well-tested by data



Partial wave amplitudes of π N scattering



Partial wave amplitudes of π N scattering



Analysis of pion electroproduction data with dynamical coupled-channels model

SXN, H. Kamano, and T. Sato, Phys. Rev. D92, 074024 (2015)

Cross section for single pion electroproduction



 x^* : variables in CM of the final πN system



 σ_X (X = T, L, LT, LT', TT) : structure functions h : electron helicity

Only $\sigma_T + \varepsilon \sigma_L$ contributes to the integrated cross section

 \rightarrow Most important structure functions

Analysis of electron-proton scattering data

Purpose : Determine Q^2 – dependence of vector coupling of p- N^* : $VpN^*(Q^2)$

Data : * 1π electroproduction



Database

- $p(e,e'\pi^0)p$
- *p(e,e'π*+)*n*
- both

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Analysis of electron-proton scattering data

Purpose : Determine Q^2 – dependence of vector coupling of p- N^* : $VpN^*(Q^2)$

- Data : * 1π electroproduction
 - * Empirical inclusive inelastic structure functions σ_T , σ_L \leftarrow Christy et al, PRC 81 (2010)



Database

- $p(e,e'\pi^0)p$
- p(e,e'π⁺)n
- both

region where inclusive $\sigma_T \& \sigma_L$ are fitted

 Q^2 =0.40 (GeV/c)²

 $\sigma_T + \varepsilon \sigma_L$ for W=1.1-1.68 GeV

 $p(e,e'\pi^0)p$

 $p(e,e'\pi^+)n$



 Q^2 =0.40 (GeV/c)²



W (MeV)

 Q^2 =1.76 (GeV/c)²

 $\sigma_T + \varepsilon \sigma_L$ for W=1.1-2.1 GeV



 $p(e,e'\pi^0)p$





 Q^2 =1.76 (GeV/c)²





 Q^2 =2.95 (GeV/c)²

 $\sigma_T + \varepsilon \sigma_L$ for W=1.11-1.67 GeV



$p(e,e'\pi^+)n$





 Q^2 =2.95 (GeV/c)²





Inclusive electron-proton scattering



Data: JLab E00-002 (preliminary)

- Reasonable fit to data for application to neutrino interactions
- Important 2π contributions for high W region

Analysis of electron-'neutron' scattering data

Purpose : Vector coupling of neutron- N^* and its Q^2 -dependence : $VnN^*(Q^2)$ (I=1/2) I=3/2 part has been fixed by proton target data

- Data : * 1π photoproduction (Q^2 =0)
 - * Empirical inclusive inelastic structure functions σ_T , σ_L ($Q^2 \neq 0$)

Christy and Bosted, PRC 77 (2010), 81 (2010)
 fitted to electron-deuteron data and electron-proton structure functions are subtracted





 $Q^2 = 0$

 $d\sigma / d\Omega$ ($\gamma n \Rightarrow \pi^0 n$) for W=1.2-1.9 GeV





Inclusive electron-deuteron scattering



• Model \rightarrow average of electron-proton and electron-neutron differential cross sections

 \rightarrow nuclear effects (Fermi motion, FSI) are ignored

• Spread of resonance width in data \rightarrow Fermi motion

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DCC vector currents has been tested by of $e^-p \& e^-$ 'n' data for whole kinematical region relevant to neutrino interactions of $E_V \le 2 \text{ GeV} \Rightarrow$ isospin separation \Rightarrow weak vector current

EM $N \rightarrow N^*$ transition form factors



• Evaluated at the resonance pole position \rightarrow form factors are complex

EM $N \rightarrow N^*$ transition form factors



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Comparison with other models

Hernandez, SXN, Nieves, Sato, Sobczyk, in preparation

The SL model

T. Sato and T. S. H. Lee, PRC 54, 2660 (1996) T. Sato and T. S. H. Lee, PRC 63, 055201 (2001)

The SL model describes $\pi N \rightarrow \pi N$ scattering and electroweak pion production in the $\Delta(1232)$ -region is reasonably described in a unified manner

The model takes account of:

- $\Delta(1232)$ -excitation mechanisms
- Meson-exchange non-resonant mechanisms
- πN channel is included
- Hadron rescattering and πN unitarity

The DCC model can be viewed as an extension of the SL model by including other meson-baryon and $\pi\pi N$ channels and resonances

The HNV model

 Hernandez, Nieves, Valverde,
 PRD 76, 033005 (2007)

 Alvarez-Ruso et al.,
 PRD 93, 014016 (2016)

 Hernandez and Nieves
 PRD 95, 053007 (2017)









- Resonance-excitation mechanisms $\Delta(1232)$ and $D_{13}(1520)$
- Non-resonant mechanisms derived from chiral Lagrangian
- Hadron rescattering is not explicitly considered
 - \rightarrow phases are multiplied to P_{33}

amplitude to satisfy

the Watson theorem (unitarity)

• *u*-channel Δ propagator is modified to fit better $v_{\mu} n \rightarrow \mu^{-} \pi^{+} n$ data

Comparison of DCC, SL, and HNV models

Hernandez, SXN, Nieves, Sato, Sobczyk, in preparation

Inclusive electron-proton scattering

E=730 MeV $\theta' = 37.1^{\circ}$ Data : PRL 53, 1627 (1984) 1.2DCĊ $d\sigma/d\Omega' dE' [\mu b/GeV sr]$ $Q^2 = 0.04 - 0.18 \text{ GeV}^2$ HN 0.80.6 0.40.2 $\left(\right)$ 1.051.151.21.251.3 1.351.1 1.4 $W_{\pi N}$ [GeV]

- Good agreement between models and data at the $\Delta(1232)$ peak
- SL (HNV) model gives smaller cross sections higher (lower) energies

Comparison (cont'd)

Single pion electroproduction

$$\frac{d\sigma_{em}}{d\Omega' dE' d\Omega_{\pi}^{*}} = \Gamma_{em} \Big\{ \sigma_{T} + \varepsilon \, \sigma_{L} + \sqrt{2\varepsilon(1+\varepsilon)} \\ \times \sigma_{LT} \, \cos \phi_{\pi}^{*} + h \sqrt{2\varepsilon(1-\varepsilon)} \, \sigma_{LT'} \, \sin \phi_{\pi}^{*} \\ + \varepsilon \, \sigma_{TT} \, \cos 2\phi_{\pi}^{*} \Big\}$$

- Good agreement between models and data for most of structure functions
- HNV model gives rather small σ_{LT}, for p (e,e' π⁰) p
 ← σ_{LT}, is from non-zero relative phase between different mechanisms and form factors
 ← DCC and SL models include meson-loops to produce additional phases; HNV does not



Parity-violating electron-nucleon scattering and axial form factors

Inclusive electron-proton scattering ($e^-p \rightarrow e^-X$)



Differential cross section with respect to lepton kinematics

$$\frac{d\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4} \left[2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right]$$

 $W^{
m em}_i(W, Q^2)$: structure functions (all information of hadron dynamics encoded in)

$$W_{1}^{\text{em}} = \frac{1}{2} \sum_{i}^{\overline{\sum}} \sum_{f} \left(\left| \left\langle f \left| J_{x}^{\text{em}} \right| i \right\rangle \right|^{2} + \left| \left\langle f \left| J_{y}^{\text{em}} \right| i \right\rangle \right|^{2} \right) \delta^{(4)}(p_{i} + q - p_{f})$$

$$W_{2}^{\text{em}} = \frac{Q^{2}}{\left| \vec{q} \right|^{2}} W_{1}^{\text{em}} + \frac{Q^{2}}{\left| \vec{q}_{c} \right|^{2}} \frac{Q^{2}}{\left| \vec{q}_{c} \right|^{2}} \sum_{i}^{\overline{\sum}} \sum_{f} \left| \left\langle f \left| J_{0}^{\text{em}} \right| i \right\rangle \right|^{2} \delta^{(4)}(p_{i} + q - p_{f})$$

+





$$A = \frac{d\sigma(h_e = +1) - d\sigma(h_e = -1)}{d\sigma(h_e = +1) + d\sigma(h_e = -1)} = -\frac{Q^2 G_F}{\sqrt{2}(4\pi\alpha)} \frac{N}{D}$$

$$D = \cos^2 \frac{\theta}{2} W_2^{\text{em}} + 2\sin^2 \frac{\theta}{2} W_1^{\text{em}}$$
$$N = \cos^2 \frac{\theta}{2} W_2^{\text{em-nc}} + 2\sin^2 \frac{\theta}{2} W_1^{\text{em-nc}} + \sin^2 \frac{\theta}{2} (1 - 4\sin^2 \theta_W) \frac{E + E'}{M} W_3^{\text{em-nc}}$$

+





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$$J_{\rm nc}^{\mu} = (1 - 2\sin^2\theta_W) J_{\rm em}^{\mu} - V_{\rm isoscalar}^{\mu} - A_3^{\mu}$$

+





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$$J_{\text{nc}}^{\mu} = (1 - 2 \sin^{2} \theta_{W}) J_{\text{em}}^{\mu} - V_{\text{isoscalar}}^{\mu} - A_{3}^{\mu} \qquad (1 - 2 \sin^{2} \theta_{W}) D + \left(\cos^{2} \frac{\theta}{2} W_{2}^{\text{em-is}} + 2 \sin^{2} \frac{\theta}{2} W_{1}^{\text{em-is}}\right)$$





$$A = -Q^2 \frac{G_F}{\sqrt{2} 4\pi\alpha} \left(2 - 4\sin^2\theta_W + \Delta_V + \Delta_A \right)$$

$$\Delta_V = \frac{\cos^2 \frac{\theta}{2} W_2^{\text{em-is}} + 2\sin^2 \frac{\theta}{2} W_1^{\text{em-is}}}{D}$$
$$\Delta_A = \frac{\sin^2 \frac{\theta}{2} (1 - 4\sin^2 \theta_W) \frac{E + E'}{M} W_3^{\text{em-nc}}}{D}$$





Parity-violating asymmetry

$$A = -Q^{2} \frac{G_{F}}{\sqrt{2} 4 \pi \alpha} \left(2 - 4 \sin^{2} \theta_{W} + \Delta_{V} + \Delta_{A} \right)$$

$$\approx 8.99 \times 10^{-5} (\text{GeV}^{-2}) \approx 1.075 \text{ (main term)}$$

$$\Delta_V = \frac{\cos^2 \frac{\theta}{2} W_2^{\text{em-is}} + 2\sin^2 \frac{\theta}{2} W_1^{\text{em-is}}}{D}$$
$$\Delta_A = \frac{\sin^2 \frac{\theta}{2} (1 - 4\sin^2 \theta_W) \frac{E + E'}{M} W_3^{\text{em-nc}}}{D}$$

 $1 - 4\sin^2\theta_W \approx 0.08$





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 $1 - 4\sin^2\theta_w \approx 0.08$

•
$$2W_3^{\text{em-nc}} = W_3^{\text{CC}}$$

 $\propto \langle f | V_{\text{iso-vector}} | i \rangle \langle f | A_{\text{iso-vector}} | i \rangle^*$

- → PV asymmetry data for backward electron kinematics can measure
 W₃ for neutrino CC process and axial matrix element (form factors)
- Δ_V is proportional to isoscalar current \rightarrow small in $\Delta(1232)$ region

$$A / Q^{2} = -89.9 \times 10^{-6} (1.075 + \Delta_{V} + \Delta_{A})$$
[1/GeV²]

$$\Delta_V = \frac{\cos^2 \frac{\theta}{2} W_2^{\text{em-1s}} + 2\sin^2 \frac{\theta}{2} W_1^{\text{em-1s}}}{D},$$
$$\Delta_A = \frac{\sin^2 \frac{\theta}{2} (1 - 4\sin^2 \theta_W) \frac{E + E'}{M} W_3^{\text{em-nc}}}{D}.$$



 $p(\vec{e}, e'), W = 1.232 GeV$

 $\Delta_{\!A}$ gives ~10% correction to A

Sensitivity of PV asymmetry to $N \rightarrow \Delta(1232)$ axial form factor



- 5% precision PV asymmetry data may discriminate 0.2 GeV difference in the axial mass
- Sensitivity to $N \rightarrow N^*$, Δ^* (higher resonances) transition axial form factors can be studied with the DCC model

Comparison with PV asymmetry data from JLab



$$A = \frac{d\sigma(h_e = +1) - d\sigma(h_e = -1)}{d\sigma(h_e = +1) + d\sigma(h_e = -1)}$$
$$A/Q^2 = -89.9 \times (1.075 + \Delta_V + \Delta_A)$$
[10⁻⁶/GeV²]

30-50% precision data for A already exist
→ Event rate measured at 0.3% precision

Deuteron target data

- → proton and neutron cross sections are simply summed in calculation
- Forward electron kinematics \rightarrow axial current hardly contribute
- Deviation from data in $\Delta(1232)$ may be from nuclear effects (FSI, Fermi motion, etc.)
- Deviations in higher W region \rightarrow calling improvement on the model (isospin separation)

Conclusion

Extracting elementary vector amplitudes (transition form factors) with DCC model from electron scattering data in resonance region

Start with DCC model developed through analyzing γN , $\pi N \rightarrow \pi N$, $\pi \pi N$, ηN , $K\Lambda$, $K\Sigma$

- → extension of vector current to $Q^2 \neq 0$ region through analysis of $e^- p$ & $e^- in'$ data for $W \leq 2$ GeV, $Q^2 \leq 3$ (GeV/c)²
- \rightarrow isospin separaton \rightarrow neutrino-induced reactions
 - $N \rightarrow N^*$ transition form factors are determined

Detailed comparison of state-of-art elementary pion production models

Structure functions for pion electroproduction from DCC, SL and HNV models are compared

- Good agreement between models and data for most of structure functions
- HNV model gives σ_{LT}, for p (e,e' π⁰) p significantly smaller than data
 DCC and SL models reproduce the data because meson-loops can generate phase

Parity-violating electron-nucleon scattering for extracting elementary axial amplitudes (transition form factors)

- PV asymmetry data for backward electron kinematics can measure W_3 for neutrino CC process and axial elementary amplitude (form factors)
- 5% precision PV asymmetry data may discriminate 0.2 GeV difference in the axial mass
- Current data precision is 30-50% for PV asymmetry (0.3% precision event rate)
- Need estimate radiative corrections

Thank you very much for your attention

Acknowledgments

- Financial support for this work
 FAPESP 2016/15618-8
 KAKENHI JP25105010
- Computing resource
 NERSC
 BEBOP, LCRC, Argonne National Lab



$\gamma n \rightarrow \pi^0 n \mid d\sigma/d\Omega$ for W < 2 GeV





: new fit

Recent MAMI data included PRL 112, 142001 (2014)





Recent JLab data included PRC 96, 035204 (2017)