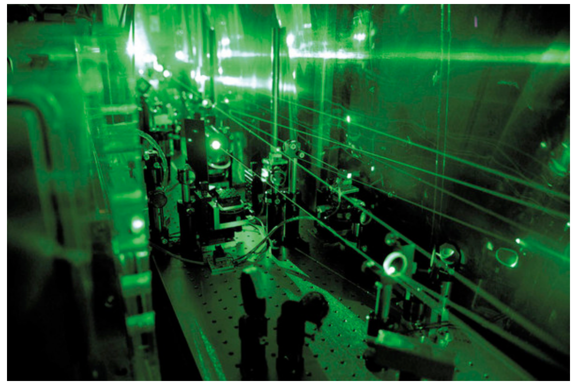
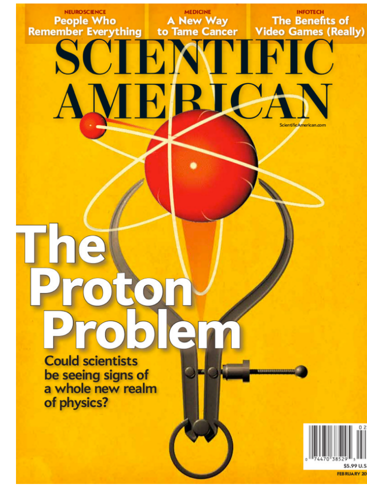
The Proton Radius Puzzle- Why we all should care

Gerald A. Miller, University of Washington

Feb. 2014

Pohl et al Nature 466, 213 (8 July 2010)





4 % Difference

$$r_p^2 \equiv -6 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

Pohl, Gilman, Miller, Pachucki (ARNPS63, 2013)

C Carlson PPNP 82,59(2015)

muon H r_p =0.84184 (67) fm Small

electron H r_p =0.8768 (69)fm Large

electron-p scattering r_p =0.875 (10)fm

PRad at JLab- lower Q^2

$$3 \times 10^4 \leq Q^2 \leq 5 \times 10^{-2}\ \mathrm{GeV}^2$$

$$7.7 \times 10^{-3} \leq Q^2 \leq 0.13\,\mathrm{fm}^{-2}$$

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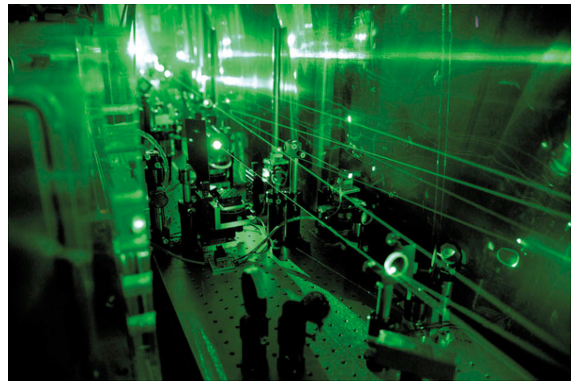

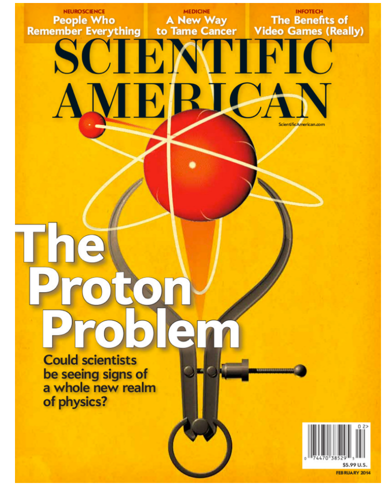

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4 % in radius: why care?

- Can't be calculated to that accuracy?

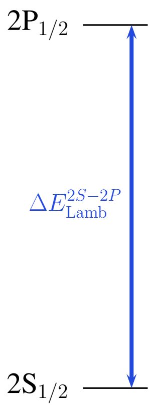
Is the muon-proton interaction the same as the electron-proton interaction?
violation of universality
connections with muon g-2?
connections with LHCb ?

Something happening here ???

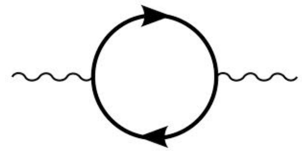
Kaons ? Kohl talk

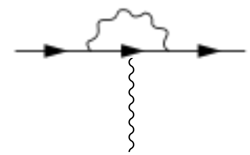
Outline - a) review history experiments
b) List & explain possible resolutions

Muonic hydrogen experiment and r_p



The Lamb shift is the splitting of the degenerate $2S_{1/2}$ and $2P_{1/2}$ eigenstates

Dominant in μH  vacuum polarization 205 of 206 meV

Dominant in eH  electron self-energy

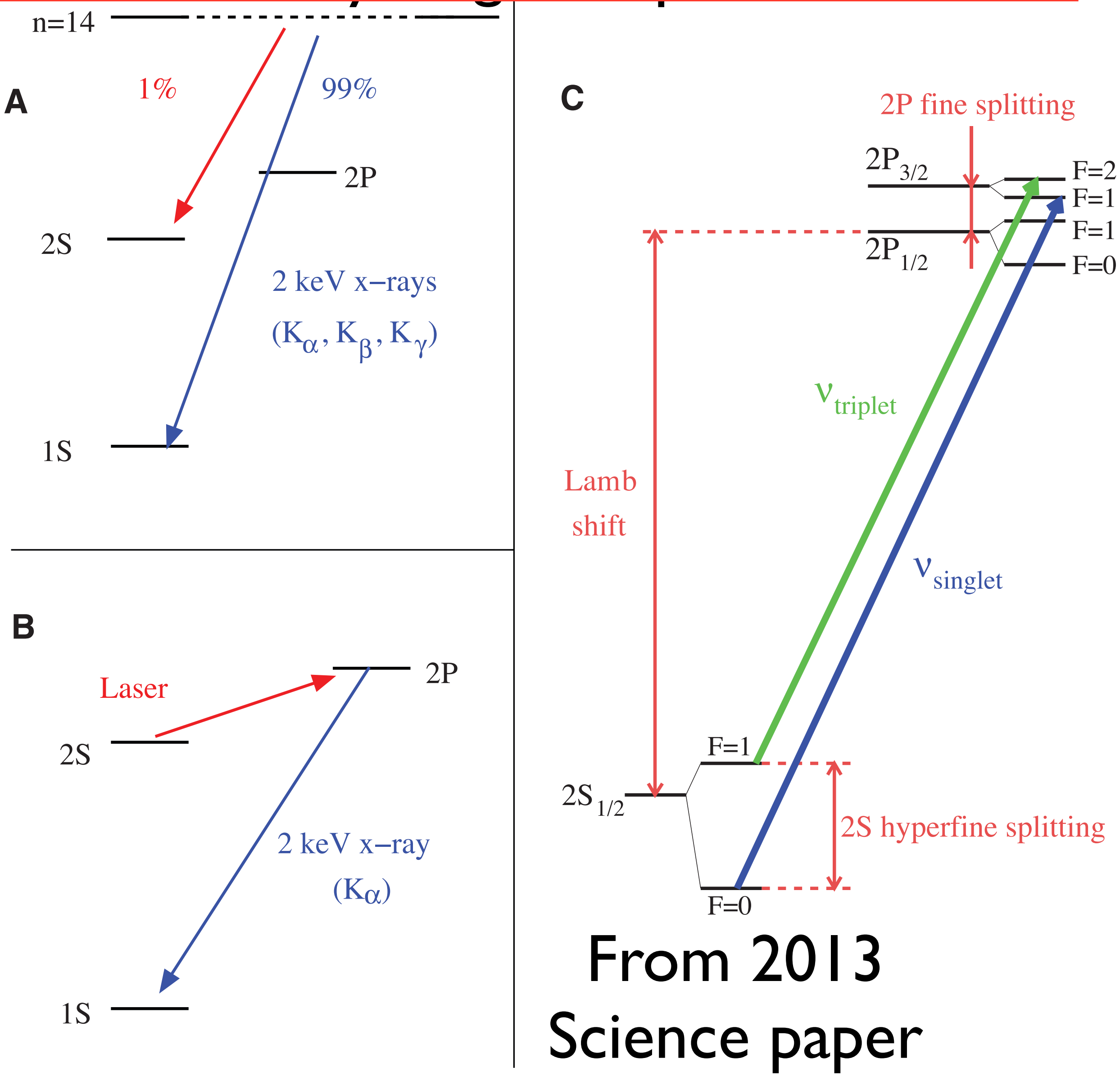
Proton radius in Lamb shift

$$\Delta E = \langle \Psi_S | V_C - V_C^{\mathrm{pt}} | \Psi_S \rangle = \frac{2}{3}\pi\alpha|\Psi_S(0)|^2 \underline{(-6G'_E(0))}$$

$= r_p^2$

Muon/electron mass ratio 205! 8 million times larger for muon

Muonic Hydrogen Experiment



From 2013 Science paper

Fig. 1. (**A**) Formation of μp in highly excited states and subsequent cascade with emission of "prompt" $K_{\alpha,\,\beta,\,\gamma}$. (**B**) Laser excitation of the 2S-2P transition with subsequent decay to the ground state with K_α emission. (**C**) 2S and 2P energy levels. The measured transitions ν_s and ν_t are indicated together with the Lamb shift, 2S-HFS, and 2P-fine and hyperfine splitting.

Randolf Pohl

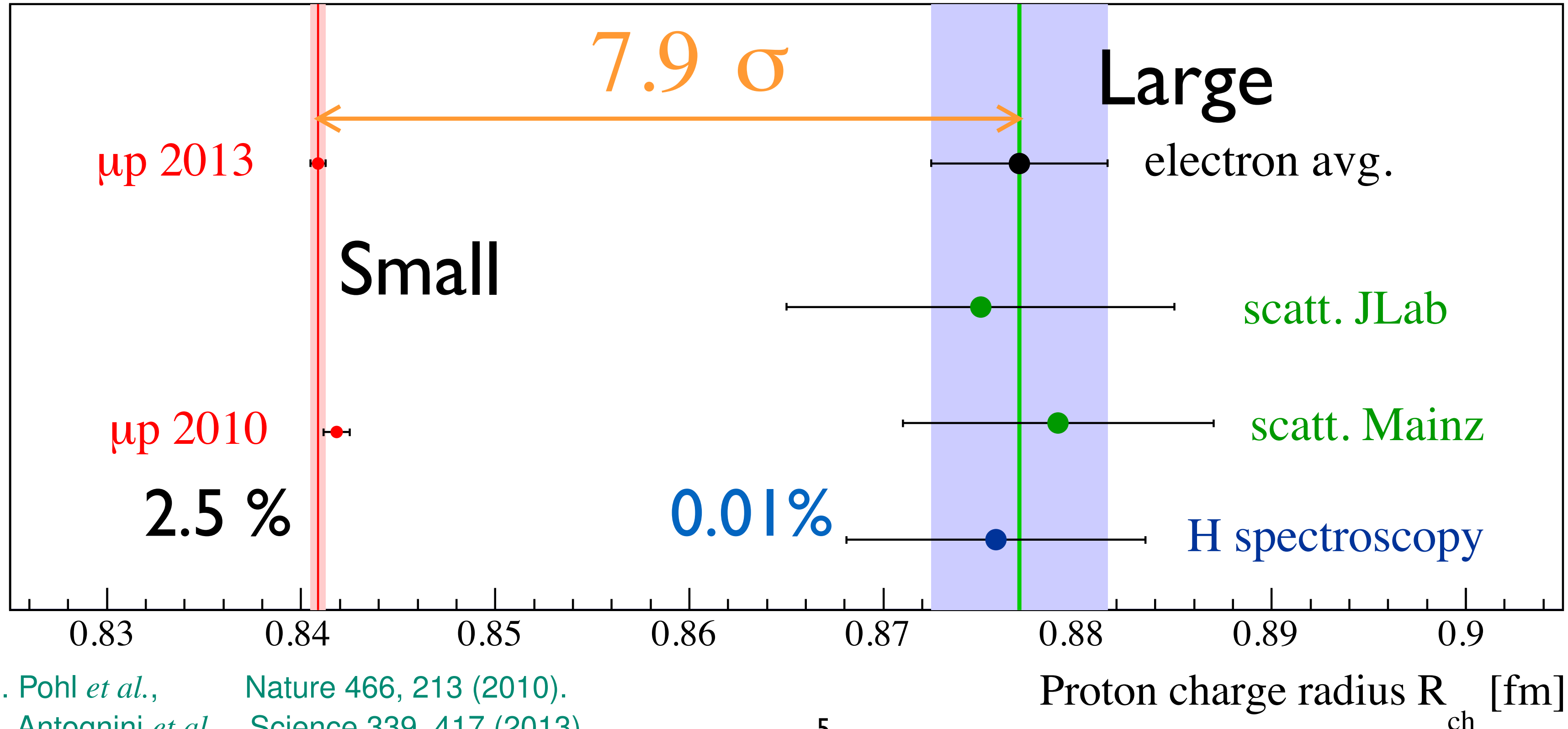
Aldo Antognini

MPQ

The proton rms charge radius measured with

electrons: 0.8770 ± 0.0045 fm

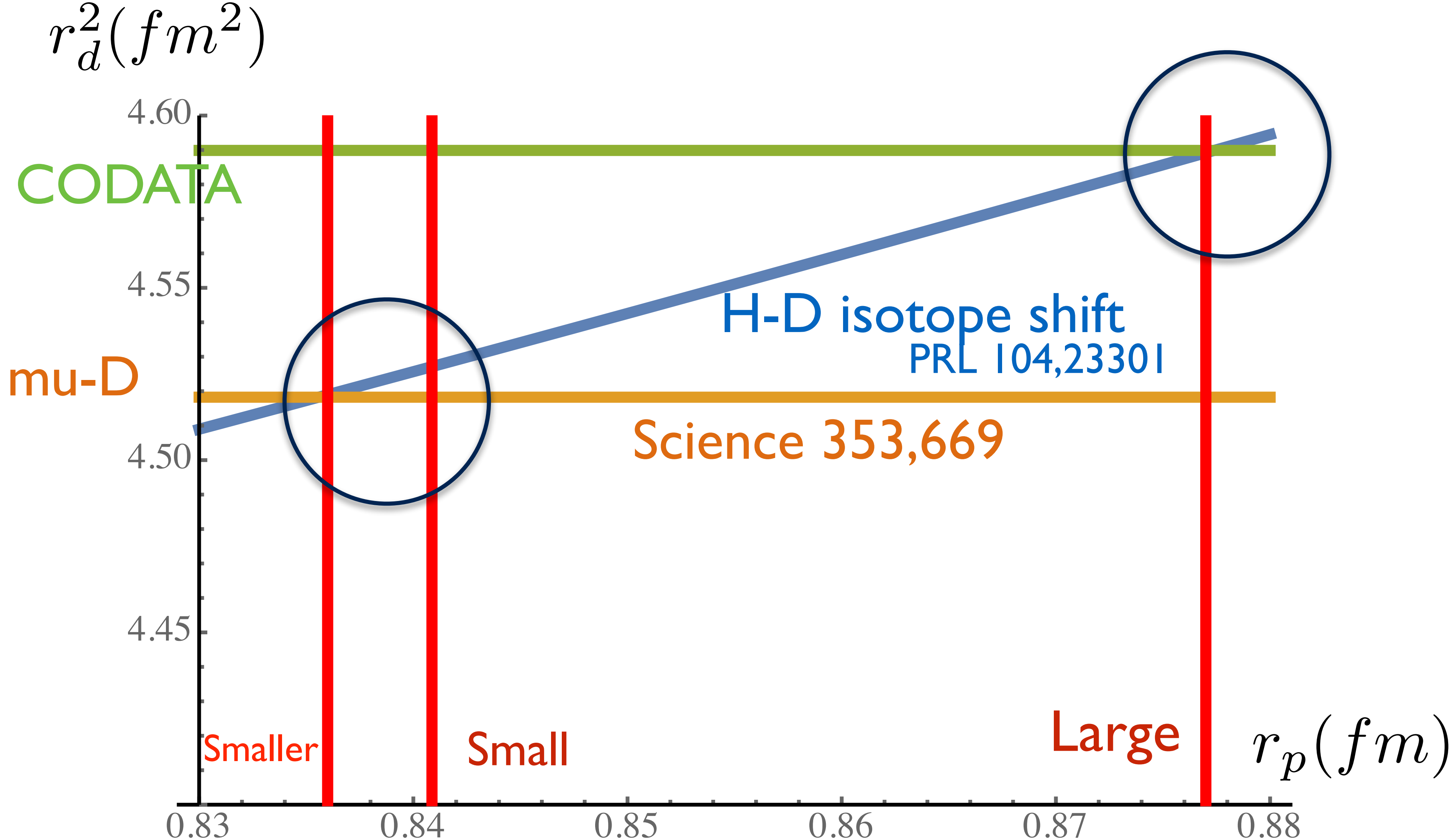
muons: 0.8409 ± 0.0004 fm



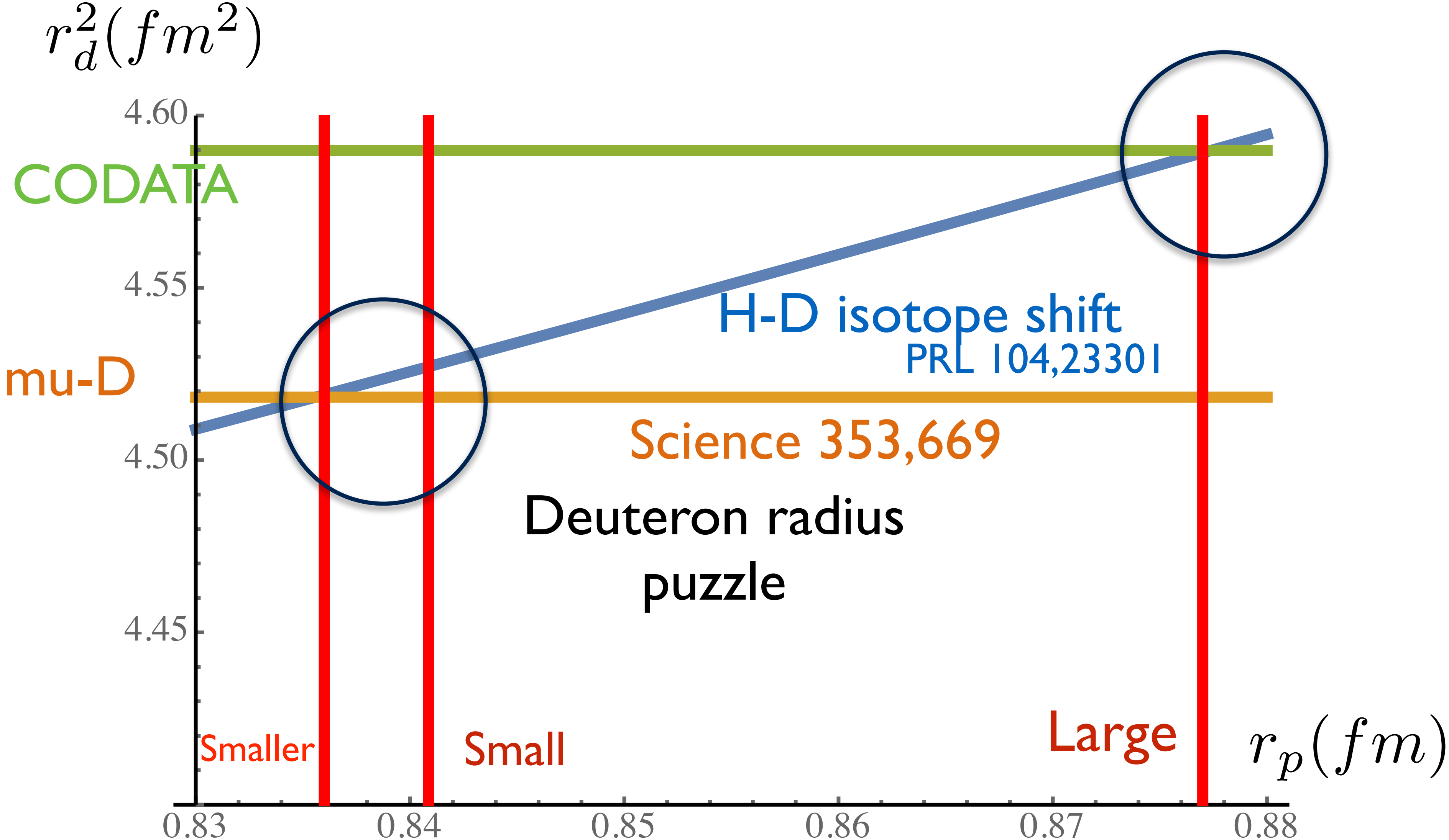
R. Pohl *et al.*, Nature 466, 213 (2010).
A. Antognini *et al.*, Science 339, 417 (2013).

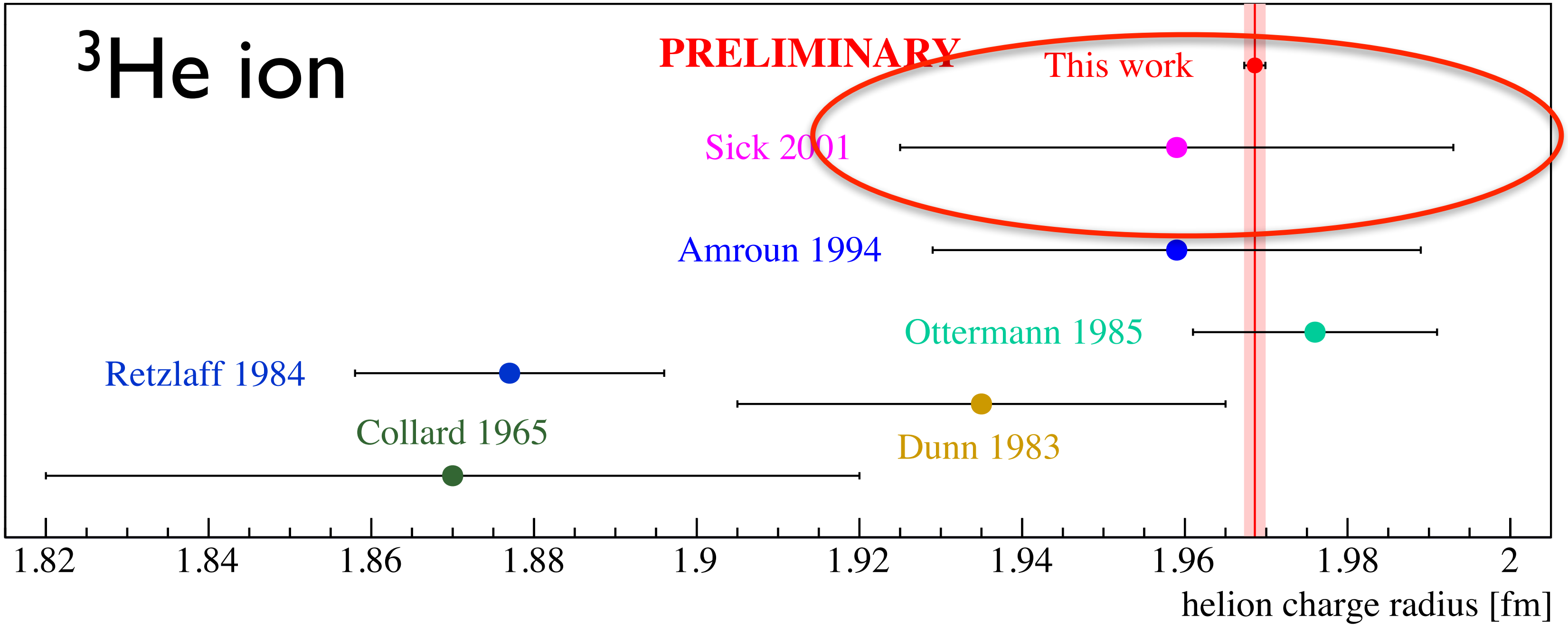
5

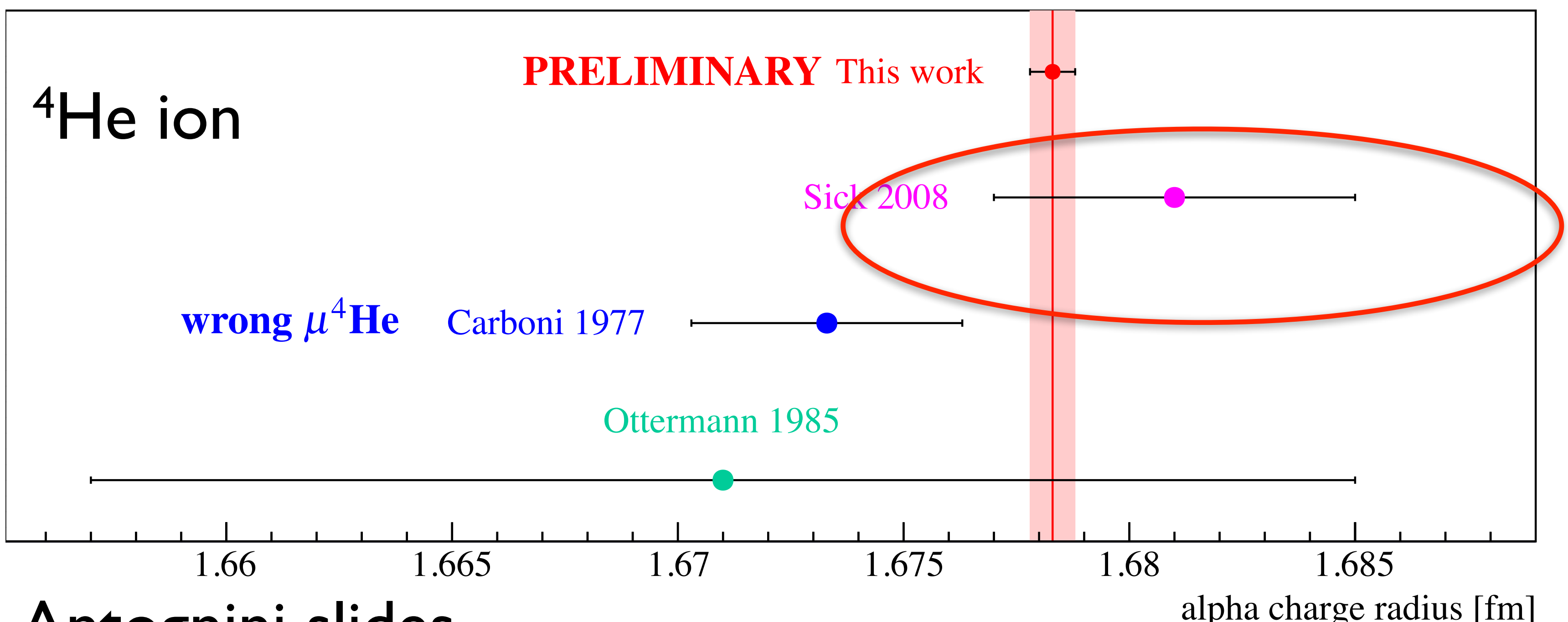
Deuteron is smaller too



Deuteron is smaller too







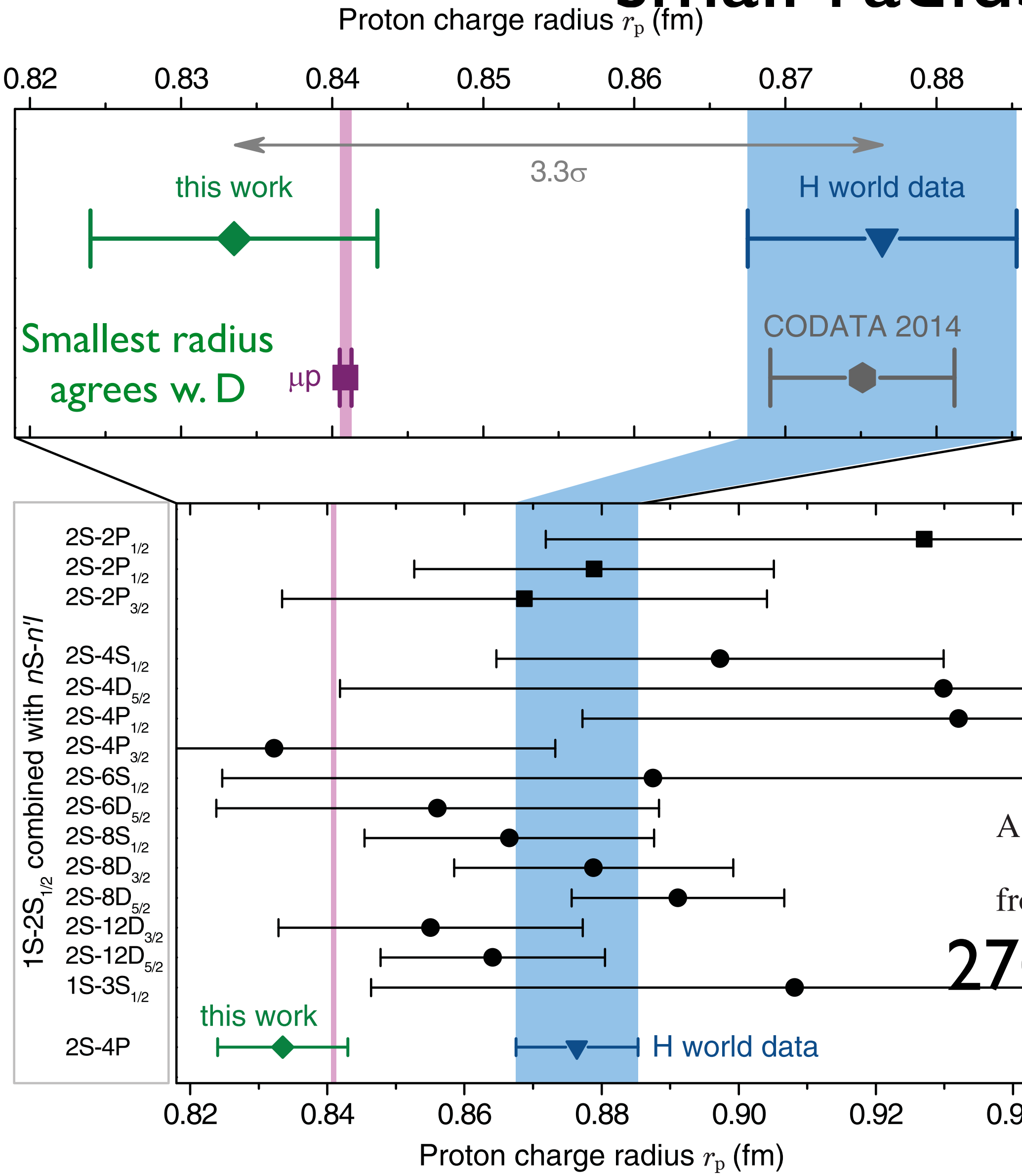
A. Antognini slides

nuclear structure Barnea et al

So far

- effect in the proton
- effect in the deuteron, implies effect on neutron
- no effect in 3He (large error bars)
- no effect in 4He (small error bars)
- any explanation must account for the above
- how would you react to this?

Beyer et al Science 358,79 electron H - small radius



2S-4P transition

take 2017 result -no puzzle

use old eH- puzzle as before

one more new eH needed

go halfway

Average over two levels

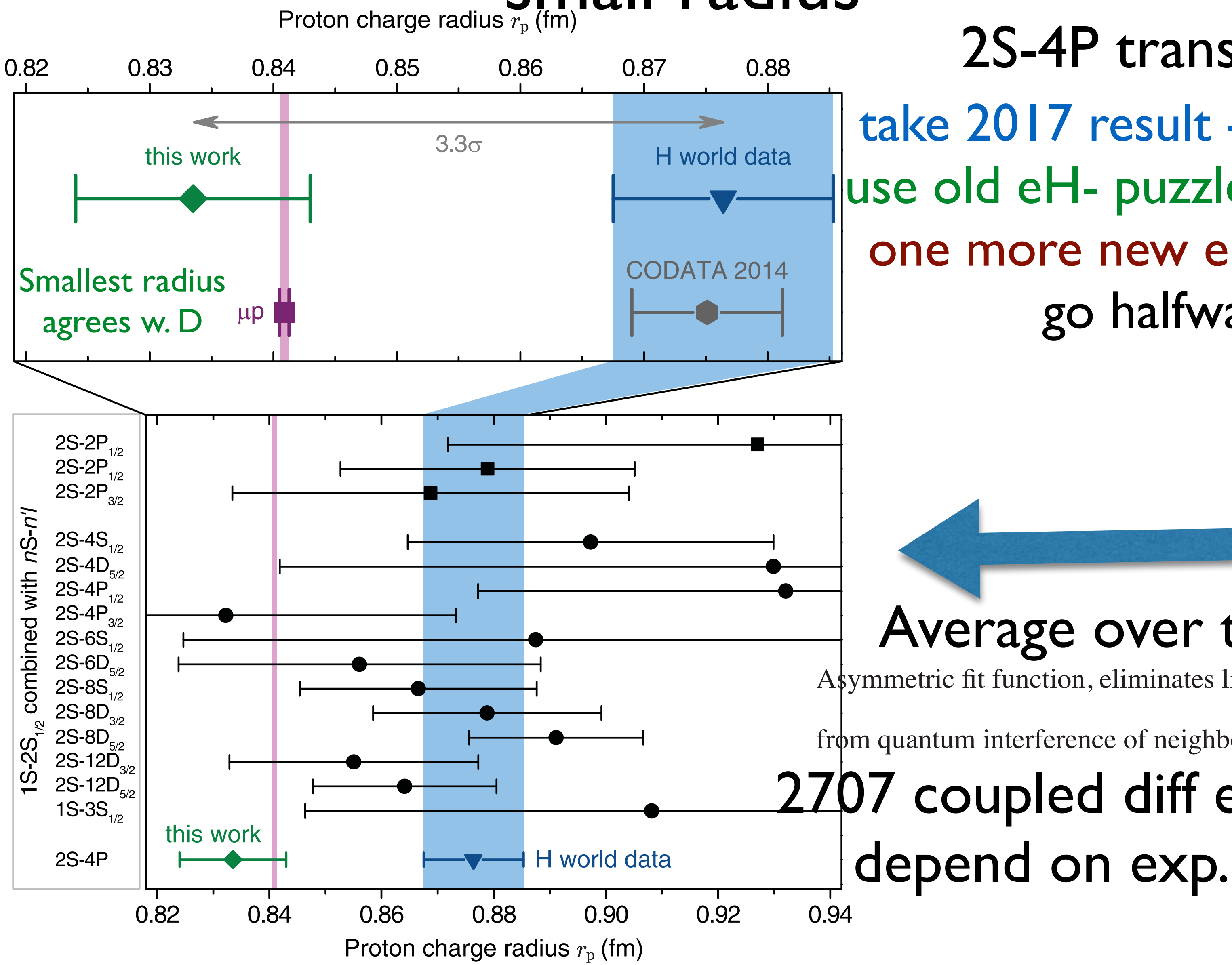
Asymmetric fit function, eliminates line shifts

from quantum interference of neighboring atomic resonanc

2707 coupled diff eqs solved

depend on exp. set up

Beyer et al Science 358,79 electron H - small radius



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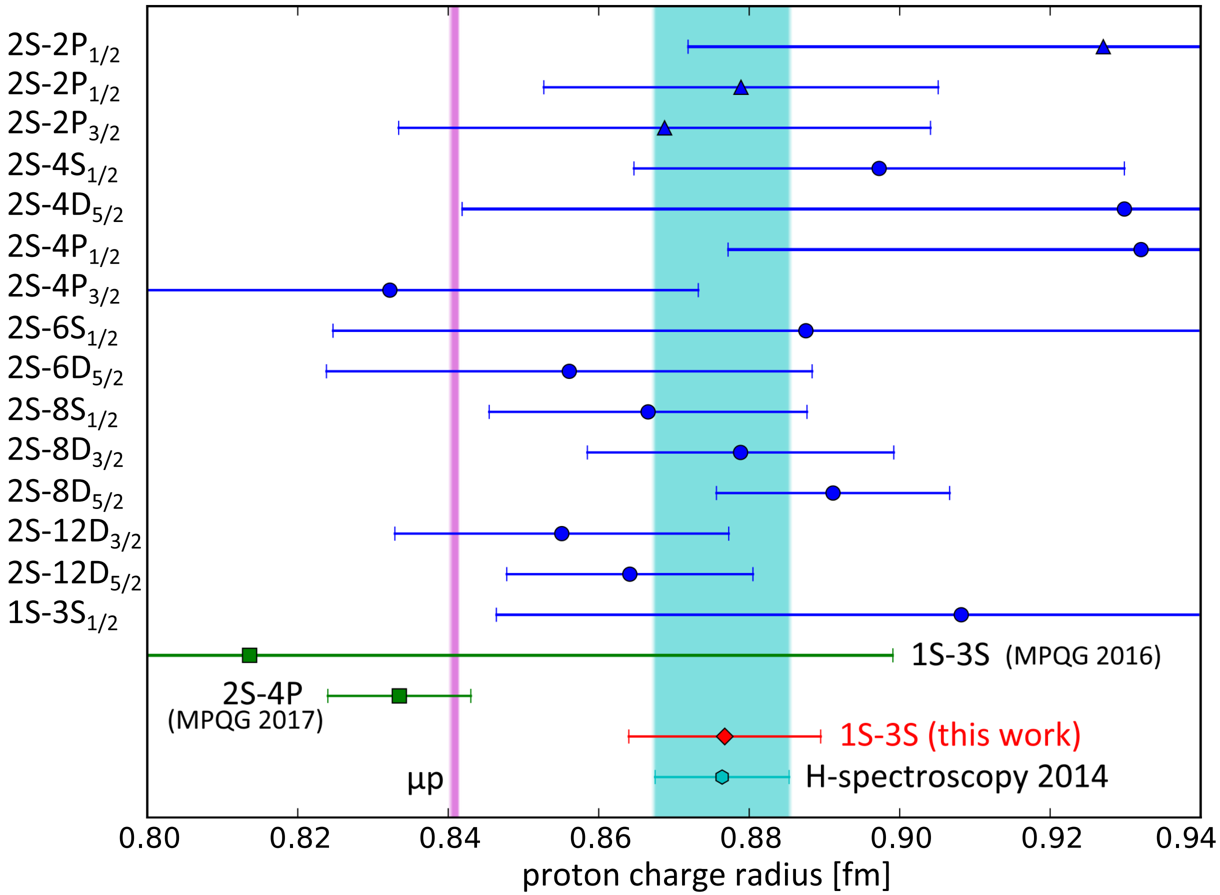
26 Jan., 2018 1801.08816 Paris
1S-3S hydrogen
Phys. Rev. Lett. 120, 183001 (2018)



Is there still a proton radius puzzle?

Is electron-hydrogen spectroscopy accurate enough?

Possible resolutions

- Electron H spectroscopy not so accurate
- Strong interaction effect in two photon exchange diagram
- Muon interacts differently than electron!- new scalar boson

The last two resolutions could be halved and not be in conflict with data

Shift from history to our efforts to explain

The search begins

- Pohl et al table 32 terms!
- Most important -HFS- measured Jan '13

muon

electron

A new effect on mu-H energy shift
must vary at least as fast as lepton mass
to the fourth power, if short-ranged

The search begins

- Pohl et al table 32 terms!
- Most important -HFS- measured Jan '13

muon

electron

A new effect on mu-H energy shift must vary at least as fast as lepton mass to the fourth power, if short-ranged

Suppose radii extracted in earlier H experiments is correct, some new muon effect is responsible

What energy difference corresponds to 4% in radius?

Measured =206.2949(32)= 206.0573(45)-5.2262 r_p^2+0.0347 r_p^3 meV

computed

Explain puzzle with radius as in earlier H atom measures: increase 206.0573 meV by 0.31 meV-attractive effect on 2S state

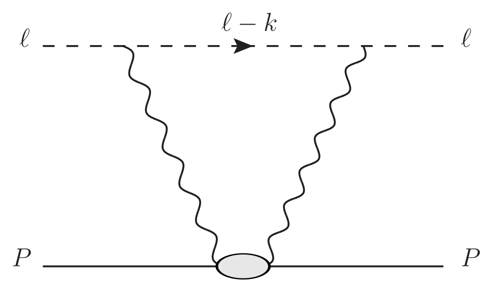
Can go half way and not disagree with data

Two photon exchange

Measured =206.2949(32)= 206.0573(45)-5.2262 r_p^2+0.0347 r_p^3 meV

computed

Miller PLB 2012



energy shift proportional to lepton mass^4

Re part of virtual Compton scattering

Im part is measured

use dispersion relations

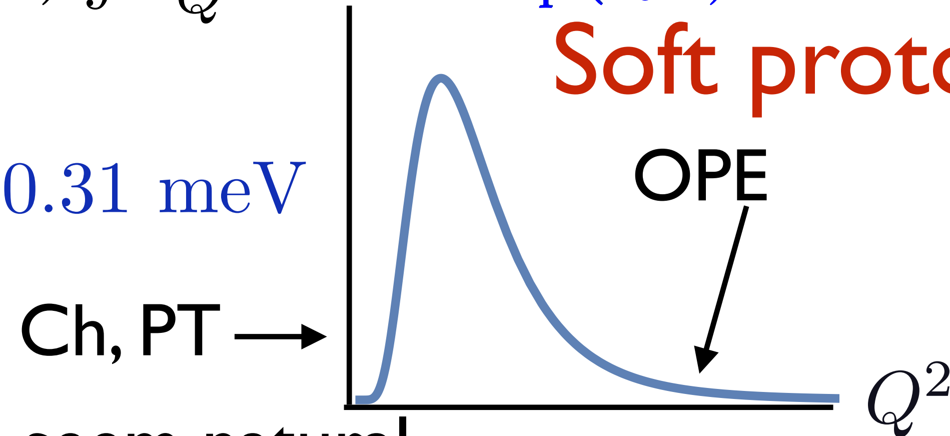
but unknown subtraction function is needed

Can account for 0.31 meV, no conflict with e-H

Almost unknown $\bar{T}_1(0, Q^2)$ Miller PLB 2012

$$\Delta E^{\rm subt} \propto \alpha^2 m \Psi_S^2(0) \int \frac{dQ^2}{Q^2} \cdots F_{\rm loop}(Q^2)$$

number of
Infinite $F_{\rm loop}(Q^2)$ give 0.31 meV
satisfy all constraints



Recast in EFT- parameters seem natural

So far size of this term cannot be determined from theory-experiment is needed

lattice QCD may disprove above sentence

Nuclear dependence of short-ranged mu-p effects

- Energy shift is proportional to square of muon wave function at the origin

GAM
1501.01036

- Suppose you have effect that gives energy shifts Ep (on proton) E_n (on neutron)

$$E_A = \left(\frac{1+\frac{m_\mu}{m_p}}{1+\frac{m_\mu}{Am_p}}\right)^3 Z^3 (ZE_p + NE_n)\left(1 - \mathcal{O}(\frac{R_A^2}{a_\mu^2})\right) \approx \left(\frac{1+\frac{m_\mu}{m_p}}{1+\frac{m_\mu}{Am_p}}\right)^3 Z^3 (ZE_p + NE_n),$$

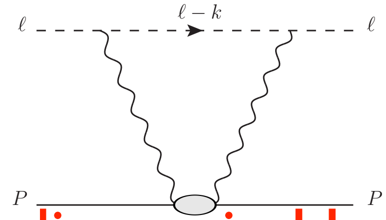
Nuclear shift

Square of wave fun

Counting

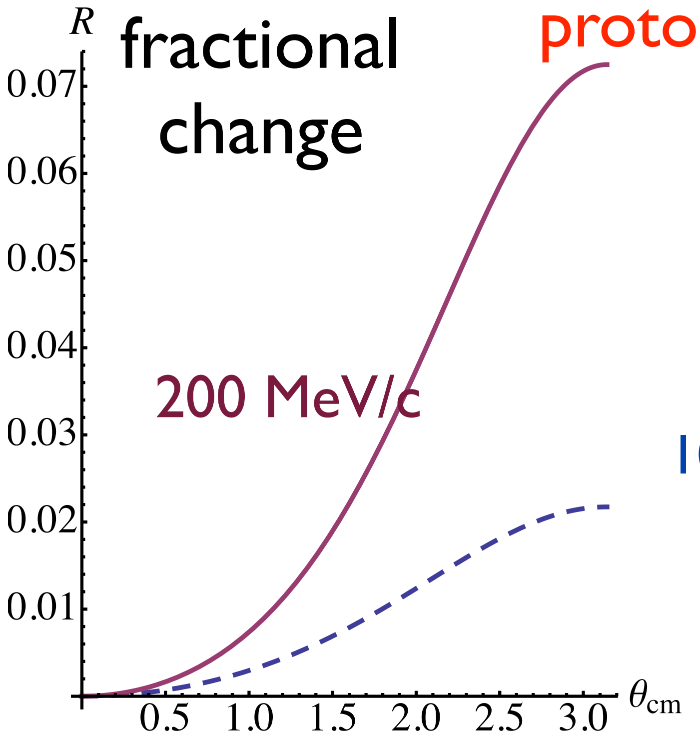
Size of nucleus

My model: ~0.3 meV (1+0.3)(8)(2) =-6.3 meV about 6 st. dev off **RIP any short range idea**



energy shift proportional to lepton mass[4]

Explain puzzle with radius as in H atom increase 206.0573 meV by 0.31 meV-attractive effect on 2S state, reproduce Lamb shift in proton, deuterium



10-20% effect in scattering

Radians

fails in 4He

$$\delta E_L^{\mu A} \propto Z^3 (Z\delta E_L^p + N\delta E_L^n)$$

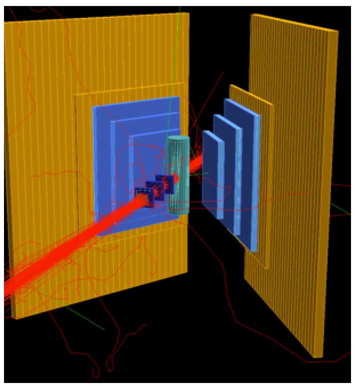
4 standard deviations off ^{4}He

or 1 st dev

e^+/e^- and μ^+/μ^- scattering on proton

So what? MUSE expt

http://www.physics.rutgers.edu/~rgilman/elasticmup/



PSI proposal R-12-01.1

- constrains two photon effect, which still survives at significant level
- if large radius correct and no two photon all leptons see large radius
- if small radius correct and no two photon all leptons see small radius
- will not see a new light particle, but all leptons see large radius

e^+/e^- and μ^+/μ^- scattering on proton

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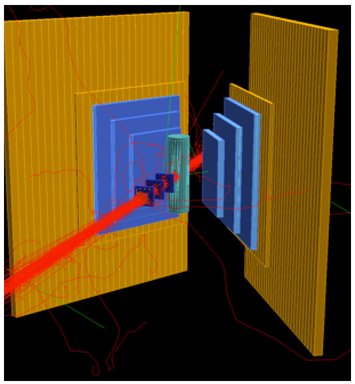

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Next - muon interacts differently than electron introduce new scalar boson

muon anomalous moment

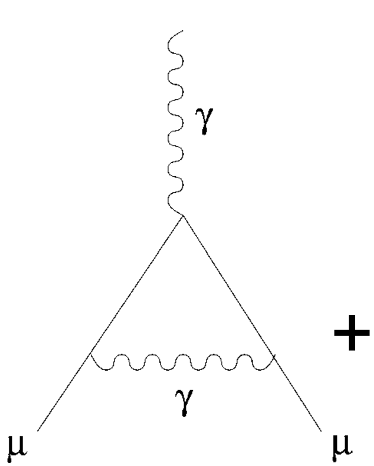


Figure 1 The first-order QED correction to g-2 of the muon.

+

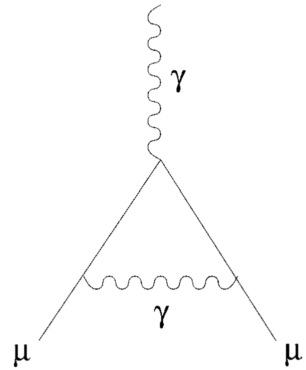


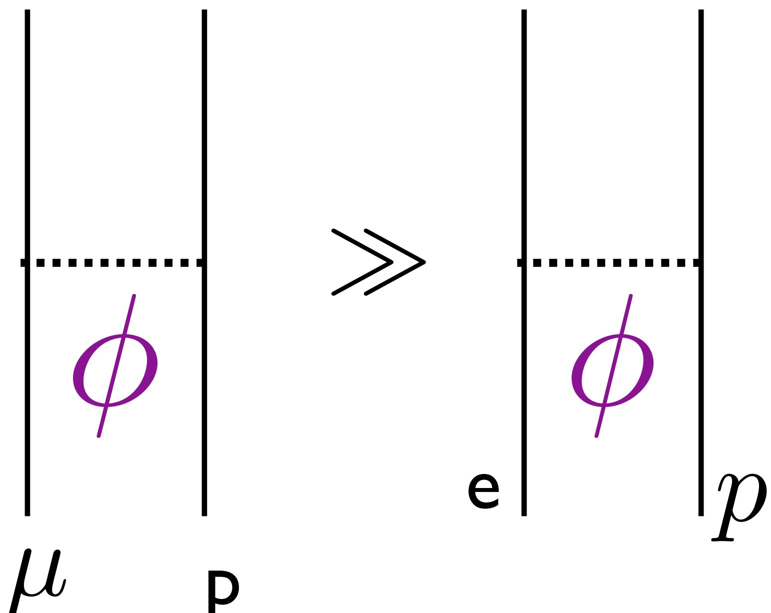
Figure 1 The first-order QED correction to g-2 of the muon.

H

3.6 st. dev anomaly now
fix add heavy photon
interacts preferentially with muon

Muon data is g-2 - BNL exp't, Hertzog- FermiLab now...

Maybe dark matter, energy particles show up in muon physics!



Postulate new scalar boson!

look for violation
of 4 momentum conservation
elastic ep scattering
Dark Light interest

New scalar bosons

assumes puzzle exists

ϕ

- give μ-p Lamb shift
- almost no hyperfine in μ proton
- small effect for D, almost no effect ^{4}He
- consistent with g-2 of μ and electron
- avoid many other constraints
- be found

PRL **117,** 101801 (2016) PHYSICAL REVIEW LETTERS week ending 2 SEPTEMBER 2016

Electrophobic Scalar Boson and Muonic Puzzles

Yu-Sheng Liu,* David McKeen,† and Gerald A. Miller‡

New scalar bosons

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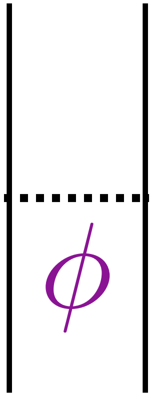
PRL **117,** 101801 (2016) PHYSICAL REVIEW LETTERS week ending 2 SEPTEMBER 2016

Electrophobic Scalar Boson and Muonic Puzzles

Yu-Sheng Liu,[*] David McKeen,[†] and Gerald A. Miller[‡]

Our approach

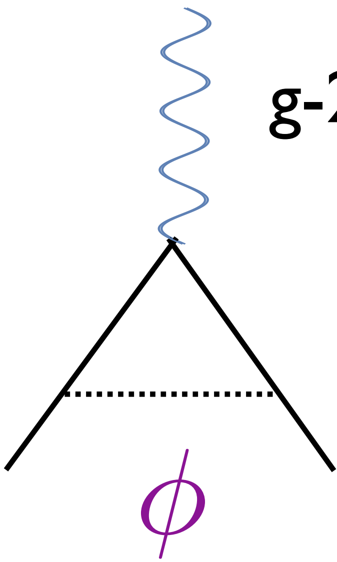


any signs

$$V(r) = -\epsilon_{f_1}\epsilon_{f_2}\alpha\frac{e^{-m_\phi r}}{r}. \qquad |\epsilon_e| \ll \epsilon_\mu$$

Bohr radius

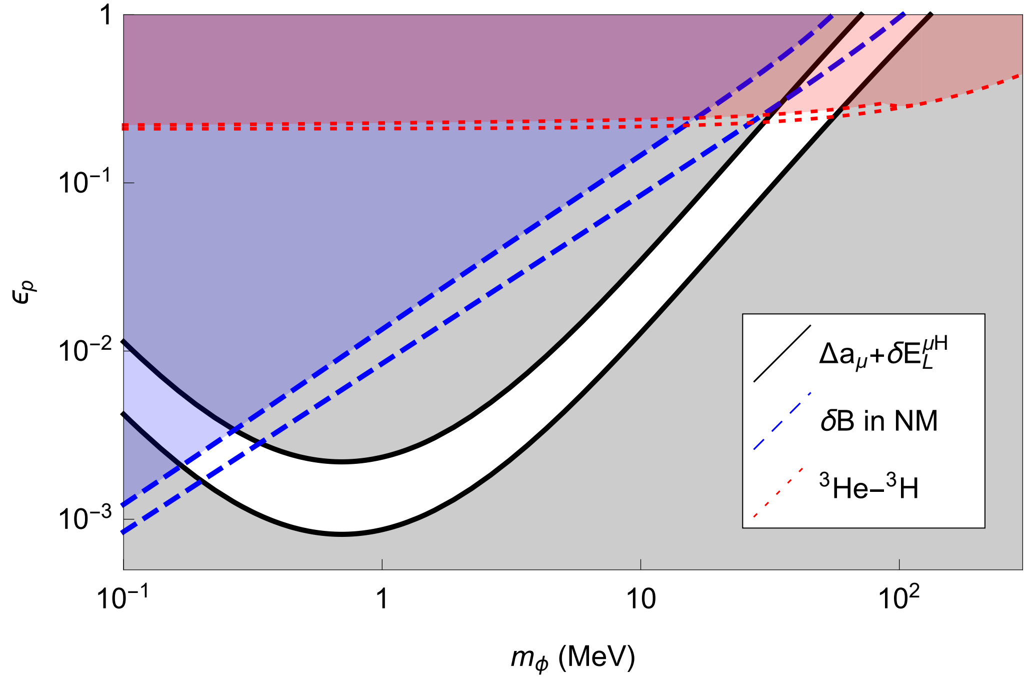
$$\delta E_L^{\ell \mathrm{N}} = -\frac{\alpha}{2a_{\ell \mathrm{N}}}\epsilon_\ell[Z\epsilon_p + (A-Z)\epsilon_n]\frac{(a_{\ell \mathrm{N}} m_\phi)^2}{(1+(a_{\ell \mathrm{N}} m_\phi))^4}$$



g-2

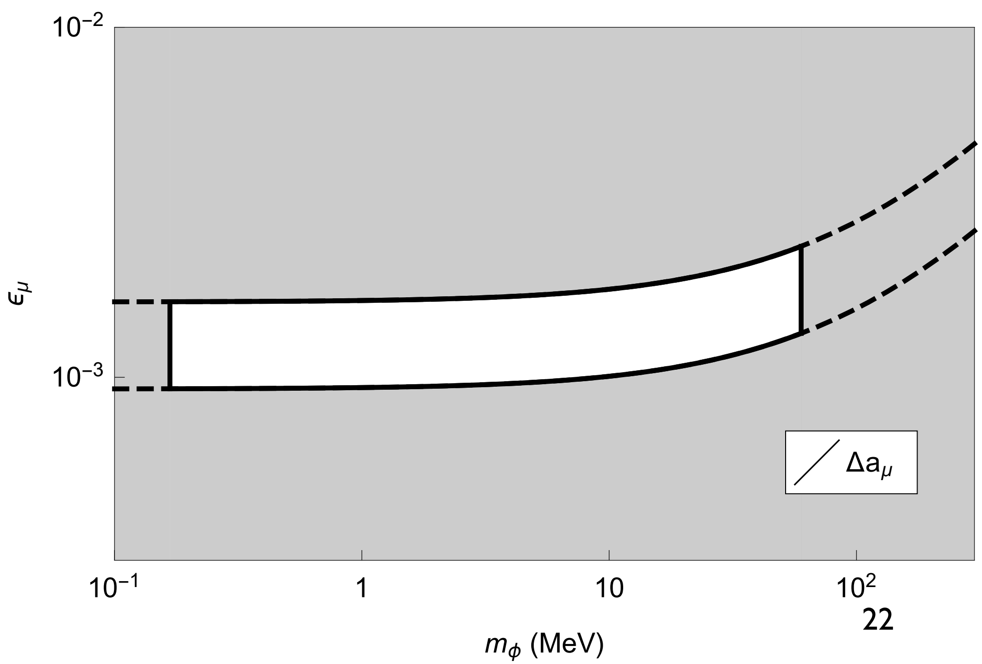
$\Delta a_\mu = 287(80) \times 10^{-11}$, $\Delta a_e = 1.5 \times 10^{-12}$ From ^{87}Rb

For light mass- evades 4He data

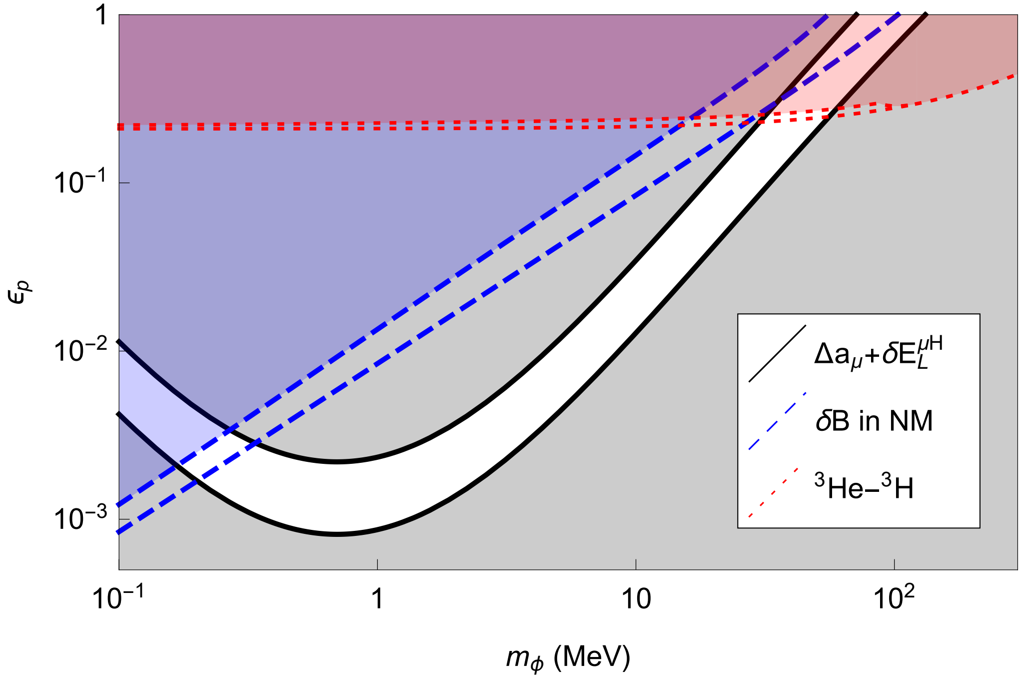


Unshaded allowed by muon g-2 and muon-p Lamb shift

Two anomalies have same scale



Unshaded allowed by muon g-2



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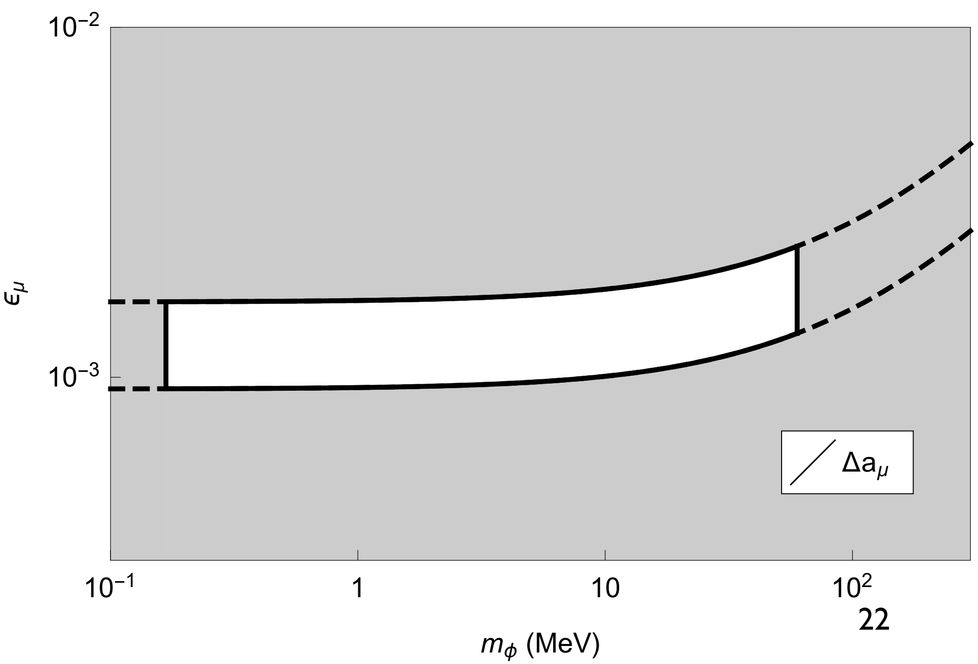


Unshaded allowed by muon g-2

Using new eH experiment
$\epsilon_\mu \epsilon_p$ is reduced by
a factor of 3:
barely visible in loglog plot

Nuclear Physics constraints

$$\epsilon_n/\epsilon_p$$

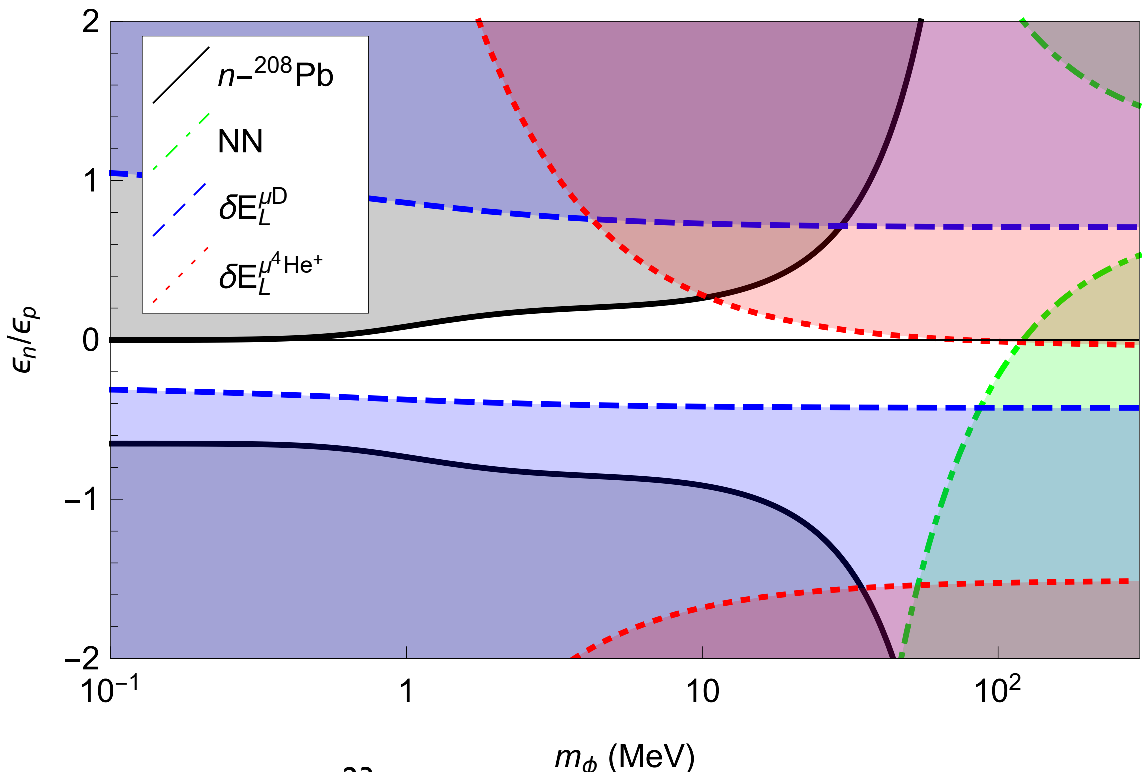
If $\epsilon_n = \epsilon_p$ scalar is ruled out by n-^{208}Pb scattering

If ϵ_n has opposite sign as ϵ_p parameter space widens

Other constraints:

NN scattering, nuclear matter & $^3\mathrm{He} - ^3\mathrm{H}$ binding

muonic D, ^{3}He

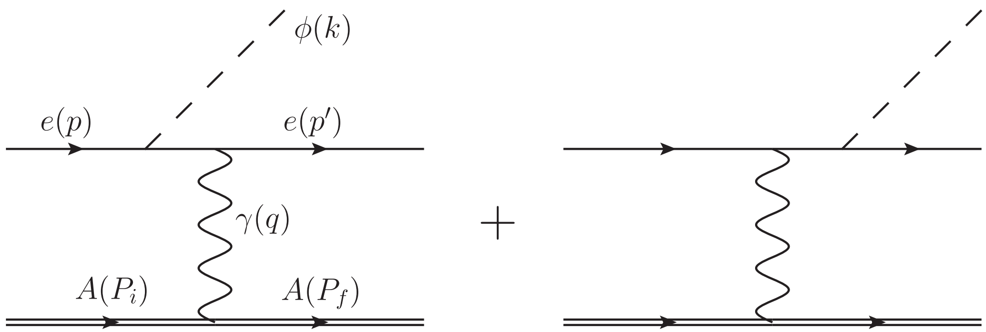


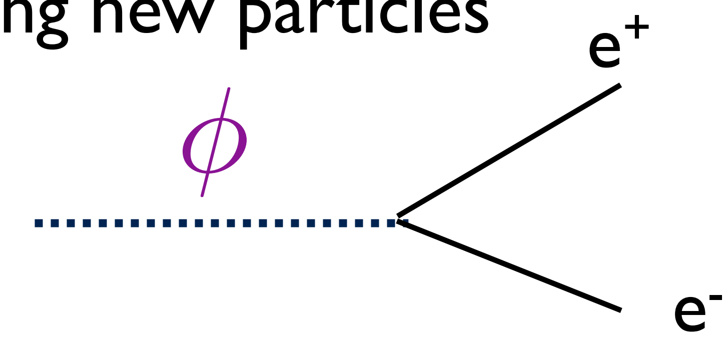
Beam dump experiments

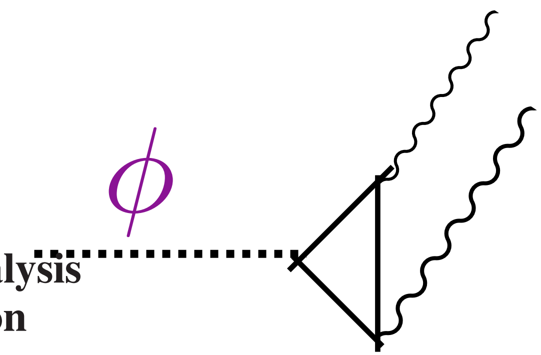
Beam dumps absorb beam of charged particles

e+^{27}Al to dissipate energy

hope e+^{27}Al makes penetrating new particles







PHYSICAL REVIEW D **95**, 036010 (2017)

Validity of the Weizsäcker-Williams approximation and the analysis of beam dump experiments: Production of a new scalar boson

Yu-Sheng Liu,* David McKeen,† and Gerald A. Miller‡

Better analysis if discovery

PHYSICAL REVIEW D **96**, 016004 (2017)

Validity of the Weizsäcker-Williams approximation and the analysis of beam dump experiments: Production of an axion, a dark photon, or a new axial-vector boson

Yu-Sheng Liu* and Gerald A. Miller†

Constraining ϕ

Beam dump experiments do not see

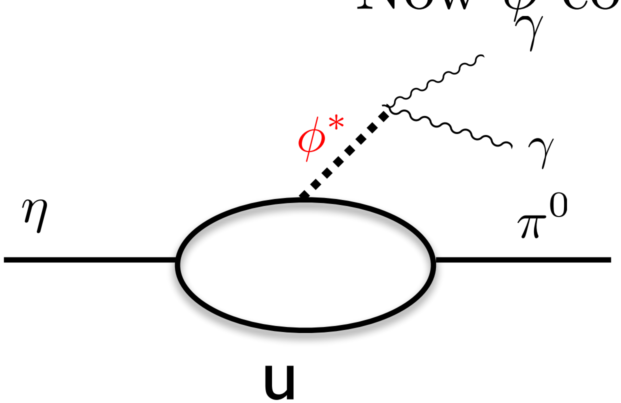
$$\phi \to e^+e^-, \ \phi \to \gamma\gamma$$

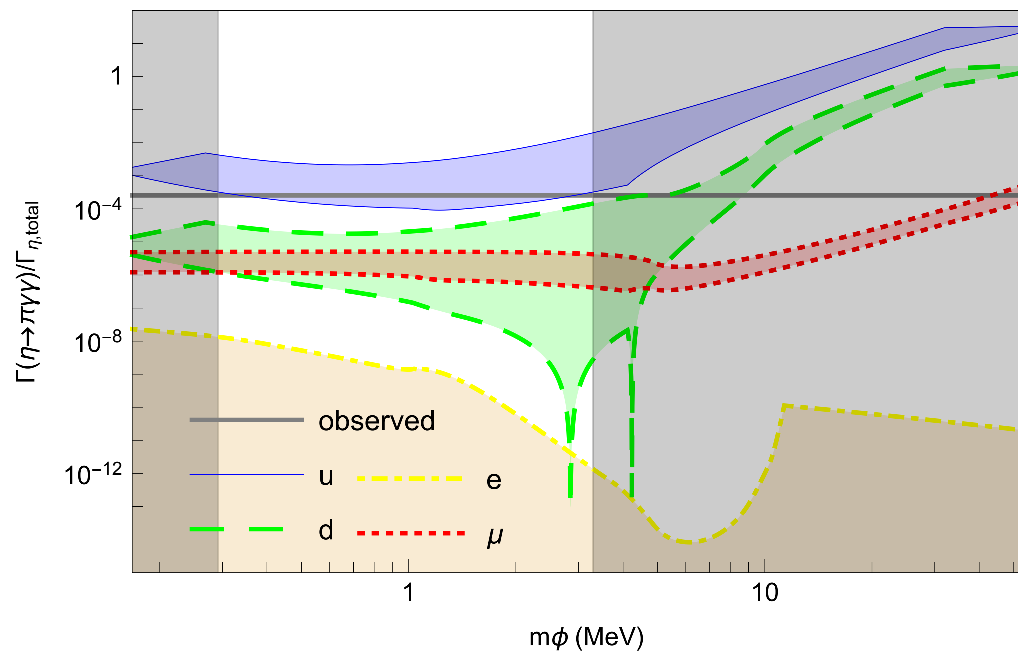
Eta decay and muonic puzzles 1805.01028

Yu-Sheng Liu, Ian Cloet, GAM

Previously ϕ couples to p, n, e, μ

Now ϕ couples to $u, d, e\mu$





Allowed mass range from 200 KeV to 3 MeV

Summary

- If the proton radius puzzle exists : new scalar boson of mass from 300 KeV to 3 MeV may exist- narrow target
- Direct detection is needed.

Does 4% matter?

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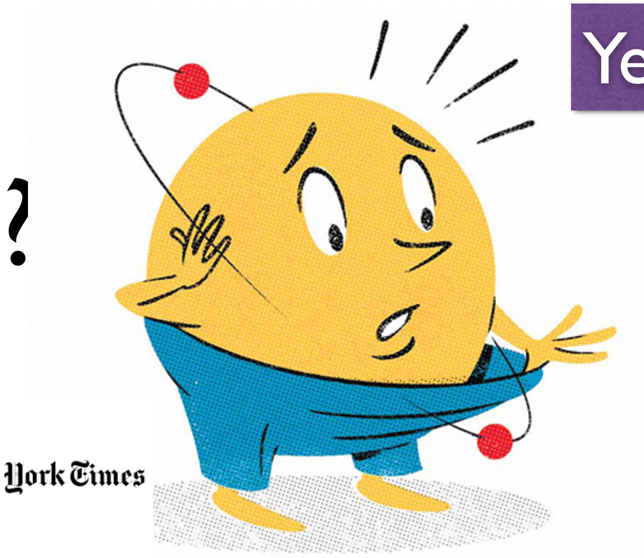
Does 4% matter?

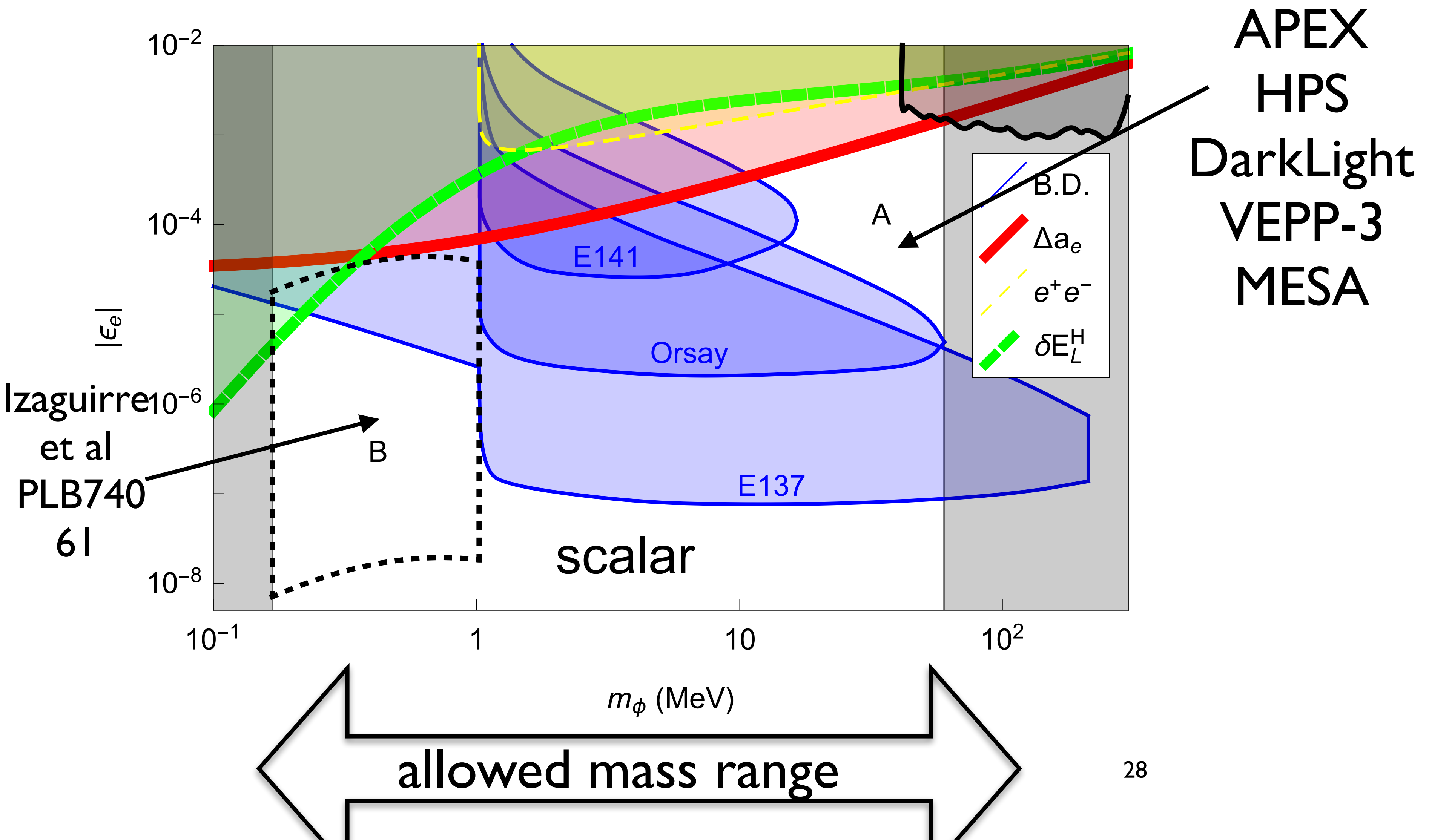


Yes

The New York Times

Spares follow

electron Exclusion plot



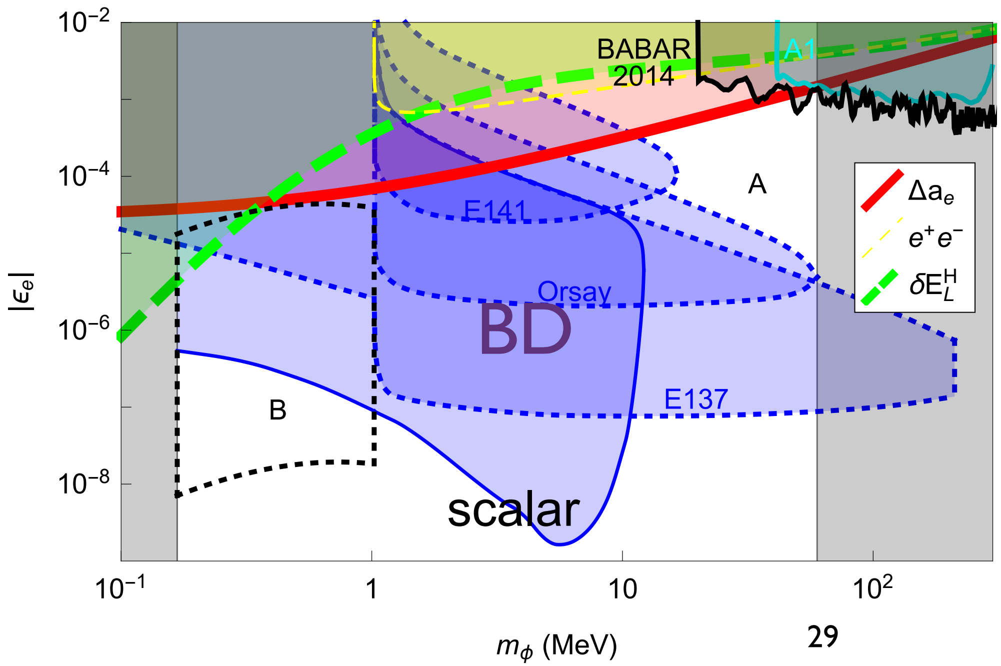
More constrains Coupling to electron

ϵ_e

Hydrogen atom Lamb shift $\delta E_L^H < 45\ \mathrm{kHz}$

e^+e^- resonant (?) scattering not seen

electron anomalous magnetic moment



BD is beam dump discussed next

Electrophobic

Science 12 Aug 2016:
Vol. 353, Issue 6300, pp. 669-673

Deuteron is smaller too

Electron (D-H) isotope shift (2S-1S) 2 photon spectroscopy PRL 104, 233001

$r_d^2 - r_p^2 = 3.82007(65)$

$\mu - D$ Lamb shift $r_d = 2.12562(78)$ fm Science 353 (2016) 669

CODATA (2010)$r_d = 2.1424(26)$ fm - mainly electron scattering

Use $r_p = 0.84087$ in $r_d^2 - r_p^2 = 3.82007(65)$ gives $r_d = 2.12769$ fm

μD and Electron (D-H) isotope shift are consistent$\rightarrow$ redo eD scattering?

If NO proton radius puzzle, there still is a missing Lamb shift

OR:

CODATA deuteron radius is too large 2.1424 vs 2.12769

remeasure deuteron ?

Nuclear Physics constraints

$$\epsilon_n/\epsilon_p$$

- Low energy scattering of neutrons on ^{208}Pb using ϕ-nucleon coupling g_N.
  $\frac{g_N^2}{e^2} \to \frac{A-Z}{A}\epsilon_n^2 + \frac{Z}{A}\epsilon_p\epsilon_n$ cancellation evades previous limits

- NN charge-independence breaking scattering length

  $\Delta a = (a_{pp} + a_{nn})/2 - a_{np}$, measured: 5.64(60) fm, theory: 5.6(5)

  Scalar boson exchange: $\Delta a_\phi \propto \int_0^\infty \Delta V \bar{u} u_{np} dr \leq 1.6$ fm (2 S.D.)

- Change in binding energy/A infinite nuclear matter: less than 1 MeV

- binding energy B(^3He) $- B(^3H)$= 763.76 keV due to Coulomb (693 keV) + strong force charge symmetry breaking (68 keV) ϕ exchange < 30 keV

Lepton-universality violating one boson exchange

- Tucker-Smith & Yavin PRD83, 101702 new particle scalar or vector coupling
- Brax & Burrage scalar particles PRD 83, 035020 &'14
- Batell, McKeen & Pospelov PRL 107, 011803 new gauge boson kinetically mixing with $F^{\mu\nu}$ plus scalar for muon mag. mom. 1401.6154 W decays enhanced
- Carlson Rislow PRD 86, 035013 fine tune scalar pseudoscalar or polar and axial vector couplings
- Barger et al PRL106,153001 - new particles ruled out but assumes universal coupling
- Kaon decays provide constraints

Looking for new scalars is not new
Low mass Higgs searches

$$p(1.88\,\mathrm{MeV}) +^{19} F \to \alpha +^{16} O^*(6.05)$$
$$^{16}O^*(6.05) \to^{16} O(GS) + \phi$$

Kohler et al PRL 33, 1628 (1974)

Freedman et al. PRL 52, 240 (1984)

$$p +^3 H \to^4 He(20.1) \to 4He(GS) + \phi$$

No Scalars found, but assumed coupling constants were much larger than what we will use

Almost unknown $\overline{T}_1(0, Q^2)$ Miller PLB 2012

$$\Delta E^{\rm subt} = \frac{\alpha^2}{m}\Psi_S^2(0)\int_0^\infty dQ^2\,\frac{h(Q^2)}{Q^2}\overline{T}_1(0,Q^2)$$

Soft proton

$$\lim_{Q^2\to\infty} h(Q^2) \sim \frac{2m^2}{Q^2},\ \text{chiral PT}:\ \overline{T}_1(0,Q^2) = \frac{\beta_M}{\alpha}Q^2 + \cdots$$

$\rightarrow$ Logarithmic divergence

Typo bleow

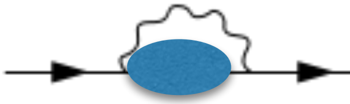
$\overline{T}_1(0,Q^2) \rightarrow \frac{\beta_M}{\alpha}Q^2 F_{\rm loop}(Q^2)$ Cuts off integral

Birse & McGovern assume dipole : $\Delta E^{\rm subt} = 0.004\,{\rm meV}$ very small

Miller $F_{\rm loop}(Q^2) = \left(\frac{Q^2}{M_0^2}\right)^n \frac{1}{(1+aQ^2)^N}, n \geq 2, N \geq$ n+3

Infinite parameter set gets needed 0.31meV, NO constraint on neutron

Choose parameters so shift in proton mass <0.5 MeV (current uncertainty)



Recast in EFT- parameters seem natural

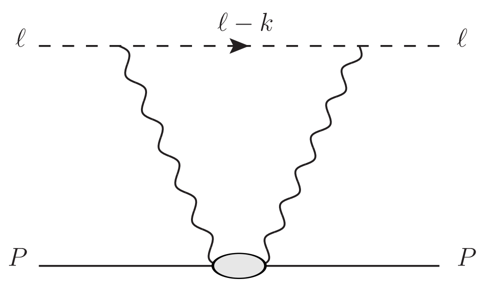
Two photon exchange

Measured =206.2949(32)= 206.0573(45)-5.2262 r_p^2+0.0347 r_p^3 meV

computed

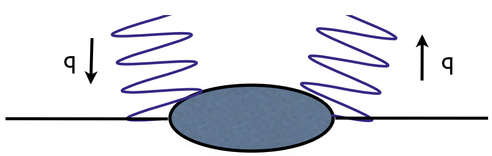
Explain puzzle with radius as in H atom increase 206.0573 meV by 0.31 meV-attractive effect on 2S state needed

Our idea



energy shift proportional to lepton mass[4]

$$T^{\mu\nu} =$$



$$= -(g^{\mu\nu} - \cdots)T_1 + (P^\mu - \cdots)(P^\mu - \cdots)T_2$$

Dispersion relation: $Im[T_1] \propto W_1$ measured

Large virtual photon energy ν, $W_1 \sim \nu$ integral over energy diverges

Subtraction function needed: $\bar{T}_1(0, Q^2)$ zero energy

Hill & Paz- big uncertainty in dispersion approach

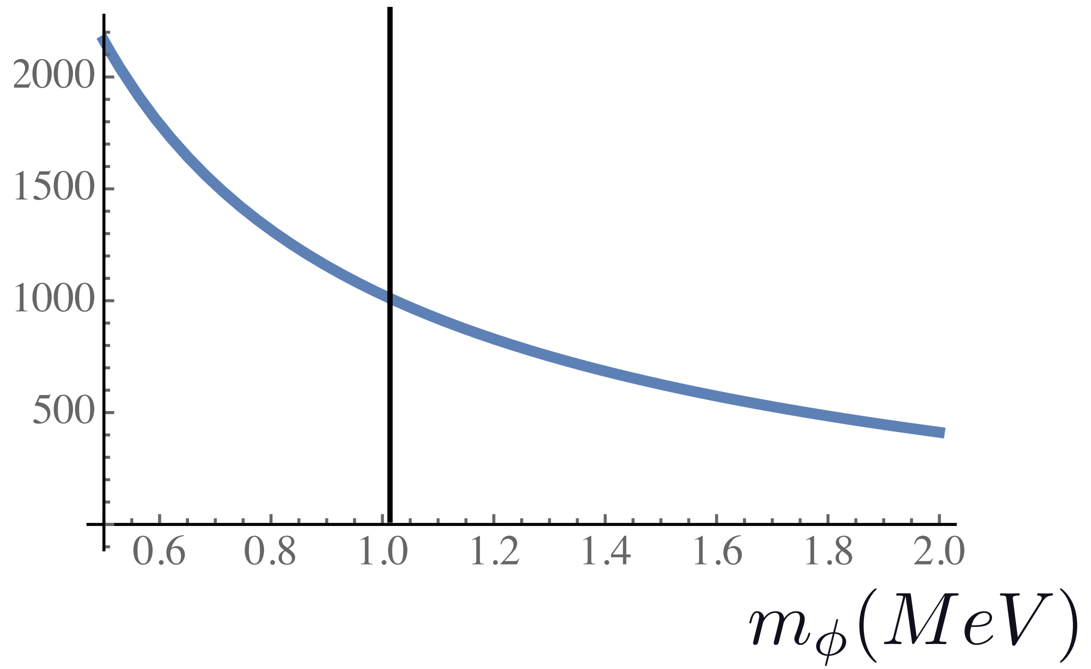
$$^{16}O(6.05, 0^+) \rightarrow ^{16}O(\mathrm{GS}, 0^+) + \phi, \quad \text{No single photon decay}$$

From electron g-2

$$\frac{\tau(A^* \rightarrow A + e^+ e^-)}{\tau(A^* \rightarrow A + \phi} = 3.3 \times 10^3 \frac{g^2_{\phi e \bar{e}}}{e^2} \left(1 - \left(\frac{m_\phi}{6\mathrm{MeV}}\right)^2\right)^{5/2}$$

$$\tau(A^* \rightarrow A + e^+ e^-): \quad \text{lifetime is } 10^{-10}\mathrm{s}$$

Decay length (m) nuclear emission of scalar boson



Several new electron spectroscopy experiments

- Independent measurement of Rydberg constant. This would change only extracted r_P nothing else
- 2S-6S UK, 2S-4P Germany, 1S-3S France
- 2S-2P classic, Canada
- Highly charged single electron ions NIST

2S-4P has reported preliminary results- small radius not yet published

Yes it really is G_E

- Non-relativistic reduction of one-photon exchange leads to the spin independent interaction being $G_E(Q^2)/Q^2$
- All recoil effects properly accounted for:Breit-Pauli Hamiltonian computed for non-zero lepton and proton momentum

Arbitrary functions

$$\overline{T}_1(0,Q^2) = \frac{\beta_M}{\alpha} Q^2 F_{\rm loop}(Q^2)\,.$$

$$F_{\rm loop}(Q^2) = \left(\frac{Q^2}{M_0^2}\right)^n \frac{1}{(1+aQ^2)^N},\ n \geq 2,\ N \geq n+3,$$

$$\overline{T}_1(0,Q^2) \sim \frac{1}{Q^4} \text{ or faster},\ \beta_M \to \beta$$

$$\Delta E^{\rm subt} \approx 3\alpha^2 m \Psi_S^2(0) \frac{\beta}{\alpha} \gamma^n B(N,n), \gamma \equiv \frac{1}{M_0^2 a}$$

3 parameters: n, N, a $\quad (M_0 = M_\beta)$

Choose parameters such that shift in proton mass < electromagnetic uncertainty of 0.5 MeV

Almost unknown $\overline{T}_1(0,Q^2)$

Miller PLB 2012

$$\Delta E^{\rm subt} = \frac{\alpha^2}{m}\Psi_S^2(0)\int_0^\infty dQ^2\,\frac{h(Q^2)}{Q^2}\overline{T}_1(0,Q^2)$$

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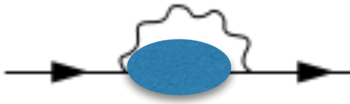
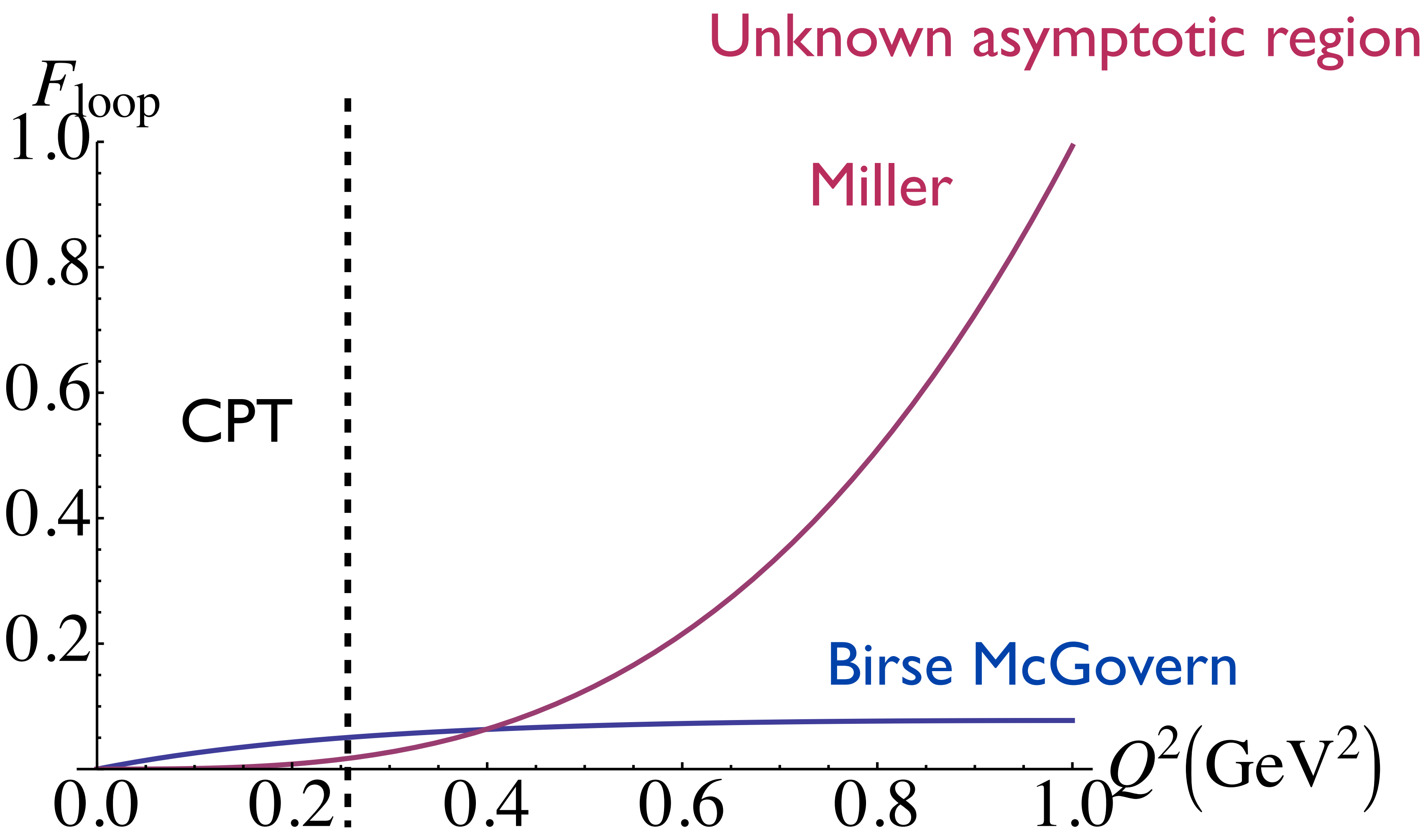


Recast in EFT- parameters seem natural

Form factors

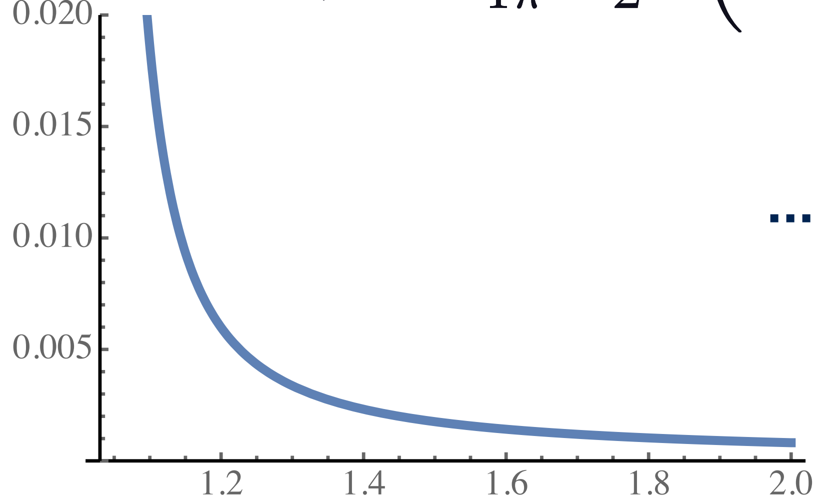


If recast into effective field theory strength seems natural

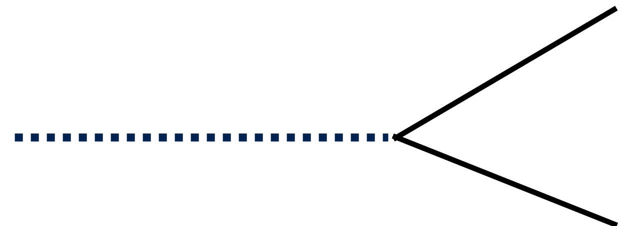
ϕ Decay modes

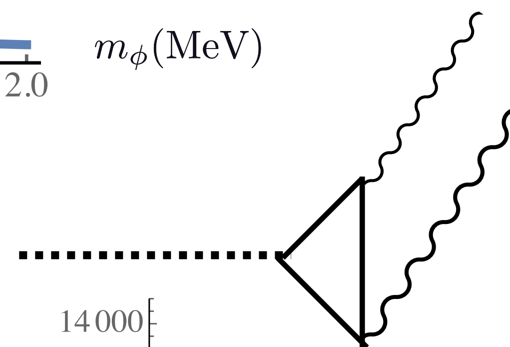
$$\Gamma(\phi \to e^+e^-) = \frac{g_{\phi e}^2}{4\pi}\frac{m_\phi}{2}\left(1 - \frac{4m_e^2}{m_\phi^2}\right)^{3/2}, m_\phi > 2m_e$$

$c\tau(\mathrm{m})$



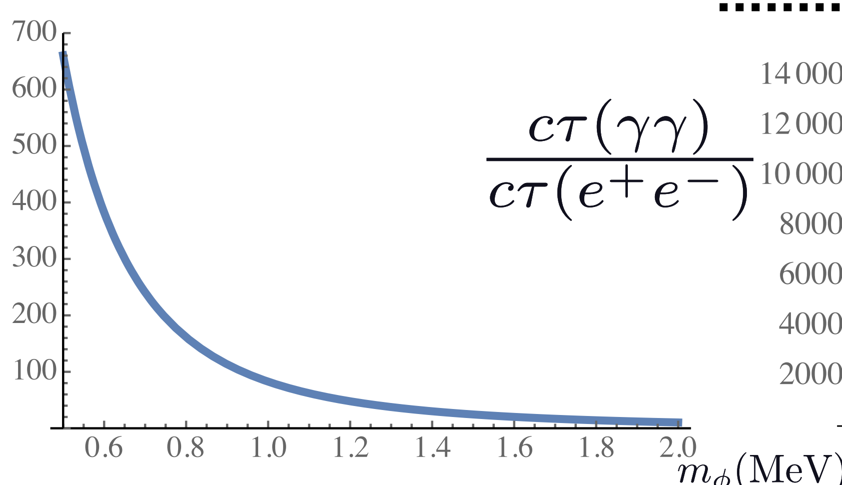
m_ϕ(MeV)



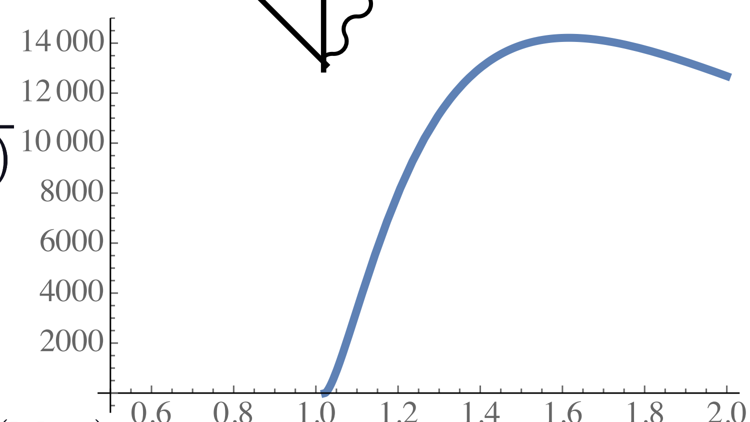


$$\Gamma(\phi \to \gamma\gamma) = g_{\phi e}^2 \frac{\alpha^2}{144\pi^3}\frac{m_\phi^3}{m_e^2}$$

$c\tau(\mathrm{m})$



$$\frac{c\tau(\gamma\gamma)}{c\tau(e^+e^-)}$$



Pohl et al. Table of calculations

Lamb shift: vacuum polarization many, many terms

#	Contribution	Ref.	Our selection Value	Unc.	Pachucki[1–3] Value	Unc.	Borie[5] Value	Unc.
1	NR One loop electron VP	1,2			205.0074			
2	Relativistic correction (corrected)	1–3,5			0.0169			
3	Relativistic one loop VP	5	205.0282				205.0282	
4	NR two-loop electron VP	5,14	1.5081		1.5079		1.5081	
5	Polarization insertion in two Coulomb lines	1,2,5	0.1509		0.1509		0.1510	
6	NR three-loop electron VP	11	0.00529					
7	Polarisation insertion in two and three Coulomb lines (corrected)	11,12	0.00223					
8	Three-loop VP (total, uncorrected)				0.0076		0.00761	
9	Wichmann-Kroll	5,15,16	−0.00103				−0.00103	
10	Light by light electron loop contribution (Virtual Delbrück scattering)	6	0.00135	0.00135			0.00135	0.00015
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	1,2	−0.00500	0.0010	−0.006	0.001	−0.005	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	17–19	−0.00150					
13	Mixed electron and muon loops	20	0.00007				0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	21–23	0.01077	0.00038	0.0113	0.0003	0.011	0.002
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	22,23	0.000047					
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	22,23	−0.000015					
17	Recoil contribution	24	0.05750		0.0575		0.0575	
18	Recoil finite size	5	0.01300	0.001			0.013	0.001
19	Recoil correction to VP	5	−0.00410				−0.0041	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	2,7	−0.66770		−0.6677		−0.66788	
21	Muon Lamb shift 4th order	5	−0.00169				−0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	2,5–7	−0.04497		−0.045		−0.04497	
23	Recoil of order α^6	2	0.00030		0.0003			
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	1,2,7	−0.00960		−0.0099		−0.0096	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2,5,22,25	0.015	0.004	0.012	0.002	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	0.00019					
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	−0.00001					
	Sum		206.0573	0.0045	206.0432	0.0023	206.05856	0.0046

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Radiative corrections to nuclear finite size effect	2,26	−0.0275	$< r_p^2 >$	−0.0282	−0.0273
Nuclear size correction of order $(Z\alpha)^6 < r_p^2 >$	1,27–29	−0.001243	$< r_p^2 >$		
Total $< r_p^2 >$ contribution		−5.22619	$< r_p^2 >$	−5.2256	−5.2244
Nuclear size correction of order $(Z\alpha)^5$	1,2	0.0347	$< r_p^3 >$	0.0363	0.0347

Table 2: All relevant radius-**dependent** contributions as summarized in Eides et al.[7], compared to Refs.[2,5]. Values are in meV and radii in fm.

Resolution 1- QED calcs not OK

α

Pohl et al. Table of calculations

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Mostly irrelevant-theory replaced by experiment

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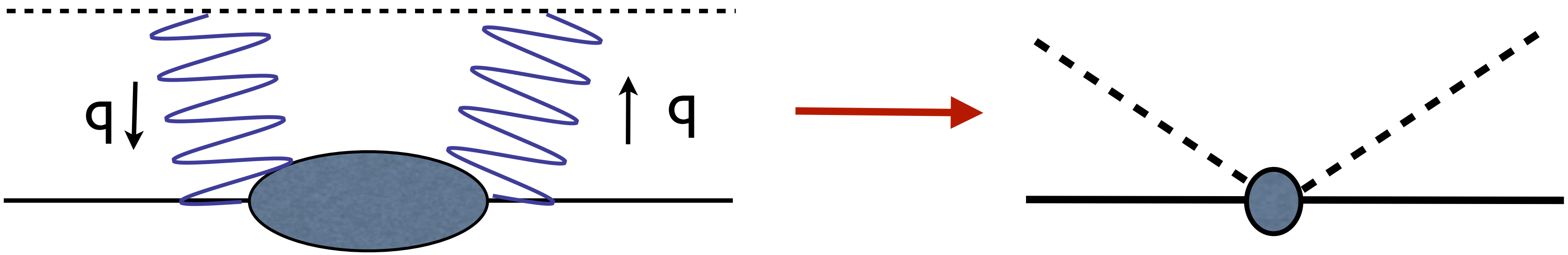
Resolution 1- QED calcs not OK

QED calcs expand in α

EFT of μp interaction

Caswell Lepage '86

- Compute Feynman diagram, remove log divergence using dimensional regularization
- include counter term in Lagrangian



$$\mathcal{M}_2^{DR} = \frac{3}{2} i\, \alpha^2 m \frac{\beta_M}{\alpha}\left[\frac{2}{\epsilon} + \log\frac{\mu^2}{m^2} + \frac{5}{6} - \gamma_E + \log 4\pi\right] \overline{u}_f u_i \overline{U}_f U_i,$$

$$= i\, \alpha^2 m \frac{\beta_M}{\alpha}(\lambda + 5/4)\, \overline{u}_f u_i \overline{U}_f U_i$$

Choose λ to get 0.31 meV shift

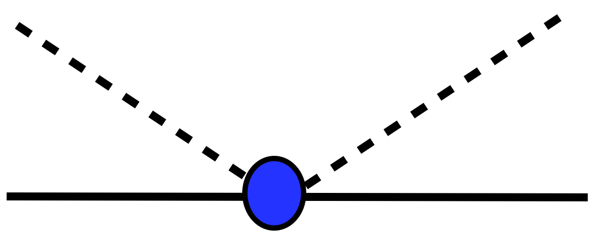
$$\Delta E^{\rm subt}(DR) = \alpha^2 m \frac{\beta_M}{\alpha} \Psi_S^2(0)(\lambda + 5/4)$$

$$\Delta E^{\rm subt}(DR) = 0.31 \text{ meV} \rightarrow \boxed{\lambda = 769}$$

β_M(magnetic polarizability) $= 3.1 \times 10^{-4} \text{fm}^3$ very small

Natural units $\beta_M/\alpha \sim 4\pi/(4\pi f_\pi)^3$ Butler & Savage '92

$$\mathcal{M}_2^{DR} = i\, 3.95\, \alpha^2 m \frac{4\pi}{\Lambda_\chi^3} \overline{u}_f u_i \overline{U}_f U_i.$$



3.95 =natural