Ab-initio calculations of electroweak response functions

Alessandro Lovato

In collaboration with:
Alessandro Baroni, Joe Carlson, Stefano Gandolfi, Winfried Leidemann, Juan Nieves,
Giuseppina Orlandini, Maria Piarulli, Steve Pieper, Noemi Rocco, and Rocco Schiavilla

INFN Istituto Nazionale di Fisica Nucleare

Argonne NATIONAL LABORATORY

The Physics case

Neutrino-oscillation and 0νββ experiments

- Charge-parity (CP) violating phase and the mass hierarchy will be measured
- Determine whether the neutrino is a Majorana or a Dirac particle
- Need for including nuclear dynamics; mean-field models inadequate to describe neutrino-nucleus interaction

Multi-messenger era for nuclear astrophysics

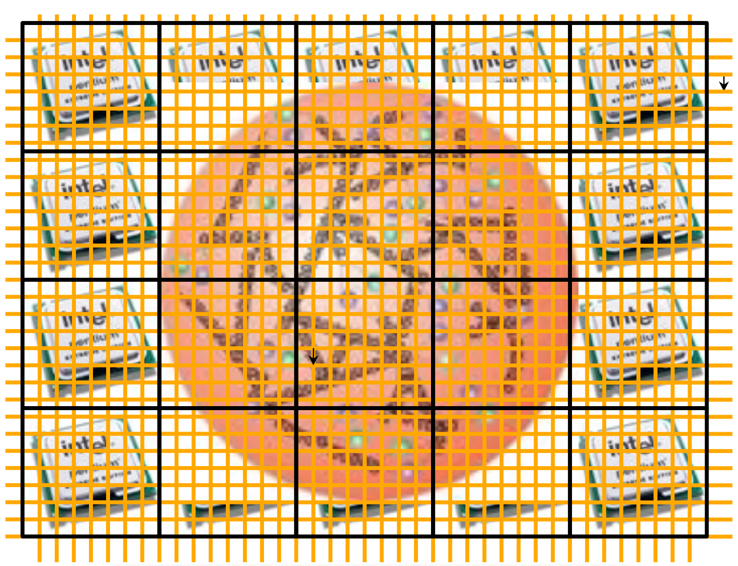
- Gravitational waves have been detected!
- Supernovae neutrinos will be detected by the current and next generation neutrino experiments
- Nuclear dynamics determines the structure and the cooling of neutron stars

The basic model of nuclear physics

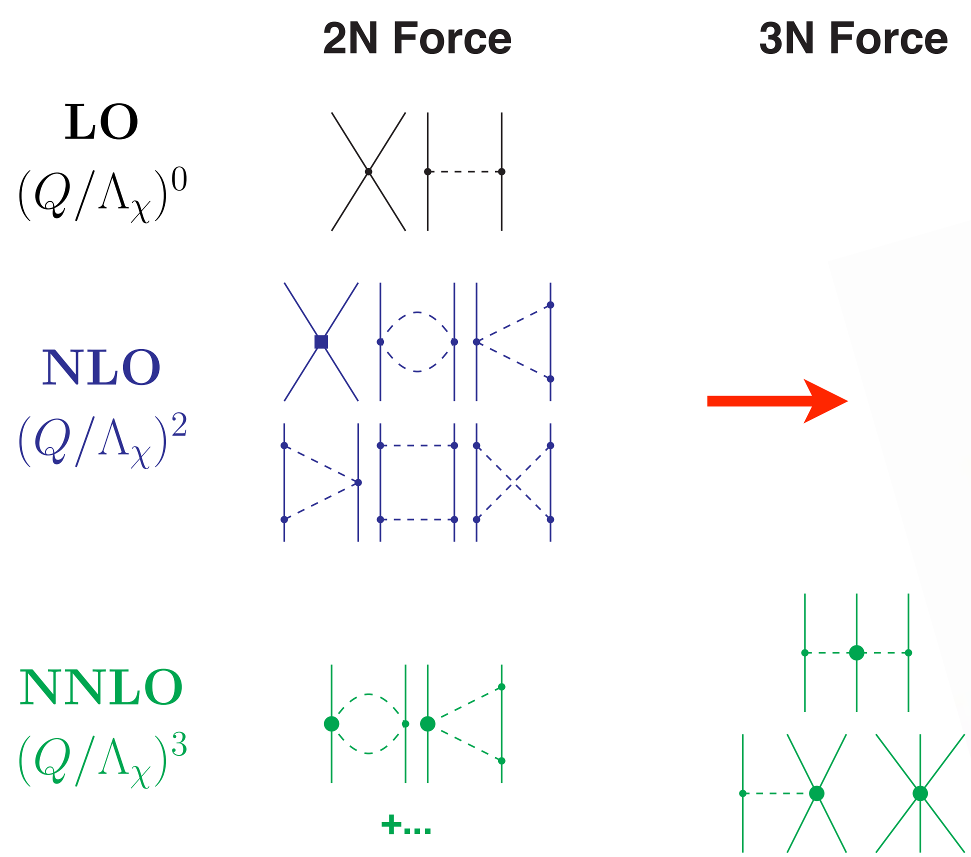
- In the low-energy regime, quark and gluons are confined inside hadrons. Nucleons can treated as point-like particles interacting through the Hamiltonian

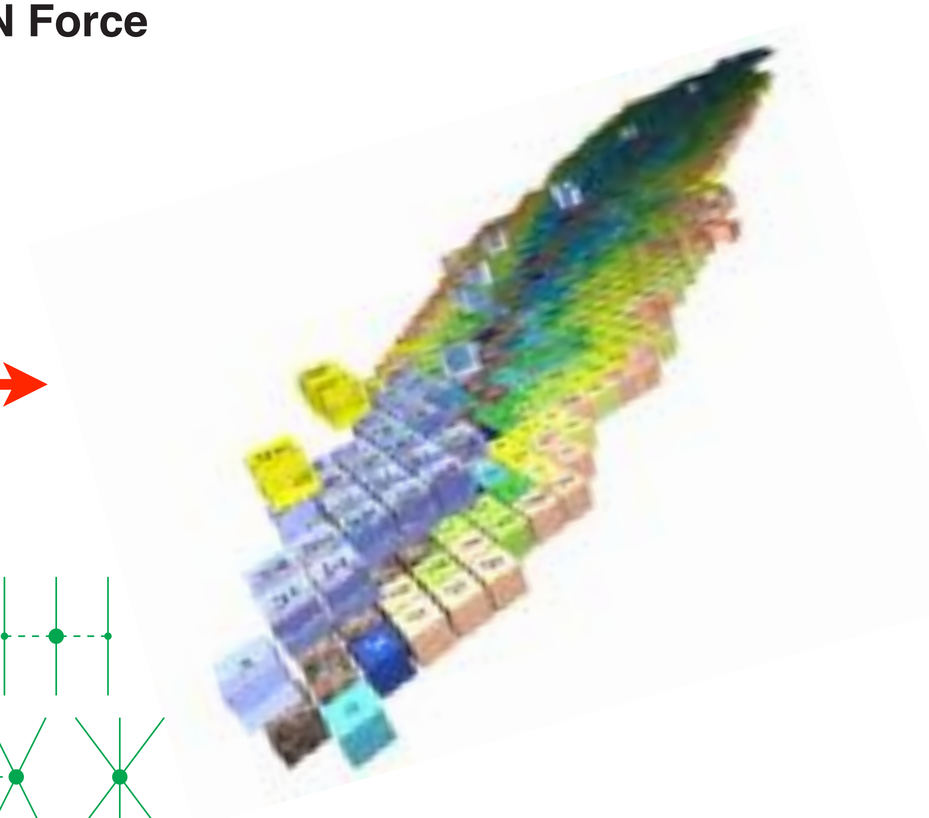
$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

- Effective field theories are the link between QCD and nuclear observables. They exploit the separation between the "hard" (M~nucleon mass) and "soft" (Q ~ exchanged momentum) scales



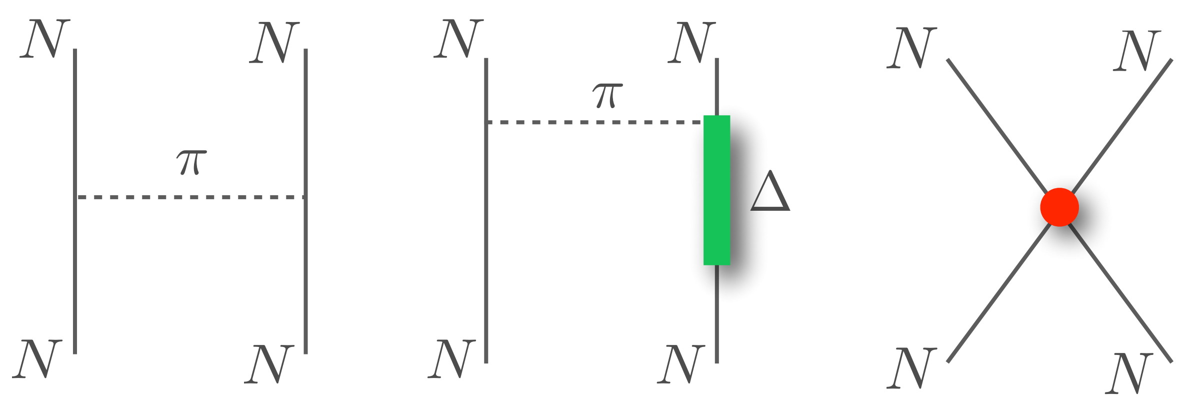
Courtesy of M. Savage



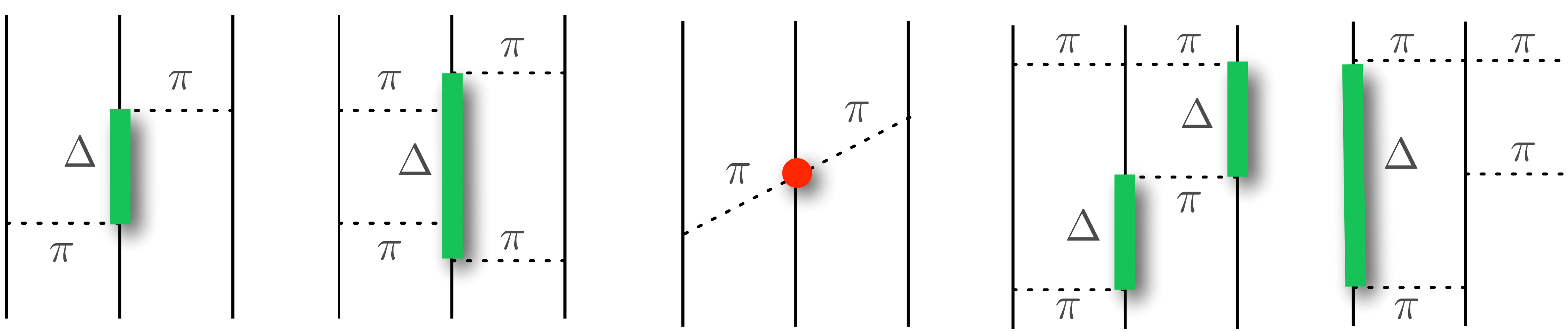


Nuclear (phenomenological) Hamiltonian

The Argonne v_{18} is a finite, local, configuration-space potential controlled by ~4300 np and pp scattering data below 350 MeV of the Nijmegen database



Three-nucleon interactions effectively include the lowest nucleon excitation, the Δ(1232) resonance, end other nuclear effects

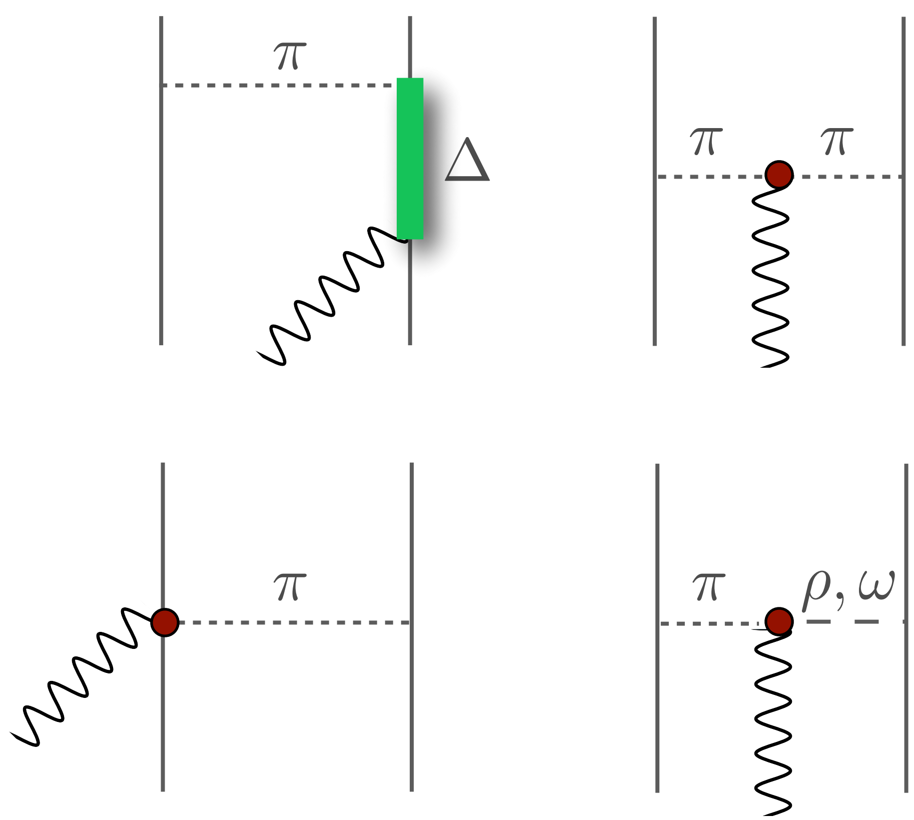


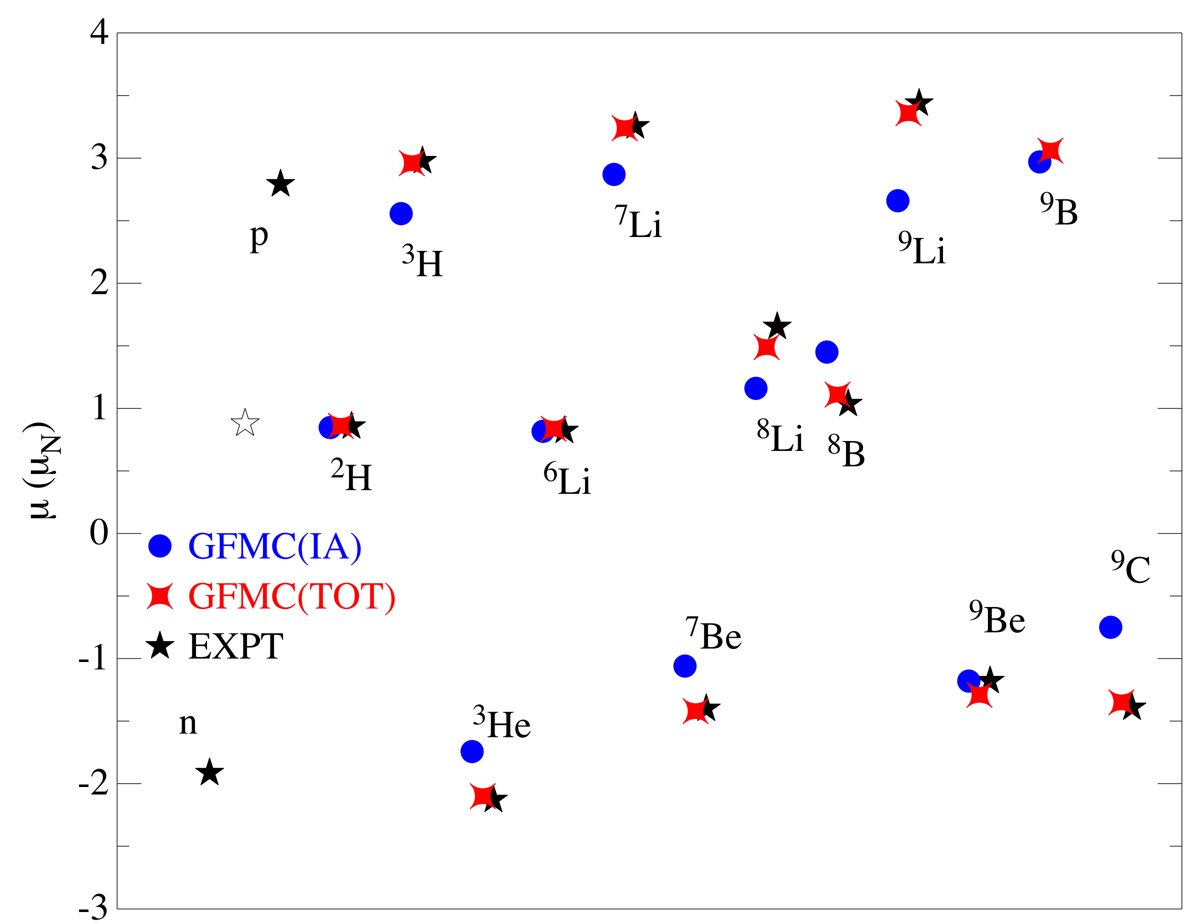
Nuclear currents

The nuclear electromagnetic current is constrained by the Hamiltonian through the continuity equation

$$\nabla \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0$$

- The above equation implies that $\mathbf{J}_{\rm EM}$ involves two-nucleon contributions.
- They are essential for low-momentum and low-energy transfer transitions.





S. Pastore at al., PRC 87, 035503 (2013)

Quantum Monte Carlo

- Diffusion Monte Carlo methods use an imaginary-time projection technique to enhance the ground-state component of a starting trial wave function.

- Any trial wave function can be expanded in the complete set of eigenstates of the the hamiltonian according to

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \qquad\qquad H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

which implies

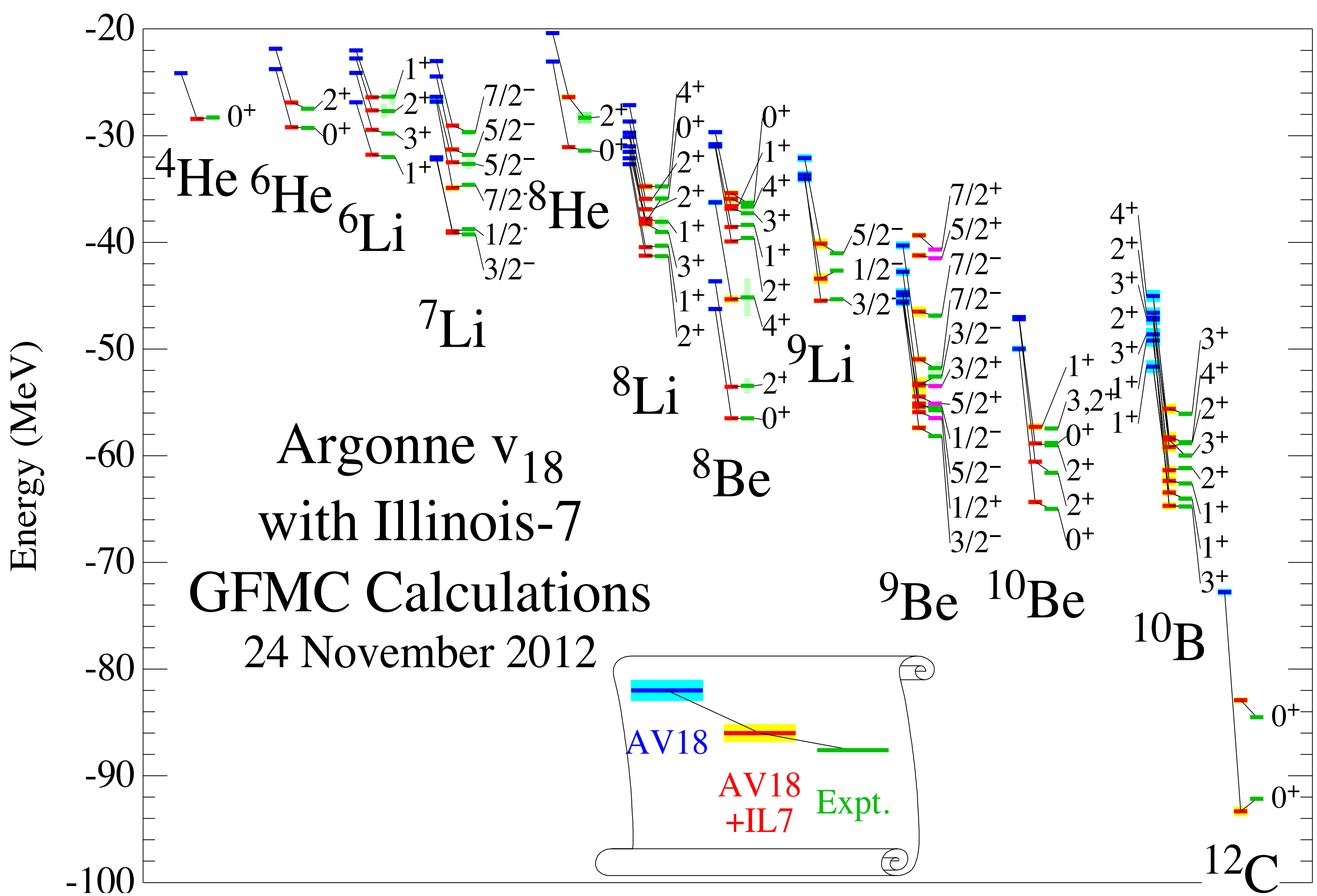
$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = \lim_{\tau\to\infty}\sum_n c_n\, e^{-(E_n-E_0)\tau}|\Psi_n\rangle = c_0|\Psi_0\rangle$$

GFMC and AFDMC project out the exact lowest-energy state, provided the trial wave function it is not orthogonal to the ground state.

Why quantum Monte Carlo?

Quantum Monte Carlo provides a way to go from the nuclear hamiltonian to nuclear properties

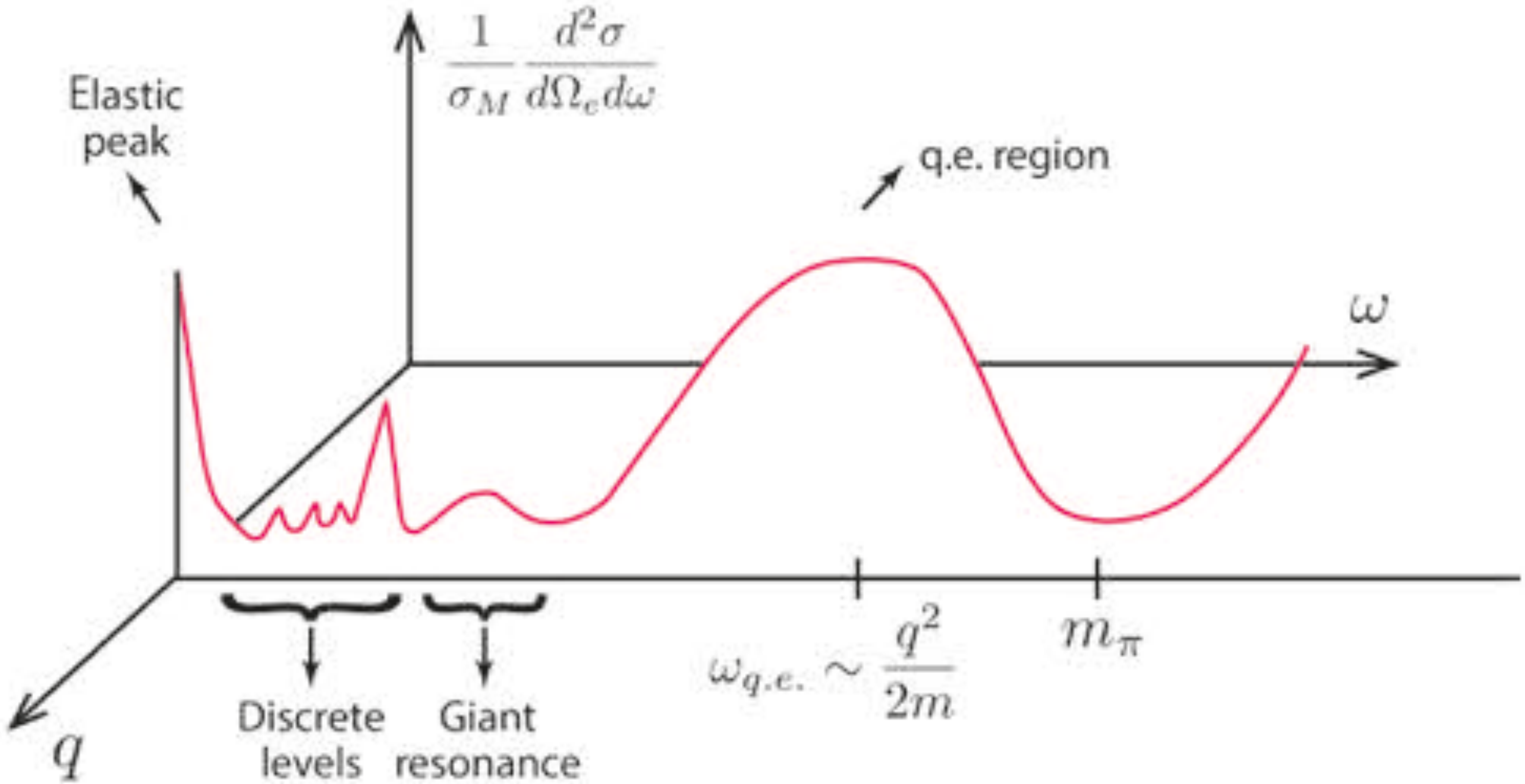
$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$



J. Carlson et al. RMP 87, 1067 (2015)

Lepton-nucleus scattering

Schematic representation of the inclusive cross section as a function of the energy loss.

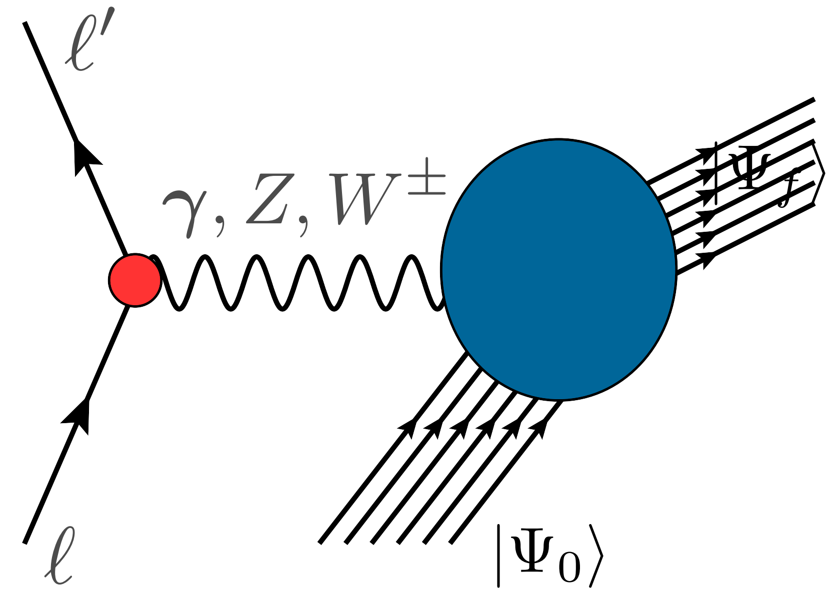


S. Bacca, S. Pastore J.Phys. G 41, 123002 (2014)

Lepton-nucleus scattering

The inclusive cross section of the process in which a lepton scatters off a nucleus can be written in terms of five response functions

$$\frac{d\sigma}{dE_{\ell'}d\Omega_\ell} \propto [v_{00}R_{00} + v_{zz}R_{zz} - v_{0z}R_{0z} + v_{xx}R_{xx} \mp v_{xy}R_{xy}]$$

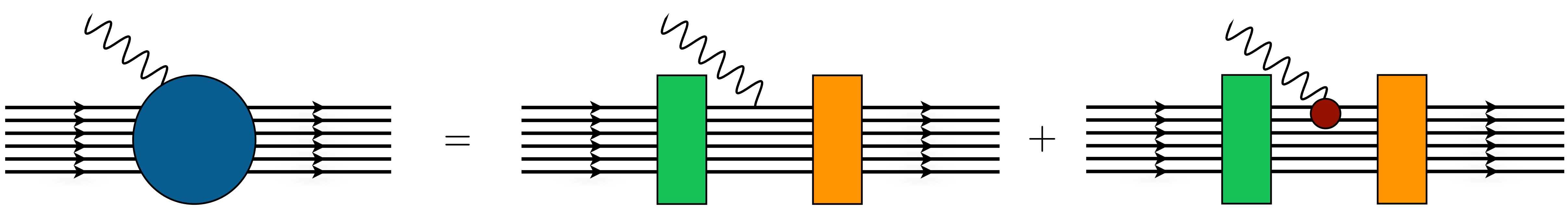


- In the electromagnetic case only the longitudinal and the transverse response functions contribute

- The response functions contain all the information on target structure and dynamics

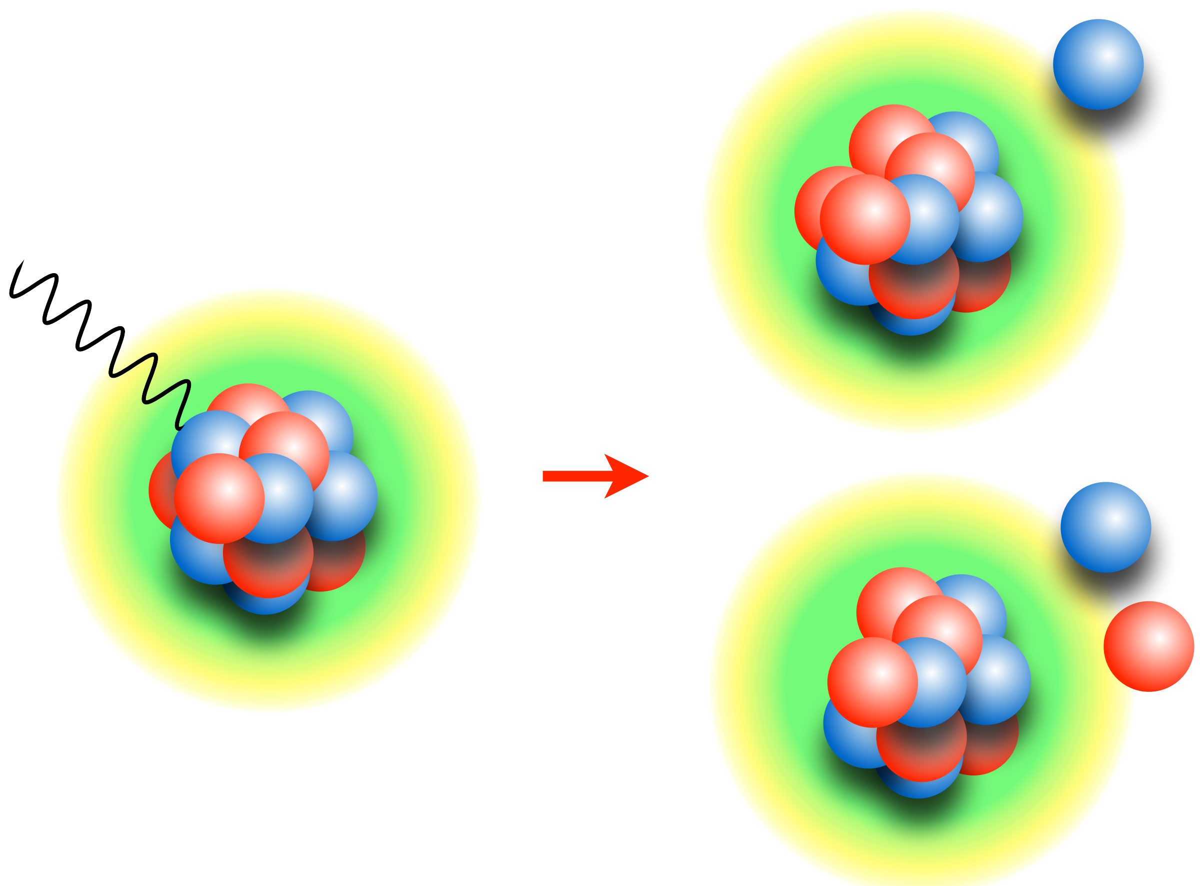
$$R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_f \langle\Psi_0|J^\dagger_\alpha(\mathbf{q})|\Psi_f\rangle\langle\Psi_f|J_\beta(\mathbf{q})|\Psi_0\rangle\delta(\omega - E_f + E_0)$$

- They account for initial state correlations, final state correlations and two-body currents



Lepton-nucleus scattering

- At (very) large momentum transfer, scattering off a nuclear target reduces to the sum of scattering processes involving bound nucleons → short-range correlations.



$$|\Psi_f\rangle \simeq |p_1\rangle \otimes |\Psi_f\rangle_{A-1}$$

$$|\Psi_f\rangle \simeq |p_1, p_2\rangle \otimes |\Psi_f\rangle_{A-2}$$

- Relativistic effects play a major role and need to be accounted for along with nuclear correlations

- Resonance production and deep inelastic scattering processes also need to be addressed

Moderate momentum-transfer regime

- At moderate momentum transfer, the inclusive cross section can be written in terms of the response functions

$$R_{\alpha\beta}(\omega,\mathbf{q})=\sum_f \langle\Psi_0|J_\alpha^\dagger(\mathbf{q})|\Psi_f\rangle\langle\Psi_f|J_\beta(\mathbf{q})|\Psi_0\rangle\delta(\omega-E_f+E_0)$$

- Both initial and final states are eigenstates of the nuclear Hamiltonian

$$H|\Psi_0\rangle=E_0|\Psi_0\rangle \qquad H|\Psi_f\rangle=E_f|\Psi_f\rangle$$

- As for the electron scattering on ^{12}C

$$|^{12}C^*\rangle, |^{11}\mathrm{B},p\rangle,\ |^{11}\mathrm{C},n\rangle,\ |^{10}\mathrm{B},pn\rangle,\ |^{10}\mathrm{Be},pp\rangle$$

- Relativistic corrections are included in the current operators and in the nucleon form factors

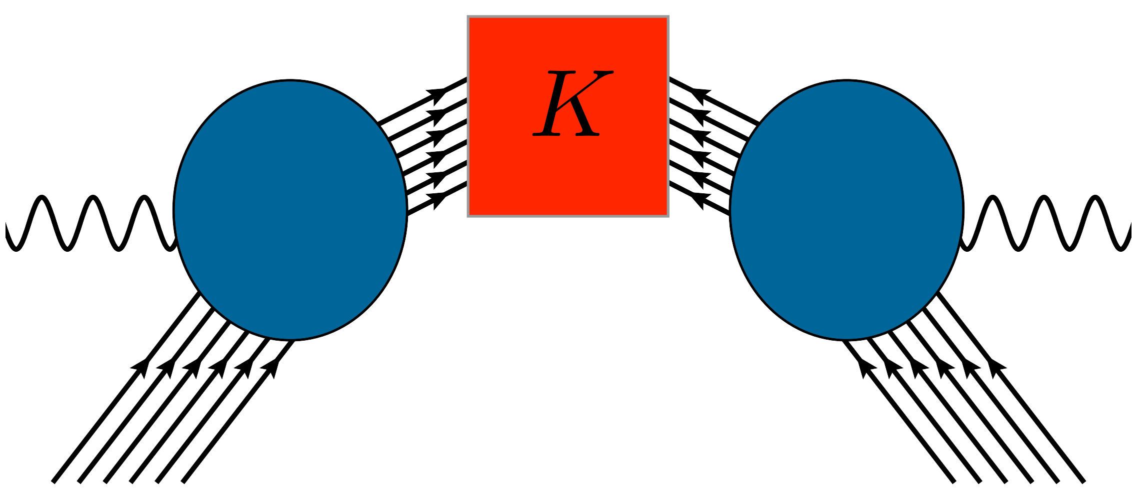
Integral transform techniques

- The integral transform of the response function are generally defined as

$$E_{\alpha\beta}(\sigma, \mathbf{q}) \equiv \int d\omega K(\sigma, \omega) R_{\alpha\beta}(\omega, \mathbf{q})$$

- Using the completeness of the final states, they can be expressed in terms of ground-state expectation values

$$E_{\alpha\beta}(\sigma, \mathbf{q}) = \langle \Psi_0 | J_\alpha^\dagger(\mathbf{q}) K(\sigma, H - E_0) J_\beta(\mathbf{q}) | \Psi_0 \rangle$$

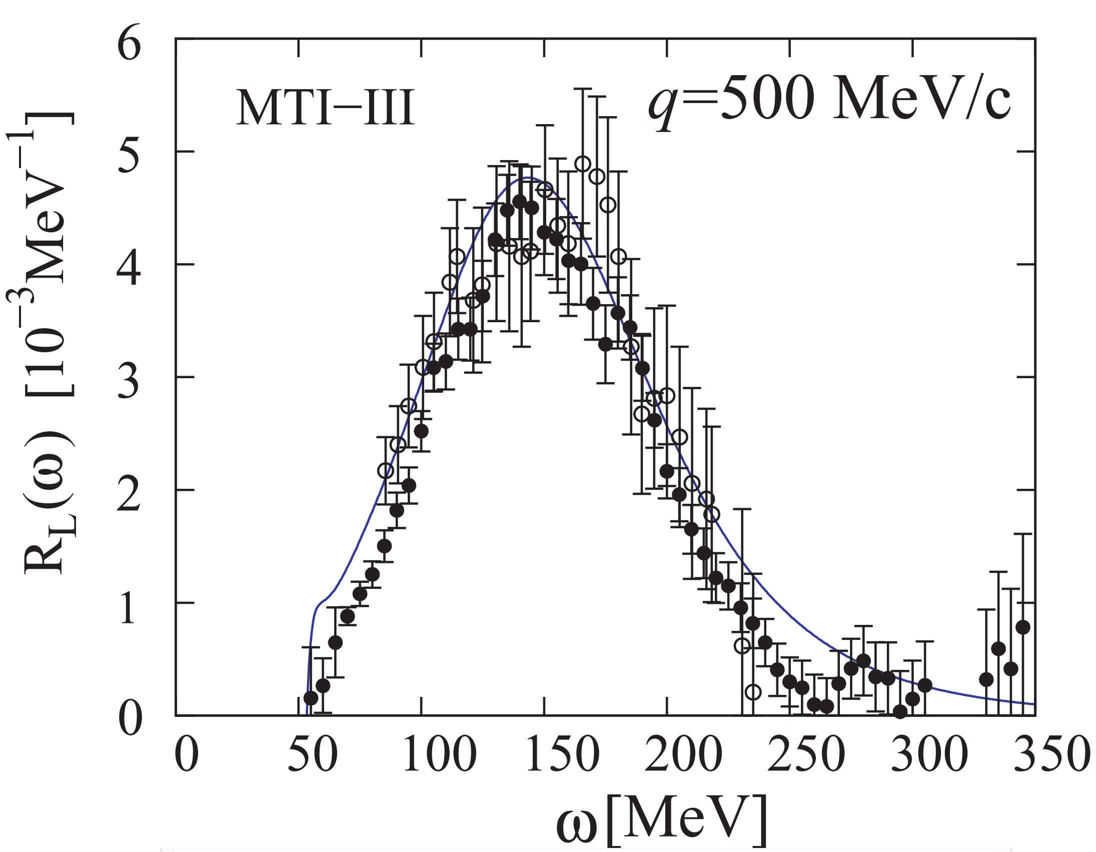


Lorentz integral transform (LIT)

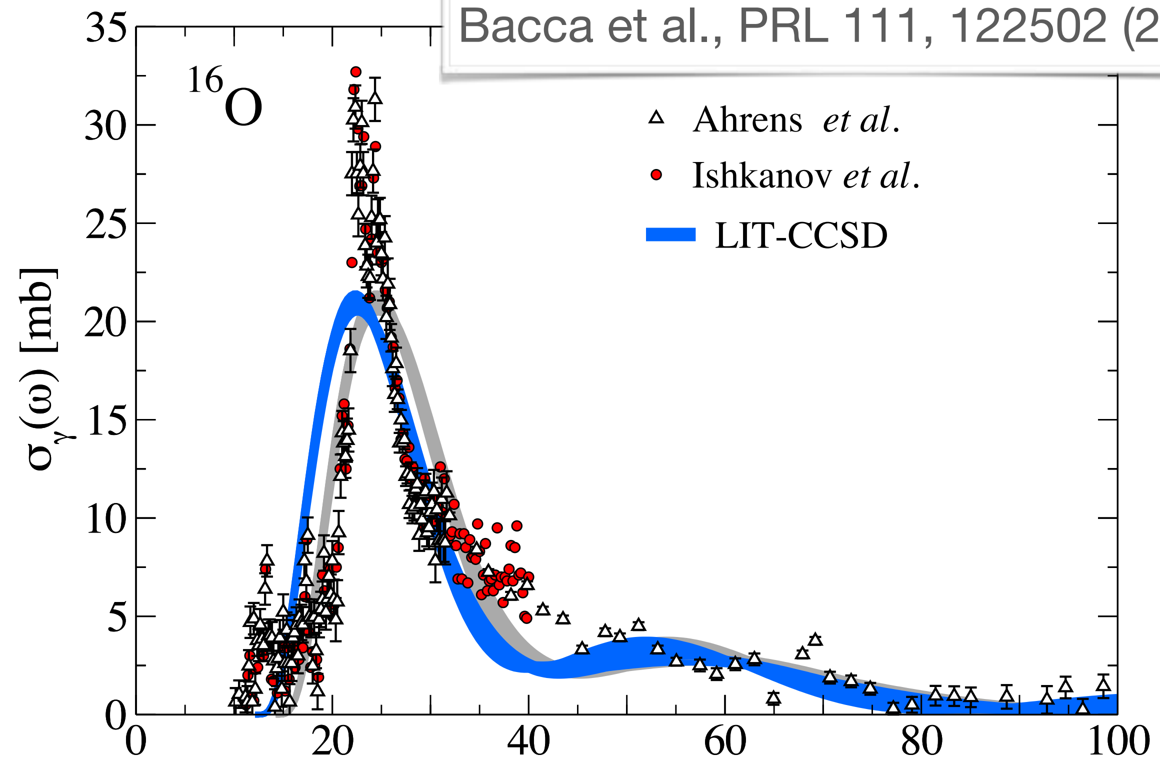
- The Lorentz integral transform

$$K(\sigma, \omega) = \frac{1}{(\omega - \sigma_R)^2 + \sigma_I^2}$$

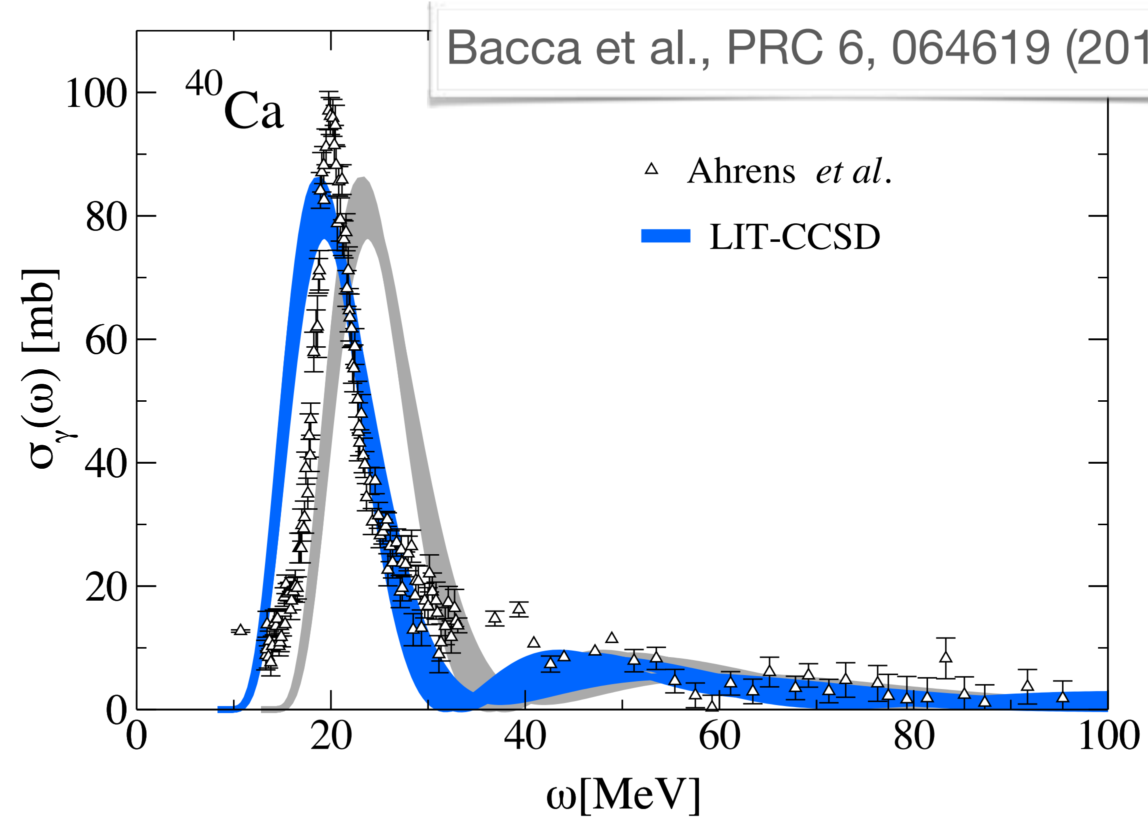
has been successfully exploited in the calculation of electromagnetic and neutral-weak responses



Bacca et al., PRC 76, 014003 (2007)



Bacca et al., PRL 111, 122502 (2013)



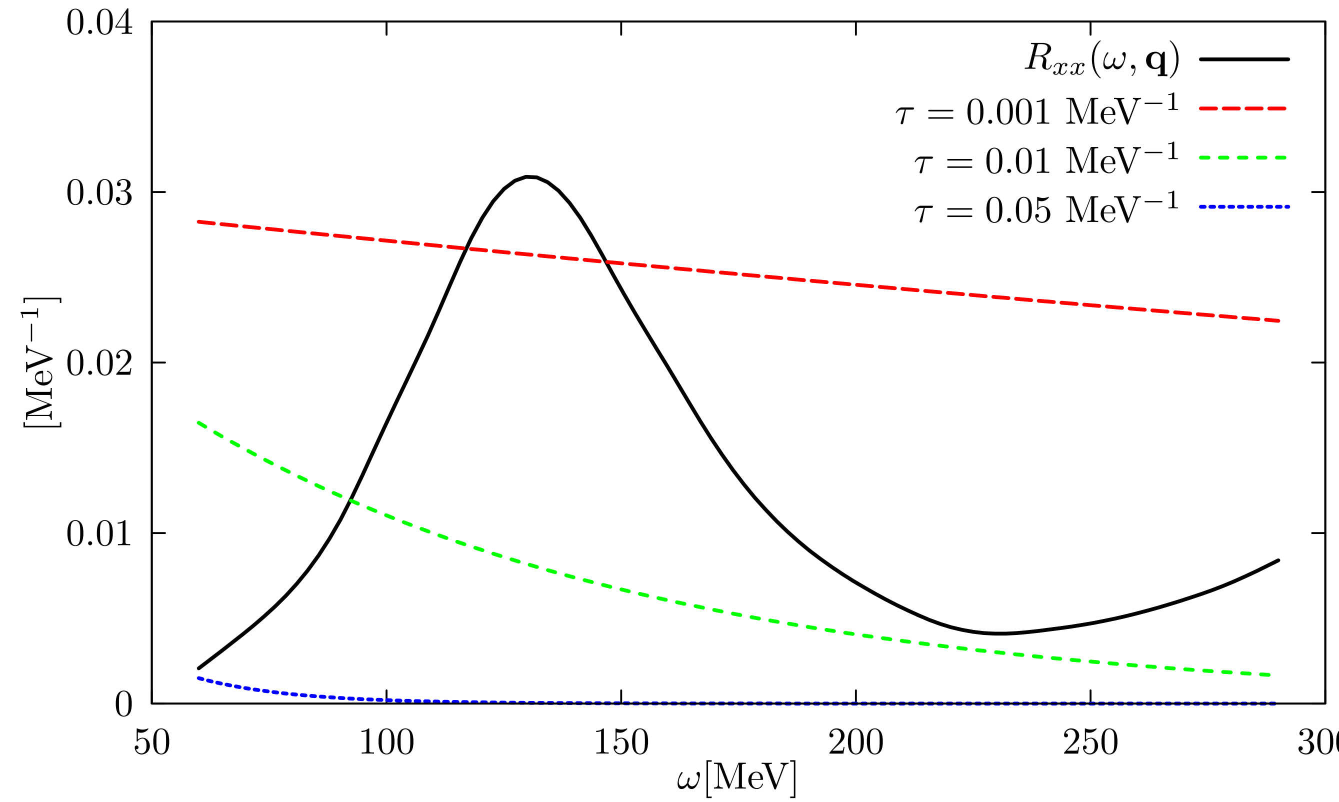
Bacca et al., PRC 6, 064619 (2014)

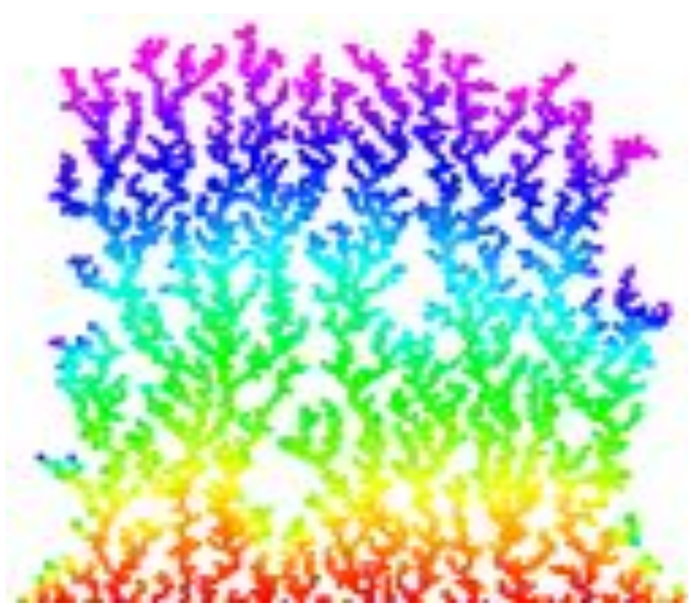
Euclidean response function

Valuable information on the energy dependence of the response functions can be inferred from their Laplace transforms

$$E_{\alpha\beta}(\tau, \mathbf{q}) \equiv \int d\omega e^{-\omega\tau} R_{\alpha\beta}(\omega, \mathbf{q})$$

At finite imaginary time the contributions from large energy transfer are quickly suppressed





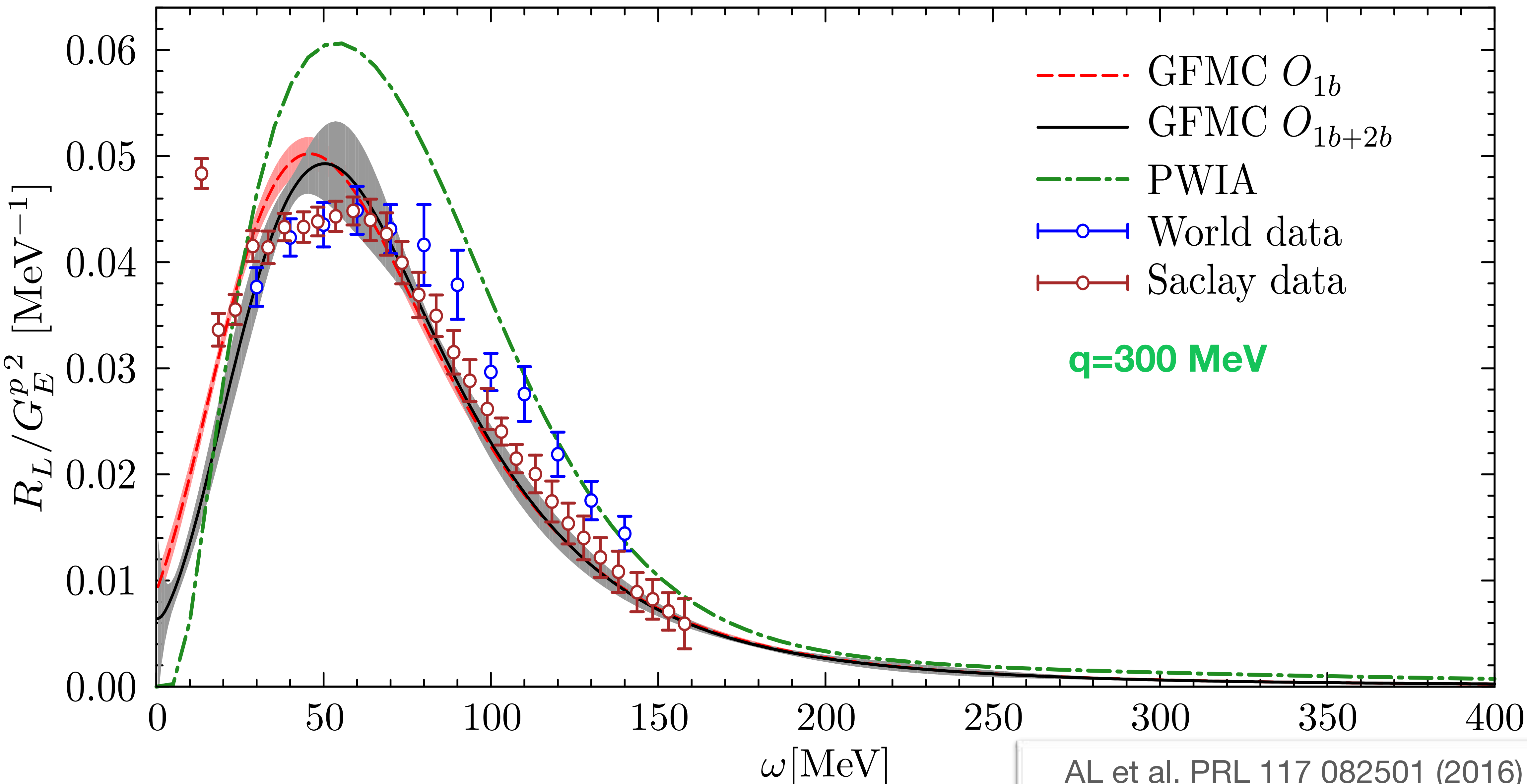
The system is first heated up by the transition operator.
Its cooling determines the Euclidean response of the system

$$E_{\alpha\beta}(\tau, \mathbf{q}) = \langle\Psi_0|J^\dagger_\alpha(\mathbf{q})e^{-(H-E_0)\tau}J_\beta(\mathbf{q})|\Psi_0\rangle$$

Same technique used in Lattice QCD, condensed matter physics…

^{12}C electromagnetic response

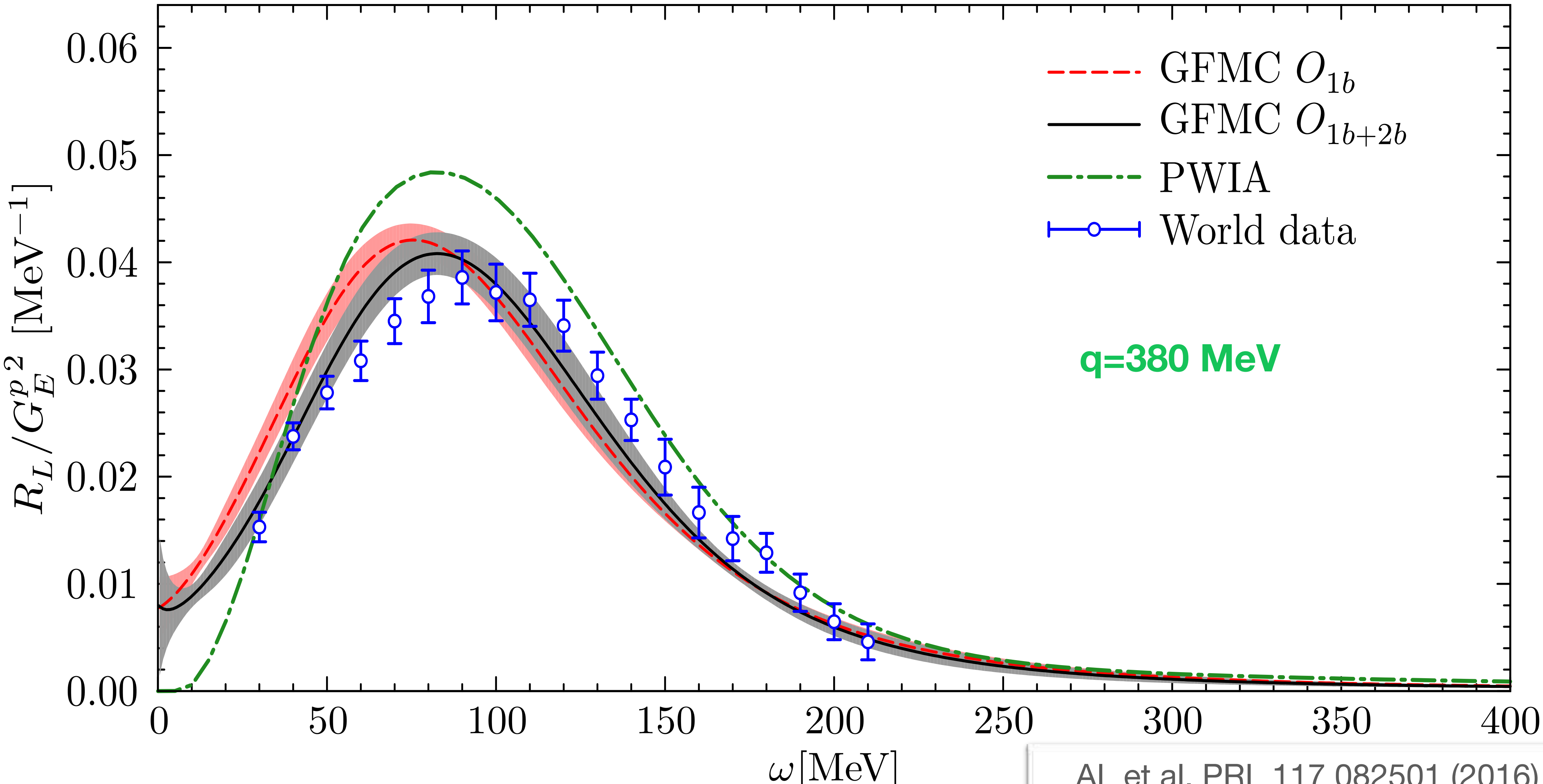
- We inverted the electromagnetic Euclidean response of ^{12}C
- Very good agreement with the experimental data. Small contribution from two-body currents.



AL et al. PRL 117 082501 (2016)

^{12}C electromagnetic response

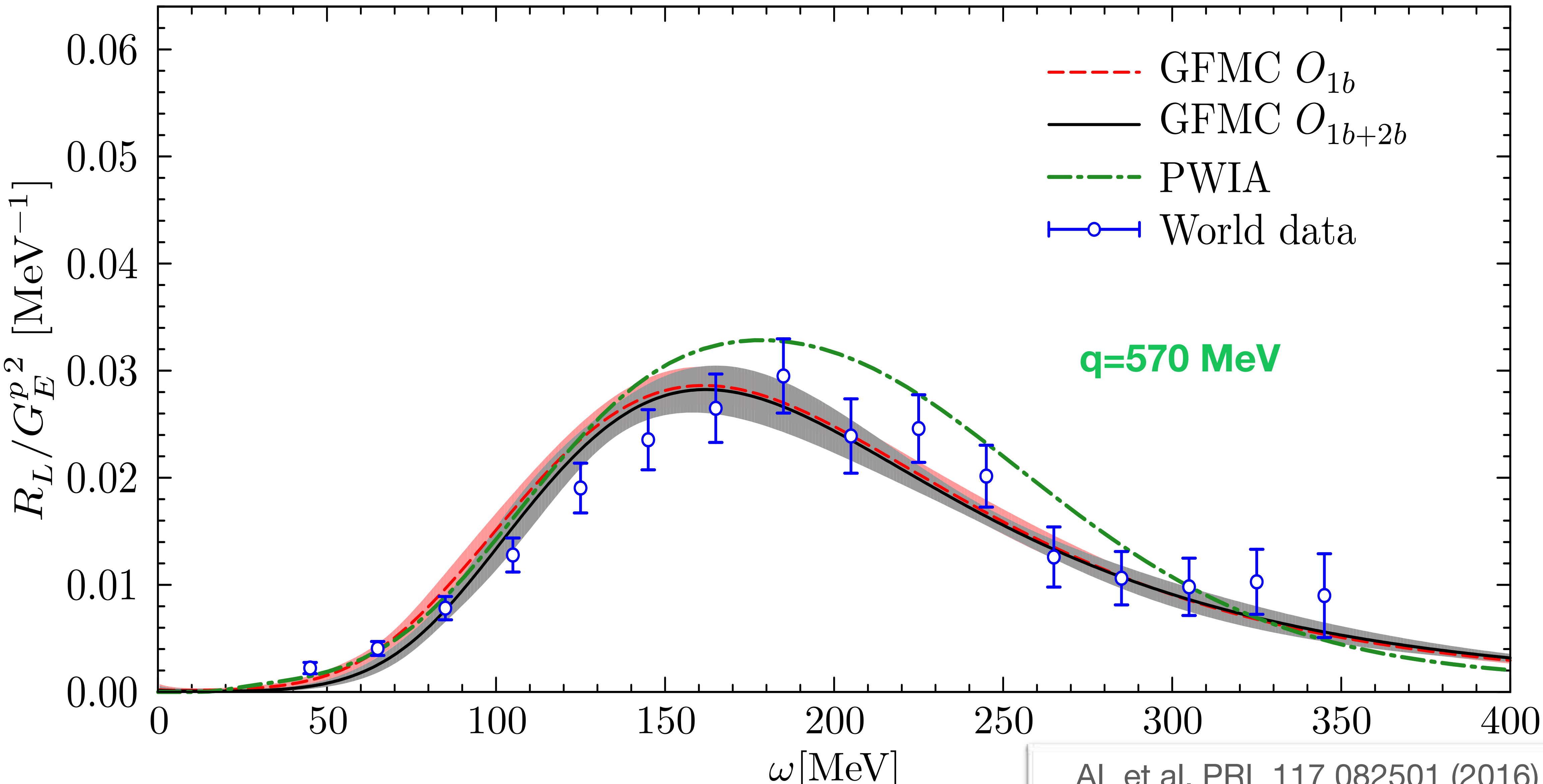
- We inverted the electromagnetic Euclidean response of ^{12}C
- Very good agreement with the experimental data. Small contribution from two-body currents.



AL et al. PRL 117 082501 (2016)

^{12}C electromagnetic response

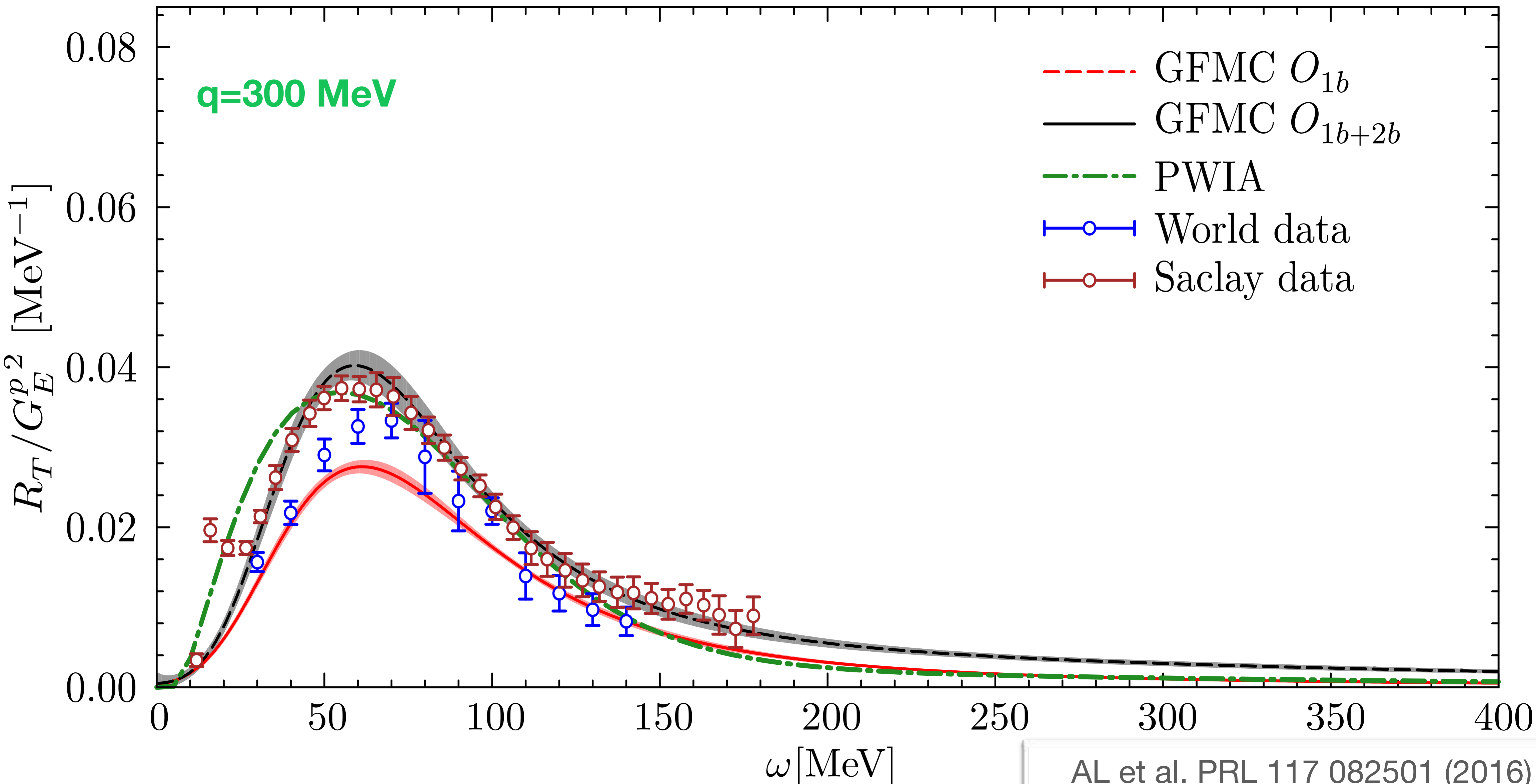
- We inverted the electromagnetic Euclidean response of ^{12}C
- Very good agreement with the experimental data. Small contribution from two-body currents.



AL et al. PRL 117 082501 (2016)

^{12}C electromagnetic response

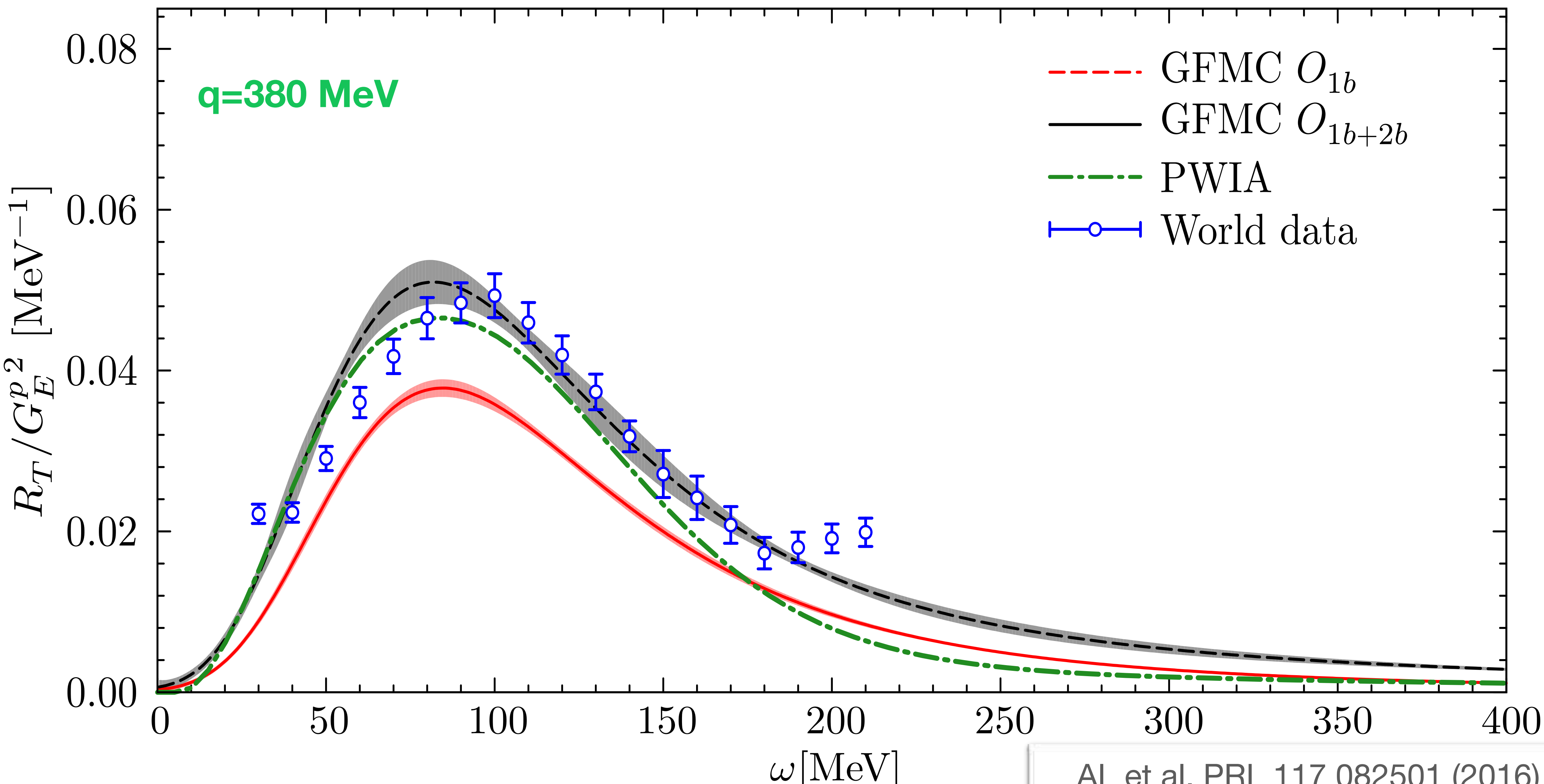
- We inverted the electromagnetic Euclidean response of ^{12}C
- Very good agreement with the experimental data once two-body currents are accounted for



AL et al. PRL 117 082501 (2016)

^{12}C electromagnetic response

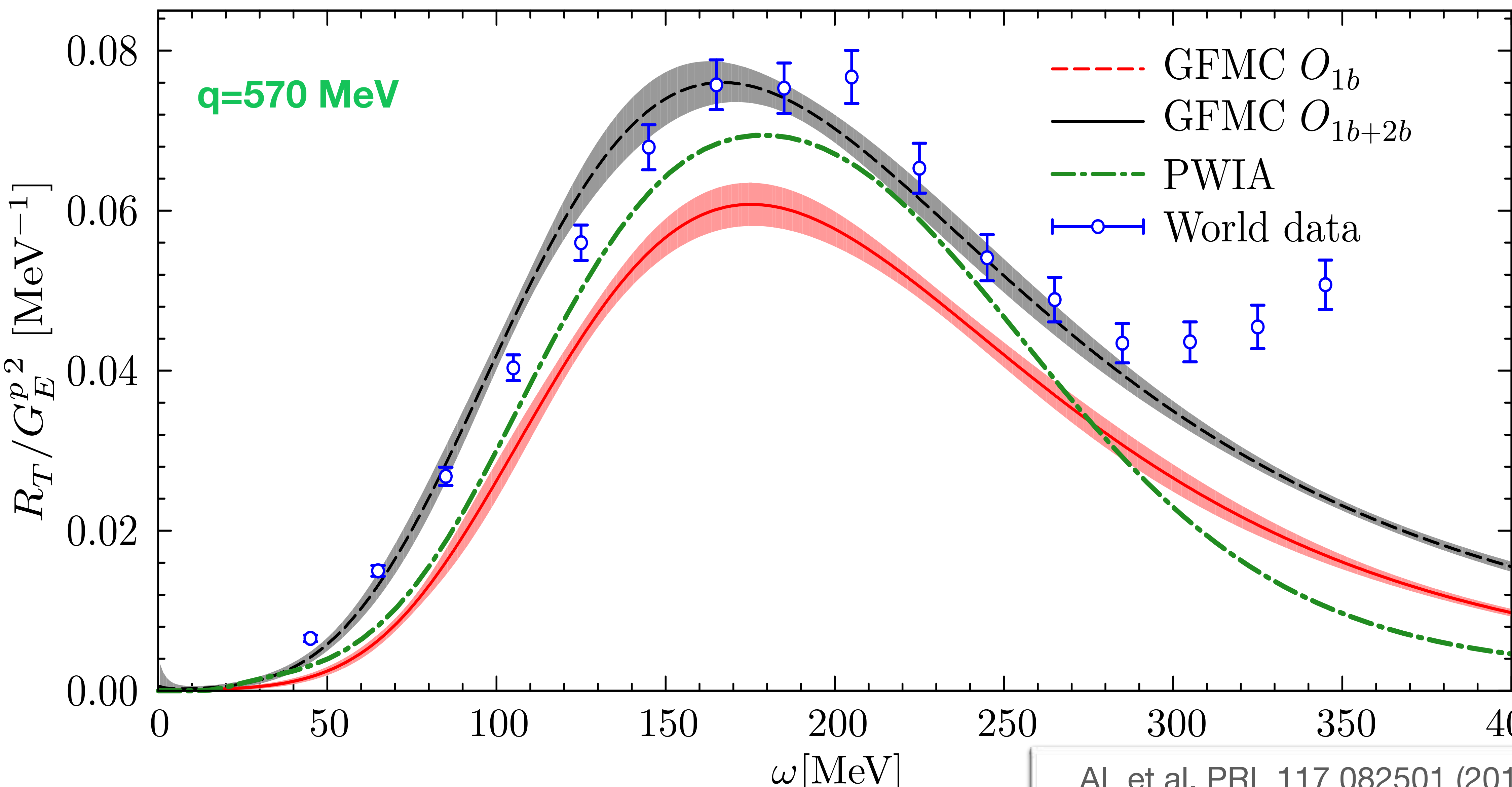
- We inverted the electromagnetic Euclidean response of ^{12}C
- Very good agreement with the experimental data once two-body currents are accounted for



AL et al. PRL 117 082501 (2016)

^{12}C electromagnetic response

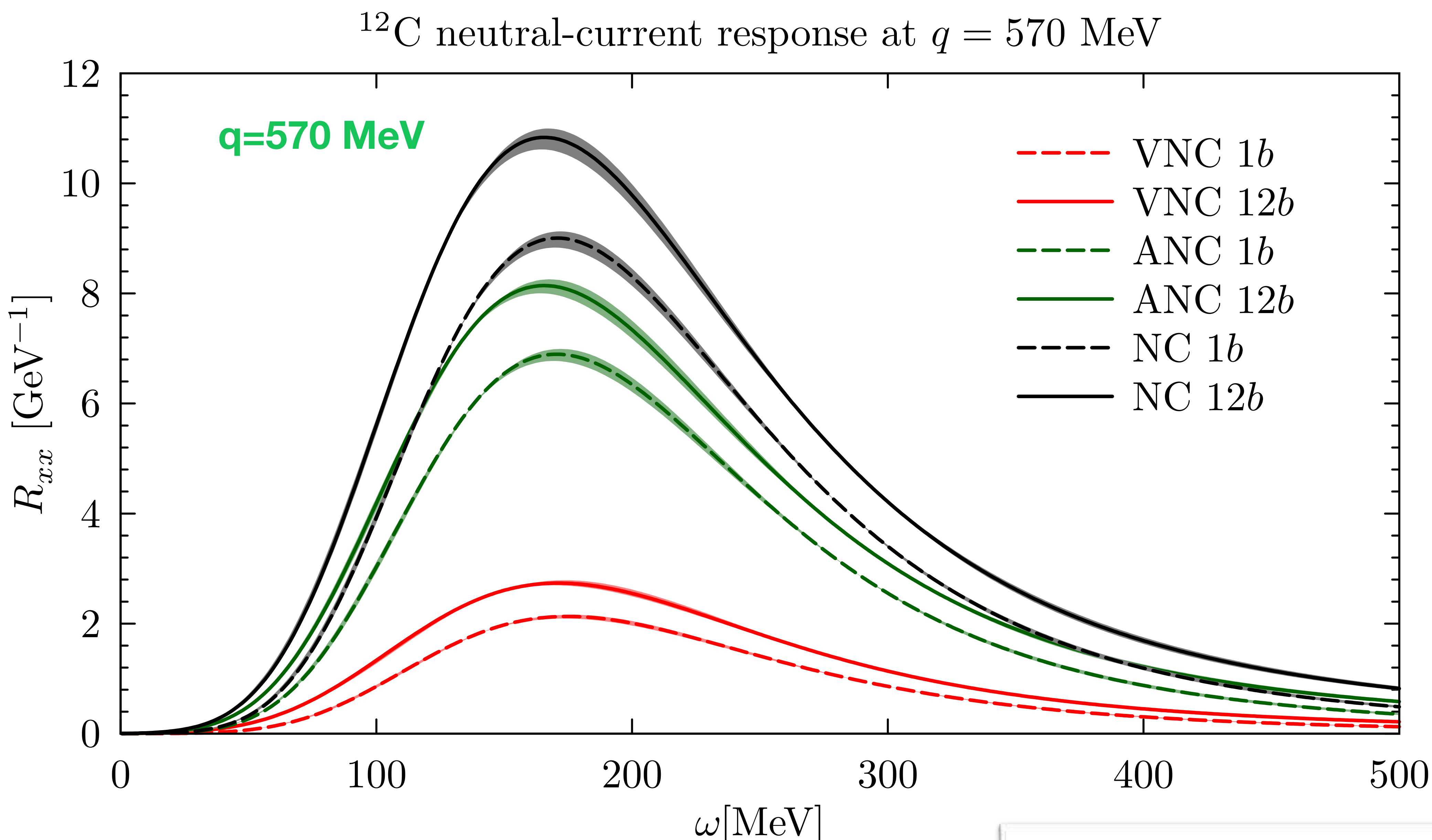
- We inverted the electromagnetic Euclidean response of ^{12}C
- Very good agreement with the experimental data once two-body currents are accounted for



AL et al. PRL 117 082501 (2016)

^{12}C neutral-current response

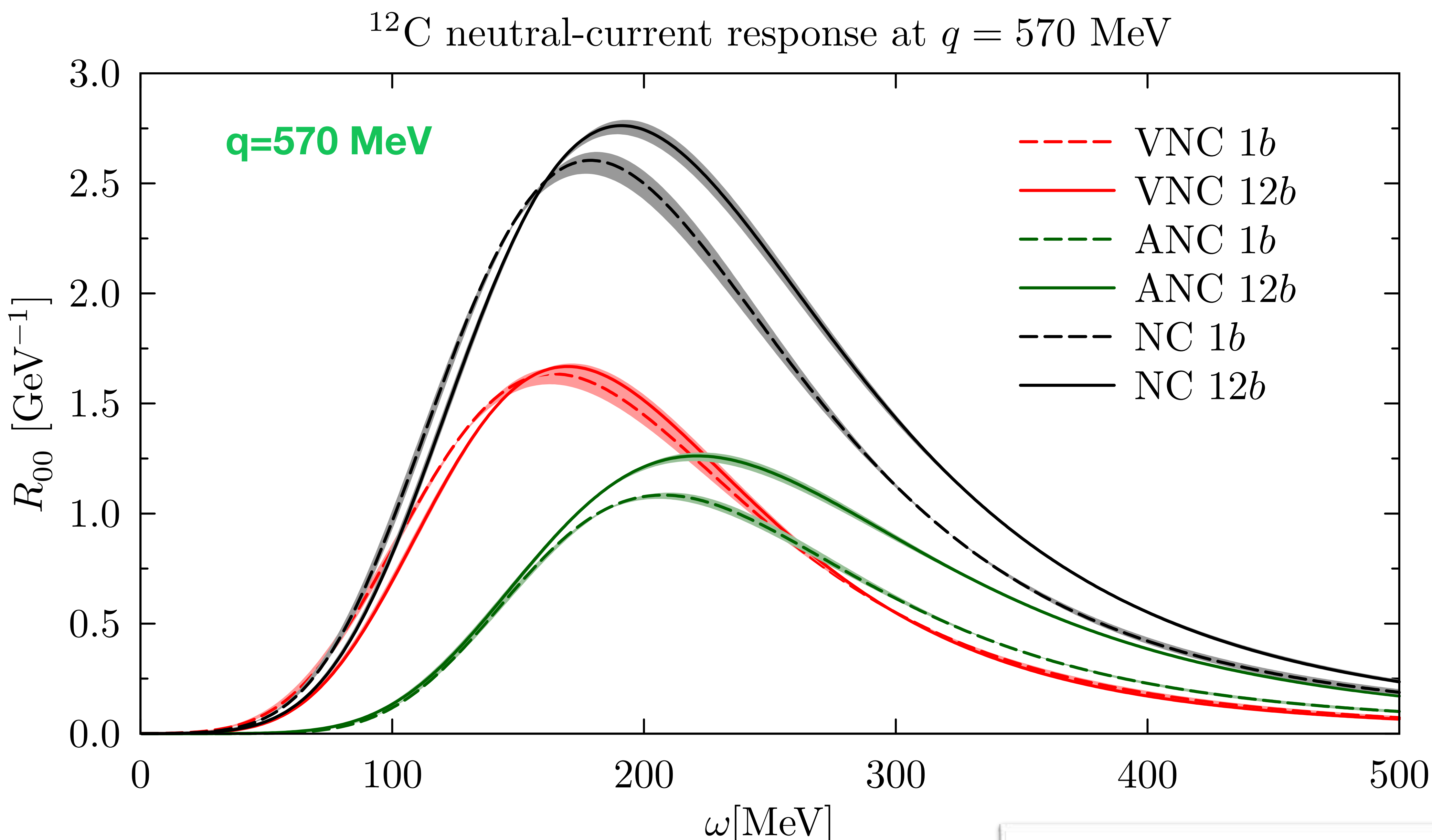
- We were recently able to invert the neutral-current Euclidean responses of ^{12}C



AL et al. PRC 97 022502 (2018)

^{12}C neutral-current response

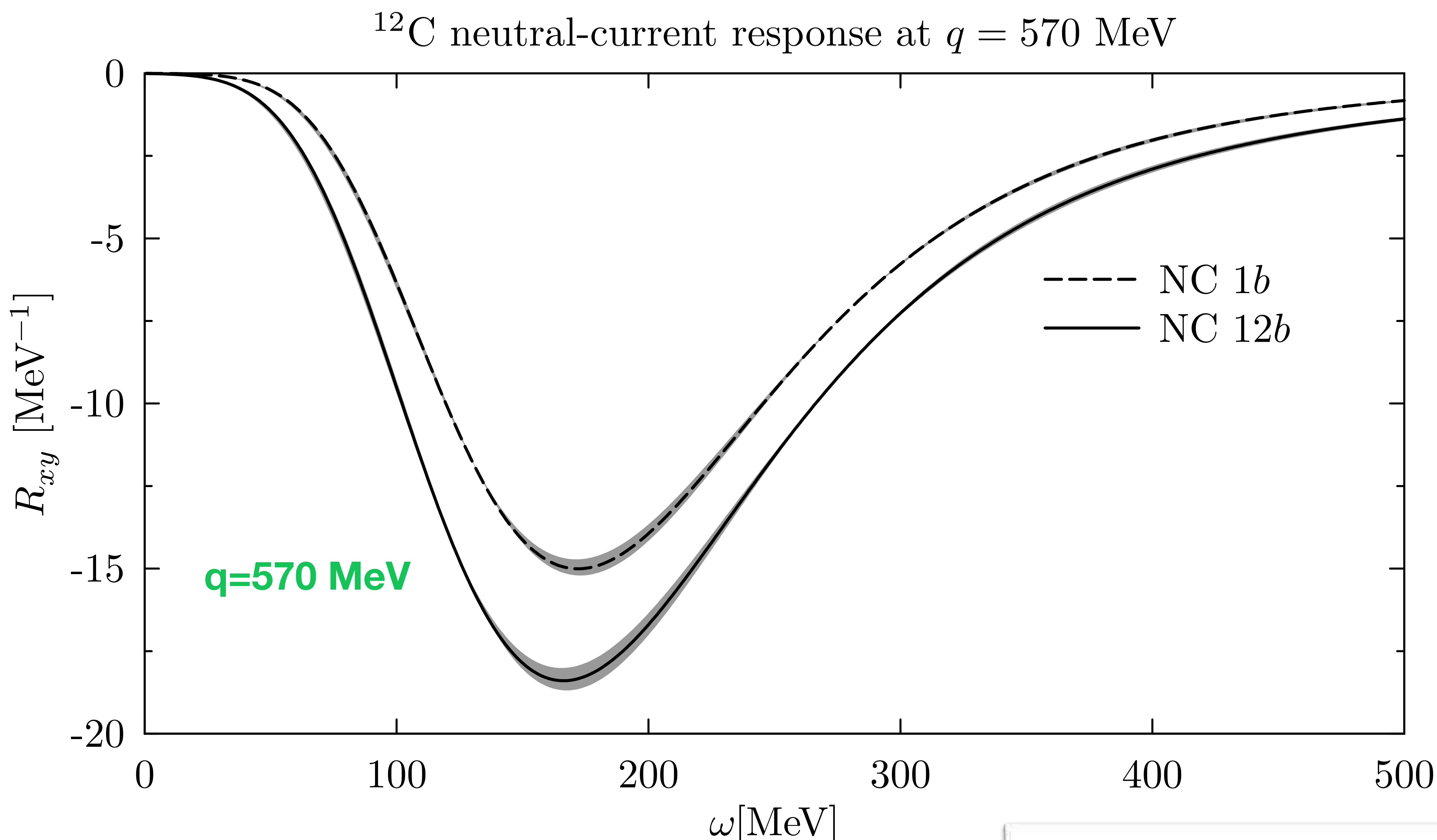
- We were recently able to invert the neutral-current Euclidean responses of ^{12}C



AL et al. PRC 97 022502 (2018)

^{12}C neutral-current response

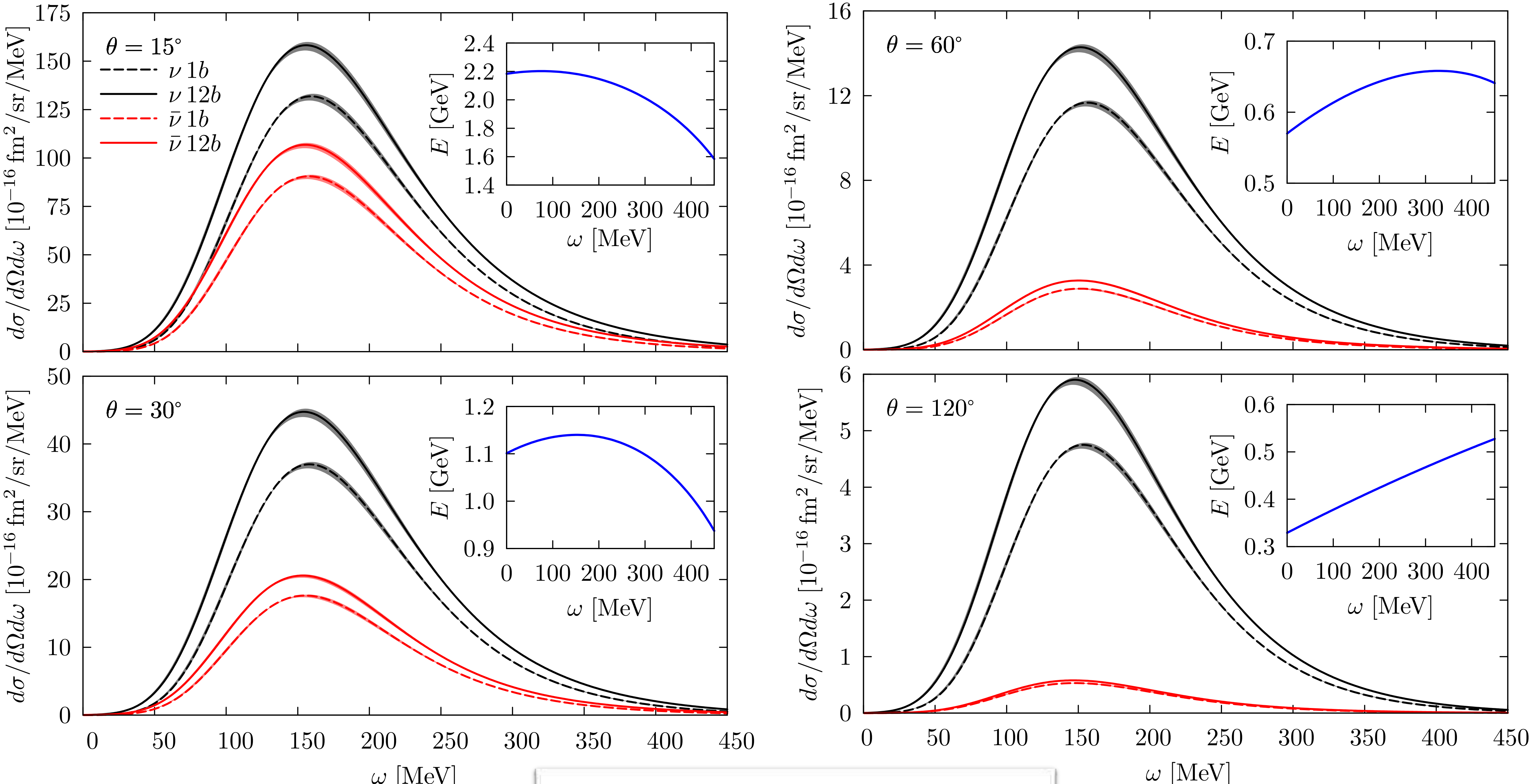
- We were recently able to invert the neutral-current Euclidean responses of ^{12}C



AL et al. PRC 97 022502 (2018)

^{12}C neutral-current cross-section

- We computed the neutrino and anti-neutrino differential cross sections for a fixed value of the three-momentum transfer as function of the energy transfer for a number of scattering angles



AL et al. PRC 97 022502 (2018)

Relativistic effects in a correlated system

- Non relativistic approaches are limited to moderate momentum transfers

- In a generic reference frame the longitudinal response reads

$$R_L^{fr} = \sum_f \left| \langle \psi_i | \sum_j \rho_j(\mathbf{q}^{fr}, \omega^{fr}) | \psi_f \rangle \right|^2 \delta(E_f^{fr} - E_i^{fr} - \omega^{fr})$$

$$\delta(E_f^{fr} - E_i^{fr} - \omega^{fr}) \approx \delta[e_f^{fr} + (P_f^{fr})^2/(2M_T) - e_i^{fr} - (P_i^{fr})^2/(2M_T) - \omega^{fr}]$$

- The response in the LAB frame is given by the Lorentz transform

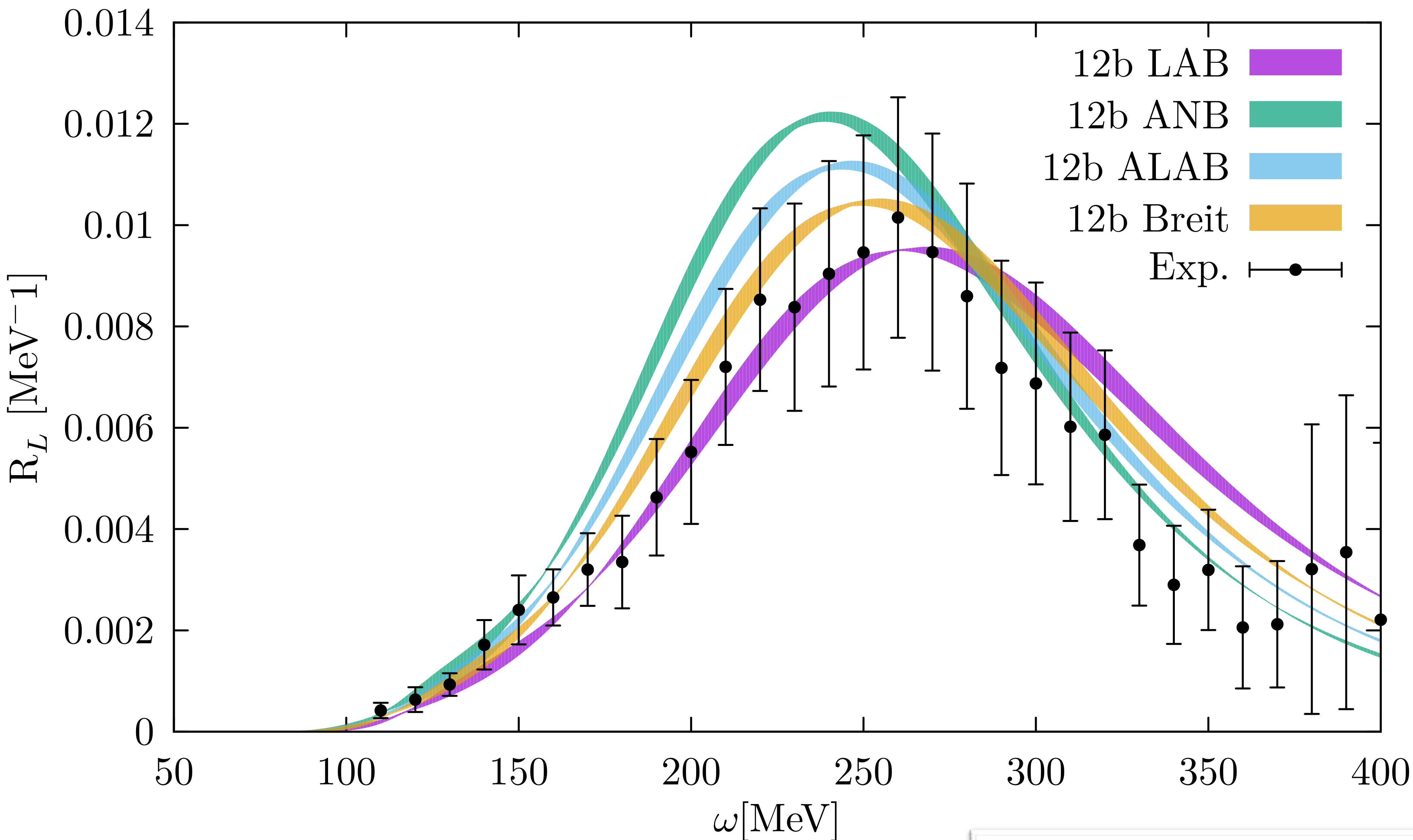
$$R_L(\mathbf{q}, \omega) = \frac{\mathbf{q}^2}{(\mathbf{q}^{fr})^2} \frac{E_i^{fr}}{M_0} R_L^{fr}(\mathbf{q}^{\mathbf{fr}}, \omega^{fr})$$

where

$$q^{fr} = \gamma(q - \beta\omega),\ \omega^{fr} = \gamma(\omega - \beta q),\ P_i^{fr} = -\beta\gamma M_0,\ E_i^{fr} = \gamma M_0$$

Relativistic effects in a correlated system

- The ^{4}He longitudinal response at q=700 MeV **strongly** depends on the original reference frame



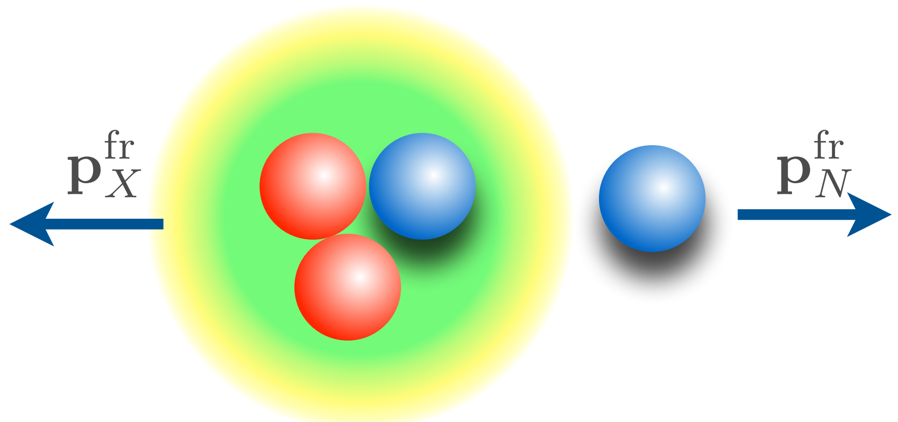
N. Rocco et al. PRC 97 055501(2018)

Relativistic effects in a correlated system

- To determine the relativistic corrections, we consider a two-body breakup model

$$\mathbf{p}^{\rm fr} = \mu\left(\frac{\mathbf{p}_N^{\rm fr}}{m} - \frac{\mathbf{p}_X^{\rm fr}}{M_X}\right) \qquad \mu = \frac{m M_X}{m + M_X}$$

$$\mathbf{P}_f^{\rm fr} = \mathbf{p}_N^{\rm fr} + \mathbf{p}_X^{\rm fr} \qquad M_X = (A-1)m + \epsilon_0^{A-1}$$



- The relative momentum is derived in a relativistic fashion

$$\omega^{\rm fr} = E_f^{\rm fr} - E_i^{\rm fr}$$

$$E_f^{\rm fr} = \sqrt{m^2 + (\mathbf{p}^{\rm fr} + (\mu/M_{A-1})\mathbf{P}_f^{\rm fr})^2} + \sqrt{M_{A-1}^2 + (\mathbf{p}^{\rm fr} - (\mu/m)\mathbf{P}_f^{\rm fr})^2}$$

- And it is used as input in the non relativistic kinetic energy

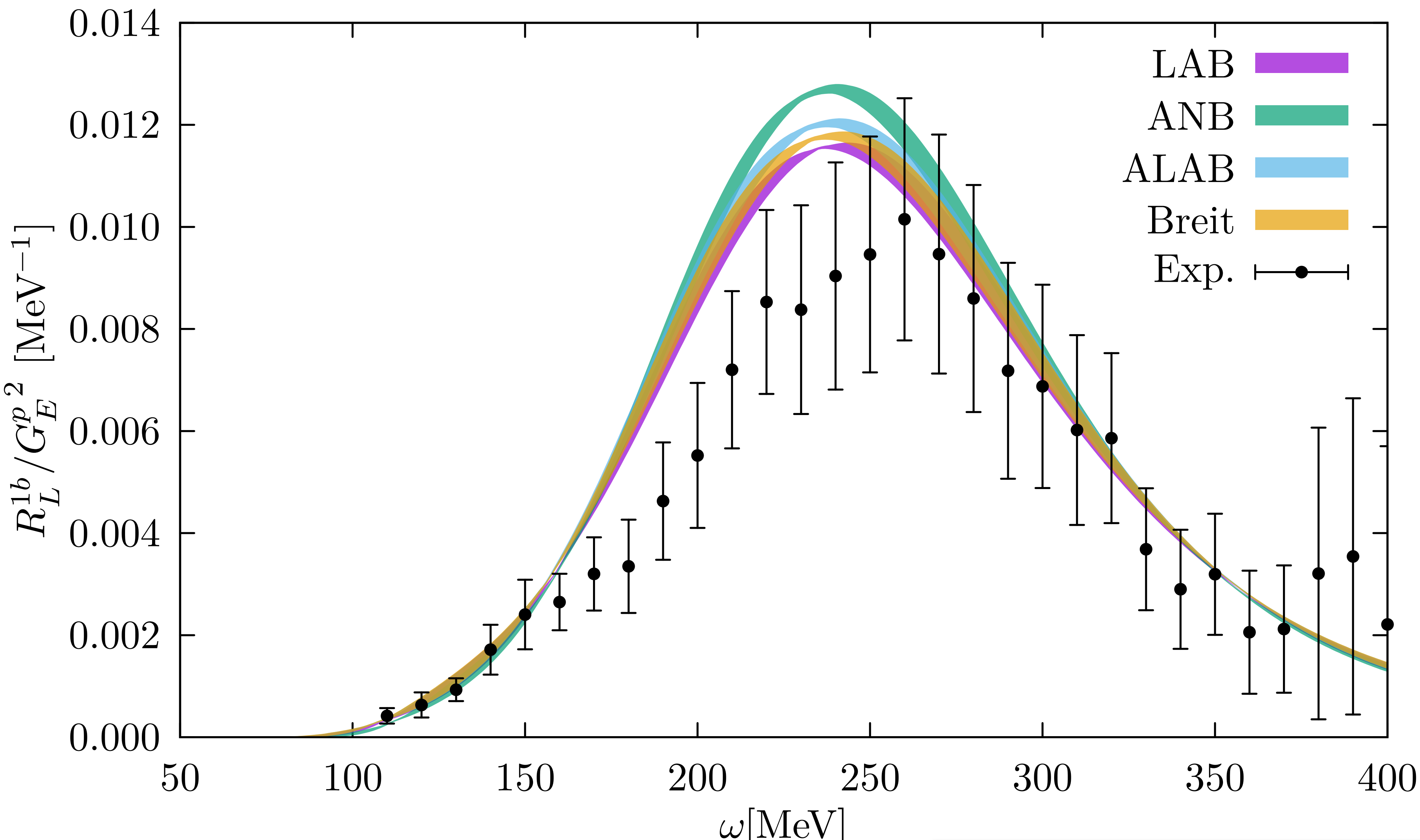
$$\epsilon_f = \frac{p_f^2}{2\mu} + \epsilon_0^{A-1}$$

- The energy-conserving delta function reads

$$\delta(\omega^{fr} - E_f^{\rm fr}(\epsilon_f) + E_0^{fr}) = \left(\frac{\partial E_f^{\rm fr}(\epsilon_f)}{\partial \epsilon_f^{\rm fr}}\right)^{-1} \delta\left(\epsilon_f - \frac{p_f^2(\omega^{\rm fr}, |\mathbf{q}^{\rm fr})}{2\mu} - \epsilon_0^{A-1}\right)$$

Relativistic effects in a correlated system

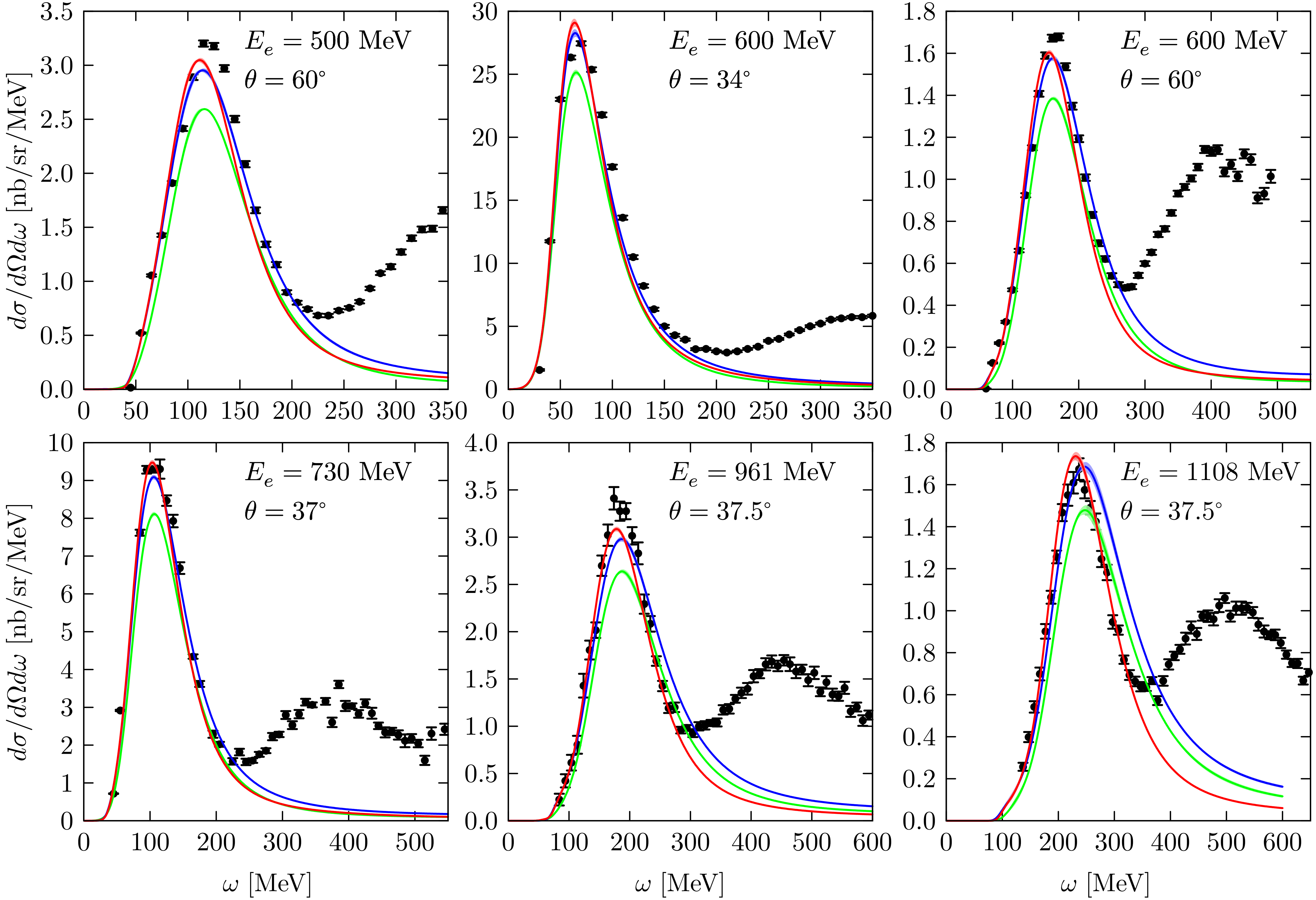
- The ^{4}He longitudinal response at q=700 MeV **mildly** depends on the original reference frame



N. Rocco et al. PRC 97 055501(2018)

N. Rocco et al. PRC 97 055501(2018)

Relativistic effects in a correlated system

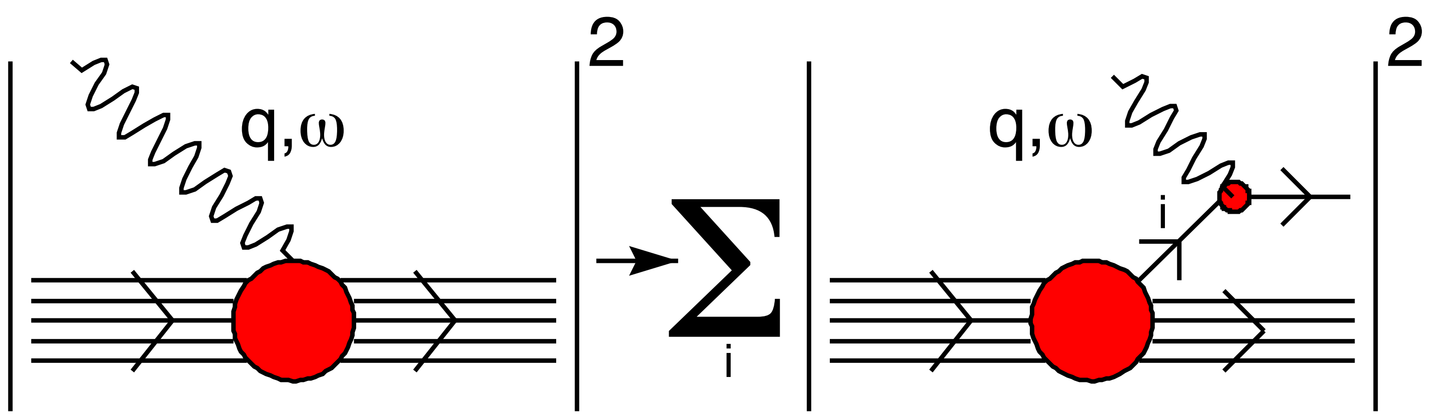


Spectral function approach

At large momentum transfer, scattering off a nuclear target reduces to the incoherent sum of scattering processes involving individual bound nucleons

$$J^\mu \to \sum_i j_i^\mu$$

$$|\Psi_f\rangle \to |\mathbf{p}\rangle \otimes |\Psi_{\tilde{f}}\rangle_{A-1}$$

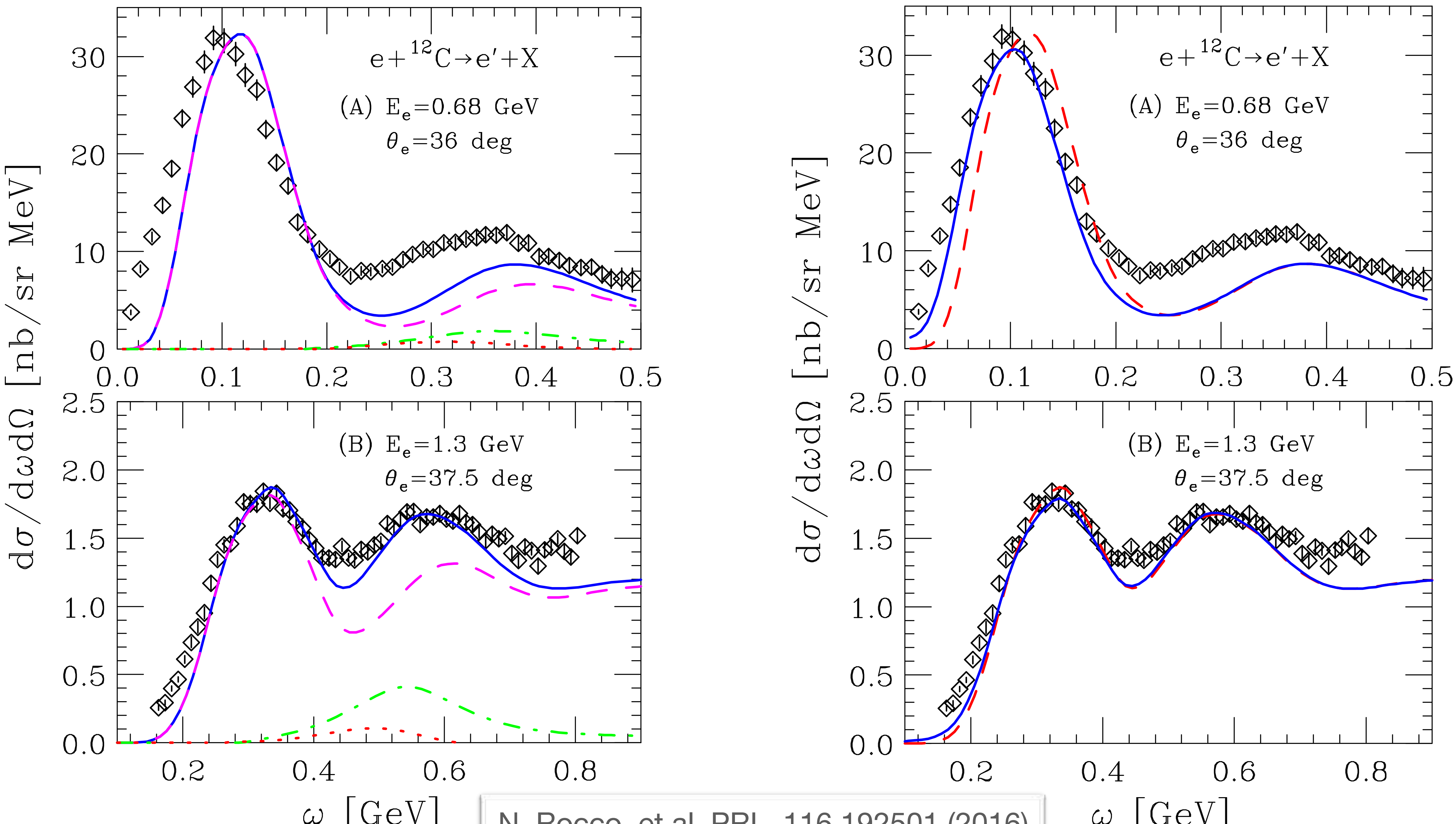


$$\frac{d\sigma_{IA}}{d\Omega_{e'} dE_{e'}} = \int d^3p\, dE\, P(\mathbf{p}, E) \left[Z \frac{d\sigma_{ep}}{d\Omega_{e'} dE_{e'}} + (A - Z) \frac{d\sigma_{en}}{d\Omega_{e'} dE_{e'}} \right]$$

The spectral function yields the probability of removing a nucleon with momentum $\mathbf{p}$ from the target ground state leaving the residual system with excitation energy E .

Electron-nucleus scattering

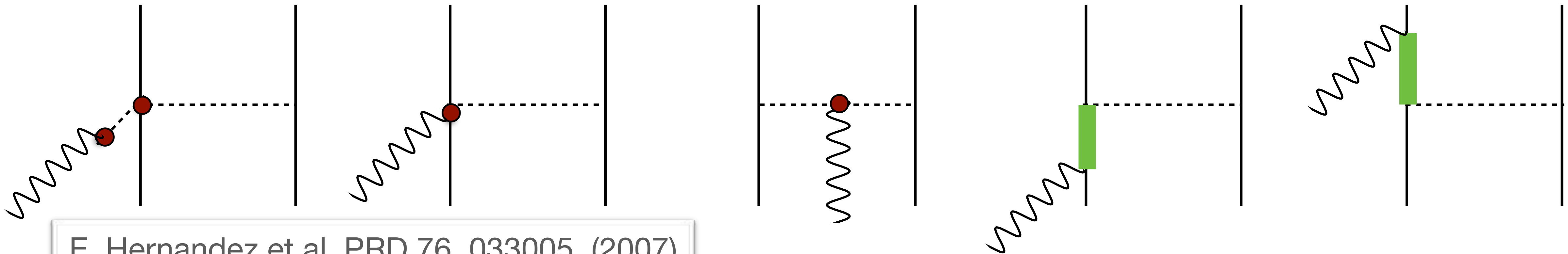
Using relativistic MEC requires the extension of the factorization scheme to two-nucleon emissions ⟷ $|\Psi_f\rangle \rightarrow |\mathbf{pp}'\rangle \otimes |\Psi_{\tilde{f}}\rangle_{A-2}$



N. Rocco, et al. PRL. 116 192501 (2016)

Neutrino-nucleus scattering

- We implemented vector and axial vector relativistic two-body currents in the factorization scheme



E. Hernandez et al. PRD 76, 033005 (2007)

We developed an highly-parallel Monte Carlo integration code

The calculation of the MEC current matrix elements is carried our automatically

→

No need to use approximations such that of the “frozen nucleons”

Simplifies the use of a different version of the MEC

- We employ the factorization of the two-body spectral function, related to

$$n(\mathbf{k}_1, \mathbf{k}_2) = n(\mathbf{k}_1) n(\mathbf{k}_2) + \mathcal{O}\left(\frac{1}{A}\right)$$

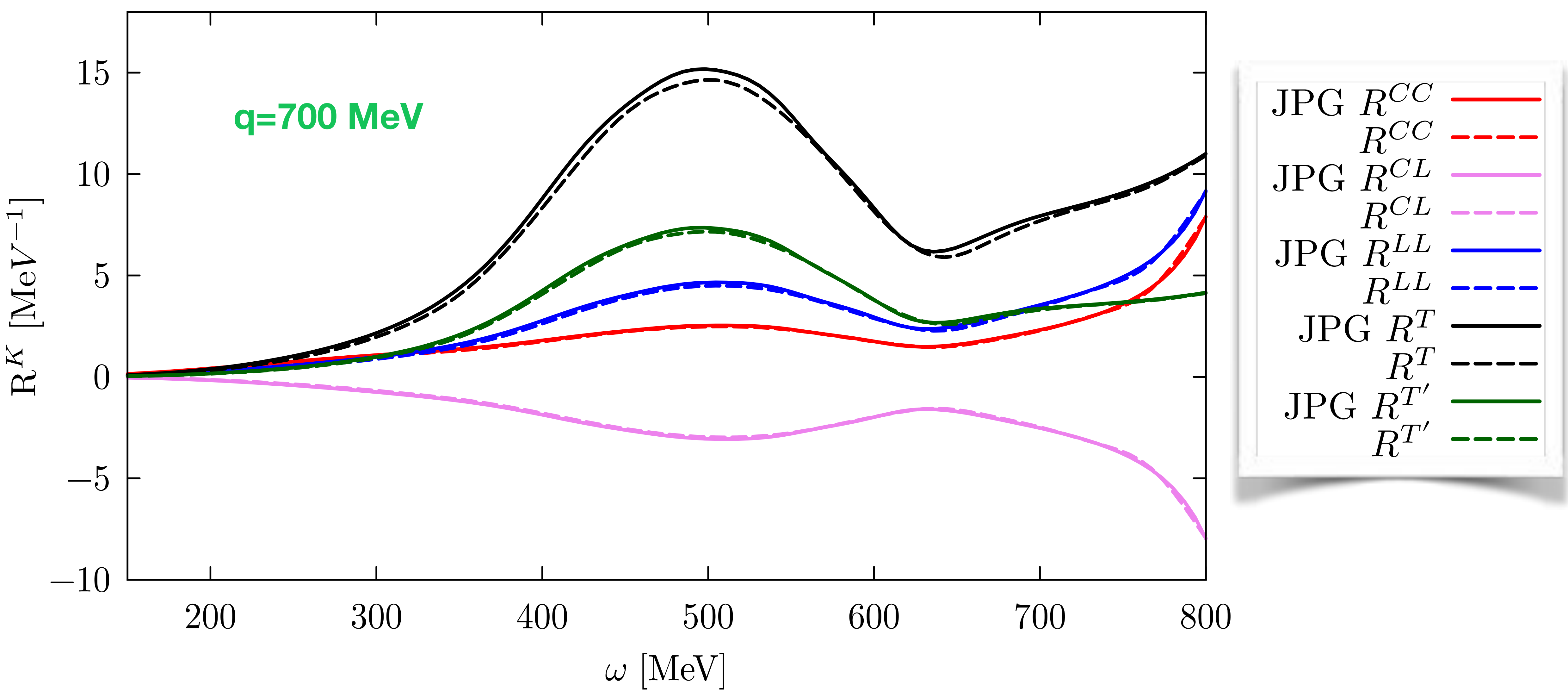
We are improving this approximation using the cluster-expansion formalism

↔

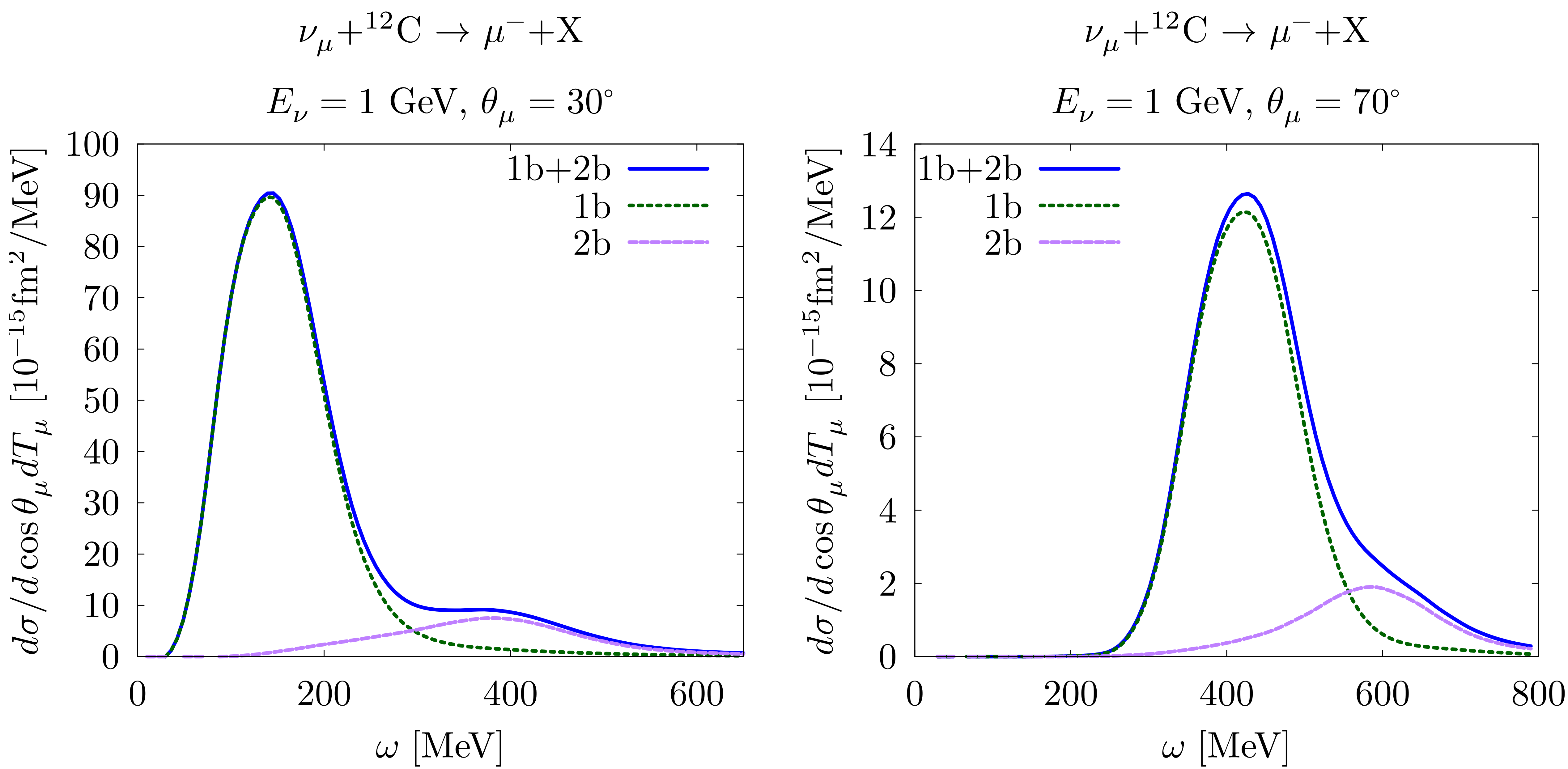
Analogy with the “short-time approximation” and the “contact formalism”

Charged responses

- We successfully compared the charged-current response functions of ^{12}C with the results of I. Ruiz Simo, et. al, Journal of Phys. G 44, no. 6 (2017)
- To this aim we approximated the two-body spectral function with that of the global relativistic Fermi gas model

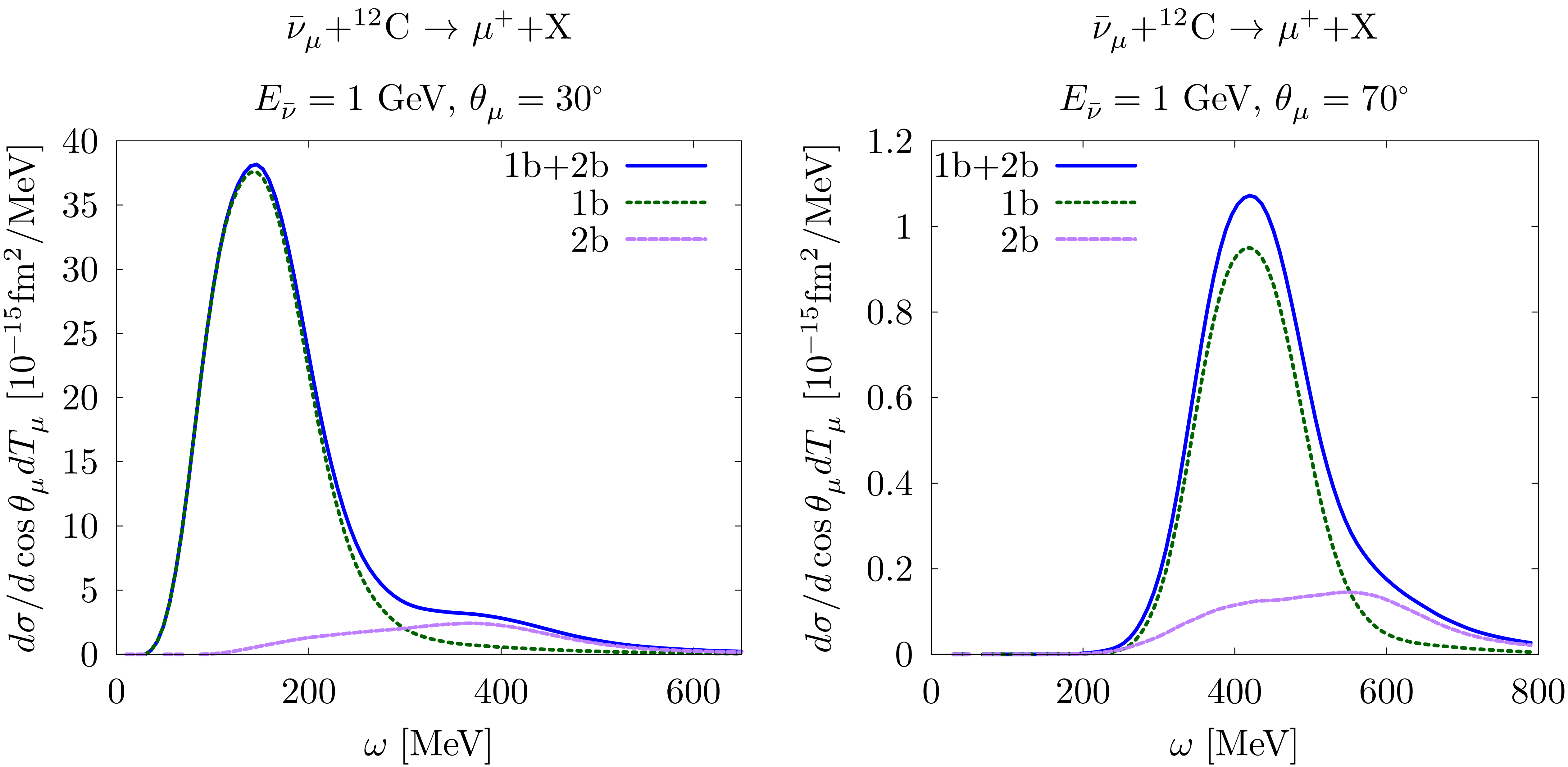


Neutrino-^{12}C cross sections



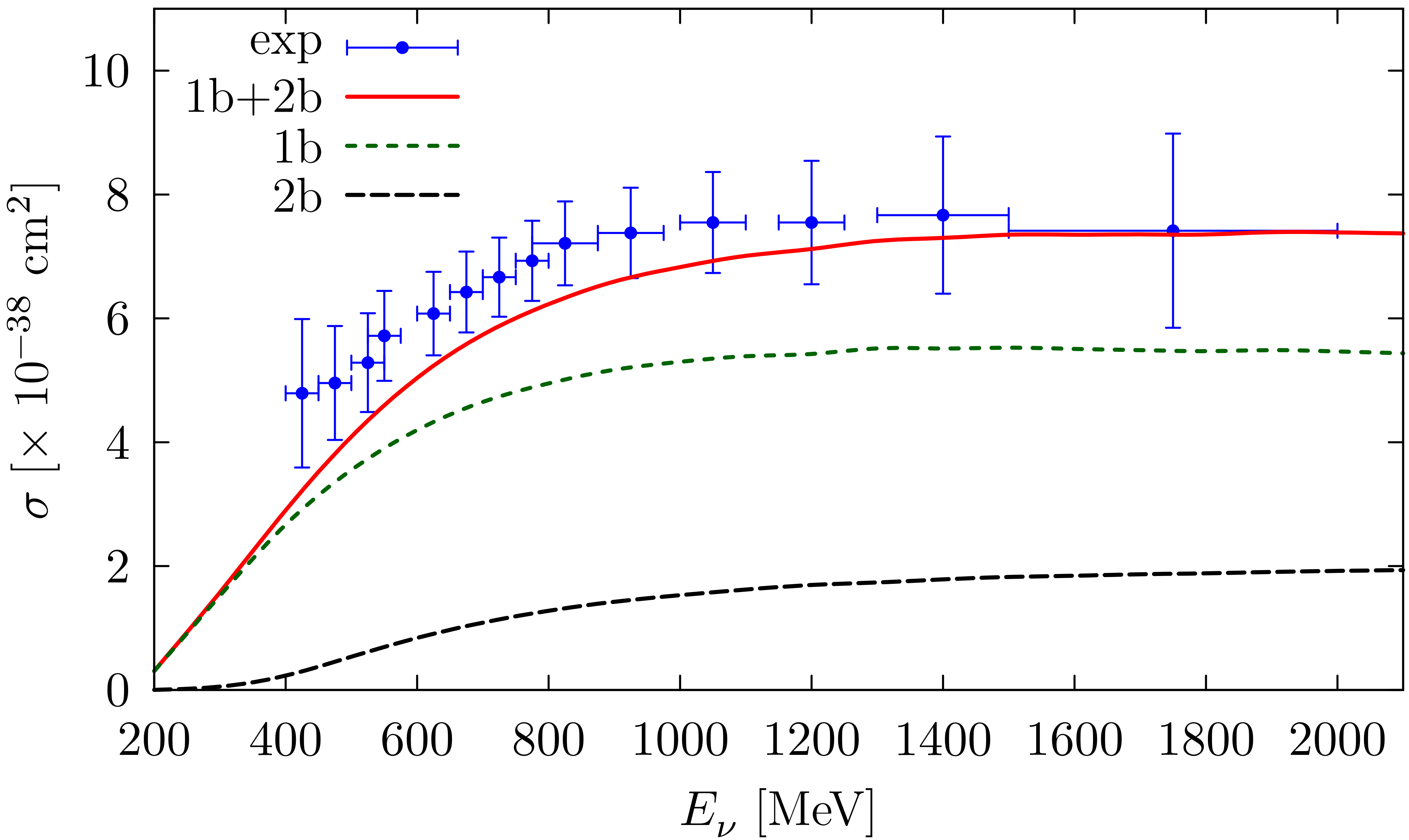
- MEC mostly affect the 'dip' region and strongly enhance the cross section for large values of the scattering angle

Antineutrino-^{12}C cross sections



- MEC mostly affect the 'dip' region and strongly enhance the cross section for large values of the scattering angle

Total cross section: MiniBooNE data



- Two-body currents affect the energy - reconstruction procedure
- Need to include interference between one- and two-body currents

(Intermediate) Conclusions

- ^{12}C electromagnetic responses are in good agreement with experiments.
- Two-body current contributions enhance the longitudinal and transverse axial responses
- Quantum Monte Carlo is suitable to compute cross-sections, not only responses
- The factorization scheme allows to include leading nuclear dynamics effect and relativist one- and two-body currents

Disclaimer

- The continuity equation only constraints the longitudinal components of the current
- The transverse component and the axial terms are phenomenological (the coupling constant is fitted on the tritium beta-decay)
- Two- and three- body forces not consistent

The theoretical error arising from modeling the nuclear dynamics cannot be properly assessed

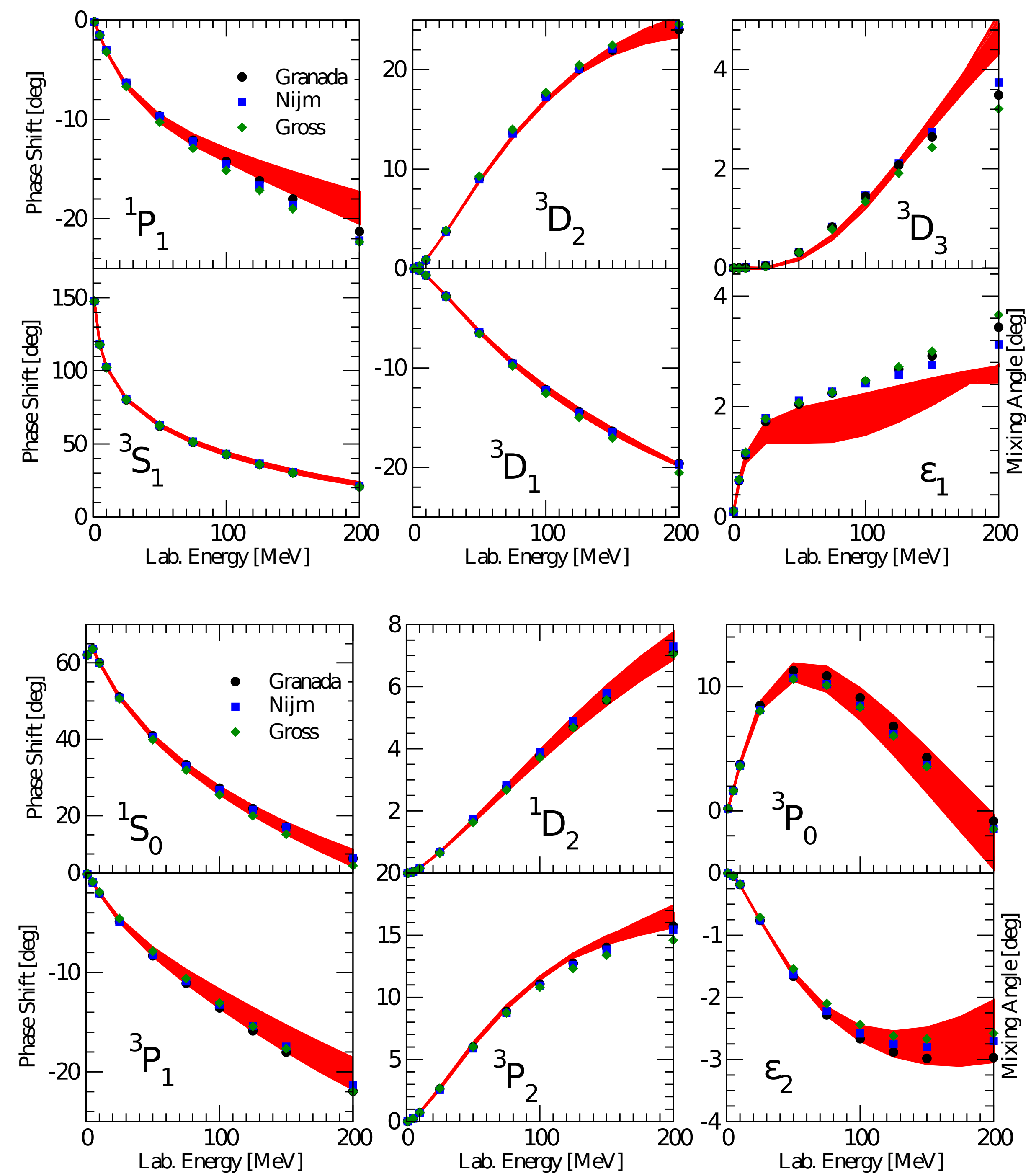
Δ-full local chiral potential

We have complemented the historical “Argonne” approach by considering a local chiral Δ -full potential giving an excellent fit to the NN scattering data that can be readily used in QMC.

- Closer connection with QCD
- Consistent MEC being constructed
- Reliable theoretical uncertainty estimation

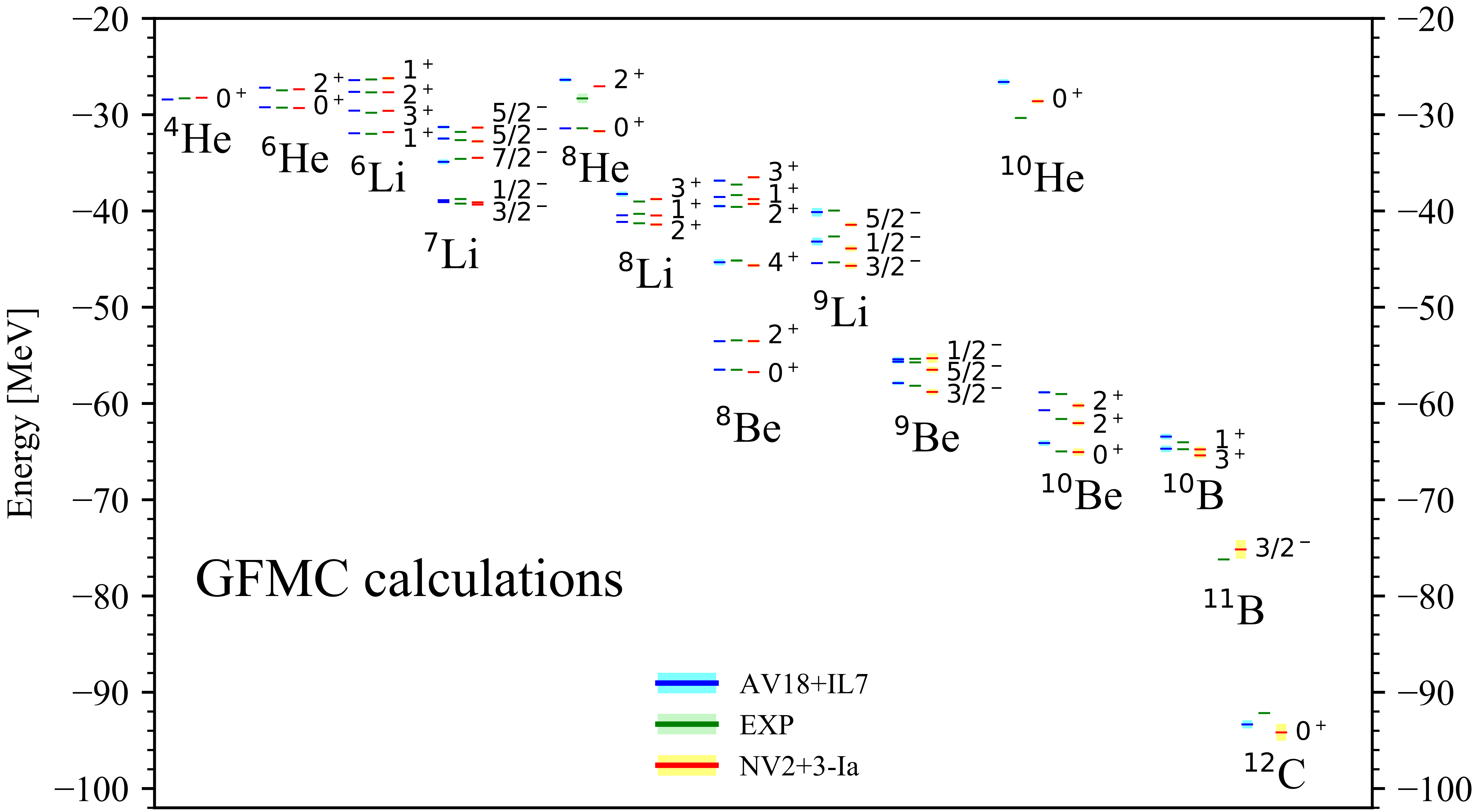
model	order	E_{Lab} (MeV)	N_{pp+np}	χ^2/datum
b	LO	0–125	2558	59.88
b	NLO	0–125	2648	2.18
b	N2LO	0–125	2641	2.32
b	N3LO	0–125	2665	1.07
a	N3LO	0–125	2668	1.05
c	N3LO	0–125	2666	1.11
$\widetilde{a}$	N3LO	0–200	3698	1.37
$\widetilde{b}$	N3LO	0–200	3695	1.37
$\widetilde{c}$	N3LO	0–200	3693	1.40

M. Piarulli et al. PRC 94 (2016) 054007



Δ-full local chiral potential

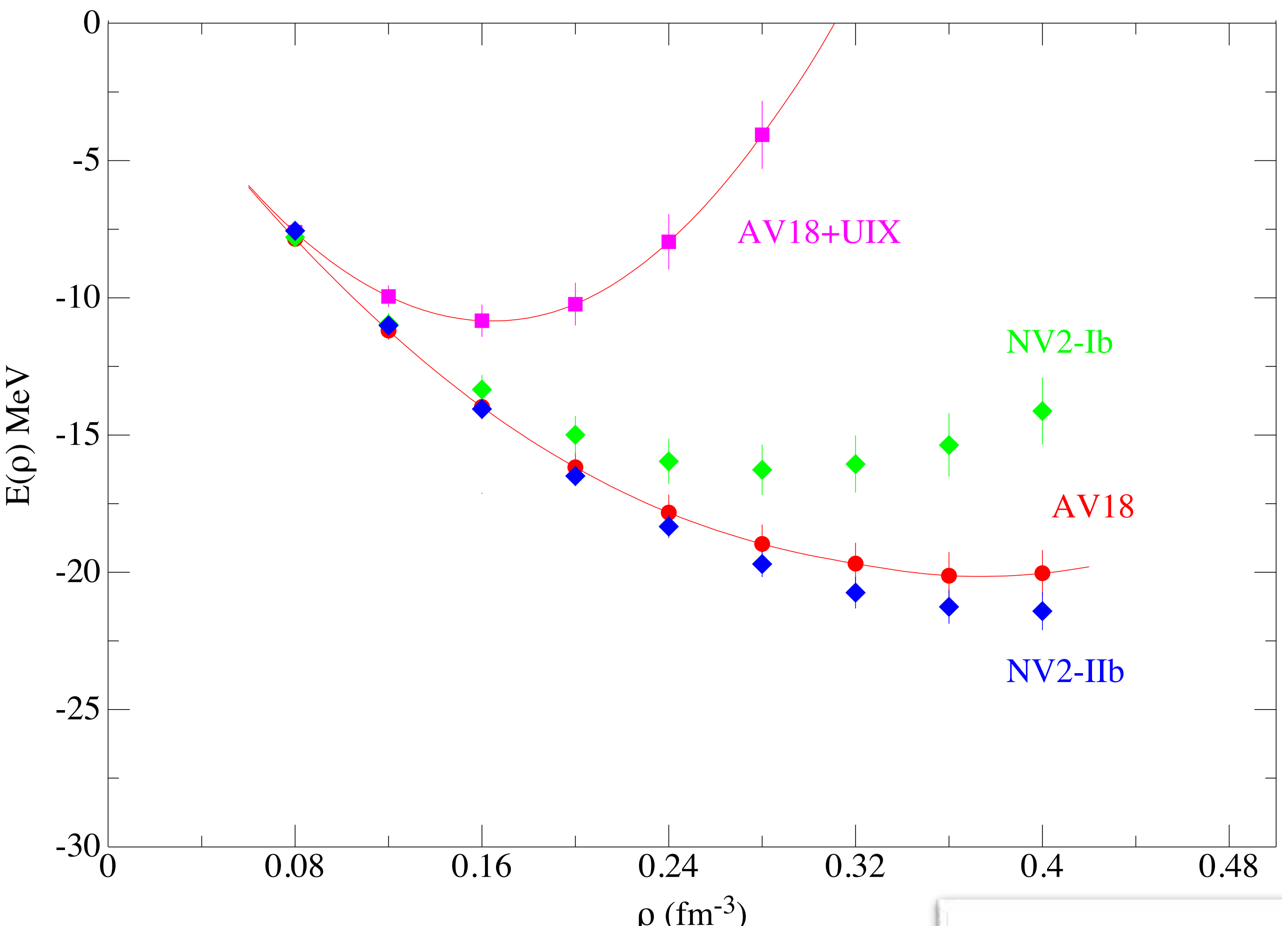
The experimental A≤12 ground- and excited state energies are very well reproduced by the local Δ-full NN+NNN chiral interaction

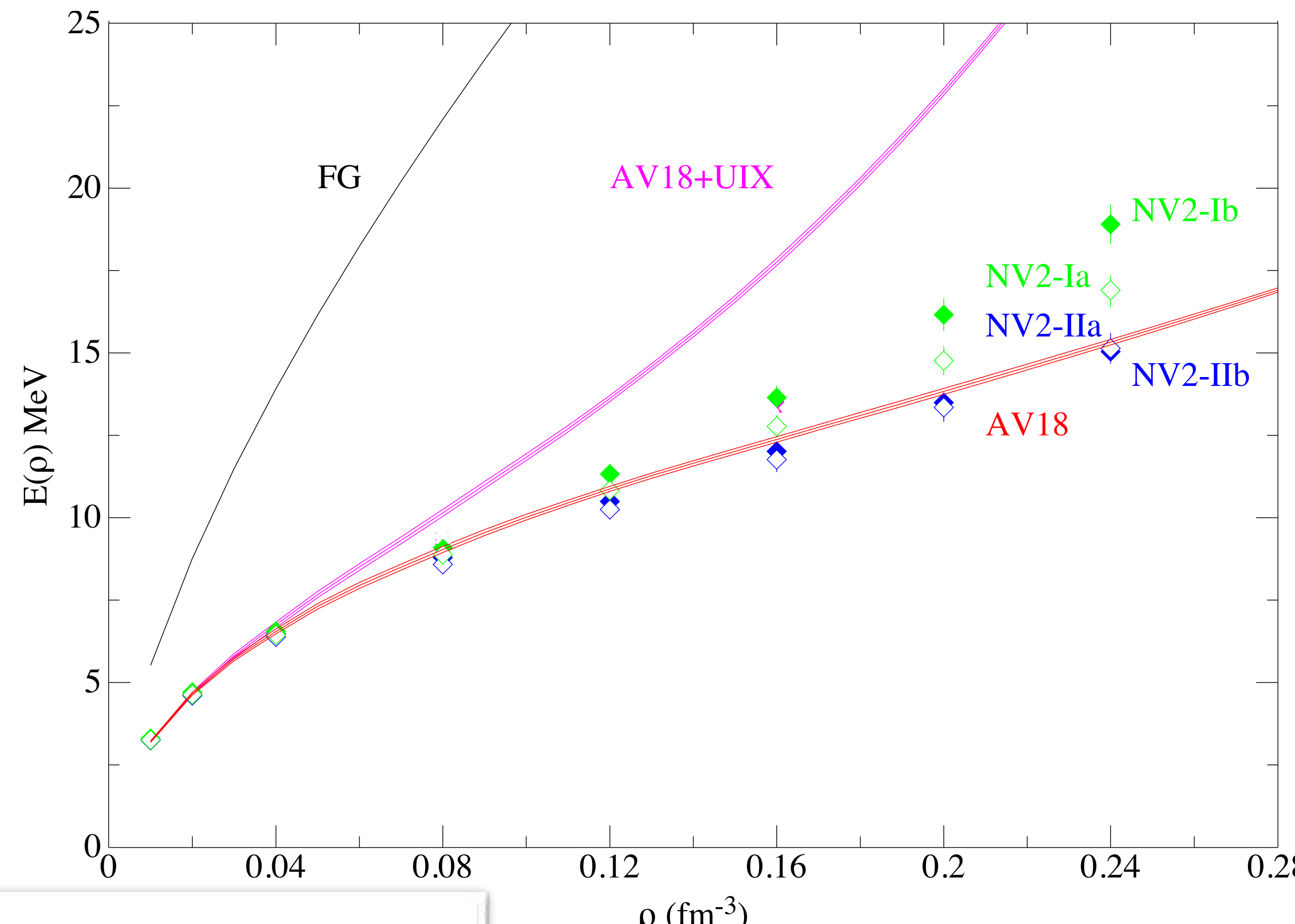


M. Piarulli, et al. PRL 120 (2018) 052503

Δ-full local chiral potential

We performed variational FHNC calculations for the energy per particle of isospin-symmetric nuclear matter and pure neutron matter

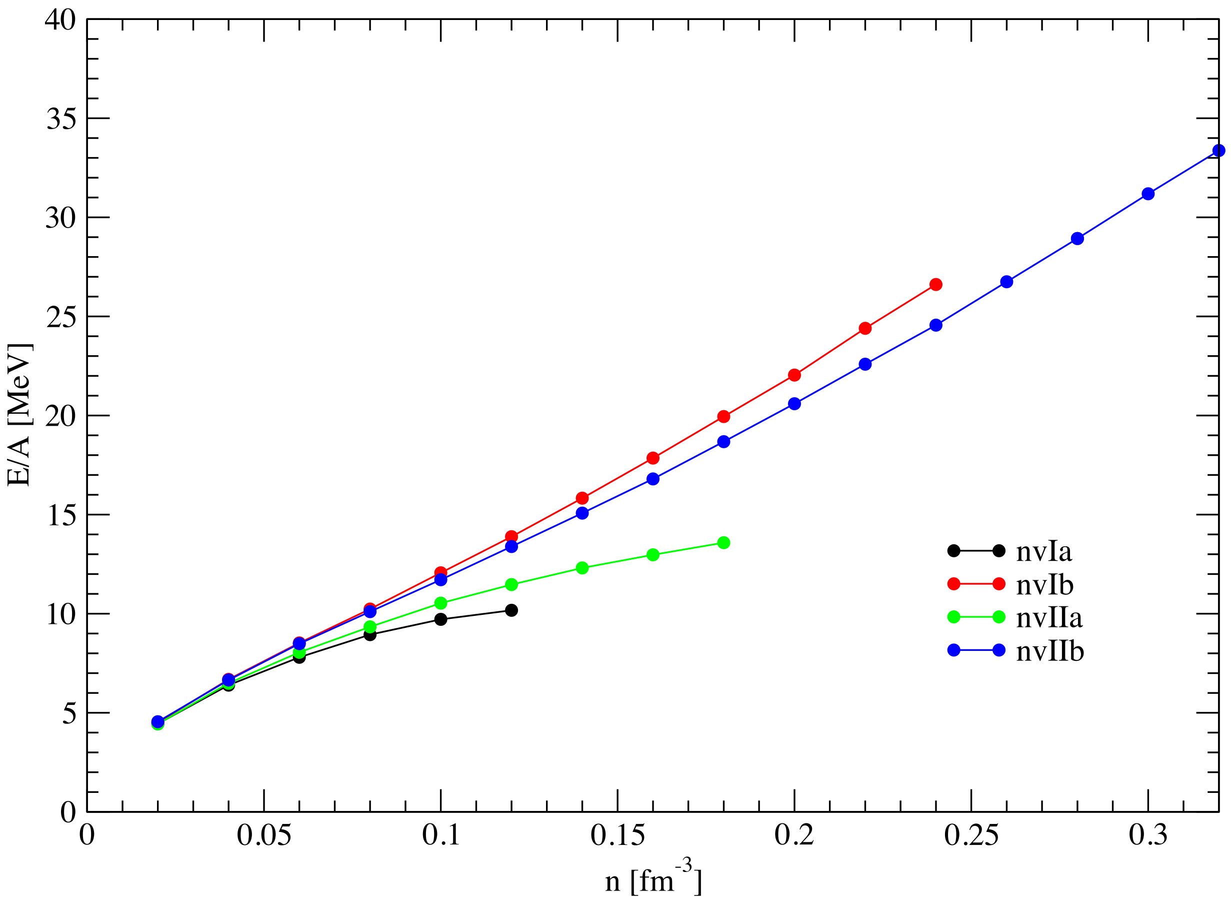




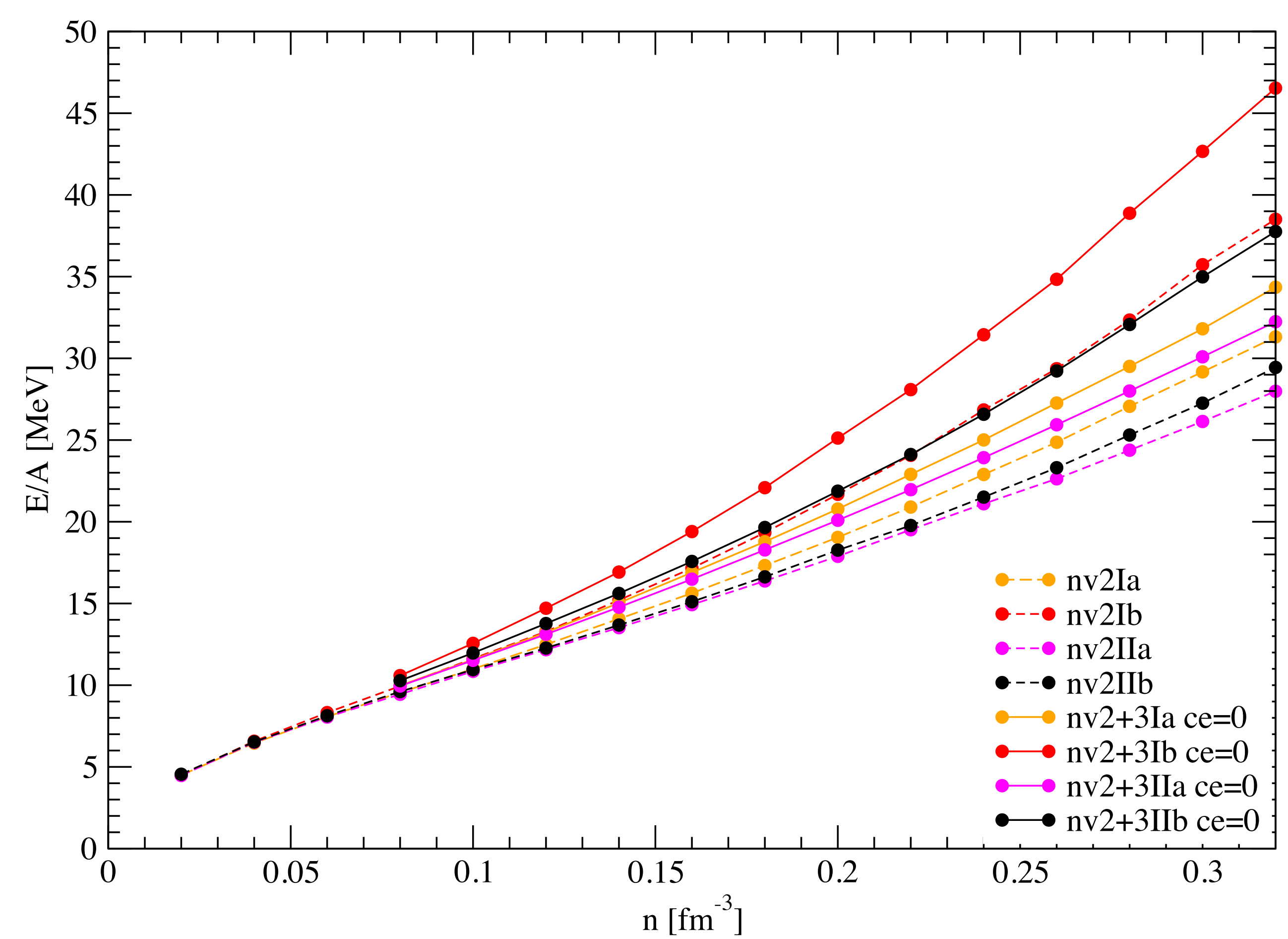
M. Piarulli, et al. in preparation

- Local two-nucleon interactions fit to NN data saturate symmetric nuclear matter (SNM) at ≈ $2\rho_0$
- Shorter-range three-body force must provide net repulsion to saturate at empirical density
- The NV2-II models fit to higher energy are closer to AV18 in both SNM and pure neutron matter

Δ-full local chiral potential



- The softest potentials fail to provide a realistic neutron matter EoS
- Overall good agreement between improved FHNC and AFDMC results

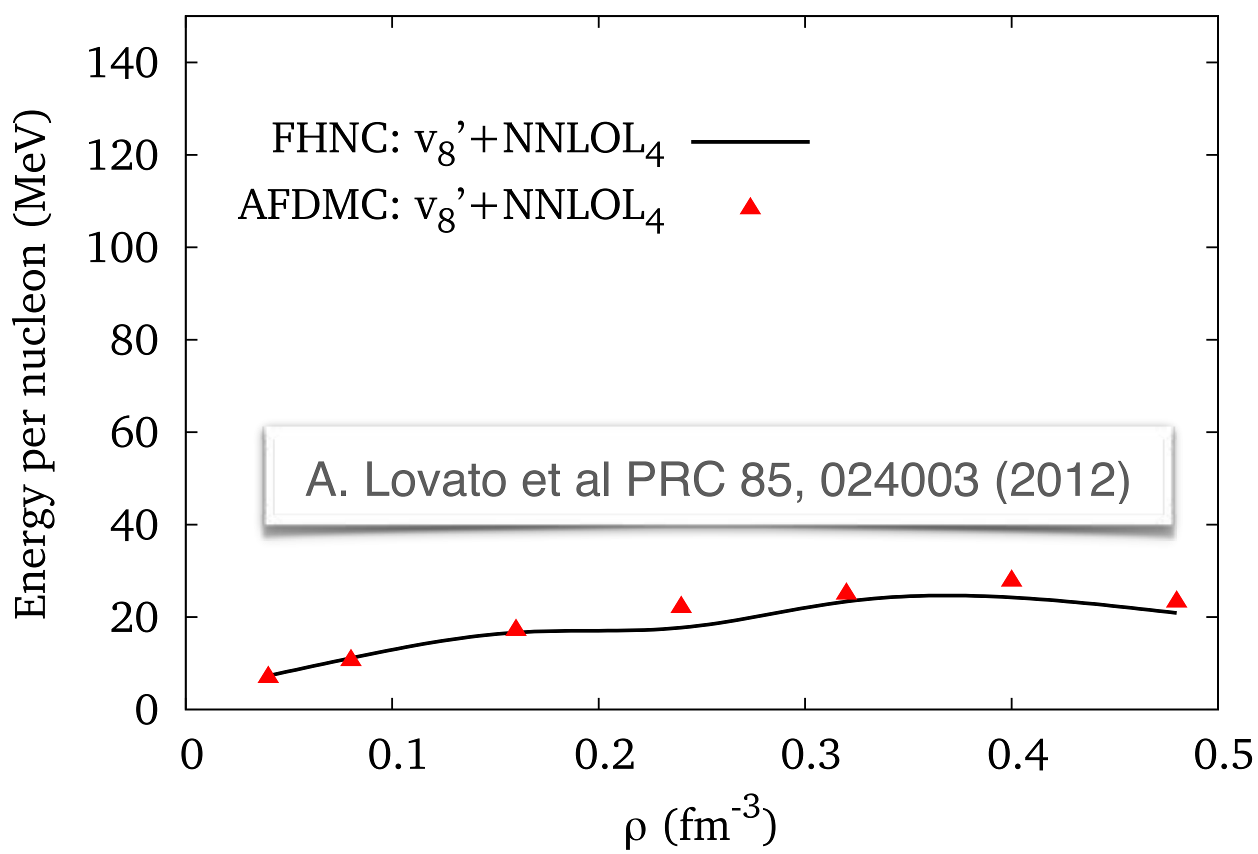


- Most of the regulator dependence comes from the c_E term in the three-body potential
- In the infinite regulator limit this term is zero in PNM (Pauli principle)

M. Piarulli, et al. in preparation

Regulator issues (over and over again)

- Regulator artifacts are **not new**

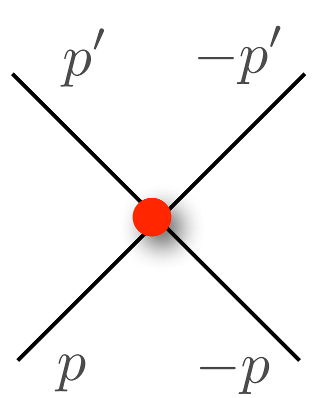


A. Lovato et al PRC 85, 024003 (2012)

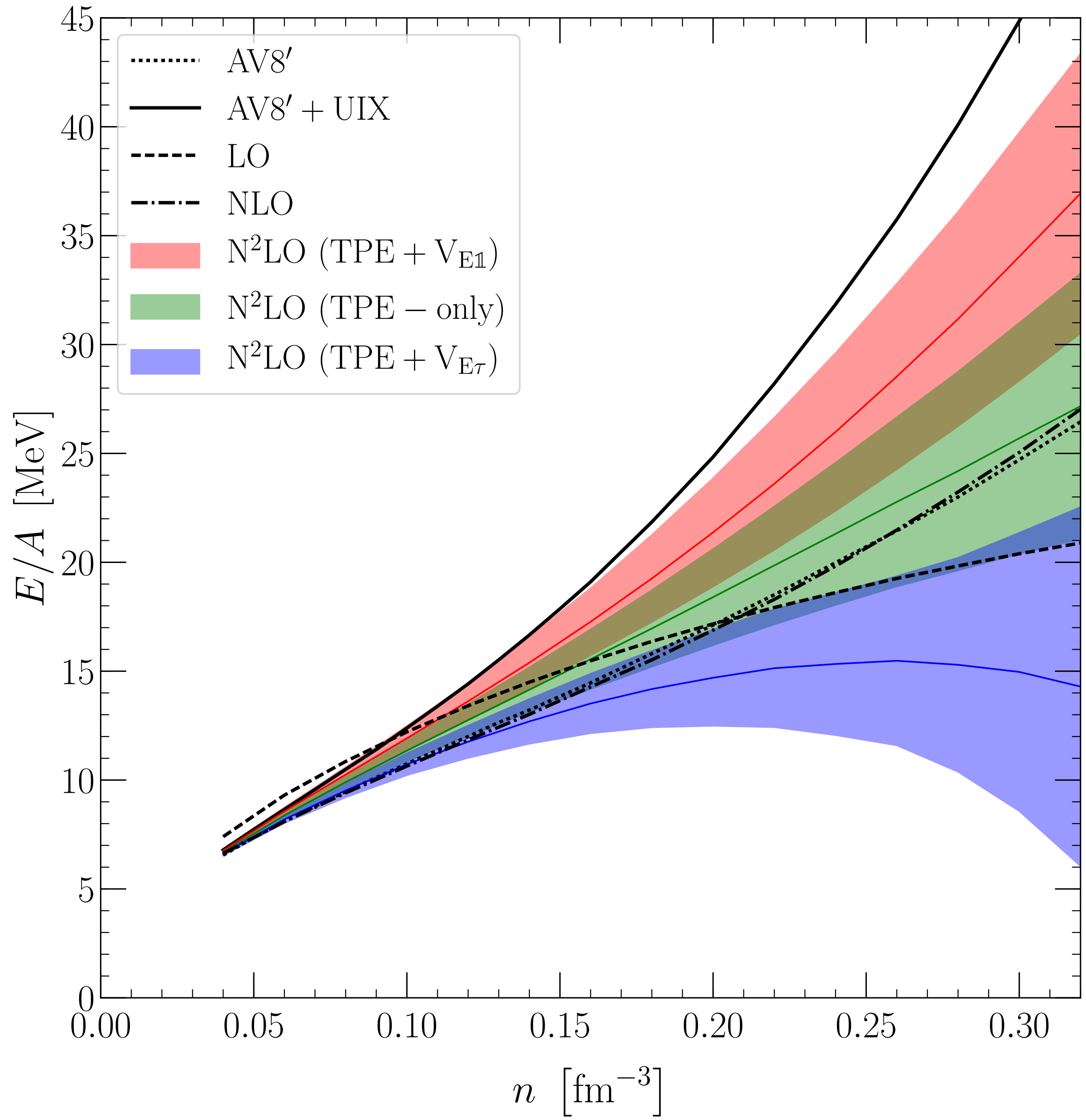
- These are essentially due to

$$P_{12} f_\Lambda(q) = f_\Lambda(k)$$

$$P_{12}\, e^{-q^2/\Lambda^2} = e^{-q^2/\Lambda^2} e^{-2p\cdot p'/\Lambda^2}$$



L. Huth et al., PRC 96, 054003 (2017)



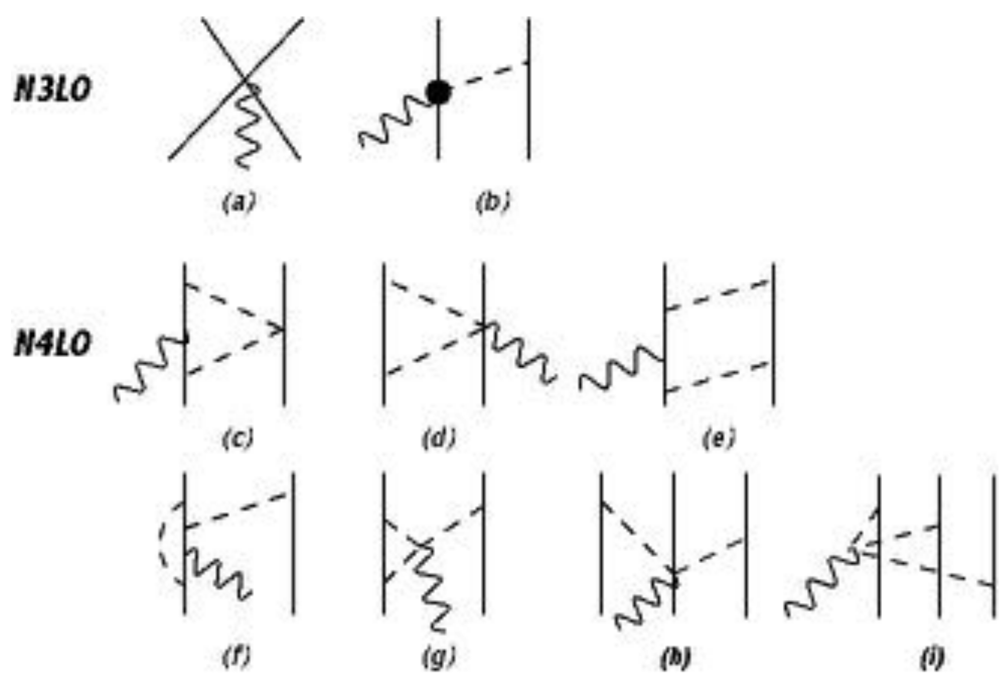
J. Lynn et al. PRL 116, 062501 (2016)

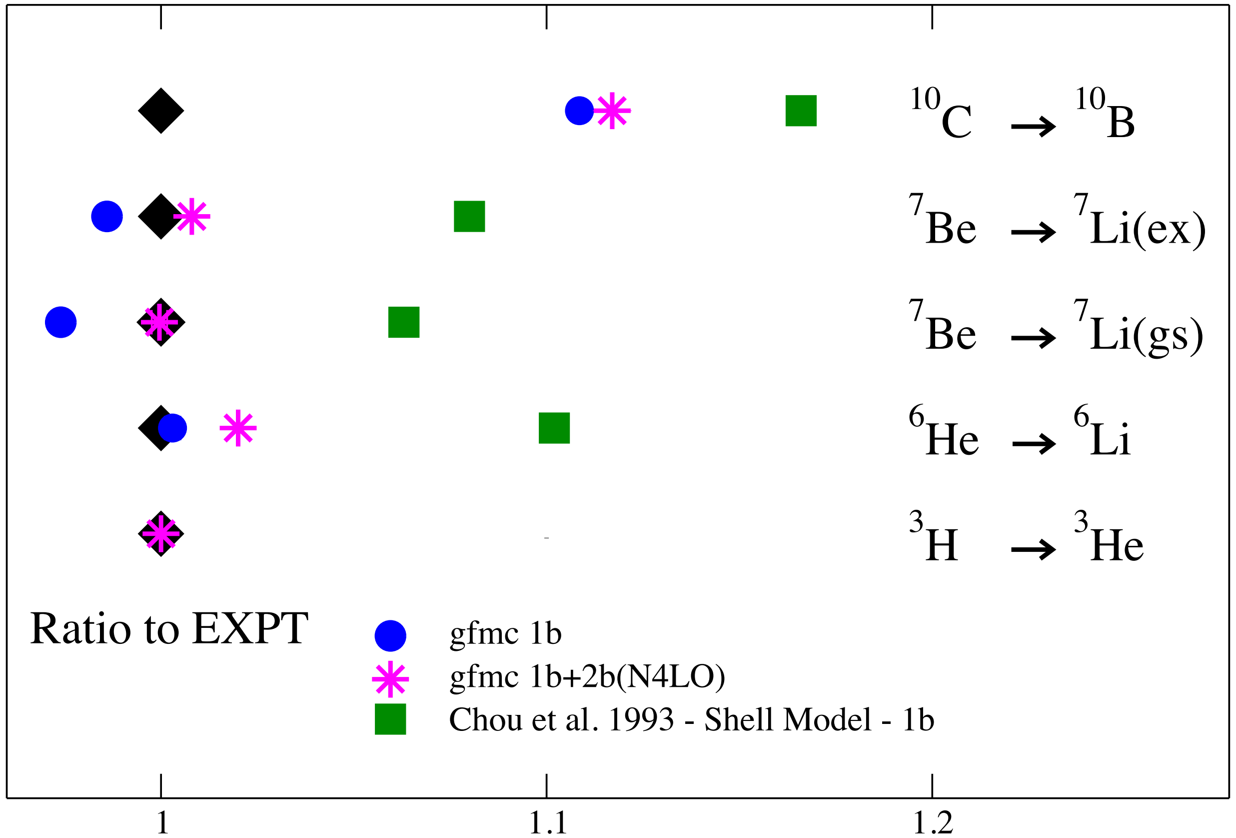
I. Tews et al. arXiv:1801.01923 (2018)

Chiral-EFT currents

- Chiral currents consistent with the Δ-full local chiral potential are being developed

- Mixed-approach calculations indicate a slight enhancement of the decay rates from MEC

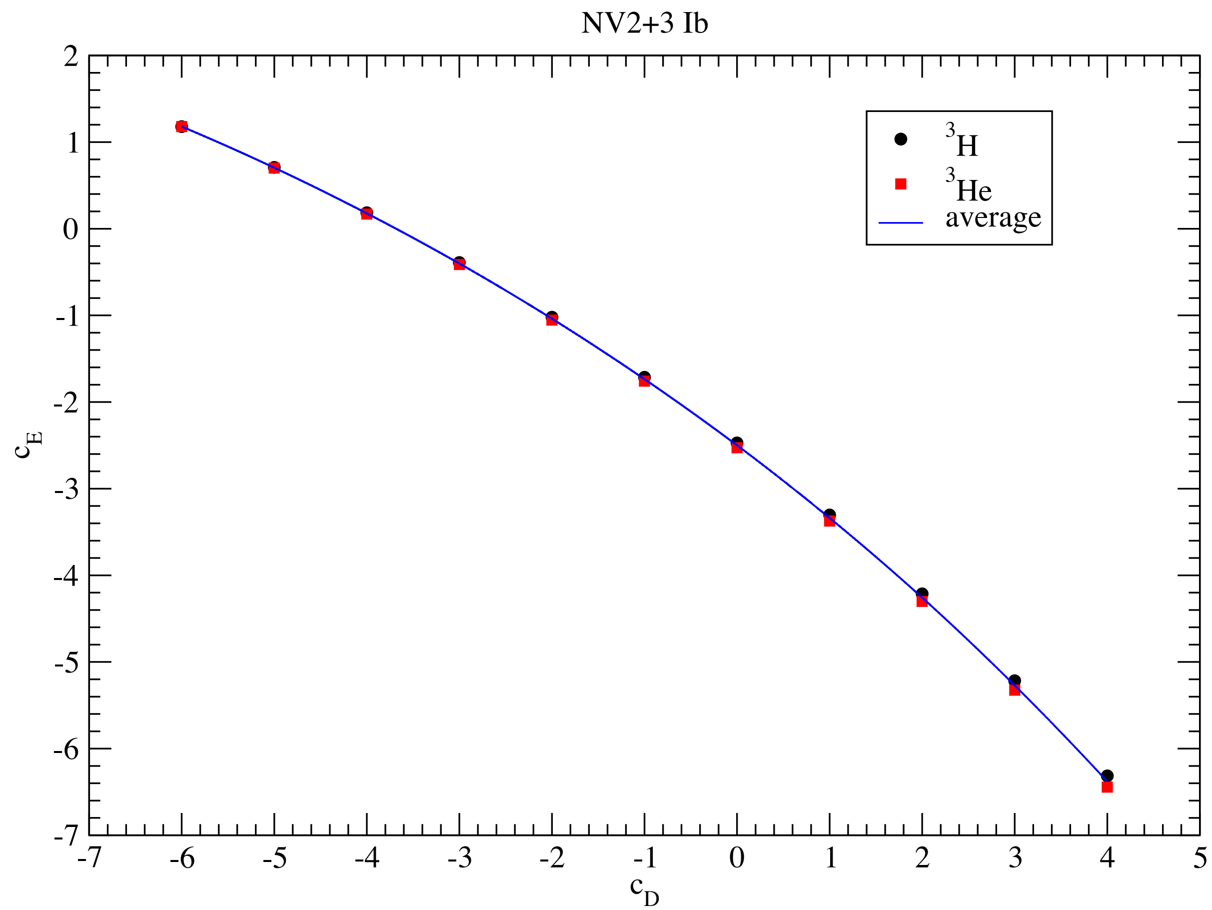




S. Pastore et al. PRC 97, 022501 (2018)

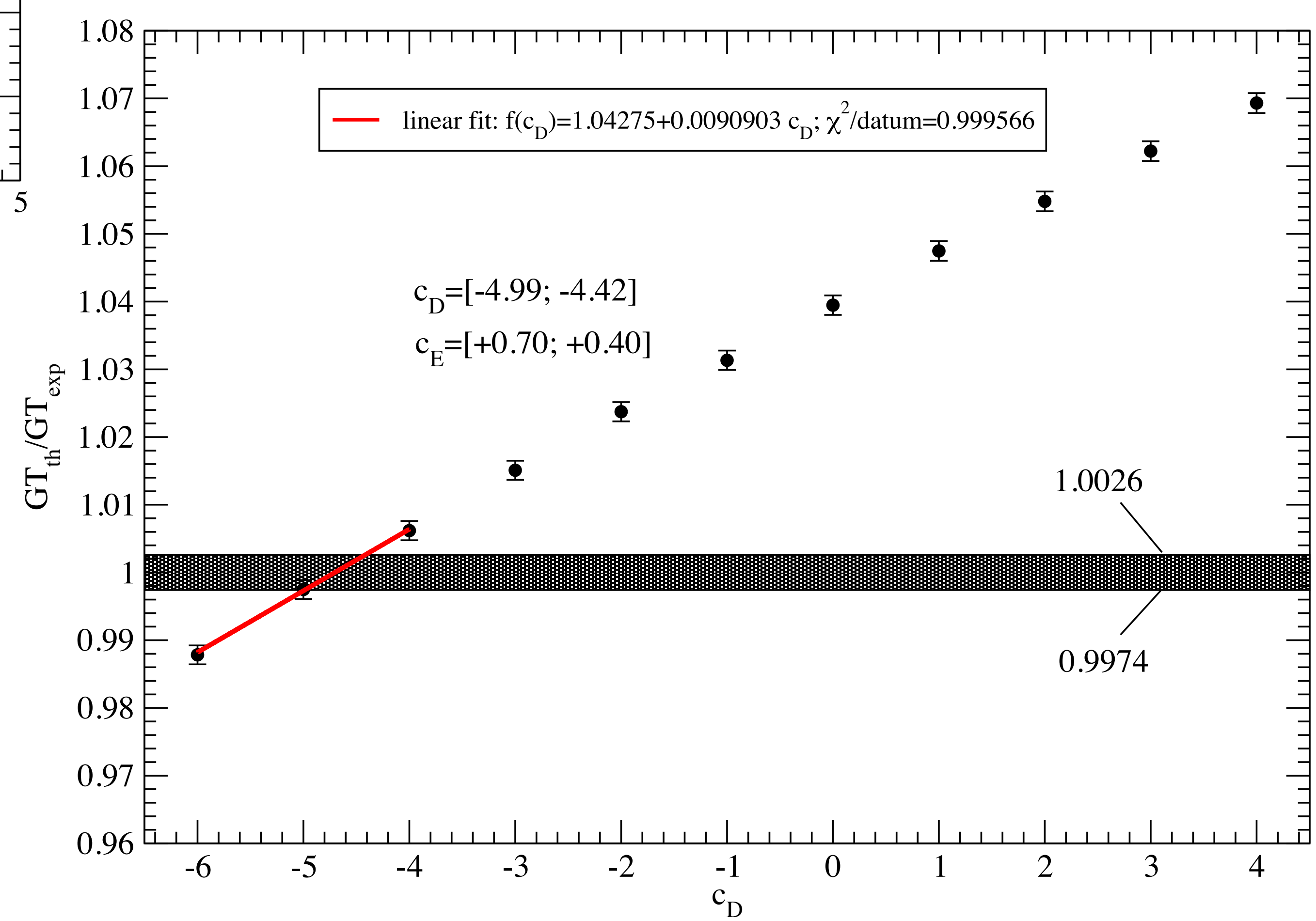
Nuclear spectra and decays

A. Baroni et al. arXiv:1806.10245

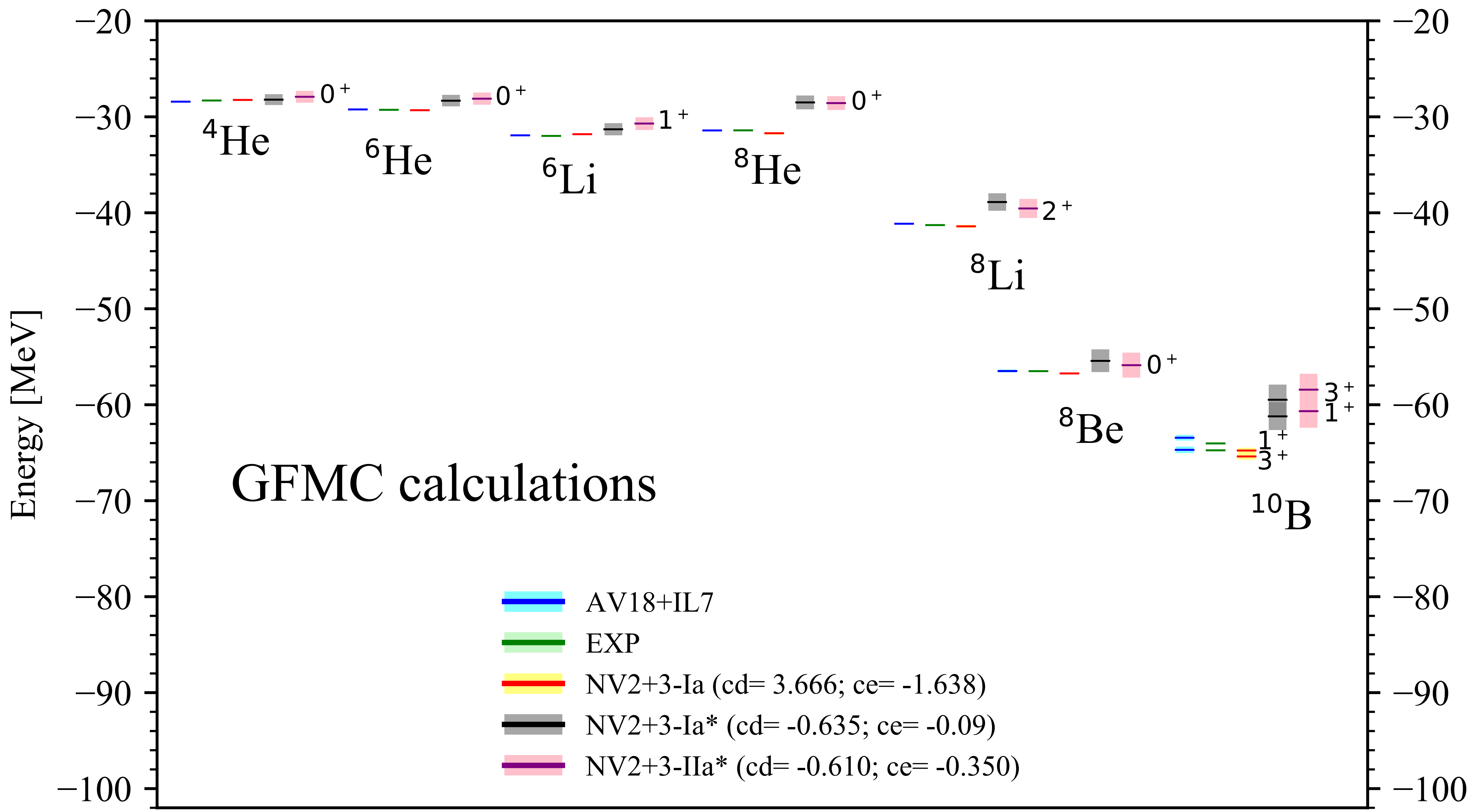


- Triton and ^{3}He average binding energies provide a correlation line between c_D and c_E

- Triton decay is most sensitive to c_D



Nuclear spectra and decays



M. Piarulli, AL et al. in preparation

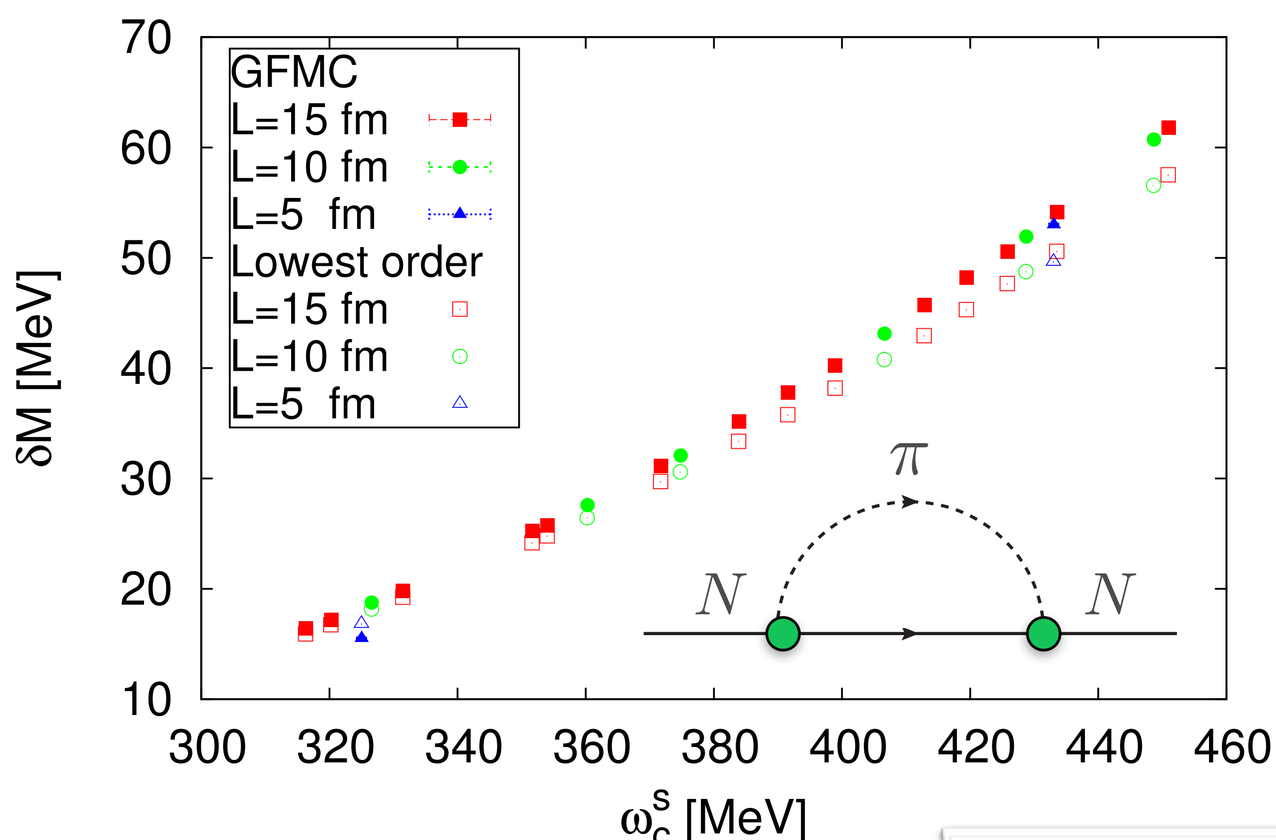
Explicit-pion QMC

The non-relativistic wave functions found solving the many-body Schrödinger equations describe the quantum-mechanical amplitudes of the nucleonic degrees of freedom.

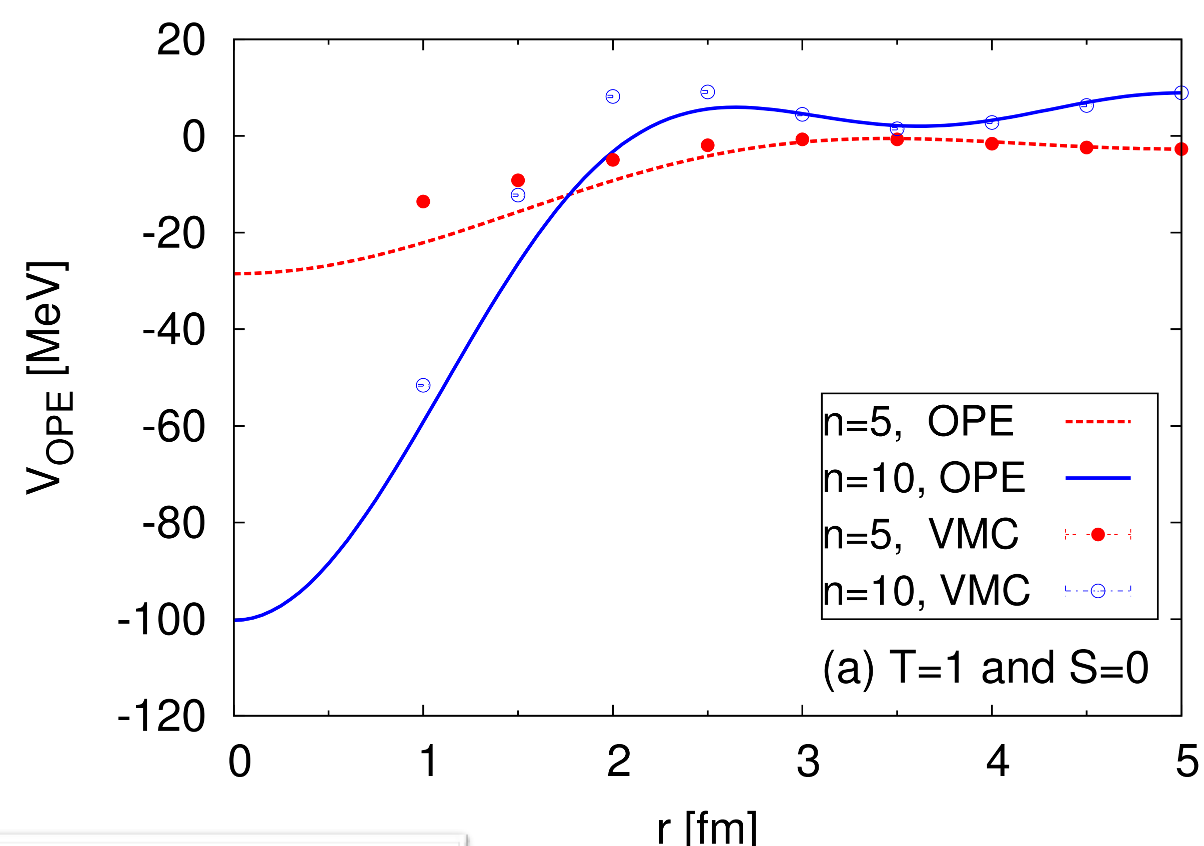
The relativistic pions and the nucleons are **both** explicitly included in the quantum-mechanical states of the system

$$\langle \mathbf{R}, S | \Psi \rangle \longrightarrow \langle \mathbf{R}, S, \boldsymbol{\pi}_k | \Psi \rangle \qquad \boldsymbol{\pi}(\mathbf{x}) = \frac{1}{\sqrt{L^3}} \sum_{\mathbf{k}} \boldsymbol{\pi}_k e^{i\mathbf{k}\cdot\mathbf{x}}$$

The **nucleon-mass renormalization** is consistent with quantum-field theory



The interaction between two static nucleons reduces to one-pion exchange at large distance



L. Madeira, et al. arXiv:1803.10725

Explicit-pion QMC

Our goal (a long way ahead) is to perform reliable predictions for **pion production** in electron- and neutrino-nucleus scattering