Electromagnetic Response of Light Nuclei with Integral Transforms

- LIT method

- Low-energy continuum observables with LIT
  - Resonances
  - S-Factor in presence of Coulomb potential

- Electron scattering at q ≥ 500 MeV/c
  - ^{3}He (LIT)
  - ^{4}He (LIT, GFMC)

LIT method

The LIT of a function $R(E)$ is defined as follows

$$\Rightarrow \quad L(\sigma) = \int dE \, \mathcal{L}(E,\sigma) \, R(E) \, ,$$

where the kernel $\mathcal{L}$ is a Lorentzian,

$$\Rightarrow \quad \mathcal{L}(E,\sigma) = \frac{1}{(E-\sigma_R)^2 + \sigma_I^2}$$

For inclusive reactions the LIT $L(\sigma)$ is calculated by solving an equation of the form

$$\boxed{(H-\sigma)\,\tilde{\Psi} = S \, ,}$$

where H is the Hamiltonian of the system under consideration and S is an asymptotically vanishing source term related to the operator inducing the specific reaction.

The solution $\tilde{\Psi}$ is localized and the LIT is given by

$$\boxed{L(\sigma) = \langle \tilde{\Psi} | \tilde{\Psi} \rangle \, .}$$

Alternative way:

$$L(\sigma) = -\frac{1}{\sigma_I} Im(\langle S | \frac{1}{\sigma_R + i\sigma_I - H} | S \rangle) \, .$$

The source term S for inclusive reactions has the form

$$\Rightarrow \quad |S\rangle = \theta|0\rangle \, ,$$

where the operator θ induces a specific electroweak reaction.

The corresponding response function is given by

$$\Rightarrow \quad R(E_f) = \int dE_f |\langle f|\theta|0\rangle|^2 \delta(E_f - E_0 - \omega)$$

Ingredients of the solution of the LIT equation via an expansion on a basis of dimension N:

N eigenstates with eigenenergies

ϕ_n $\qquad$ E_n

LIT states

and strength

$$S_n = |\langle \phi_n|\theta|0\rangle|^2 \quad \Leftarrow$$

strength to a LIT state

leading to the following LIT

$$\Rightarrow \quad L(\sigma) = \sum_{i=1}^{N} \frac{S_n}{(\sigma_R - E_n)^2 + \sigma_I^2}$$

Inversion of the LIT

- LIT is calculated for a fixed σ_I in many σ_R points

- Express the searched response function formally on a basis set with M basis basis functions $f_m(E)$ and open coefficients c_m with correct threshold behaviour for the $f_m(E)$ (e.g., $f_m = f_{thr}(E) \exp(-\alpha E/m)$)

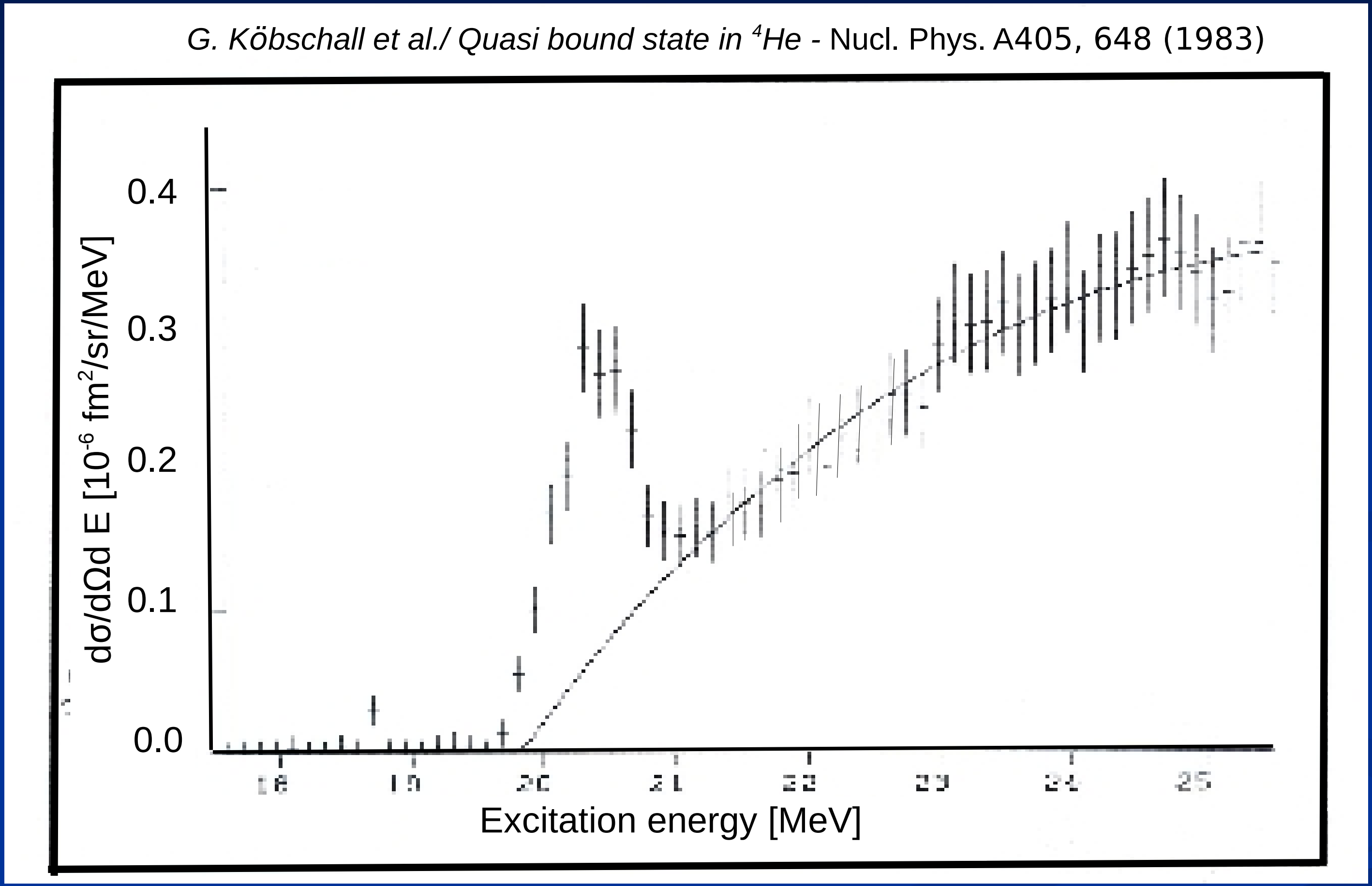
- Make a LIT transform of the basis functions and determine coefficents c_m by a fit to the calculated LIT

- Increase M up to the point that a sufficient convergence is obtained (uncontrolled oscillations should not be present)

A regularization method is needed for the inversion

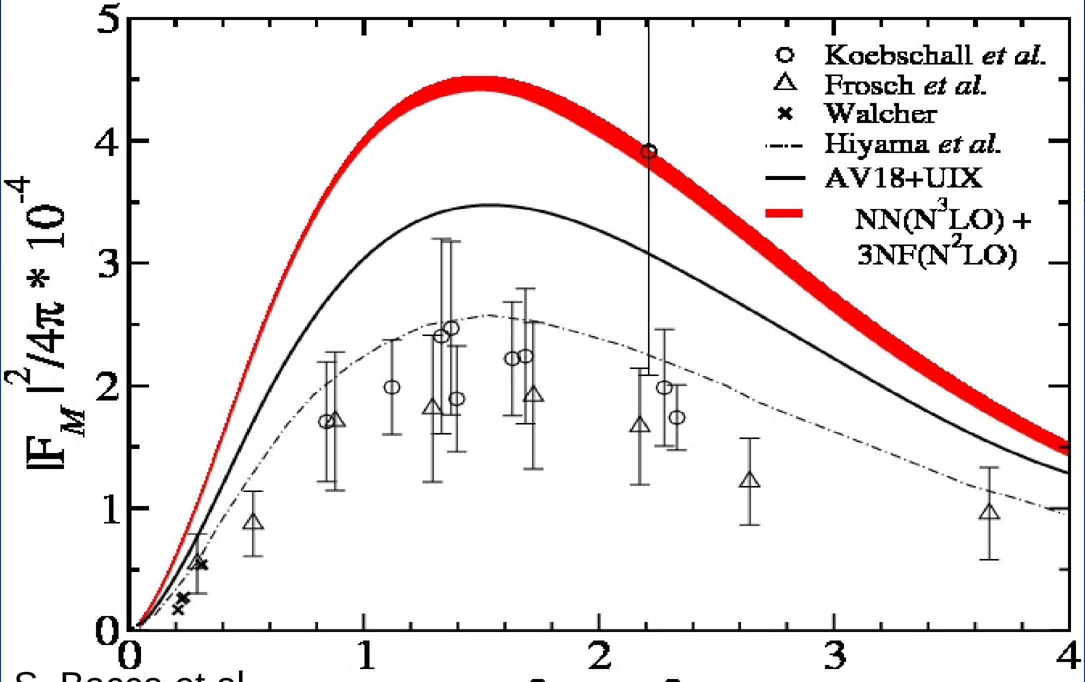
Resonances

0^+ Resonance in the ^{4}He compound system



Resonance at E_R = -8.2 MeV, i.e. above the ^{3}H-p threshold. Strong evidence in electron scattering off ^{4}He, Γ = 270±50 keV

Comparison to experimental results

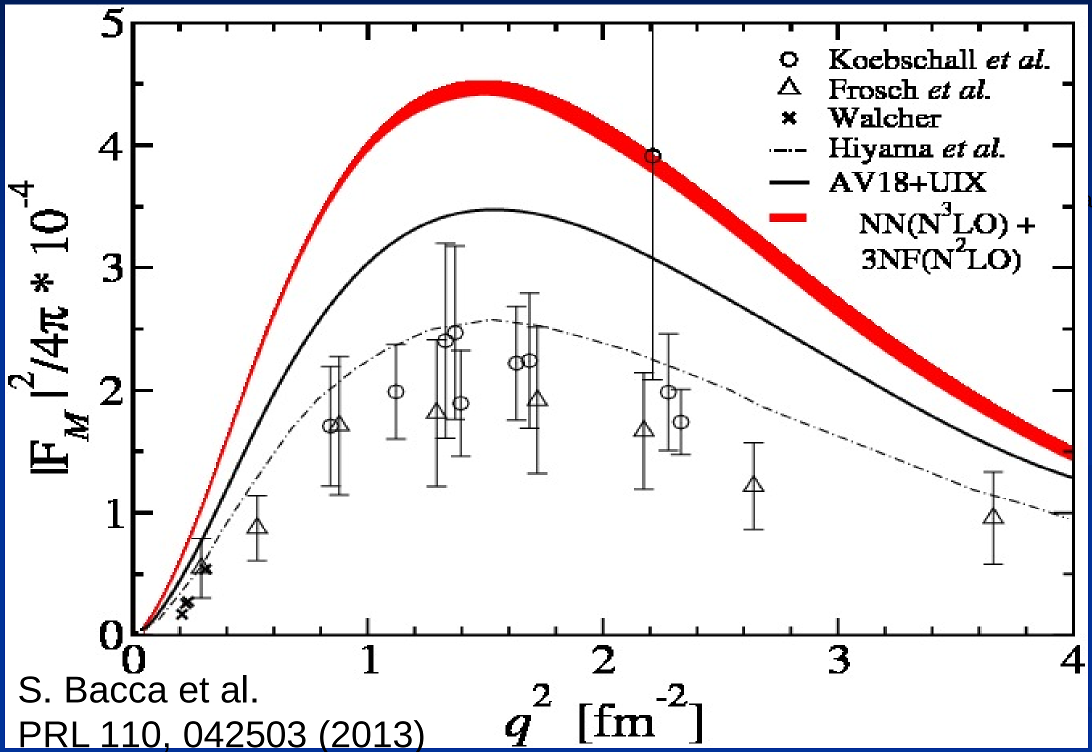


S. Bacca et al.
PRL 110, 042503 (2013)

LIT/EIHH Calculation for AV18+UIX and Idaho-N3LO+N2LO

dash-dotted: AV8' + central 3NF (Hiyama et al.)

Comparison to experimental results



Observable is strongly dependent on potential model

Breathing Mode? (S. Bacca et al., PRC 91, 024303 (2015))

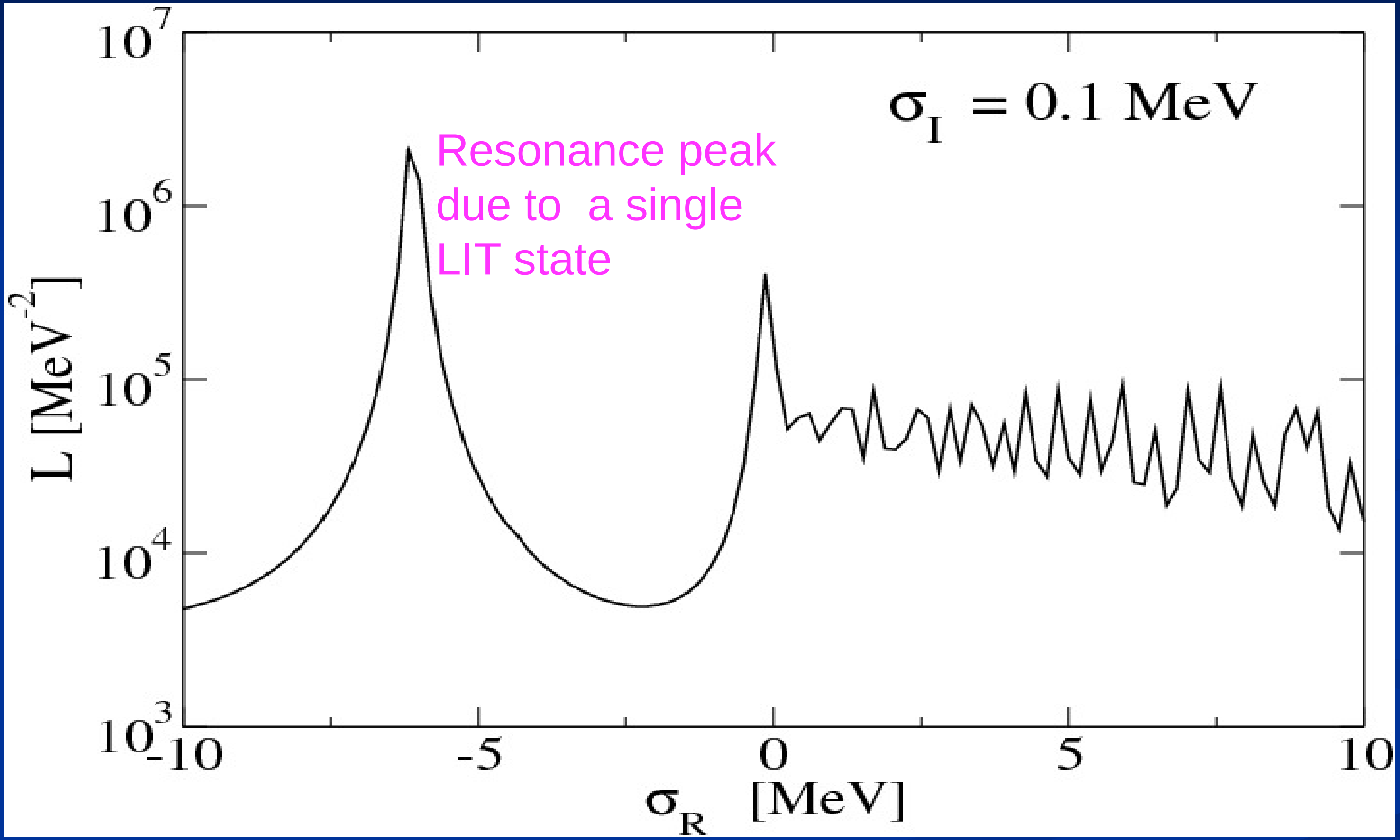
Why were we unable to determine the width of the 4He isoscalar monopole resonance?

To answer this let us check our very first LIT calculation from 1997:

^{4}He(e,e') inelastic longitudinal response function

with a central NN potential

Unpublished result from a CHH calculation with the TN potential (V. Efros, WL, G. Orlandini, PRL 78,432 (1997))



To study the problem better let us consider first instead of a four-body reaction a **simpler three-body reaction**:

$$^3\mathrm{He} + \gamma \longrightarrow d + p$$

at low energies

LIT calculation with central MTI/III NN potential in unretarded dipole approximation

Aim: Increase low-energy density of LIT states

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$$^{3}He + \gamma \longrightarrow d + p$$

at low energies

LIT calculation with central MTI/III NN potential in unretarded dipole approximation

How: Increase number of basis states, both, hyperradial and hyperspherical

To study the problem better let us consider first instead of a four-body reaction a simpler three-body reaction:

$$^{3}\mathrm{He} + \gamma \longrightarrow \mathrm{d} + \mathrm{p}$$

at low energies

LIT calculation with central MTI/III NN potential in unretarded dipole approximation

Also note: hyperradial basis states consist in an expansion on Laguerre polynomials times a spatial cutoff $\exp(-\rho/b)$

Increase of b shifts spectrum to lower energies

To study the problem better let us consider first instead of a four-body reaction a simpler three-body reaction:

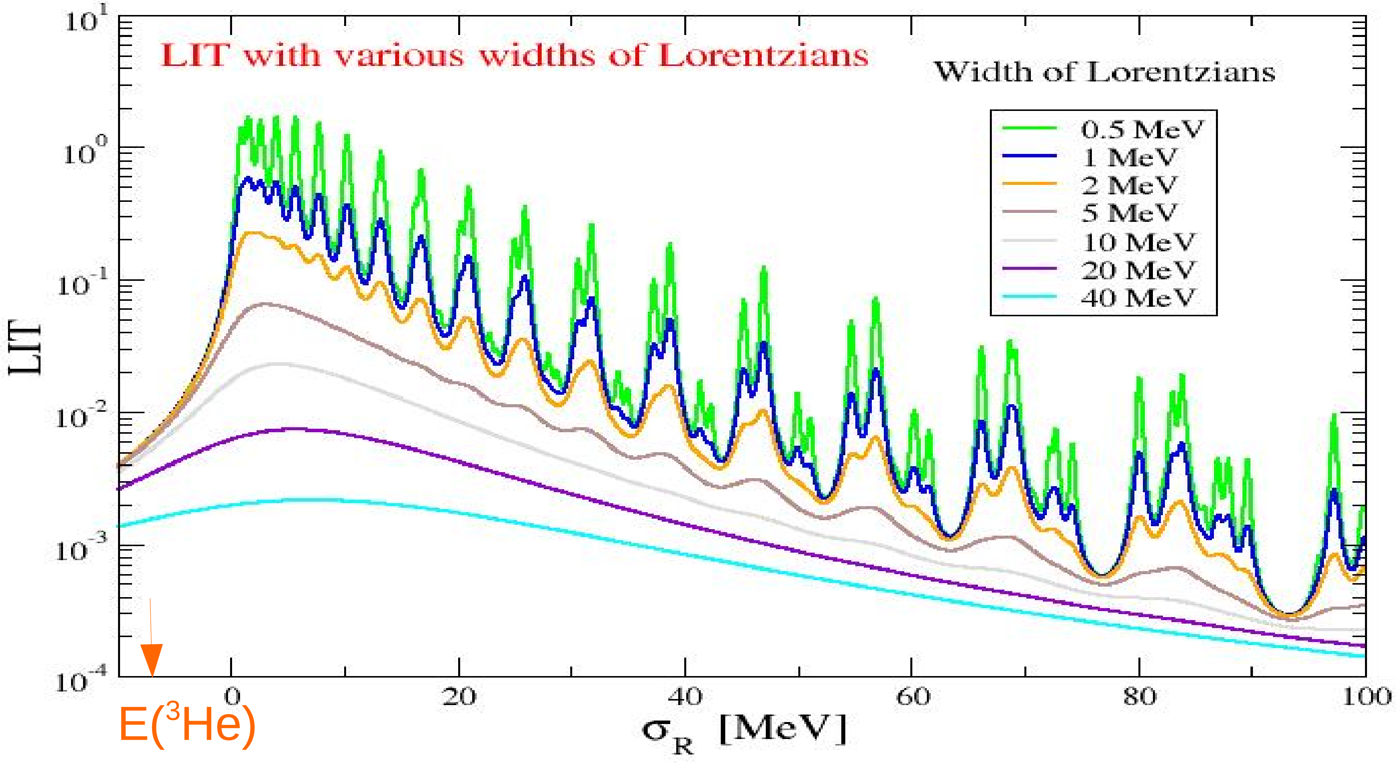
$$^3\text{He} + \gamma \longrightarrow d + p$$

at low energies

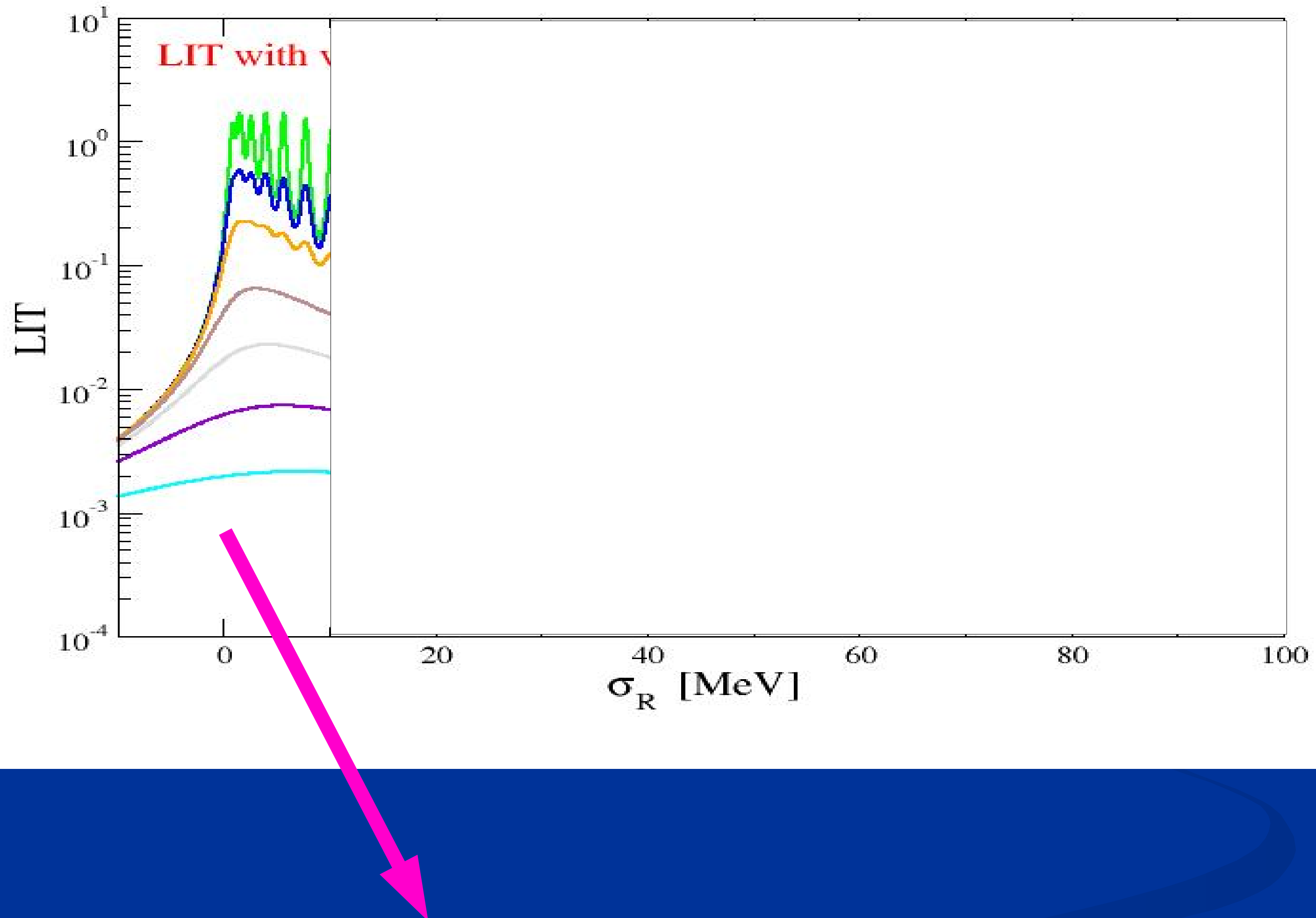
LIT calculation with central MTI/III NN potential in unretarded dipole approximation

Next slide: LIT with 30 hyperspherical and 31 hyperradial basis functions ⇒ **930 basis states** with b = 0.6 fm

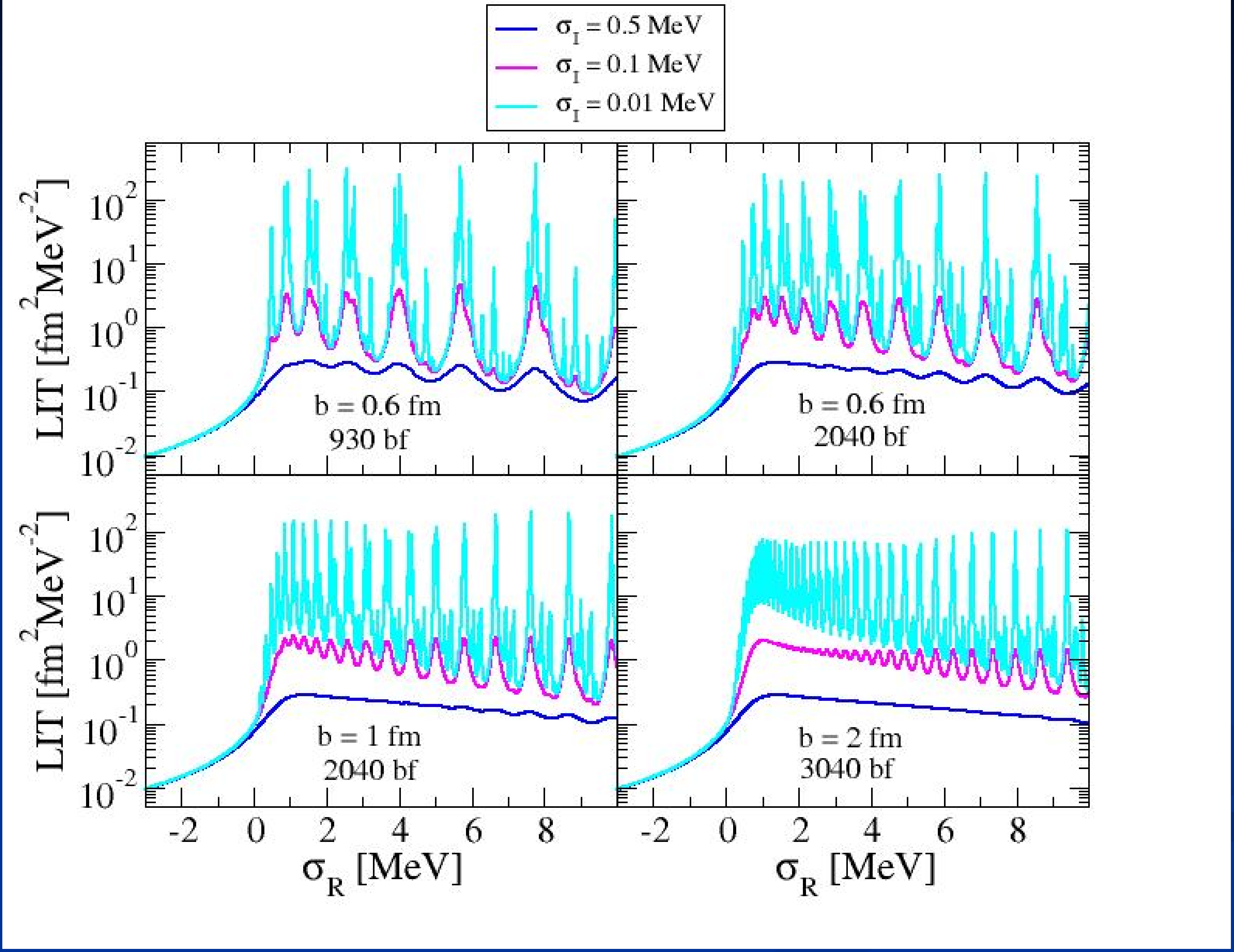
LIT with various widths of Lorentzians



30 hyperspherical and 31 hyperradial basis functions
⇒ 930 basis states
b = 0.6 fm



Increase LIT state density and zoom in



$\sigma_I = 0.5$ MeV
$\sigma_I = 0.1$ MeV
$\sigma_I = 0.01$ MeV
LIT [fm^{2}MeV^{-2}]
b = 0.6 fm
930 bf
b = 0.6 fm
2040 bf
b = 1 fm
2040 bf
b = 2 fm
3040 bf
σ_R [MeV]
σ_R [MeV]

Observation

The LIT is a method with a controlled resolution

But not a single LIT state below three-body breakup threshold
In present LIT calculation! Similar problem as in the previous
four-body case

Solution: use instead of the HH basis a somewhat modified basis

New A-body basis

Note one of the (A-1) Jacobi vectors can be written in the following form:

$$\boldsymbol{\eta} = \mathbf{r}_A - \mathbf{R}_{cm}(1,2,\ldots,A\text{-}1)$$

This is the coordinate one would use for the scattering of a nucleon with a (A-1)-nucleon system. In other words the relevant coordinate for a two-body breakup. Therefore

A-body HH basis → (A-1)-body HH basis times expansion on $\boldsymbol{\eta}$

radial part: Laguerre polynomials

angular part: $Y_{LM}(\theta_\eta,\phi_\eta)$

Four-body system: HH for 3 particles plus 4-th particle coordinate $\boldsymbol{\eta}$

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A-body HH basis → (A-1)-body HH basis times expansion on $\boldsymbol{\eta}$

radial part: Laguerre polynomials

angular part: $Y_{LM}(\theta_\eta,\phi_\eta)$

Three-body system: pair coordinate for two particles plus 3rd particle coordinate $\boldsymbol{\eta}$

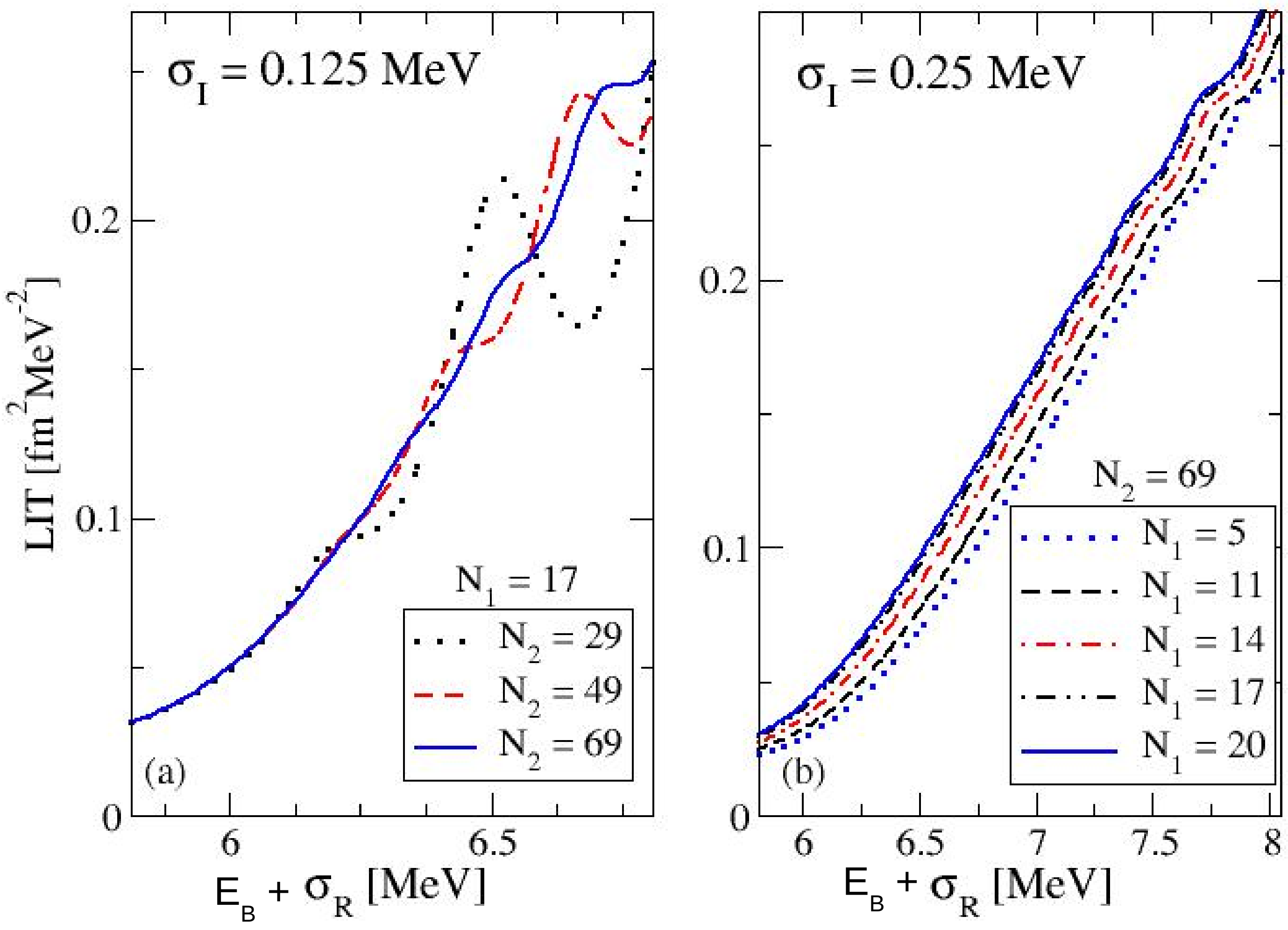
First three-body case

$$^3He + \gamma \longrightarrow d + p$$

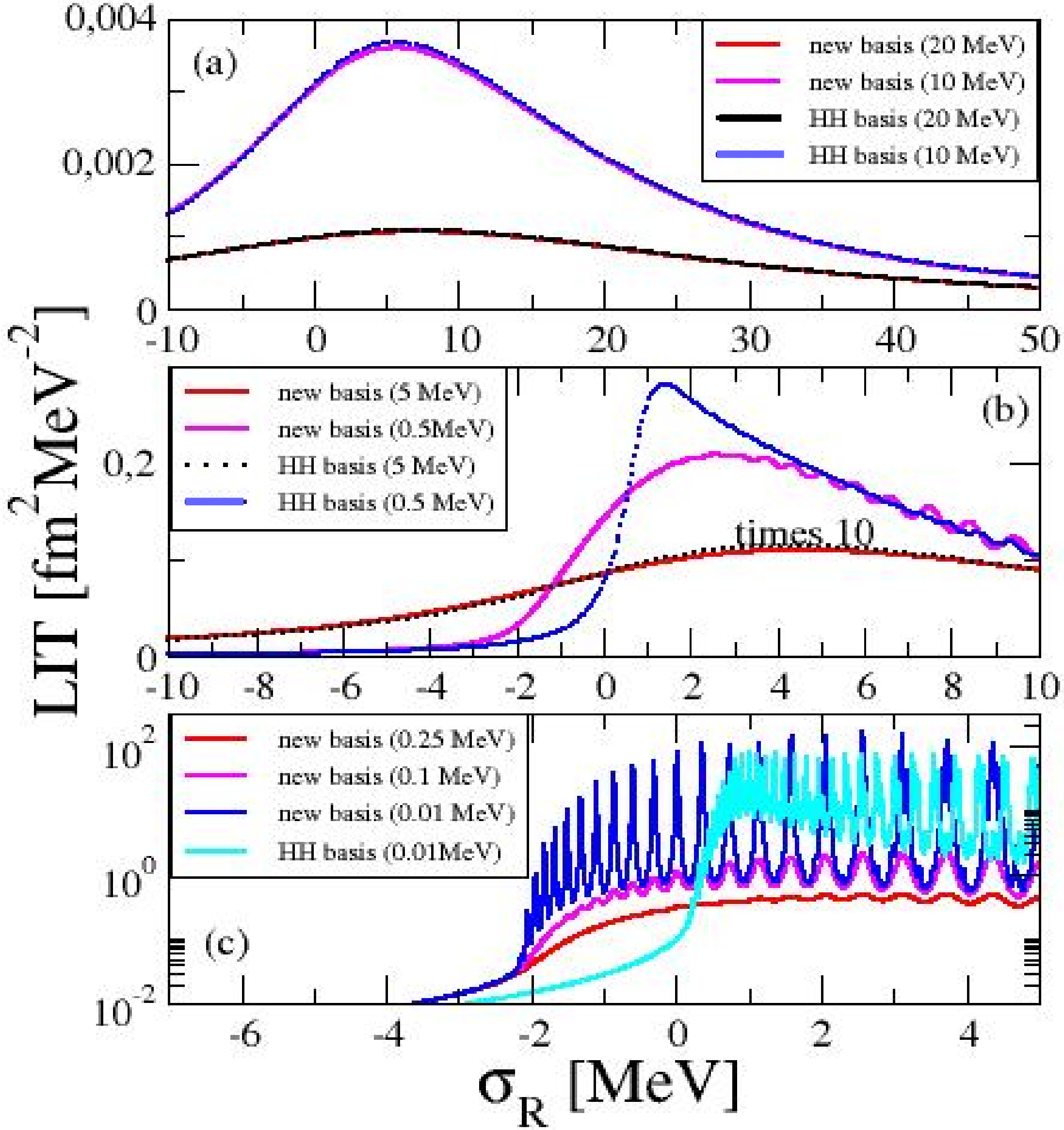
With convergence for expansions in pair and third particle coordinate

Third particle coordinate

Pair coordinate



LIT results with HH and new basis

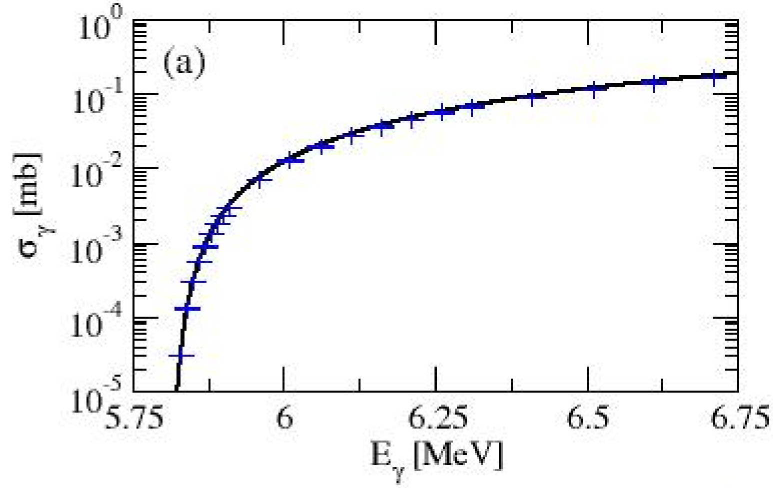


Inversions

Implement correct threshold behaviour for $^3He + \gamma \longrightarrow d + p$

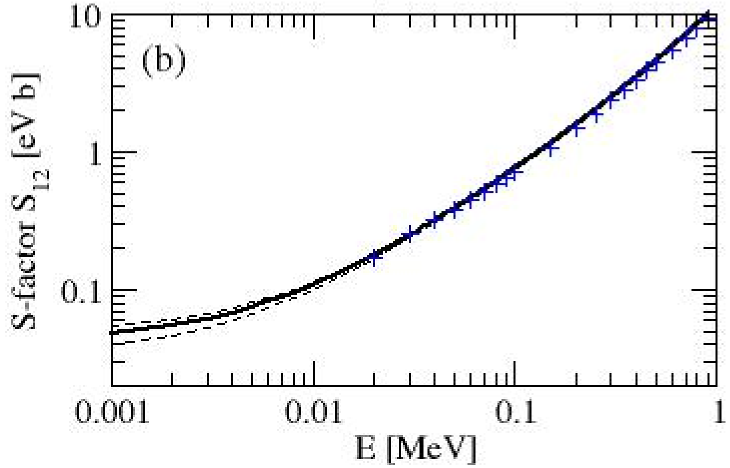
Due to Coulomb potential: usual Gamow factor

Comparison with explicit calculation of continuum state



Cross section
$^3He + \gamma \longrightarrow d + p$

LIT: full curves
cont. wf: +



S-factor
$d + p \longrightarrow {}^3He + \gamma$

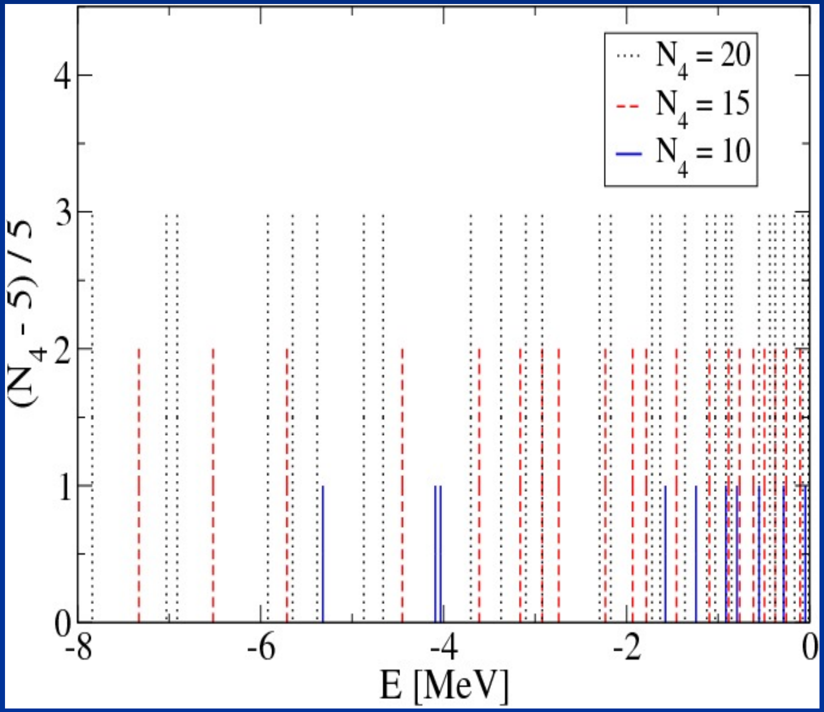
Error due to
inversion: dashed
(Standard deviation from inversions with 11-18 basis functions)

S. Deflorian, V. Efros, WL, FBS 58:3 (2017)

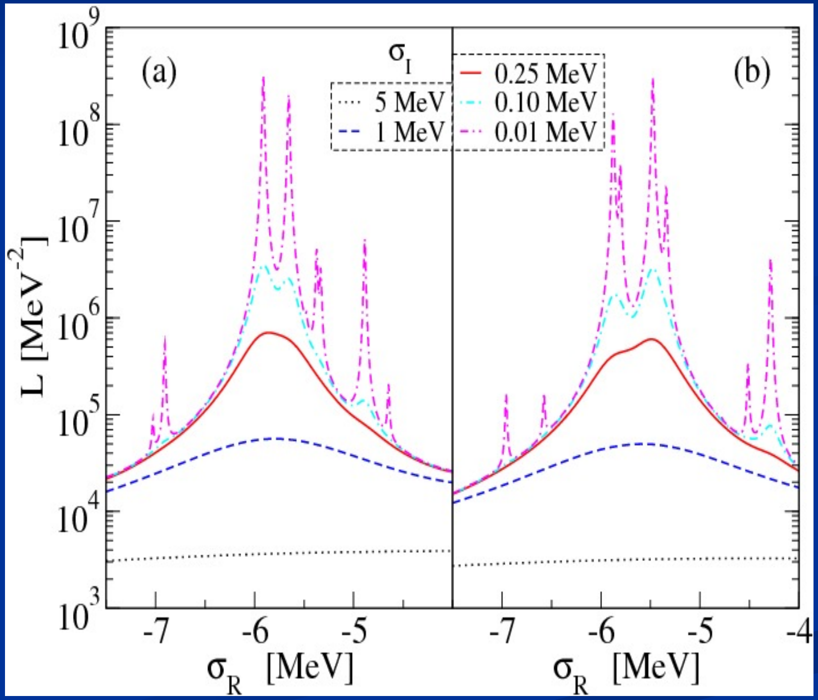
Back to the ^{4}He resonance

Results with new basis

Number of basis functions in 4-th particle coordinate

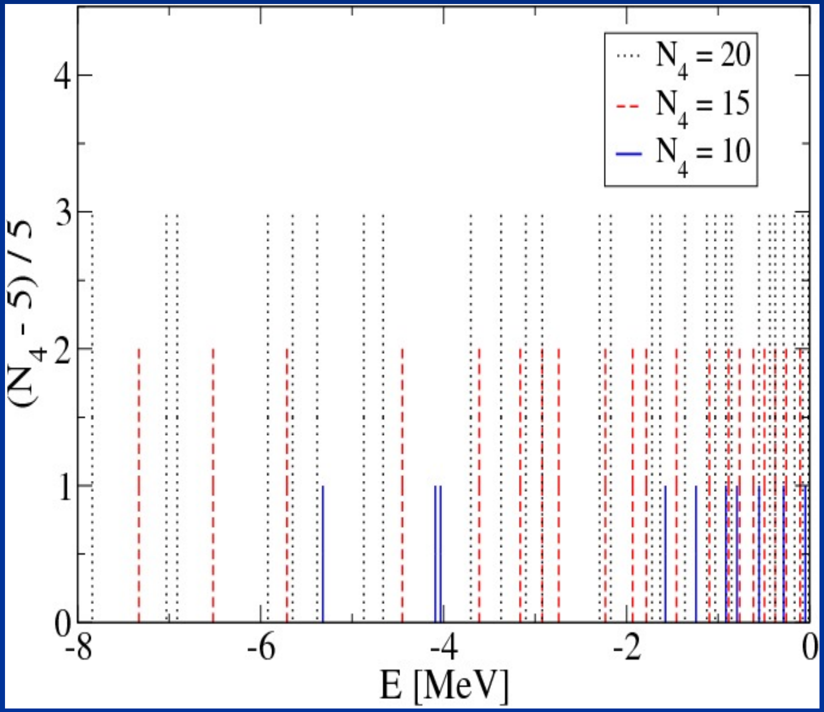


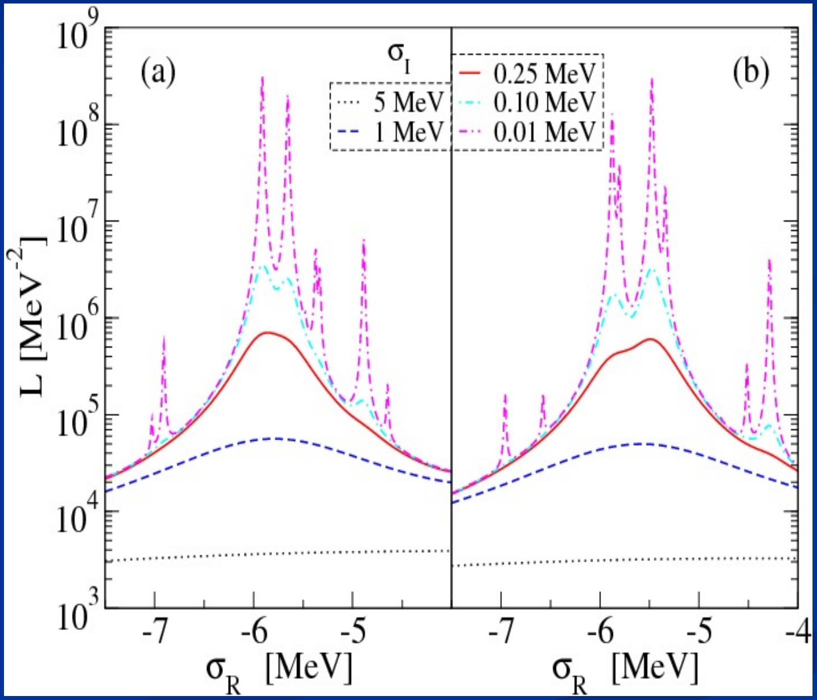
$N_4 = 20$ $N_4 = 21$



LIT

Results with new basis





Inversion: Γ = 180(70) keV

WL, PRC 91, 054001 (2015)

^{3}He (e,e') Response Functions in the Quasielastic Region

V.D. Efros, W.L., G. Orlandini, E.L. Tomusiak

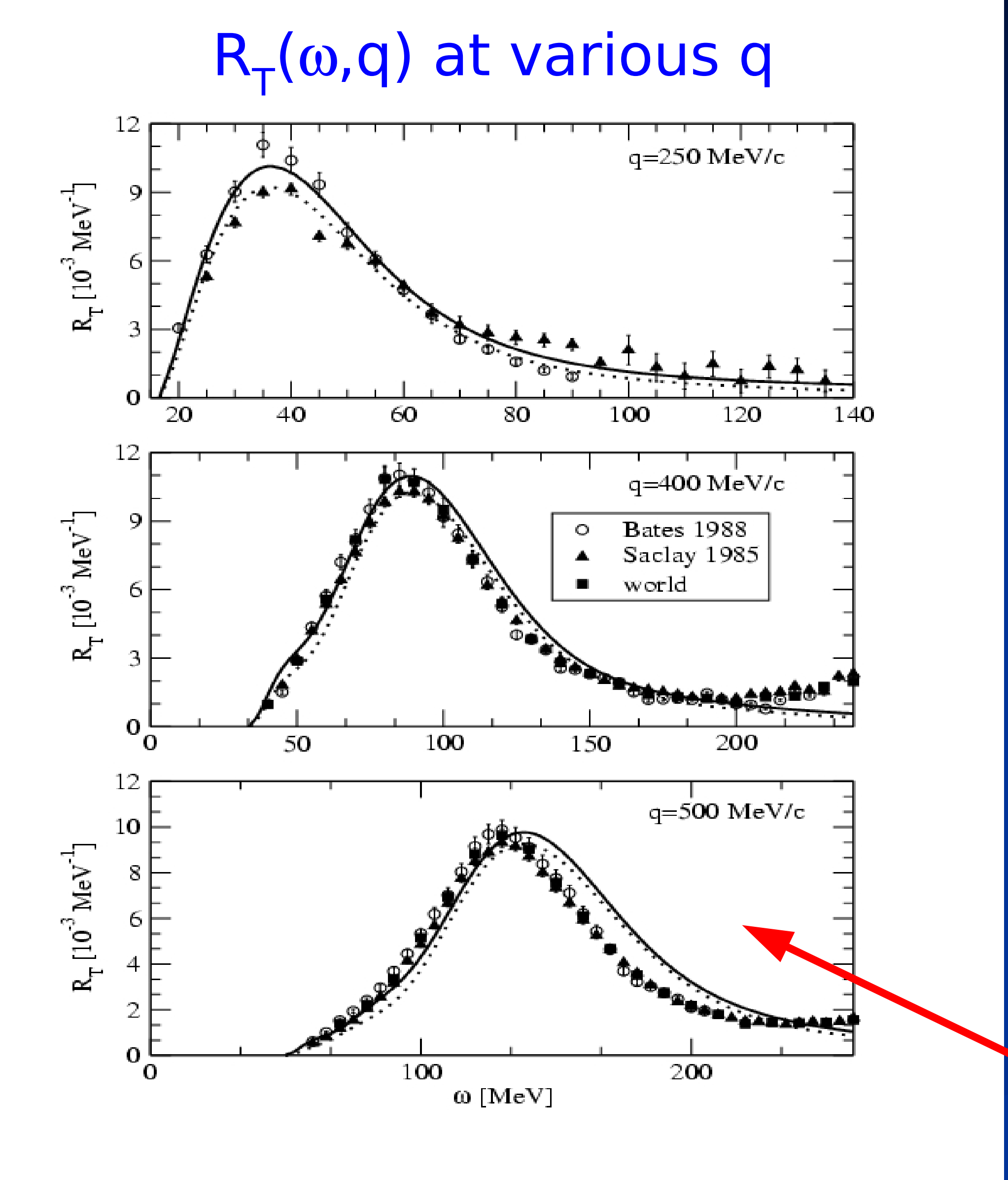
The unpolarized (e,e') cross section is governed by the longitudinal and transverse response functions $R_L(\omega,q)$ and $R_T(\omega,q)$ induced by operators for nuclear charge ρ and current **J**, respectively

The quasielastic region is dominated by the one-body parts of ρ and **J**, but relativistic contributions become increasingly important with growing momentum transfer q

calculation: non-rel. + rel. corrections

with realistic nuclear forces

Motivation



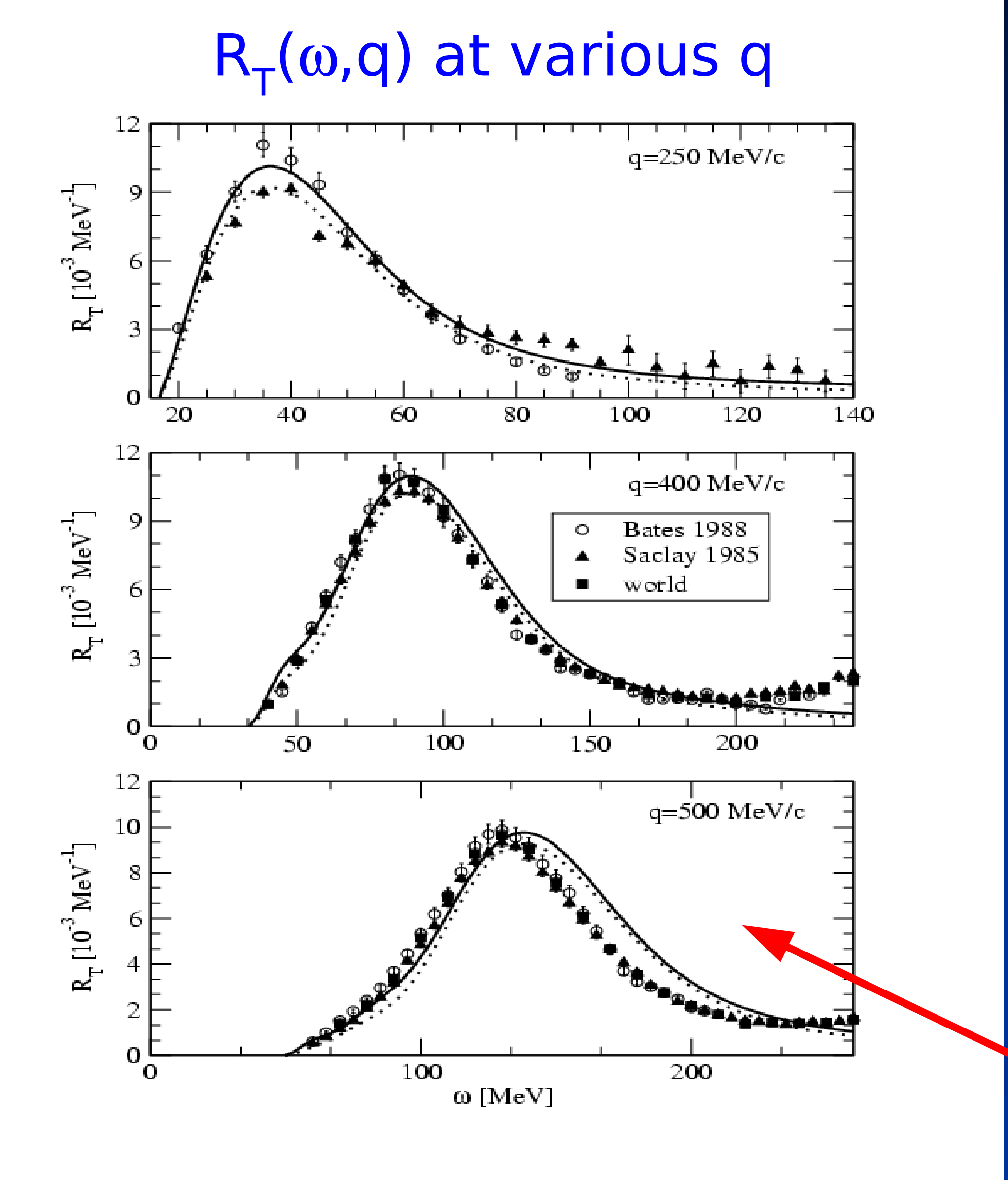
Potential: BonnRA +TM'

one-body current: dashed
one+two-body current: full

(S. Della Monaca et al., PRC 77, 044007 (2008))

Bad agreement between theory and experiment because of non considered relativistic effects

Motivation



Potential: BonnRA +TM'

one-body current: dashed
one+two-body current: full

Quasi-elastic kinematics (q=500 MeV/c),
Kinetic energy of outgoing nucleon:

non-rel. : $T = q^2/2m = 133$ MeV
rel.: $T = (m^2+q^2)^{1/2} - m = 125$ MeV

Bad agreement between theory and experiment because of non considered relativistic effects

We already considered this problem for R_L and studied R_L in various reference frames:

Laboratory: $P_T = 0$

Breit: $P_T = -q/2$

Anti-Lab: $P_T = -q$

Active Nucleon Breit: $P_T = -Aq/2$

non-rel.: $\omega_{\text{frame}} + (P_T)^2/2Am = \omega_{\text{internal}} + (P_T+q)^2/2Am$

$R_L(\omega,q)$ at higher q

Frame dependence

calculation in various frames:

Laboratory: $P_T = 0$

Breit: $P_T = -q/2$

Anti-Lab: $P_T = -q$

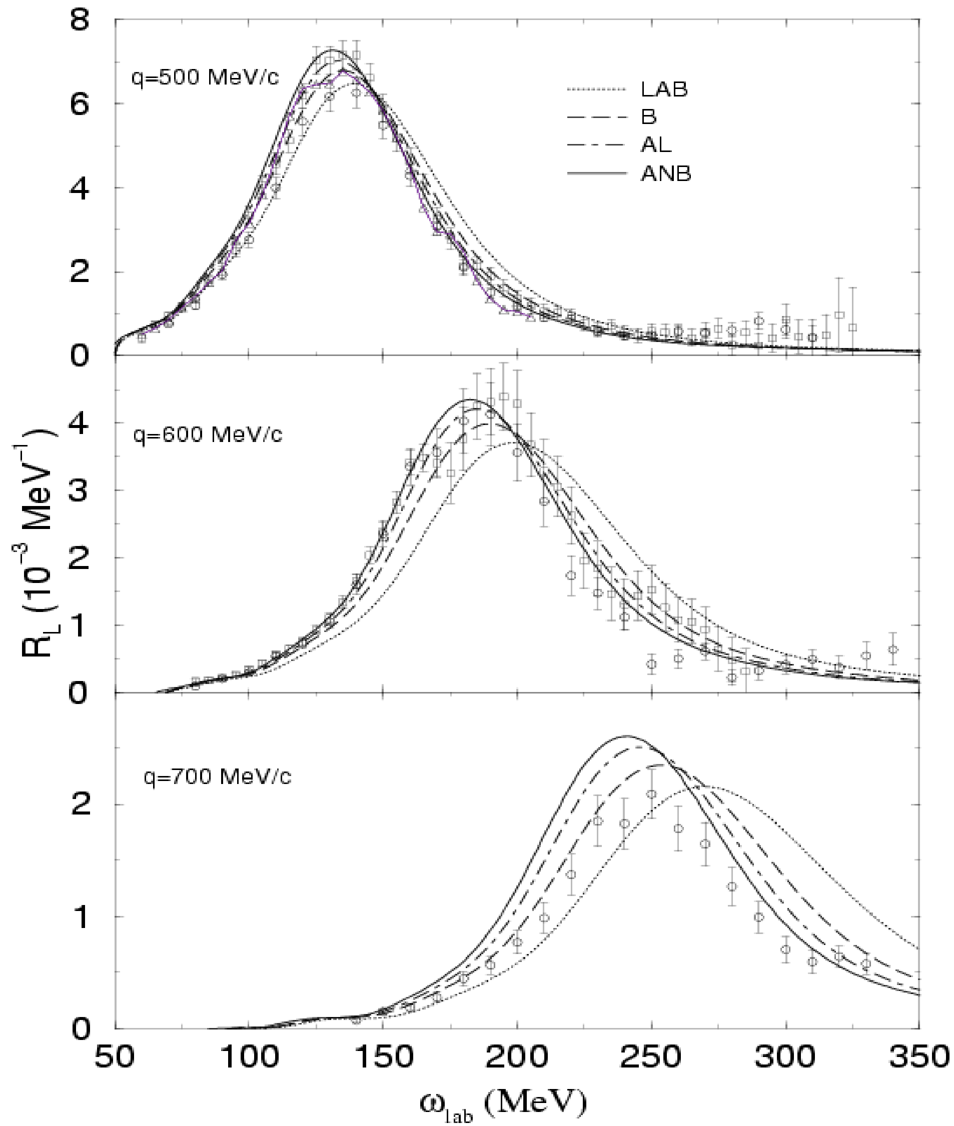
Active Nucleon Breit: $P_T = -Aq/2$

Potential: AV18+UIX

Result in LAB frame

$$R_L(\omega,q) = \frac{q^2}{(q_{fr})^2}\frac{E_T^{fr}}{M_T}\; R_L^{fr}(\omega^{fr},q^{fr})$$

V. Efros, W.L., G. Orlandini, E. Tomusiak
PRC 72 (2005) 011002(R)



Exp: Marchand 1985, Dow 1988, Carlson 2002

How to get more frame independent results?

Two fragment model: Assume quasi-elastic kinematics

whole energy and momentum transfer taken by the knocked out nucleon (residual two-body system is in its lowest energy state)

$\Rightarrow$ Effective two-body problem
Treat kinematics relativistically correct

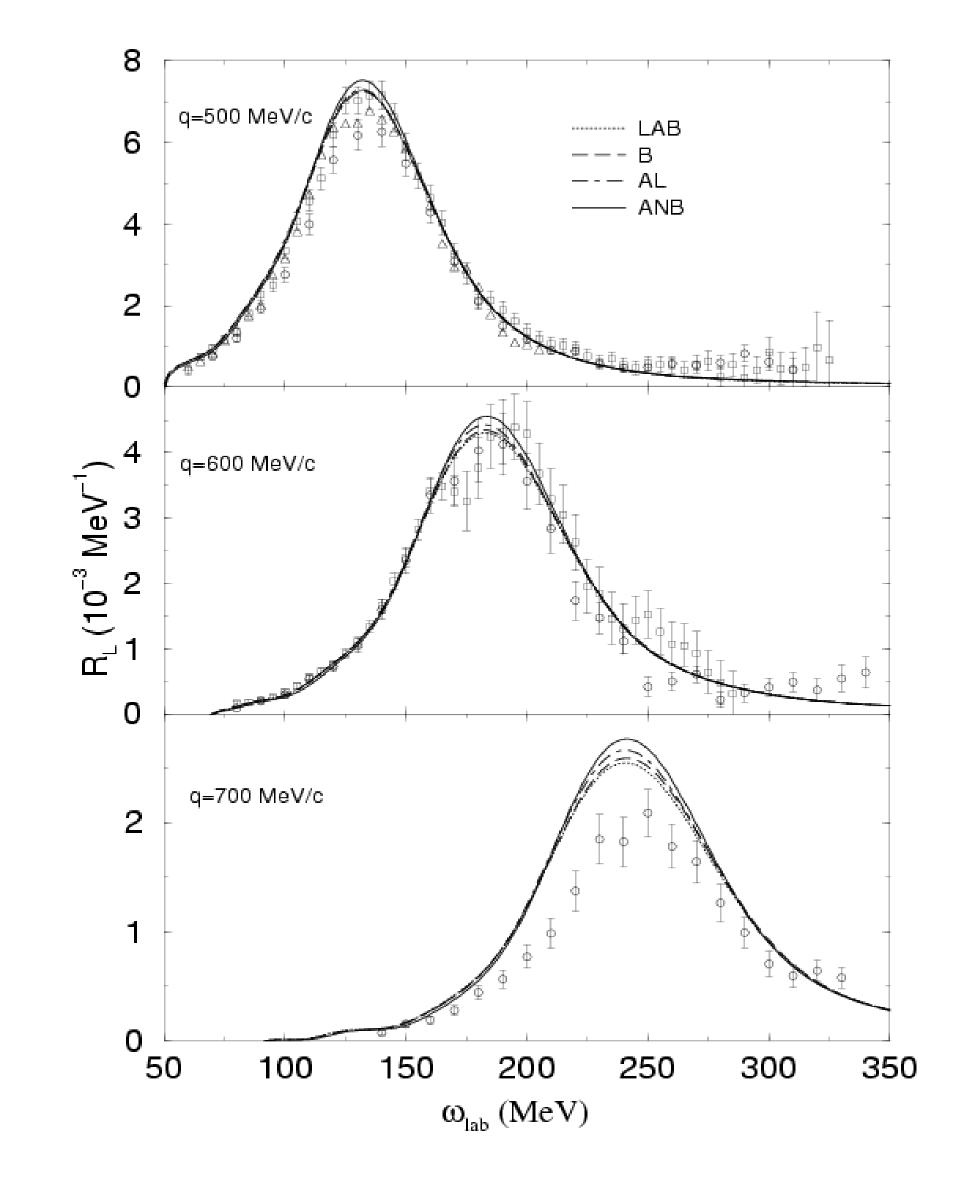
Take the correct relativistic relative momentum k_{rel} and calculate the corresponding non-relativistic relative energy

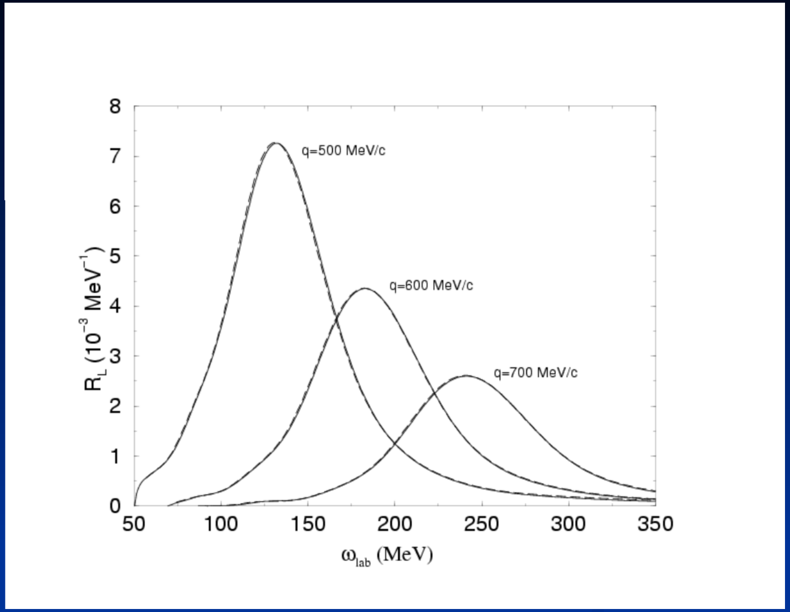
$$E_{nr} = (k_{rel})^2/2\mu$$

with reduced mass μ of nucleon and residual system

use $E_{nr} - E_0(A) + E_0(A-1)$ as $\omega_{internal}$ in the calculation

$R_L(\omega,q)$ at higher q





R_L calculated in ANB frame with (dashed) and without (full) assumption of a two-body break-up

Quasielastic region: assume two-body break-up and use the correct relativistic relative momentum

R_T calculation

Further calculation details

The current operator **J**

$\mathbf{J} = \mathbf{J}^{(1)} + \mathbf{J}^{(2)}$

$\mathbf{J}^{(1)} = \mathbf{J}^{(1)}(q,\omega,P_T) = \mathbf{J}_{spin} + \mathbf{J}_p + \mathbf{J}_q + (\omega/M)\,\mathbf{J}_\omega$

for instance spin current

$\mathbf{J}_{spin} = \exp(i\mathbf{q}\cdot\mathbf{r})\; i\,\boldsymbol{\sigma}\times\mathbf{q}/2M\;[G_M(1-q^2/8M^2) - G_E\,\kappa^2 q^2/8M^2]$

with $\kappa = 1+2P_T/Aq$

Further calculation details

The current operator $\mathbf{J}$

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$$\mathbf{J}_{spin} = \exp(i\mathbf{q}\cdot\mathbf{r})\; i\,\boldsymbol{\sigma}\times\mathbf{q}/2M\,[G_M(1-q^2/8M^2) - G_E\,\kappa^2 q^2/8M^2]$$

with $\kappa=1+2P_T/Aq$

Transformation from ANB frame to LAB frame

$$R_T^{LAB}(\omega^{LAB},q^{LAB}) = R_T^{ANB}(\omega^{ANB},q^{ANB})\;\; E_T^{ANB}/M_T$$

Results

Comparison of

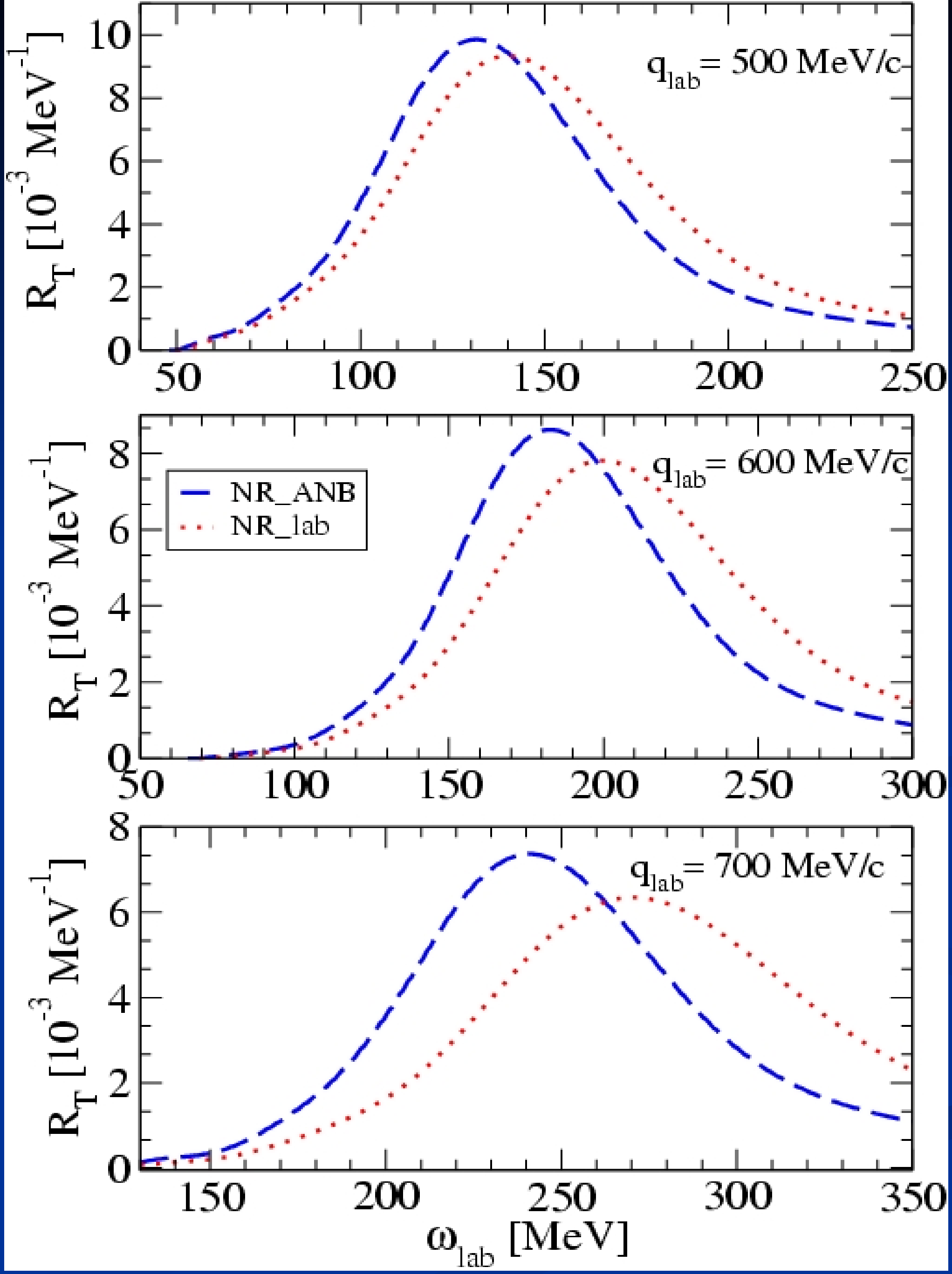
ANB and LAB calculation:

strong shift of peak

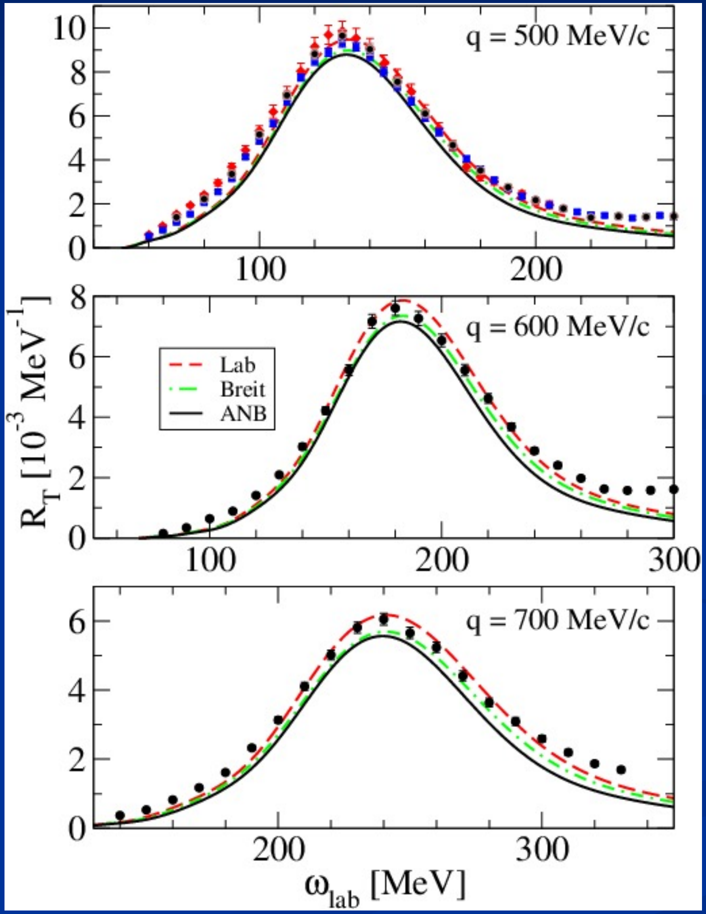
to lower energies!

(8.7, 16.7, 29.3 MeV at

q=500, 600, 700 MeV/c)



With two-fragment model: peak positions agree

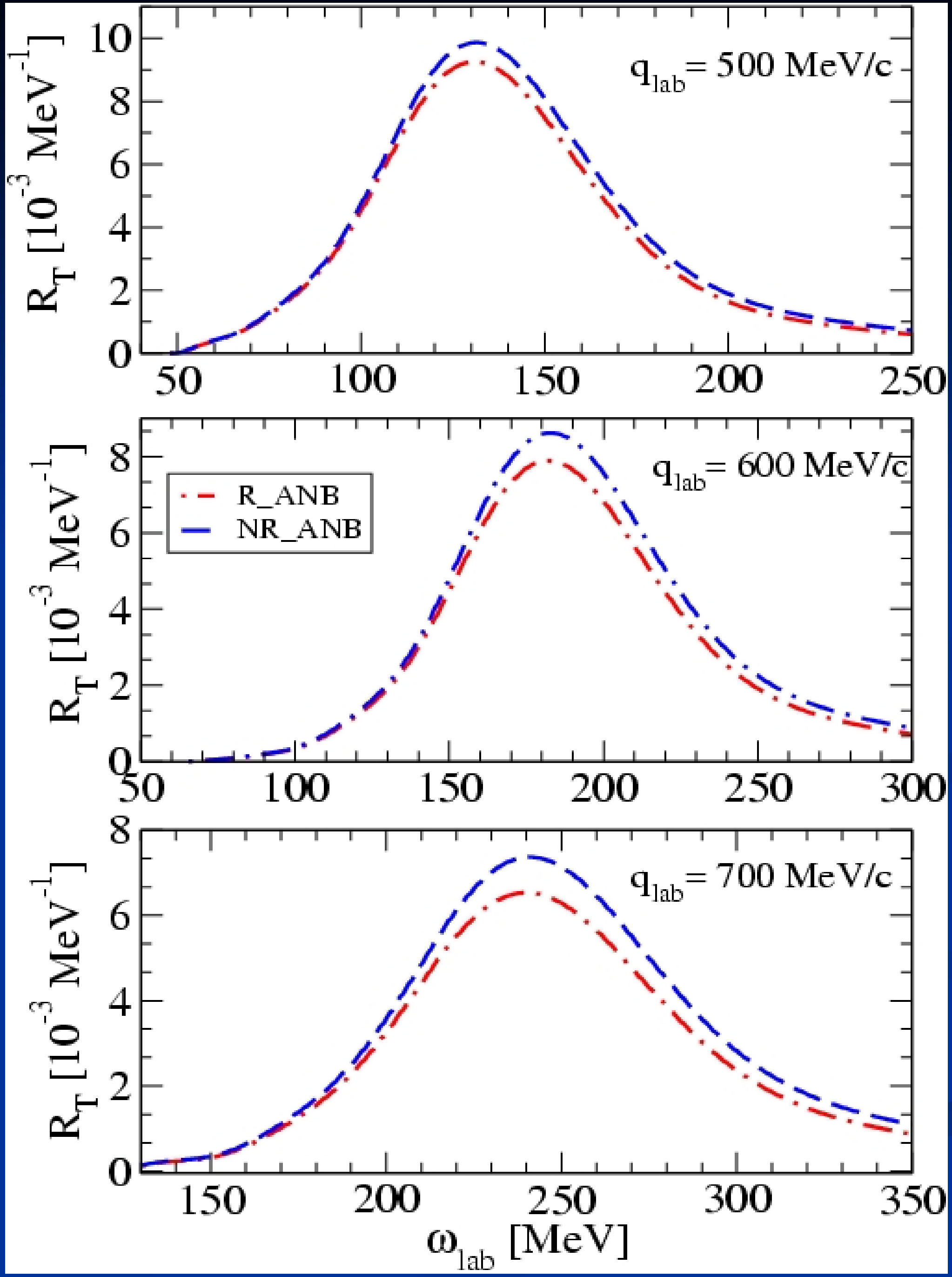


Results

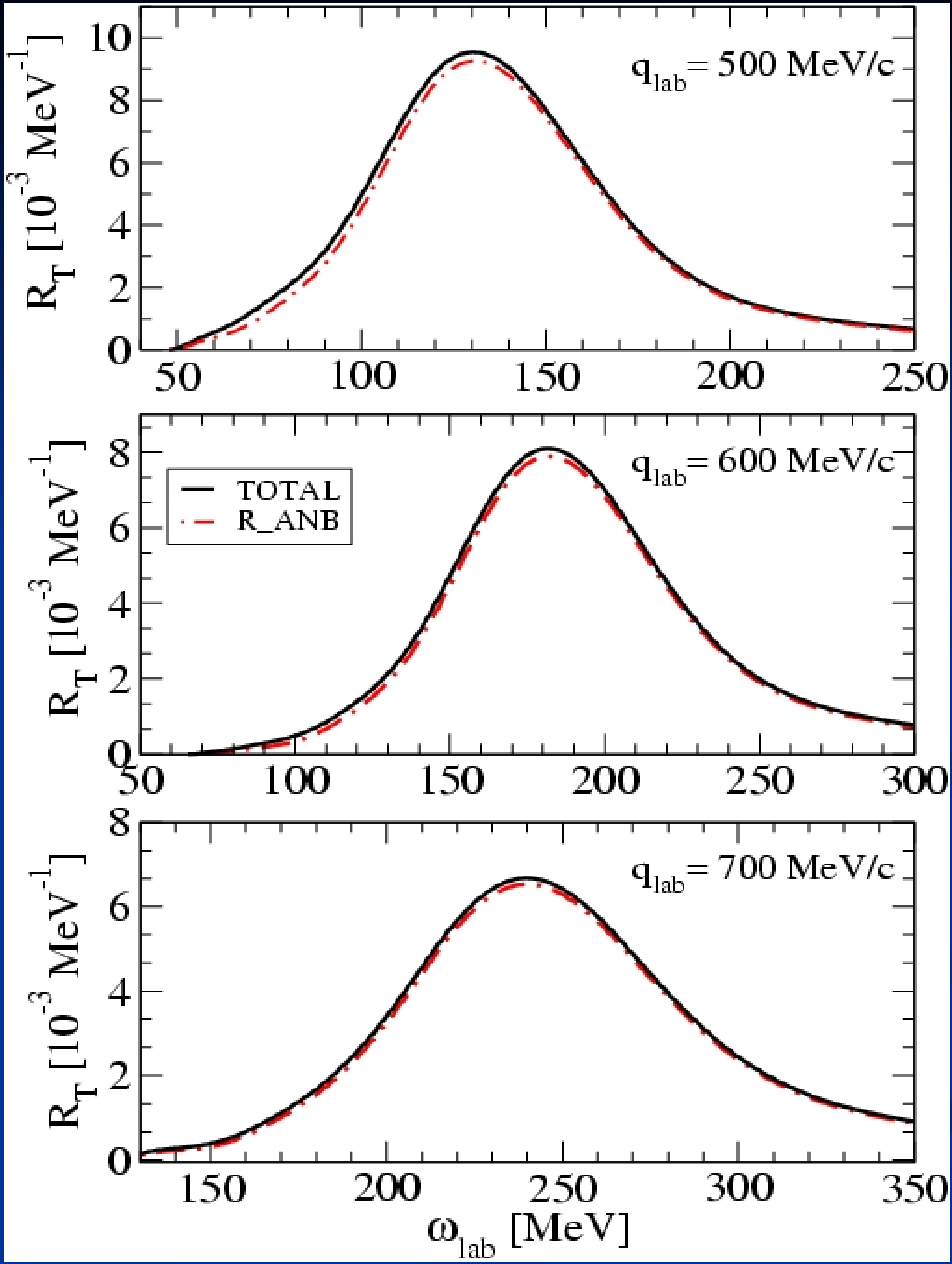
Rel. contribution:

reduction of peak height

(6.2%, 8.5%, 11.3 % at q=500, 600, 700 MeV/c)



Results



MEC:

small increase of

peak height

(3.2%, 2.7%, 2.2% at

q=500, 600, 700 MeV/c)

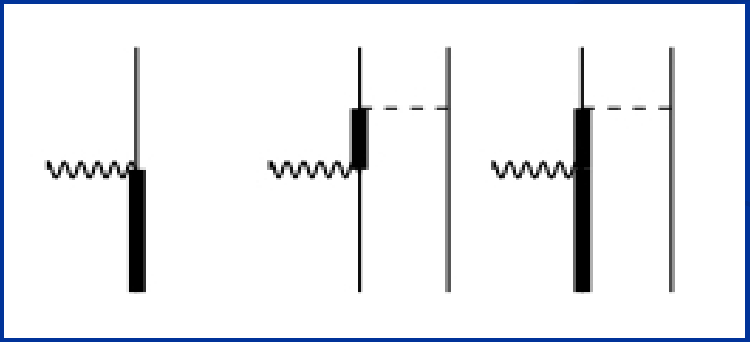
Δ Isobar degrees of freedom in the ^{3}He transverse (e,e') Response Function

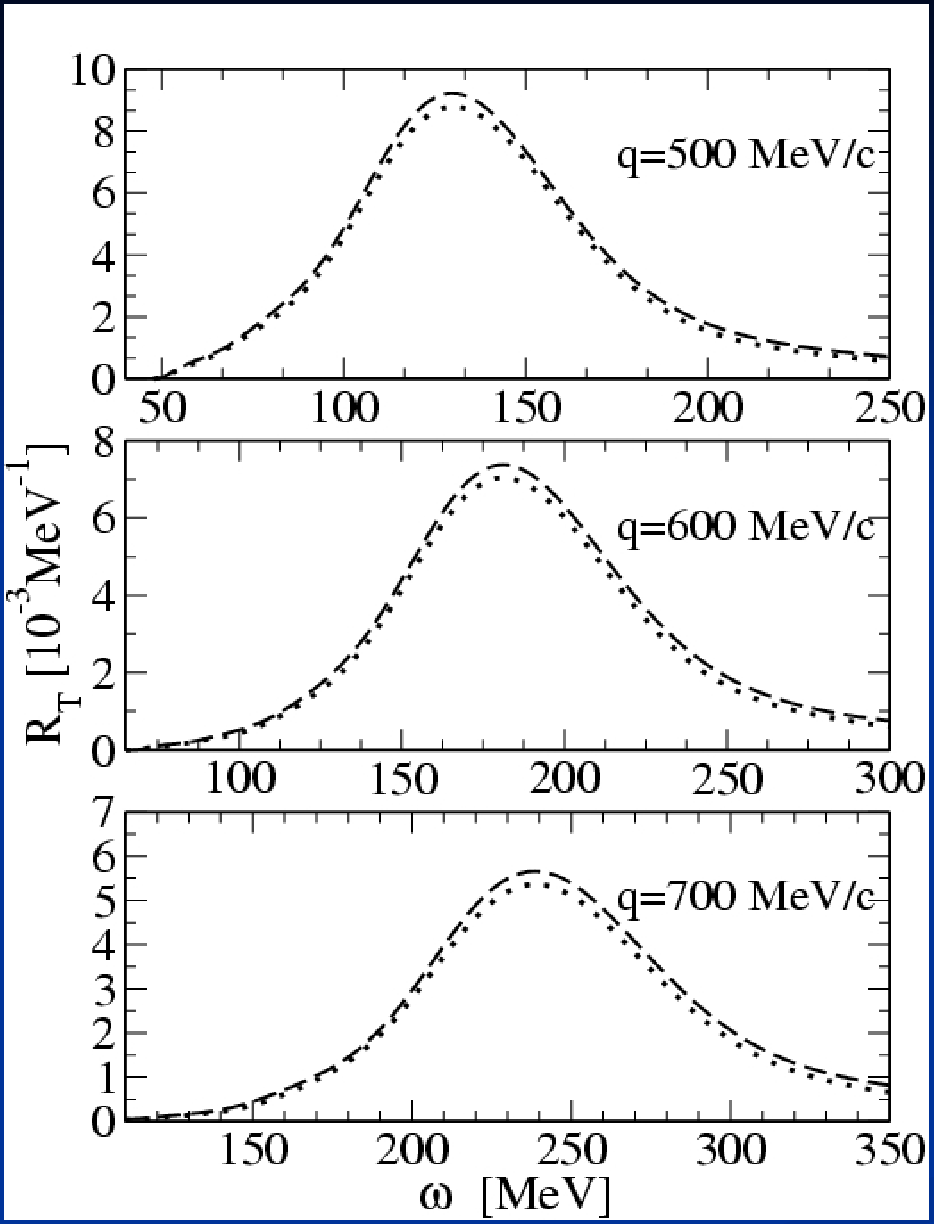
L. Yuan, W.L., V.D. Efros, G. Orlandini, E.L. Tomusiak
PLB 706, 90 (2011)

LIT equation with Δ degrees of freedom

$$(T_N + V_{NN} - \sigma)\,\tilde{\Psi}_{\mathbf{N}} = -V_{NN,N\Delta}(H_\Delta - \sigma)^{-1}(O_{\Delta N}\,\Psi_{0,N} + O_{\Delta\Delta}\,\Psi_{0,\Delta})$$

$$+\,O_{NN}\,\Psi_{0,N} + O_{N\Delta}\,\Psi_{0,\Delta}$$



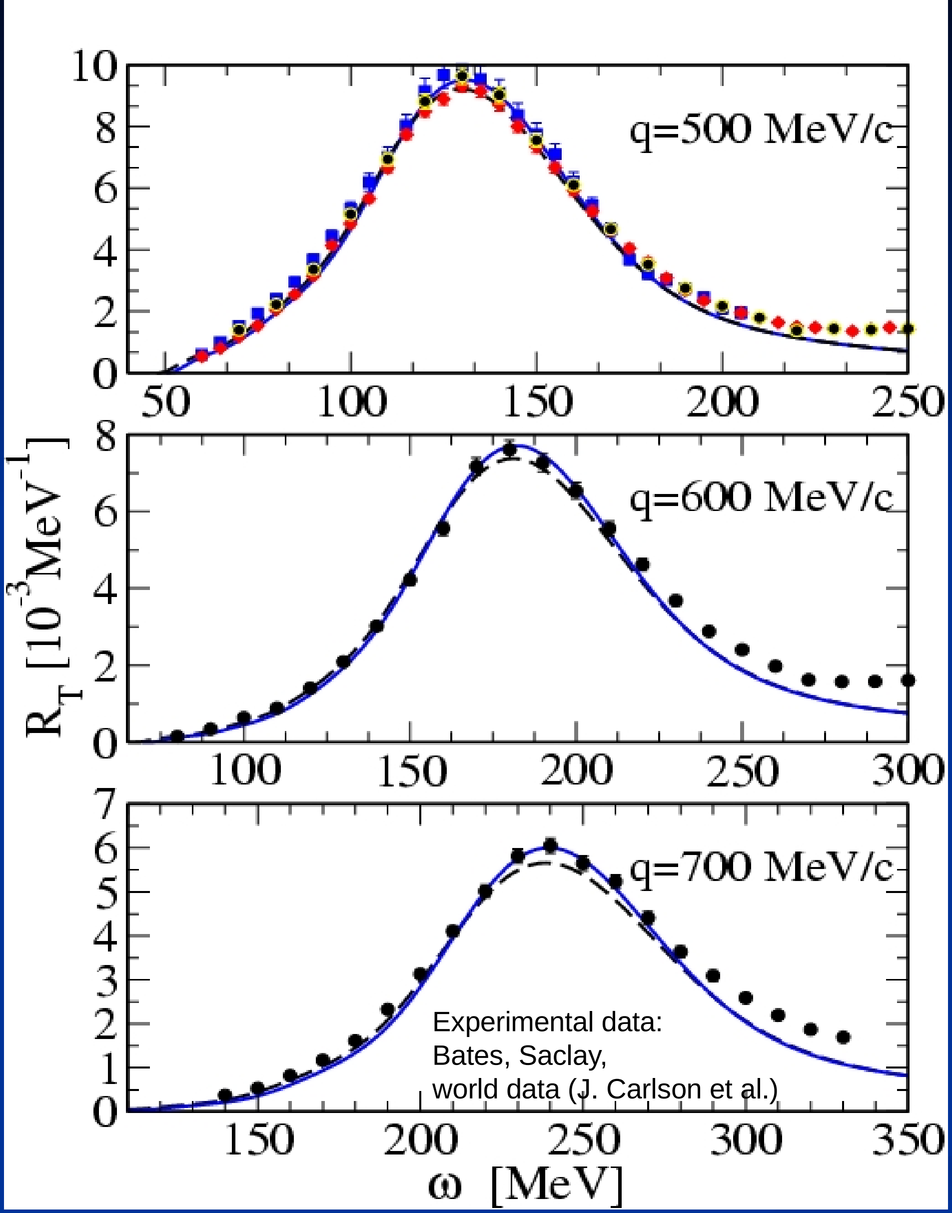


Δ-IC contribution

Dotted: without Δ

Dashed with Δ

Effect of two-fragment model



Dashed: with Δ (as before)
Solid: same but with two-fragment model

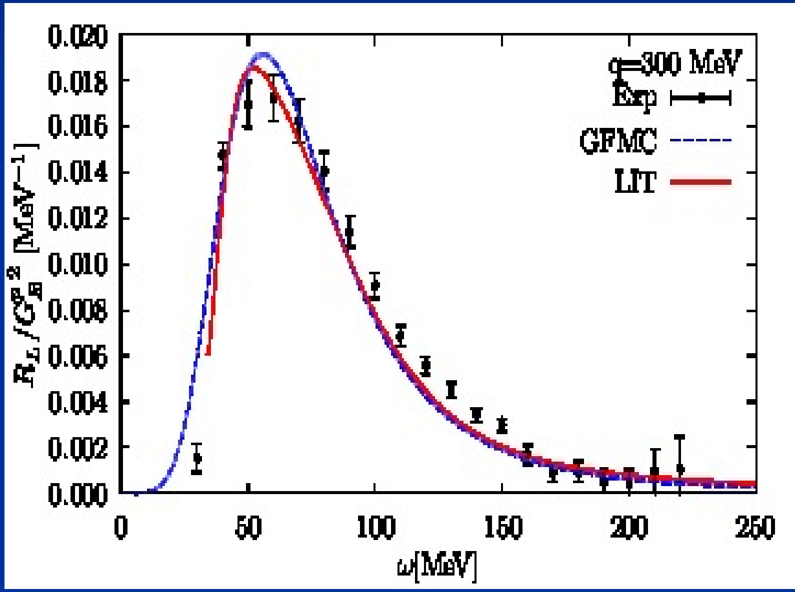
$^4He(e,e')$ with GFMC

N. Rocco, WL, A. Lovato, G. Orlandini, PRC 97, 055501 (2018)

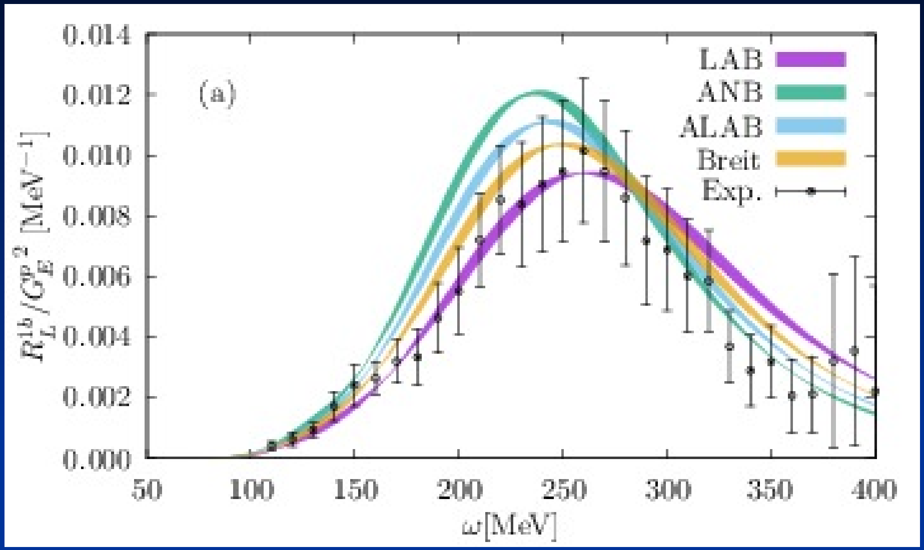
- Inversion of Euclidean response (Laplace transform of response)
- Calculation includes relativistic corrections for charge but not for current operator
- MEC and IC included
- Interaction: AV18 + IL7-3NF

Comparison LIT-Euclidean response

LIT from S. Bacca et al., PRC 80,064001 (2009)



R_L

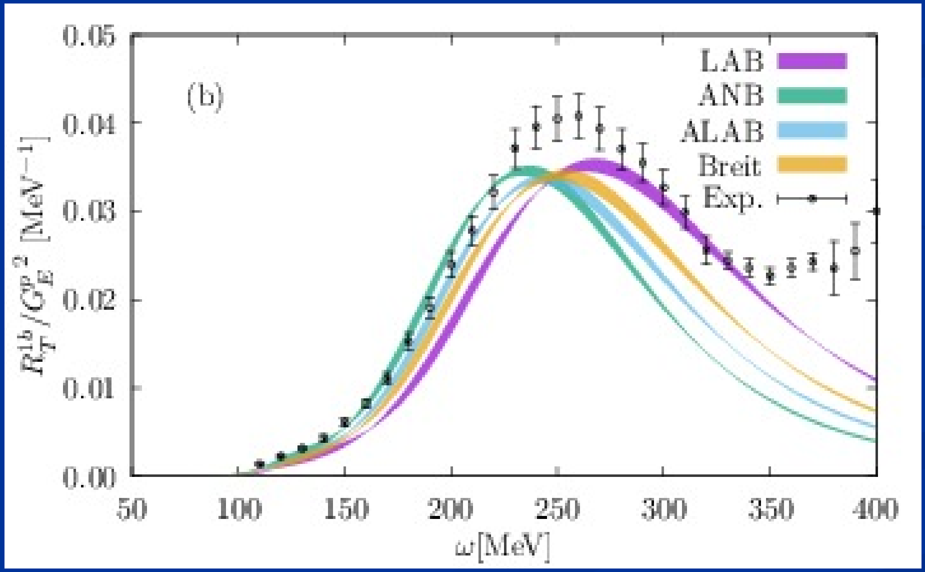


^{4}He(e,e')

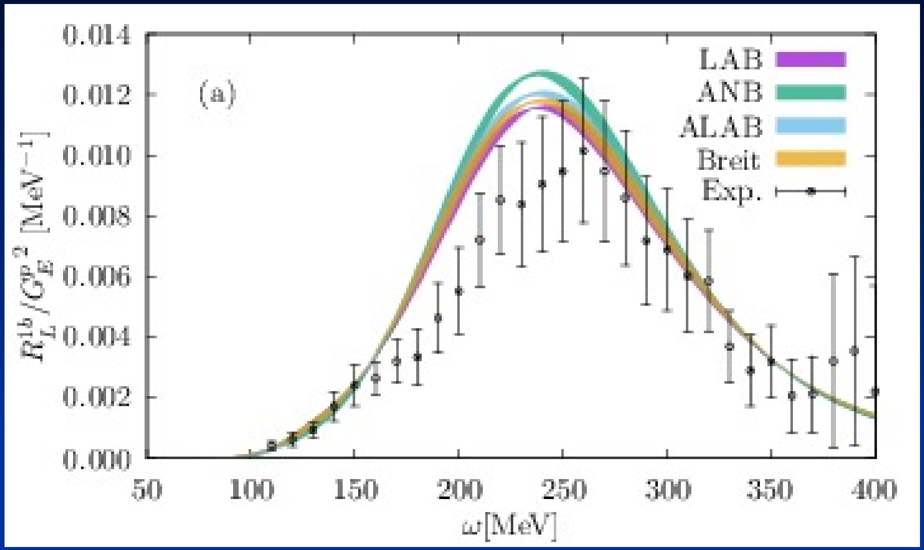
q=700 MeV/c

frame dependence

R_T



R_L



^{4}He(e,e')

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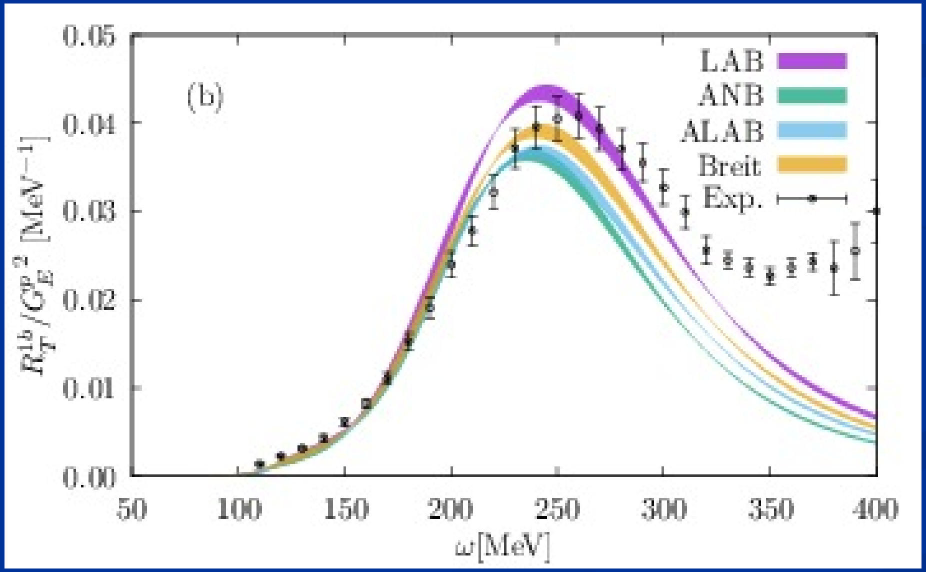
R_T



frame dependence
with two-fragment model

Cross sections