

Sum rules from the *ab initio* symmetry-adapted NCSM and LIT

Kristina Launey

... LSU Team ...

Jerry Draayer, Tomas Dytrych,

Alexis Mercenne

Robert Baker, Ali Dreyfuss,

David Kekejian, Grigor Sargsyan,

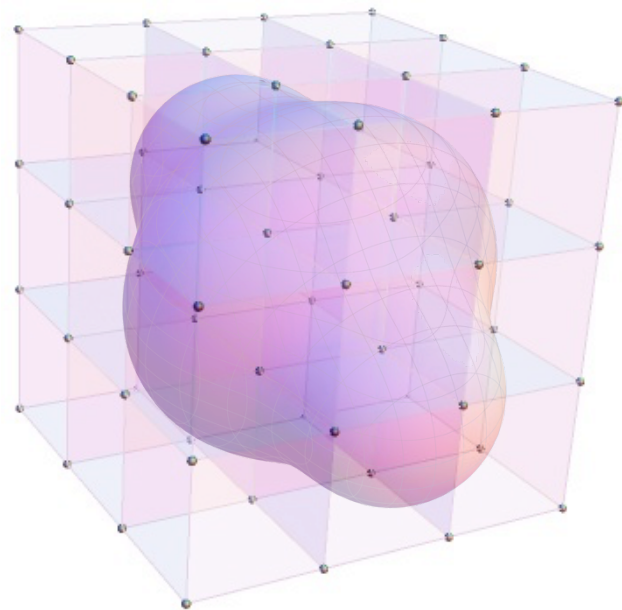
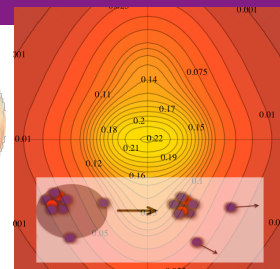
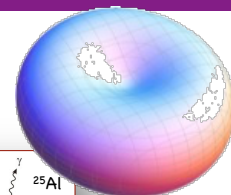
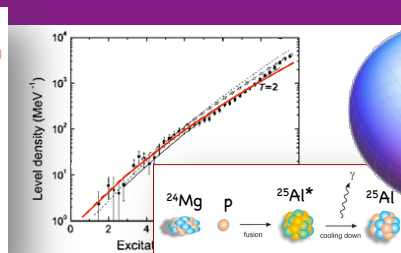
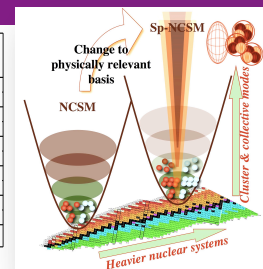
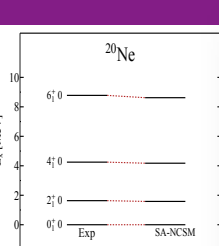
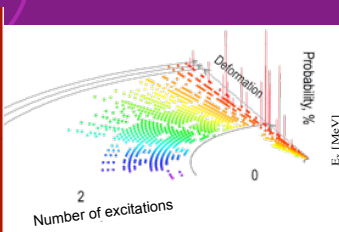
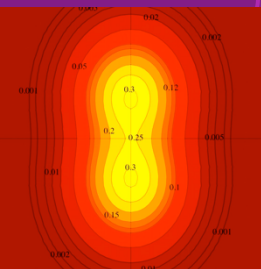
Madeleine Miora

In collaboration with
Sonia Bacca & Nir Nevo Dinur
Princeton U. – W. Tang & B. Wang
Czech Republic – D. Langr & T. Oberhuber

HPC Resources
NSF/U. of Illinois ...*BlueWaters*
LSU...*SuperMike-II*

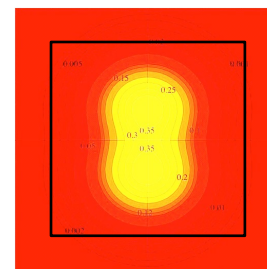
Supported by NSF & DOE





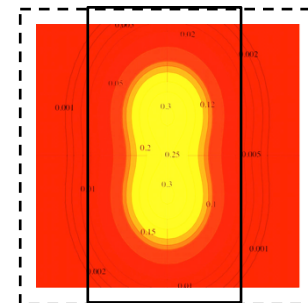
NCSM

Total HO quanta
 N_{\max}



SA-NCSM

Total HO quanta
 $N_{\max} +$
Distribution:
z, x, y

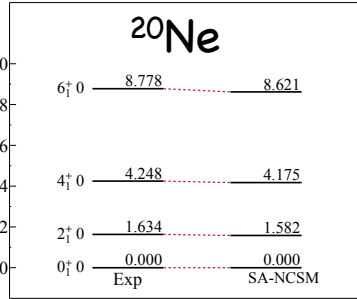
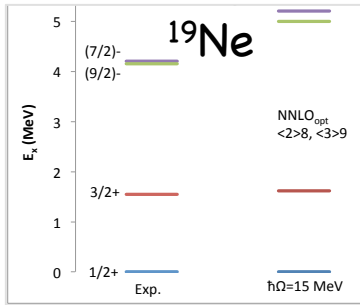
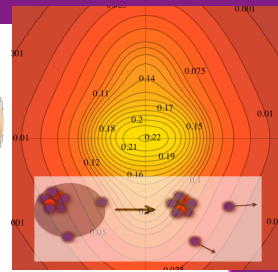
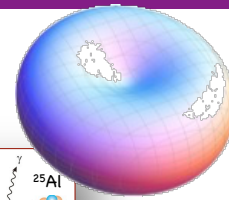
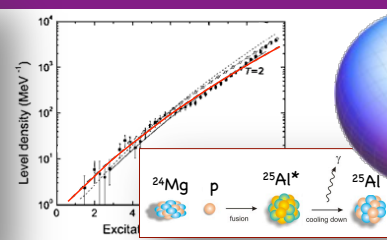
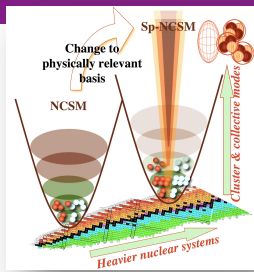
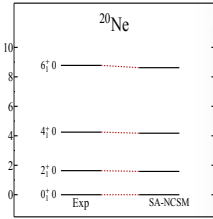
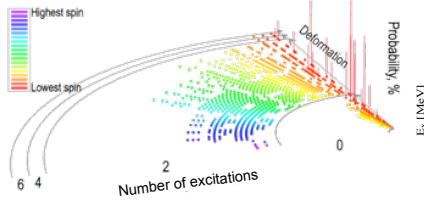


LSU code (LSU3shell): sourceforge.net/projects/lsu3shell

Dytrych et al., Phys. Rev. Lett. 111 (2013) 252501

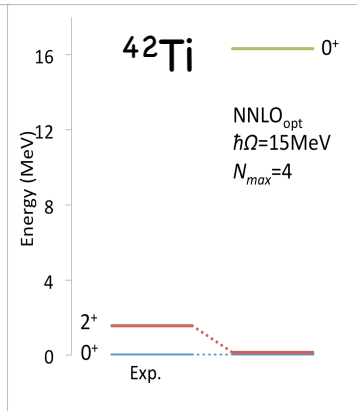
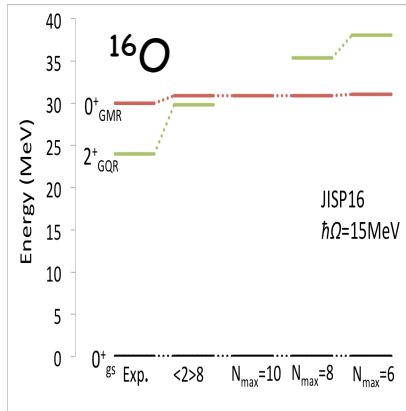
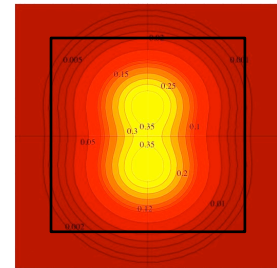
Launey et al., Prog. Part. Nucl. Phys. 89 (2016) 101





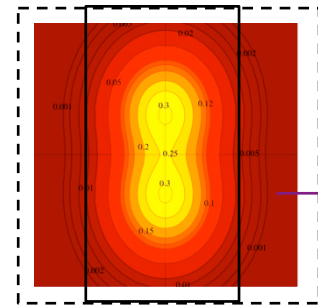
SU(3) basis

NCSM
Total HO quanta
 N_{max}



Symplectic
Sp(3,R) basis

SA-NCSM
Total HO quanta
 $N_{\text{max}} +$
Distribution:
 z, x, y



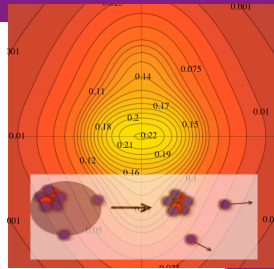
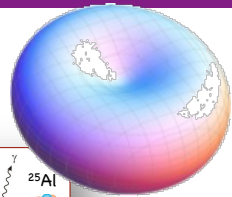
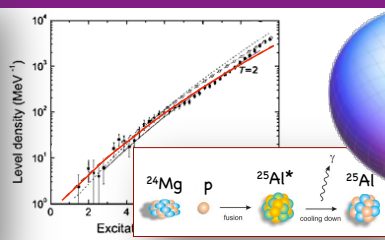
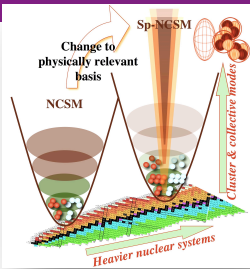
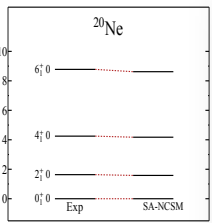
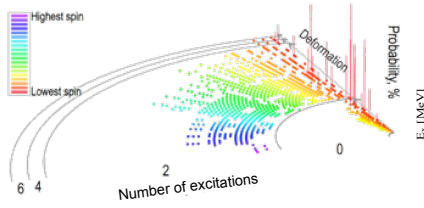
Deformation, rotations... & vibrations
Symmetry-adapted: **SU(3), Sp(3,R)**
Guided by Symplectic symmetry

LSU code (LSU3shell): sourceforge.net/projects/lsu3shell

Dytrych et al., Phys. Rev. Lett. 111 (2013) 252501

Launey et al., Prog. Part. Nucl. Phys. 89 (2016) 101

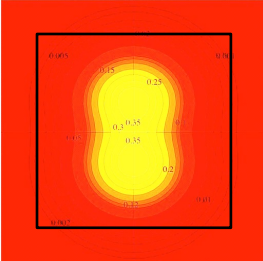




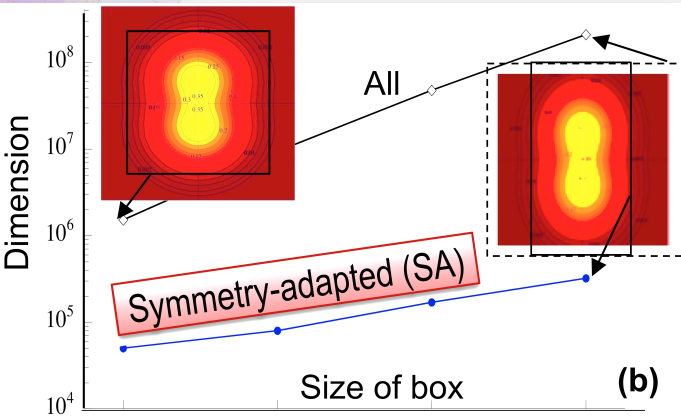
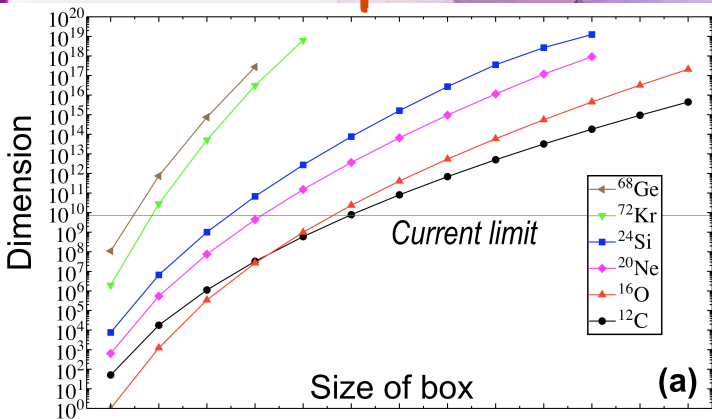
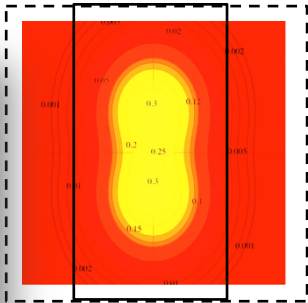
Unitary transformation to collective basis (SA)

NCSM

Total HO quanta N_{max}



SA-NCSM



LSU code (LSU3shell): sourceforge.net/projects/lsu3shell

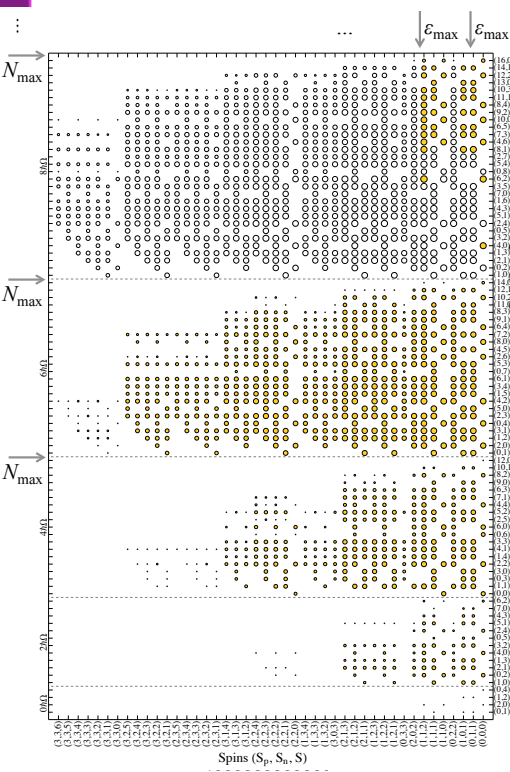
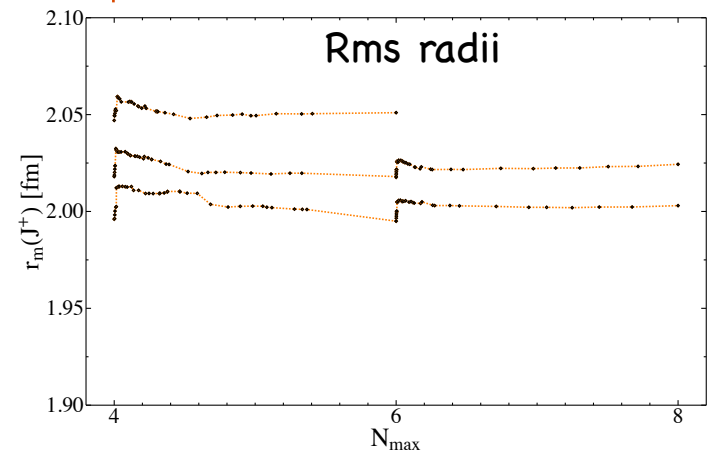
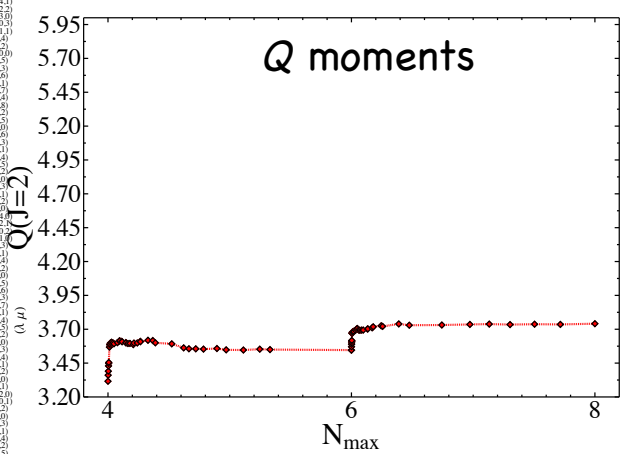
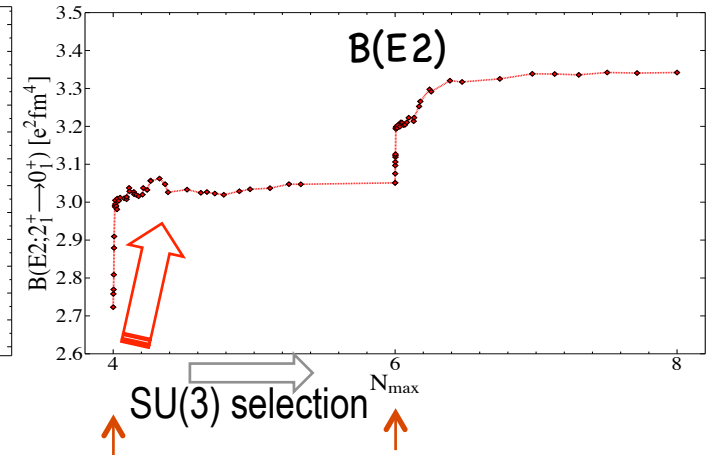
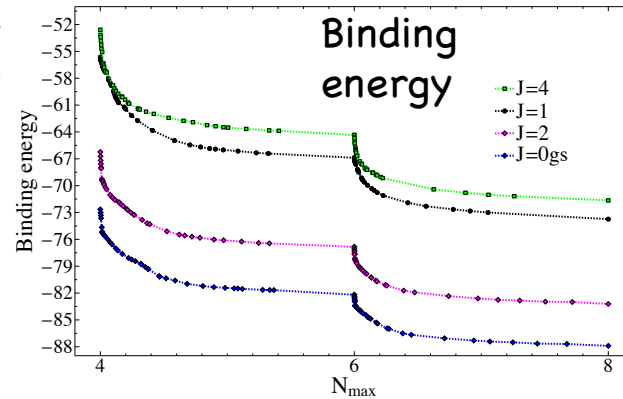
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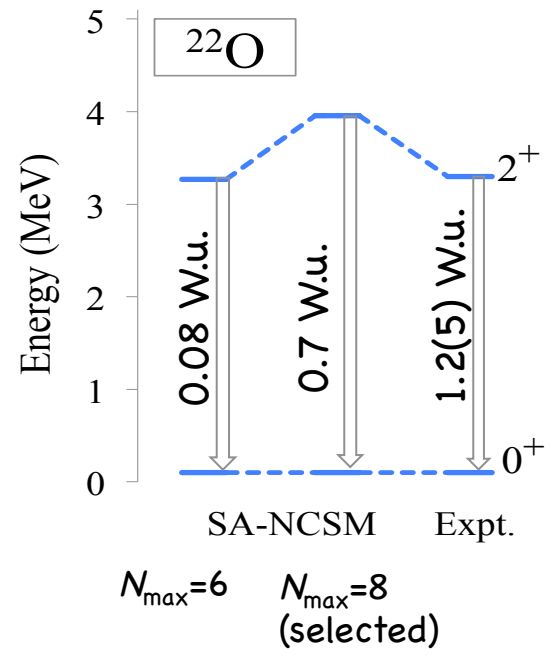
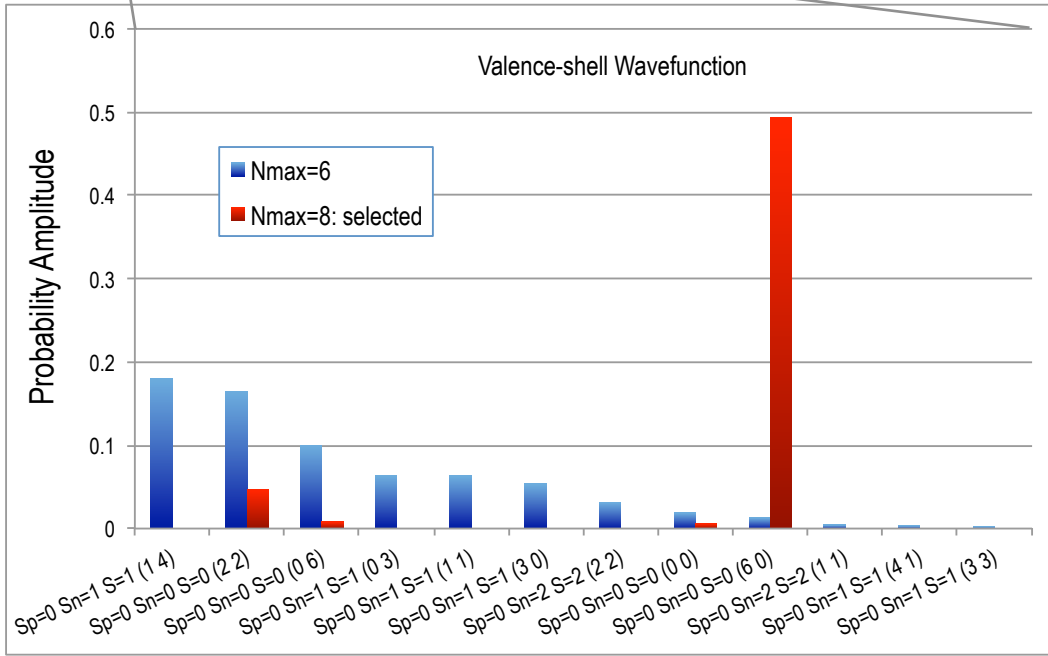
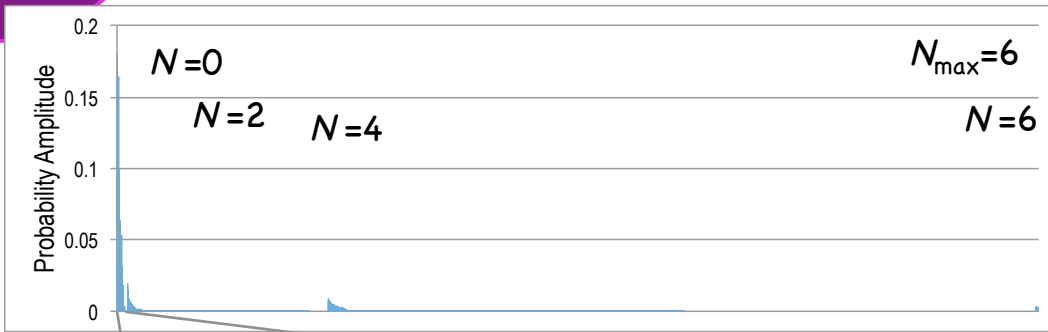
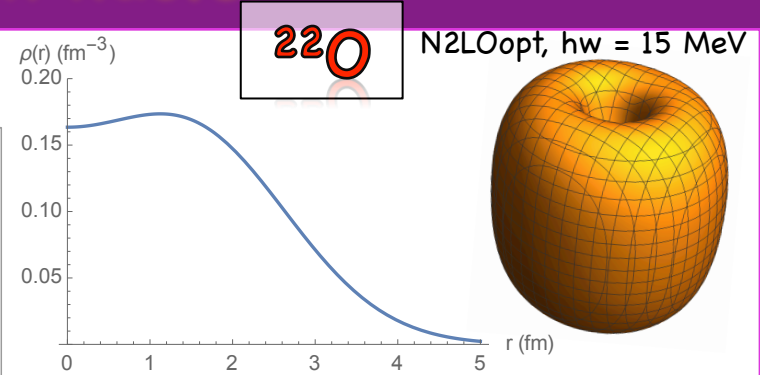
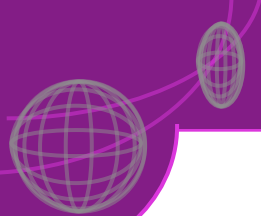


SA Selection

¹²C

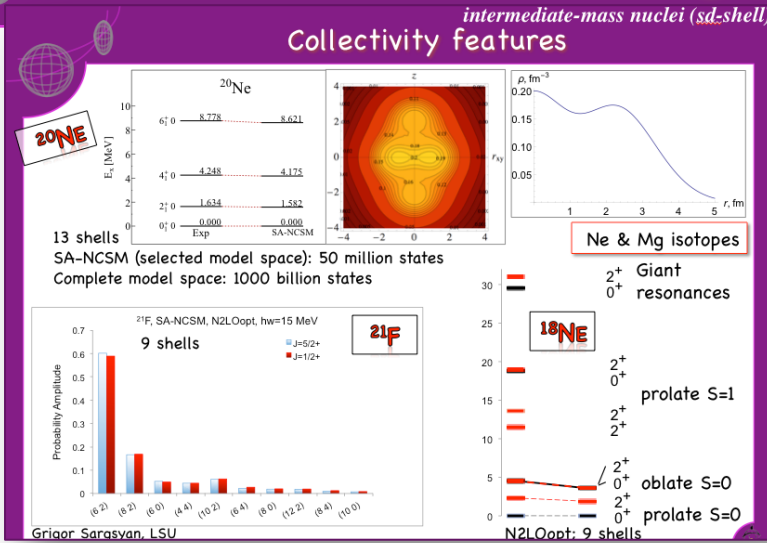


Collectivity in nuclei

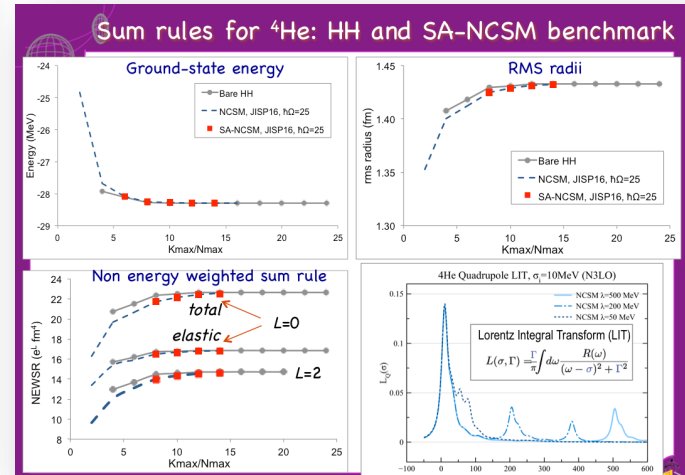


Grigor Sargsyan, PhD student, LSU

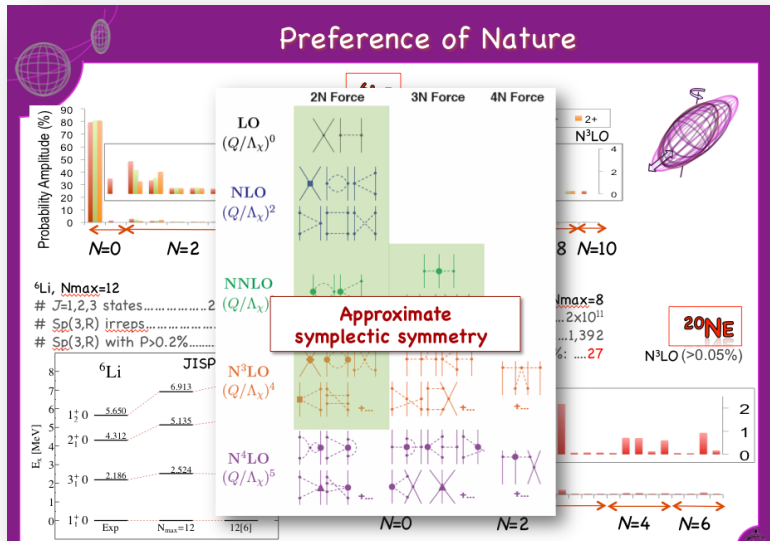
Outline



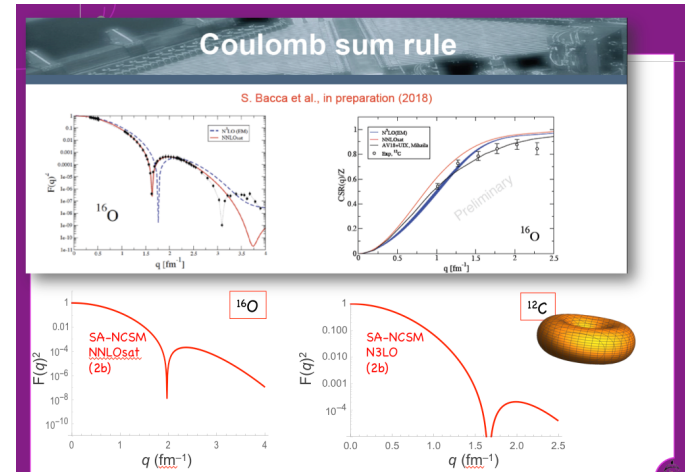
Collectivity in nuclei from first principles?



SA-NCSM+LIT (with S. Bacca):
sum rules and response functions



Nature's preference



Fundamental Physics with Electroweak Probes of Light Nuclei
INT-18-2a, July 2018

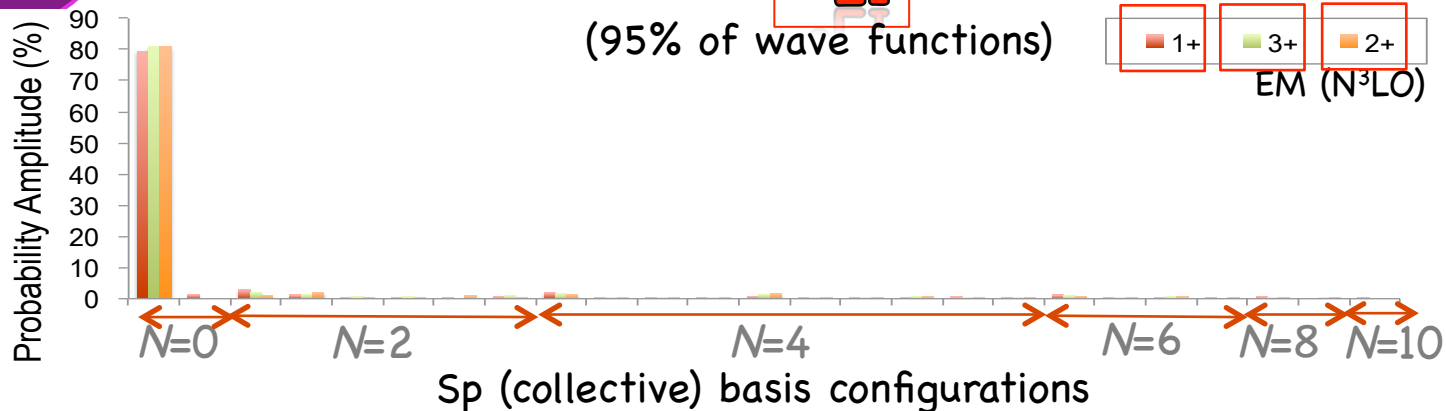
KD Launey
Louisiana State University

Nature's Preference

${}^6\text{Li}$

(95% of wave functions)

■ 1+
■ 3+
■ 2+
 EM ($N^3\text{LO}$)

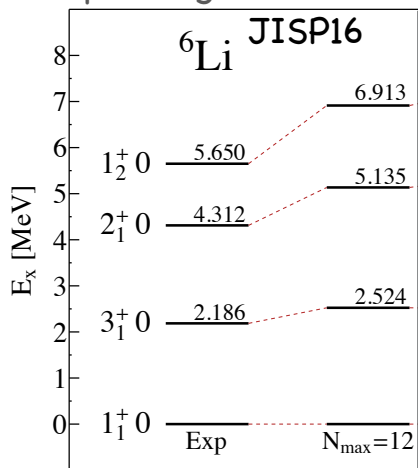


${}^6\text{Li}$, $N_{\text{max}}=12$

$J=1,2,3$ states..... 2×10^7

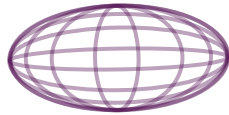
Sp configurations.....528

Sp configurations with $P > 0.2\%$**25**

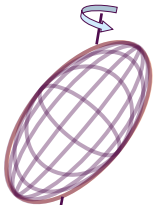


What physics can we learn from Sp basis?

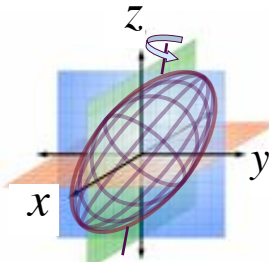
Sp (collective) basis configuration:



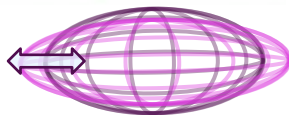
one equilibrium deformation ("shape")



rotations



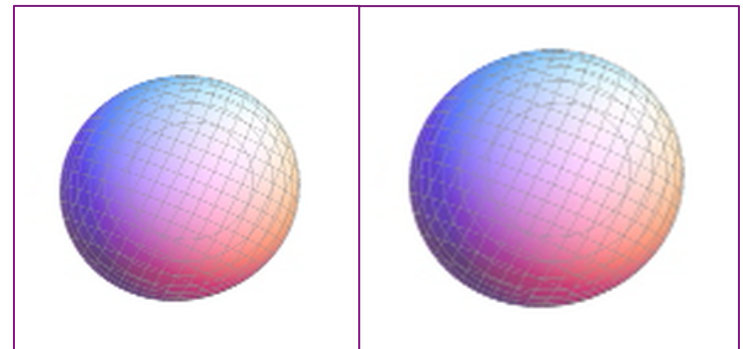
space orientation



Vibrations
(of the giant resonance monopole (r^2)/ quadrupole (Q) type)

All states preserve the equilibrium shape...

Symmetry?



Symplectic Sp(3,R) Symmetry!

Formal definition

All linear canonical transformations of the single-particle phase-space observables

$$x_{i\alpha} \rightarrow \sum_{\beta=x,y,z} a_{\alpha\beta} x_{i\beta} + b_{\alpha\beta} p_{i\beta}$$

$$p_{i\alpha} \rightarrow \sum_{\beta=x,y,z} c_{\alpha\beta} x_{i\beta} + d_{\alpha\beta} p_{i\beta}$$

that **preserve the canonical commutation relation**

$$[x_{i\alpha}, p_{j\beta}] = i\hbar \delta_{ij} \delta_{\alpha\beta}$$

Generators: $Q_{ij} = \sum_n x_{ni} x_{nj},$

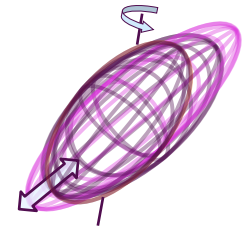
$$S_{ij} = \sum_n (x_{ni} p_{nj} + p_{ni} x_{nj}),$$

$$L_{ij} = \sum_n (x_{ni} p_{nj} - x_{nj} p_{ni}),$$

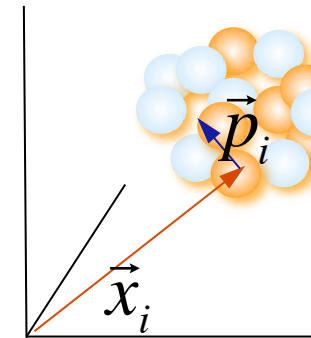
$$K_{ij} = \sum_n p_{ni} p_{nj},$$

SU(3)
in a HO shell
(Elliott, 1958)

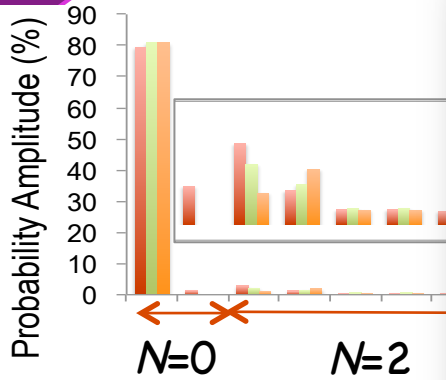
Rowe, Rosensteel, Draayer, Hecht, Suzuki, Escher, Bahri, ...



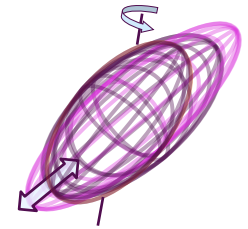
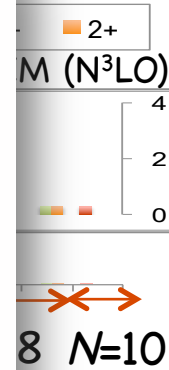
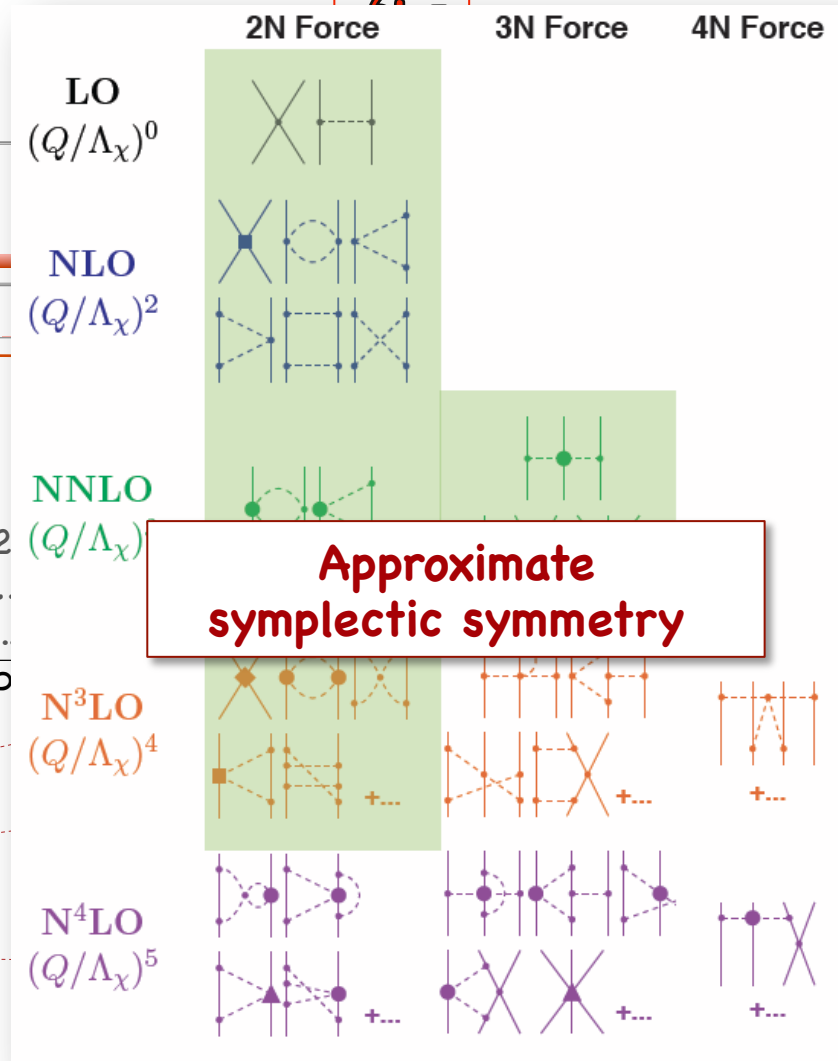
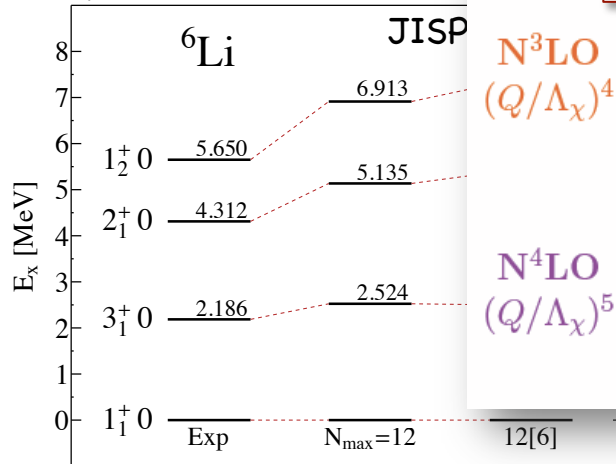
Nucleus with A nucleons



Novel Approximate Symmetry



${}^6\text{Li}$, $N_{\text{max}}=12$
 # $J=1,2,3$ states.....2
 # $\text{Sp}(3,\text{R})$ irreps.....
 # $\text{Sp}(3,\text{R})$ with $P>0.2\%$

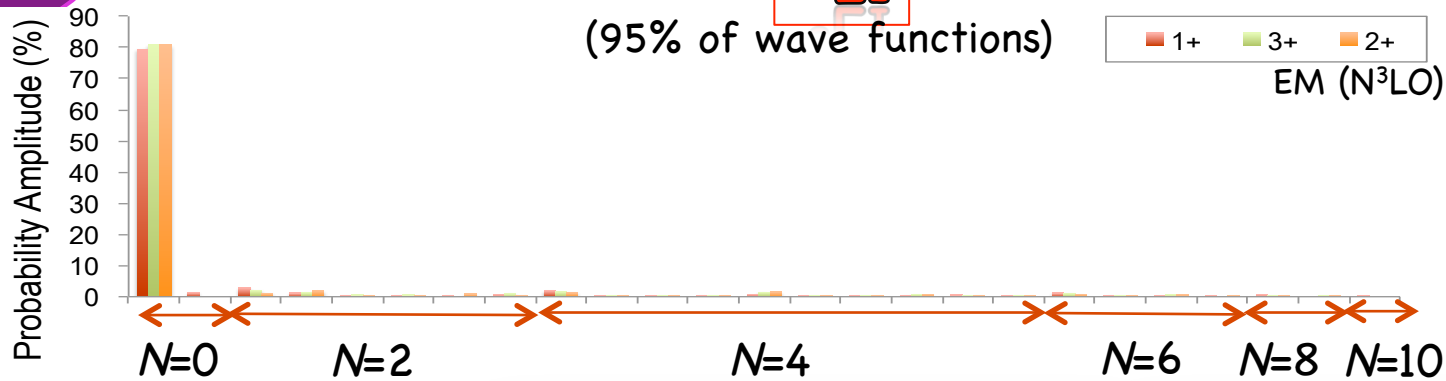
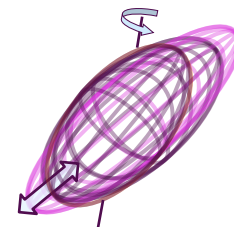


SA-NCSM with Sp(3,R) basis

${}^6\text{Li}$

(95% of wave functions)

EM ($N^3\text{LO}$)

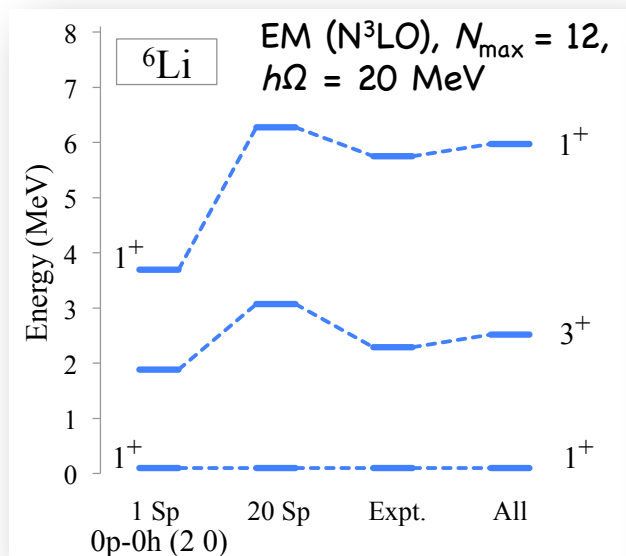


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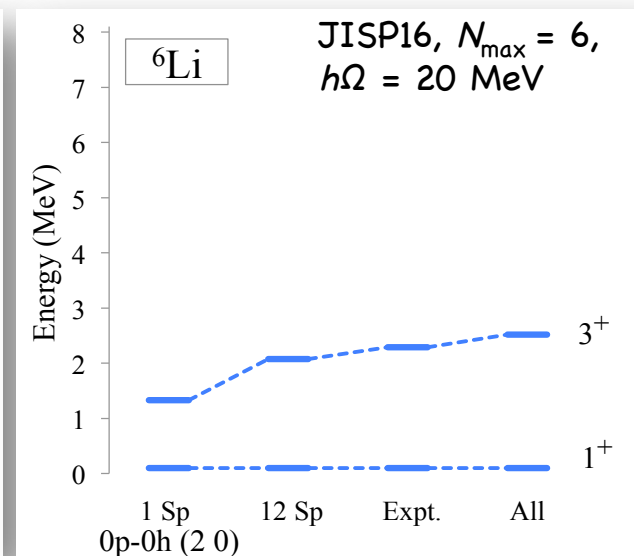
Sp(3,R) irreps.....528

Sp(3,R) with $P > 0.2\%$**25**



Single Sp(3,R) irrep

20 Sp(3,R) irreps

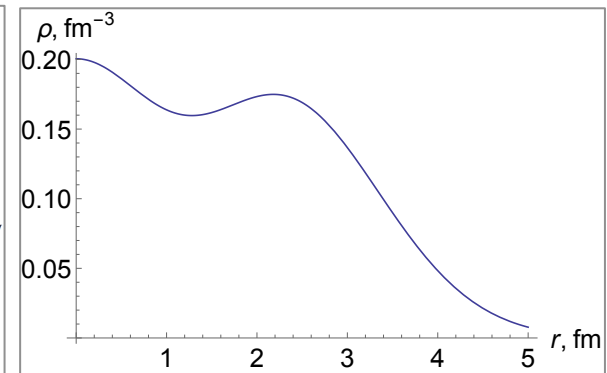
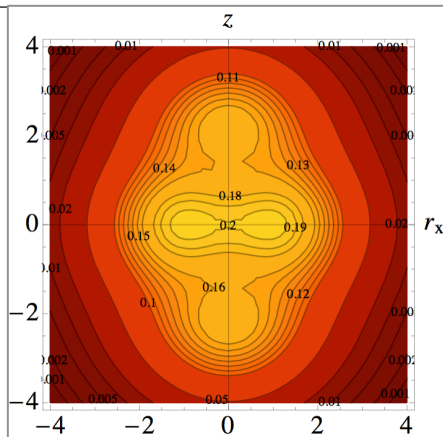
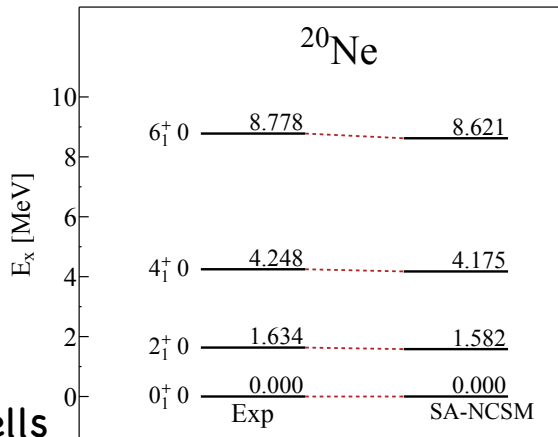


Single Sp(3,R) irrep

12 Sp(3,R) irreps

Collectivity features

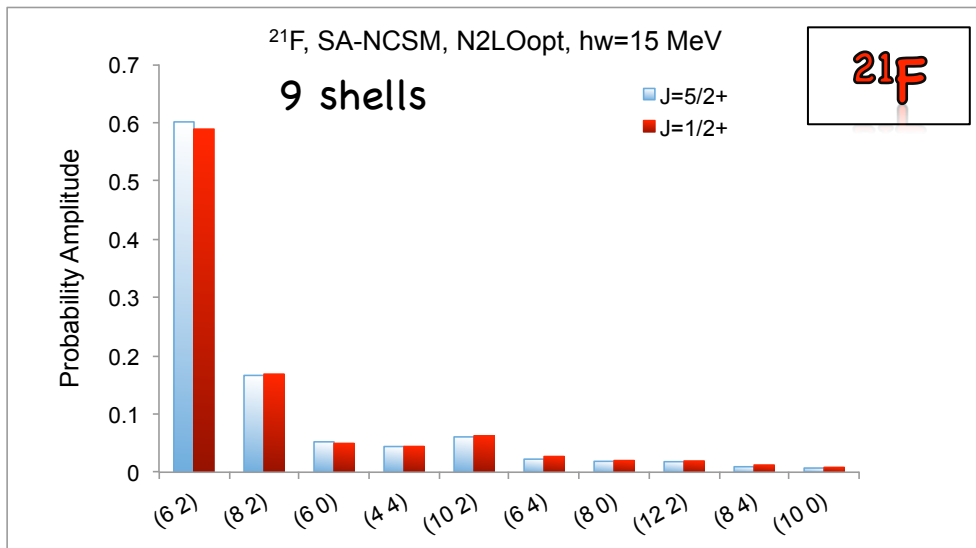
20Ne



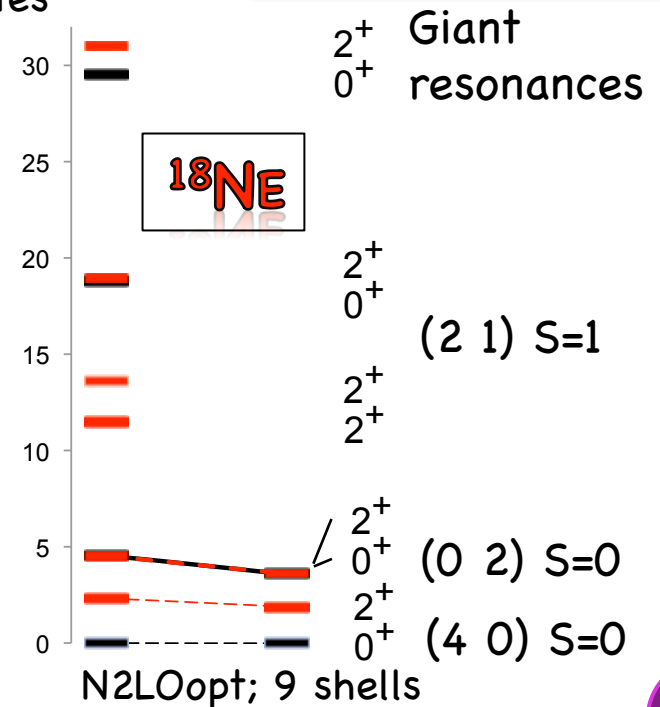
13 shells

SA-NCSM (selected model space): 50 million SU(3) states
 Complete model space: 1000 billion states

Ne & Mg isotopes

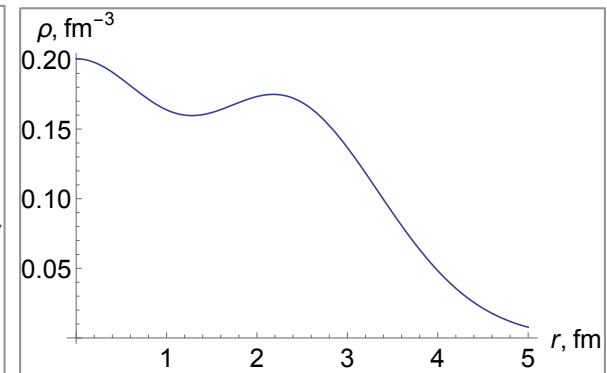
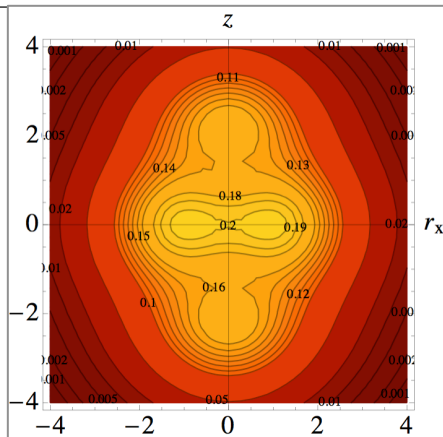
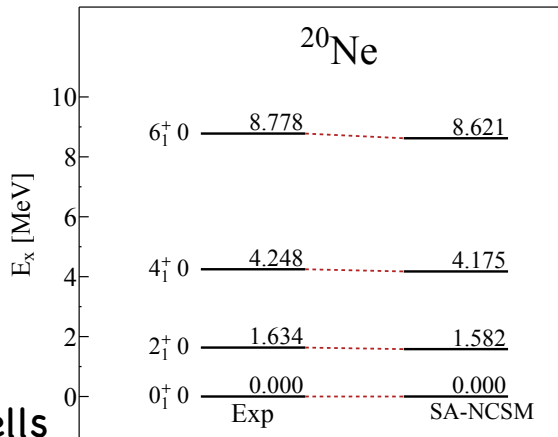


Grigor Sargsyan, PhD student, LSU



Collectivity features

20Ne



13 shells

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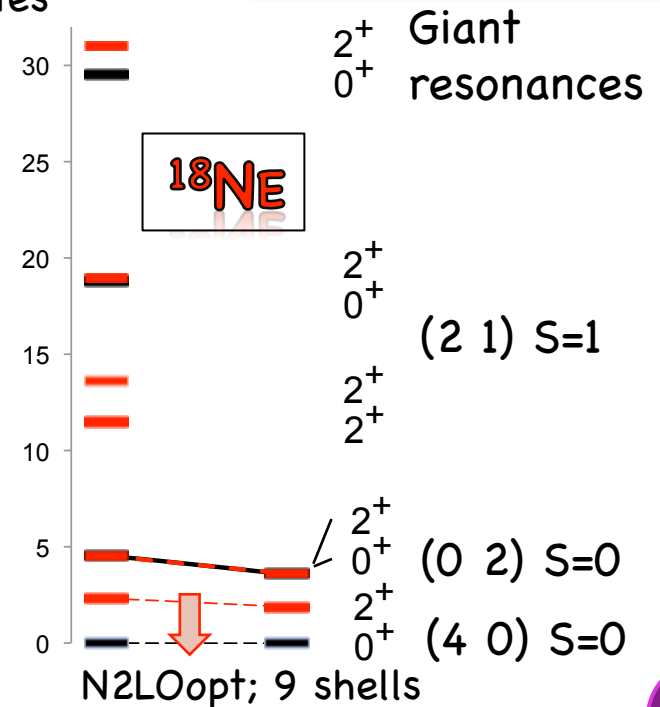
Ne & Mg isotopes

^{18}Ne , $B(E2: 2^+ \rightarrow 0^+)$

 Experiment..... 17.7(18) W.u.

9 shells 1.13 W.u.

33 shells 13.0(7) W.u.
 (no effective charges)



Grigor Sargsyan, PhD student, LSU

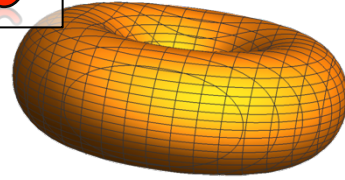
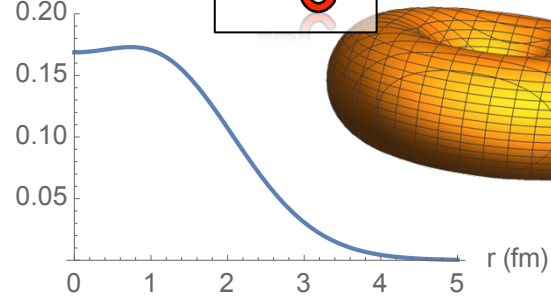


Carbon isotopes

$\rho(r)$ (fm⁻³)

¹²C

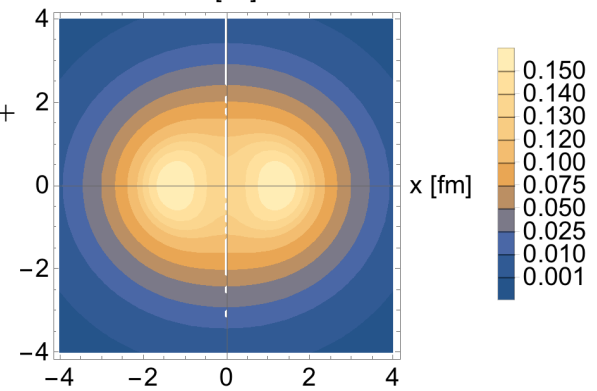
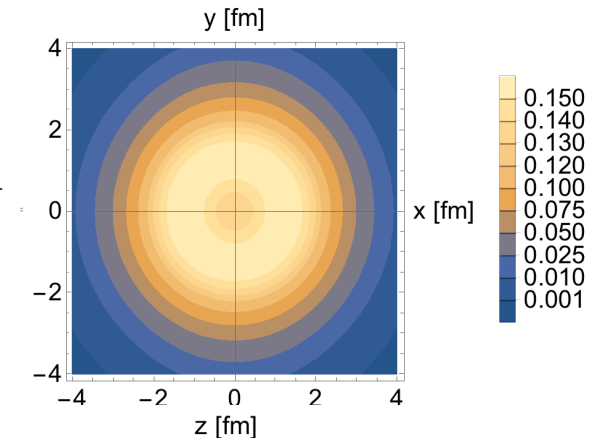
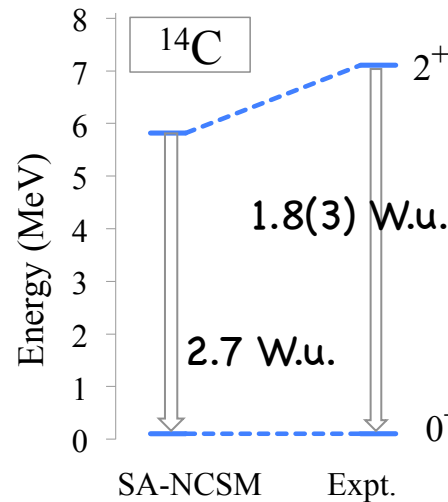
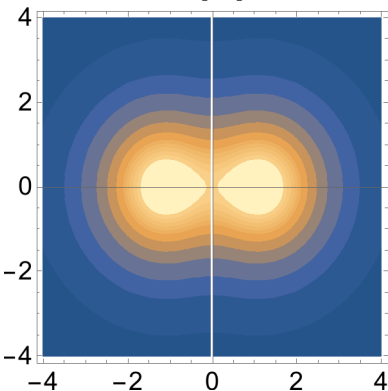
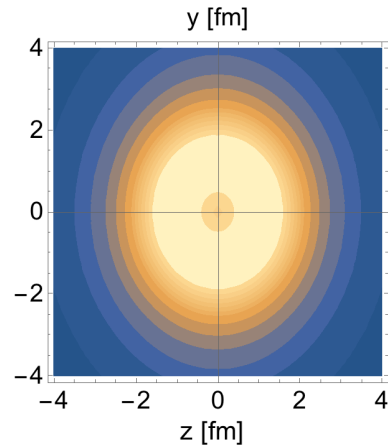
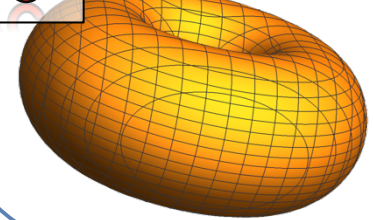
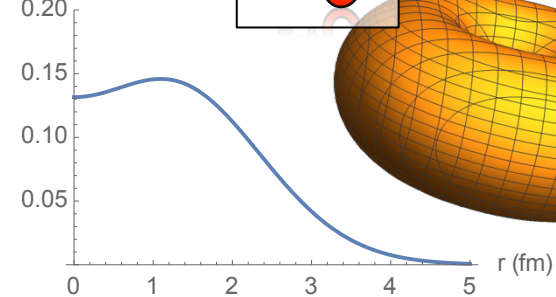
JISP16, hw = 18 MeV



$\rho(r)$ (fm⁻³)

¹⁴C

N2LOopt, hw = 15 MeV



Grigor Sargsyan, PhD student, LSU

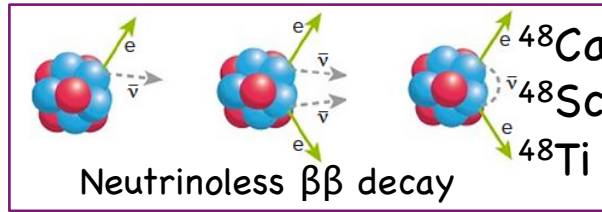
Structure of Ca-48 and Ti-48

^{48}Ca

8 shells, N2LOopt
 0^+

SA-NCSM (selected):966,152
 Complete model space:3,162,511,819

2^+
 SA-NCSM (selected):3,055,554
 Complete model space: ...14,522,234,982



^{48}Ti

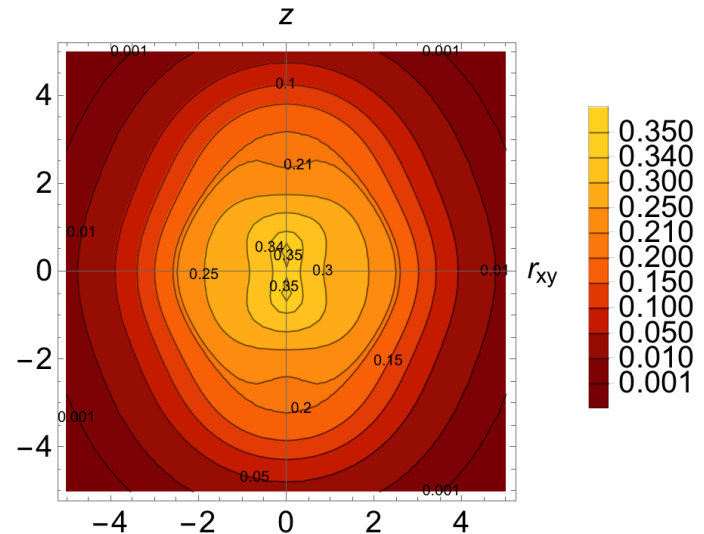
8 shells, N2LOopt
 0^+

SA-NCSM (selected):602,493
 Complete model space:24,694,678,414

2^+
 SA-NCSM (selected):1,178,834
 Complete model space: ...113,920,316,658

^{48}Ti , $Q(2^+)$ [$e \text{ fm}^2$]

 Experiment..... -17.7
 8 shells -19.3
 (no effective charges)



Grigor Sargsyan, PhD student, LSU



Treating NN and $3N$ in SA-NCSM

3-body interaction

$$H(3) = \binom{A}{3} \mathcal{H}^{(3)}(0) + \binom{A-1}{2} \mathcal{H}^{(3)}(1) + (A-2) \mathcal{H}^{(3)}(2) + \mathcal{H}^{(3)}(3).$$

Average energy per triplet

For given isospin: $H_{mon} \equiv$

$$\begin{aligned} & \frac{W_{2,0} + 3W_{2,1}}{4} \binom{\hat{n}}{2} + \frac{W_{2,1} - W_{2,0}}{2} (\mathbf{T}^2 - \frac{3}{4}\hat{n}) & NN \\ & + \frac{W_{3,\frac{1}{2}} + W_{3,\frac{3}{2}}}{2} \binom{\hat{n}}{3} + \frac{W_{3,\frac{3}{2}} - W_{3,\frac{1}{2}}}{3} (\hat{n} - 2)(\mathbf{T}^2 - \frac{3}{4}\hat{n}). & NNN \end{aligned}$$

p shell

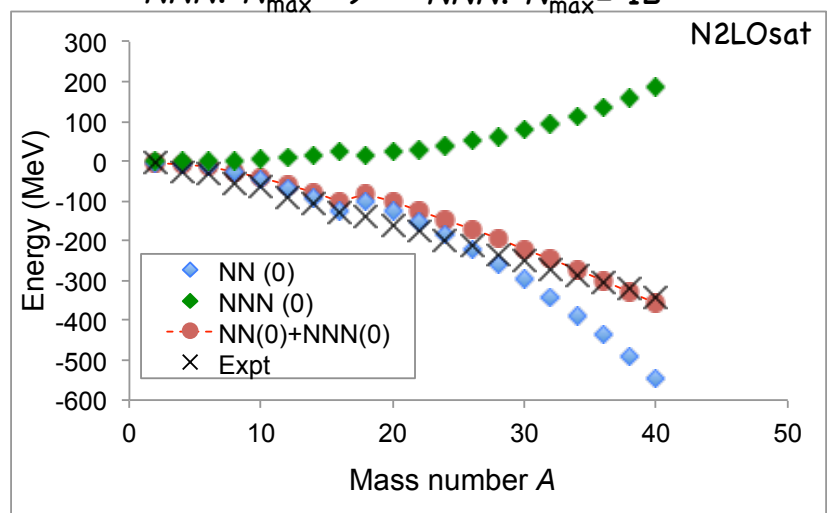
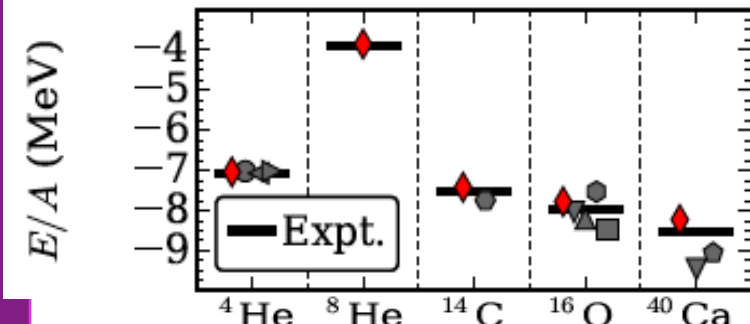
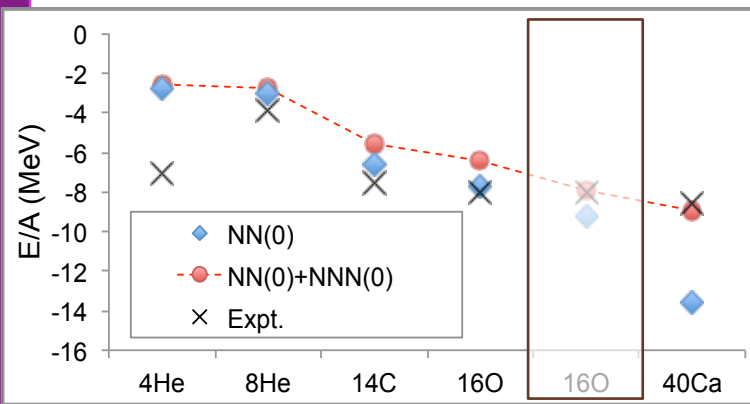
$NN: N_{\max} = 6$

$NNN: N_{\max} = 9$

sd shell

$NN: N_{\max} = 8$

$NNN: N_{\max} = 12$



Ekström et al., Phys. Rev. C 91, 051301(R) (2015)



Treating NN and $3N$ in SA-NCSM

3-body interaction

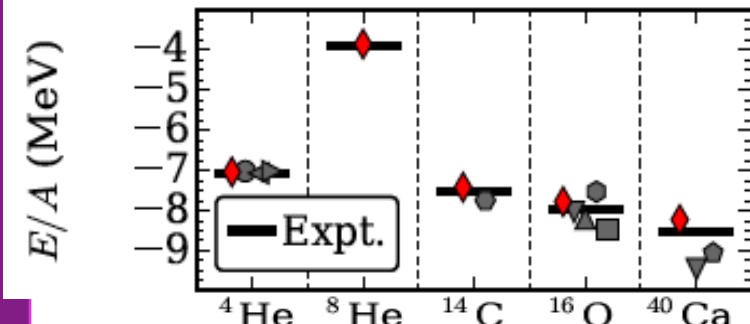
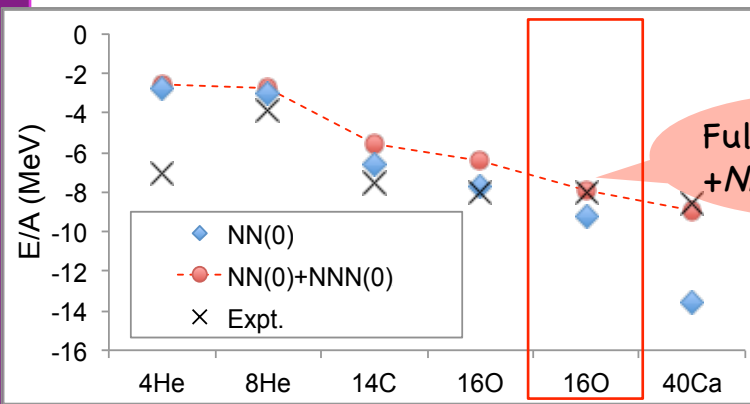
$$H(3) = \binom{A}{3} \mathcal{H}^{(3)}(0) + \binom{A-1}{2} \mathcal{H}^{(3)}(1) + (A-2) \mathcal{H}^{(3)}(2) + \mathcal{H}^{(3)}(3).$$

Average energy per triplet

For given isospin: $H_{mon} \equiv$

$$\frac{W_{2,0} + 3W_{2,1}}{4} \binom{\hat{n}}{2} + \frac{W_{2,1} - W_{2,0}}{2} (\mathbf{T}^2 - \frac{3}{4}\hat{n}) \quad NN$$

$$+ \frac{W_{3,1/2} + W_{3,3/2}}{2} \binom{\hat{n}}{3} + \frac{W_{3,3/2} - W_{3,1/2}}{3} (\hat{n} - 2)(\mathbf{T}^2 - \frac{3}{4}\hat{n}). \quad NNN$$



Using spectral distribution theory (French, Hecht, Draayer,...)

p shell

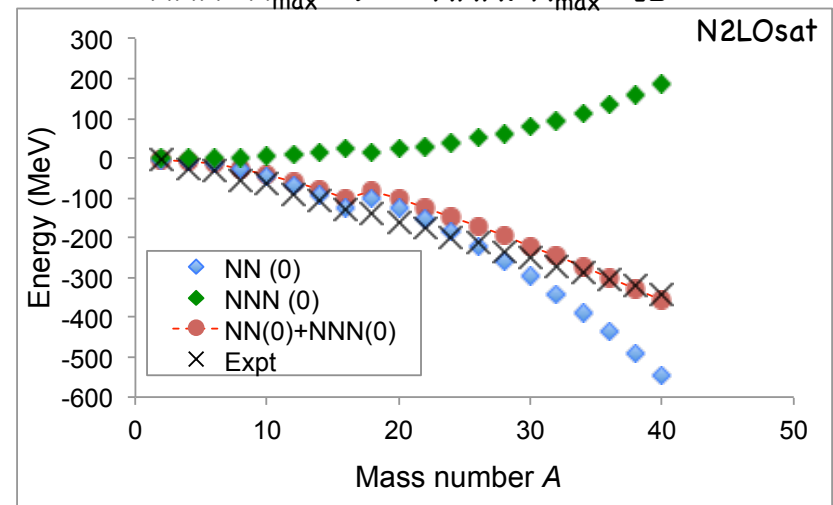
NN: $N_{max} = 6$

NNN: $N_{max} = 9$

sd shell

NN: $N_{max} = 8$

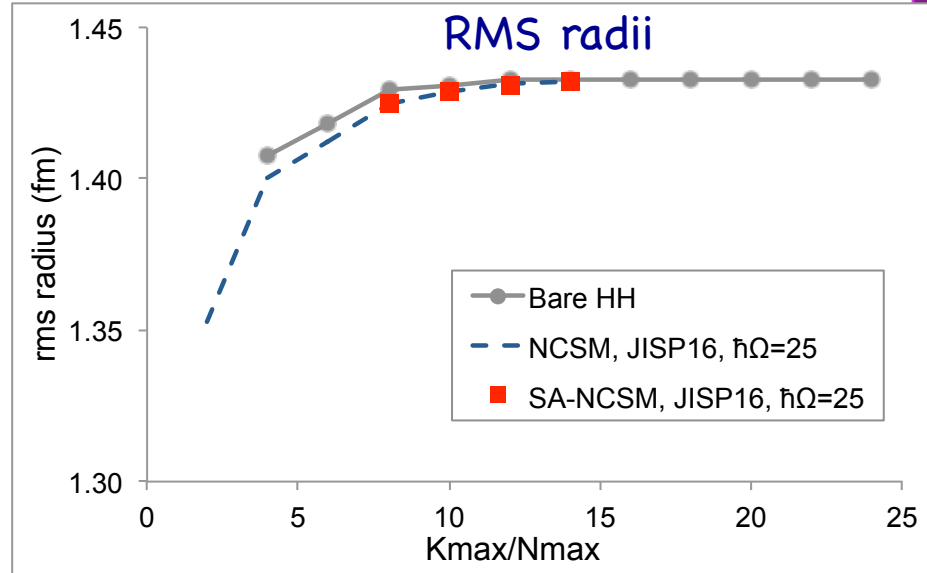
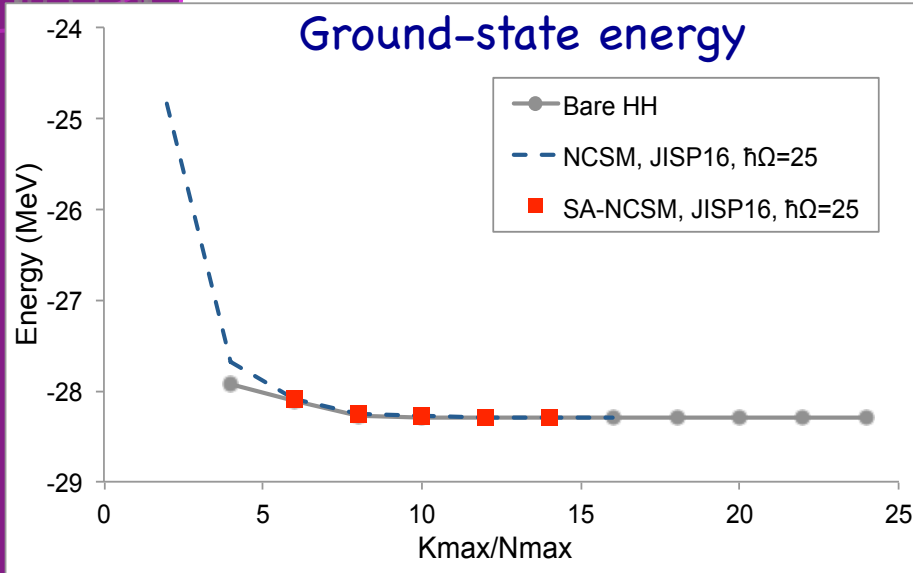
NNN: $N_{max} = 12$



Ekström et al., Phys. Rev. C 91, 051301(R) (2015)

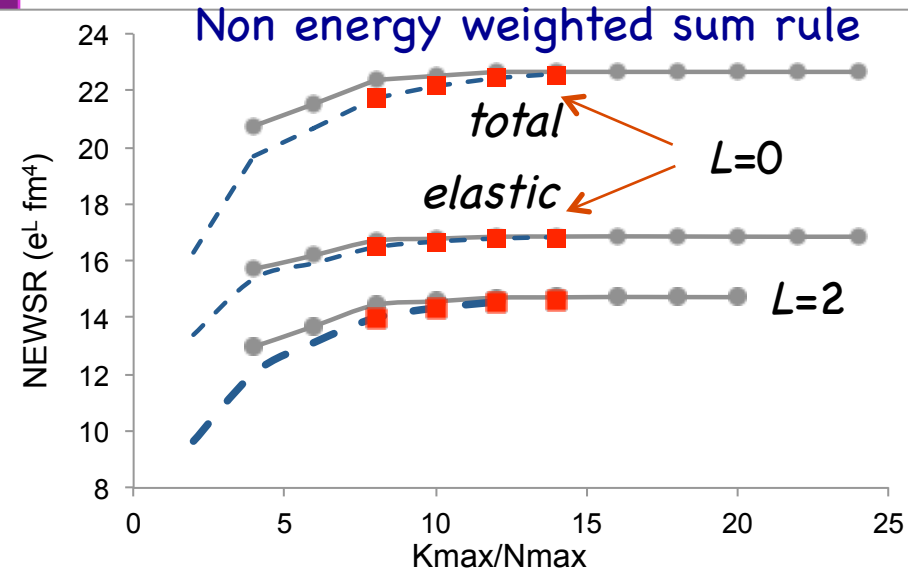
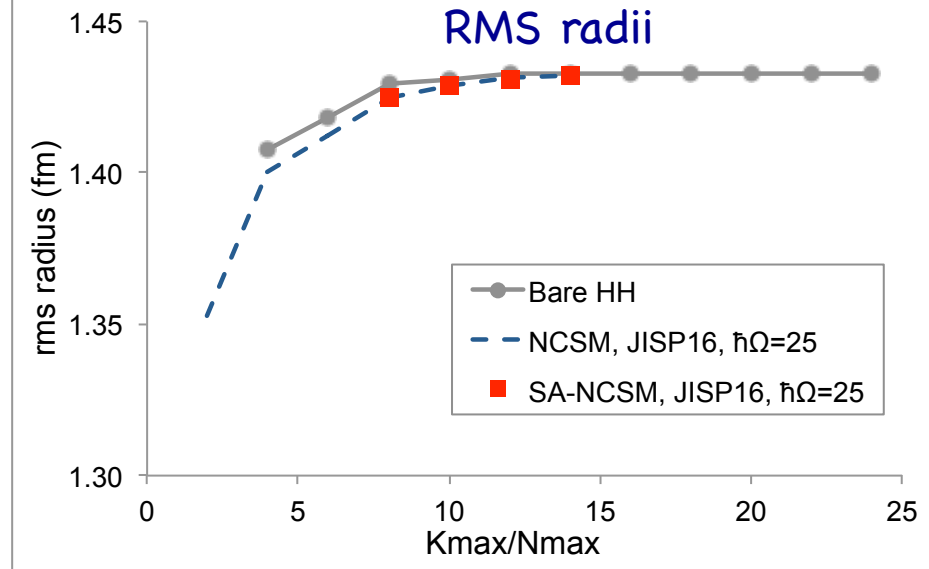
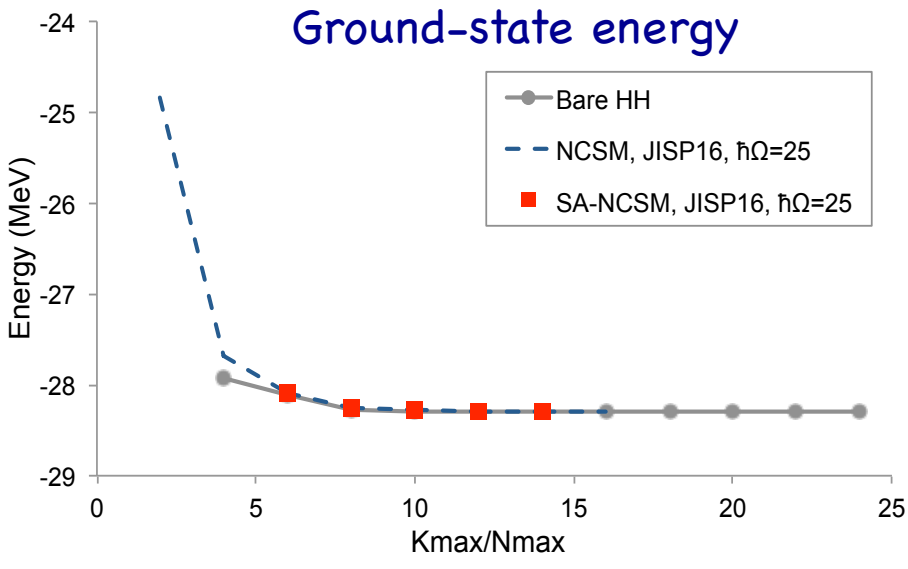


Merging SA-NCSM and LIT: benchmark for ${}^4\text{He}$



Baker et al., in preparation (2018)

Sum rules for ^4He : HH and SA-NCSM benchmark



Response function

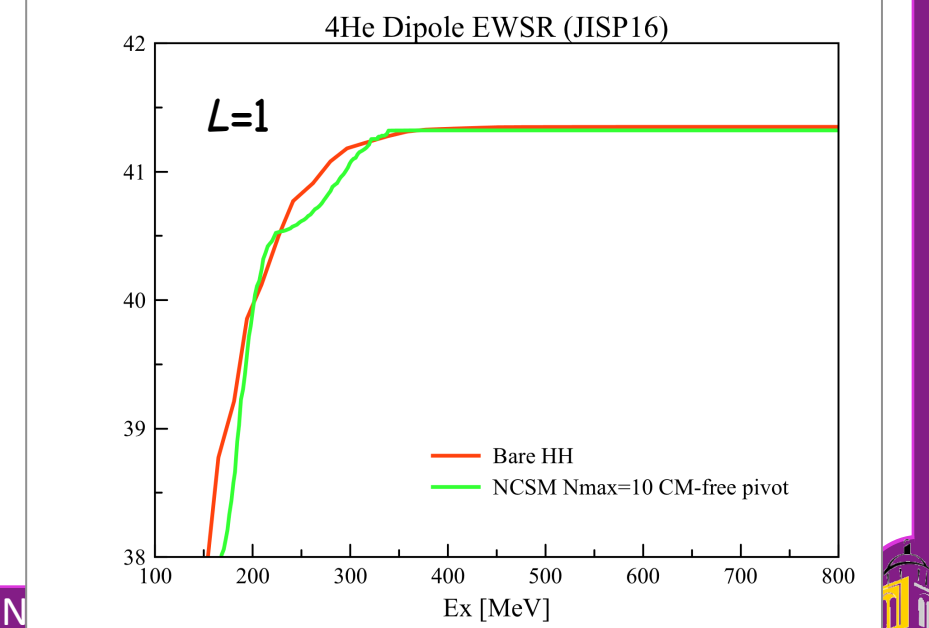
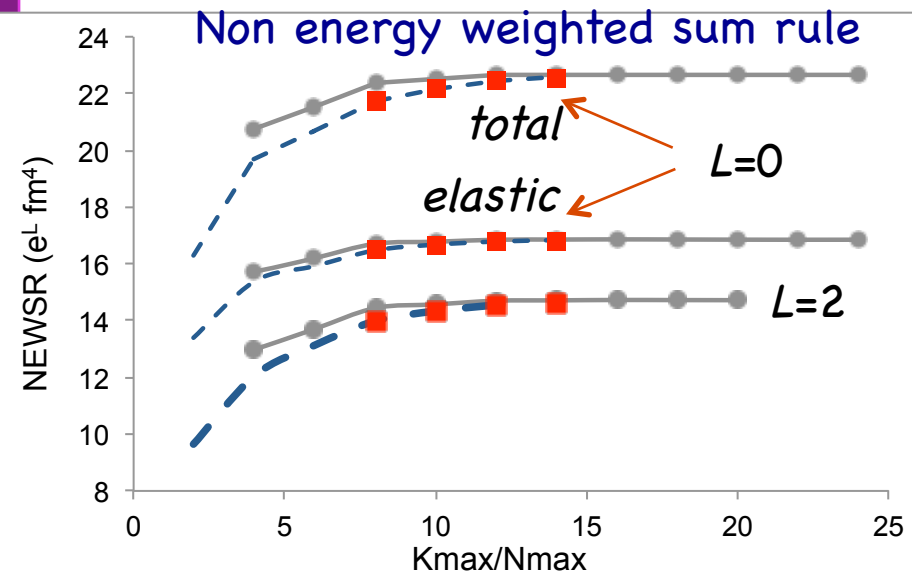
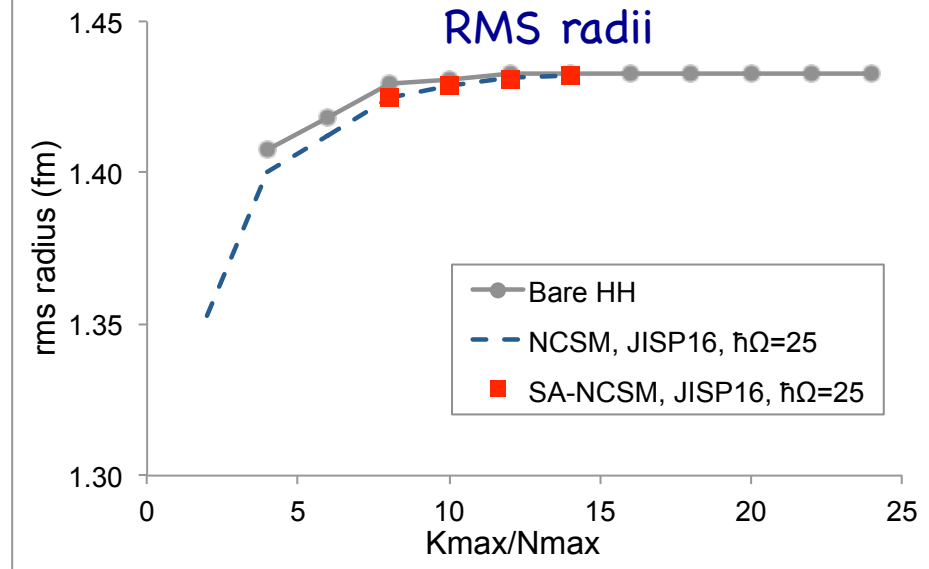
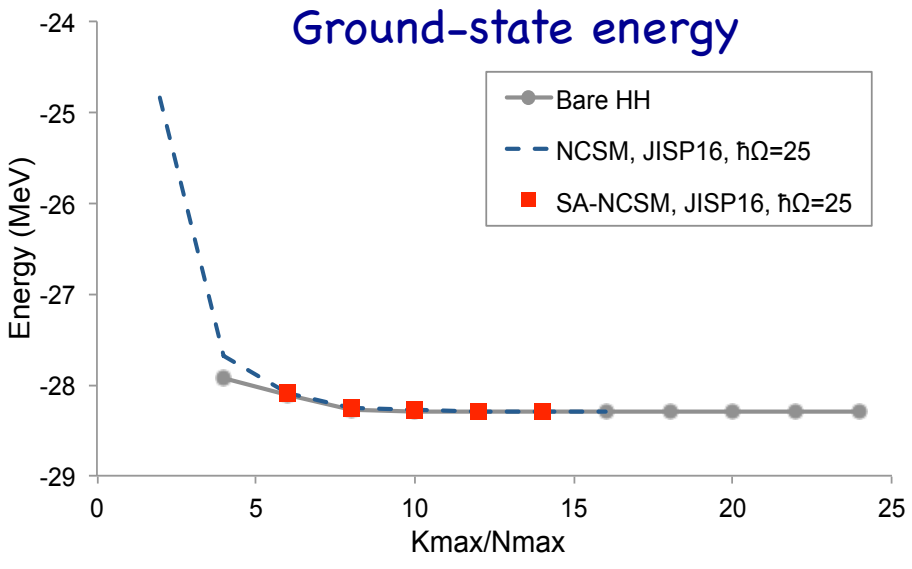
$$R(\omega) = \sum_f |\langle \psi_f | \Theta | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$

Sum rules

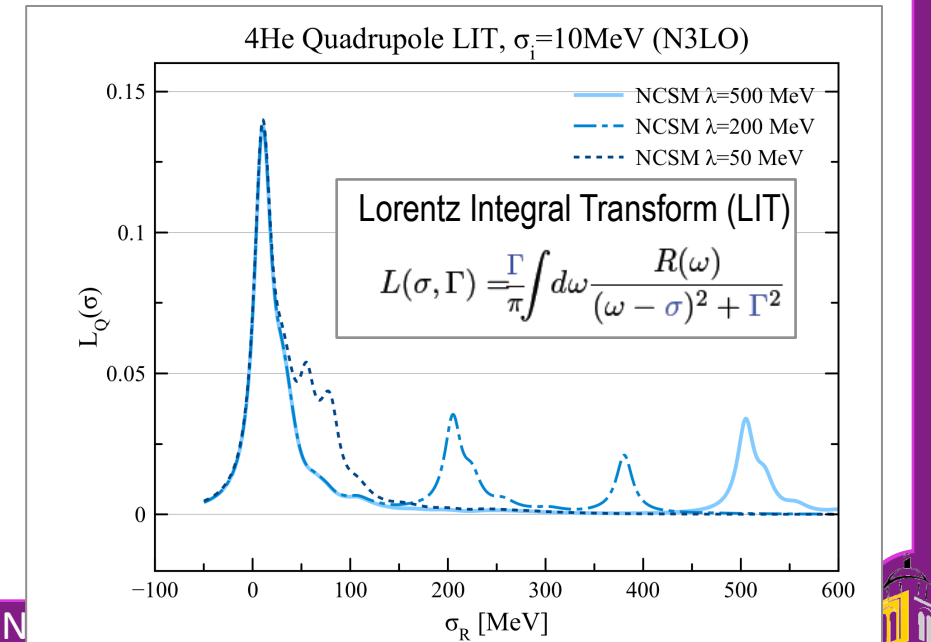
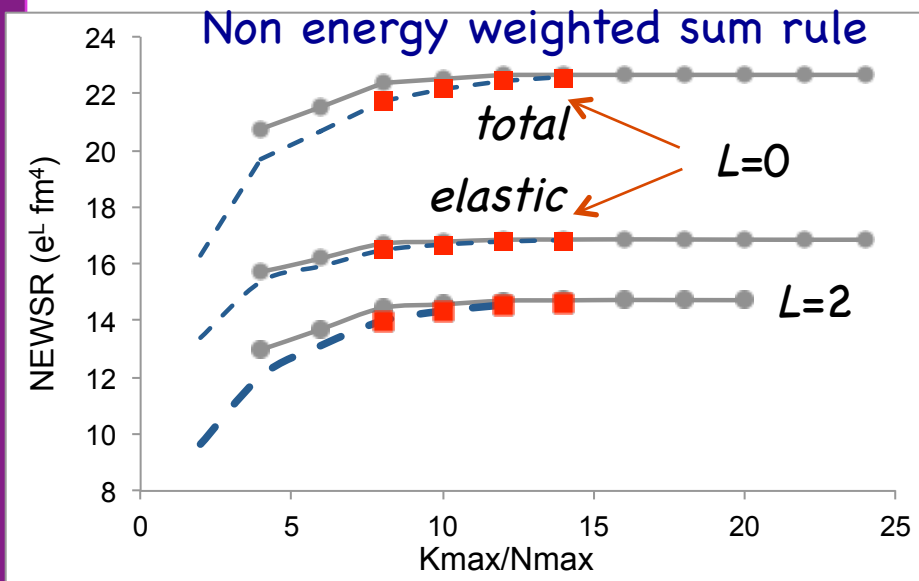
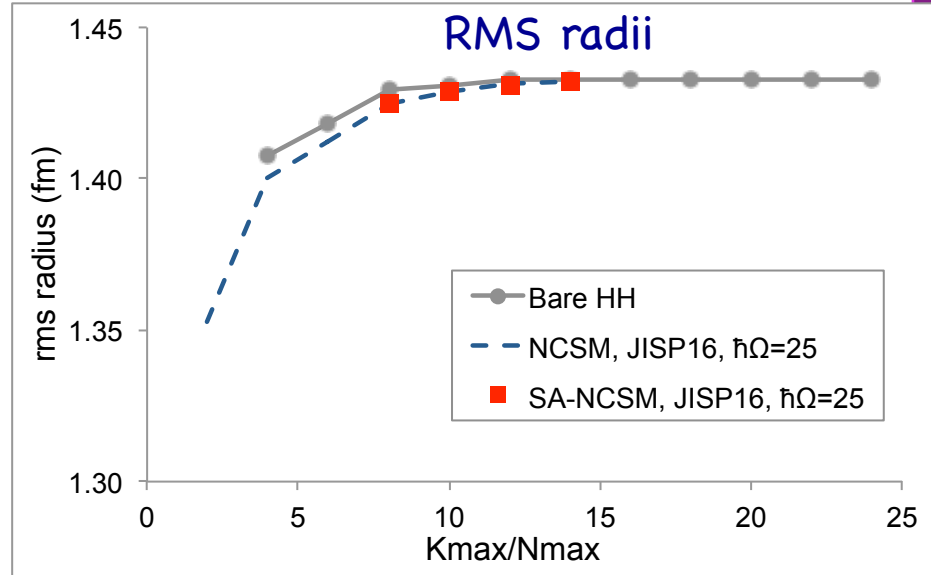
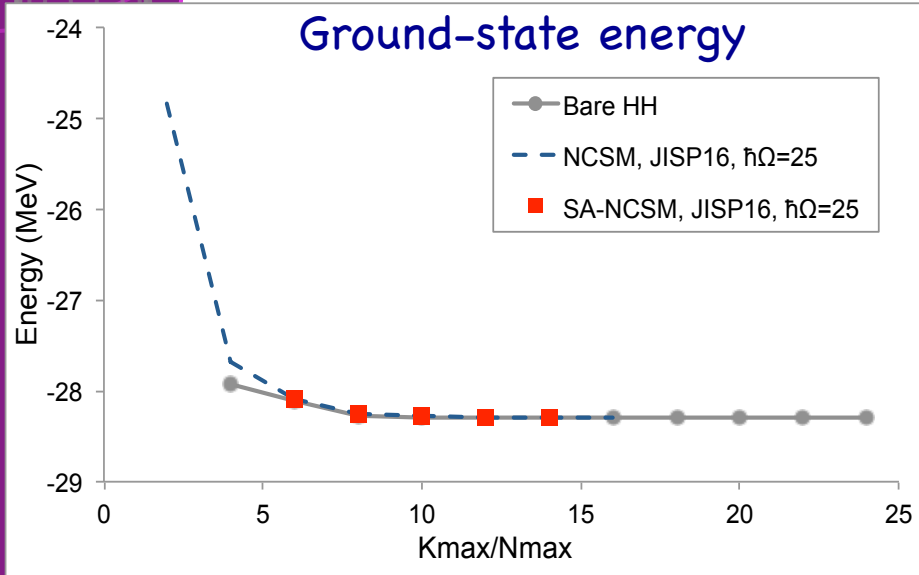
$$m_n = \int_0^\infty d\omega \omega^n R(\omega)$$

Baker et al., in preparation (2018)

Sum rules for ^4He : HH and SA-NCSM benchmark



Sum rules for ^4He : HH and SA-NCSM benchmark



Efficacy of SA-NCSM

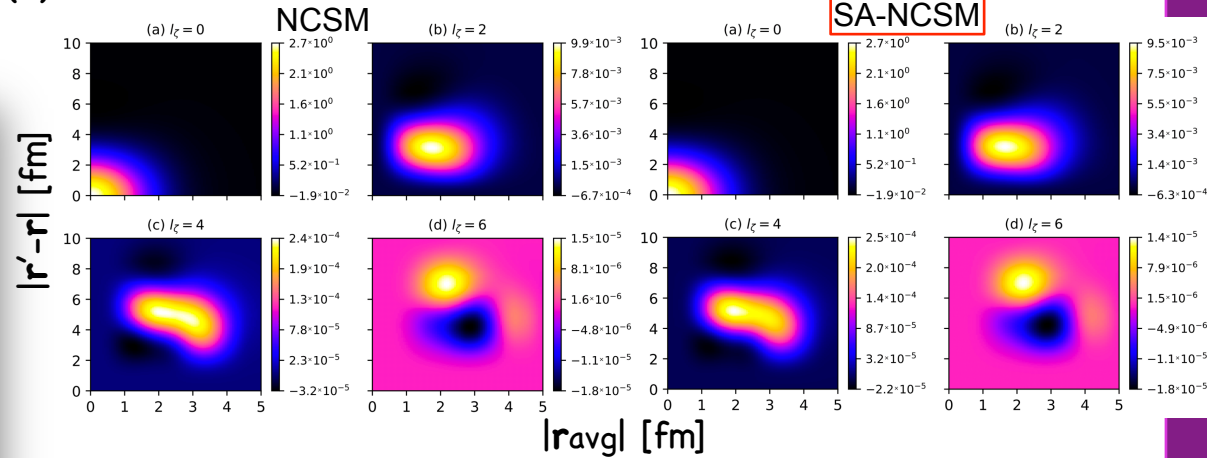
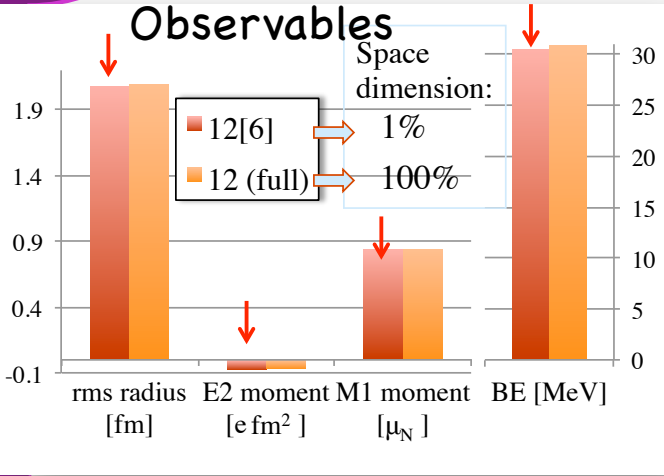
⁶Li

SA-NCSM, SU(3)

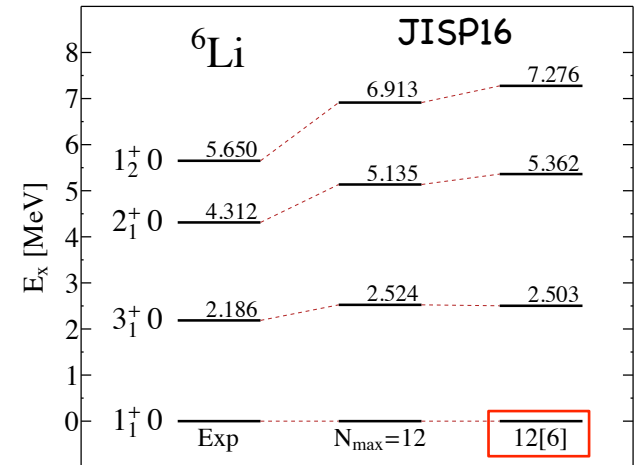
Non-local densities

SA-NCSM

N2LOopt



Burrows, Elster, Popa, Launey, Nogga, Maris, Phys. Rev. C 97 (2018) 024325



Launey et al., Prog. Part. Nucl. Phys. 89 (2016) 101;
 Dytrych et al., Phys. Rev. C 91 (2015) 024326



Efficacy of SA-NCSM

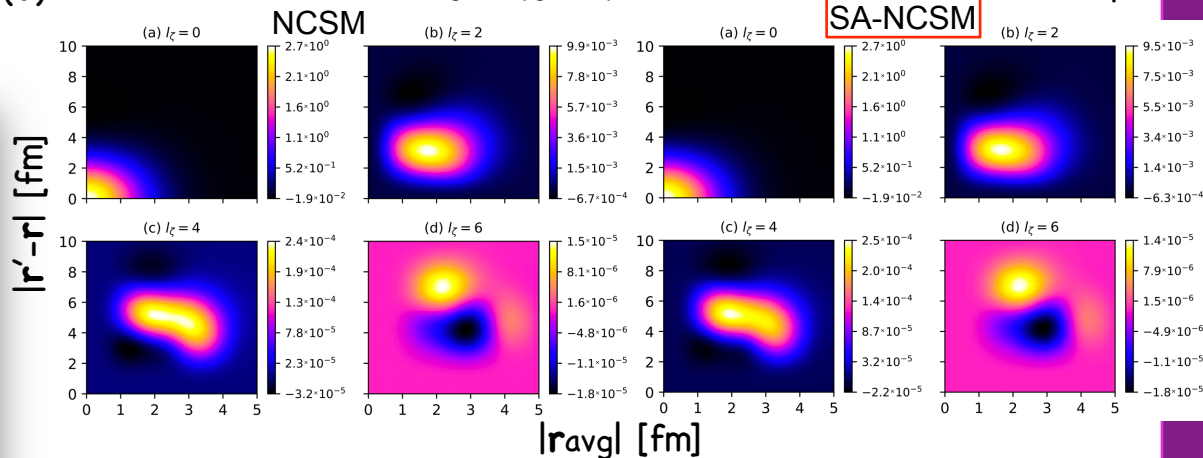
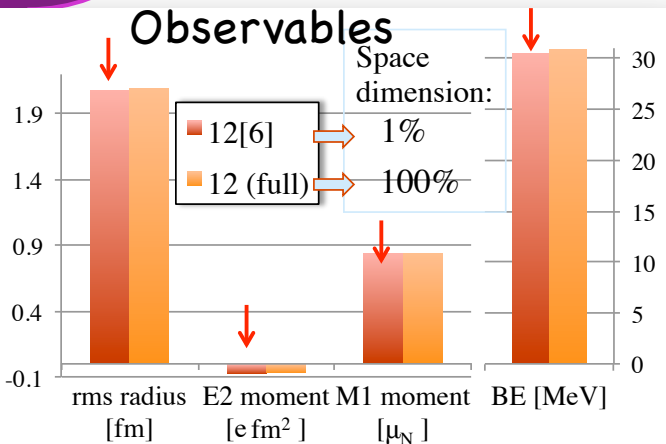
⁶Li

SA-NCSM, SU(3)

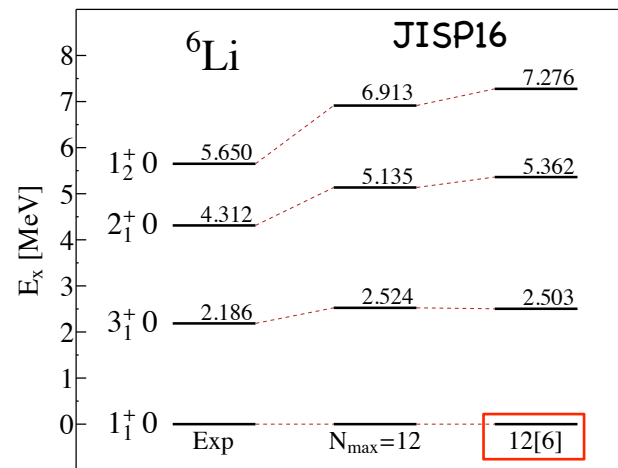
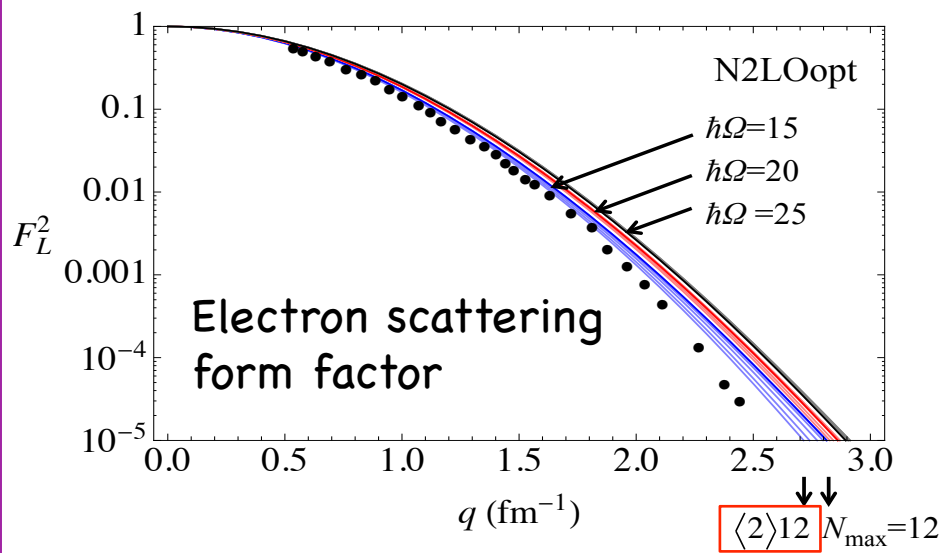
Non-local densities

N2LOopt

SA-NCSM



Burrows, Elster, Popa, Launey, Nogga, Maris, Phys. Rev. C 97 (2018) 024325



Launey et al., Prog. Part. Nucl. Phys. 89 (2016) 101; Dytrych et al., Phys. Rev. C 91 (2015) 024326



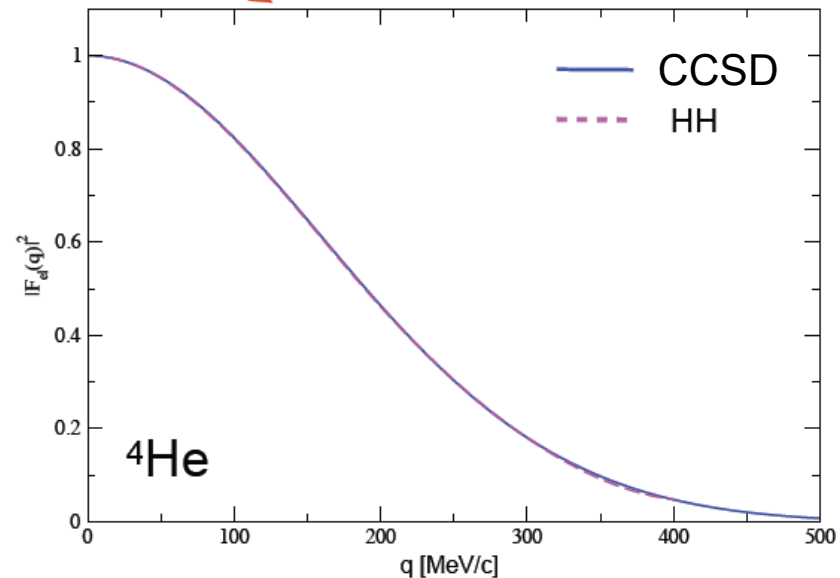
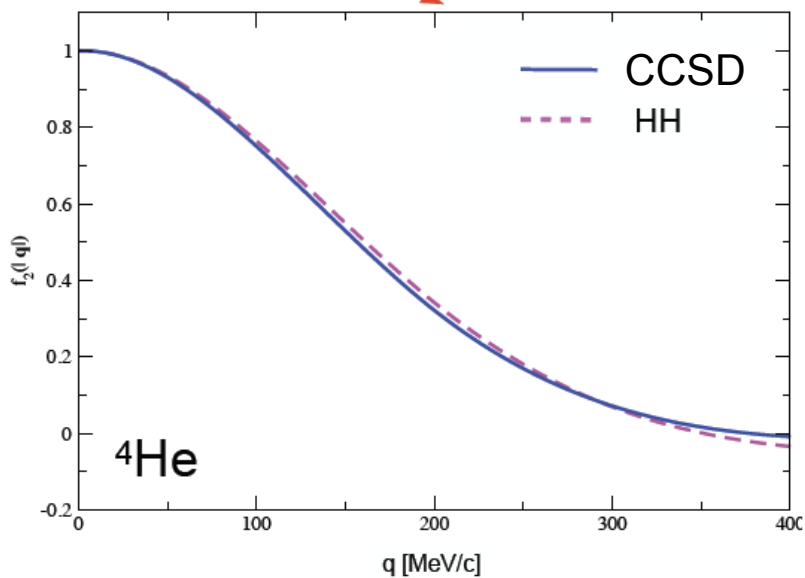
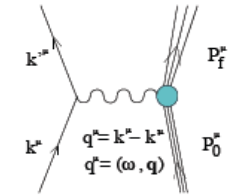
Coulomb sum rule

Total strength of inelastic longitudinal response function

$$\text{CSR}(q) = \int d\omega R_L^{in}(\omega, \mathbf{q}) \quad R_L^{in}(\omega, \mathbf{q}) = \sum_f |\langle f | \rho(\mathbf{q}) | 0 \rangle|^2 \delta(\omega - \mathbf{E}_f + \mathbf{E}_0)$$

$$\text{CSR}(q) = Z + \langle 0 | \sum_{i \neq j} e^{i\mathbf{q} \cdot (\mathbf{r}_i - \mathbf{r}_j)} | 0 \rangle - |F_{el}(\mathbf{q})|^2 Z^2$$

||
Z(Z-1)f₂(|q|)



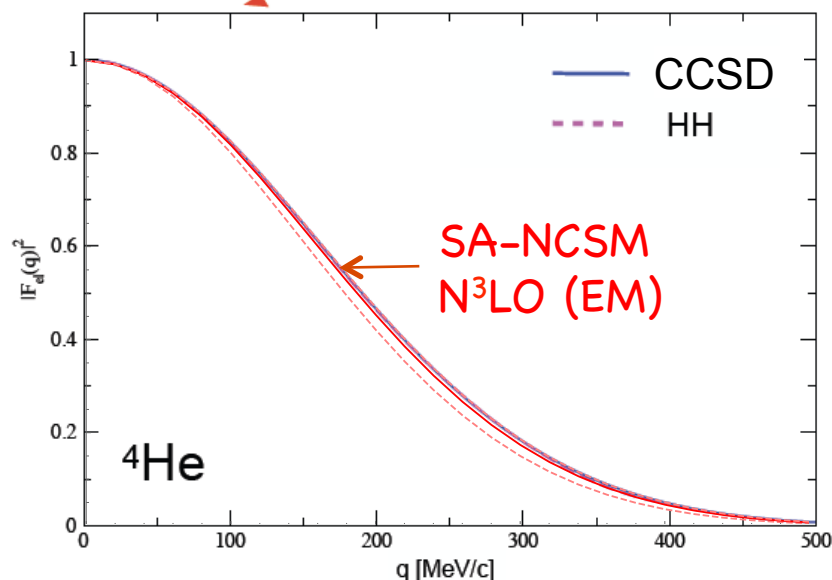
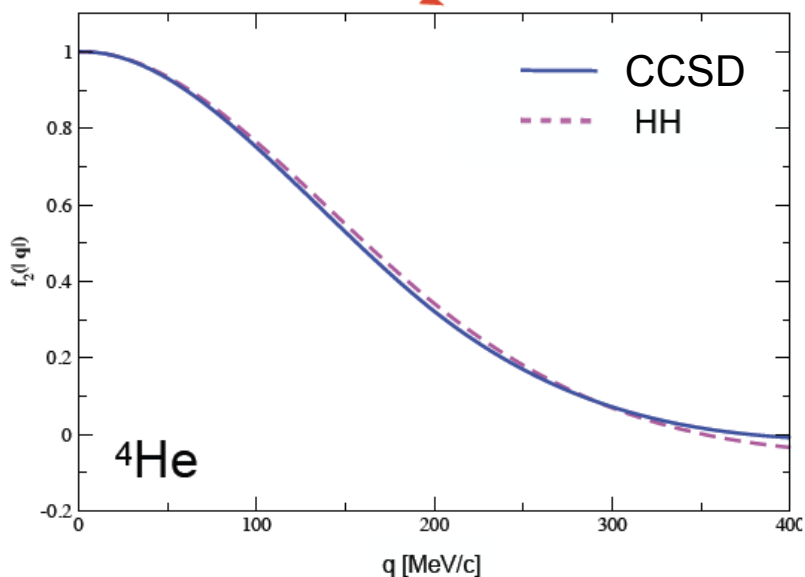
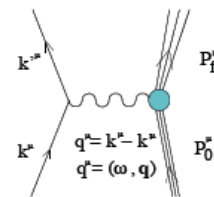
Coulomb sum rule

Total strength of inelastic longitudinal response function

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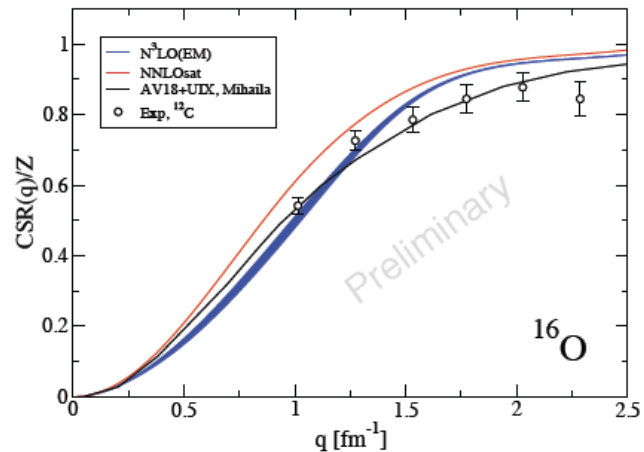
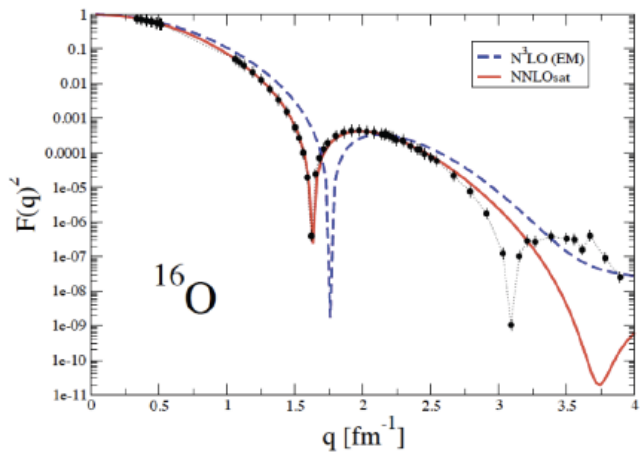
$$\text{CSR}(q) = Z + \langle 0 | \sum_{i \neq j} e^{i\mathbf{q} \cdot (\mathbf{r}_i - \mathbf{r}_j)} | 0 \rangle - |F_{el}(\mathbf{q})|^2 Z^2$$

||
Z(Z-1)f₂(|q|)



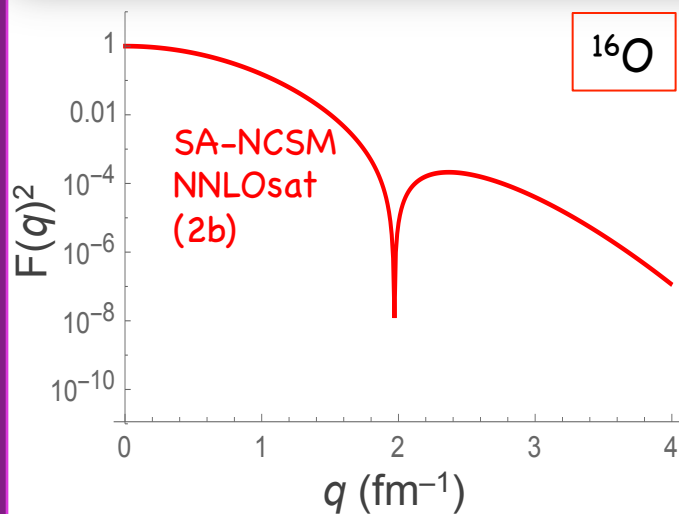
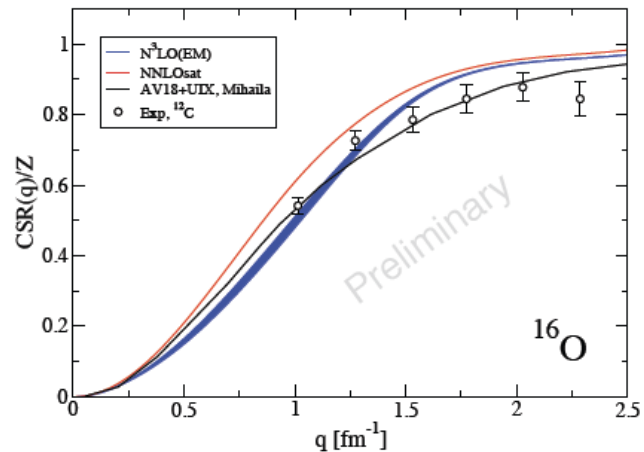
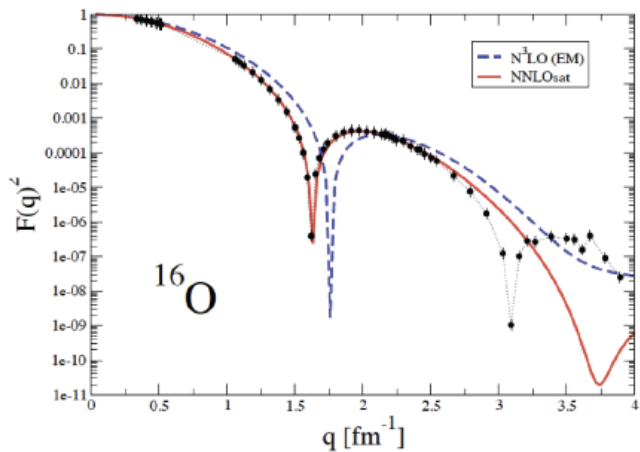
Coulomb sum rule

S. Bacca et al., in preparation (2018)



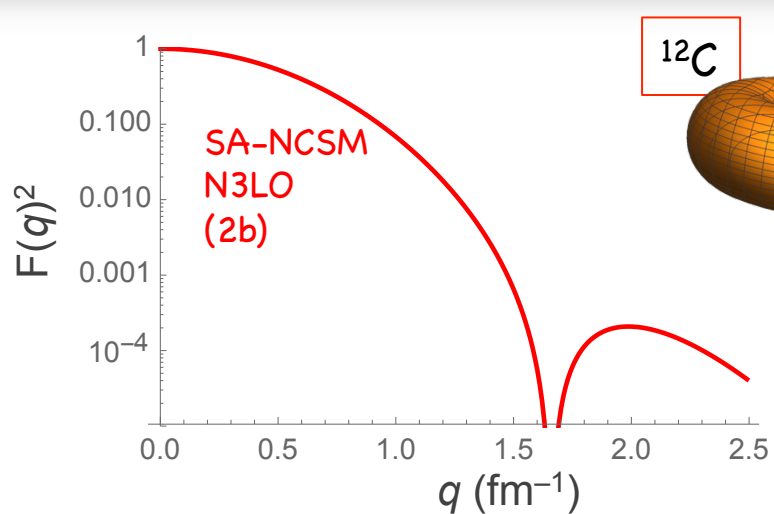
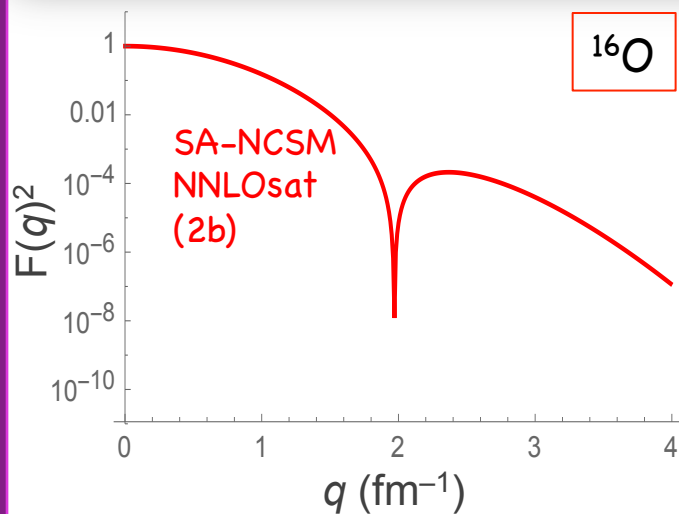
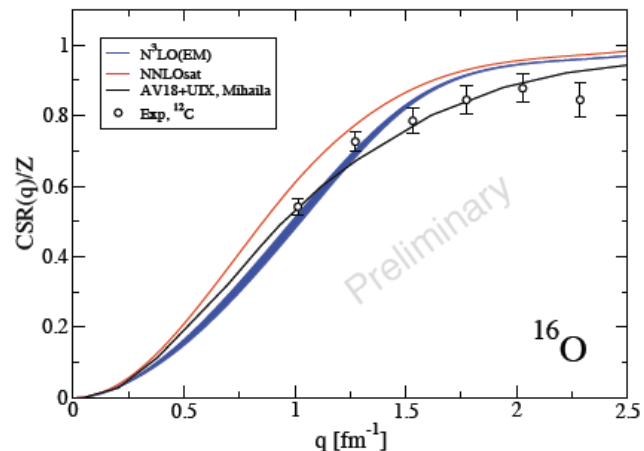
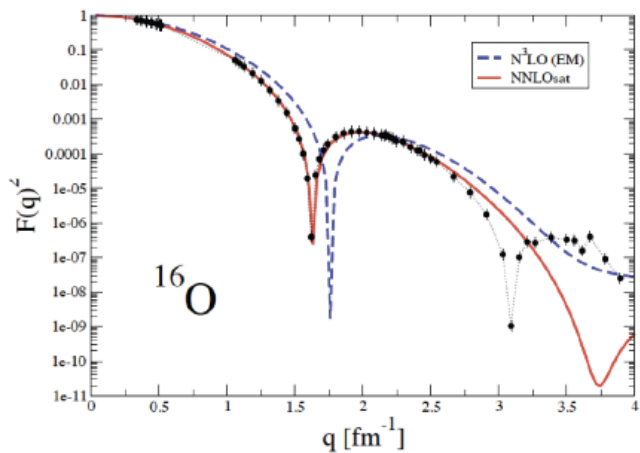
Coulomb sum rule

S. Bacca et al., in preparation (2018)

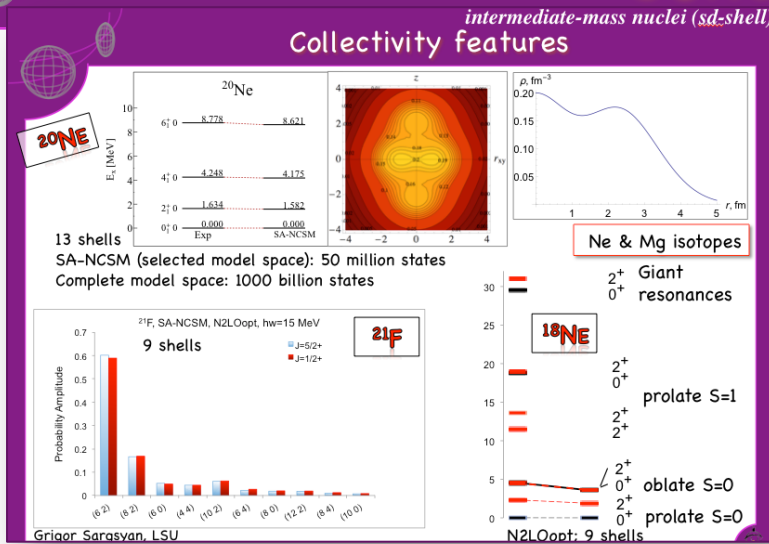


Coulomb sum rule

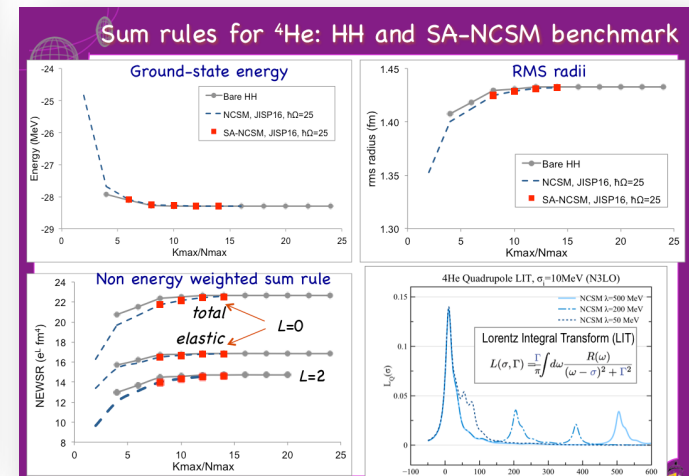
S. Bacca et al., in preparation (2018)



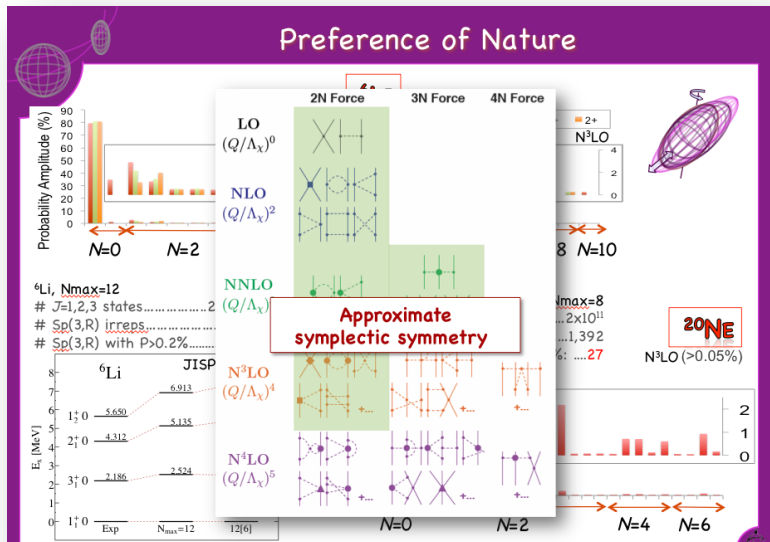
Conclusions



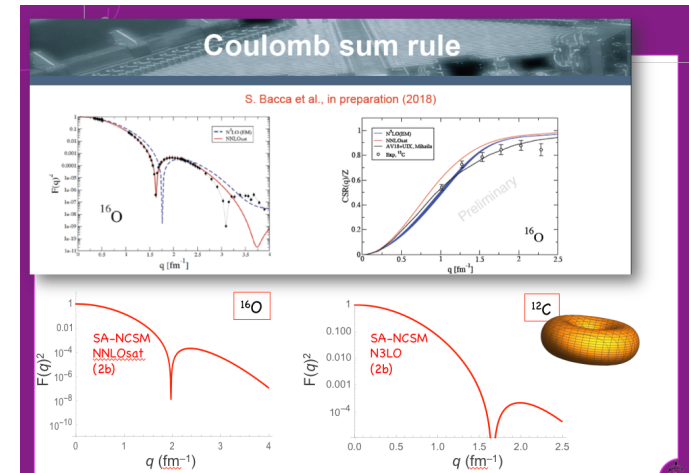
Collectivity in nuclei from first principles



SA-NCSM+LIT (with S. Bacca): sum rules and response functions



Simple physics: "shape" + vibrations + rotations



Fundamental Physics with Electroweak Probes of Light Nuclei
INT-18-2a, July 2018

KD Launey
Louisiana State University

