Elementary Amplitudes

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From Nucleons to Nuclei | INT
June 25, 2018

Neutrino Physics $\subset$ Particle Physics

- Neutrinos have mass, change flavor (“oscillate”), and may violate CP.
- Taxpayers are eager to find out about CP and to test the three-family paradigm:
  - $\$10^9$ for LBNF/DUNE;
  - numerous other experiments.

- Oscillation experiments make a(n) (anti)neutrino beam and aim it at nuclei such as ^{12}C, ^{16}O, ^{40}Ar.
- A nucleus can be thought of as a collection of weakly-interacting nucleons.
- “A riddle, wrapped in a mystery, inside an enigma”—
  - a flavor change, wrapped in a nucleon, inside a nucleus.

Pontecorvo–Maki–Nakagawa–Sakata Matrix

- The PMNS matrix is to neutrino (lepton-flavor) physics as the Cabibbo-Kobayashi-Maskawa (CKM) matrix is to quark-flavor physics:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = [U]_{\ell j}$$

  where $\ell \in \{e, \mu, \tau\}$labels flavor and $i \in \{1,2,3\}$ labels mass eigenstate.

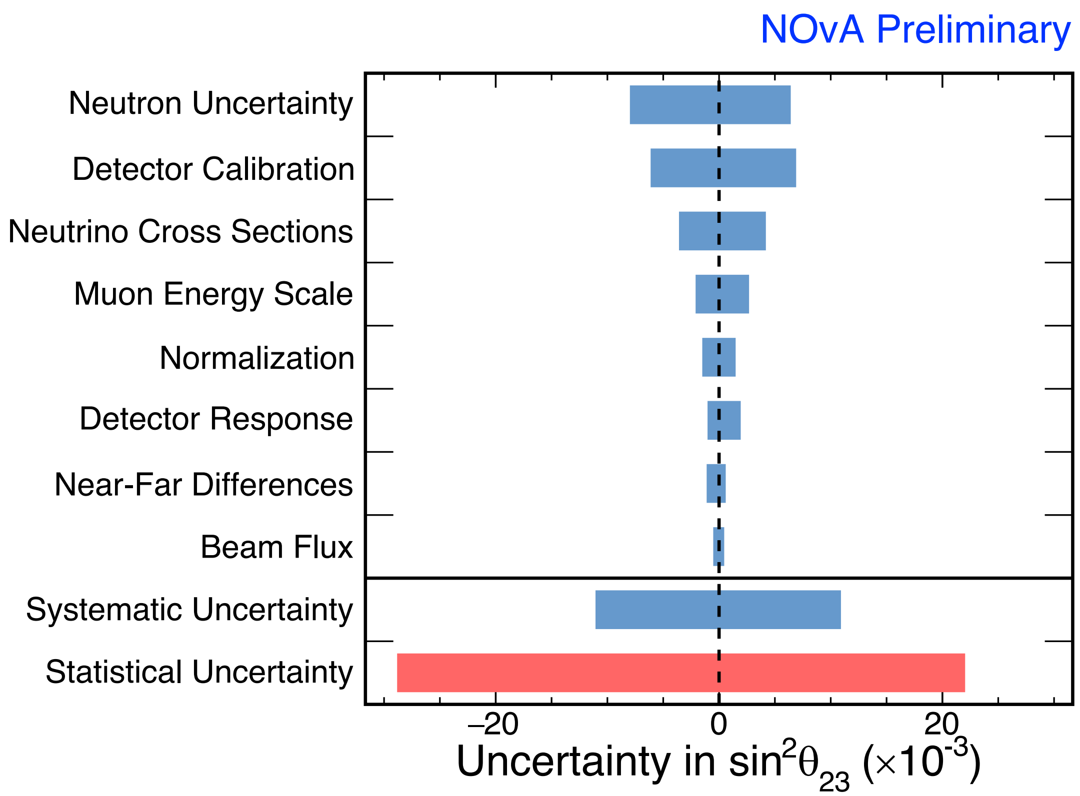
- Like CKM, PMNS has three mixing angles and a CP-violating phase.

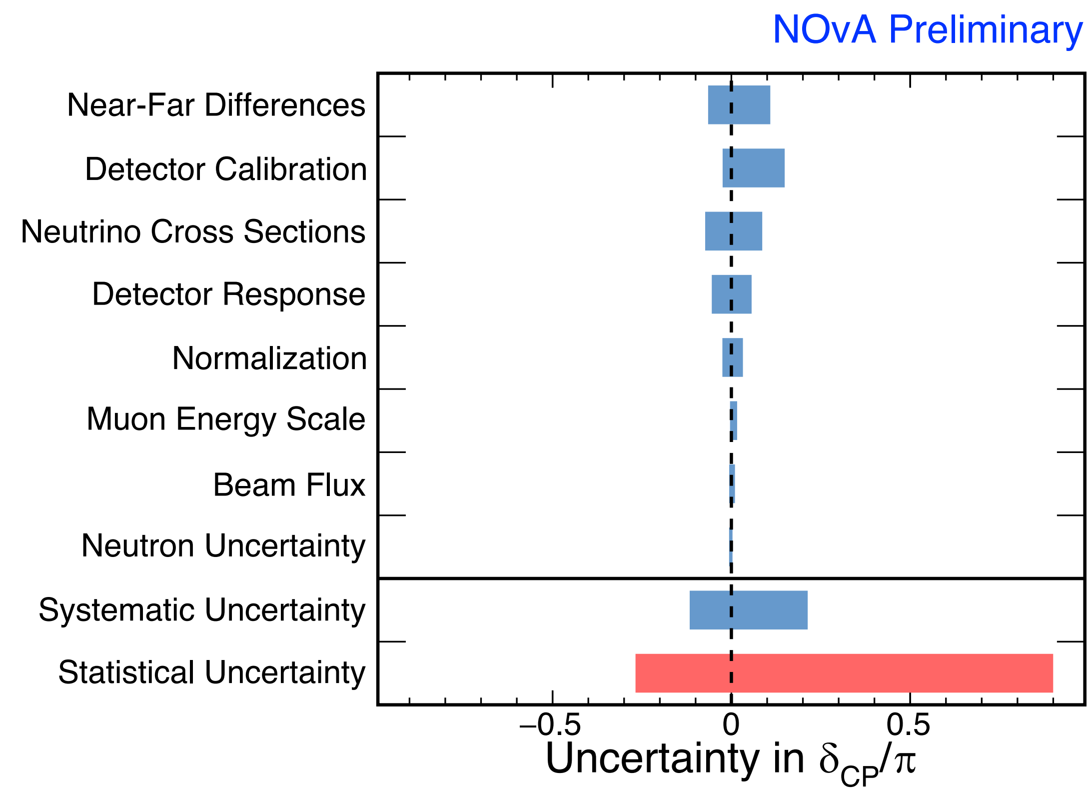
- Because no symmetry forbids a Majorana mass for right-handed neutrinos, two more CP-violating phases arise in PMNS ← $0\nu\beta\beta$.

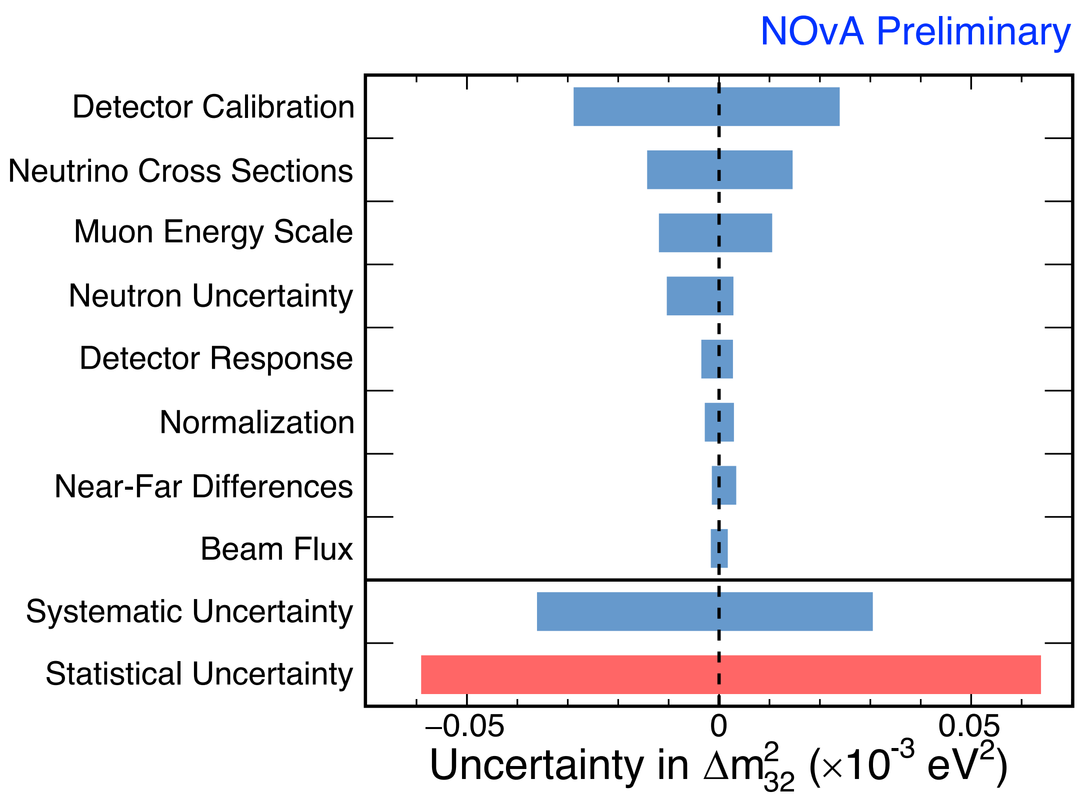
Particle Physics Questions

- Mediator in direct dark matter detection (if detected)?
- Mediator in lepton flavor violation (if detected)?
- How many sterile neutrinos (within reach)?
- Do leptons violate CP?
- What values do the neutrino masses take?
- What values do the PMNS matrix elements take?
- What, if any, is the relation of PMNS to CKM?

Systematic Uncertainties





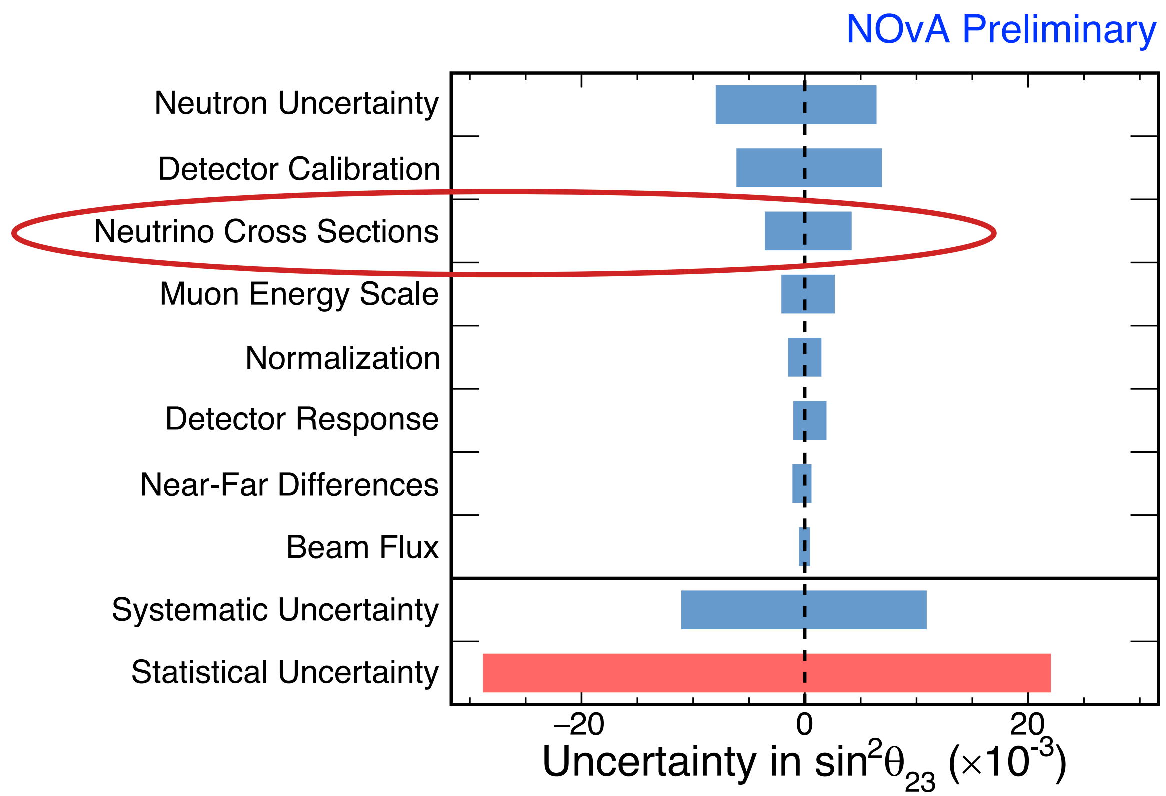


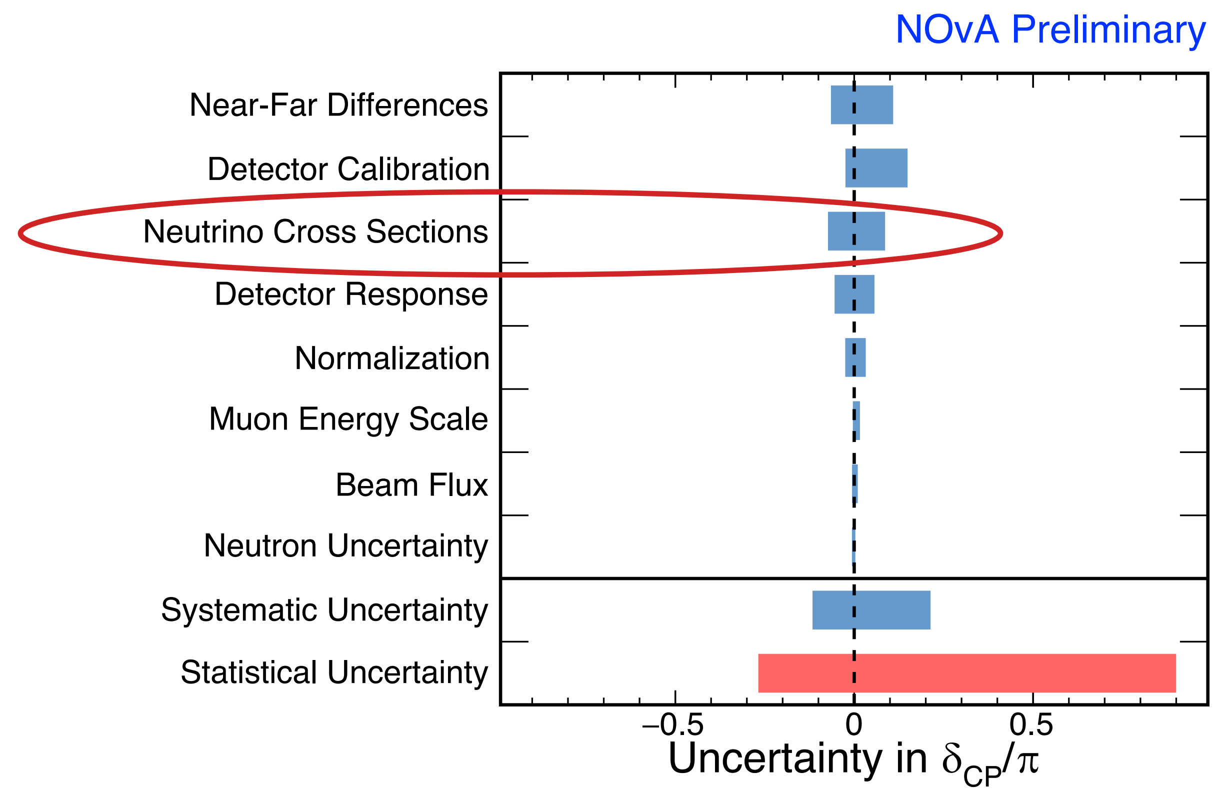
Most important systematics:

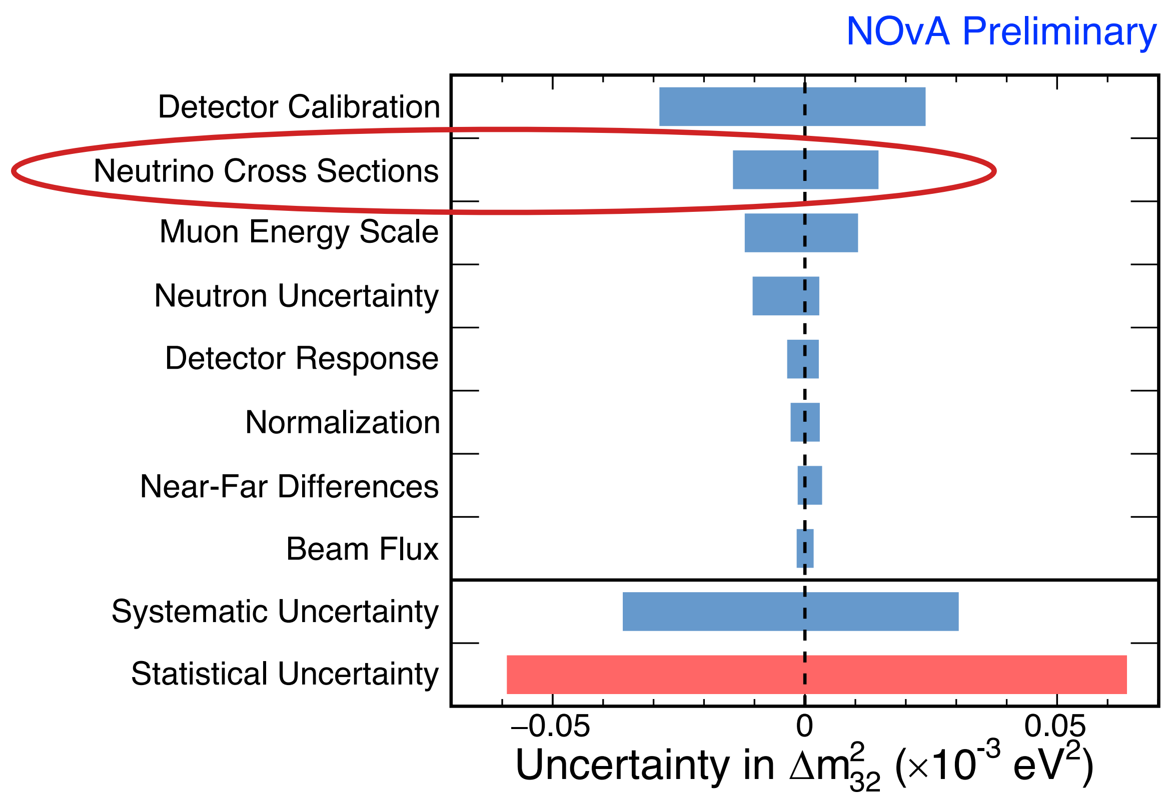
- Detector Calibration
  - Will be improved by the 2019 test beam program
- Neutrino cross sections
  - Particularly nuclear effects (RPA, MEC)
- Muon energy scale
- Neutron uncertainty – **new** with $\bar{\nu}$'s

Alex Himmel, June 15 @ Fermilab

Systematic Uncertainties







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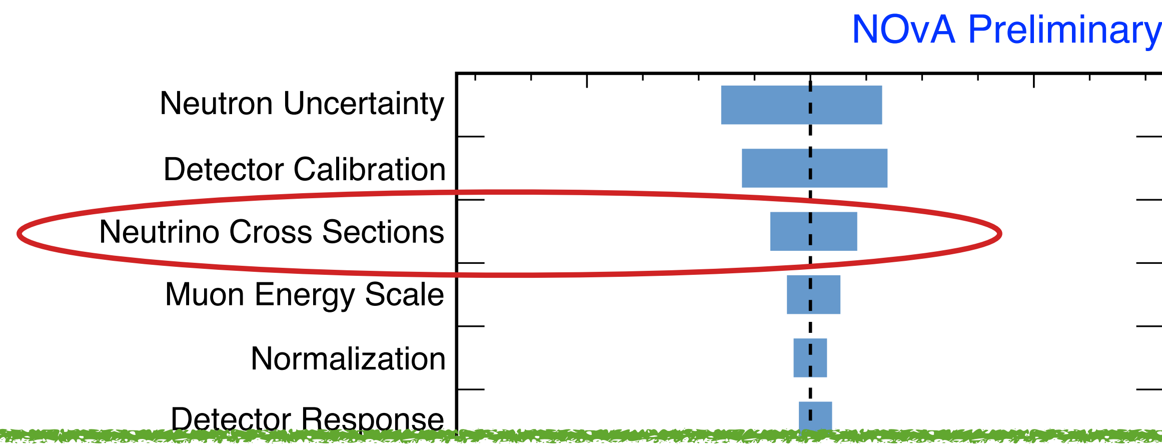
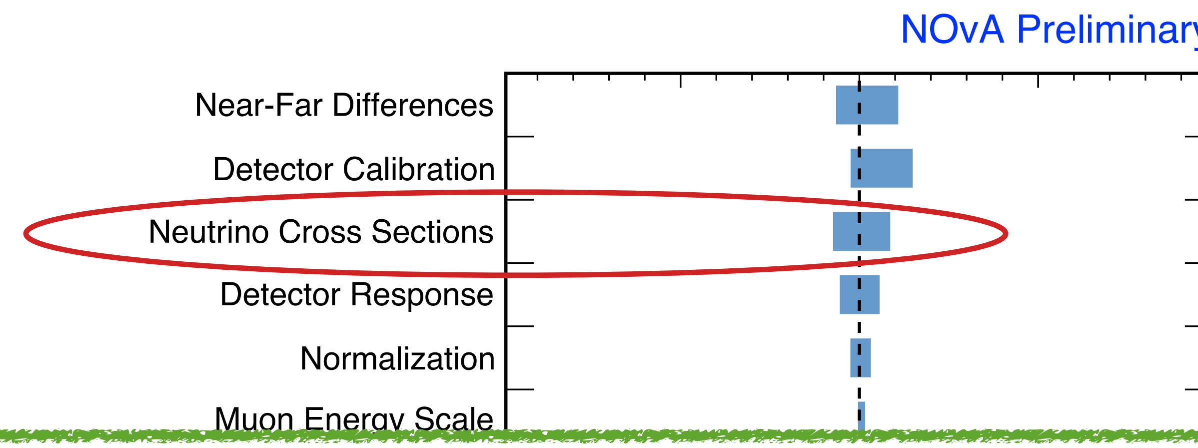
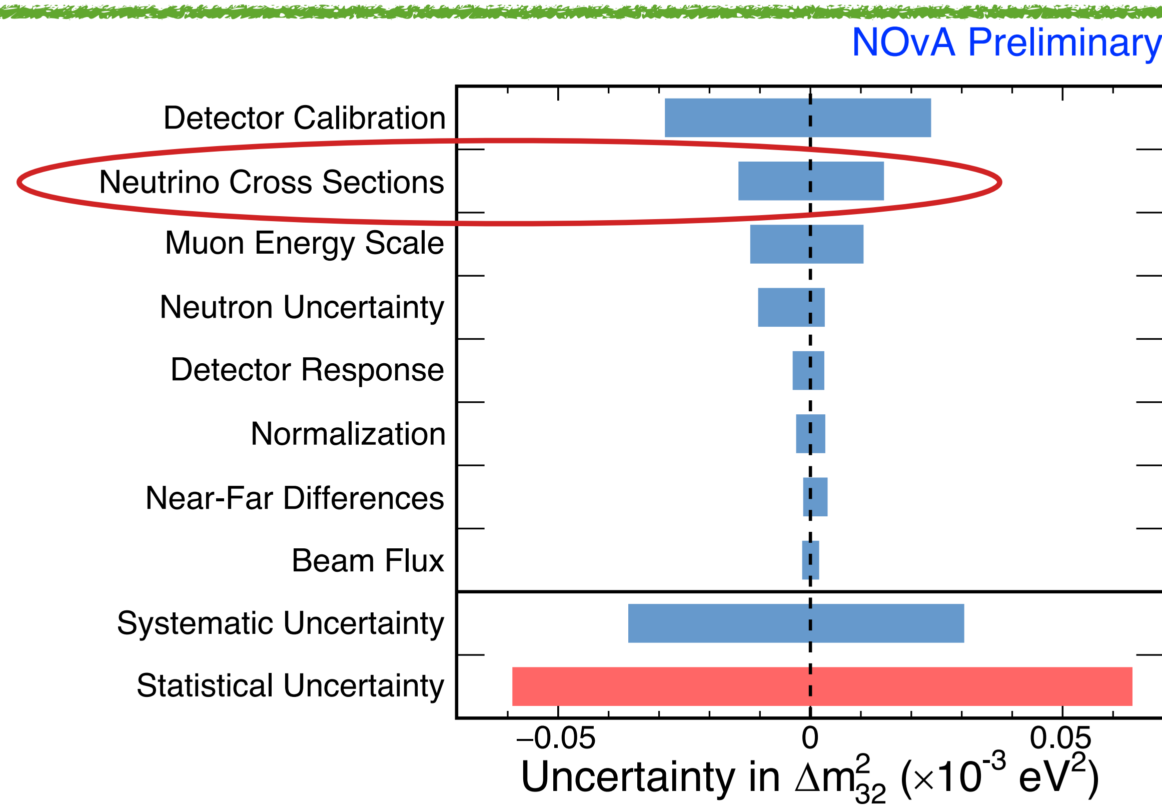
Systematic Uncertainties





Uncertainty in Δm^2_{lj}, θ_{lj}, δ_{CP}, α_i owing to imperfect knowledge of nucleus?



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Nuclear Structure

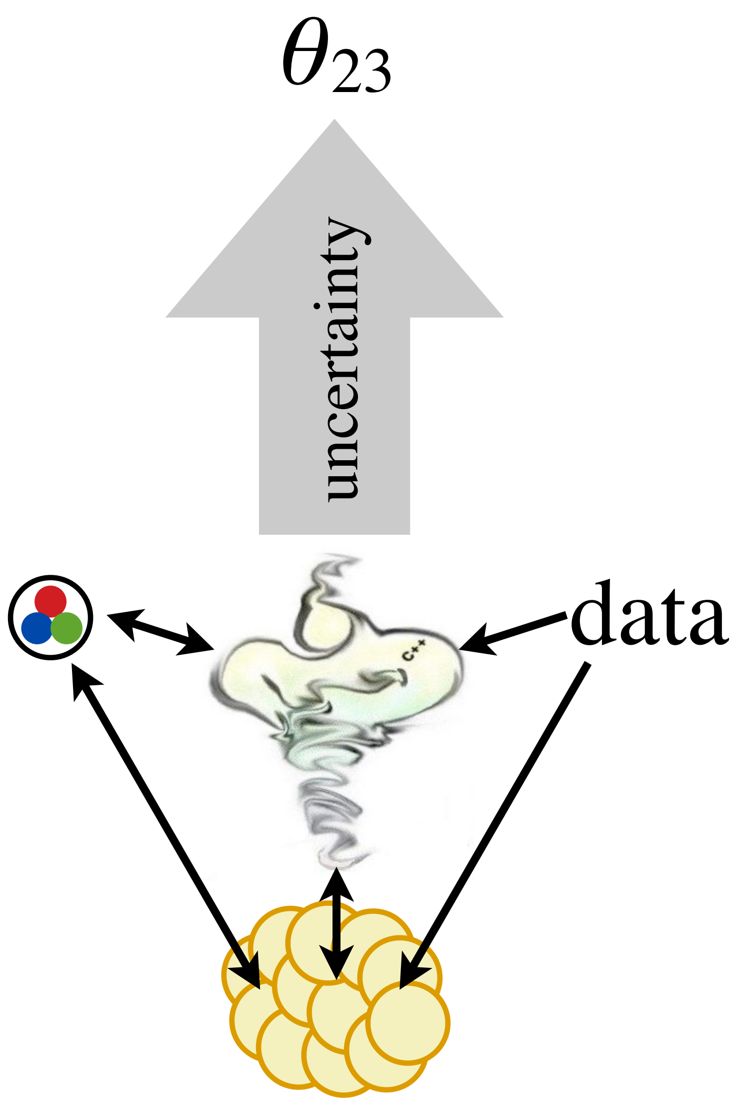
- A huge challenge:
  - many approaches, some called "*ab initio*"—
  - means that a unified formalism describes many things and can be systematically improved—
    - can be cast as a chiral effective theory (nucleons in a pion cloud);
    - not QCD though.
- Full error budgets are yet to come and may be impossible.
- Measure "data over here" to constrain nuclear model; then trust validated models "over there" where oscillation parameters are determined.

Hadronic Physics—νN Scattering

- Hard, but easy compared to nuclear theory.
- Avoid trap: nuclear model with nucleonic ingredients inconsistent with QCD:
    - even if tuned up “here” such a thing would be scary “over there”.
- Therefore, aim to get scattering amplitudes from first principles.
- Two approaches:
    - $\bar{\nu} p$ or νd scattering experiments in the Tokai, NuMI, or LBNF beam;
    - lattice QCD with error budgets as comprehensive as those for CKM.

Generating Events for Neutrino Interaction Experiments

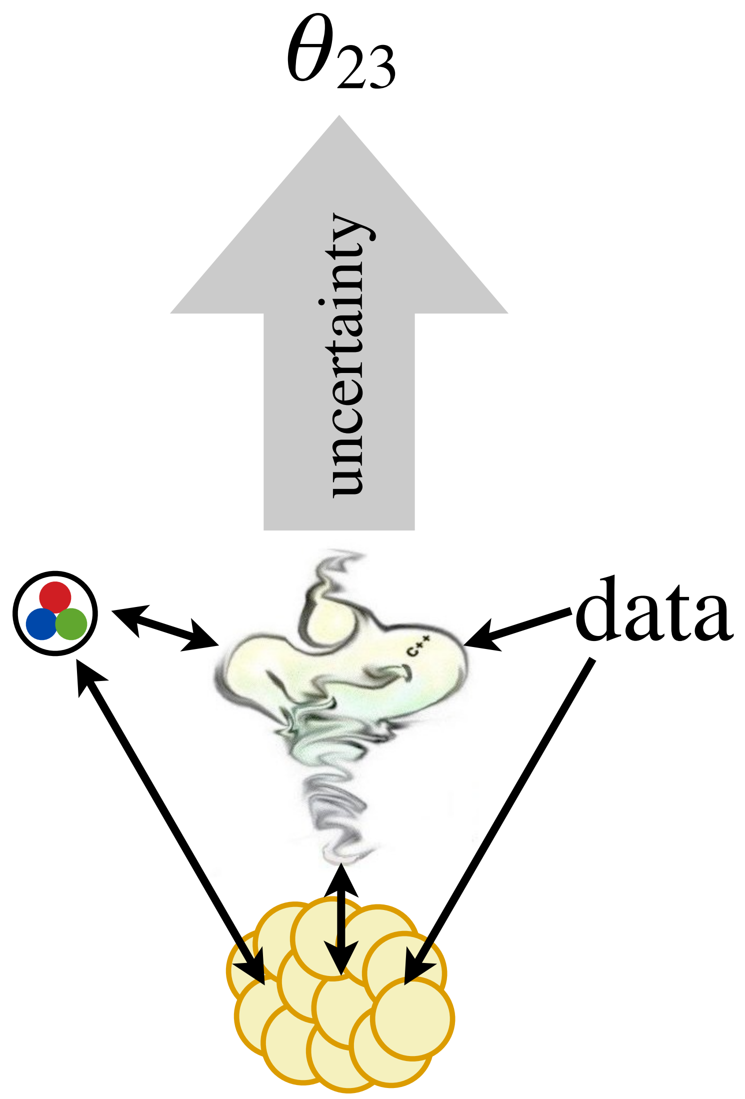
- GENIE starts with nucleon-level description and adds nuclear effects.

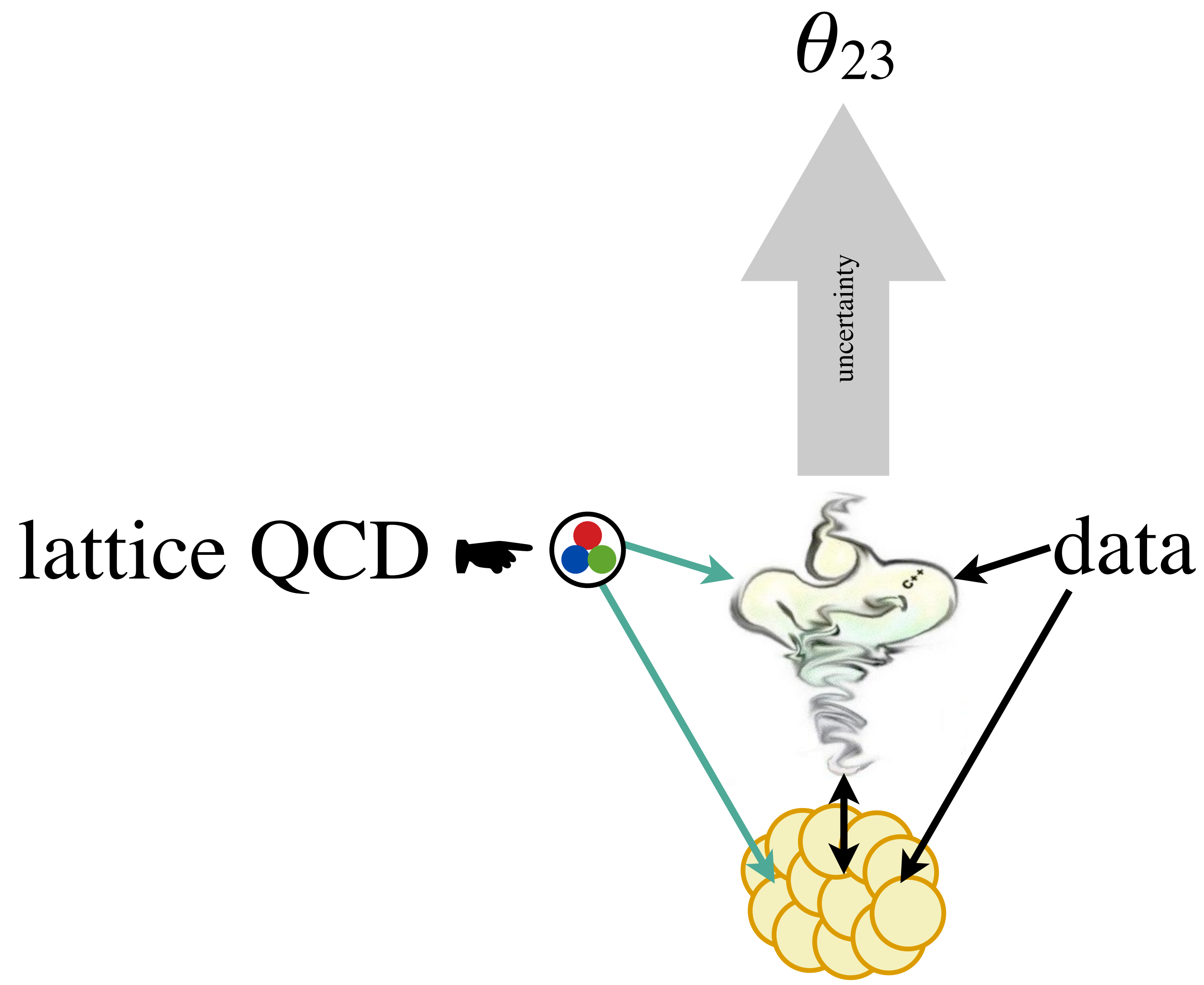


- If the nucleon-level description is not QCD, what does it mean to compare the generator output with experiment?

Neutrino Experiments

- The situation would change with *ab-initio* nucleon-level QCD information.



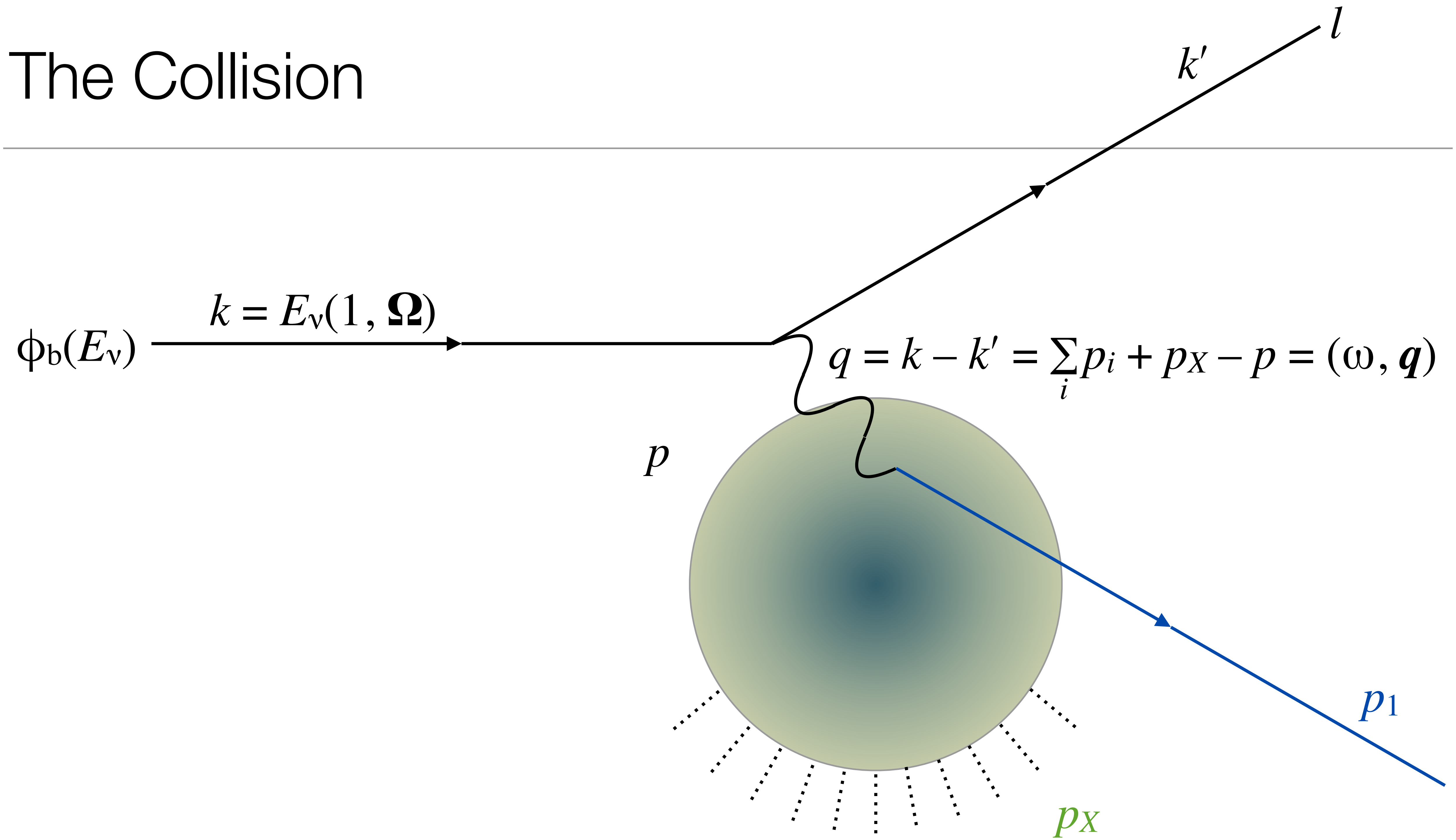


Outline

- Introduction
- My view of the problem
- A little nuclear theory (of initial state)
- Lattice QCD and the CKM matrix—a few highlights
- Lattice QCD for beginners
- Status of target calculations, attend & listen: Phiala Shanahan, Weds 9 AM
- Outlook

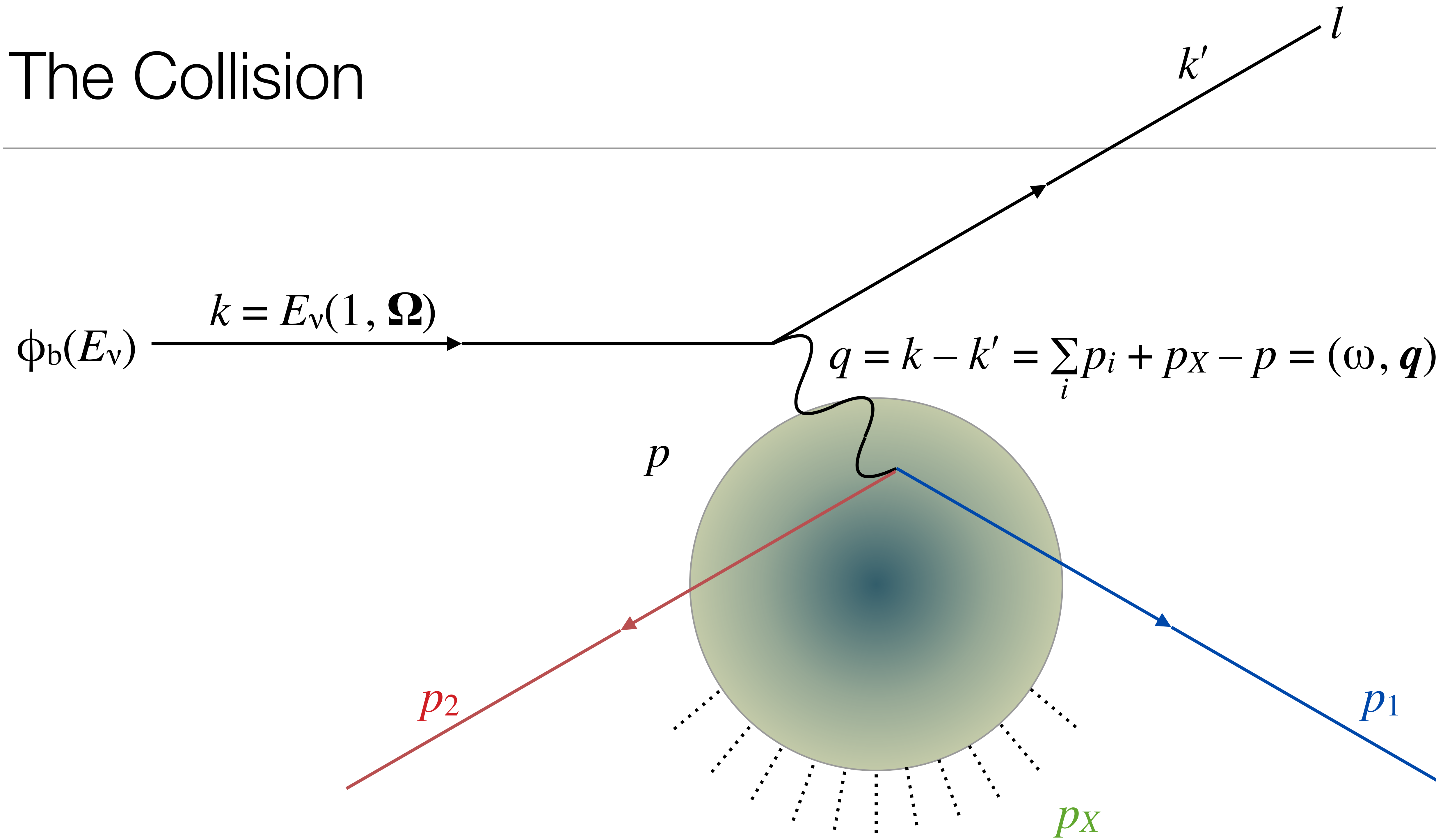
νA Scattering

The Collision



Useful discussion of kinematics by Donnelly

The Collision



knock out 2nd nucleon:
sign of "2p2h"

Useful discussion of
kinematics by Donnelly

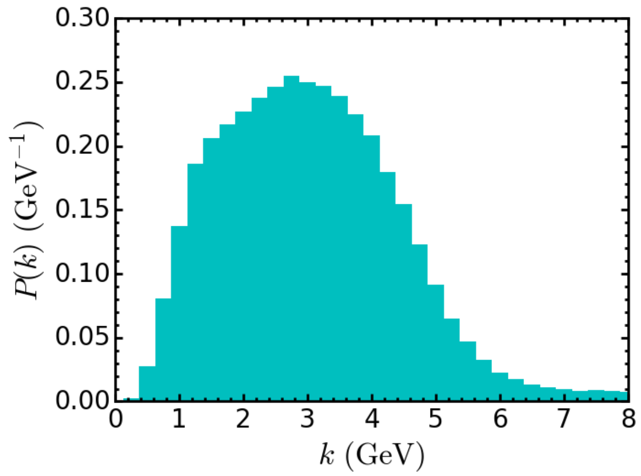
Statement of the Problem I

- Neutrino-nucleus (A) scattering: $\nu_{\mathrm{b}}A \to \mu^- X_p, \bar{\nu}_{\mathrm{b}}A \to \mu^+ X_n$, where X_p can be A, an excitation of A, $A'p$, $A'p\pi$, …, smithereens.

- Measure 4-momentum of muon, probably of proton, maybe of a few more charged particles; just consider muon:

$$\frac{d^2\sigma(P_\nu)}{dE_\mu d\cos\theta} \equiv \int \frac{d^2\sigma(E_\nu)}{dE_\mu d\cos\theta} P_\nu(E_\nu) dE_\nu$$

muon energy & direction



DUNE flux as a probability density.

- Cross section proportional to lepton & hadron tensors:

$$d\sigma(E_\nu) \propto L^{\mu\nu} W_{\mu\nu}$$

$$W_{\mu\nu} \propto \sum_{X_p} \langle A|\hat{J}^\dagger_\nu|X_p\rangle \langle X_p|\hat{J}_\mu|A\rangle$$

- The hadronic tensor depends on the four-vectors q^μ and $p^\nu = M_T v^\nu$:

$$W^{\mu\nu}(q,p) = W_1(q^2, q\cdot v)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + W_2(q^2, q\cdot v)\left(v^\mu - \frac{q\cdot v}{q^2}q^\mu\right)\left(v^\nu - \frac{q\cdot v}{q^2}q^\nu\right)$$

neutral vector current here

- HEP calls the W_i "structure functions" (especially when $A = N$); while NP calls them "nuclear responses" (especially when $A \neq N$).

- HEP (Bjorken) writes $\nu = q\cdot v$; NP denotes this "energy transfer" ω.

- Everyone writes $Q^2 = -q^2 > 0$.

- NP sometimes writes q^2 for 3-momentum aka $|\boldsymbol{q}|^2$.

- Lab frame is (essentially) nucleus rest frame; use v to make invariant.

Energy Scales

- Thermal, molecular motion: $k_B T = 7.5$ meV, $16/A$ meV;
- Energy transfer $\omega = E_\nu - E_l$: 200 MeV $< \omega < E_\nu - m_l$;
- Beam energy E_ν: 1 GeV $< E_\nu <$ 6 GeV (DUNE);
- Binding energy (per nucleon): 8.6 MeV (^{40}Ar);
- Pion mass: 140 MeV;
- Fermi motion of nucleons: 250 MeV;
- Nuclear mass: $0.94A$ GeV, $A \in \{1, 2, 12, 16, 40\}$.

Statement of the Problem II

- So we have to know the W_i reliably—for me this means a curve plus an error band:

- An empirical error band will do (for HEP).

- The energies cover a range in which both nuclear and nucleonic effects are important:

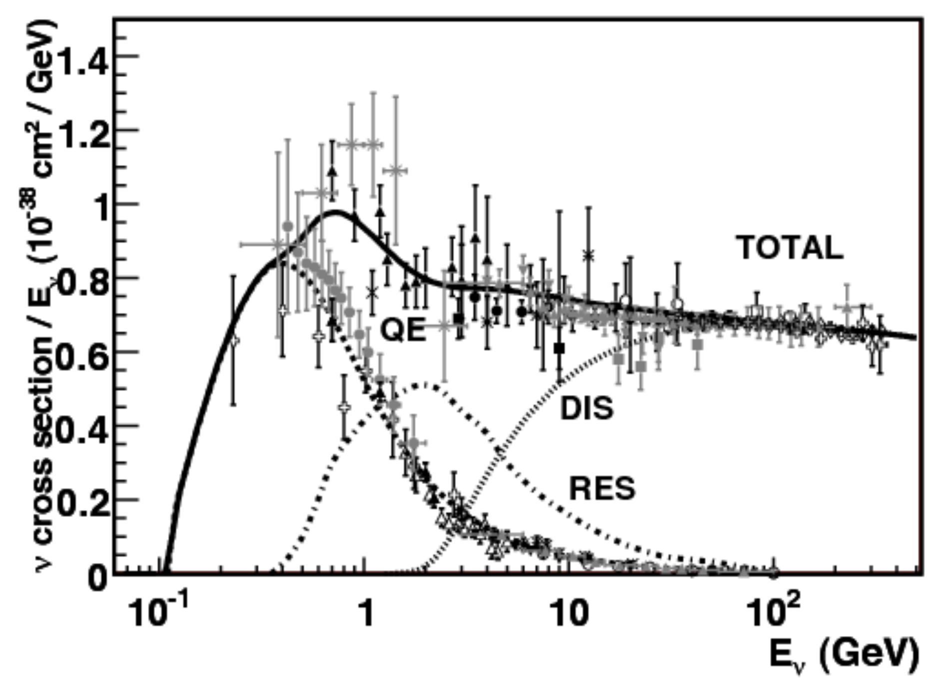
  - quasielastic—scatter off "whole nucleon";

  - resonance—enough energy for Δ, $N\pi$;

  - shallow inelastic scattering—many pions, but not enough for OPE;

  - deep inelastic scattering—but Q^2 not huge in oscillation experiments.

Formaggio & Zeller, arXiv:1305.7513



Neutrino-Nucleus Collisions

- At the energies used by experiments measuring oscillation parameters, the nucleus is treated as a collection of weakly interacting nucleons:
  - use nuclear theory for nuclear structure;
  - use *ab initio* nucleon-level calculations for neutrino-nucleon transition matrix elements including, eventually—
    - $\nu N \to eX$, $X = N'$ or $N\pi$ or Δ or $\ldots$, i.e., matrix element $\langle N|V-A|N\rangle$ &
    - two-body effects: $\langle NN|V-A|NN\rangle$;
  - use nuclear transport (e.g., GiBUU) theory for post-collision absorption/secondary production of pions.

Nuclear Theory

Nuclear Hamiltonian

- The A nucleons in a nucleus are described by the Hamiltonian:

nucleon kinetic energy;
nucleon mass M_N

two-body potentials;
e.g., Argonne v_{18}

$$H = -\frac{1}{2M_N}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

three-body potentials;
e.g., Illinois 7

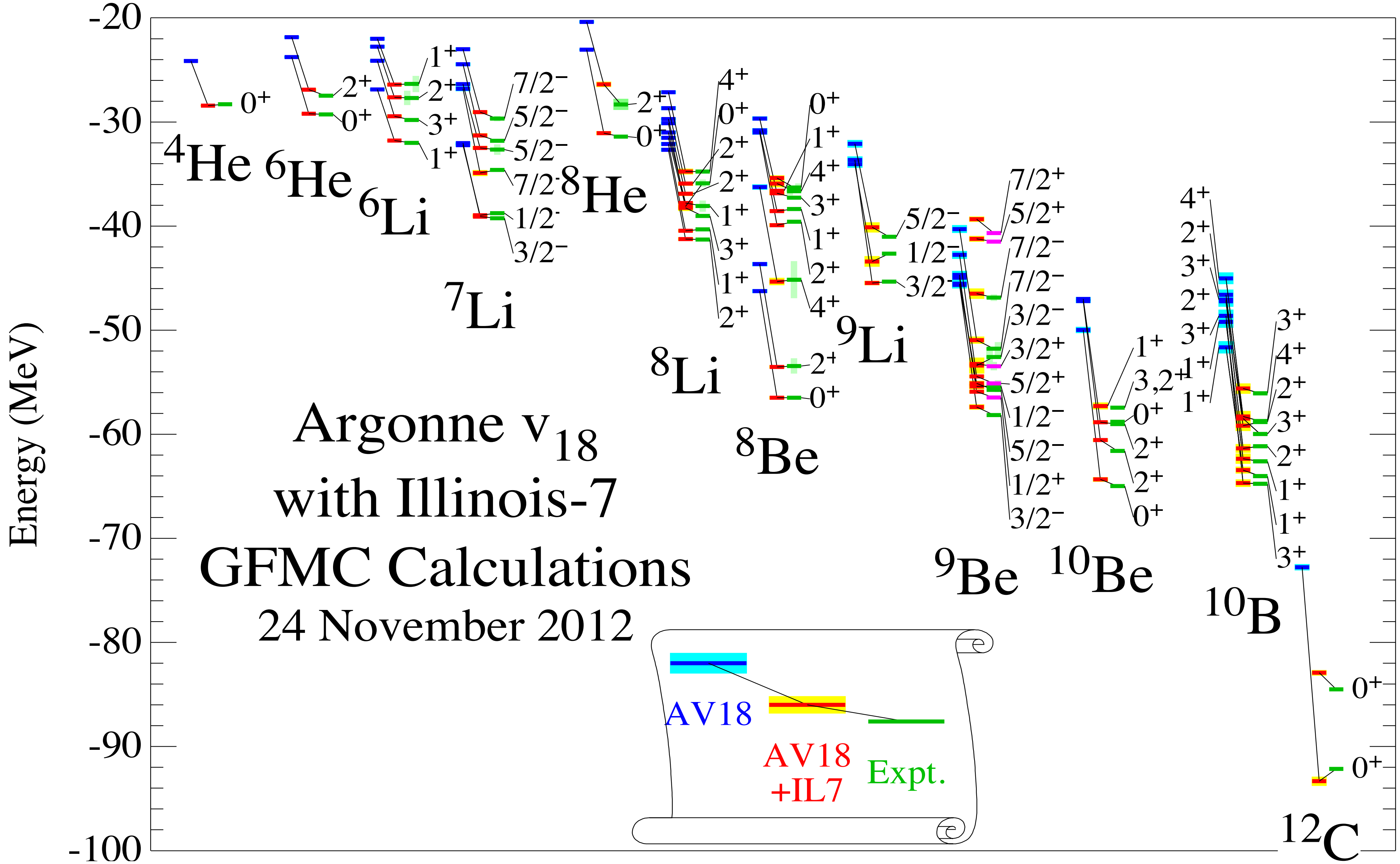
- Fit to scattering data.
- Reproduced by πN EFT.
- Could also be extracted from lattice QCD.

Analogs in Atomic & Particle Physics

- Consider an atom:
  - weak interaction unless Z is too big;
  - even so set up NRQED or pNRQED;
  - solve a bound state problem with a nonperturbative technique.
- Consider quarkonium:
  - “strong” interaction but short distance, therefore asymptotically weak;
  - derive effective field theory (NRQCD or potential NRQCD);
  - solve a bound state problem with a nonperturbative technique.

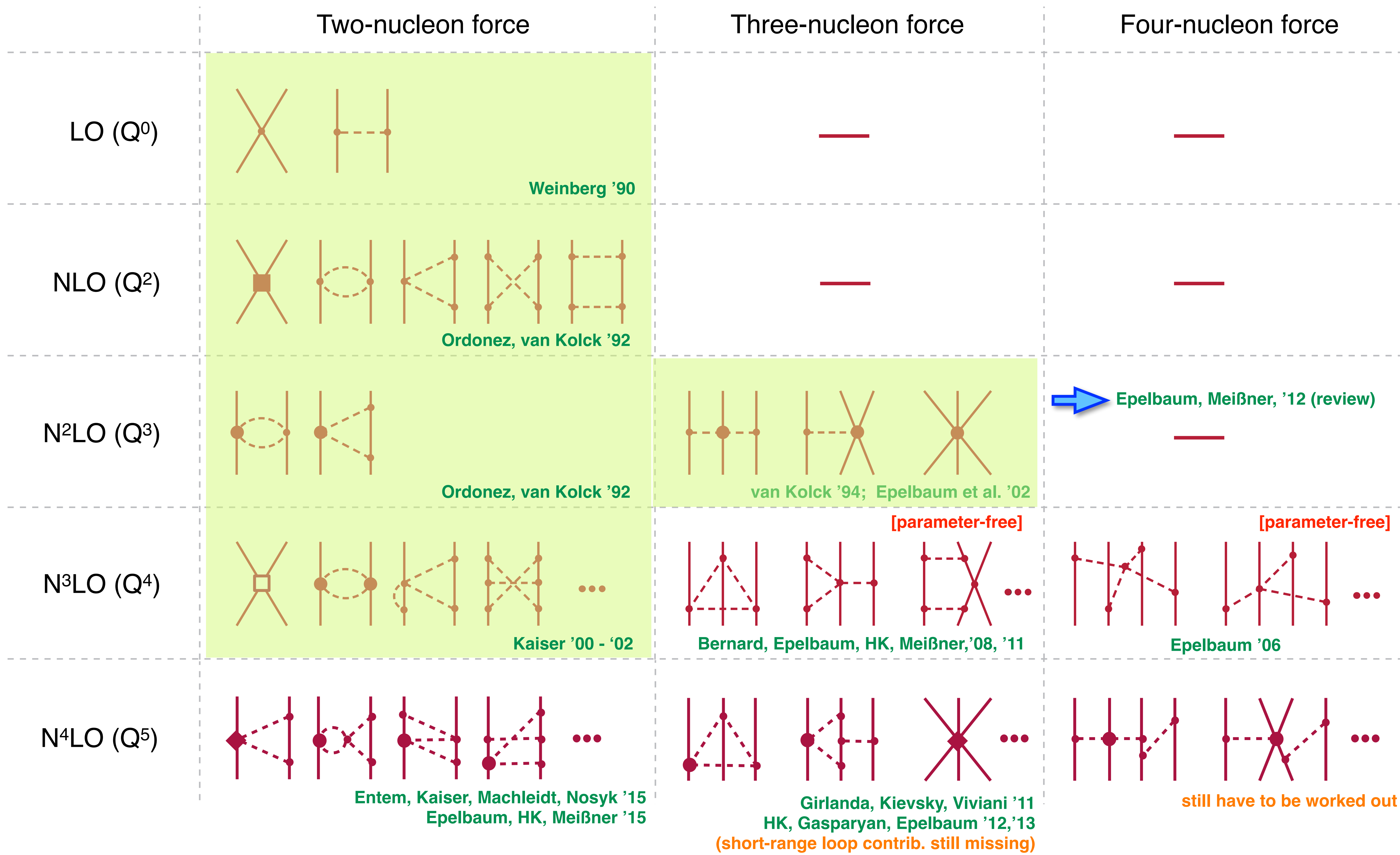
Big Success

- Compute level splittings in nuclei up to ^{12}C:



Hermann Krebs, June 12 @ INT

Chiral Expansion of the Nuclear Forces



Neutrino-Nucleus Interaction

- The A nucleons interact with the neutrino via an electroweak current:

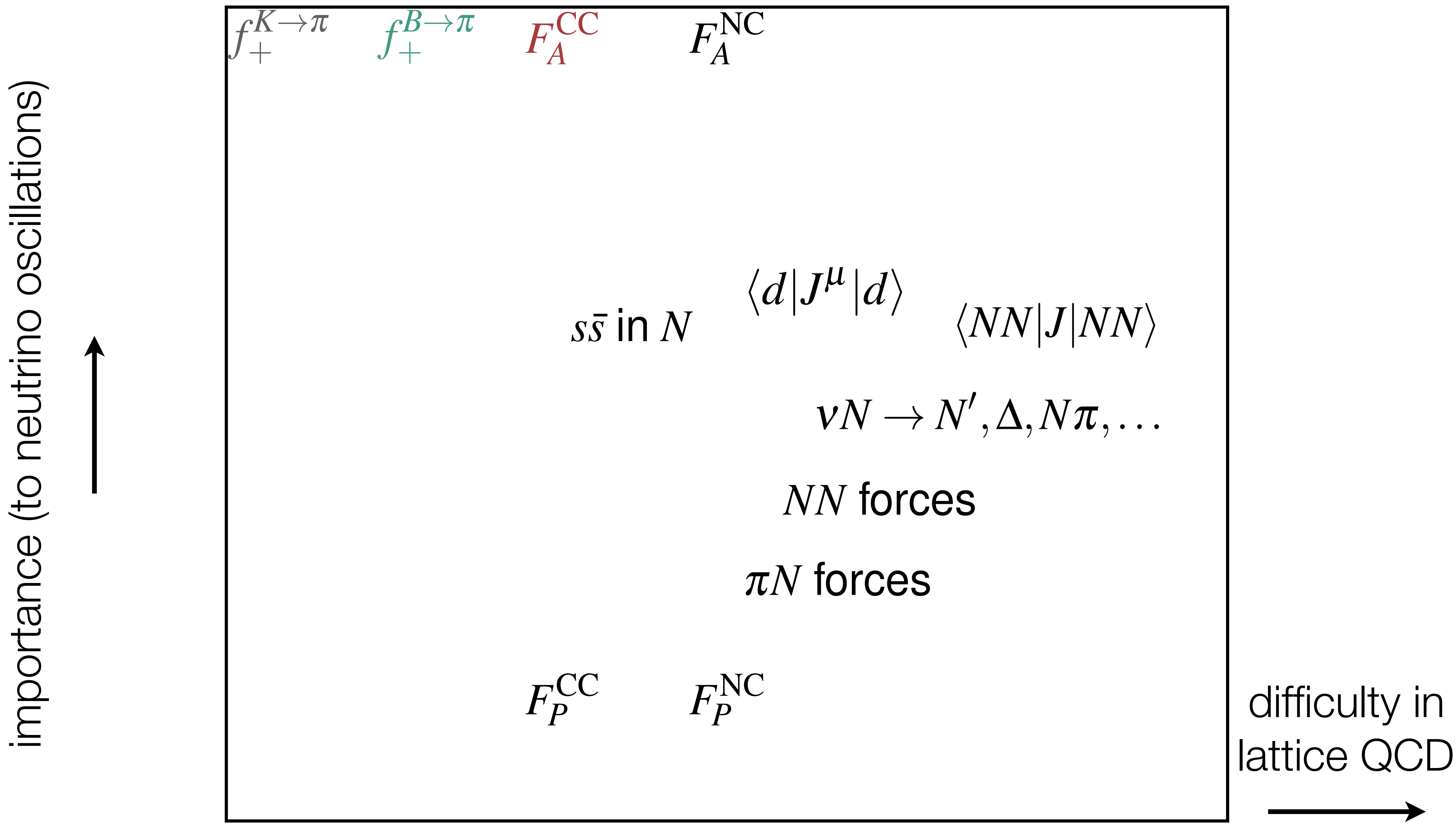
"one-body" currents;
aka form factors

"two-body" current;
i.e., ff of $\langle NN | J^\mu | NN \rangle$

$$J^\mu = \sum_i j_i^\mu + \sum_{i<j} j_{ij}^\mu$$

- Extract one-body currents from $\bar{\nu} p$ or νd scattering, or—
    - computed with lattice QCD.
- Two-body currents can be described by meson-exchange in πN EFT.

Priority vs Difficulty

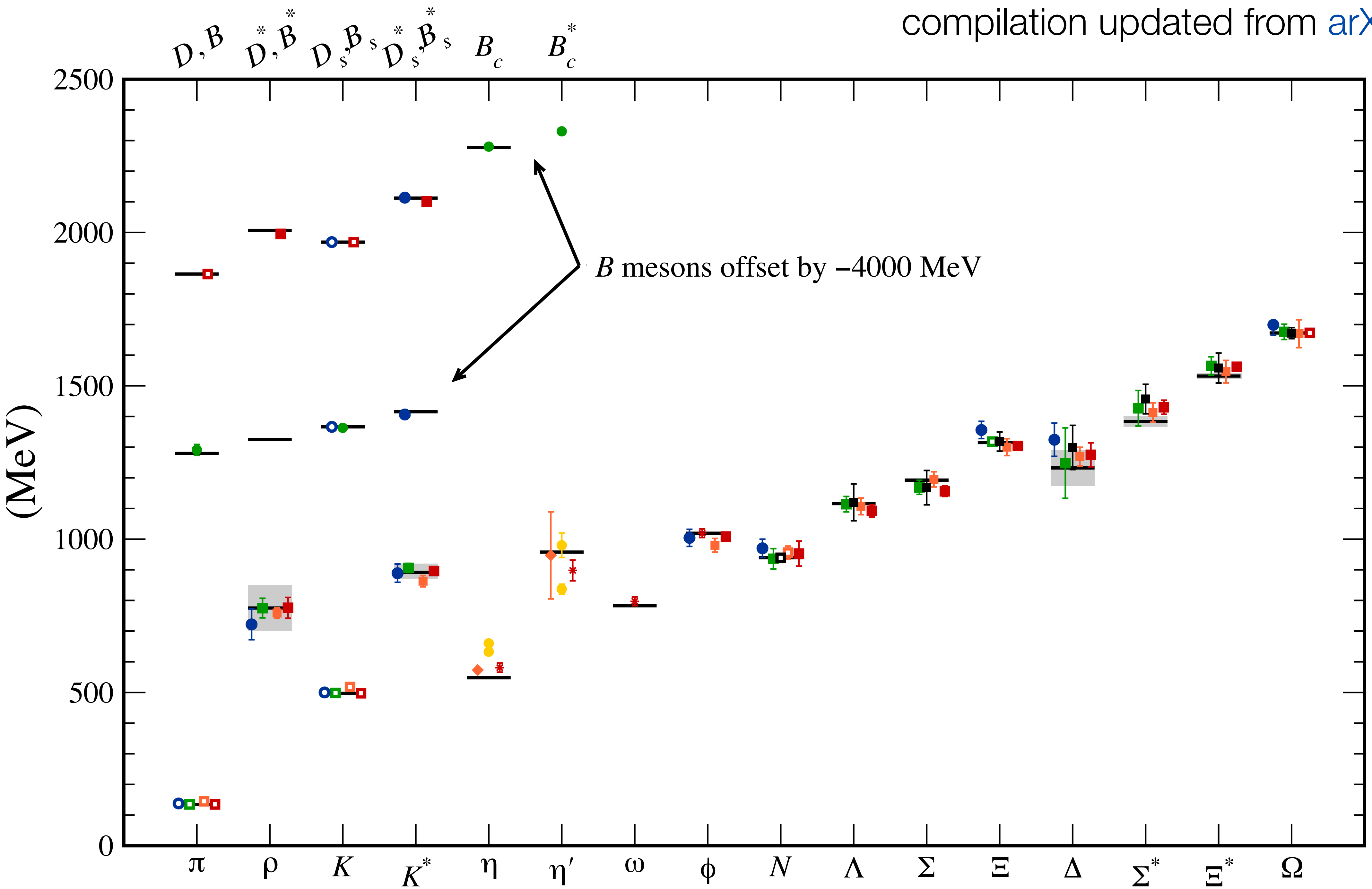


Lattice QCD and CKM

QCD Hadron Spectrum

π…Ω: BMW, MILC, PACS-CS, QCDSF; ETM (2+1+1);
η-η′: RBC, UKQCD, Hadron Spectrum (ω);
D, *B*: Fermilab, HPQCD, Mohler&Woloshyn

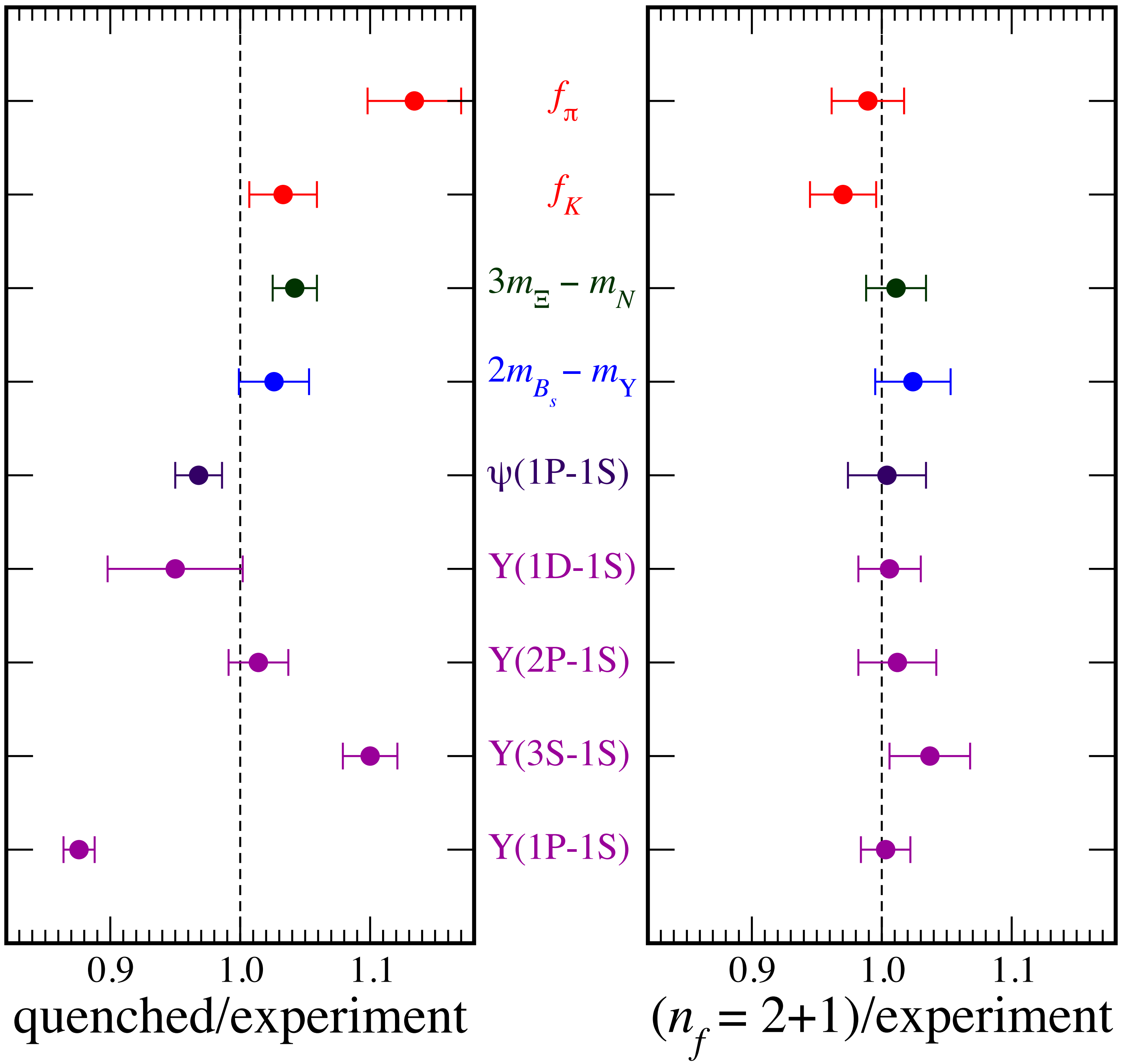
compilation updated from arXiv:1203.1204



numerous quarkonium omitted

Postdictions

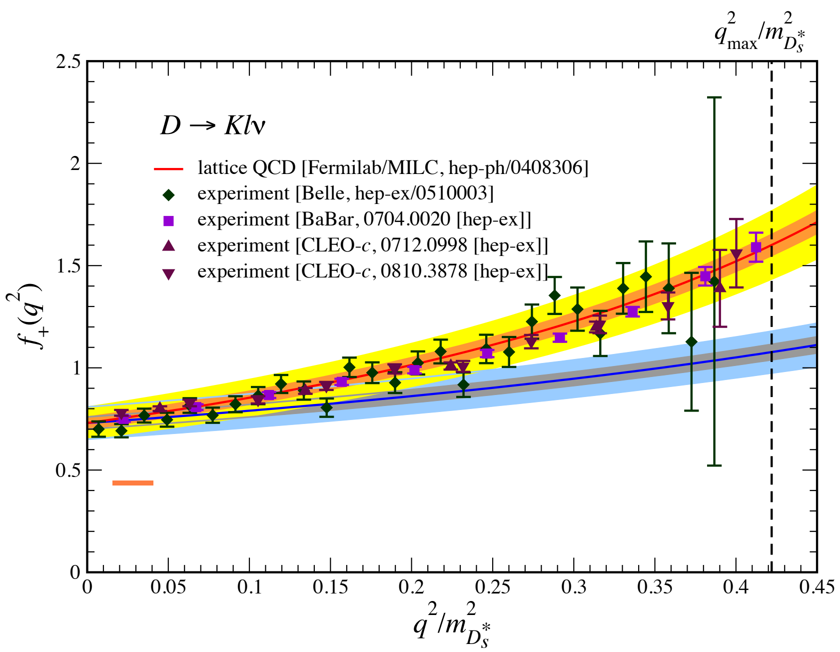
HPQCD, MILC, Fermilab Lattice, hep-lat/0304004

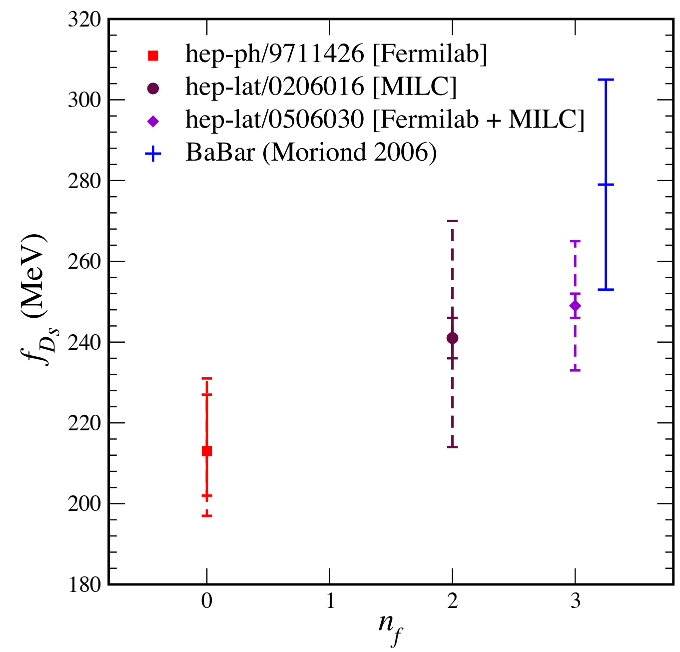


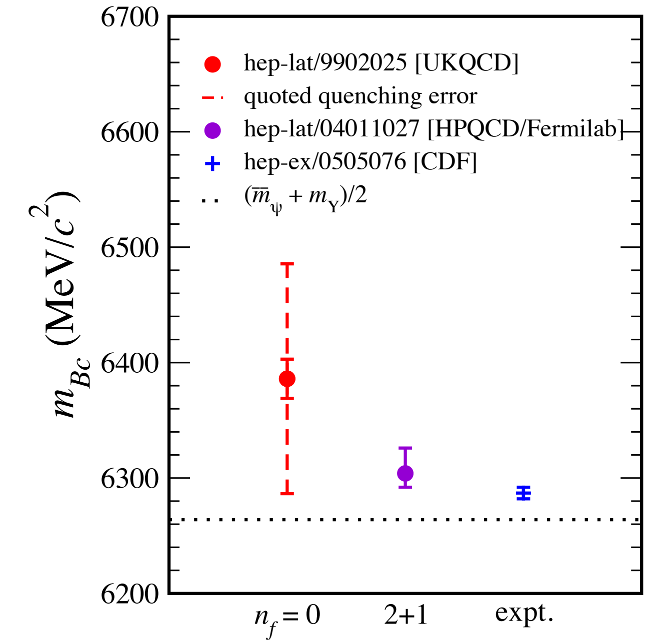
- From 2003!
- $a = 0.12$ & 0.09 fm;
- $O(a^2)$ improved: asqtad;
- FAT7 smearing;
- $2m_l < m_q < m_s$;
- π, K, $\Upsilon(2S\text{-}1S)$ input.

Predictions

Fermilab Lattice, MILC, HPQCD



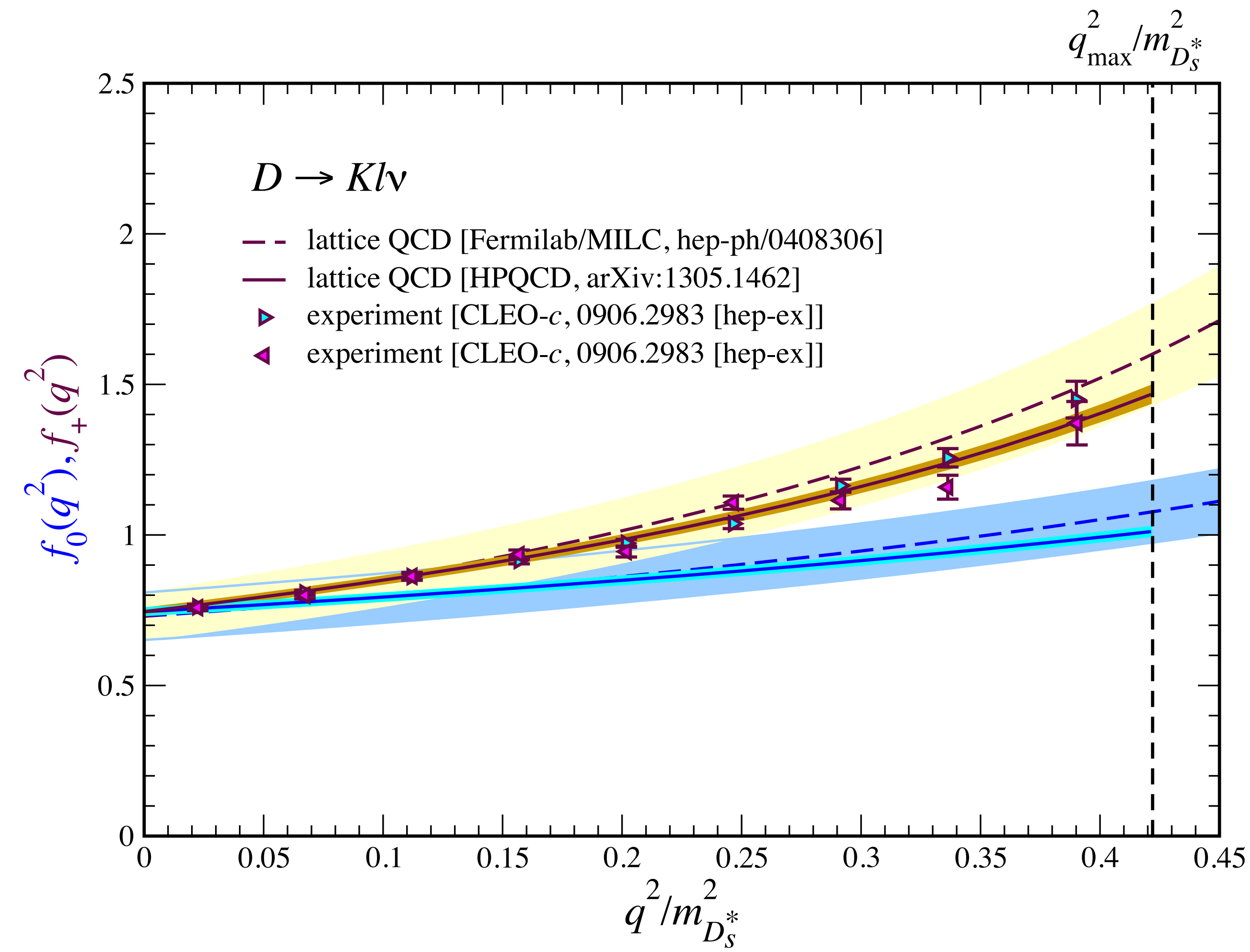




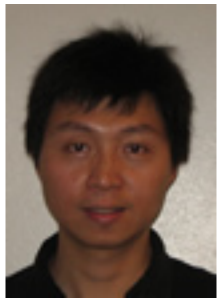
- Semileptonic form factor for $D \to K l \nu$: hep-ph/0408306—updates: arXiv:1008.4562 (normalization), arXiv:1305.1462 (shape);
- Charmed-meson decay constants: hep-lat/0506030—update below;
- Mass of B_c meson: hep-lat/0411027—updates: arXiv:0909.4462 ($M_{B^*} = 6330 \pm 9$ GeV), arXiv:1010.3848.

Semileptonic Decays

- Even when the narrative is simple, the best lattice QCD calculation is a moving target.
- FOCUS, Belle, BaBar, & CLEO validated 2004 lattice QCD.
- Lattice QCD is more precise now:
- Instead of check, one can determine $|V_{cs}|$ [arXiv:1305.1462].
- Similarly, determine $|V_{ub}|$, $|V_{cb}|$.

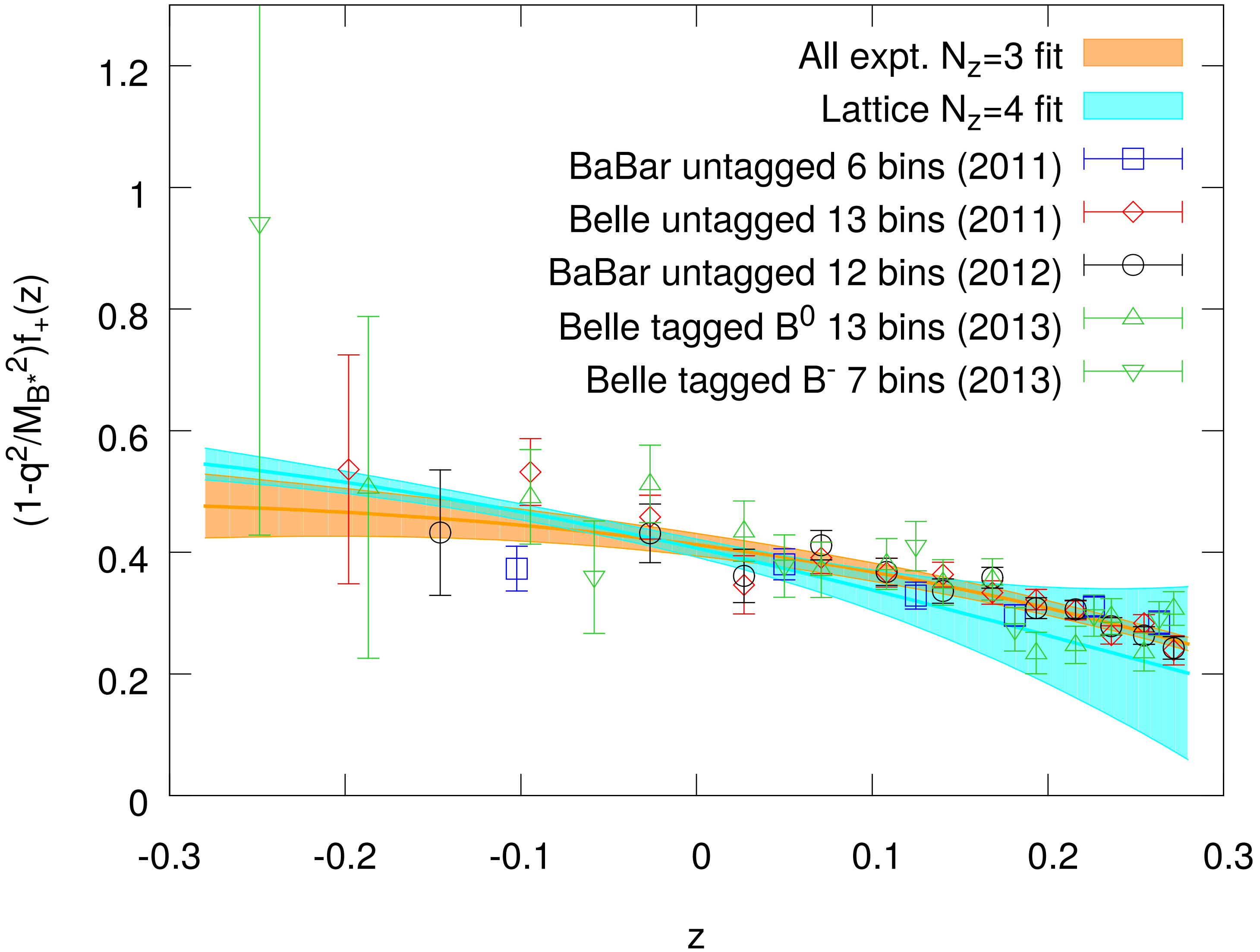


Semileptonic $B \to \pi l \nu$ for $|V_{ub}|$

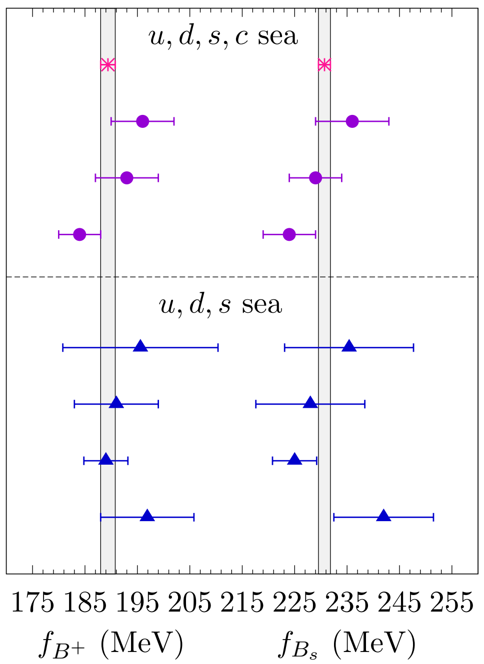


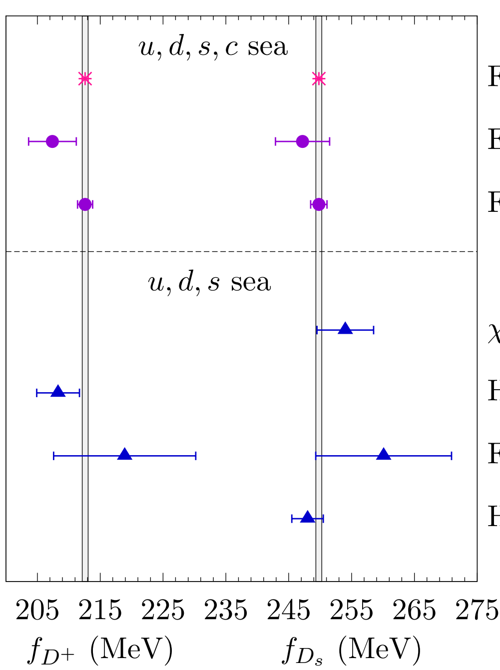
$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

- Much more precise than 2008.
- z variable extends range.
- Functional fitting method.
- Relative norm'n yields $|V_{ub}|$.
- Total error on $|V_{ub}|$ is 4.1%:
  - $10^3|V_{ub}| = 3.72 \pm 0.16$



Results for Decay Constants





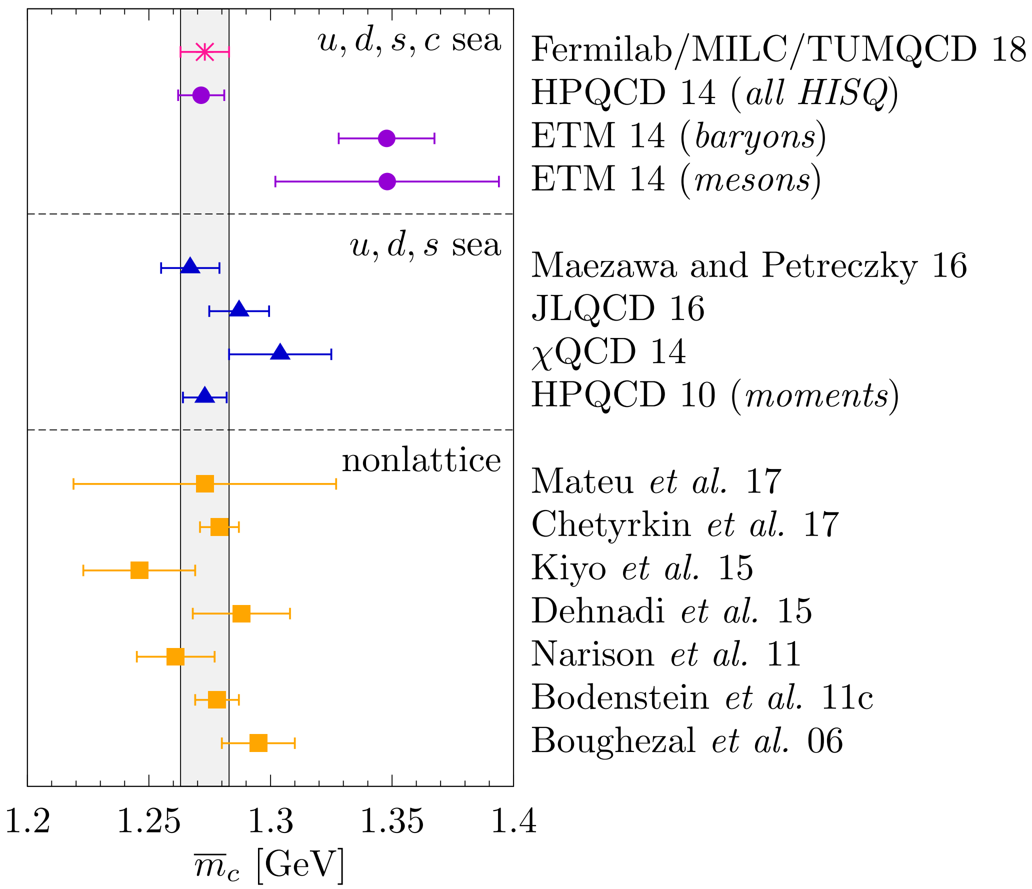
- Fermilab Lattice & MILC [arXiv:1712.09262]:

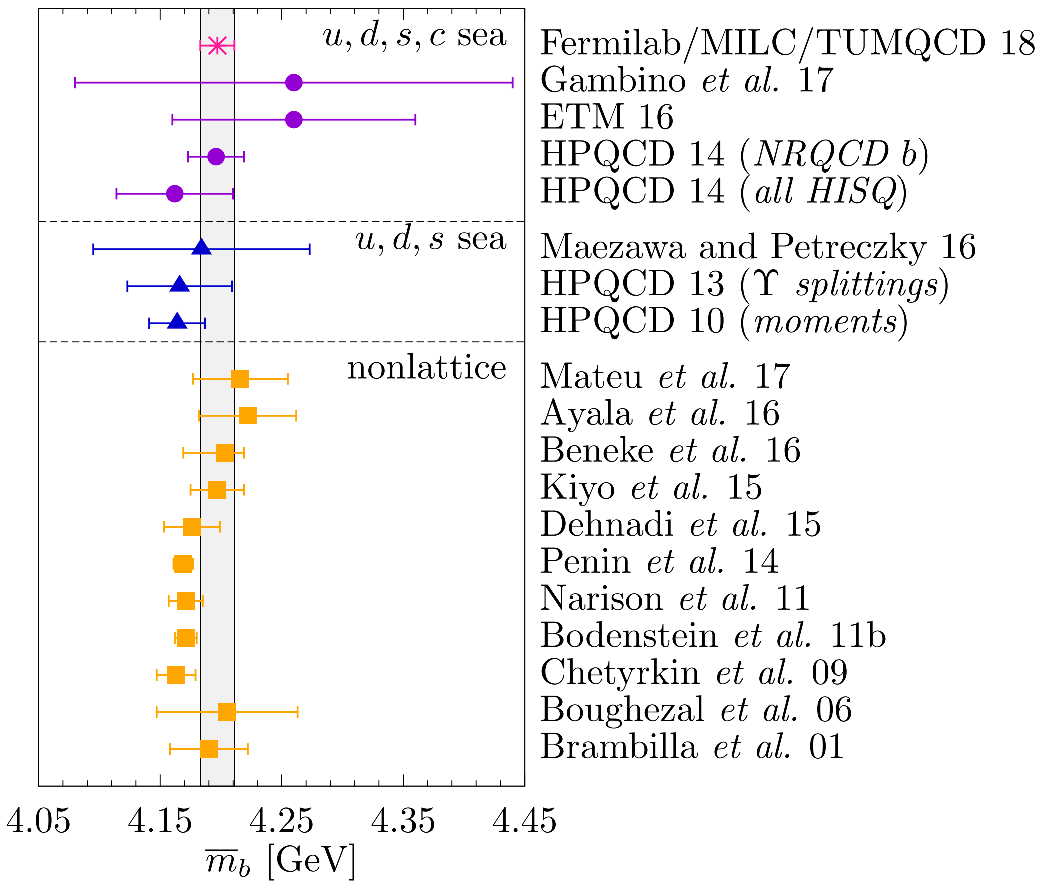
$$f_{D^0} = 211.5(0.3)_{\text{stat}}(0.3)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}}\ \text{MeV}$$
$$f_{D^+} = 212.6(0.3)_{\text{stat}}(0.3)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}}\ \text{MeV}$$
$$f_{D_s} = 249.8(0.3)_{\text{stat}}(0.3)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}}\ \text{MeV}$$
$$f_{B^+} = 189.4(0.8)_{\text{stat}}(1.1)_{\text{syst}}(0.3)_{f_{\pi,\text{PDG}}}\ \text{MeV}$$
$$f_{B^0} = 190.5(0.8)_{\text{stat}}(1.0)_{\text{syst}}(0.3)_{f_{\pi,\text{PDG}}}\ \text{MeV}$$
$$f_{B_s} = 230.7(0.8)_{\text{stat}}(0.8)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}}\ \text{MeV}$$

- Overall uncertainty: ~0.2% for D mesons,
  ~0.7% for B mesons.

Results & Comparisons

- Results form arXiv:1802.04248:

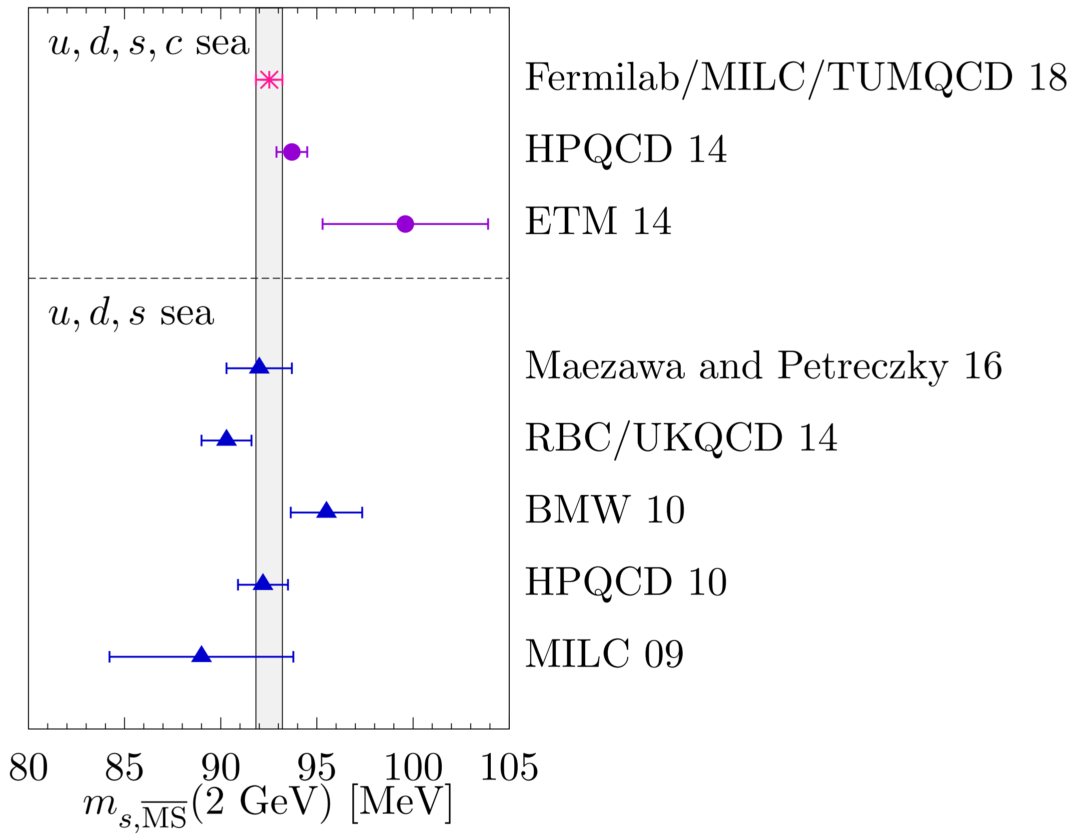


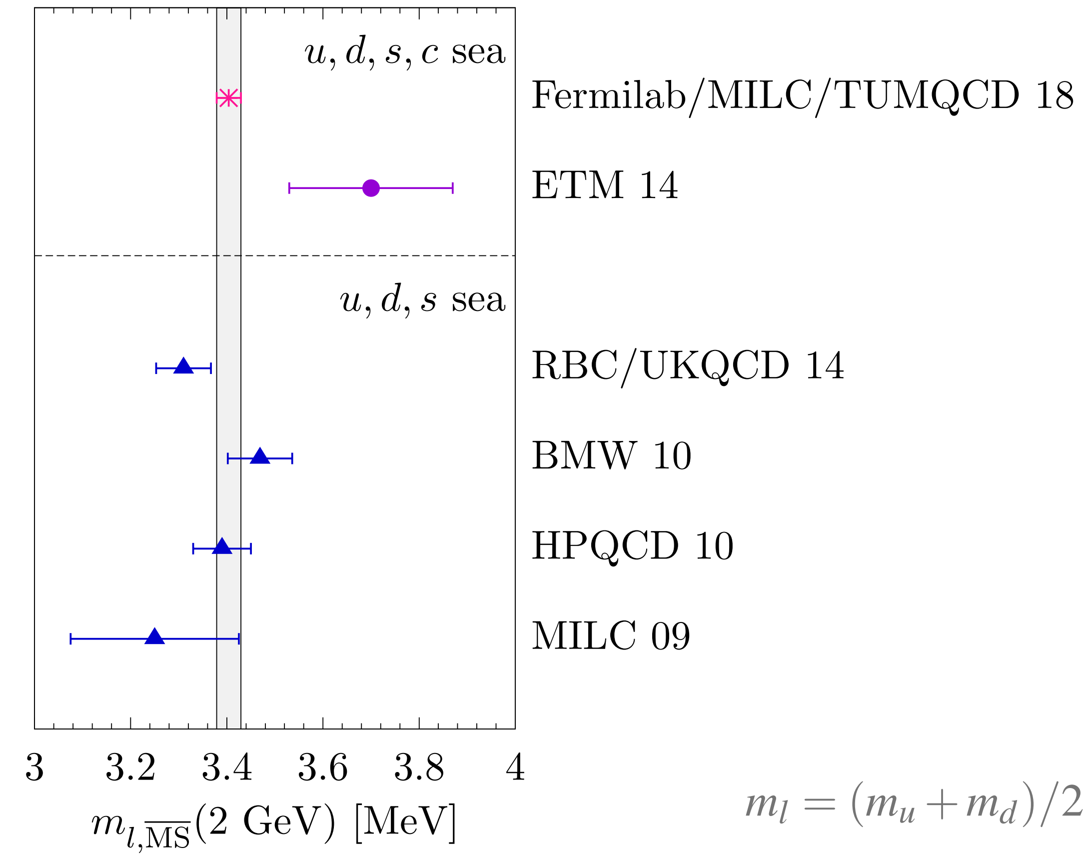


- To our knowledge, first results w/ order-α_s^5 running & order-α_s^4 matching.
- Precision: 0.3% for bottom to 0.5% for charm.

Results & Comparisons 2

- With mass ratios from light pseudoscalar mesons:





$m_l = (m_u + m_d)/2$

- Most precise s-, d-, and u-quark masses to date; e.g., m_u to 2%.
- All quarks except top.

Results & Comparisons 3

- Masses in numerical form:

$$m_{l,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 3.404(14)_{\mathrm{stat}}(08)_{\mathrm{syst}}(19)_{\alpha_s}(04)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$
$$m_{u,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 2.118(17)_{\mathrm{stat}}(32)_{\mathrm{syst}}(12)_{\alpha_s}(03)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$
$$m_{d,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 4.690(30)_{\mathrm{stat}}(36)_{\mathrm{syst}}(26)_{\alpha_s}(06)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$
$$m_{s,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 92.52(40)_{\mathrm{stat}}(18)_{\mathrm{syst}}(52)_{\alpha_s}(12)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$
$$m_{c,\overline{\mathrm{MS}}}(3\ \mathrm{GeV}) = 984.3(4.2)_{\mathrm{stat}}(1.6)_{\mathrm{syst}}(3.2)_{\alpha_s}(0.6)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$
$$m_{b,\overline{\mathrm{MS}}}(m_{b,\overline{\mathrm{MS}}}) = 4203(12)_{\mathrm{stat}}(1)_{\mathrm{syst}}(8)_{\alpha_s}(1)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$

- Mass ratios:

$$m_c/m_s = 11.784(11)_{\mathrm{stat}}(17)_{\mathrm{syst}}(00)_{\alpha_s}(08)_{f_{\pi,\mathrm{PDG}}}$$
$$m_b/m_s = 53.93(7)_{\mathrm{stat}}(8)_{\mathrm{syst}}(1)_{\alpha_s}(5)_{f_{\pi,\mathrm{PDG}}}$$
$$m_b/m_c = 4.577(5)_{\mathrm{stat}}(7)_{\mathrm{syst}}(0)_{\alpha_s}(1)_{f_{\pi,\mathrm{PDG}}}$$

Lattice Gauge Theory
for Beginners

The QCD Lagrangian

- SU(3) gauge symmetry and $1 + n_f + 1$ parameters:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QCD}} &= \frac{1}{g_0^2}\,\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}] \\
&- \sum_f \bar{\psi}_f(\not{D} + m_f)\psi_f \\
&+ \frac{i\theta}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}[F_{\mu\nu}F_{\rho\sigma}]
\end{aligned}
$$

- Observable CP violation $\propto \vartheta = \theta - \arg\det m_f$ (if all masses nonvanishing):
    - neutron electric-dipole moment sets limit $\vartheta \lesssim 10^{-11}$;
    - bafflingly implausible cancellation called the strong CP problem.

The QCD Lagrangian

- SU(3) gauge symmetry and $1 + n_f + 1$ parameters:

$$\mathcal{L}_{\rm QCD} = \frac{1}{g_0^2}\,\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}] \qquad r_1 \text{ or } m_\Omega \text{ or } \Upsilon(2S\text{-}1S), \ldots.$$

$$- \sum_f \bar{\psi}_f(\not{D} + m_f)\psi_f \qquad m_\pi, m_K, m_{J/\psi}, m_\Upsilon, \ldots.$$

$$+ \frac{i\theta}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}[F_{\mu\nu}F_{\rho\sigma}] \qquad \theta = 0.$$

- Observable CP violation $\propto \vartheta = \theta - \arg\det m_f$ (if all masses nonvanishing):
  - neutron electric-dipole moment sets limit $\vartheta \lesssim 10^{-11}$;
  - bafflingly implausible cancellation called the strong CP problem.

Lattice Gauge Theory

K. Wilson, *PRD* **10** (1974) 2445

- Invented to understand asymptotic freedom without the need for gauge-fixing and ghosts [Wilson, hep-lat/0412043].
- Gauge symmetry on a spacetime lattice:
    - mathematically rigorous definition of QCD functional integrals;

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \, \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left(-S\right) [\bullet]$$

    - enables theoretical tools of statistical mechanics in quantum field theory and provides a basis for constructive field theory.
- Lowest-order strong coupling expansion demonstrates confinement.

Numerical Lattice QCD

- Nowadays "lattice QCD" usually implies a numerical technique, in which the functional integral is integrated numerically on a computer.
- A big computer.
- Some compromises:
  - finite human lifetime ⇒ Wick rotate to Euclidean time: $x^4 = ix^0$;
  - finite memory ⇒ finite space volume & finite time extent;
  - finite CPU power ⇒ light quarks often heavier than up and down.

Lattice Gauge Theory

- Infinite continuum: uncountably many d.o.f. (⇒ UV divergences);

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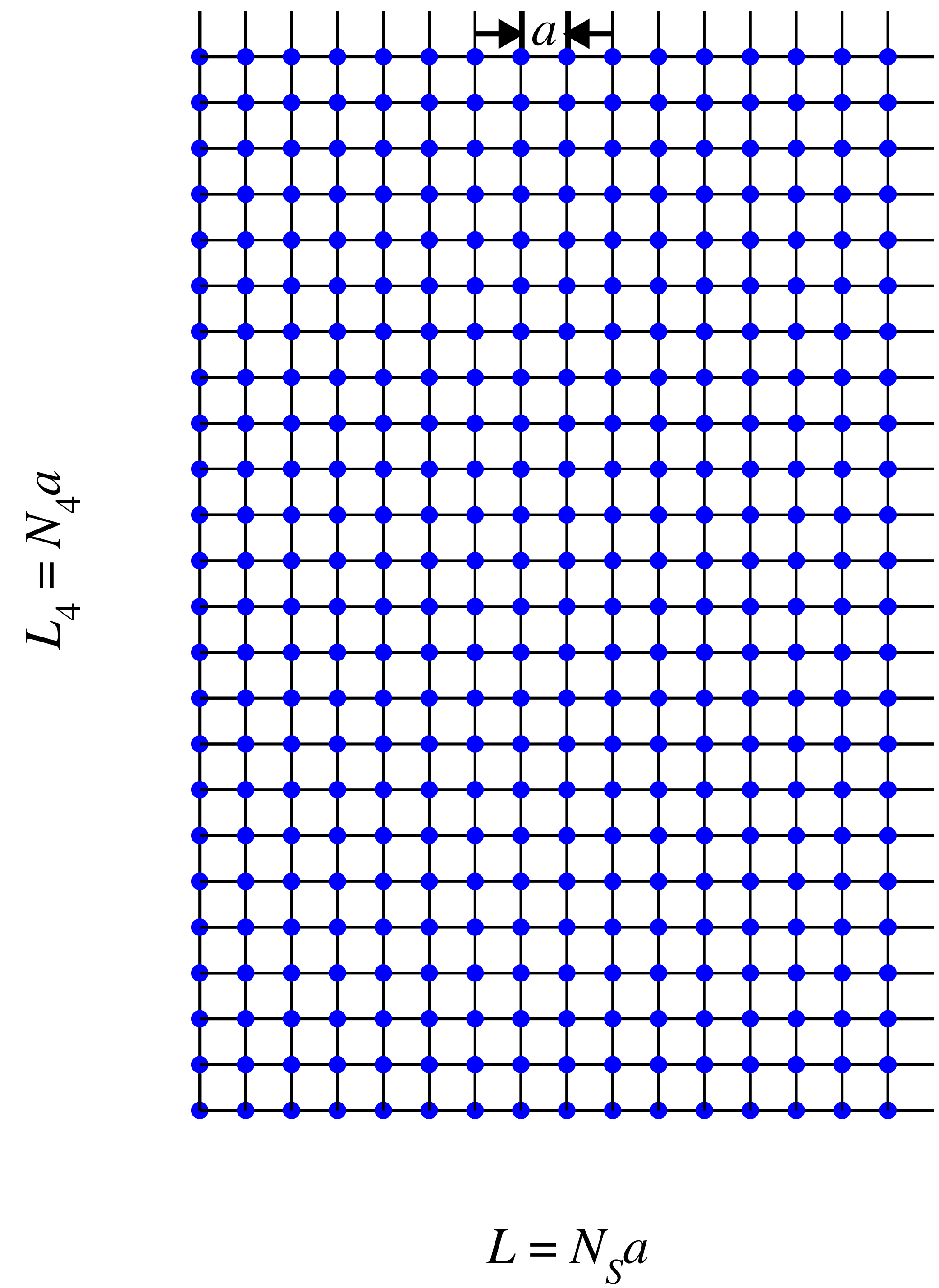
- Infinite lattice: countably many; used to define QFT;

- Finite lattice: finite dimension ~ 10^8, so compute integrals numerically.



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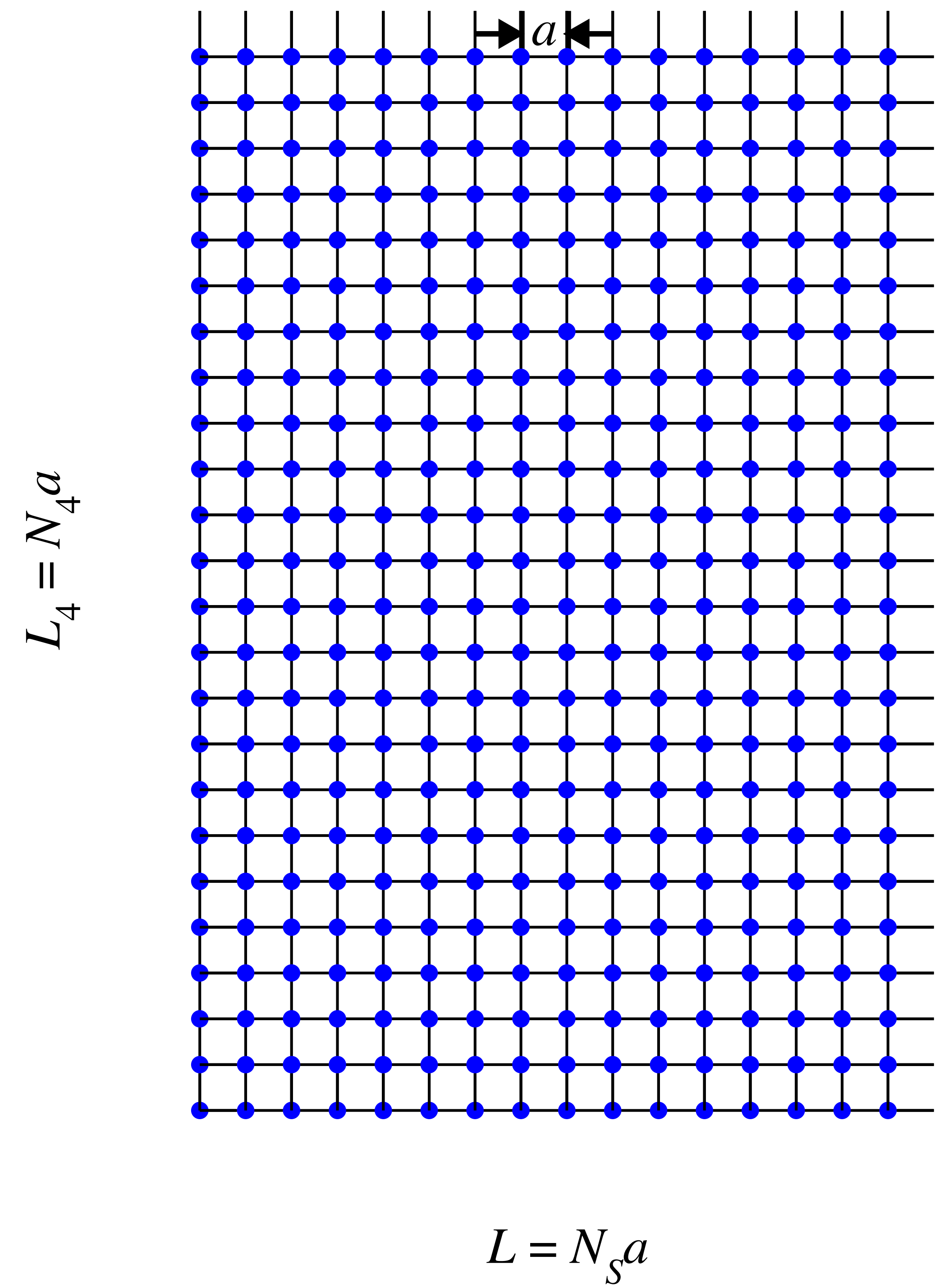

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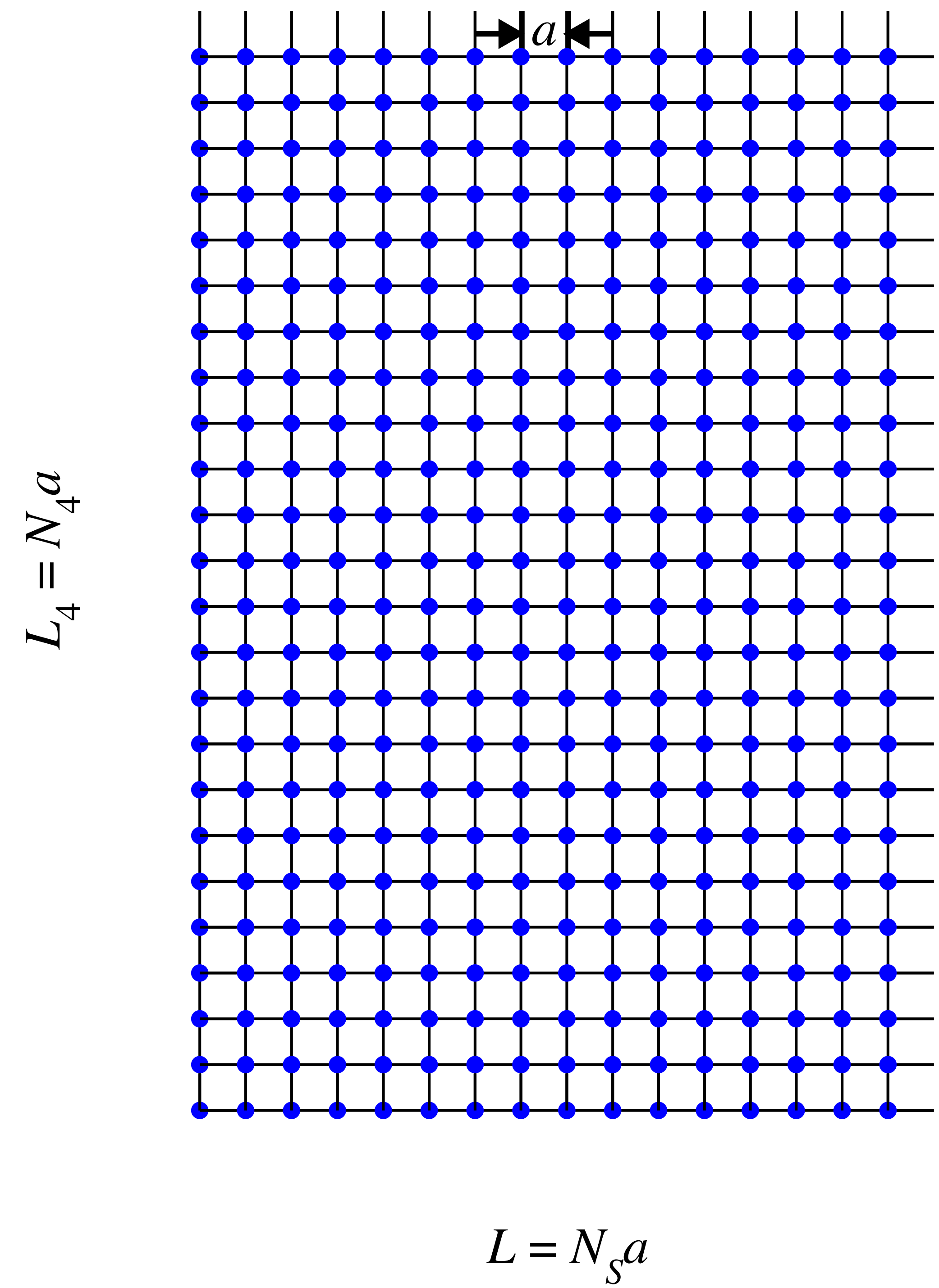

The Steps

- Use random number generator to create gauge fields distributed $\sim e^{-S}$.
- Solve $(\not{D} + m)_{xz} G_{zy} = b_x$ for quark propagators in these gauge fields.
- Fit correlation functions to get masses and matrix elements.
- Repeat several times while varying bare gauge coupling and bare masses.
- Find a trajectory with constant pion, kaon, D_s, B_s, masses (one for each quark) in dimensionless but physical units and obtain the continuum limit.
- Convert units to MeV.
- Comprehensive analysis of uncertainties!

Error Budget

TABLE I. Error budget for strange-, charm- and bottom-quark masses, their ratios, and the HQET matrix element $\overline{\Lambda}_{\text{MRS}}$. See the text for the description.

Error (%)	m_b/m_c	m_c/m_s	m_b/m_s	$m_{s,\overline{\text{MS}}}$(2 GeV)	$\overline{m}_c$	$\overline{m}_b$	$\overline{\Lambda}_{\text{MRS}}$
Statistics and EFT fit	0.10	0.09	0.12	0.43	0.30	0.29	4.5
Two-point correlator fits	0.08	0.02	0.10	0.13	0.08	0.03	1.1
Scale setting and tuning	0.02	0.08	0.10	0.12	0.03	0.02	0.2
Finite-volume corrections	0.02	0.04	0.06	0.07	0.02	0.01	0.1
Topological charge distribution	0.00	0.00	0.00	0.00	0.00	0.00	0.0
Electromagnetic corrections	0.11	0.11	0.01	0.02	0.08	0.01	0.1
α_s	0.01	0.00	0.01	0.56	0.75	0.18	2.9
$f_{\pi,\text{PDG}}$	0.03	0.07	0.10	0.13	0.04	0.02	0.3

- Many results compiled by Flavor Lattice Averaging Group (FLAG)
- With meson form factors, mixing matrix elements, etc., it is now common to publish correlation matrices as well as error budgets.

Outlook

Form Factors

- Hadronic matrix elements of the weak currents are decomposed into Lorentz covariant forms, multiplied by form factors:

$$\langle p(\boldsymbol{p})|\mathcal{V}^{\mu}_{\bar{u}d}|n(\boldsymbol{k})\rangle = \bar{u}_p(\boldsymbol{p})\gamma^{\mu}u_n(\boldsymbol{k})\,F_1(q^2) + \frac{q_\nu}{2M_N}\bar{u}_p(\boldsymbol{p})i\sigma^{\mu\nu}u_n(\boldsymbol{k})\,F_2(q^2),$$

$$\langle p(\boldsymbol{p})|\mathcal{A}^{\mu}_{\bar{u}d}|n(\boldsymbol{k})\rangle = \bar{u}_p(\boldsymbol{p})\gamma^{\mu}_{\perp}\gamma^5u_n(\boldsymbol{k})\,F_A(q^2) + \frac{2M_Nq^{\mu}}{q^2}\bar{u}_p(\boldsymbol{p})\gamma^5u_n(\boldsymbol{k})\,F_P(q^2),$$

$$q^{\mu} = k^{\mu} - p^{\mu}$$

- The q^2 dependence is constrained by unitarity and analyticity, leading to a model-independent parametrization known as the “z expansion”.

- Combining lattice-QCD calculations with the z expansion is a key ingredient in CKM determinations, *e.g.*, $|V_{ub}|$.

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$$2M_N q^\mu$$

Use lattice QCD to compute $F_1(z(q^2))$, $F_A(z(q^2))$, … and then the model-independent z expansion to get the shape.

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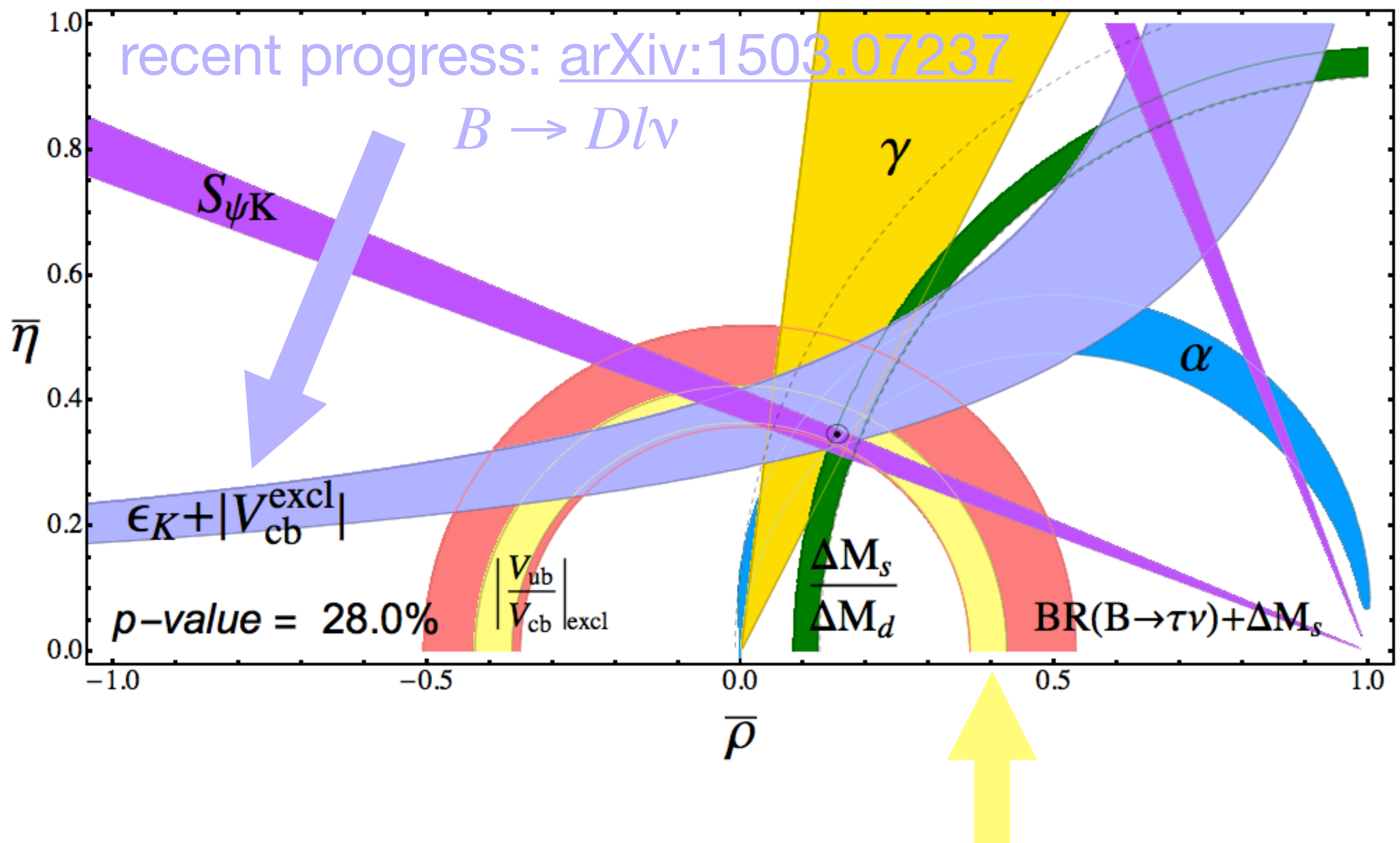
- Combining lattice-QCD calculations with the z expansion is a key ingredient in CKM determinations, *e.g.*, $|V_{ub}|$.

Outlook

- Neutrino-nucleus scattering is a difficult problem.
- Understanding it better is of paramount importance for neutrino exp'ts.
- A "parton model" approach:
  - compute the elementary amplitudes from first principles;
  - use models, parametrizations, data to constrain nucleus.
- The amplitudes can be computed in lattice QCD.

CKM Unitarity Triangle

- Decades of conversation and collaboration among experimenters and QCD theorists led to



recent progress: arXiv:1602.03560
$B^0_{(s)} \leftrightarrow \bar{B}^0_{(s)}$

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Lattice QCD calculations of hadronic transitions

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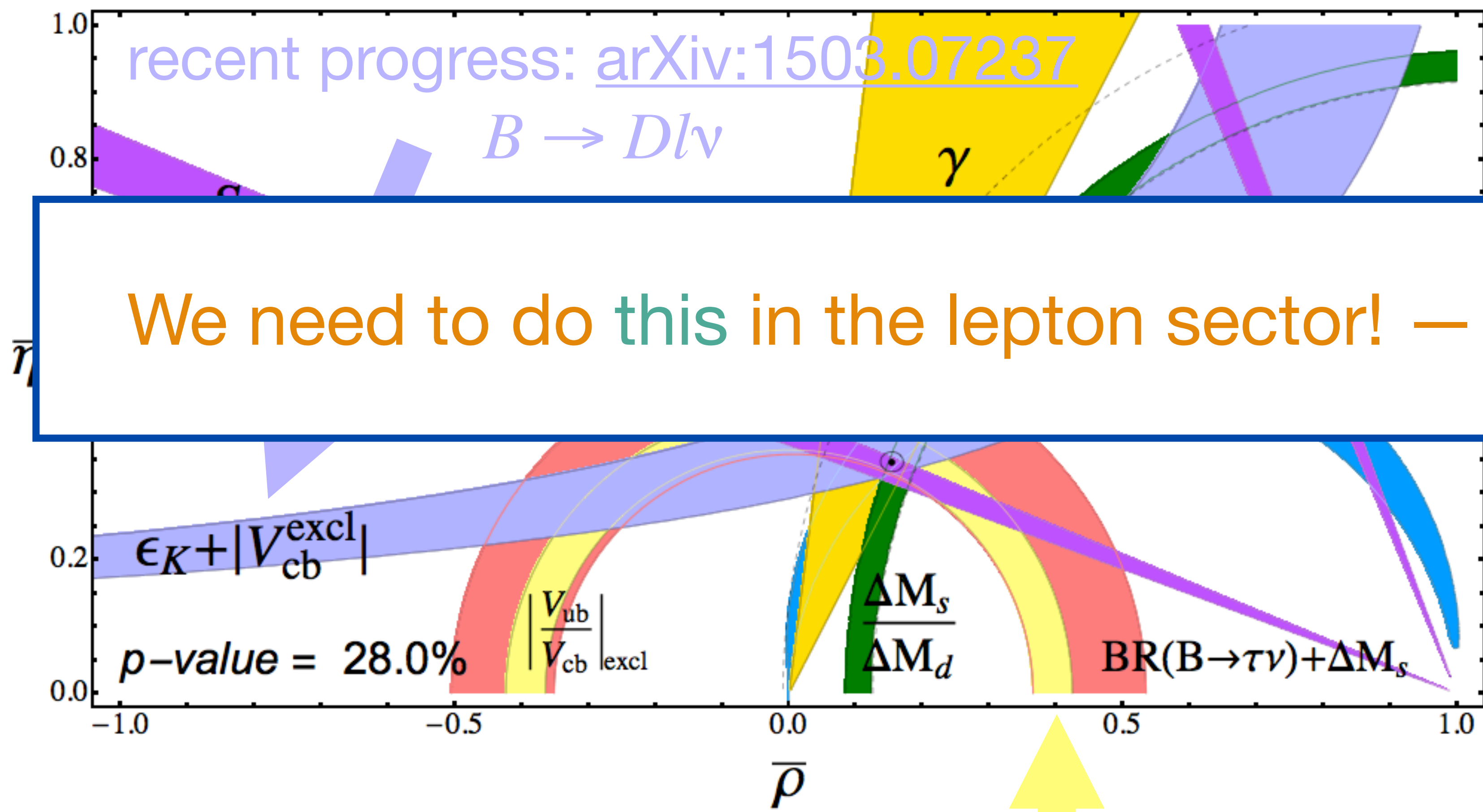

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We need to do this in the lepton sector! — André de Gouvêa

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Lattice QCD calculations of hadronic transitions

Thank you!